Chapter 2

Methodical fundamentals

In this chapter, the principle theory on which magnetotellurics (MT) and geomagnetic depth sounding (GDS) are based is presented. External time-dependent electromagnetic fields induce currents in the conducting earth, and relations between components of the total electromagnetic variation field measured at the earth's surface are functions of the subsurface conductivity distribution. Analytical and numerical modelling aims at reproducing these functions. In the period range that is considered in this study (between 10 s and $2 \cdot 10^4$ s), the natural sources that is taken advantage of originate from time varying current systems within the ionosphere and magnetosphere of the earth, exited mainly by solar radiation. These currents are supposed to have dimensions > 1000 km, so that the external variation fields at the earth's surface can be regarded as not too far from being uniform, a circumstance which is essential to quantitatively infer conductivities of the subsurface from their induced currents in the conductive earth.

According to the theory of classical electrodynamics, electromagnetic fields in a non or slowly accelerated reference frame can completely be described with Maxwell's equations, here in differential form:

$$\nabla \cdot \mathbf{E} = \frac{q}{\varepsilon_0} \tag{2.1a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.1b}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.1c}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{2.1d}$$

Within materials, to macroscopically describe the interaction of the field with the host (i.e. polarization and magnetization effects), eq. 2.1a and 2.1d are commonly written as

$$\nabla \cdot \mathbf{D} = q' \tag{2.1a$'$}$$

$$\nabla \times \mathbf{H} = \mathbf{j}' + \frac{\partial \mathbf{D}}{\partial t} \tag{2.1d$'$}$$

$$\text{with} \quad \mathbf{j}' = \sigma \mathbf{E} \tag{2.2}$$

$$\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E} \quad \text{and} \quad \mathbf{B} = \mu_r \mu_0 \mathbf{H} \tag{2.3}$$

where the relative permittivity ε_r, the relative permeability μ_r and the electrical conductivity σ are assumed to be scalars (in fact, in highly anisotropic material, they all can be tensors, too). Here, q' denotes the density of free charge carriers (i.e. total charge density q minus a charge density of polarization), and $\mathbf{j}'$ denotes the total current density $\mathbf{j}$ minus current densities of polarization and magnetization currents. For the earth, the relative permeability μ_r can be regarded as approximately 1, whereas ε_r can have values between 1 (vacuum) and ~80 (water).

For time intervals considered in this thesis (≥10 s) and conductivities observable in the earth, the so-called displacement currents $\varepsilon_r\varepsilon_0\partial\mathbf{E}/\partial t$ can be neglected with respect to $\sigma\mathbf{E}$. For time-harmonic variations of period T the term $\partial\mathbf{E}/\partial t$ is of magnitude E/T. With $T \gg \varepsilon_r\varepsilon_0/\sigma$, $(\sigma \neq 0)$ we have:

$$\frac{\left|\varepsilon_r\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right|}{\sigma\left|\mathbf{E}\right|} \ll 1 \tag{2.4}$$

This approximation is called *quasi stationary.* For the following, all equations are Fourier transformed to the frequency domain and the field quantities are their corresponding Fourier amplitudes. With the above approximations, we now write down the basic equations of electromagnetic induction in the earth:

$$\nabla\cdot\mathbf{E} = \frac{q}{\varepsilon_0} \tag{2.5a}$$

$$\nabla\times\mathbf{E} = -i\omega\mathbf{B} \tag{2.5b}$$

$$\nabla\times\mathbf{B} = \mu_0\sigma\mathbf{E} \tag{2.5c}$$

Because displacement currents are neglected, the relative permittivity is not further considered for simplicity. Still, if the electric field has a component parallel to the gradient of conductivity, it actually cannot really be discarded, as charge densities q will appear.
The next section will introduce the theory of transfer functions, following the approach of *Egbert and Booker* [1989] which is strictly based on linear algebra mathematics and leads directly to a multivariate analysis of electromagnetic array data, as it has been used here.

2.1 The physical model

At the earth's surface, the curl of the magnetic field vanishes, and it can be regarded as the gradient of a scalar potential $\mathbf{B} = \nabla\Phi$. The fundamental postulation now is that the external magnetic field is a linear combination of a defined number p of linear independent source potentials, which span the space $\mathbf{\Phi}$ of all possible source potentials:

$$\Phi^e = \sum_{j=1}^{p}\alpha_j\Phi_j \in \mathbf{\Phi} = \ll\Phi_1,\ldots,\Phi_p\gg \tag{2.6}$$

For true uniform sources (p=2), we would have $\Phi_1^e = x$, $\Phi_2^e = y$ and $\mathbf{B}_1^e = (1,0,0)$, $\mathbf{B}_2^e = (0,1,0)$. The potential Φ of the total magnetic field at the earth's surface can be written as the sum of the source potential of the external magnetic field and a potential of the induced internal magnetic field: $\Phi = \Phi^e + \Phi^i$. *Egbert* [1987] showed that Φ_i is a linear function of the external potential, which depends on the frequency and the conductivity of the subsoil:

$$\Phi = \Phi^e + L^B_{\omega,\sigma}(\Phi^e) \tag{2.7}$$

Dmitriev and Berdichevsky [1979] proved that the electric field $\mathbf{E}$ is related to the horizontal magnetic field by a linear operator, also depending on ω and σ:

$$\mathbf{E} = L^E_{\omega,\sigma}(\mathbf{B}_{hor}) \tag{2.8}$$

Taking eqs. 2.6, 2.7 and 2.8 together, we can write:

$$\mathbf{B} = \nabla\left[\Phi^e + L^B_{\omega,\sigma}(\Phi^e)\right] = \sum_{j=1}^{p} \alpha_j \left[\Phi^e + L^B_{\omega,\sigma}(\Phi^e)\right] = \sum_{j=1}^{p} \alpha_j \mathbf{B}_j \tag{2.9a}$$

$$\mathbf{E} = L^E_{\omega,\sigma}\mathbf{B}_{hor} = \sum_{j=1}^{p} \alpha_j L^E_{\omega,\sigma}(\mathbf{B}_{hor}) = \sum_{j=1}^{p} \alpha_j \mathbf{E}_j \tag{2.9b}$$

where $\mathbf{B}_j$, $\mathbf{E}_j$ are the magnetic and electric fields associated with the source potential Φ_j. Now the electromagnetic field $\mathbf{F} = (\mathbf{B}, \mathbf{E})$ is regarded as a function of position $\mathbf{r}$ and supposed to be measured at n stations. Then, if the source potential is equal at all field stations, i.e. translationally invariant, the complex coefficients α in eqs. 2.9 are the same at all sites and express the *actual* source field configuration. Composing vectors of all m measured field components, we can write:

$$\mathbf{b} = \begin{pmatrix} \mathbf{F}(\mathbf{r}_1) \\ \vdots \\ \mathbf{F}(\mathbf{r}_n) \end{pmatrix} = \sum_{j=1}^{p} \alpha_j \begin{pmatrix} \mathbf{F}(\mathbf{r}_1) \\ \vdots \\ \mathbf{F}(\mathbf{r}_n) \end{pmatrix} = \sum_{j=1}^{p} \alpha_j \mathbf{u}_j = \mathbf{U}\boldsymbol{\alpha} \tag{2.10}$$

where $\mathbf{U} = (\mathbf{u}_1, \ldots, \mathbf{u}_p)$. Thus, the principle assumption of eq. (2.6) in combination with the translationally invariance of Φ^e leads to the conclusion, that all measurable data vectors have to lie in a p-dimensional subspace of $\mathbf{C}_m$, the *response space*

$$\mathcal{R} = \ll \mathbf{u}_1, \ldots, \mathbf{u}_p \gg \tag{2.11}$$

The task of careful field data analysis is equivalent with the task of finding the response space $\mathcal{R}$ (see section 4.1).

2.2 Calculation of transfer functions

We postulate from now on true uniform source excitation, i.e. $p = 2$ and $\Phi^{\mathbf{e}}_{\mathbf{1}} = x$, $\Phi^{\mathbf{e}}_{\mathbf{1}} = y$ $\Rightarrow$ $\mathbf{B}^e_1 = (1, 0, 0)$, $\mathbf{B}^e_2 = (0, 1, 0)$ (for $p \geq 2$ see *Egbert and Booker* [1989]). As it is the total field that is measured, the determination of the response vectors $\mathbf{u}_{1,2}$ is naturally impossible. But suppose the data processing succeeded in finding $\mathcal{R}$ and thus found two basis vectors $\mathbf{v}_1, \mathbf{v}_2$, then there is an unknown regular (2×2) matrix $\mathbf{A}$ with $\mathbf{V} = \mathbf{UA}$. Now rearranging the lines of eq. (2.10) we group the field components into $m-2$ channels (index 2) that shall be predicted from the two remaining channels (index 1), and calculate

$$\begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{pmatrix} \mathbf{U}_1^{-1} \mathbf{U}_1 \boldsymbol{\alpha} = \begin{pmatrix} \mathbf{I}_2 \\ \mathbf{U}_2 \mathbf{U}_1^{-1} \end{pmatrix} \mathbf{b}_1 \tag{2.12}$$

where $\mathbf{I}_2$ is the two-dimensional identity matrix. The matrix $\mathbf{T} = \mathbf{U}_2 \mathbf{U}_1^{-1}$ is consisting of inter-component transfer functions that relate the $m-2$ channels of index 2 to the two channels of index 1. Remembering that from data analysis a matrix $\mathbf{V} = (\mathbf{v}_1, \mathbf{v}_2)$ has been determined instead of $\mathbf{U}$, it is important to notice that the matrix of transfer functions $\mathbf{T}$ is independent of the basis:

$$\mathbf{T} = \mathbf{U}_2 \mathbf{U}_1^{-1} = (\mathbf{V}_2 \mathbf{A})(\mathbf{V}_1 \mathbf{A})^{-1} = \mathbf{V}_2 \mathbf{V}_1^{-1} \tag{2.13}$$

so that all possible inter-component transfer functions are determined by the knowledge of the response space $\mathcal{R}$.
Also the commonly analyzed *local* magnetotelluric impedance tensor $\mathbf{Z}$ and the local geomagnetic 'tipper' transfer functions T_x and T_y can be calculated:

$$\mathbf{b}_2 = \begin{pmatrix} B_z \\ E_x \\ E_y \end{pmatrix} = \begin{pmatrix} T_x & T_y \\ Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \mathbf{V}_2 \mathbf{V}_1^{-1} \mathbf{b}_1 \tag{2.14}$$

For the calculation of inter-station geomagnetic transfer function, we chose the horizontal magnetic field of a preferred station as reference. To correspond with the notation of Schmucker's perturbation matrix $\mathbf{W}$ (*Schmucker* [1970]):

$$\mathbf{B} - \mathbf{B}^0_{hor} = \begin{pmatrix} h_H & h_D \\ d_H & d_D \\ z_H & z_D \end{pmatrix} \mathbf{B}^0_{hor} = \mathbf{W}\mathbf{B}^0_{hor} \tag{2.15}$$

we have:

$$\mathbf{b}_1 = \begin{pmatrix} B^0_x \\ B^0_y \end{pmatrix} \qquad \mathbf{b}_2 = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \qquad \mathbf{V}_2 \mathbf{V}_1^{-1} = \mathbf{W} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{2.16}$$

where $\mathbf{b}_2$ can be expanded to all stations, so that all measured geomagnetic field components are referred to one horizontal reference field. With some simple linear algebra, the formalism also allows to construct synthetic references, for example a spatially averaged horizontal magnetic field (see *Egbert and Booker* [1989]).

2.3 Analysis of transfer function tensors

The dimension of the subsurface conductivity distribution, i.e. the number of spatial coordinates from which σ is dependent, is reflected in the transfer function tensors. The first task in data analysis is usually to detect this dimensionality, which is called *dimensionality analysis* (only isotropic structures are considered below).

• One-dimensional conductivity distributions

The conductivity distribution is a function of depth only ($\sigma = \sigma(z)$). As $\mathbf{E} \perp \nabla\sigma$, we have $\nabla \cdot \mathbf{E} = 0$ (see also: eq. 2.31). From the formulas of geomagnetic induction (eqs. 2.5), with $\nabla \times \nabla \times \mathbf{F} = \nabla\nabla \cdot \mathbf{F} - \triangle\mathbf{F}$, $\mathbf{F} = \mathbf{E}, \mathbf{B}$, we can derive the Helmholtz equations:

$$\triangle\mathbf{E} - i\omega\mu_0\sigma\mathbf{E} = 0 \tag{2.17}$$

$$\triangle\mathbf{B} - i\omega\mu_0\sigma\mathbf{B} = 0 \tag{2.18}$$

Within an homogenous area, solutions of these equations are diffusing up- or downgoing waves

$$\mathbf{F} = \mathbf{F}_0^{\pm}(z)e^{i\omega t \pm \frac{z}{C(\omega)}} \qquad C(\omega)^2 = \frac{1}{i\omega\mu_0\sigma} \tag{2.19}$$

with the *complex penetration depth* $C(\omega)$ — its doubled real part is called *skin depth.* Maxwell's equations additionally require that $\mathbf{E} \perp \mathbf{B}$, i.e. no electrical field component parallel to the magnetic field is induced. The impedance tensor takes the form:

$$\mathbf{Z} = \begin{pmatrix} 0 & Z \\ -Z & 0 \end{pmatrix} \tag{2.20}$$

For a homogeneous half-space, we have just a downgoing wave and C is directly referred to the impedance from the fields measured at the surface ($Z = i\omega C$), so that the resistivity of the earth can be determined

$$\rho = \frac{\mu_0}{\omega}\,|Z|^2 \tag{2.21}$$

and the complex phase $\phi = arg(Z)$ of the impedance is equal to $\pi/4$ or $45°$ (*Cagniard* [1953]).

Weidelt [1972] showed that for an arbitrary layered earth model, the real part of $Z(\omega)/(i\omega)$ represents the depth of the "center of gravity" of the in-phase induced current system. The *apparent resistivity* $\rho_a(\omega)$ calculated from eq. (2.21) can therefore be regarded as a representative conductivity for that depth, the phase $\phi(\omega)$ giving additional structural information. The *magnetotelluric method* (MT) is concerned with the evaluation of these functions (*Cagniard* [1953]), which are often constructed for all tensor elements, though not having a clear physical meaning in the multidimensional case.

For 1-D distributions, no lateral anomalous electric currents are induced and thus all tipper and perturbation transfer functions are zero ($T_{x,y} = 0, \mathbf{W} = \mathbf{O}$).

• Two-dimensional conductivity distributions

The conductivity distribution is a function of depth and one lateral spatial coordinate (e.g., y). The induction equations (eq. 2.5) separate into two independent modes of polarization, with currents flowing parallel to the conductivity contrast (*TE-mode*) or perpendicular to it (*TM-mode*).

$$\begin{array}{rcl|rcl} & \text{TE-mode} & & & \text{TM-mode} & \\ \hline \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} &=& \mu_0 \sigma E_x & \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &=& -i\omega B_x \\ \frac{\partial E_x}{\partial z} &=& -i\omega B_y & \frac{\partial B_x}{\partial z} &=& \mu_0 \sigma E_y \\ -\frac{\partial E_x}{\partial y} &=& -i\omega B_z & -\frac{\partial B_x}{\partial y} &=& \mu_0 \sigma E_z \end{array} \qquad (2.22)$$

In the TM-mode, no vertical magnetic field is induced, and the horizontal magnetic field is spatially constant along the earth's surface (as $\sigma = 0$ at $z = -0$). Therefore, the investigation of geomagnetic anomalies due to 2-D conductivity contrasts is always dealing with the TE-mode. The tensors take the form:

$$\mathbf{Z} = \begin{pmatrix} 0 & Z_{\|} \\ Z_{\perp} & 0 \end{pmatrix} \qquad \mathbf{W} = \begin{pmatrix} 0 & 0 \\ 0 & d_D \\ 0 & z_D \end{pmatrix} . \qquad (2.23)$$

In general, $Z_{\|} \neq -Z_{\perp}$. Local tipper functions are: $T_x = 0$, $T_y = B_z/B_y$.
Suppose the measurements have not been carried out in the coordinate system of the conductivity contrast. Data analysis then yields the tensors

$$\mathbf{Z}' = \mathbf{R}_{\alpha}^T \mathbf{Z} \mathbf{R}_{\alpha} \qquad \mathbf{W}' = \mathbf{R}_{B,\alpha}^T \mathbf{W} \mathbf{R}_{\alpha} \qquad (2.24)$$

$$\mathbf{R}_{\alpha} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \qquad \mathbf{R}_{B,\alpha} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with α being the rotation angle between the measured coordinate system and the system of the 2-D strike. All transfer functions of **Z** and **W** are in general $\neq$ zero. Local geomagnetic transfer functions are often illustrated as *induction vectors* IV

$$\Re(IV) = \Re(T_x)\mathbf{e}_x + \Re(T_y)\mathbf{e}_y \qquad \Im(IV) = \Im(T_x)\mathbf{e}_x + \Im(T_y)\mathbf{e}_y \qquad (2.25)$$

with the unity vectors $\mathbf{e}_{x,y}$ in north and east direction, respectively. From a combination of tipper and perturbation tensor analysis, it can easily be shown that for a single 2-D contrast $\tan\alpha_{tp} = T_y/T_x$, i.e. the induction vector is oriented perpendicular to the strike direction, real parts pointing away from the better conductor (Wiese-convention, *Wiese* [1962]).

To find the strike direction from the impedance tensor (with an ambiguity of ± 90 degrees), *Swift* [1967] derived from the request $|Z'_{xx}|^2 + |Z'_{yy}|^2 \rightarrow \min!$ (zero for true 2-D structures):

$$\alpha_{sw} = \frac{1}{4} \cdot \arctan \frac{2 \cdot \Re[(Z'_{xx} - Z'_{yy})(Z'_{xy} + Z'_{yx})]}{\left|Z'_{xx} - Z'_{yy}\right|^2 - \left|Z'_{xy} + Z'_{yx}\right|^2} \tag{2.26}$$

Analogous to Swift, *Siemon* [1991] derived a geomagnetic strike direction from the request $|h'_D|^2 + |d'_H|^2 \rightarrow \min!$

$$\alpha_{si} = \frac{1}{4} \cdot \arctan \frac{2 \cdot \Re[(h'_H - d'_D)(h'_D + d'_H)]}{\left|h'_H - d'_D\right|^2 - \left|h'_D + d'_H\right|^2} \tag{2.27}$$

To quantitatively specify the dimensionality of the subsurface, Swift and Siemon additionally defined rotationally invariant *skewnesses*, which vanish under true 2-D conditions, but are *not* necessarily unequal zero above 3-D structures:

$$\kappa_{sw} = \frac{\left|Z'_{xx} + Z'_{yy}\right|}{\left|Z'_{xy} - Z_{yx}\right|} \qquad \kappa_{si} = \frac{|h'_D + d'_H|}{\left|h'_H + d'_D + 2\right|}. \tag{2.28}$$

Eggers [1982] had a more formal approach to the recovery of the 2-D impedance tensor in strike direction coordinates, presenting a generalized eigenstate formulation of the magnetotelluric impedance tensor. From the request $\mathbf{E}^i \cdot \mathbf{B}^i = 0$ for each polarization resp. eigenvector, he derived the eigenstate problem

$$\left(\mathbf{Z} - \Lambda^i\right)\mathbf{B}^i = 0 \qquad \Lambda^i = \begin{pmatrix} 0 & \lambda^i \\ -\lambda^i & 0 \end{pmatrix} \tag{2.29}$$

with rotationally invariant solutions resp. eigenvalues:

$$\lambda^{\pm} = Z_1 \pm \sqrt{Z_1^2 - \det \mathbf{Z}} \tag{2.30}$$

($Z_1 = (Z_{xy} - Z_{yx})/2$, $\det \mathbf{Z} = Z_{xx}Z_{yy} - Z_{xy}Z_{yx}$). It is easy to show that for 2-D structures the eigenvalues $\lambda^{\pm}$ are identical with the impedances $Z_{\parallel}$ and $Z_{\perp}$ from the unrotated impedance tensor. This formulation of the problem can also be helpful if the structural setting is not truly two-dimensional.

- Three-dimensional conductivity distributions

The electrical conductivity is a function of all spatial coordinates. All transfer functions of $\mathbf{Z}, \mathbf{W}, \mathbf{IV}$ are in general $\neq$ zero, independent from the coordinate system of observation. Compared to the investigation of 1-D and 2-D structures, a full 3-D approach to field data that are obviously three-dimensional can be a significantly more difficult task and is rarely performed extensively due to bad data coverage of the investigation area and high computational costs.

However, data often look more three-dimensional than they actually are:
Heterogeneities of small extent close to the earth's surface, which are *not* the target of investigation, can act as local scatterers and distort the (eventually one- or two-dimensional)

regional electromagnetic field, i.e. the field that would be observed without superficial heterogeneities. As is sketched below, the effect of scattering can be described by means of real matrices acting on the regional field, which implies that *if* the regional field is one- or two-dimensional, the information from the undistorted transfer function tensor can be partly recovered.

As $\nabla \cdot (\nabla \times \mathbf{B}) = 0$, the quasi stationary approximation leads to $\nabla \cdot \mathbf{j} = \nabla \cdot (\sigma \mathbf{E}) = 0$ and with $\nabla \cdot (\sigma \mathbf{E}) = \sigma \nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla \sigma$ and eq. (2.5a) finally to:

$$q = -\frac{\varepsilon_0}{\sigma}(\nabla \sigma) \cdot \mathbf{E} \tag{2.31}$$

Thus, as soon as the electric field has a component $E_\perp$ perpendicular to the conductivity strike direction, it is no longer free of sources and charge densities do accumulate (also for the 2-D TM-mode). The densities arise *in phase* with the primary electric field and the factor between q and $E_\perp$ is *independent of frequency.* Because of the permittivity term, the charge densities and the corresponding secondary currents are minute. The secondary electric field, however, which can be calculated by Coulomb's law

$$\mathbf{E}_s(\mathbf{r}_0) = \frac{1}{4\pi\varepsilon_0} \int_{V_S} \frac{q_i}{r_{i0}^2} \frac{\mathbf{r}_{i0}}{r_{i0}} dv' \tag{2.32}$$

$(\mathbf{r}_{i0} = \mathbf{r}_i - \mathbf{r}_0)$ can be of the same order as the primary electric field (*Price* [1973]). Depending on geometry and conductivity of the structures, the total electric field – the sum of the primary and secondary electric fields – is enhanced or diminished in the vicinity of anomalies, resulting in channelling or deflection of currents (*Jiracek* [1990]).

For confined superficial bodies of anomalous conductivity with dimensions small compared to the skin depth of the embedding host for the periods considered, the described effect, which will be referred to as 'quasi-static' in the following, can exceed inductive effects by far — the body acts as a distorter. As is suggested from the above description, and as can be derived from a full treatment of the problem, the effect of scattering can be described by means of *real* matrices (*Groom and Bahr* [1992]; *Habashy et al.* [1993]; *Chave and Smith* [1994]):

$$\mathbf{E} = \mathbf{C}\mathbf{E}^r \qquad\qquad \mathbf{B} = \mathbf{B}^r + \mathbf{D}\mathbf{E}^r \tag{2.33}$$

where the telluric distortion (2×2) matrix $\mathbf{C}$ is non-dimensional and the magnetic distortion (3×2) matrix $\mathbf{D}$ has the units of an admittance ([s/m]; or [A/V], if $\mathbf{H}$ instead of $\mathbf{B}$ is used). We can derive:

$$\mathbf{Z} = \mathbf{C}\mathbf{Z}^r \cdot (\mathbf{I}_2 + \mathbf{D}_{hor}\mathbf{Z}^r)^{-1} \quad \text{or:} \quad \mathbf{Z} = \mathbf{C}\mathbf{Z}^r - \mathbf{Z}\mathbf{D}_{hor}\mathbf{Z}^r \tag{2.34}$$

If the dimensions of the scattering body are small enough, then the currents related to distortion and the corresponding secondary magnetic fields can be neglected ($\mathbf{Z} = \mathbf{C}\mathbf{Z}^r$). If then the regional fields are due to a one-dimensional structure or due to a two-dimensional structure and the tensor is in the coordinate system of the strike, the effect of distortion shifts the logarithmic ρ_a curves by a factor independent of the frequency, while impedance phases ϕ remain unchanged (*static shift*).

Decomposition techniques resp. transfer function analyses mainly aim at quantifying, if the model of quasi-static distortion can be applied to the data and, if yes, at finding the strike direction of the regional structures, if there is any (*Zhang et al.* [1987]; *Bahr* [1988]; *Groom and Bailey* [1989]). In doing so, most authors only consider electric distortion, which is substantially easier. More recently, also magnetic quasi-static distortion effects are examined *Chave and Smith* [1994]; *Ritter* [1996].

For example, *Bahr* [1988] derived a rotationally invariant *phase sensitive skew*, which measures the phase differences between elements of the same column of the impedance tensor, and thus is a measure how appropriate the quasi-static telluric distortion model is with respect to the data[1]:

$$\eta_b = \frac{\sqrt{[D_1, S_2] - [S_1, D_2]|}}{D_2} \tag{2.35}$$

If the model is appropriate, i.e. if $\eta_b = 0$, the strike direction α_b of the regional 2-D structure can be calculated via

$$\tan 2\alpha_b = \frac{[S_1, S_2] - [D_1, D_2]}{[S_1, D_1] + [S_2, D_2]} \tag{2.36}$$

For cases where the superposition model is not valid, i.e. $\eta_b \neq 0$ and the phase differences between elements of the same column do not vanish in any coordinate system, *Bahr* [1991] (corrected by *Prácser and Szarka* [1999]) found a formula to find an angle α that at least minimizes these phase differences, what the above α_b does not (*phase deviation method*).

[1] $[S_1, D_2] = \Re[S_1]\Im[D_2] - \Im[S_1]\Re[D_2]$,
$S_1 = Z_{xx} + Z_{yy}, \quad S_2 = Z_{xy} + Z_{yx}, \quad D_1 = Z_{xx} - Z_{yy}, \quad D_2 = Z_{xy} - Z_{yx}$