Appendix B

Derivation of an approximate solution for a two level system

Using the Magnus series [119] we can write the time ordered product Eq. [2.8] for the Liouville operator $\hat{Z}(t)$ ($\hbar = 1$) as:

$$\rho(t) = \exp(-i\Omega(t))\rho_0, \tag{B.1}$$

where

$$\Omega(t) = \int_0^t \hat{Z}(s_1)ds_1 + \frac{1}{2}\int_0^t \Big[\hat{Z}(s_1), \int_0^{s_1} \hat{Z}(s_2)ds_2\Big]ds_1 + \tag{B.2}$$
$$+\frac{1}{4}\int_0^t \Big[\hat{Z}(s_1), \int_0^{s_1} \Big[\hat{Z}(s_2), \int_0^{s_2} \hat{Z}(s_3)ds_3\Big]ds_2\Big]ds_1 + ...$$

Our assumption that $\rho = \rho(\theta, t)$ means that we neglect all terms in the series Eq. [B.2] except the first one. The condition that this will be a good approximation is:

$$\|\hat{Z}(s_1)\| \gg \Big\|\Big[\hat{Z}(s_1), \int_0^{s_1} \hat{Z}(s_2)ds_2\Big]\Big\|, \tag{B.3}$$

with $s_1 \in [0, T]$. Here the sign $\Big\|\Big\|$ denotes the norm of a matrix. Now let us use the mean value theorem, that for a function $f(x)$ (if one can define $f'(x)$ on the interval (a, b)) the following equality holds $f(b) - f(a) = f'(c)(b - a)$, where $c \in (a, b)$. Applying the theorem to the term $\int_0^{s_1} \hat{Z}(s_2)ds_2$ in Eq. [B.3] we obtain:

$$\|\hat{Z}(s_1)\| \gg \Big\|\Big[\hat{Z}(s_1), \hat{Z}(c)s_1\Big]\Big\|, \tag{B.4}$$

where $c \in (0, s_1)$. Let us know apply the mean value theorem to the term $\hat{Z}(c)$ again that gives:

$$\|\hat{Z}(s_1)\| \gg \Big\|\Big[\hat{Z}(s_1), \Big(\hat{Z}(s_1) + \frac{\partial \hat{Z}(c_1)}{\partial t}(s_1 - c)\Big)s_1\Big]\Big\|, \tag{B.5}$$

where $c_1 \in (c, s_1)$. Replacing the terms $s_1 - c$ and s_1 by their maximum value T and taking into account that $\left[\hat{Z}(s_1), \hat{Z}(s_1)\right] = 0$ we write Eq. [B.5] as

$$\left|\hat{Z}(s_1)\right| \gg T^2 \left\| \left[\hat{Z}(s_1), \frac{\partial \hat{Z}(c_1)}{\partial t}\right)\right] \right\|. \tag{B.6}$$

Then, using the explicit form of the operator $\hat{Z}$ we finally obtain:

$$\left|V(t)\right| \gg T^2 \left|\frac{\partial V(t_1)}{\partial t}\gamma_i\right|, i = 1, 2. \tag{B.7}$$

Here $t, t_1 \in (0, T)$. For a two level system under the RWA the Liouville operator $\hat{Z}$ reads (see Eq. [2.13]) as

$$\hat{Z}(t) = \begin{pmatrix} 0 & i\gamma_1 & -V(t) & V(t) \\ 0 & -i\gamma_1 & V(t) & -V(t) \\ -V(t) & V(t) & -i\gamma_2 & 0 \\ V(t) & -V(t) & 0 & -i\gamma_2 \end{pmatrix}. \tag{B.8}$$

While the initial conditions for the density matrix ρ_0 we set as

$$\rho(t_0) = \begin{pmatrix} \rho_{11}(t_0) \\ \rho_{22}(t_0) \\ \rho_{12}(t_0) \\ \rho_{21}(t_0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{B.9}$$

Using Eq. [B.8] and truncating the series Eq. [B.2] after the first term, it is easy to obtain an approximate analytical solution for the occupation $\rho_{22}(t)$:

$$\begin{aligned} \rho_{22}(t) = 2\theta^2(t)F^{-1}\Big(1 - \cosh(H)\exp(-(\gamma_1 + \gamma_2)t/2) \\ +(\gamma_1 + \gamma_2)t\ \sinh(H)\exp(-(\gamma_1 + \gamma_2)t/2)H^{-1}\Big), \end{aligned} \tag{B.10}$$

where

$$H = \sqrt{((\gamma_1 - \gamma_2)^2 t^2 - 16\theta^2(t))}/2,$$

and

$$F = \gamma_1\gamma_2 t^2 + 4\theta^2(t).$$