Chapter 3

The Allocation of Authority in a Joint Project under Limited Liability

3.1 Introduction

Whenever several persons or institutions undertake a joint project, decisions that influence all involved parties have to be made. Who should and who will make such a decision in a world of incomplete contracts? This question arises in very different applications. For example, consider two firms or two departments within one firm working on a new product. While one firm or department designs the product, the other one is working on a marketing strategy. Decisions about the quality of the product or the included features have an impact on both parties. A high-quality or complex product is harder to develop and more difficult to explain to the customers. On the other hand, such a quality decision also influences the expected sales. Further examples are two firms forming a research joint venture, and coauthorship by two researchers.

This chapter analyzes the allocation of authority in a project jointly undertaken by two agents. Authority is viewed as the right to undertake a noncontractible project-oriented decision. This decision right is contractually assigned to one of the agents. The decision itself remains unverifiable ex post. It affects not only the decision-maker but also the other parties involved. It determines all parties' private costs as well as the project's expected outcome, independently of who makes the decision. To keep things simple, we assume a binary project outcome. In case of success, a positive output is generated; in case of failure, the project output is zero. Further, we assume a linear success probability and quadratic cost functions. The success of the project is verifiable, so that transfer payments can condition on it. The agents share the available surplus according to generalized Nash bargaining. The focus of the chapter is on the effect of limited liability constraints.

As long as transfers are unrestricted, the first best outcome, which maximizes overall surplus, is reached. But limited liability creates some distortion due to a trade off between surplus maximization and rent extraction. The decision-maker can no longer be compensated for a surplus-maximizing decision. In general, a project different from first best is implemented. Two factors are decisive. First, the differences between the agents' cost functions determine the externality the decision-maker exerts on the other agent. Second, the relationship between the decision-maker's costs and her bargaining power describes the severity of the trade off. The trade off vanishes if and only if bargaining power perfectly reflects the cost structure. If authority is allocated exogenously to the agent whose marginal costs increase faster, the surplus is larger than in the alternative allocation of authority. But if the agents bargain over authority, the other agent receives the decision right if her bargaining power is large and the differences in cost parameters are not too large.

Our restriction to contracts that simply allocate authority seems justified in that their optimality is often robust to the introduction of message games (see Aghion and Rey (2002)). The main results hold true if we allow for more general cost functions. The allocation of authority turns out to be independent of the size of the project output in case of success. This finding does not hold for more general cost functions, but it is not crucial for our main results. Under limited liability, a distortion is created if bargaining power does not reflect marginal costs. This result does not rely on the quadratic functions.[1] If the agent with the steeper marginal cost function receives authority, a project closer to the first best project is implemented than in the alternative allocation. As long as overall surplus is symmetric with respect to the decision under consideration, this allocation creates a larger surplus than the alternative allocation. While this clear result might rest on the symmetry assumption, our general argumentation concerning the allocation of authority should remain valid even in the case of asymmetries. If the agents have similar cost functions, their incentive constraints are also similar and the allocation does not have much effect on the surplus. Rent extraction is favored, and bargaining power is the decisive variable. If the agents are very different in costs, bargaining power plays a minor role, because both agents benefit from an increase in surplus reached through allocating authority to the one with the more efficient decision behavior.

Under limited liability, there is only one instrument – the payment in case of success – to solve two problems, surplus maximization and distribution of surplus. Adding another instrument should cure the resulting inefficiencies. Therefore, one might argue that our results rely on the binary character of the project outcome. To see that this is not the case, assume the project has three instead of two possible outcomes, so that the payment scheme contains an additional payment carried out if the third outcome occurs. If the probabilities are further assumed to be linear, the situation with three possible project outcomes can be reduced to a situation with only two possible outcomes, but possibly with a strict positive project outcome in case of failure. The third payment does not enable us to optimize surplus and distribution independently; it is not an effective instrument.

[1] For details, see section 3.4.1.

If we extend our model to include the case of a small but positive output in case of failure, the limited liability constraints relax. At least a small payment is possible even if the project fails. The larger the project output in case of failure, the larger is the parameter range resulting in a first best efficient outcome. The details are discussed in section 3.5.

The concept of the allocation of authority is widely used in the literature.[2] The allocation may be enforced through asset ownership, so that a transfer of decision rights is a transfer of property rights. The property rights approach according to Grossman and Hart (1986) usually assumes a decision that is not describable at the contracting stage but verifiable at the bargaining stage, while the choice of action(s) in Aghion and Bolton (1992), Hart and Holmstrom (2002), Schmitz (2005), and Bester (2005) remains unverifiable ex post. We follow the latter approach and assume that the project-oriented decision is unverifiable at any point in time. The complexity of a scientific experiment and the detailed quality properties of a new product are too hard to identify for a third party. The enforcement of an allocation of authority is not specified explicitly in our model.

All parties incur costs from undertaking the project. These costs might be disutility from work, but the decision analyzed here is different from the usual effort choice,[3] which influences the decision-maker's costs but not anybody else's costs.[4] In contrast, the decision in our model determines the costs of all parties involved. For example, the introduction of a new product implies more difficult work for the product designer as well as for the marketing specialist if the product has high quality – the latter has to explain a more complex product to the customers. This is no individual effort choice, but a decision that influences both agents' workload. The decision-maker exerts an externality on the other parties. In contrast with our model, Schmitz (2005)

[2]See, for example, Grossman and Hart (1986), Aghion and Bolton (1992), Bester (2005), or Schmitz (2005).

[3]See, for example, Aghion and Tirole (1997).

[4]The assignment of tasks in Holmstrom and Milgrom (1991) is – with respect to the costs – similar to an effort choice.

models the allocation of control rights in a two-stage hidden action problem with an effort choice at every stage. Aghion and Bolton (1992) analyze financial contracting in which one party has private benefits from a chosen action no matter who has chosen the action. Bester (2005) analyzes a model of externalities and the allocation of authority in a firm, which is therefore similar to our model, but he deals with asymmetric information. Whereas Aghion and Tirole (1997) describe how authority influences the agent's incentive to acquire information, Bester (2005) and some other papers analyze the revelation of a given piece of private information, which might be viewed as communication.[5] In our model, the information structure is given as well, but information is completely symmetric.

Most of the papers mentioned above model the allocation of authority as the job of a principal who makes a take-it-or-leave-it offer to an agent. This is reasonable if one party has full commitment power and can credibly threaten not to accept any other contract. In such a setting, the allocation of authority is often referred to as the delegation of decision rights. There is a principal who initially owns the decision right but may transfer it to an agent. Instead, we consider the broader approach of two agents who share the expected overall surplus according to generalized Nash bargaining. The principal agent model is contained as a special case, and we also allow our agents to have partial instead of full commitment power, reflected by their bargaining power. There is no initial owner of decision rights who decides to delegate them; the agents bargain over the decision rights. Therefore, we prefer to talk about the allocation of authority instead of the delegation of decision rights, even though the concepts are very close. Bargaining in Grossman and Hart (1986) takes place ex post. In Aghion and Rey (2002), ex post bargaining is used to induce the decision-maker to make a decision different from his preferred one. This requires the decision-maker to credibly commit to such a decision at this stage, which is impossible in our model. In

[5]Bester (2005) supports truthful revelation of the agent's private information through the possibility of trading authority. In Dessein (2002), the principal chooses either to delegate the decision rights to the better informed agent or to keep authority and communicate noisily with the agent.

contrast, Gans (2005) uses bargaining to determine the ex ante allocation of decision rights. Similarly, our model uses ex ante bargaining at the contracting stage. The terms of contract and in particular the allocation of authority are not the starting point, but the result of the bargaining.

Since payments condition on project outcome, they are carried out ex post after the project is done. A net payment might be unenforceable due to wealth constraints or because the agent could break the contract and walk away instead of paying. It seems reasonable to restrict the set of possible transfers to those that implement a sharing of the realized output. Limited liability is assumed. It creates a trade off between rent extraction and surplus maximization, which leads to an overall surplus lower than first best. Whereas Aghion and Bolton (1992), Pitchford (1998), and Aghion and Rey (2002) analyze parties with different wealth, Schmitz (2005) assumes both parties' liability to be limited completely. We follow the latter approach. In line with these papers, we further assume that the whole output is given to the agents and there is no possibility to threaten to throw away part of the output or give it to a third party. Otherwise, it would always be possible to implement the first best efficient project. If each agent is required to report the project characteristic ex post and the output is destroyed if the reports do not match, the agents report truthfully and the noncontractibility is overcome. Therefore, the budget balance condition is crucial for the analysis.

Grossman and Hart (1986), Aghion and Bolton (1992), and Pitchford (1998) consider the influence of bargaining power on the surplus without linking bargaining power and cost structure directly. This is different in our model: we allow changes in both cost differences and bargaining power, so that we can derive how surplus depends not only on bargaining power itself but on the relationship between bargaining power and costs.

The rest of the chapter is structured as follows: Section 3.2 describes a formal model of the allocation of authority in a joint project. The benchmark case of unlimited liability is analyzed in section 3.3. In section 3.4, the allocation of authority under limited liability is examined. Section 3.5 concludes. The proofs can be found in the section 3.6.

3.2 The Model

The timing of the contracting game is as follows: In the initial stage, the agents bargain and sign the contract. The project choice is noncontractible, but the contract specifies the decision-maker, who chooses one of the possible projects after the contract is signed. The project is undertaken, private costs occur, and project output is realized. The payment scheme is executed. The details are given in the rest of this section.

Two agents $i = 1, 2$ jointly undertake a project. The agents could be, for example, a scientist developing a new product and an advertising director creating the marketing strategy for it. There is a set of possible projects $\mathcal{D} = [0, 1]$. The project characteristic $d \in \mathcal{D}$ could describe, for example, the size of the project or the level of complexity. The project can succeed or fail. If it succeeds, an exogenously given output $X > 0$ is generated. If it fails, the output is zero. The realized output is verifiable. While the two possible outputs are independent of the project's characteristic, the success probability is given by d. A large project is more likely to succeed than a small one, and a project is more likely to succeed if it is undertaken with a high intensity level. For example, a new product is more likely to succeed on the market if it has a high quality. If a project d is undertaken, each agent incurs private costs. Agent i's cost function is $C_i(d) = c_i\, d^2$ with $c_i > 0$. Without loss of generality we assume $c_1 \leq c_2$.

The contract specifies a payment scheme (w_h, w_l). In the case of failure, there is a transfer w_l from agent 1 to agent 2. If the transfer is negative, it is in fact a payment from agent 2 to agent 1. In the case of success, the agents share the output X and may carry out an additional transfer. Agent 1 gets $X - w_h$, while agent 2 gets w_h. As long as $0 \leq w_h \leq X$, the two agents simply share the output. In any other case, one agent receives the whole output plus an additional payment. A slightly different interpretation considers w_l as a transfer independent of success, while payments $w_h - w_l$ and $X - w_h + w_l$, respectively, are in addition received in the case of success only. While the notation might suggest that agent 1 receives the output and compensates agent 2 via w_h, this is a possibly misleading interpretation. Agent 1 does not own the project or have any other privilege that agent 2 does not. Further note that our model incorporates a budget balance condition. It is impossible to throw away part of the output. Under limited liability, we restrict the payments to a sharing of the realized output, that is, $w_l = 0$ and $w_h \in [0, X]$.

The agents are risk-neutral, and their payoffs are composed of their expected benefits and their private costs, resulting in the payoff functions

$$\begin{aligned} U_1(w_h, w_l, d) &= d(X - w_h) + (1-d)(-w_l) - c_1 d^2 \\ &= d(X - w_h + w_l) - w_l - c_1 d^2 \quad , \\ U_2(w_h, w_l, d) &= d w_h + (1-d) w_l - c_2 d^2 \\ &= d(w_h - w_l) + w_l - c_2 d^2 \quad . \end{aligned} \tag{3.1}$$

Each agent's outside option gives a zero payoff. A project d is called (first best) efficient if and only if it maximizes the overall expected surplus $U_1 + U_2$.

The project characteristic d is noncontractible, but the right to choose a project is contractible. This decision right is called authority. The contract specifies the allocation of authority, denoted by r. If $r = 1$, agent 1 receives authority over the project choice, whereas $r = 2$ gives it to agent 2. The agent who receives authority is called the decision-maker. After the contract has been signed, the decision-maker chooses the project to be undertaken, i.e., the project characteristic d. She chooses a project that fulfills her incentive constraint

$$d \in \arg\max_{d' \in \mathcal{D}} U_r(w_h, w_l, d') \quad . \tag{3.2}$$

The agents share the available expected surplus $U_1 + U_2$ according to the generalized Nash bargaining solution.[6] Let $\alpha \in [0, 1]$ indicate agent 1's exogenously given bargaining power, and $1-\alpha$ the bargaining power of agent 2. The agents sign a contract that maximizes

$$B(w_h, w_l, r) = U_1^{\alpha}(w_h, w_l, d)\, U_2^{1-\alpha}(w_h, w_l, d) \quad , \tag{3.3}$$

subject to $d \in \mathcal{D}$, the participation constraints $U_1, U_2 \geq 0$, the decision-maker's incentive constraint (3.2), and $w_l = 0$, $w_h \in [0, X]$ under limited liability. Such a contract is called optimal. If $\alpha = 0$ ($\alpha = 1$), the generalized Nash bargaining results in a principal–agent model with agent 2 (agent 1) as a principal. An allocation of authority is called optimal if there is an optimal contract that implements it. We assume the costs to be large enough to avoid corner solutions; a sufficient condition is $2c_1 + c_2 > X$.[7]

3.3 Unlimited Liability

Throughout this section, take liability to be unlimited.

Proposition 3.1 *There are exactly two optimal contracts; one gives the authority to agent* 1*, and the other one gives it to agent 2. Both optimal contracts implement the first best efficient project*

$$d_e := \frac{X}{2(c_1 + c_2)} \in \mathcal{D} \quad , \tag{3.4}$$

and the agents share the resulting surplus according to their bargaining power, that is, $U_1 = \alpha(U_1 + U_2)$ *and* $U_2 = (1 - \alpha)(U_1 + U_2)$*.*

Proof: See section 3.6.

[6] Nash (1950) introduced this concept for the case of equal bargaining power. It can easily be generalized to allow for agents with different bargaining power.

[7] $2c_1 + c_2 > X$ implies $c_1 + 2c_2 > X$ and $2(c_1 + c_2) > X$. While the latter condition ensures that the first best efficient project and the bargaining outcome under unlimited liability are interior solutions, the first two conditions guarantee that the bargaining outcomes under limited liability are interior for any bargaining power and even if the allocation of authority is exogenously fixed at $r = 1$ or $r = 2$. The conditions can be relaxed in several situations.

Depending on the allocation of authority, the decision-maker's incentive constraint is

$$d = \frac{X - w_h + w_l}{2c_1} \quad \text{or} \quad d = \frac{w_h - w_l}{2c_2} \quad . \tag{3.5}$$

While the appropriate $w_h - w_l$ ensures the implementation of the first best efficient project, the appropriate w_l enforces a distribution of overall surplus according to bargaining power. A contract has the two effective instruments w_h, w_l to determine surplus and distribution. Therefore, surplus and distribution may be optimized independently, and the first best solution is reached.[8] We have $w_l > 0 \iff \alpha < c_1/(c_1+c_2)$. The agent who has a large bargaining power compared to the cost structure *receives* a payment in case of project failure. Such an agent is called *powerful*. Under unlimited liability, there are no inefficiencies, and the allocation of authority has no effect. Bargaining power determines the optimal distribution of surplus, but does not play any further role. However, the efficiency result does not extend to the case of limited liability.

3.4 Limited Liability

From now on, we assume limited liability, so that $w_l = 0$ and $0 \leq w_h \leq X$.

3.4.1 Exogenous Allocation of Authority

Lemma 3.1 *Let agent 1 be the decision-maker, that is, $r = 1$. The optimal contract given this allocation of authority implements the project*

$$d_1 := \frac{(1+\alpha)X}{2(2c_1 + c_2)} \quad , \tag{3.6}$$

[8] In the case of contractible project choice, we have three effective instruments w_h, w_l, and d to control two problems, surplus and distribution. Therefore, we have infinitely many optimal contracts, that is, infinitely many bargaining solutions.

which is the first best efficient project d_e if and only if $\alpha = c_1/(c_1 + c_2)$. One has

$$d_1 \gtreqless d_e \quad \Longleftrightarrow \quad \alpha \gtreqless c_1/(c_1 + c_2) \quad . \tag{3.7}$$

The payment in case of success is

$$w_h = \frac{c_2 + (1 - \alpha)c_1}{2c_1 + c_2} X \quad . \tag{3.8}$$

Proof: See section 3.6.

In the case $r = 1$, the first best efficient project d_e is implemented if and only if bargaining power reflects cost structure, so that $\alpha = c_1/(c_1 + c_2)$. If the decision-maker agent 1 is powerful, that is, $\alpha > c_1/(c_1 + c_2)$, she implements an inefficiently large project $d_1 > d_e$. Since $X - w_h$ is increasing in α, a powerful agent 1 can extract a large share of the output in case of success. She benefits from a large project choice. Instead, if $\alpha < c_1/(c_1 + c_2)$, it does not pay to choose a large and costly project, since the extra rent from success is small. A project smaller than first best is implemented. If agent 2 is the decision-maker, the condition that ensures first best efficiency is again $\alpha = c_1/(c_1 + c_2)$. A powerful decision-maker – which now means one for whom $1 - \alpha > c_2/(c_1 + c_2)$, that is, $\alpha < c_1/(c_1 + c_2)$ – implements a project larger than first best, whereas if $\alpha > c_1/(c_1 + c_2)$, a project smaller than first best is implemented. Giving the decision right to the agent with bargaining power "too large" to reflect the cost structure will lead to a project "too large," while giving it to the other agent will result in a project "too small" compared to the first best efficient one. Under limited liability, $w_l = 0$ is fixed and a contract has only one effective instrument to determine surplus and distribution. The wage w_h determines both the project choice (which fixes the surplus) and the distribution of surplus. Those cannot be optimized simultaneously.

3.4.2 Allocation of Authority via Bargaining

If the allocation of authority is a result of the bargaining process, the trade off between rent extraction and surplus maximization concerns not only the payment scheme but also the allocation of authority. As a benchmark, Lemma 3.2 describes the effect of authority on the overall surplus.

Lemma 3.2 *Overall surplus depends on the allocation of authority if and only if* $\alpha \neq c_1/(c_1+c_2)$*. The distortions relative to first best for the two allocations can be compared by comparing*

$$\begin{aligned} |d_1 - d_e| &= d_e \frac{|\alpha(c_1+c_2) - c_1|}{2c_1 + c_2} \quad , \\ |d_2 - d_e| &= d_e \frac{|\alpha(c_1+c_2) - c_1|}{c_1 + 2c_2} \quad . \end{aligned} \tag{3.9}$$

If the agent with the larger cost parameter receives the decision right, the overall surplus is larger than in the alternative allocation of authority. Since we have $c_1 \leq c_2$ *by assumption, the surplus is larger if authority is allocated to agent 2.*
Proof: See section 3.6.

Since the overall expected surplus $U_1 + U_2$ is a symmetric function with its unique maximum at d_e, the distance $|d - d_e|$ is an appropriate measure for the loss of surplus (compared to first best) occurring if the project d is implemented. While limited liability is the only source of distortion in our model, there are still two different effects at work. The decision-maker's project choice is distorted away from the first best efficient project because bargaining power does not reflect cost structure. This is measured by $|\alpha(c_1+c_2)-c_1|$, which might be viewed as a measure of the severity of the trade off. Since w_h does not only determine the surplus (as is the case under unlimited liability) but is also used for rent extraction, some distortion is created. We call this effect the *rent extraction effect.* It is independent of the allocation of authority. If any other source of distortion is eliminated by assuming agents who differ only in bargaining power, that is, assuming $c_1 = c_2$, then both possible allocations of authority result in the same amount of distortion. The amount by which one agent chooses "too much" is exactly the amount by which the other one chooses "too little." Still, the size of the distortion depends on bargaining power. If $c_1 \neq c_2$, there is an additional source of distortion measured by the terms $2c_1 + c_2$ and $c_1 + 2c_2$. The decision-maker cares about her own costs, but does not fully internalize the externalities exerted on the other agent. She puts too little weight on the other agent's costs. We call this effect the *externality effect*. It is reflected by the denominators in (3.9), showing that the distortion is less drastic if authority is given to the agent with the larger cost parameter.

Looking at the proofs, it turns out that first best is implemented if bargaining results in $w_h = C_2' \circ (C_1' + C_2')^{-1}(X)$. Note that w_h depends on α, since it is a bargaining outcome, so that this is fulfilled if bargaining power reflects marginal costs. The rent extraction effect vanishes. For $C_i(d) = c_i d^2$, one has $\alpha = c_1/(c_1 + c_2)$. A change in α changes the bargaining outcome w_h and creates some distortion. The agents' incentive constraints are, in general, $w_h = C_2'(d)$ and $X - w_h = C_1'(d)$. A project closer to first best is implemented if the agent with the steeper marginal cost function is the decision-maker. The externality effect is minimized.

If there is unlimited liability or the bargaining power perfectly reflects the cost structure, the decision-maker does not face a trade off. The rent extraction effect vanishes, the first best efficient project is implemented, and the externality effect is idle. On the other hand, if the agents are identical so that there is no externality effect, the rent extraction effect is still at work. While the rent extraction effect is independent of the allocation of authority, the externality effect brings about that a higher surplus is generated if agent 2 (who has the steeper marginal cost function) is the decision-maker. But the bargaining might as well result in agent 1 being the decision-maker.

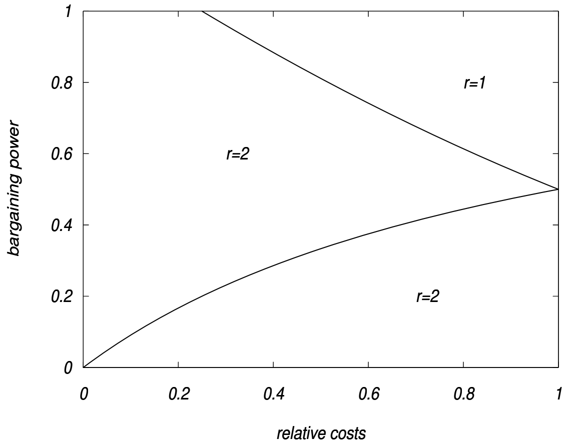

Figure 3.1: Contour Plot of $\phi(k, \alpha)$ Describing the Allocation of Authority r

To state the following proposition, we define

$$\begin{aligned}\phi(k, \alpha) := & (2k + 1)^{1+\alpha} (2 - \alpha)^{2-\alpha} \alpha^{\alpha} \\ & - (k + 2)^{2-\alpha} k^{\alpha} (1 + \alpha)^{1+\alpha} (1 - \alpha)^{1-\alpha}\end{aligned} \tag{3.10}$$

with $k := c_1/c_2$.

Proposition 3.2 *Generalized Nash bargaining results in a unique optimal contract if and only if* $\phi(k, \alpha) \neq 0$*. If* $\phi(k, \alpha) < 0$*, agent 1 receives authority, while if* $\phi(k, \alpha) > 0$ *it is allocated to agent 2. If* $\phi(k, \alpha) = 0$*, there are two optimal contracts, which differ in the allocation of authority.*
Proof: See section 3.6.

Figure 3.1 shows a contour plot of ϕ with k on the horizontal axis and α on the vertical axis. The plot approximates the sets of (k, α) with $\phi(k, \alpha) > 0$ and with $\phi(k, \alpha) < 0$. The upper right corner, which is characterized by a large k and a large α, describes the parameter constellations that result in agent 1 being the decision-maker. If agent 1's bargaining power is large and simultaneously her cost parameter nearly equals agent 2's cost parameter, she receives authority. The allocation of authority is not unique along the curve separating this area from the remaining parameter constellations. This curve describes a switch from $\phi(k, \alpha) > 0$ to $\phi(k, \alpha) < 0$, which is a switch in the allocation of authority. In addition, the allocation is not unique along the curve $\alpha = k/(k+1)$, which in fact is $\alpha = c_1/(c_1 + c_2)$. This curve does not describe a switch in the allocation; one has $\phi(k, \alpha) > 0$ above as well as below it. Below this curve, the decision-maker agent 2 is powerful (having bargaining power too large to reflect cost structure) and chooses a project larger than the first best project, while above it she is still the decision-maker, but chooses a project smaller than first best.

If the agents have similar cost functions, then the externality effect is small and the allocation of authority does not have much influence on the surplus. Rent extraction becomes important. In the extreme case $c_1 = c_2$ and $k = 1$, the overall expected surplus is even independent of the allocation of authority. For each agent, getting the decision right increases the payoff, since it allows her to extract a larger share of the surplus. The agent with the larger bargaining power receives authority. Consider a small decrease in k. The allocation of authority has two effects: If agent 1 is the decision-maker, the overall surplus is smaller than if agent 2 is. But she can extract a larger share of overall surplus. An increase in α increases the effect of the allocation on the share of the rent.[9] If agent 1 is the decision-maker, an increase in α decreases the overall surplus (due to the rent extraction effect) but increases her share of the rent. The larger α, the more agent 1 benefits from having the decision right. If α is above a certain threshold, agent 1 is better off if she receives authority instead of agent 2. In addition, the large α enables her to get the desired authority. If k is quite small instead, the externality effect is large and the allocation has a strong effect on the size of the surplus. No matter how powerful agent 1 is, she does not receive authority, but benefits from the large surplus realized by the decision of agent 2. Therefore, the cost structure plays a decisive role in allocating decision rights, which – in some parameter constellations – even outweighs the influence of bargaining power.

[9] This can be seen from direct calculation of the rent share.

3.5 Conclusion

In this chapter, we have developed a simple model of the allocation of authority in a joint project. Generalized Nash bargaining allocates authority to the agent with the steeper marginal cost function as long as the other agent has a significantly different cost function or is not too powerful. This allocation results in a larger surplus than the alternative allocation. If the cost functions differ a lot, it might be optimal for someone with high bargaining power not to have authority, as in Aghion and Tirole (1997).

Under unlimited liability, the optimal allocation of authority is not unique and first best efficiency is always reached. This is in line with the findings in Aghion and Bolton (1992) and Aghion and Rey (2002) for wealthy agents. Imposing limited liability constraints now creates some distortion. The resulting loss compared to the first best surplus is smaller if authority is allocated to the agent with the steeper marginal cost function, independent of the agents' bargaining power. This result, on first sight, looks different from that of Aghion and Rey (2002). In their model, payments needed to induce a first best efficient outcome depend on the allocation of authority, while in our model, the payments under unlimited liability are independent of the allocation. In Aghion and Rey (2002), there are no incentive payments to influence the decision-maker, but the agents bargain directly over the action to be taken, so that the allocation of authority is the starting point of the bargaining, not the bargaining outcome.

In our model, bargaining power influences the surplus in two ways. First, for an exogenously given allocation of authority, bargaining power influences decision-making. The loss compared to the first best surplus depends on how the agents' bargaining power reflects the cost structure, that is, the rent extraction effect. This can be viewed as a generalization of Pitchford (1998) who analyzes a principal who has no private costs related to the decision and a decision-making agent who has private costs. He shows that the efficiency of the implemented decision increases in the agent's bargaining power.

Since the agent bears all costs in his model, bargaining power reflects cost structure if the agent has all bargaining power. Second, the distortion of the decision-maker's choice also depends on the relationship between the two agent's costs, the externality effect. For a given allocation, it is independent of the bargaining power. But since the allocation is a bargaining outcome, it clearly depends on bargaining power. Therefore, bargaining power influences who receives authority and how this party executes authority. Similar to Gans (2005), the bargaining in our model can result in an allocation which leads to a smaller surplus than the alternative allocation.

Throughout the chapter, we have assumed that a failing project does not generate any output. Now consider the more general case that a failing project generates an output $x \in [0, X]$. The limited liability constraints relax to $0 \leq w_l \leq x$ and $0 \leq w_h \leq X$ so that the trade off between surplus maximization and rent extraction is less severe, decreasing the importance of the rent extraction effect. Under unlimited liability, the optimal payment scheme is

$$\begin{aligned} w_l &= (1-\alpha)x + \frac{(X-x)^2}{4(c_1+c_2)^2}\left[-\alpha c_2 + (1-\alpha)c_1\right] \quad , \\ w_h - w_l &= \frac{c_2(X-x)}{c_1+c_2} \quad . \end{aligned} \tag{3.11}$$

If bargaining power perfectly reflects the cost structure, so that $\alpha = c_1/(c_1 + c_2)$, the agents receive a share of the realized project output according to their bargaining power, so that $w_l = (1-\alpha)x$ and $w_h = (1-\alpha)X$. The more α deviates from $c_1/(c_1+c_2)$, the more the payments deviate from such a sharing of output. But as long as the deviation is not too large, the limited liability constraints hold. As a result, there is an interval $[\underline{\alpha}, \bar{\alpha}]$ such that for each $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, the first best efficient project is implemented even if limited liability is imposed. For $x = 0$, this interval shrinks to the single point $\alpha = c_1/(c_1 + c_2)$. The length of the interval is increasing in x, and we have $[\underline{\alpha}, \bar{\alpha}] = [0, 1]$ if and only if $x = X$.

Possible extensions of our model include the division of tasks in multitask projects, the introduction of effort incentives, and third-party involvement with collusion. A crucial assumption for our results is the contractibility of the project output. Otherwise, ex ante bargaining over the output would be impossible. The agent who receives the output could only pay a flat transfer to the other agent, which is limited to the output in case of failure, because this agent would always claim that the project failed. If this agent could not take away the output without the other agent's approval, we would also have ex post bargaining over the output. These extensions are left for future research.

3.6 Proofs

Proof of Proposition 3.1:

It is straightforward that $d_e \in \mathcal{D}$ is the first best efficient project. A contract is optimal if and only if it solves

$$\max_{w_h, w_l, r} \; U_1^{\alpha}(w_h, w_l, d)\, U_2^{1-\alpha}(w_h, w_l, d) \tag{3.12}$$

subject to $U_1, U_2 \geq 0$ and

$$d \in \arg\max_{d' \in \mathcal{D}} \; U_r(w_h, w_l, d') \quad . \tag{3.13}$$

We calculate the optimal contract(s) given r and find the optimal one(s) among those by evaluating the bargaining function. Let $r = 1$. As long as $d^* = (X - w_h + w_l)/2c_1$ is an element of $\mathcal{D}$, it is the unique project maximizing U_1. Assume for the moment that $d^* \in \mathcal{D}$. Plugging in d^* gives

$$\begin{aligned} U_1 &= \frac{(X - w_h + w_l)^2}{4c_1} - w_l \quad , \\ U_2 &= \frac{X - w_h + w_l}{2c_1}(w_h - w_l) + w_l - \frac{c_2(X - w_h + w_l)^2}{4c_1^2} \quad . \end{aligned} \tag{3.14}$$

The first-order conditions for maximizing B are

$$\begin{aligned}\alpha U_1^{\alpha-1}U_2^{1-\alpha}\frac{\partial U_1}{\partial w_l}+(1-\alpha)U_1^{\alpha}U_2^{-\alpha}\frac{\partial U_2}{\partial w_l}&=0 \quad ,\\ \alpha U_1^{\alpha-1}U_2^{1-\alpha}\frac{\partial U_1}{\partial w_h}+(1-\alpha)U_1^{\alpha}U_2^{-\alpha}\frac{\partial U_2}{\partial w_h}&=0 \quad .\end{aligned} \tag{3.15}$$

For the moment, assume there is at least one contract with $B > 0$ fulfilling the constraints, so that each optimal contract satisfies $B > 0$ as well. Then we have either $U_1, U_2 > 0$, or $U_1 > 0$, $U_2 = 0$, $\alpha = 1$, or $U_1 = 0$, $U_2 > 0$, $\alpha = 0$. The last two cases imply $B = U_1 + U_2$. Some straightforward calculations show that the maximum of $B = U_1 + U_2$ is reached at $w_h - w_l = [c_2/(c_1 + c_2)]X$. Using (3.14), it follows that $\alpha U_2 = (1 - \alpha)U_1$, so that $U_1 = \alpha(U_1 + U_2)$ and $U_2 = (1 - \alpha)(U_1 + U_2)$.

If $U_1, U_2 > 0$, then $U_1^{\alpha-1}U_2^{-\alpha} > 0$. Dividing the first-order conditions by $U_1^{\alpha-1}U_2^{-\alpha}$ results in

$$\begin{aligned}\alpha U_2\frac{\partial U_1}{\partial w_l}+(1-\alpha)U_1\frac{\partial U_2}{\partial w_l}&=0 \quad ,\\ \alpha U_2\frac{\partial U_1}{\partial w_h}+(1-\alpha)U_1\frac{\partial U_2}{\partial w_h}&=0 \quad .\end{aligned} \tag{3.16}$$

Since $\partial U_1/\partial w_l = -1 - \partial U_1/\partial w_h$ and $\partial U_2/\partial w_l = 1 - \partial U_2/\partial w_h$, the conditions in the case of $U_1, U_2 > 0$ are equivalent to

$$\begin{aligned}\alpha U_2 &= (1-\alpha)U_1 \quad ,\\ 0 &= \frac{\partial U_1}{\partial w_l}+\frac{\partial U_2}{\partial w_l} \quad .\end{aligned} \tag{3.17}$$

The first equation implies $\alpha \in (0, 1)$. Rearranging the second equation again leads to $w_h - w_l = c_2/(c_1 + c_2)X$ and $d^* = d_e$. The assumption $d^* \in \mathcal{D}$ is justified. The implementation of the project results in a surplus $U_1 + U_2 = X^2/[4(c_1 + c_2)]$. On combining this with $\alpha U_2 = (1 - \alpha)U_1$, it follows that $U_1 \geq 0$ with equality if and only if $\alpha = 0$, and $U_2 \geq 0$ with equality if and only if $\alpha = 1$. The ad hoc assumption $B > 0$ is justified. Some straightforward calculations give the unique payment scheme

$$\begin{aligned} w_l &= \frac{X^2}{4(c_1+c_2)^2}\left[-\alpha c_2 + (1-\alpha)c_1\right] \quad , \\ w_h &= \frac{X^2}{4(c_1+c_2)^2}\left[-\alpha c_2 + (1-\alpha)c_1\right] + \frac{c_2}{c_1+c_2}X \quad . \end{aligned} \tag{3.18}$$

The unique optimal contract given $r = 1$ is described by (3.18).

Now assume $r = 2$, and proceed analogously to the case $r = 1$. Note that the two problems are symmetric under $\alpha \leftrightarrow (1-\alpha)$, $c_1 \leftrightarrow c_2$, $w_l \leftrightarrow -w_l$, and $w_h \leftrightarrow X - w_h$. The payoff U_2 is maximized by the unique project $d^{**} = (w_h - w_l)/2c_2$. Maximizing the bargaining function B results in exactly the same payment scheme and project as if $r = 1$.

To summarize, there are two contracts that are candidates for an optimal contract, one with $r = 1$ and one with $r = 2$. Both contracts implement the project d_e and the payment scheme (3.18). Hence the payoffs U_1 and U_2 as well as the bargaining function B take the same values in both cases. The two candidates turn out to be the optimal contracts. ■

Proof of Lemma 3.1:
An optimal contract solves

$$\max_{w_h} U_1^{\alpha}(w_h, 0, d) U_2^{1-\alpha}(w_h, 0, d) \tag{3.19}$$

subject to $0 \leq w_h \leq X$, $U_1, U_2 \geq 0$, and $d \in \arg\max_{d' \in \mathcal{D}} U_1(w_h, d')$. As long as $d_1 := (X - w_h)/2c_1$ is an element of $\mathcal{D}$, it is the unique project maximizing U_1. For the moment, assume $d_1 \in \mathcal{D}$. Plugging d_1 in yields

$$\begin{aligned} U_1 &= \frac{(X-w_h)^2}{4c_1} \quad , \\ U_2 &= \frac{(X-w_h)w_h}{2c_1} - \frac{c_2(X-w_h)^2}{4c_1^2} \end{aligned} \tag{3.20}$$

and

$$\frac{\partial U_1}{\partial w_h} = -\frac{X - w_h}{2c_1} \quad , \qquad \frac{\partial U_2}{\partial w_h} = \frac{c_1 + c_2}{2c_1^2}(X - w_h) - \frac{w_h}{2c_1} \quad . \tag{3.21}$$

The remaining proof is similar to the proof of Proposition 3.1. Assume that there is at least one contract fulfilling $B > 0$ and the required constraints, so that each optimal contract satisfies $B > 0$ as well. Since $U_1 = 0$ implies $w_h = X, U_2 = 0$, and $B = 0$, necessarily $U_1 > 0$ and $w_h < X$, and we have $U_1, U_2 > 0$ or $U_1 > 0$, $U_2 = 0$, $\alpha = 1$.

Consider $U_1, U_2 > 0$. Dividing $\partial B/\partial w_h = 0$ by $U_1^{\alpha-1}U_2^{-\alpha} > 0$ leads to the first-order condition

$$\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1-\alpha)U_1 \frac{\partial U_2}{\partial w_h} = 0 \quad , \tag{3.22}$$

which by (3.20) and (3.21) is

$$\alpha \left[-\frac{(X - w_h)^2 w_h}{4c_1^2} + \frac{c_2(X - w_h)^3}{8c_1^3} \right] + (1-\alpha) \left[\frac{(c_1 + c_2)(X - w_h)^3}{8c_1^3} - \frac{(X - w_h)^2 w_h}{8c_1^2} \right] = 0 \quad . \tag{3.23}$$

Since $w_h = X$ is already ruled out, the unique solution of (3.23) is

$$w_h = \frac{c_2 + (1-\alpha)c_1}{2c_1 + c_2} X \quad . \tag{3.24}$$

Solving $U_2 = 0$ for w_h gives exactly the same payment for the case $U_1 > 0$, $U_2 = 0$, $\alpha = 1$. This payment fulfills the limited liability constraints and leads to

$$d_1 = \frac{(1+\alpha)X}{2(2c_1 + c_2)} \quad , \tag{3.25}$$

which is in $\mathcal{D}$, since $1+\alpha \leq 2$ and $2c_1 + c_2 > X$ by assumption. Using (3.20), (3.24), and (3.25) gives

$$U_1 = \frac{(1+\alpha)^2 X^2 c_1}{4(2c_1+c_2)^2} \ > \ 0 \tag{3.26}$$

and

$$U_2 = \frac{(1-\alpha)(1+\alpha)X^2}{4(2c_1+c_2)} \ \geq \ 0 \tag{3.27}$$

with equality if and only if $\alpha = 1$. The ad hoc assumption $B > 0$ is justified. ■

Proof of Lemma 3.2:
Note that $U_1 + U_2 = dX - (c_1 + c_2)d^2$ is a parabola open below with its maximum at d_e. To put it differently, $U_1 + U_2$ is strictly decreasing in $|d - d_e|$. It is straightforward to calculate

$$|d_1 - d_e| = d_e \frac{|\alpha(c_1+c_2) - c_1|}{2c_1 + c_2} \tag{3.28}$$

and

$$|d_2 - d_e| = d_e \frac{|\alpha(c_1+c_2) - c_1|}{c_1 + 2c_2} \quad . \tag{3.29}$$

We have $|d_1 - d_e| = |d_2 - d_e| = 0$ if and only if $\alpha = c_1/(c_1 + c_2)$. The allocation of authority does not influence the overall surplus in this case. If $\alpha \neq c_1/(c_1 + c_2)$, we have

$$|d_1 - d_e| \geq |d_2 - d_e| \iff c_1 \leq c_2 \quad . \tag{3.30}$$

The surplus is increased by allocating authority to the agent with the larger cost parameter, who is agent 2 by the assumption $c_1 \leq c_2$. ■

Proof of Proposition 3.2:
Consider $r = 1$. Using U_1 and U_2 from the proof of Lemma 3.1, the value of

the bargaining function is calculated to be

$$B = \frac{X^2}{4}\left(\frac{1}{2c_1+c_2}\right)^{1+\alpha} c_1^{\alpha}\,(1+\alpha)^{1+\alpha}\,(1-\alpha)^{1-\alpha} =: B_1 \quad . \tag{3.31}$$

If $r = 2$, analogous calculations lead to

$$B = \frac{X^2}{4}\left(\frac{1}{c_1+2c_2}\right)^{2-\alpha} c_2^{1-\alpha}\,(2-\alpha)^{2-\alpha}\,\alpha^{\alpha} =: B_2 \quad . \tag{3.32}$$

If and only if $B_1 = B_2$, there are two optimal contracts that differ in the allocation of authority. Otherwise, the optimal contract is unique, with agent 1 being the decision-maker if $B_1 > B_2$, and agent 2 if $B_2 > B_1$.

Define $k := c_1/c_2$, so that

$$B_1 = \frac{X^2}{4}(2k+1)^{-1-\alpha}k^{\alpha}c_2^{-1}\,(1+\alpha)^{1+\alpha}\,(1-\alpha)^{1-\alpha} \tag{3.33}$$

and

$$B_2 = \frac{X^2}{4}(k+2)^{-2+\alpha}c_2^{-1}\,(2-\alpha)^{2-\alpha}\,\alpha^{\alpha} \quad . \tag{3.34}$$

It follows that

$$B_2 \gtreqless B_1 \quad \Longleftrightarrow \quad \phi(k,\alpha) \gtreqless 0 \tag{3.35}$$

with

$$\begin{aligned}\phi(k,\alpha) \;&:=\; (2k+1)^{1+\alpha}\,(2-\alpha)^{2-\alpha}\,\alpha^{\alpha} \\ &-\; (k+2)^{2-\alpha}k^{\alpha}\,(1+\alpha)^{1+\alpha}\,(1-\alpha)^{1-\alpha} \quad .\end{aligned} \tag{3.36}$$

■