Introduction

Dynamical processes of phase separation and related spatial pattern formation are characteristic for many two component (binary) systems in which phase separation can be induced by rapid cooling ("quick quenching") the system. Thus, if a two component system which is spatially uniform at temperature T_1 is rapidly cooled to a second sufficiently lower temperature T_2, then the cooled system will separate itself out into regions of higher and lower concentrations. A standard description of such systems can be obtained by energy minimization arguments. The claim would be that there exists some critical temperature T_{crit}, so that for $T > T_{crit}$ the appropriate thermodynamical quantity, the *free energy*, is single welled whereas for $T < T_{crit}$ the free energy is double welled. A system of average concentration u_{av} which was spatially uniform at temperature T_1, when cooled to temperature T_2 would find it energetically preferrable to subdivide itself into two systems, one at concentartion u_1 and one at concentration u_2, where $u_1 + u_2 = 1$. The time evolution of this subdivision is usually described by the classical Cahn-Hillard model [7]. Meanwhile there are two big types of Cahn-Hillard equations, whereas every type also has various types of generalizations. The *local* or standard Cahn-Hillard equations belong to the first group whereas the *nonlocal* Cahn-Hillard equations belong to the second one. They have all in common that the starting point of their derivation is to establish the corresponding free energy functional.

This work is devoted to a nonlocal viscous Cahn-Hillard model. It is devided into six chapters. We start by introducing the different existing models, which describe phase separation. After establishing the corresponding free energy functional for every model, we derive the Cahn-Hillard equations. In Chapter 1.1 this is done for the standard Cahn-Hillard equation and the viscous Cahn-Hillard equation, which is a common generalization of the standard one. Both of these equations have been studied intensively since the paper of Cahn and Hillard [7]. In Chapter 1.2 we introduce the nonlocal Cahn-Hillard model, which is based on a nonlocal free energy functional (1.10). We derive the new nonlocal viscous Cahn-Hillard equation, which we investigate mathematically in this work. In Chapter 2 we establish the mathematical problem. We formulate two Theorems, which we prove in the following Chapters 3 and 4. Our Theorems ensure the existence and uniqueness of a solution to the nonlocal viscous Cahn-Hillard equation under different regularity conditions on the given data. To prove existence we start in Chapter 3 and 4 with a special regularization of the nonlinearity and a truncation of the nonlocal term. For the regularized equation we prove existence via a semidiscrete scheme together with

Schauder's fixed-point method. To get the existence for the original problem we establish a priori estimates, which are uniform with respect to the regularization parameter. In Chapter 5 we prove a special regularity result by Moser iteration in the form of Alikakos (see [3]). The regularity result ensures that the concentration of the system relies strictly between zero and one ($0 < u < 1$), provided this is true for the initial concentration. The key role in our proof plays a special choice of a testfunction, which allows us to analyze the behaviour of solutions u on subsets of the domain of definition of u. In chapter 6 we study the longtime behaviour by using the energy estimate. We are able to prove the convergence of the solution trajectory to an equilibrium point. But, accordingly to the fact that equilibrium points cannot expected to be unique, this convergence is shown only along subsequences.

To conclude the introduction let us mention some literature in this field from which we started our work. There is the main article [15] by Gajewski and Zacharias where the nonlocal phase separation problem without viscosity is discussed. Then there is the work of Griepentrog [19], which however deals with multicomponent nonlocal phase separation.

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