Appendix A

The adiabatic approximation

The essence of the adiabatic approximation is as follows. Following [118] we suppose that we a have system of linear differential equations:

$$\frac{d}{dt}\rho(t) = \hat{Z}(t)\rho(t), \tag{A.1}$$

where $\rho(t)$ is the n-dimensional vector, while $\hat{Z}(t)$ is the n by n matrix. We choose initial conditions: $\rho(t)|_{t_0} = \rho_0$. If eigenvectors $a_n(t)$ and eigenfrequencies $\omega_n(t)$ of the matrix $\hat{Z}(t)$ vary slowly with t:

$$\frac{d}{dt}\omega_n(t) \ll (\omega_n(t))^2,\ \frac{d}{dt}|a_n(t)| \ll \omega_n(t)|a_n(t)|, \tag{A.2}$$

then the adiabatic approximation of the solution of the set of linear equations Eq. [A.1] may be represented in the form:

$$\rho_n(t) = a_n(t)\exp\left(\int_{t_0}^{t} \omega_n(t')dt'\right). \tag{A.3}$$

Indeed, let us substitute Eq. [A.3] in Eq. [A.1] and get

$$\dot{a}_n(t)\exp\left(\int_{t_0}^{t} \omega_n(t')dt'\right) + \omega_n(t)a_n(t)\exp\left(\int_{t_0}^{t} \omega_n(t')dt'\right) = \hat{Z}(t)a_n\exp\left(\int_{t_0}^{t} \omega_n(t')dt'\right). \tag{A.4}$$

Under the slowness condition Eq. [A.2], the first term on the left-hand side of Eq. [A.4] is small compared to the second one and may by omitted. Then Eq. [A.4] only keeps the terms yielding the identity that determines the eigenfrequency ω_n.