# Classes of Banach spaces connected with the Lyapunov convexity theorem

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# Preface

Vector measures with values in finite-dimensional and infinite-dimensional spaces play an important role in various questions of analysis, operator theory, optimization theory, games and statistical decisions theory and represent an interesting object for study. Works of well known mathematicians as R. G. Bartle, N. Dunford, J. T. Schwartz, B. J. Pettis, I. Gelfand, A. Grothendieck and others were dedicated to vector measure theory. The present state of the theory is reflected in the monographs "Vector measures" by J. Diestel and J. J. Uhl [7] and "Vector measures and control systems" by I. Kluvanek and G. Knowles [20].

One of the classical results of vector measure theory is A. A. Lyapunov's theorem ([25], [7, p. 264], [29, theorem 5.5]) which states that the range of every countably additive finite nonatomic vector measure valued in a finite dimensional space is a compact and convex set. This result, first obtained by A. A. Lyapunov in 1940 attracted the attention of many mathematicians. Different versions of this theorem appeared in the fifties and sixties and in 1966 J. Lindenstrauss [23] gave the shortest proof of the statement.

It is known that the Lyapunov theorem is false in the infinite-dimensional case: for every infinite-dimensional Banach space there exists a nonatomic countably additive measure of bounded variation, with values in this space and defined on the measurable subsets of $[0,1]$, with nonconvex range ([7, p. 266], [16]). Nevertheless by additional restrictions on the measure and the space this theorem has some nontrivial generalizations that have been obtained by G. Knowles [21], D. Pecherskii [28], J. Elton [8], J. J. Uhl [7], V. M. Kadets [14], V. M. Kadets and M. M. Popov [16], V. M. Kadets and G. Schechtman [17].

However all known infinite-dimensional generalizations of the Lyapunov convexity theorem have a different nature.

The aim of this thesis is to develop infinite-dimensional generalizations of the Lyapunov theorem to clarify the situation.

The present work consists of three chapters. General information on vector measures, different approaches to generalizations of the Lyapunov convexity theorem and the three-space problem for the Lyapunov property are presented in Chapter 1. In Chapter 2 the notions of a Lyapunov tree and Lyapunov $C$-convexity are introduced. It is proved that Banach spaces having this property have the Lyapunov property. In Chapter 3 we find out which of the known Banach spaces have the Lyapunov property and which do not. Most of the constructed examples fit into the scheme: if a Banach space has no isomorphic copies of $l_2$ then it has the Lyapunov property. However, we present an example of a Banach space that does not fit into this scheme.

The main results of the dissertation were published in the works [18], [35], [34]. The results included in the thesis were communicated in the Kharkov seminar on Banach space theory and on the following conferences: XXVIII scientific-technical conference of the Kharkov Academy of Municipal Economy [36] and VI international conference in honor of academician M. Kravchuk [37].

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