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# Essays on Monetary Policy and the European Monetary Union

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## Abstract

The thesis consists of four essays of independent interest which make theoretical and empirical contributions to the fields of monetary economics and economic integration. The first essay studies the implications of measurement bias in inflation for the conduct of monetary policy. In a business cycle model with product entry and a stabilization role for monetary policy, measurement bias in inflation originates from a failure of the statistical authority to account for new products in time. Measurement bias depends systematically on the state of the business cycle and dampens inflation volatility but increases inflation persistence. If not accounted for by monetary policy, inflation mismeasurement results in too little inflation stabilization. The second essay points to a tension between stylized facts and the standard monetary model concerning money demand. Whereas the evidence for dynamic money demand is overwhelming, money demand in the standard model remains static. I reconcile the standard model with dynamic money demand and revisit the optimal monetary policy problem in the modified model. Even though dynamic money demand implies that money matters a lot for social welfare, monetary policy should pay little attention to money. This result relates to the ongoing debate on the monetary policy strategy of the European Central Bank. The third essay considers the catch up process of new European Union (EU) member states. New EU member states experience real exchange rate appreciation and, at the same time, terms of trade which improve vis á vis the euro area. Whereas the two-country two-goods real business cycle model cannot explain both facts simultaneously, I show that it can once one accounts for endogenous product variety. This finding suggests that the fundamental driving force behind the sustainable catch up process in new EU member states is a form of productivity which boosts product variety rather than product quantity. Essay number four empirically uncovers the effect of the common European currency on trade. Empirical models set up to analyze the trade effect of the euro are often restrictive. Jointly with Helmut Herwartz, we pursue a more general approach to estimate the When, How Fast and by How Much of adjustment in euro area trade. We find gradual trade creation between the years 2000 and 2003 and document that assumptions with respect to the timing of the change in trade matter for conclusions about the size of this change.

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Economics is full of measurement issues, and I am sure that also this thesis has its share of mismeasurement and error. I take sole responsibility for this.

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# Chapter 1

## Introduction

Monetary policy and its conduct in a monetary union raises multiple macroeconomic policy questions. In my thesis, I deal with four of them. Each question is independently examined in one essay.

The first essay analyzes measurement bias in consumer price inflation over the business cycle. Measurement bias in inflation is caused by the failure of statistical authorities to track the introduction of new products in time. Inflation is a core target variable for monetary policy and my framework traces repercussions of inflation mismeasurement for the performance of monetary policy. Recent evidence suggests that mismeasurement at business cycle frequency is substantial which underlines the empirical relevance of the approach.

In the second essay I point to a tension between emerging stylized facts about empirical money demand and money demand in the standard monetary model. Accordingly, I propose to generalize the standard model by dynamic money demand behavior. I revisit the optimal monetary policy problem and demonstrate the implication of the modified money demand function for the ongoing debate about the monetary policy strategy of the European Central Bank.

The third essay takes a closer look at the forces driving real exchange rates and terms of trades in the new member states of the European Union (EU) which are candidates to join the Economic and Monetary Union (EMU) at some point in the future. New EU member states undergo a sustained period of transition from formerly planned economies into distinct market economies. The essay develops a theoretical model that describes the factors which determine the path of the new members' international competitiveness during the catch up process. The model emphasizes sources of productivity, which increase product variety, and demand factors.

Essay number four estimates the change in trade costs in the euro area after the

introduction of the common currency. In much of the existing empirical literature, the introduction of the euro is associated with falling trade costs which translate into trade creation. However, researchers frequently rely on a fairly uniform empirical model which is restrictive in a number of dimensions. Jointly with Helmut Herwartz I propose and estimate a more general empirical model. Our results suggest that several assumptions typically adopted are at odds with the data.

Methodologically, all essays but the fourth rely on dynamic stochastic general equilibrium (DSGE) models which have become the standard theoretical approach for policy analysis of this kind. DSGE models, introduced by Kydland and Prescott (1982), were first applied to study nonmonetary economies. More recently, Kimball (1995) and others have combined these models with Keynesian elements to what is known as the New Keynesian model. Today, the methodology is state of the art for both academic researchers and policy institutions, such as central banks, and is being applied on a large variety of macroeconomic topics.

DSGE models derive relationships among macro variables from decision problems of economic agents formulated at the micro level. Agents solve their decision problems in an environment perturbed by random events. Quite naturally, some of these decision problems are inherently forward looking, such as investment or the life time consumption plan, so that agents formulate (rational) expectations about future states of the economy when solving their problems. Structural relations among variables therefore depend on these expectations. From a conceptual point of view, microfounded DSGE models are resistant against the influential Lucas (1976) critique of econometric policy evaluation.

The empirical exercise underlying the fourth essay applies the Kalman filter. The Kalman filter allows to efficiently estimate first and second moments of unobserved states conditional on a vector of observed variables. We apply this method within a panel framework with large time dimension to control for time variant unobserved heterogeneity. We handle a large cross section dimension by functional regression. The combination of Kalman filtering and functional regression delivers a parsimonious specification which is essential considering the short (data) history of the euro. In the following, I provide an introductory review of each essay.



## 1.1 Review of Chapter 2 on Inflation Mismeasurement

The aggregate price level is defined as the money price of a particular consumption basket of products. According to standard economic theory, monetary policy is in the position to manipulate the path of this price level. According to conventional wisdom, monetary policy is successful if it manages to implement a period by period increase in prices which is moderate. Controlling inflation is a complicated task, among others because measuring inflation is challenging. A main difficulty for statistical authorities when measuring the aggregate price level is the arrival of new products. Recent empirical evidence in Broda and Weinstein (2007) documents that the late inclusion of new products into the consumption basket maintained by the statistical authority induces sizeable and cyclical measurement bias. Cyclical bias in the price level implies that the bias also contaminates the measurement of inflation. My essay in chapter 2 analyzes measurement bias in inflation and its repercussions for the conduct of monetary policy.

I integrate measurement bias in inflation into a DSGE model with product entry and a stabilization role for monetary policy. In the model, a statistical authority computes consumer price inflation based on Laspeyres's formula. When measuring the price level the statistical authority is constrained not to observe newly arrived products for a certain number of periods. The model delivers a decomposition of measured inflation into actual inflation and measurement bias and allows to study the time series properties of the bias. The measurement bias correlates negatively with measured inflation and exhibits considerable persistence. A main prediction of the model is that measured inflation overstates actual inflation when productivity growth is strong. This prediction corroborates a concern of policy makers that inflation mismeasurement hides periods of defacto deflation.

The model lends itself to studying repercussions of measurement bias for the conduct of monetary policy and for social welfare. I analyze two different monetary policy regimes. In one regime, monetary policy is assumed to perfectly commit to future action. Commitment serves to manage private sector expectations which reduce the tradeoff between stabilizing inflation and stabilizing measures of real activity, thereby improving welfare. In contrast, under discretionary policy the central bank is not in the position to affect private sector expectations. Across the two monetary policy regimes unaccounted measurement bias always implies that inflation is stabilized too little, whereas real activity is stabilized too much. Measurement bias always deteriorates welfare under committed monetary policy because it implicitly reshuffles the stabilization focus from inflation to real activity.

Surprisingly, however, measurement bias may improve welfare under discretionary monetary policy despite too little inflation stabilization. The reason is that even though the private sector does not believe in commitments issued by a discretionary central bank, it understands that the way how the statistical authority constructs measured inflation is not easily modified. Monetary policy is necessarily formulated in terms of measured inflation because actual inflation remains unobserved. The central bank's reference to measured inflation implicitly induces a reference to past actual inflation rates because inflation mismeasurement puts more weight on past rather than on current actual price dynamics. However, responding to past economic states represents defacto commitment. The fact that this more subtle form of commitment cannot be manipulated by the central bank itself makes it credible for the private sector. Therefore, as long as a central bank has difficulties to commit, inflation mismeasurement may actually be a social asset rather than a liability because mismeasurement lets the central bank borrow commitment from the statistical authority.

The vast empirical literature which assesses measurement bias in the index of consumer prices has mainly delivered estimates of average bias and has tried to qualify these estimates by attaching standard errors to them. Shapiro and Wilcox (1996), Lebow and Rudd (2003), Gordon (2006) and Lebow and Rudd (2006) contain very useful reviews of this literature. Quite natural, the main constraint in assessing the accuracy of consumer prices has been data availability. In contrast, the study of Broda and Weinstein (2007) relies on a unique micro data set of prices and quantities for the entire universe of consumer products in a large sector of the U.S. economy. This data set has been extremely useful to infer the cyclicity of measurement bias in consumer price indices. However, such data is rarely collected so that economic models appear a useful and accessible stand in to uncover measurement biases.

## 1.2 Review of Chapter 3 on Dynamic Money Demand

In modern monetary macro models, money demand is a contemporaneous function of aggregate income and the interest rate differential between the illiquid and the liquid asset. Empirically, it proves difficult to reconcile contemporaneous money demand with monthly or quarterly data. Goldfeld (1973) is among the first to work with dynamic money demand to improve data fit. Goldfeld interprets contemporaneous money demand to represent desired money balances of private agents and argues that portfolio adjustment costs prevent immediate adjustment of actual to desired money demand. Partial adjustment then introduces a lagged money term into other-

wise contemporaneous money demand.

Since Goldfeld (1973) numerous econometric studies have confirmed that dynamic specifications of money demand deliver superior data fit (see footnote 3.1 for references). However, implications of dynamic money demand for optimal monetary policy have remained largely unexplored up to date. Therefore, my main objective in chapter 3 is to reconcile the typical New Keynesian model with dynamic money demand and to revisit the optimal monetary policy problem for the modified model.

Money demand in the New Keynesian model (Clarida, Galí, and Gertler (1999), Woodford (2003), Galí (2008)) often follows from assuming that households derive utility directly from holding real money balances. I argue that introducing habit formation of households with respect to their money holdings is a parsimonious and plausible way to derive a money demand function which is dynamic. Congdon (2005) and Mäki-Franti (2008) present empirical evidence for habit persistence with respect to the holding of liquid assets.

Frequently, the New Keynesian model features a representative household so that the natural criterion to rank different monetary policies is the representative utility level. Welfare maximizing monetary policy then should stabilize money demand to the extent to which fluctuations in money demand reduce representative welfare. When households exhibit habit formation with respect to the money stock, a particular money index achieves considerably more weight in the welfare based loss function of the central bank compared to the case of no habit with respect to the money stock.

Based on the modified loss function I derive the optimal target criterion for two polar assumptions about the ability of the central bank to commit itself, namely full discretion and perfect commitment. I find that across policy regimes the large weight in the loss function does not justify a large response to the money index in the optimal target criterion. The reason is that short run money demand elasticities with respect to aggregate income and the interest rate differential fall the more dynamic money demand is. Stabilizing the money index then involves larger costs in terms of variation in other target variables since the leverage monetary policy can exert on money demand decreases.

Woodford (2003) shows in chapter 6 that stabilizing money demand can also be achieved by individually stabilizing both of its arguments in case they enter money demand contemporaneously. In the canonical New Keynesian model, stabilizing income and the interest rate differential is already desirable for reasons such as maintaining a smooth path of consumption and concerns about the lower bound on nominal interest rates. Therefore, as long as money demand is contemporaneous the monetary authority does not need to put much additional emphasis on stabilizing income and the

interest rate differential to comply with the household's preference for stable money demand. Accordingly, research on optimal monetary policy in microfounded models either suppresses stabilization of money demand in the welfare criterion or omits the explicit account of money entirely by resorting to the limit of a cashless economy.<sup>1</sup> Results obtained in chapter 3 show that stabilizing income and the interest rate differential alone does not encompass optimal monetary policy when money demand is dynamic because dynamic money demand also depends on expected money terms.

In recent years the prominent role of money in the monetary policy strategy of the European Central Bank has been under debate (Galí (2003), Gerlach (2004), Woodford (2007), Berger, Harjes, and Stavrev (2008)). Whereas opponents of an important role for monetary aggregates point to money being superfluous to determine the aggregate price level uniquely, proponents point to an important indicator role of money for the stance of aggregate demand. For instance, Nelson (2002) incorporates money demand dynamics into an otherwise standard model and shows that money growth co-varies stronger with deviations of output from trend.

In chapter 3 I obtain the same result for intermediate degrees of dynamics in money demand in the New Keynesian model augmented with habit in money balances. However, in the model with habit in money balances the correlation between money growth and the output gap fades out quickly when the degrees of dynamics in money demand is large. On the basis of this model one is therefore bound to conclude that a substantial correlation between money growth and the output gap is likely injected by monetary policy itself if money demand is found to be strongly dynamic. This cautions against statements on the indicator role of money growth derived from historical correlations because such correlations may change when the policy regime changes.

### 1.3 Review of Chapter 4 on Productivity Catch Up

A sustained convergence process is on its way in the majority of new EU member states in Eastern Europe. The convergence process concerns the transformation of formerly planned economies into distinct market economies. The essay of chapter 4 aims to identify driving forces of convergence which are consistent with outstanding stylized facts in the data. These stylized facts are real exchange rate appreciation

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<sup>1</sup>Rotemberg and Woodford (1997), Onatski and Williams (2004) and Giannoni and Woodford (2005) analyze optimal monetary policy in microfounded models without money. Woodford (1998) establishes the limit of a cashless economy which obtains when money velocity tends to infinity.

and improving terms of trade of new EU member states vis á vis the euro area. In particular, I set up a theoretical framework to simulate two different scenarios of productivity catch up and one scenario in which convergence is demand driven. I then infer the extent to which dynamics of relative international prices are consistent with the empirical facts.

The first scenario assumes that production efficiency in the traded good sector of transition countries grows fast relative to production efficiency in the nontraded good sector, a situation which resembles the prominent Balassa Samuelson effect. This scenario induces real exchange rate appreciation but predicts deteriorating rather than improving terms of trade. Therefore, in light of the empirical evidence, relative efficiency expansion in the traded good sector is likely not a major driving force of the convergence process. Egert (2007) obtains a similar conclusion from empirical estimates.

The second scenario simulates a sustained increase in a type of productivity which reduces the barriers to market entry of new firms. Falling entry barriers induce real exchange rate appreciation jointly with improving terms of trade. Both predictions are consistent with the data. Moreover, falling entry barriers increase product variety. There is considerable empirical evidence that product variety is an important and dynamic margin in exports of new EU member states in particular (Kandogan (2006)) and in international trade more generally, which makes this type of productivity catch up a prominent candidate driver of the transition process in new EU member states.

The third scenario infers dynamics due to expanding government consumption. This scenario also turns out to be consistent with the stylized facts on international relative prices between new EU member states and the euro area. However, the demand driven explanation has considerably different implications for product variety which may help to discriminate the latter two scenarios in future empirical research.

Several reasons underscore the relevance to identify main driving forces behind transition dynamics in new EU member states. First, results in Broda and Weinstein (2004), Broda and Weinstein (2006) and Corsetti, Martin, and Pesenti (2007) suggest that a transition process due to sustained reductions in barriers to firm entry has welfare implications substantially different from a transition process driven by a catch up in production efficiency because the former boosts product variety substantially whereas the latter does so only moderately. Hence, conclusions about social costs and benefits of the convergence process are potentially sensitive to with respect to conclusions about the main driving forces at work.

Second, medium run model based projections of core economic indicators, such as inflation, are likely to be sensitive with respect to the composition of the fundamen-

tal forces that drive transition. For instance, it makes a difference for projections of medium term euro area inflation if terms of trade as the ratio of export over import prices are predicted to rise rather than to fall in new EU member states. Inflation projections are pivotal for the conduct of monetary policy and their quality ensures the quality of monetary policy decisions.

Finally, conclusions regarding the fundamental forces driving convergence are vital to evaluate alternative monetary policies and to propose reasonable ones. Natalucci and Ravenna (2005) show that monetary policy allocates the real exchange rate appreciation among an increase in inflation in the nontraded good sector and an appreciation of the nominal exchange rate if productivity catch up concerns production efficiency in the traded good sector. This would confront new EU member states with the tradeoff to comply with the Maastricht inflation criterion on the one hand and to limit movements of the nominal exchange rate vis á vis the euro as required by the Exchange Rate Mechanism II on the other. By now it is far from clear if a similar tradeoff exists for productivity catch up which manifests itself in simplified market entry of new firms.

## 1.4 Review of Chapter 5 on Euro Area Trade Costs

Empirical work that assesses the impact of the euro on euro area trade has been accumulating rapidly since enough data points of the new regime are available to run meaningful regressions. Despite the fact that the literature is rich on remarkably different estimates of the euro's trade effect, it is almost monolithic when it comes to the empirical model specification (Baldwin (2006a) and Baldwin (2006b) review this literature). Empirical work mostly departs from Gravity theory which links trade to trade costs, competitiveness and economic activity (Anderson (1979), Anderson and van Wincoop (2003)). The main purpose of the essay in chapter 5 is to assess to what extent substantive restrictions typically imposed when implementing Gravity theory are decisive for conclusions about the trade effect of the euro. The essay is joint work with Helmut Herwartz from Christian Albrechts Universität in Kiel.

In the majority of cases the basic framework applied to infer the trade creation effect of the euro is a linear panel data model with homogenous coefficients and time fixed effects. The cross section comprises euro area countries and, as reference group, industrialized countries with national currency. For all countries data on trade flows are fit to a set of measurable trade costs, measures of competitiveness and economic activity. Among the proxies for trade costs is a time dummy which is zero before and

unity after the introduction of the euro in 1999. This euro dummy is interacted once with a cross section dummy which selects euro area countries and a second time with a cross section dummy which selects the reference group. In spirit of a difference-in-difference estimator the trade effect of the euro is then derived as the difference between the level shift in euro area trade costs and the corresponding level shift in the reference group. The conventional empirical model setup also accounts for the fact that the bulk of trade costs and measures of competitiveness are either poorly measured or entirely unobserved by using a set of time dummies common to the entire cross section as stand in for more accurate or explicit information.

The typical empirical model appears restrictive along three dimensions. First, the euro dummy restricts the trade effect of the euro to materialize immediately and exhaustively. Second, homogenous coefficients suppress any heterogeneity specific to a particular trade relationship. Third, the set of time dummies common to all trade relationships does not comply with the prediction of Gravity theory that time variation in trade costs is specific to a particular trade relationship.

We attempt to estimate the 'When, How Fast and by How Much' of adjustment in trade costs. We do so by implementing a more general notion of transition which allows for gradual adjustment of trade costs to a new plateau and for the possibility of medium to long run effects. Moreover, our approach allows for effects of trade costs on trade and for unobserved variation in trade costs that are specific to a particular trade relationship. We find gradual adjustment in trade costs during the years 2000 and 2003 by 10 to 20 percent but not much scope for medium to long run effects to develop. Our results suggest that a euro dummy with break point in 1999 is misleading with respect to the size of the trade effect of the euro because the timing of the change in trade costs matters for conclusions about the size of this change.

Reliable estimates of the trade effect of the euro are vital both for countries that contemplate adopting the euro and countries that have already done so. For the first group of countries the challenge is to weigh expected benefits from adopting the euro against expected costs from doing so. Abandoning independent monetary and fiscal policies due to the fixing of exchange rates and compliance with the Stability and Growth Pact are commonly expected to be costly in terms of less appropriate stabilization. However, the European monetary union seemingly increases trade among its members. Such fostered trade intensity is booked as a benefit because one expects it to integrate good markets more tightly and to offer a larger array of goods to consumers.

I recall two arguments to motivate interest in the trade effect of the euro from the perspective of the euro area and its policy authorities. First, to make the case for the common currency many advocates argued that the euro would decrease trade

costs and thereby increase trade intensity. It is of interest to verify this argument in retrospective to hold European decision making bodies accountable for the quality of their decisions. Second, euro area wide stabilization policy is simplified if economic shocks affect different countries of the euro area to a similar degree. Frankel and Rose (1998) argue that the monetary union endogenously homogenizes the response of countries within the euro area to shocks to the extent to which the union fosters trade linkages because trade serves as prime shock transmitter. Lane (2006) contrasts this centripetal force with a centrifugal force. Based on the finding in Micco, Stein, and Ordonez (2003) that jointly with trade among euro area countries trade of the euro area with third countries has increased, he points to an extended scope for external shocks to affect the euro area asymmetrically. Evaluating the relative importance of these arguments requires inference about the euro's effect on internal and external trade.



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## Chapter 2

# Mind the Gap – Mismeasured Inflation and Monetary Policy

Recent empirical evidence documents measurement bias in U.S. inflation at business cycle frequency due to product entry and exit. In a New Keynesian model with product entry and exit I study measurement bias in inflation which originates from a failure of the statistical authority to account for new products in time. I find that measurement bias depends systematically on the state of the business cycle. In particular, measured inflation overstates actual inflation in times of strong productivity growth. More generally, measured inflation is more persistent and less volatile than actual inflation. Applying the model to U.S. inflation reveals a volatile measurement bias. Across different monetary policy regimes measurement bias implies less than intended inflation stabilization. However, whereas measurement bias always deteriorates welfare under committed monetary policy, it may actually improve welfare under discretionary policy despite too little inflation stabilization.

### 2.1 Introduction

Economic analysis asserts that for monetary policy to maximize welfare it should stabilize actual inflation (Woodford (2002)). Rather than actual inflation, however, policy makers observe measured inflation which is the change of the consumer price index.

The consumer price index reflects expenditure required to afford a basket of products at current prices relative to expenditure required to afford the same basket at prices in some previous period. Actual inflation is the growth rate of the price level which arises in a utility maximizing framework. This price level represents minimum expenditure required to afford a given level of utility.

Recently, Broda and Weinstein (2007) document by means of a unique micro data set that the failure to account for new products in time induces severe measurement bias into the U.S. consumer price index. Importantly, the authors provide evidence for a strong cyclical measurement bias. I introduce cyclical measurement bias in consumer prices into a dynamic general equilibrium model to study the implications of this bias for monetary policy. In line with the evidence, measurement bias in the model originates from a failure of the statistical authority to account for new products in time. Greenwood and Uysal (2005) argue that economic models provide a useful laboratory for assessing the performance of alternative price index measures. Equivalently, economic models may serve as a handle on cyclical measurement bias. This is the basic idea pursued in this paper.

For monetary policy knowledge about the gap between measured and actual inflation matters. Conceptually, there is easy account of a constant gap between measured and actual inflation by adjusting policy targets appropriately and many central banks indeed associate price stability with positive instead of zero measured inflation. However, a cyclical and volatile measurement bias complicates the assessment of the current situation of monetary policy (Shapiro and Wilcox (1996)). Unfortunately, there is no easy solution to address volatile measurement bias, a point stressed by Issing (2001), p.2 and 4. "The present scenario of rapid changing technology combined with low inflation makes the issue of measurement biases in price indices of the utmost relevance for monetary policy. [...] A near constant or low volatile bias is not of major concern [...]. A high volatility of the bias is a matter of much more serious concern, and makes achieving price stability more difficult. There is no clear indication on the size of this volatility. There is a need therefore to enhance our knowledge on how best to conduct monetary policy in the presence of a highly volatile measurement bias."

My analysis augments a New Keynesian model with product entry and exit and introduces a statistical authority into the model which compiles the consumer price index based on Laspeyres' formula. Two measurement biases arise in this setup. One bias is the well known substitution bias. It arises because optimizing consumers react to changes in relative prices by adjusting quantities. The consumer price index cannot capture such substitution because in Laspeyres' formula prices are weighted by quantities which are constant over time. The second bias is due to the failure of

the statistical authority to account for new products in time. Therefore, the consumer price index is computed with reference to a nonrepresentative basket of products. As it turns out, this nonrepresentative basket implies price dynamics which are different from those of the representative basket.

I find that the new product bias depends systematically on the state of the business cycle whereas the substitution bias is irrelevant up to a first order approximation. The fundamental source of endogenous variation in the new product bias originates from its nonrepresentative array of products. Nonrepresentativeness matters when prices of new products take values different from the average price. The basic assumption to generate a difference between prices of new products and the average price is that firms which assemble new products are free to choose their price optimally, whereas firms which assemble established products are subject to price adjustment constraints. Since pricing depends on economic fundamentals such as productivity growth the new product bias varies over the business cycle. State dependent measurement bias implies that measured inflation overstates actual inflation in times of strong productivity growth.

Technically, measured inflation depends on contemporaneous actual inflation and on lags of actual inflation. Measurement bias thus can be thought of as a filter that maps actual inflation into measured inflation. The frequency domain reveals that measurement bias shifts weight of the spectrum of measured inflation to high frequencies and, at the same time, increases the variance of measured inflation across all frequencies. Therefore, measured inflation is more persistent and less volatile than actual inflation. The model lends itself to recover historic paths of actual inflation. I demonstrate this application for U.S. consumer price inflation for which I obtain a cyclical measurement bias which correlates negatively with measured U.S. inflation. I obtain measures of real growth from deflating nominal expenditure with the measured rather than the actual price level. In line with Eldridge (1999) and Bils (2004), measurement bias in inflation therefore shows up as opposite bias in measures of real growth.

Implications of measurement bias for monetary policy, if it responds to measured rather than actual quantities, are surprising. First, measurement bias implies less than intended inflation stabilization which holds true across different monetary policy regimes. The reason is that measurement bias mistakenly attributes some inflation variation to real activity. As long as inflation receives more weight in the central bank loss function than does real activity, bias acts to downgrade inflation stabilization. Whereas measurement bias always deteriorates welfare under committed monetary policy due to this distortion, it may actually improve welfare under discretionary policy despite too little inflation stabilization. The reason is subtle. Since measured infla-

tion is expressed as lag polynomial of actual inflation a central bank that responds to measured inflation effectively responds to current and past values of actual inflation. Thereby, its targeting rule exhibits history dependence so that, even though the central bank itself cannot commit, it effectively borrows commitment from the statistical authority.

Finally, I contrast the notion of measurement bias to the notion of measurement error. Whereas measurement bias depends on the state of the economy, measurement error is meant to capture a difference between measured and actual variables which is independent of the state of the economy but fluctuates randomly. Measurement error is a frequently applied device to investigate robustness of monetary policy rules with respect to data uncertainty (for instance, Rudebusch (2001), Orphanides (2003), Aoki (2006)). Contrary to measurement bias, measurement error is shown neither to distort the effective weight attached to inflation nor to introduce history dependence into central banks' targeting rule. Rather, measurement error in variables observed by the central bank has effects similar to an exogenous central bank control error.

In related work, Bilbiie, Ghironi, and Melitz (2007) develop a business cycle model with endogenous product entry and imperfect price adjustment to study the role of entry for monetary policy. Whereas endogenous product entry certainly is a step towards realism compared to my setup with exogenous entry, their formulation of imperfect price adjustment implies a collapsed price distribution. That is, all product prices are identical in their model because the equilibrium is symmetric in firms. This rules out a priori any type of measurement bias which originates from a nonrepresentative array of products so that their framework cannot be brought to bear for the question here. Price distributions in the three papers discussed next are all collapsed. Elkhoury and Mancini-Griffoli (2007) augment the real business cycle model featuring endogenous entry in Bilbiie, Ghironi, and Melitz (2006) with a role for monetary policy by introducing nominal rigidities exclusively in entry costs. Lewis (2008) introduces monopoly power in wage setting to have a layer for monetary policy. Bergin and Corsetti (2005) analyze monetary stabilization policy in an analytically solvable model with firm entry. Finally, when it comes to price index measurement Diewert (1998) argues for the prominence of the new product bias in the U.S. consumer price index and Balk (1999) proposes an index based on a modified CES utility function which cures substitution and new product bias.



## 2.2 Model

The business cycle model considered here is a minimal setup to convey implications of time variant measurement bias in consumer price inflation. In the model, products enter and exit the market in every period at exogenously given rate. The statistical authority measures consumer price inflation as the change in expenditure required to afford a particular basket of products. This basket omits new products because the statistical authority accounts for new products only with delay. In contrast, a representative household consumes the entire universe of products available in a given period. The change in minimum household expenditure required to afford a given level of utility thus is a natural benchmark against which to judge the accuracy of the inflation measure produced by the statistical authority. Firms which assemble products set prices at random points in time according to Calvo (1983) and sell their products on monopolistically competitive markets. The government conducts monetary and fiscal policy. As a special case the model reduces to the standard New Keynesian model without product entry and exit treated extensively in Woodford (2003) and Galí (2008).

### 2.2.1 Statistical Authority

The statistical authority measures consumer price inflation  $\pi_t^m = P_{t,t}^m / P_{t-1,t}^m$  by referring to Laspeyre's index. Measured price levels are defined as

$$P_{t,t}^m = \int_{\mathcal{N}(t,\ell)} P_t(j)Q(j) dj \quad , \quad P_{t-1,t}^m = \int_{\mathcal{N}(t,\ell)} P_{t-1}(j)Q(j) dj . \quad (2.1)$$

Price  $P_t(j)$  of product  $j$  at time  $t$  is weighted by the quantity  $Q(j)$  which is held constant by the statistical authority. Measured price levels in consecutive periods refer to the same set of products  $\mathcal{N}(t,\ell)$  which represents the market basket maintained by the statistical authority. This set comprises all products which have a sufficiently long life time,

$$\mathcal{N}(t,\ell) = \{ \text{all products } j \text{ available in period } t \text{ with life time greater than } \ell \} .$$

The lag  $\ell \geq 1$  denotes the number of periods for which new products remain undetected by the statistical authority. It reflects the average time which elapses between market and basket introduction of a new product. In practice, the time lag derives from delays in obtaining survey data on consumer expenditure to determine representative weights for new products, from averaging consumer expenditure data over several years to mitigate seasonality and idiosyncratic shocks or, more generally, from

a limited availability of funds and resources which prohibits tracking market developments more closely. For instance, large scale introduction of price scanning is likely to reduce  $\ell$ . The time lag implies that the market basket maintained by the statistical authority comprises only a subset of the universe of products available to consumers in any given period.

Time subscripts of measured price levels indicate that in each period  $t$  two observations of the measured price level exist, namely  $P_{t,t}^m$  and  $P_{t,t+1}^m$ . Each price level refers to a different definition of the market basket. Redefining the market basket over time is unavoidable if products enter and exit the market place. In period  $t + 1$ , the market basket is updated for products that exist since  $\ell + 1$  periods and, therefore, are no longer subject to the observability constraint. At the same time, the market basket in period  $t + 1$  excludes those products that left the market at the end of period  $t$ . To maintain a constant market basket between periods so that measured inflation compares consecutive aggregate prices which refer to the same basket of products, updating of the market basket takes place within periods.

This period by period updating scheme is similar to the permanent revisions of the market basket conducted by statistical authorities in practice. Statistical authorities face exactly the same challenge to maintain coverage of their market basket when new products arrive, and have developed methods to account more promptly for new products between base periods.<sup>1</sup> The main method implemented by the U.S. Bureau of Labor Statistics (BLS) for the U.S. CPI is sample rotation. In every year, the market basket is updated for about 20% of the geographic area so as to better reflect recent developments in consumer expenditure in this particular area. The entire market basket then is updated once every five years (Armknrecht, Lane, and Stewart (1997)). Sample rotation ensures that a fraction of new products is phased in each period which is similar to the updating scheme considered here. Measured inflation rewrites as

$$\pi_t^m = \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^m(j) \pi_t(j) dj \quad (2.2)$$

with measured weights corresponding to

$$w_{t-1,t}^m(j) = \frac{P_{t-1}(j)Q(j)}{\int_{\mathcal{N}(t,\ell)} P_{t-1}(j)Q(j) dj} \quad , \quad 1 = \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^m(j) dj . \quad (2.3)$$

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<sup>1</sup>In base periods the market basket is updated for new products but these periods are infrequent events. In base periods weights are determined according to which prices of available products enter the index. Historically, several years pass between two consecutive base periods of the CPI. For instance, for the U.S. the historical record of revisions is 1940, 1953, 1964, 1978, 1987 and 1998. Moreover, until 1998 weights were usually outdated by roughly three years when they entered the base period revision. Starting with 2002 weights are updated biannually (table 1 in BLS (1997), chapter 17).

The variable  $\pi_t(j) = P_t(j)/P_{t-1}(j)$  captures inflation of product  $j$  and  $w_{t-1,t}^m(j)$  denotes the fraction of measured nominal consumption expenditure spent on product  $j$ . Weights integrate to unity by construction.

### 2.2.2 Product Entry and Exit

Let  $j \in [0,1]$  index products available in the market. There is a one to one match between a product and a firm so that firm  $j$  assembles product  $j$ . In each period the unit interval comprises  $\delta \in [0,1)$  new firms and  $(1 - \delta)$  established firms. New firms assemble a product for the first time whereas established firms assemble their product at least for the second time. No firm ever liquidates owing to bad aggregate shocks. However, all firms are subject to shocks at the micro level which cause a random fraction  $\delta$  out of new and established firms to leave the market at the end of each period. At the beginning of each period  $\delta/(1 - \delta)$  startups enter the economy. Startups coexist with operating firms and start production in the period after entry. Once they start production startups are counted as new firms. Only  $\delta$  of all startups become new firms and replace the  $\delta$  firms that exited the market at the end of the previous period because startups also are subject to the exit shock in their entry period. This setup is a special case of the entry and exit mechanism applied in Bilbiie, Ghironi, and Melitz (2006).

### 2.2.3 Household

The representative household is endowed with time, its ownership of firms and last period bond holdings. It maximizes expected discounted life time utility

$$\max_{\{C_t(j), B_t, L_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t, \xi_t) - h(L_t, \xi_t)] , \quad \beta \in (0, 1) . \quad (2.4)$$

$E_0$  denotes the expectation operator conditional on period zero information. Utility  $u(\cdot)$  ( $h(\cdot)$ ) is twice continuously differentiable, increasing in its argument and concave (convex) for each value of the vector  $\xi_t$  which comprises zero mean preference shocks. Let  $C_t$  denote period  $t$  consumption,  $L_t$  labor and  $B_t$  bonds. Utility maximization is subject to the budget constraint

$$\int_0^1 P_t(j)C_t(j) dj + B_t = (1 + i_{t-1})B_{t-1} + (1 - \tau)W_tL_t + D_t + T_t . \quad (2.5)$$

The household receives returns from last period bond holdings  $(1 + i_{t-1})B_{t-1}$ , after tax labor income  $(1 - \tau)W_tL_t$ , nominal dividends  $D_t$  from firm ownership and a governmental lump sum subsidy  $T_t$ .  $W_t$  and  $i_t$  denote the nominal wage rate and the

nominal interest rate, respectively. The household bundles intermediate products according to the Dixit and Stiglitz (1977) index

$$C_t = \left( \int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \quad (2.6)$$

The utility based or actual price level measures minimum household expenditure required to afford a marginal unit of the consumption composite  $C_t$ ,

$$P_t = \left( \int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (2.7)$$

Actual inflation which serves as reference to judge the accuracy of measured inflation below is defined as the change in the actual price level,  $\pi_t = P_t/P_{t-1}$ . Cost minimization determines the demand for product  $j$  as  $C_t(j)/C_t = (P_t(j)/P_t)^{-\theta}$ . Appendix A.1.1 collects conditions which ensure optimal intertemporal household choices.

## 2.2.4 Firms

Provided all products sell at identical price and deliver identical utility to the household price level measurement is a trivial task. It suffices to observe the single price to reveal the actual price level. In practice, however, price level measurement is a challenging task because the distribution of prices is not degenerated. I introduce a nondegenerated price distribution into the model by assuming that established firms adjust prices in a given period with probability  $(1 - \alpha)$  having  $\alpha \in [0, 1)$ . New firms choose with probability one the price which maximizes discounted profits. Thus, they face a problem identical to those established firms which happen to reoptimize their price in the current period. In subsequent periods former new firms adjust their price with probability  $(1 - \alpha)$ . This is a variant of the price setting mechanism in Calvo (1983).

Firm  $j$  produces quantity  $Y_t(j)$  with labor input  $L_t(j)$  and with a technology of the form  $Y_t(j) = A_t f(L_t(j))$ . Technology  $f(\cdot)$  is increasing and concave and productivity  $A_t > 0$  is an exogenous stochastic process with mean  $\bar{A} = 1$ . The firm obtains labor on competitive factor markets and sells its product on monopolistically competitive product markets. Moreover, it operates on its demand function thus ensuring  $Y_t(j) = C_t(j)$ . The firm's problem amounts to

$$\max_{P_t(j)} E_t \sum_{s=t}^{\infty} (\kappa\beta)^{s-t} \Omega_{s,t} \left[ P_t(j) Y_s(j) - W_s f^{-1} \left( \frac{Y_s(j)}{A_t} \right) \right] \quad \text{s.t.} \quad \frac{Y_s(j)}{Y_s} = \left( \frac{P_t(j)}{P_s} \right)^{-\theta}. \quad (2.8)$$

The discount factor for future profits includes the exit probability  $\delta$  with  $\kappa = \alpha(1 - \delta)$  and applies the household discount factor for nominal payoffs  $\Omega_{s,t} = u_c(C_s, \xi_s)P_t / (u_c(C_t, \xi_t)P_s)$ . The optimality condition (A.4) to this problem in appendix A.1.2 reveals that all optimizing firms chose the same price  $P_t^*$ .

A crucial assumption is that the firm sets its price with reference to the actual price level  $P_t$ . Certainly, in the real world a firm is extremely well informed about prices of products which serve as close substitutes for its product, because such prices will figure predominately for the firm's market share and thereby for realized profits. In contrast, firm profits are likely to react much less sensitively to prices of remote substitutes. In the model, each product  $j'$  serves equally well as close substitute for product  $j$  so that the actual price level is best interpreted as price index of close substitutes. This motivates the reference to  $P_t$  in the firm's price setting problem.

## 2.3 Analysis

In this section, I analyze measurement bias as the ratio of measured over actual inflation. Measured inflation as compiled by the statistical authority is the change in expenditure required to afford a particular basket of products. Actual inflation is the change in minimum expenditure required to afford a given level of utility and derives from the household's utility criterion.

Actual inflation is a natural benchmark against which to judge the accuracy of measured inflation for several reasons. First, the bulk of normative conclusions about optimal monetary policy derived in microfounded models implies (or is even formulated as) a particular path of actual inflation. It should therefore be of considerable interest to establish a mapping between actual and measured inflation. Second, statistical authorities discriminate between Cost of Living and Cost of Goods indices. Whereas the Cost of Goods index is meant to track the price of a representative product basket over time, the Cost of Living index aims to track the price to afford a given level of utility.<sup>2</sup> In the model, measured inflation corresponds to a Cost of Goods index whereas actual inflation corresponds to a Cost of Living index. Finally, my definition of measurement bias ensures compatibility with a large bulk of empirical literature on medium to long run measurement bias in the consumer price index (Shapiro and Wilcox (1996), Gordon (2006), Lebow and Rudd (2003) and Lebow and Rudd (2006)

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<sup>2</sup>For instance, whereas the U.S. BLS intends to provide a Cost of Living index (Armknrecht, Lane, and Stewart (1997), p.388), the Office for National Statistics in the United Kingdom intends to provide a Cost of Goods index (ONS (2005), chapter 10.10).

provide reviews).

To simplify interpretation of the measurement bias I introduce a third measure of inflation on top of actual and measured inflation. Similar to measured inflation the additional inflation rate is bound to record price changes of established products only. Different from measured inflation, however, the additional inflation rate weights price changes of established products in a way consistent with cost minimization of the household.

### 2.3.1 Inflation of Established Products

Let  $\pi_t^{eg}$  ('established goods') denote inflation of established products  $\pi_t^{eg} = P_{t,t}^{eg} / P_{t-1,t}^{eg}$  with

$$P_{t,t}^{eg} = \left( \int_{\mathcal{N}(t,\ell)} P_t^{1-\theta}(j) dj \right)^{\frac{1}{1-\theta}}, \quad P_{t-1,t}^{eg} = \left( \int_{\mathcal{N}(t,\ell)} P_{t-1}^{1-\theta}(j) dj \right)^{\frac{1}{1-\theta}}. \quad (2.9)$$

Here  $P_{t,t}^{eg}$  is the period  $t$  cost minimal price of one unit of a consumption index equivalent to (2.6) except that this index integrates only over a subset  $\mathcal{N}(t, \ell)$  out of the universe of existing products. Similarly,  $P_{t-1,t}^{eg}$  collects period  $t - 1$  prices in a cost minimal way but refers to the same subset of products  $\mathcal{N}(t, \ell)$ . Because the subset of products remains constant across the two subsequent periods one can rewrite  $\pi_t^{eg}$  in a way analog to  $\pi_t^m$ ,

$$\pi_t^{eg} = \left( \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^{eg}(j) \pi_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (2.10)$$

After substituting household demand for product  $j$ , weights obey

$$w_{t-1,t}^{eg}(j) = \frac{P_{t-1}(j)C_{t-1}(j)}{P_{t-1,t}^{eg}C_{t-1,t}^{eg}}, \quad 1 = \int_{\mathcal{N}(t,\ell)} w_{t-1,t}^{eg}(j) dj. \quad (2.11)$$

Here  $C_{t-1,t}^{eg}$  indicates the Dixit Stiglitz consumption index which underlies  $P_{t-1,t}^{eg}$ . Thus, weights represent relative consumption expenditure for established products and integrate to unity by construction.

### 2.3.2 Measurement Bias

I define total measurement bias as  $\mathcal{B}_t = \pi_t^m / \pi_t$ . Augmenting this definition with inflation for established products one obtains

$$\mathcal{B}_t = [\pi_t^m / \pi_t^{eg}] \times \{\pi_t^{eg} / \pi_t\} = [\mathcal{B}_t^{sub}] \times \{\mathcal{B}_t^{new}\}. \quad (2.12)$$

The substitution bias  $\mathcal{B}_t^{sub}$  compares two inflation rates which both omit new products. However, both inflation rates differ with respect to their weighting scheme. Inflation  $\pi_t^{eg}$  maintains weights which respond to consumed quantities  $C_{t-1}(j)$ , which in turn depend on relative prices by household's demand function for product  $j$ . These weights are consistent with household cost minimization in that cheap products are substituted for expensive ones. In contrast,  $\pi_t^m$  maintains weights based on quantities which the statistical authority holds constant over time. The substitution bias thus results because the statistical authority weights established products not in a cost minimal way.

The new product bias  $\mathcal{B}_t^{new}$  compares two measures of inflation which both weight product specific inflation in a way consistent with household cost minimization. However, whereas  $\pi_t^{eg}$  records only price changes of established products,  $\pi_t$  records price changes of the entire universe of products available to the household. The new product bias arises because the statistical authority tracks a nonrepresentative basket of products. As it turns out this basket has price dynamics that are different from the representative basket.

I now approximate total measurement bias (2.12) accurate up to first order. The approximation is taken around a steady state in which firms set flexible prices and actual gross inflation is set to unity. Evaluating the optimality condition of the firm's problem (2.8) for the case of flexible prices reveals that all products sell at identical price. Symmetry in prices jointly with the assumption of homogenous quantities  $Q(j) = 1/(1 - \delta)^\ell$  for all  $j$  implies that the steady state is free of measurement bias because all measures of inflation coincide in this case. A hat on top of a variable indicates percentage deviation from steady state.

### Substitution Bias

I obtain the following proposition on the substitution bias.

**Proposition 1:** *The substitution bias defined in equation (2.12) is zero up to first order.*

First order approximation of  $\pi_t^m$  and  $\pi_t^{eg}$  decouples product specific inflation rates from corresponding weights. Because weights integrate to unity by construction their percentage deviations from steady state integrate to zero. The remaining difference between  $\pi_t^m$  and  $\pi_t^{eg}$  in  $\mathcal{B}_t^{sub}$  is curvature which is suppressed by the first order approximation. Appendix A.3.1 provides details on the derivation. In a similar vein, Hausman (2003) mentions the second order character of the substitution bias. One

immediate implication of proposition 2 is that the new product bias equals total measurement bias,  $\hat{\mathcal{B}}_t = \hat{\mathcal{B}}_t^{new}$ , up to first order.

### New Product Bias

To derive the new product bias results on the actual price level are required. I obtain the following proposition for the actual price level.

**Proposition 2:** *The actual price level (2.7) has recursive representation*

$$P_t^{1-\theta} = (1 - \kappa)(P_t^*)^{1-\theta} + \kappa P_{t-1}^{1-\theta} \quad (2.13)$$

with  $P_t^*$  denoting the optimal price at date  $t$  and  $\kappa = \alpha(1 - \delta)$ .

New firms replace firms which left the market in the previous period. Whereas new firms set the price  $P_t^*$  optimal in their first production period, liquidated firms perfectly replicate the entire price distribution because they have been drawn randomly from the entire firm interval. Therefore, firm entry and exit  $\delta > 0$  increases the fraction  $(1 - \alpha(1 - \delta))$  of products with optimal price. Appendix A.2 provides a formal derivation. With this recursion of the actual price level I obtain the following proposition on the new product bias.

**Proposition 3:** *The new product bias defined in equation (2.12) is a finite order invertible lag polynomial of actual inflation up to first order,*

$$\hat{\mathcal{B}}_t^{new} = (a(L) - 1)\hat{\pi}_t .$$

with  $a(L)$  defined as

$$a(L) = \begin{cases} \frac{1-\alpha}{1-\alpha(1-\delta)}L^0 + \frac{(1-\alpha)\delta}{1-\alpha(1-\delta)}\sum_{k=1}^{\ell-1}(\alpha L)^k & \text{if } \ell \geq 2 \\ \frac{1-\alpha}{1-\alpha(1-\delta)}L^0 & \text{if } \ell = 1 \end{cases}$$

and  $L$  denoting the lag operator.

Appendix A.3.2 provides details on the derivation of  $\hat{\mathcal{B}}_t^{new}$  and appendix A.3.3 shows invertibility of  $a(L)$ . Measurement bias is a function of current and lagged actual inflation rates. Therefore, whenever actual inflation changes over time so does the measurement bias. This is true despite the fact that product entry and exit is exogenous and time invariant. Moreover, the time lag with which the statistical authority accounts for new products determines the order of  $a(L)$ . There are two informative



special cases when measurement bias is absent. The first case appears when shutting down the extensive margin setting  $\delta$  to zero. Without product entry and exit the observability constraint of the statistical authority is irrelevant. The second case corresponds to a situation of flexible prices with  $\alpha$  equal to zero. With prices flexible all products sell at identical price so that the price distribution collapses. Observing the price of any product is sufficient to infer the actual price level.

The expression for  $\hat{\mathcal{B}}_t^{new}$  implies a simple mapping between measured and actual inflation. Because the substitution bias is zero up to first order  $\hat{\pi}_t^{eg} = \hat{\pi}_t^m$  so that the new product bias rewrites as  $\hat{\pi}_t^m = \hat{\mathcal{B}}_t^{new} + \hat{\pi}_t$ . Substituting for  $\hat{\mathcal{B}}_t^{new}$  delivers

$$\hat{\pi}_t^m = a(L)\hat{\pi}_t. \quad (2.14)$$

Measured inflation is a weighted average of current and lagged actual inflation rates. The top panel of figure 2.1 plots  $a(L)$  for  $\alpha$  equal to 0.8,  $\delta$  equal to 0.0625 and  $\ell$  equal to 12 (calibration is described in section 2.4). The lag polynomial redistributes weight from current actual inflation to past actual inflation. To see why consider the relationship between measured and actual price level established in appendix A.3.2,

$$\hat{P}_{t,t}^m = (1 - \alpha) \sum_{k=0}^{\ell-1} \alpha^k \hat{P}_{t-k}^* + \alpha^\ell \hat{P}_{t-\ell}, \quad \hat{P}_t = (1 - \kappa) \sum_{k=0}^{\infty} \kappa^k \hat{P}_{t-k}^*.$$

Whereas the measured price level attributes the weights  $(1 - \alpha)\alpha^k$  to the  $\ell$  most recent prices, the actual price level attributes larger weights  $(1 - \kappa)\kappa^k$  to these prices having  $\kappa > \alpha$  with product entry and exit.

The bottom panel of figure 2.1 plots both weighting schemes for different values of  $k$ . Evidently, the statistical authority underestimates the relevance of most recently optimized prices thereby shifting weight to past optimal prices. However, recently optimized prices are central to the dynamics of actual inflation. The gap between actual and measured inflation thus arises because recently optimized prices remain underrepresented in measured inflation. To sum up, measurement bias in inflation arises because the statistical authority tracks a nonrepresentative basket of products which has price dynamics different from the representative basket.

### 2.3.3 Measured Output and Output Gap

According to Bils (2004) any measurement bias in U.S. consumer price inflation constructed by BLS ends up as opposite bias in growth rates of real activity. The reason is that the U.S. National Income and Product Accounts (NIPA) produced by the Bureau of Economic Analysis (BEA) derive growth in real activity by subtracting inflation

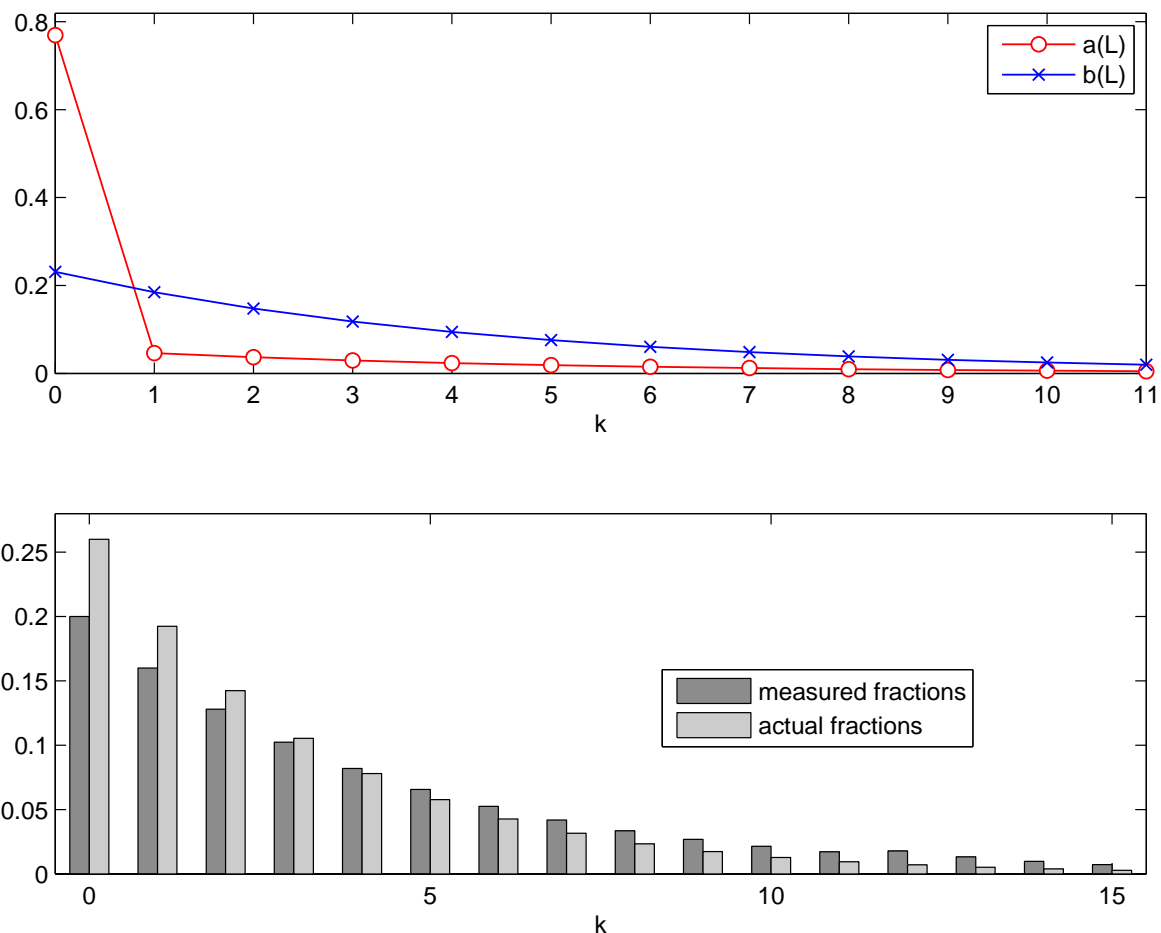


Figure 2.1: Top panel shows lag polynomials  $a(L)$  and  $b(L)$ . Bottom panel shows measured and actual fractions of current and past optimal prices. The first joint bar indicates that statistical authority estimates 20% of product to sell at the optimal price in the current period whereas actually 25% of products sell at this price. Calibration is described in section 2.4.

rates from growth in nominal expenditure where utilized inflation rates are to a large extent based on consumer price inflation constructed by the BLS.

Eldridge (1999) explains that BEA makes use of several indices including BLS consumer price indices, producer price indices and export and import price indices among others to deflate components of real output and stresses that the accuracy of output measures is contingent upon these price indices to be appropriately measured. Eldridge's estimate of the relative importance of each index in the construction of GDP includes 49.7% for consumer prices, 11.8% for producer prices and 8.5 (-9.9)% for export (import) prices. The BEA primarily uses consumer price indices as deflator for components of personal consumption expenditure.

Equivalently, to the BEA approach I deflate nominal consumption expenditure using the measured price level.<sup>3</sup> Let  $Y_t$  denote actual real output which aggregates individual products according to an index identical to equation (2.6). In a closed economy without government consumption and absent investment, output equals consumption and the actual GDP deflator coincides with the actual price level. Measured output then is nominal output deflated by the measured price level,  $Y_t^m = P_t Y_t / P_t^m$ . Linearizing this expression and substituting for actual and measured price levels using terms derived in appendix A.3.2 one obtains

$$\hat{Y}_t^m = \hat{Y}_t + b(L)\pi_t, \quad b_k = \frac{\alpha^{k+1}\delta}{1-\alpha(1-\delta)}, \quad k = 0, \dots, \ell - 1. \quad (2.15)$$

Here  $b_k$  denotes the coefficients of the finite order invertible lag polynomial  $b(L)$  the order of which again is determined by the observability constraint of the statistical authority (appendix A.3.3 proves invertibility of  $b(L)$ ). The top panel of figure 2.1 plots  $b(L)$ . Contemporaneously,  $a_0 + b_0 = 1$  so that measurement bias in inflation indeed triggers the opposite bias in measured output.

For policy analysis it is convenient to convert measured output into the measured output gap. The output gap is the difference between output under sticky prices and the efficient or natural level of output  $Y_t^n$  which realizes in a situation of flexible prices. Subtracting natural output as percentage deviation from steady state from equation (2.15) delivers the measured output gap as percentage deviation from steady state,

$$x_t^m = x_t + b(L)\hat{\pi}_t, \quad (2.16)$$

defining  $x_t^m = \hat{Y}_t^m - \hat{Y}_t^n$  and  $x_t = \hat{Y}_t - \hat{Y}_t^n$ . Uncovering the output gap, which provides important information for the conduct of monetary policy, is difficult. For instance, Orphanides (2001) shows that flash estimates of macroeconomic data often undergo major revisions as statistical authorities account for more accurate information over time. Such data revisions then result in revisions of the output gap which economists attempt to now-cast. In contrast, the measurement bias in  $x_t^m$  originates in the nonrepresentative market basket that underlies the compilation of consumer price inflation and transmits into measures of real activity when such measures are derived from nominal expenditure data. Thus, even with finally revised data the measurement bias in  $x_t^m$  continues to exist.

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<sup>3</sup>It makes no difference in the model if one transforms nominal expenditure growth or the level of nominal expenditure.

### 2.3.4 Aggregate Supply and Demand

In this section I consider aggregate supply (AS) and aggregate demand (AD) relationships. Appendix A.1.2 provides details to the derivation of the AS relationship which relates inflation to inflation expectations and the output gap,

$$\hat{\pi}_t = \phi x_t + \beta E_t \hat{\pi}_{t+1} + u_t, \quad \phi = \frac{[1-\alpha(1-\delta)\beta][1-\alpha(1-\delta)]}{\alpha(1-\delta)} \zeta, \quad \zeta = \frac{\omega + \sigma^{-1}}{1 + \theta \omega_p}. \quad (2.17)$$

The AS relationship is augmented by the adhoc disturbance  $u_t$  which represents exogenous variation in inflation not triggered by the output gap, such as changes in production costs or taxes. The natural level of output  $\hat{Y}_t^n$  that underlies the output gap  $x_t$  is the level of output under flexible prices absent  $u_t$  disturbances. Parameters that make up  $\zeta$  are described in appendix A.1.2.

It turns out that  $\phi$  is increasing in  $\delta$  but decreasing in  $\alpha$ . The more flux of firms the more important becomes the output gap for inflation dynamics because in every period a larger fraction of firms sets prices optimally as a function of current marginal cost. Higher  $\alpha$  in turn increases the extent to which firms are forward looking and thus reduces the effect of current marginal cost.<sup>4</sup>

One important implication of product entry and exit is that the average censored price duration  $(1 - \alpha(1 - \delta))^{-1}$  decreases in the exit probability since product exit increases the likelihood for price contracts to terminate in the future. This is in line with Bils and Klenow (2004) who find that the rate of product turnover is a robust predictor of more frequent price changes for a large array of consumer goods.

Once linearized the household Euler equation delivers a standard AD equation which relates the current output gap to its expectation and the gap between the ex ante real interest rate and the natural rate of interest,

$$x_t = E_t x_{t+1} - \sigma(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n).$$

The natural rate of interest  $\hat{r}_t^n = -\sigma^{-1} E_t(1 - L^{-1})(\hat{Y}_t^n - g_t)$  corresponds to the natural rate of output and is the rate that reveals in a situation of flexible prices absent  $u_t$  disturbances. The parameter  $\sigma = -\frac{u_c}{Y u_{cc}} > 0$  governs the intertemporal elasticity of substitution. Notation  $u_c$  indicates marginal utility of consumption evaluated at steady state and  $g_t = -\frac{u_{c\zeta}}{Y u_{cc}} \hat{\zeta}_t$  represents a shock to the marginal utility of consumption.

<sup>4</sup>To highlight the implications of firm entry and exit for the duration of price contracts consistent with a particular estimate of  $\phi$ , totally differentiate

$$d\phi = \frac{\partial \phi}{\partial \delta} d\delta + \frac{\partial \phi}{\partial \alpha} d\alpha = 0 \quad \text{or} \quad \frac{d\alpha}{d\delta} = -\frac{\frac{\partial \phi}{\partial \delta}}{\frac{\partial \phi}{\partial \alpha}} = \frac{\alpha}{1-\delta} \geq 0.$$

Higher firm entry and exit requires a larger uncensored price duration  $(1 - \alpha)^{-1}$  to maintain a particular  $\phi$ . The reason is that entry and exit provides an additional source for price contracts to terminate. To compensate for this effect  $\alpha$  has to increase.

## 2.4 Equilibrium and Parametrization

In equilibrium the statistical authority produces measured inflation, the representative household maximizes life time utility (2.4) subject to (2.5) and (2.6), firms set prices according to (2.8), product markets clear,  $C_t(j) = Y_t(j)$  for all  $j$ , the labor market clears,  $L_t = \int_0^1 L_t(j) dj$ , the bond market clears,  $B_t = 0$ , the government fulfills its budget constraint  $T_t = \tau W_t L_t$  and the monetary authority conducts policy as specified below.

It is straightforward to show existence and uniqueness of the flexible price steady state. The dynamic solution accurate up to first order is obtained with the numerical methods described in Sims (2002). I solve the model numerically because reasonable choices of  $\ell$  induce a significant state vector. For all monetary policies considered the equilibrium recursive law of motion is determinate and unique. The linear model is summarized as

$$\begin{aligned}
 \hat{\pi}_t^m &= a(L)\hat{\pi}_t \\
 x_t^m &= x_t + b(L)\hat{\pi}_t \\
 \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \phi x_t + u_t \\
 x_t &= E_t x_{t+1} - \sigma(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n)
 \end{aligned} \tag{2.18}$$

conditional on  $\hat{r}_t^n$  and  $u_t$  and initial conditions for actual inflation.

I calibrate the model to quarterly data. The core parameters which shape persistence and volatility of the measurement bias are  $\delta$ ,  $\alpha$  and  $\ell$ . I set  $\delta$  to 0.0625 which implies an annual entry and exit rate of 25%. In survey data from household purchases of products with barcode, Broda and Weinstein (2007) report median entry and exit rates of 25% and 24% per year, respectively. Broda and Weinstein find that the extent of median product entry amounts to 9% per year when measured as the value of new products relative to the value of all products in the market. The corresponding number for product exit is 3%. Whereas in the data entry (exit) rate and relative entry (exit) value diverge, both statistics coincide in the symmetric steady state of the model. Obviously, this mismatch reflects that the model is stylized by postulating that the sole difference between new and established products concerns prices. As this assumption probably wipes out important differences between new and established products such as quality, fashion and life cycle effects, the model is likely to understate measurement bias. This concern motivates the calibration of  $\delta$  to the entry rate rather than to the relative entry value.

I set  $\alpha$  equal to 0.8 which implies a mean price duration of the uncensored price distribution of 5 quarters. This value lies at the upper end of recent micro evidence in Nakamura and Steinsson (2007) but is at the lower end of estimates obtained from

medium scale DSGE models such as Smets and Wouters (2003). Jointly with the value of  $\delta$  the mean censored price duration amounts to slightly above 3 quarters which aligns with micro evidence. I set the observability lag of the statistical authority  $\ell$  equal to 12 quarters. This appears a conservative estimate given that sample rotation for the U.S. consumer price index takes 4 to 5 years to update the market basket once (Armknrecht, Lane, and Stewart (1997)). According to those numbers the statistical authority samples a fraction  $(1 - \delta)^\ell \approx 0.46$  of the entire universe of products. The evidence in Broda and Weinstein (2007) suggests that this is a crude overstatement of the fraction of products actually sampled by the BLS.

A subjective discount rate of 0.99 is in line with an annual real interest rate of about three percent on average. The intertemporal elasticity of substitution  $\sigma$  is set to 1 which corresponds to logarithmic utility of consumption. I set the steady state markup of firms to 20 % so that  $\theta$  equals 6. Comparable to Giannoni and Woodford (2005),  $\omega_w$  is set to 0.3 and  $\omega_p$  equals 0.5. Both numbers are consistent with a Cobb-Douglas technology with a labor coefficient of 2/3 and a labor supply elasticity with respect to the real wage  $\nu$  equal to 0.2. This calibration implies  $\phi$  equal to 0.039 which is well in line with estimates of this coefficient in Linde (2005) and Altig, Christiano, Eichenbaum, and Linde (2005) for U.S. data. Disturbances  $a_t$  and  $g_t$  follow AR(1) processes with AR coefficient 0.95 and  $u_t$  is AR(1) with AR coefficient equal to 0.5. The  $\bar{L}_t$  disturbance is observationally equivalent to  $a_t$  and is thus omitted. Numerical results derived below are independent of the exact size of shock variances as long as these variances remain positive so that I abstain from calibrating these parameters.

## 2.5 Results on Mismeasured Inflation

In this section, I illustrate the effect of measurement bias in the frequency domain. Further, I apply the model of measurement bias constructed here to U.S. consumer price inflation to uncover actual inflation from a time series of measured inflation. Finally, I discuss impulse response functions of measured and actual quantities. Repercussions of measurement bias for monetary policy are taken up in section 2.6.

### 2.5.1 Properties of Measurement Bias

The lag polynomial  $a(L)$  which maps actual inflation into measured inflation can be understood as a particular filter that, once applied to actual inflation, recovers measured inflation. Conversely, filtering measured inflation with the inverse lag polynomial  $a(L)^{-1}$  recovers actual inflation. Knowledge of the filter and a time series of

measured inflation thus is sufficient to construct a time series of actual inflation. Provided  $a(L)$  is invertible, the spectra of measured and actual inflation relate according to (Hamilton (1994), chapter 6)

$$S_{\pi^m}(\omega) = a(e^{-i\omega})a(e^{i\omega}) S_{\pi}(\omega) .$$

Here  $S_{\pi^m}$  ( $S_{\pi}$ ) denotes the spectrum of measured (actual) inflation,  $\omega$  displays a particular frequency and  $i = \sqrt{-1}$ . It is convenient to discuss the properties of the filter in terms of the ratio of spectra  $a(\cdot)a(\cdot)$  because this discussion does not require any assumption regarding the underlying inflation processes and hence is general in terms of  $S_{\pi^m}$  ( $S_{\pi}$ ).

Figure 2.2 plots the inverse of  $a(\cdot)a(\cdot)$  which equals the ratio of the spectrum of actual inflation over measured inflation. Evidently, the variance of actual inflation exceeds the variance of measured inflation because the inverse filter exceeds unity over all frequencies. Furthermore, the filter downgrades low frequency variation in measured inflation and shifts weight to variation in high frequencies. With quarterly data horizons larger than roughly three years will be affected by the downgrade. As a result measured inflation is less volatile and more persistent than actual inflation.

Figure 2.4 shows measurement bias when the filter  $a(L)$  is applied to quarterly annualized U.S. consumer price inflation. Measurement bias varies over time and correlates negatively with measured inflation. In the model, inflation which deviates strongly from steady state implies that prices set by optimizing firms are very different from the average price in the economy. Therefore, high measured inflation indicates that optimal prices have increased by a large amount. In such a situation underestimating the fraction of products with recently optimized price understates the actual extent of inflation. Thus, measured inflation is below actual inflation and measurement bias is negative. Indeed, the contemporaneous correlation between measured inflation and measurement bias is negative and large in absolute value with -0.63. Measurement bias is fairly persistent with autocorrelation 0.77. For comparison, the autocorrelation coefficient of U.S. consumer price inflation is 0.90.

Figure 2.4 plots measured U.S. inflation jointly with actual U.S. inflation as implied by the model of measurement bias put forth here. Measured inflation is less volatile than actual inflation and both measures differ in particular when deviations from the mean are large and in times when inflation changes drastically. The mean absolute difference between measured and actual inflation amounts to 0.29% and from a times series perspective the difference between the two measures of inflation may appear small. However, presuming a simple monetary policy rule which makes the nominal interest rate a function of inflation one can provide a back of the envelope assessment

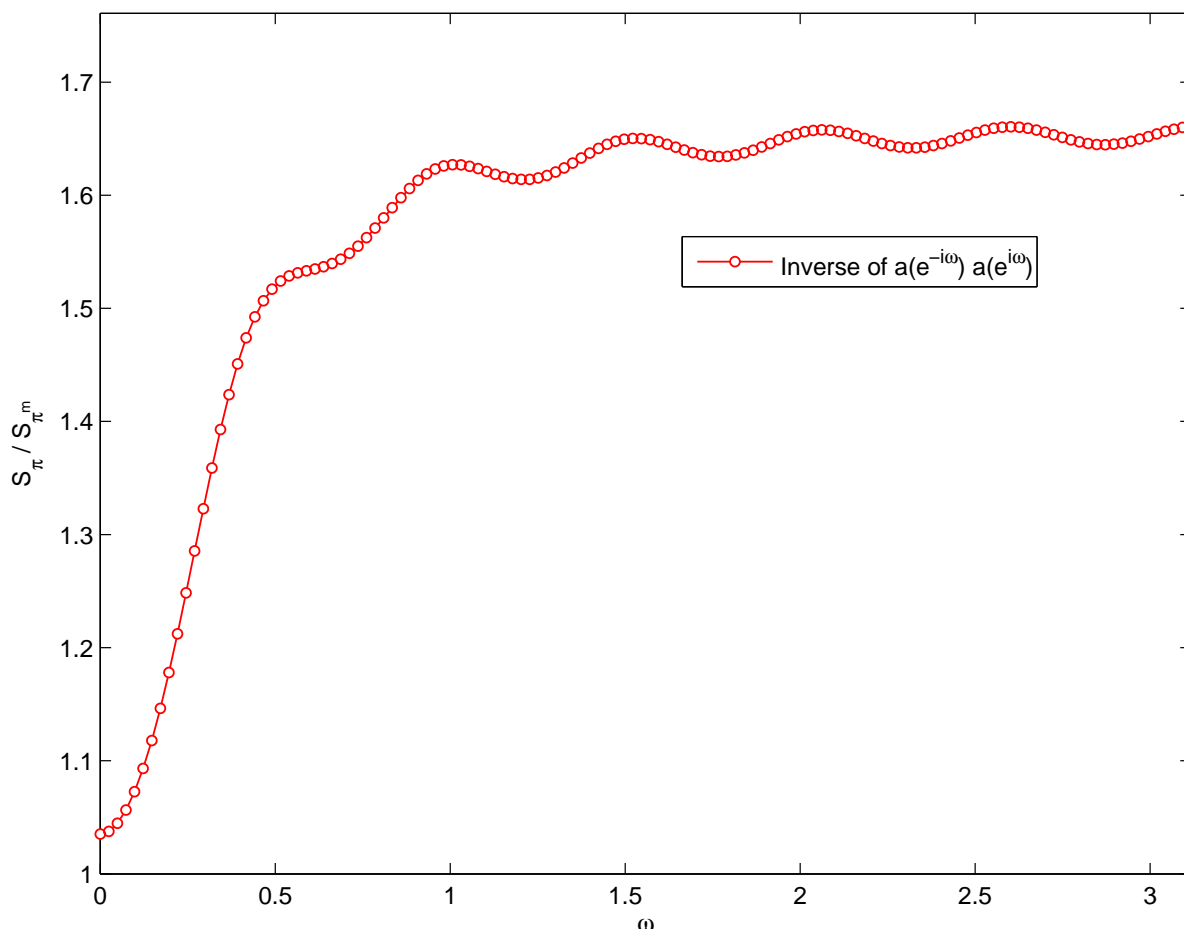


Figure 2.2: Ratio of the Spectrum of Actual Inflation over Measured Inflation.

of the implied difference in nominal interest rates. Comparing a central bank which sets its rate according to measured inflation,  $\hat{i}_t^m = 1.5\hat{\pi}_t^m$ , with a central bank which refers to actual inflation,  $\hat{i}_t = 1.5\hat{\pi}_t$ , one obtains a mean absolute difference between interest rates equal to 0.44%. Presuming feedback rules which also refer to mismeasured variables of real activity implies larger differences between nominal interest rates.

## 2.5.2 Dynamic Adjustment to Shocks

Figure 2.5 shows adjustment of measured and actual quantities to a positive productivity shock when monetary policy follows to the rule

$$\hat{i}_t = 1.5\hat{\pi}_t + 0.5x_t .$$



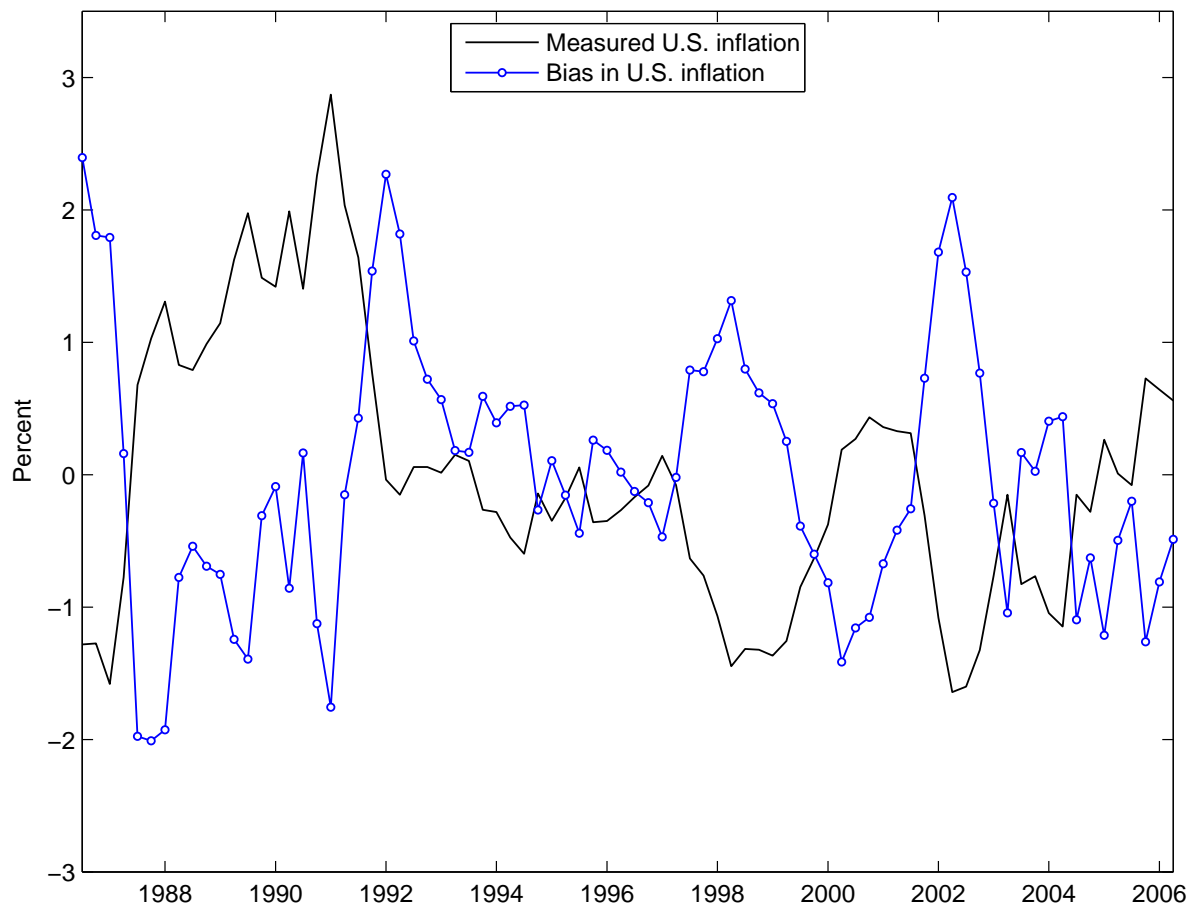


Figure 2.3: U.S. Consumer Price Inflation and Measurement Bias. Bias is defined as measured inflation minus actual inflation. The underlying time series is quarterly CPI ALL-ITEMS from the IFS database for the sample 1986:02 to 2006:01. U.S. Consumer price inflation is demeaned and initial conditions of the filter are set to zero.

I assume that monetary policy responds to actual variables in order to separate the effect of mismeasurement in variables from a situation in which measurement bias affects paths of actual variables because it is introduced into the policy process. High productivity induces firms to lower prices because marginal costs fall so that output increases. However, because prices adjust slowly sticky price output increases less than the natural level of output which makes the output gap fall.

Importantly, in times of high productivity measured inflation overstates actual inflation. It is an important concern of policy makers that measured inflation overstates actual inflation when productivity grows fast, and the model of measurement bias

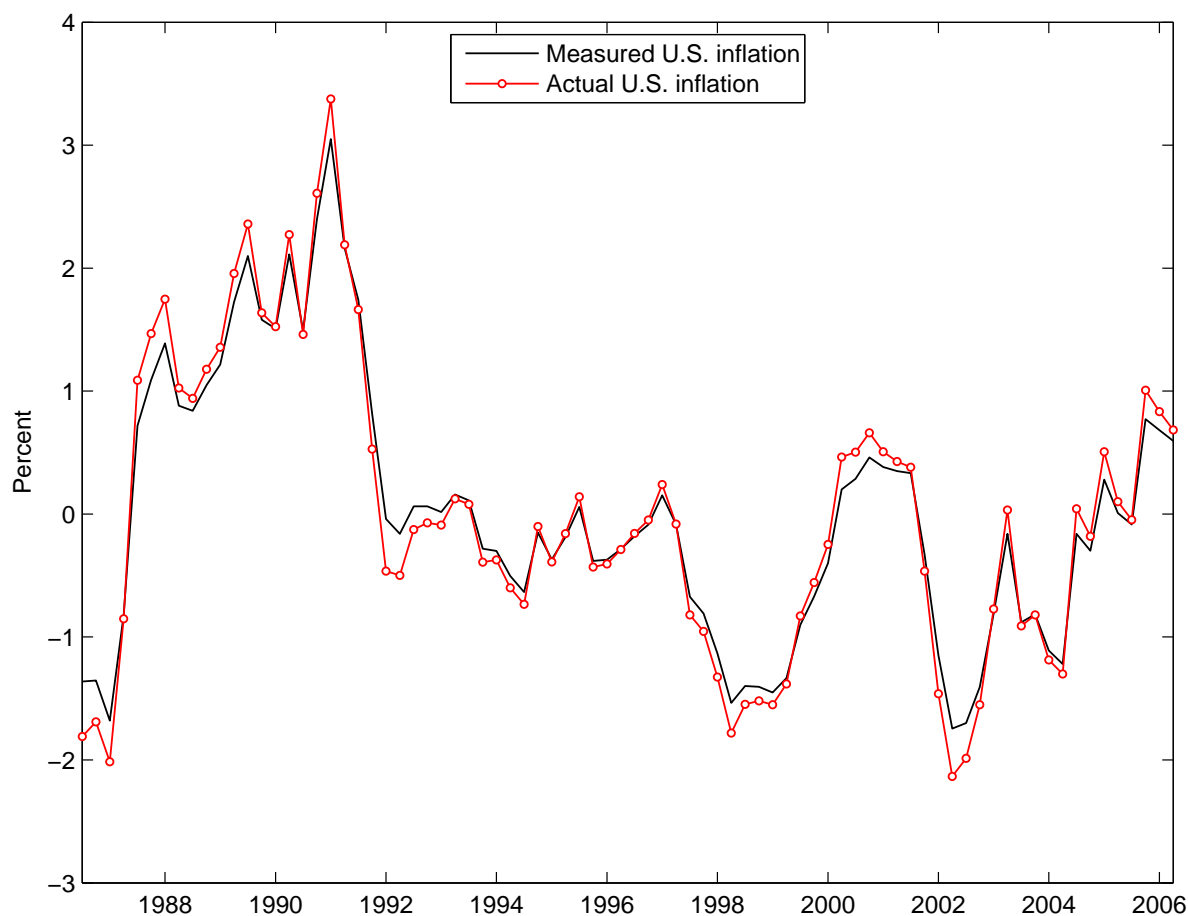


Figure 2.4: Demeaned U.S. Consumer Price Inflation and Actual Inflation.

outlined here features exactly this prediction. Measured inflation above actual inflation translates into a measured output gap that falls more than the actual output gap because measurement bias in inflation implies opposite mismeasurement in the output gap. Finally, measurement bias introduces persistence into both inflation and output gap dynamics. Whereas inflation mismeasurement dies out after about two years output gap mismeasurement remains substantial through the entire adjustment path.

## 2.6 Mismeasured Inflation and Monetary Policy

Mismeasurement in variables important for monetary policy raises the question of how monetary policy can accommodate measurement bias. In the model considered

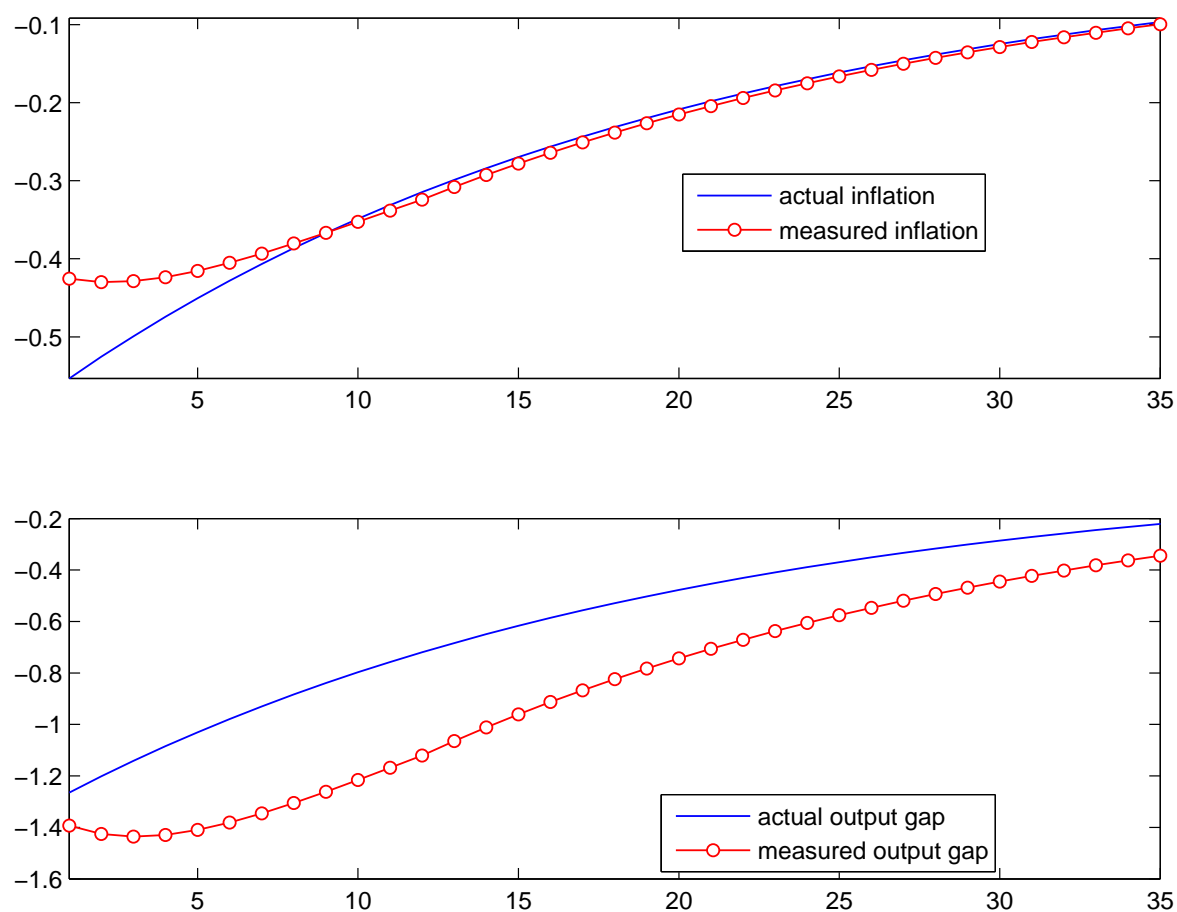


Figure 2.5: Impulse Response Function to a Positive Productivity Shock.

here the answer is straightforward. First, the central bank recovers actual inflation and the actual output gap from measured quantities by applying filters  $a(L)^{-1}$  and  $b(L)$ . Second, the central bank applies its targeting rule to actual quantities and achieves stabilization outcomes identical to the case of no measurement bias.<sup>5</sup> The remaining effect of measurement bias is that observers of the economy see inflation evolving more persistent and less volatile and the output gap evolving more persistent and more volatile.

In this section, I assess the implication of measurement bias for stabilization outcomes if the central bank does not recover actual inflation and the actual output gap before operating its targeting rule. Rather, it operates the targeting rule on measured

<sup>5</sup>Woodford (2003) derives optimal targeting rules in the model considered here absent measurement bias.

variables directly. This exercise delivers conclusions about how unaccounted measurement bias distorts central bank actions and how costly unintended stabilization is.

This exercise also serves to contrast policy implications of measurement bias to those of measurement error. Measurement *bias* captures a systematic difference between measured and actual variables which depends on the state of the economy. Measurement *error* is meant to capture a difference between measured and actual variables which is independent of the state of the economy but fluctuates randomly. Frequently, measurement error is taken to mimic mismeasurement in real world data in the literature that assesses robustness of monetary policy rules.

### 2.6.1 Measurement Bias

It is easiest to illustrate effects of unaccounted measurement bias by comparing two economies. In the *Benchmark* economy monetary policy operates on actual quantities and implements the targeting rule

$$\hat{\pi}_t + \frac{\lambda}{\phi} x_t = 0, \quad (2.19)$$

with  $\lambda > 0$  and  $\phi > 0$  as the slope of the AS relationship. In the *Agnostic* economy monetary policy remains agnostic with respect to measurement bias and operates on measured quantities while maintaining the same functional form of the targeting rule,

$$\underline{\hat{\pi}}_t^m + \frac{\lambda}{\phi} \underline{x}_t^m = 0. \quad (2.20)$$

In general, both measured and actual variables in the Agnostic economy evolve differently from their correspondents in the Benchmark economy because monetary policy operates differently in each case. I signify this difference by underlining actual and measured variables in the Agnostic economy.

Implicitly, I have chosen a targeting rule which minimizes expected discounted losses in the Benchmark economy,

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [\hat{\pi}_t^2 + \lambda x_t^2], \quad (2.21)$$

assuming that monetary policy cannot commit to future action. That is, when choosing an infinite sequence  $\{x_t, \hat{\pi}_t\}$ , targeting rule (2.19) minimizes  $\mathcal{L}$  subject to equation (2.17) and conditional on private sector expectations. Inferring consequences of measurement bias conditional on the optimal targeting rule rules out the possibility that measurement bias improves outcomes merely because it drives effective coefficients

of the targeting rule closer to optimal coefficients.<sup>6</sup> Losses in the Agnostic economy are evaluated according to

$$\underline{\mathcal{L}} = E_0 \sum_{t=0}^{\infty} \beta^t [\hat{\pi}_t^2 + \lambda \underline{x}_t^2]. \quad (2.22)$$

A convenient way to uncover effects of measurement bias is to vary the entry and exit rate  $\delta$ . Without entry and exit ( $\delta = 0$ ) there is no measurement bias so that both economies share the same limit. Increasing  $\delta$  makes delayed recording of new products and thereby measurement bias increasingly important. Measurement bias then drives a wedge between measured and actual quantities so that monetary policies in the two economies diverge. At the same time, however, changing  $\delta$  affects the slope coefficient of the AS relationship  $\phi$ . To isolate effects of measurement bias from those that results from structural changes in the economy I compute the relative loss as percentage deviation of the loss in the Agnostic economy from the loss in the Benchmark economy,  $\mathcal{L}_R = (\underline{\mathcal{L}} - \mathcal{L})/\mathcal{L}$ , for each value of  $\delta$ .

The left panel of figure 2.6 plots  $\mathcal{L}_R$  when monetary policy acts discretionary for increasing degrees of measurement bias and three values of the information lag  $\ell$ . I set  $\lambda = \phi/\theta$  which corresponds to the weight of the output gap in the welfare based loss function as derived in Woodford (2003), chapter 6, so that  $\lambda$  equals 0.0064 for the calibration in section 2.4. If the information lag of the statistical authority amounts to one quarter, relative losses increase monotonically in the degree of measurement bias. However, if the information lag is extended to four quarters or three years, relative losses actually fall so that measurement bias improves performance of discretionary monetary policy in the Agnostic economy relative to the benchmark in these cases.

To understand this result substitute for  $\underline{\pi}_t^m$  and  $\underline{x}_t^m$  in equation (2.20) by equations (2.14) and (2.16). Use the relationship  $a_0 + b_0 = 1$  and define  $q_i = [a_i + \frac{\lambda}{\phi} b_i]/[a_0 + \frac{\lambda}{\phi} b_0]$  to obtain

$$\begin{aligned} \hat{\pi}_t + \frac{\Lambda}{\phi} \underline{x}_t &= 0 & \text{if } \ell = 1 \\ \hat{\pi}_t + \sum_{i=1}^{\ell-1} q_i \hat{\pi}_{t-i} + \frac{\Lambda}{\phi} \underline{x}_t &= 0 & \text{if } \ell \geq 2. \end{aligned} \quad (2.23)$$

The effective relative weight attached to stabilizing the output gap  $\Lambda$  is the intended weight  $\lambda$  multiplied by a factor that changes with the extent of measurement bias,

$$\Lambda = \lambda [a_0 + \frac{\lambda}{\phi} (1 - a_0)]^{-1}.$$

<sup>6</sup>The loss function is proportional to household utility up to second order so that rule (2.19) implements welfare optimal discretionary policy if  $\lambda = \phi/\theta$ . Woodford (2003) derives the welfare based loss function in chapter 6 for a model equivalent to the one considered here absent measurement bias.

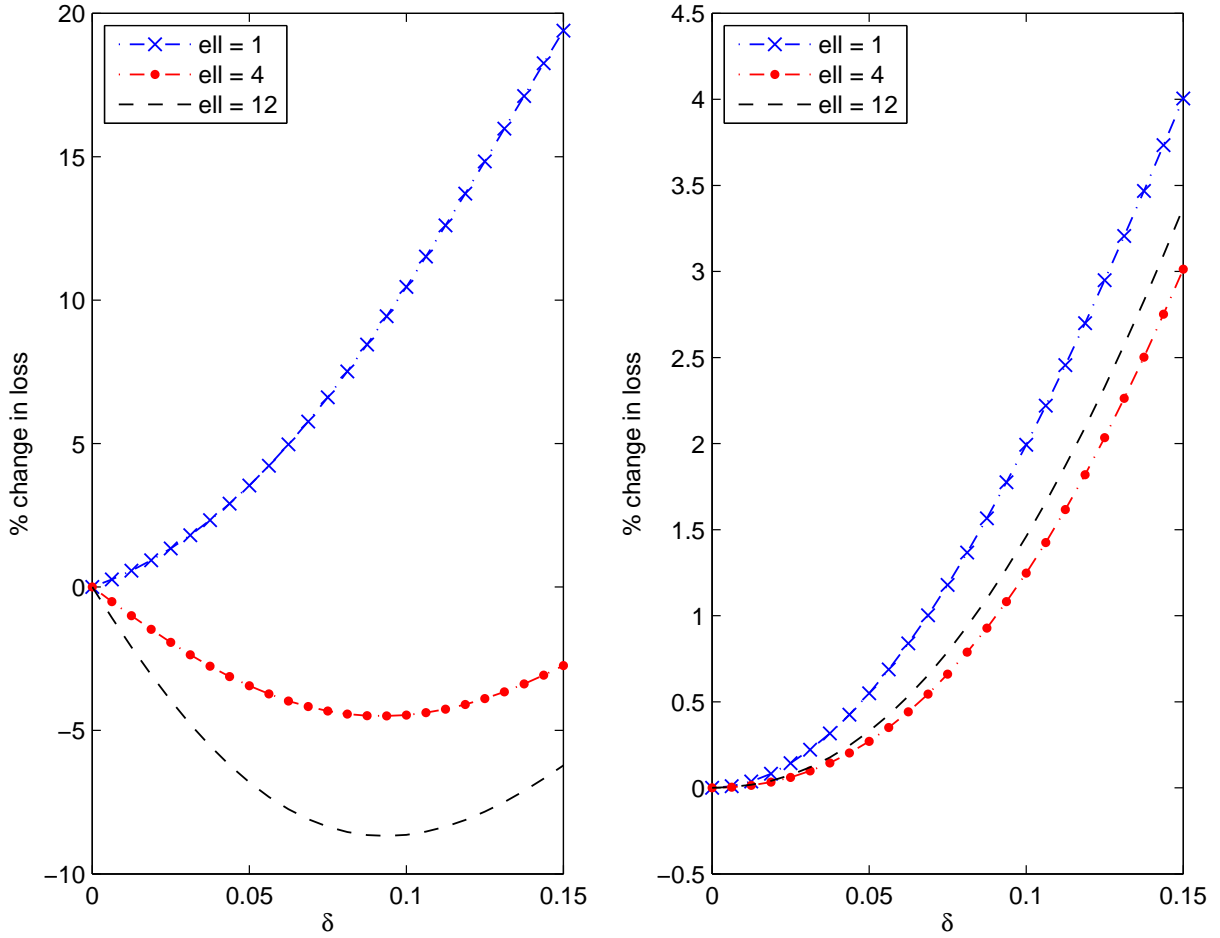


Figure 2.6: Percentage Change in Loss under Discretion and Commitment. The figure shows  $\mathcal{L}_R = 100 (\underline{\mathcal{L}} - \mathcal{L}) / \mathcal{L}$  as a function of  $\delta$ . The left panel plots  $\mathcal{L}_R$  for discretionary monetary policy. The right panel plots  $\mathcal{L}_R$  for committed monetary policy. Notation ell indicates  $\ell$ .

If  $\ell = 1$ , measurement bias distorts the relationship between  $\hat{\pi}_t$  and  $x_t$  in the targeting rule by changing the effective weight attached to stabilizing the output gap. The factor  $\frac{\lambda}{\phi}$  in the targeting rule  $\hat{\pi}_t + \frac{\lambda}{\phi} x_t = 0$  represents the rate at which the central bank optimally trades off inflation for the output gap absent measurement bias. Measurement bias mistakenly attributes a fraction  $b_0 > 0$  of actual inflation to the output gap,  $x_t^m = x_t + b_0 \hat{\pi}_t$ , while subtracting the same fraction from inflation,  $\hat{\pi}_t^m = (1 - b_0) \hat{\pi}_t$ . Thereby a fraction  $b_0$  of actual inflation is weighted by  $\frac{\lambda}{\phi}$  rather than by the inflation weight of unity which distorts the overall rate at which monetary policy trades off actual inflation for the actual output gap.

Thus, whenever  $\frac{\lambda}{\phi} > 1$ , the effective weight on actual inflation increases if measure-

ment bias exists so that more than intended inflation stabilization results. Weighting actual inflation more is equivalent to weighting the actual output gap less,  $\Lambda < \lambda$ . In turn, if  $\frac{\lambda}{\phi} < 1$  the effective weight on actual inflation falls if measurement bias exists so that less than intended inflation stabilization results. Measurement bias does not distort stabilization of inflation and the output gap in case  $\frac{\lambda}{\phi} = 1$ . The factor  $[a_0 + \frac{\lambda}{\phi}(1 - a_0)]^{-1}$  equals 1.2 for the calibration assumed here. Measurement bias therefore implies that a central bank which is agnostic about the existence of the bias attaches significantly less weight to stabilizing inflation than it actually intended.

If  $\ell > 1$ , the striking observation in the left panel of figure 2.6 is that a monetary policy which is agnostic with respect to measurement bias actually pushes losses below those implied by a monetary policy which accommodates measurement bias perfectly. As can be taken from equation (2.23) measurement bias introduces a reference to past actual inflation rates into central bank's effective targeting rule while the distortive weighting of inflation and the output gap continues to exit. Lagged inflation rates make the targeting rule history dependent despite the fact that monetary policy acts under discretion because the central bank implicitly borrows commitment from the statistical authority.

The statistical authority constructs measured inflation as a weighted average of actual current and past inflation rates,  $\hat{\pi}_t^m = a(L)\hat{\pi}_t$ , due to its failure to track prices of newly arrived products in time. By responding to this weighted average, monetary policy imports history dependence in actual variables into its rule. This implies that price level increases due to a positive  $u_t$  shock are offset later to some extent by negative inflation rates (Woodford (2003)). Price setting firms anticipate this reversion and restrain initial price increases so that inflation increases less initially. Accordingly, the output gap falls less initially.<sup>7</sup>

To check robustness of these mechanisms with respect to the policy regime I repeat the same experiment for monetary policy which perfectly commits to future action. Minimizing  $\mathcal{L}$  over  $\{x_t, \hat{\pi}_t\}$  subject to equation (2.17) and accounting for monetary policy's impact on private sector expectations implies a targeting rule in the Benchmark economy equal to

$$\hat{\pi}_t + \frac{\lambda}{\phi}\Delta x_t = 0.$$

Here  $\Delta$  indicates the difference operator. The corresponding rule in the Agnostic economy again exchanges actual quantities with measured quantities. The right panel of

<sup>7</sup>Even though history dependence here is displayed in terms of inflation rates it is straightforward to represent the targeting rule (2.20) in terms of lagged output gaps,  $\pi_t + \frac{\lambda}{\phi}g(L)x_t = 0$  having  $g(L) = [a(L) + \frac{\lambda}{\phi}b(L)]^{-1}$ .

figure 2.6 plots relative losses under commitment as function of  $\delta$  and for different  $\ell$ . As under discretion, measurement bias that is exclusively contemporaneous is detrimental to stabilization outcomes. Different from discretionary monetary policy measurement bias continues to generate suboptimal stabilization when the information lag increases. Suboptimal stabilization indicates that history dependence injected into the targeting rule by measurement bias disturbs history dependence that monetary policy optimally brings in due to commitment.

## 2.6.2 Measurement Error

Measurement bias and measurement error have different implications for monetary policy. Different from measurement bias, measurement error in observed variables is independent of the state of the economy and fluctuates randomly. I contrast both notions of mismeasurement by repeating the exercise of the previous section for the case of measurement error. In particular, I compare an economy free of measurement error in variables with one in which the central bank is exposed to error prone variables on which it operates its policy. I restrict attention to discretionary policy. A typical formulation of measurement error adds exogenous random noise to actual quantities,

$$\begin{aligned}\hat{\pi}_t^{\zeta} &= \tilde{\pi}_t + \zeta_{\pi t} \quad , \quad \zeta_{\pi t} \sim (0, \sigma_{\pi}^2) \quad , \\ x_t^{\zeta} &= \tilde{x}_t + \zeta_{xt} \quad , \quad \zeta_{xt} \sim (0, \sigma_x^2) \quad .\end{aligned}\tag{2.24}$$

Here,  $\hat{\pi}_t^{\zeta}$  and  $x_t^{\zeta}$  denote observed variables and  $\zeta_{\pi t}$  and  $\zeta_{xt}$  represent white noise components which are mutually independent, independent from  $u_t$  and have zero mean and known variances. Actual variables consistent with equilibrium under measurement error are  $\tilde{\pi}_t$  and  $\tilde{x}_t$ . Monetary policy is bound to target observed variables and does so by the rule  $\hat{\pi}_t^{\zeta} + \frac{\lambda}{\phi} x_t^{\zeta} = 0$  analog to the case of measurement bias. This targeting rule nests the benchmark of no measurement error as special case. Parameters which govern the degree of measurement error are  $\sigma_{\pi}^2$  and  $\sigma_x^2$  so that  $\delta$  which governed the degree of measurement bias before is held constant now. Substituting equations (2.24) the targeting rule is alternatively expressed as

$$\tilde{\pi}_t + \frac{\lambda}{\phi} \tilde{x}_t = \mu_t\tag{2.25}$$

having  $\mu_t = -(\zeta_{\pi t} + \frac{\lambda}{\phi} \zeta_{xt})$ . Measurement error does not introduce systematic distortions into the targeting rule as is the case in equation (2.23) where measurement bias modifies the effective weight attached to the output gap. Rather, measurement error functions similar to an unsystematic central bank control error. Different from



measurement bias in its general form, measurement error also does not introduce a reference to past endogenous states into the targeting rule.

It is straightforward to show how measurement error affects losses under discretionary monetary policy. In parallel to the case of measurement bias losses are evaluated according to

$$\mathcal{L}^{\xi} = E_0 \sum_{t=0}^{\infty} \beta^t [\tilde{\pi}_t^2 + \lambda \tilde{x}_t^2].$$

The system of equilibrium conditions comprising the targeting rule (2.25) and the aggregate supply relationship (2.17) is easily solved by the method of undetermined coefficients. Equilibrium paths of inflation and the output gap depend on all shock processes and evolve according to

$$\tilde{\pi}_t = f_{\pi} u_t + \gamma_{\pi} \xi_{\pi t} + \gamma_x \xi_{x t}, \quad \tilde{x}_t = \frac{-\phi f_{\pi}}{\lambda} u_t + \frac{\gamma_{\pi}}{\phi} \xi_{\pi t} + \frac{\gamma_x}{\phi} \xi_{x t}$$

with coefficients  $f_{\pi} = \frac{\lambda}{(\lambda(1-\beta\rho_u)+\phi^2)}$ ,  $\gamma_{\pi} = \frac{-\phi^2}{(\lambda+\phi^2)}$  and  $\gamma_x = \frac{-\lambda\phi}{(\lambda+\phi^2)}$ . For  $\beta \rightarrow 1$  the loss function rewrites in terms of unconditional variances  $\mathcal{L}^{\xi} = \text{var}(\tilde{\pi}_t) + \lambda \text{var}(\tilde{x}_t)$  and equilibrium paths of inflation and the output gap serve to express these variances in terms of fundamental shock variances. The loss as a function of  $\sigma_{\pi}^2$  and  $\sigma_x^2$  then obtains as

$$\mathcal{L}^{\xi} = \frac{\lambda+\phi^2}{\lambda} \left( \frac{f_{\pi}^2 \sigma_u^2}{1-\rho_u^2} \right) + \frac{1}{\lambda+\phi^2} [\phi^2 \sigma_{\pi}^2 + \lambda^2 \sigma_x^2].$$

Absent measurement error in observed variables, i.e.  $\sigma_u^2$ ,  $\sigma_{\pi}^2$  and  $\sigma_x^2$  equal zero, the loss reduces to the first term on the right hand side. Therefore, this part of the loss function represents the benchmark loss for the situation in which the central bank responds to perfectly observed actual variables. The second term indicates losses that result from the existence of measurement error in inflation and the output gap. Because this second term is positive whenever measurement error is present the overall loss increases for any combination of measurement error in observed variables relative to the benchmark situation. Thus, under measurement error a situation in which losses fall relative to the benchmark of perfect measurement never occurs. This contrasts implications of measurement bias which reduces the loss under discretion for reasonable calibrations of the information lag.

## 2.7 Conclusion

Recent evidence from micro data points to a substantial cyclical new product bias in U.S. consumer prices. Evidently, micro data on prices and quantities for the entire

universe of products is extremely useful to infer measurement biases in consumer price indices constructed by statistical authorities. However, up to now such data is nonexistent for many countries, rare in others and difficult to collect over long periods of time. Until micro data is routinely available economic models appear a useful stand-in to identify measurement biases. To this end, I have provided a stylized model of measurement bias which uncovers macro time series more in line with the ideal indices economists often have in mind. The model studied here remains a minimal setup to convey basic implications of inflation mismeasurement for measures of real activity and for the conduct of monetary policy.

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## Chapter 3

# Dynamic Money Demand and Optimal Monetary Policy

Estimated money demand is often intrinsically persistent beyond accounting for transaction volume and opportunity costs. I integrate dynamic money demand into the typical New Keynesian model by means of habit formation in the money stock. The weight for stabilizing a particular money index in the welfare based loss function increases in the degree of habit in money and is roughly three times the one attached to the output gap for a reasonable calibration. The substantial weight does not justify an equally substantial response to the money index in the optimal target criterion because short run money demand elasticities decrease when money demand is dynamic rather than static. Stabilizing the money index thus involves large costs in terms of variation in other target variables because monetary policy has small leverage on money demand.

### 3.1 Introduction

Estimated money demand is often intrinsically persistent beyond accounting for transaction volume and opportunity costs. This holds true across monetary aggregates, countries, empirical methods and for different periods of time.<sup>1</sup> In this paper, I derive

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<sup>1</sup>Heller and Khan (1979) find for U.S. data on M1 (M2) coefficients on lagged money in the range of 0.85–0.99 (0.7–0.8). More recently, Ball (2002) estimates a coefficient on lagged money of 0.8 using

a model in which money demand exhibits this property and explore the implications of dynamic money demand for optimal monetary policy.

In standard treatments of the New Keynesian model as for instance in Woodford (2003), chapter 3, money demand is a contemporaneous function of transaction volume, opportunity costs and disturbances to the marginal willingness to hold money. Persistence in money demand exists only to the degree to which driving variables are persistent on their own which appears at odds with empirical evidence. The modification pursued here is to introduce habit persistence in the stock of money. With habit in the money stock households derive utility from money balances only after putting them into relation to money balances held in the past. As a result, current money demand is partly driven by past money balances which increases persistence in money demand beyond what is contained in other driving variables.

Mansoorian and Michelis (2005) argue that formulations of habit should reflect the habitual standard of living. Beyond consumption, habit thus should extend to money balances as a means to simplify the purchase of consumption goods because the standard of living depends not exclusively on the level of consumption but also on how difficult it is to obtain this level. Transaction services of money are often considered complementary to the quality of money as asset (see for instance Lucas (1988)). Accordingly, one may accept habit persistence in money at the same time as the assumption that households assess the value of a liquid asset relative to past amounts of this asset in their financial portfolio. Habit persistence in money then is consistent with the idea that households have reference values for liquid assets which derive from past experience. Mäki-Fränti (2008) finds some empirical support for habit formation with respect to liquid assets.

Money demand is also driven by expectations about future money balances once habit formation extends to the money stock. The increase of money balances in the current period reduces the benefit of future money balances because high current balances drive up future habit levels. Expectations in money demand indicate that the household accounts for this effect on future habit levels when adjusting current money balances. Nelson (2002) shows that, once forward looking, money demand depends on a measure of long term interest rates. Nelson (2003) emphasizes that

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U.S. data for M1. Tin (1999) finds lagged coefficients mostly between 0.25–0.45 using U.S. micro data for monetary assets. Stracca (2003) estimates a respective coefficient of 0.92 for M1 euro area data derived as  $1-0.08$  from equation (14) in that paper due to its error-correction framework. Coenen and Vega (2001) estimate a coefficient on lagged money of roughly 0.87 using euro area data for M3. Andrés, López-Salido, and Nelson (2008) estimate money demand that includes forward and backward looking terms and report significantly positive coefficients on both terms for euro area and U.S. data.



this dependency on long term interest rates enhances money's ability to signal the stance of aggregate demand since long term interest rates are also a main driving force of aggregate demand. Under limited information on the side of the monetary authority Nelson shows that optimal monetary policy under commitment attributes more weight to money growth if money demand is forward looking rather than static. Money growth does not enter the loss function of the central bank that underlies these results.

One may wonder, however, if money growth (or transformations thereof) should enter the welfare based loss function to the extent that forward looking money demand reflects agents' willingness to stabilize money growth. In this case it is less clear a priori how money's role as indicator variable interacts with the objective to stabilize money growth (or transformations thereof). It therefore seems warranted to explore optimal monetary policy in an environment of dynamic money demand abstracting from informational frictions. Besides the role of money as indicator variable, the evidence that dynamics are a salient feature of empirical money demand justifies independent interest in optimal policy responses in such an environment.

I find that the loss function which approximates agents' utility function indeed puts a non-trivial weight to stabilizing the quasi growth rate of money adjusted for money demand disturbances if habit in the money stock exists. The weight attached to stabilizing this money index is increasing in agents' habit. For instance, without habit the weight is about one tenth of the weight attached to stabilizing the output gap when the model is calibrated to euro area data. However, the weight is almost three times larger than the one attached to stabilizing the output gap when habit in the money stock is calibrated such as to match estimates of persistence in money demand beyond what is contained in transaction volume and opportunity costs.

Surprisingly, the substantial weight in the loss function does *not* justify an equally substantial response to money under optimal monetary policy. The reason is that habit formation in the money stock reduces impact or short run elasticities of transaction volume and opportunity costs in money demand. Large habit makes smoothing money balances the predominant objective of the household so that transaction and opportunity cost motives become negligible. Attempts to stabilize the money index then trigger large fluctuations in inflation and the output gap because the influence monetary policy can exert on money demand is small. As it turns out the large weight in the loss function and the low short run elasticities exactly offset each other in the optimal target criterion under both discretionary and committed monetary policy. The effective weight attached to stabilizing the money index coincides with the one that prevails under static money demand.

Disturbances in the natural real rate generate a tradeoff between stabilizing inflation and the output gap versus stabilizing the money index if money demand is static.<sup>2</sup> Dynamics in money demand affect this tradeoff. For discretionary monetary policy the tradeoff is less severe when money demand is dynamic because short run money demand elasticities of transaction volume and opportunity costs fall. Due to low elasticities, money demand decouples from movements in the nominal rate so that monetary policy can stabilize inflation and the output gap better without inducing more variation in money demand. As a result nominal rates vary more unless the additional variation conflicts with the lower bound on nominal rates. To prevent such a conflict more weight has to be put on stabilizing nominal rates under dynamic money demand. The fact that the weight of the money index in the loss function increases when short run money demand elasticities decrease does not undermine that the relation between money demand and transaction and opportunity cost variables becomes weaker. It is this weaker relation that reduces the tradeoff.

Under committed monetary policy this mechanism has a slightly different twist. Committed monetary policy manipulates private sector expectations by referring to past states of the economy (Woodford (1999)). Due to the forward looking nature of dynamic money demand the leverage over expectations makes money demand respond more to changes in transaction and opportunity cost variables if such changes persist. Therefore, forward looking money demand reinforces the tradeoff between stabilizing inflation and the output gap versus stabilizing the money index relative to discretion. Effectively, history dependence offsets some of the benefits due to low short run money demand elasticities under dynamic money demand.

Turning to the role of money as information variable I demonstrate that the unconditional correlation between money growth and the output gap may indeed increase for intermediate degrees of habit in the money stock relative to the case of static money demand and in line with results in Nelson (2002). However, this correlation fades out quickly for large degree of habit because in this case short run elasticities of money demand are negligible so that the structural reason for money growth to co-move with the output gap disappears.

The paper is related to Amato and Laubach (2004) who analyze implications of habit formation in consumption for optimal monetary policy and find that habit in

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<sup>2</sup>For the case of static money demand the loss function is often expressed in terms of output gap, inflation and the nominal interest rate. Then the tradeoff exists between stabilizing inflation and the output gap versus stabilizing the nominal interest rate because perfect stabilization of inflation and the output gap achieves if the nominal rate equals the natural rate, see for instance Woodford (2003), chapter 6, p. 425.

consumption affects endogenous dynamics and conclusions about the conduct of policy. Woodford (2003) is a main reference for optimal monetary policy in New Keynesian models as considered here. The literature which formulates alternative models for dynamic money demand is discussed in section 3.2.2.

## 3.2 Model

The model considered here is a version of the model in Woodford (2003), chapter 3 and 5, and Giannoni and Woodford (2005) modified by introducing habit formation in the stock of real money. The model features a representative household, many firms and a government. The household values money, consumes all of the infinitely many goods, decides its financial portfolio and supplies homogenous labor. Firms produce with labor as sole production input, are subject to a price setting constraint as in Calvo (1983) and sell their goods in monopolistically competitive markets. The government implements monetary policy, operates a tax scheme and issues bonds. The model's equilibrium is inefficient for three reasons. First, output is inefficiently low as a result of monopolistically competitive goods markets. Second, the constraint on firms' price setting drives a wedge between prices that maximize firms period profit and constrained optimal prices. This wedge then distorts the allocation. Third, despite the fact that money is supplied at zero costs a non-negative price for money liquidity prevents the economy from reaching money satiation.

### 3.2.1 Household

The representative household maximizes the objective

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t - \eta C_{t-1}, \xi_t) + q(\frac{M_t}{P_t} - \phi \frac{M_{t-1}}{P_{t-1}}, \xi_t) - h(N_t, \xi_t)] . \quad (3.1)$$

$E_0$  denotes the expectation operator conditional on period zero information. Let  $C_t$  denote period  $t$  consumption,  $M_t \geq 0$  money holdings at the end of period  $t$ ,  $N_t$  labor, and  $P_t$  the money price of consumption. The vector  $\xi$  comprises mean zero shocks. For each value of  $\xi$  the functions  $u(\cdot)$  and  $q(\cdot)$  ( $h(\cdot)$ ) are increasing and concave (convex) in their arguments. Table 5.3 describes parameters and summarizes restrictions on parameters. I posit separability between consumption and real money balances in line with results in Andrés, López-Salido, and Nelson (2008) and Ireland (2004) who show that non-separability between consumption and real money balances is quantitatively

negligible for euro area and U.S. data, respectively. Utility is non-separable over time due to internal habit in consumption and real money balances.

Two complementary arguments motivate the assumption of habit in the money stock. First, Mansoorian and Michelis (2005) argue that habit should refer to the habitual standard of living which includes benefits derived from the transaction services provided by money.<sup>3</sup> The second argument concerns the quality of money as an asset. Liquid or short run assets such as money are little exposed to changes in expected long term interest rates. Accordingly, positive money balances may reflect a desire to hold a financial portfolio a fraction of which is insensitive to expected long term interest rates. In that vein one may take habit persistence in money as the assumption that households assess the value of the liquid asset relative to past amounts of this asset in their financial portfolio. In equilibrium habit in money balances introduces persistence into the process of money. This persistence is consistent with household's having reference values for liquid assets which are derived from past experience. Congdon (2005) argues that the amount of liquid assets is particularly stable for institutional investors and Mäki-Fränki (2008) finds some evidence for habit with respect to liquid assets.<sup>4</sup>

Aggregate consumption is a Dixit-Stiglitz composite of intermediate goods,

$$C_t = \left( \int_0^1 C_t(j)^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)} .$$

By definition the price level  $P_t$  is the cost minimal money price of  $C_t$ . The household enters period  $t$  with last period money and bond holdings at interest rates  $i_t^m \geq 0$  and  $i_t$ , respectively. It receives after tax labor income  $(1 - \tau)W_t N_t$ , an equal share of aggregate firm profits  $\int_0^1 D_t(j) dj$  and pays a lump sum tax  $T_t$ . A balanced budget requires that the sum of consumption expenditure plus end of period financial assets equals beginning of period financial wealth plus income net of taxes,

$$P_t C_t + B_t + M_t = (1 + i_{t-1})B_{t-1} + (1 + i_{t-1}^m)M_{t-1} + (1 - \tau)W_t N_t + \int_0^1 D_t(j) dj - T_t .$$

<sup>3</sup>In light of this argument it also seems natural to consider habit formation in labor. I do not pursue this extension here to keep the analysis focused on implications of dynamic money demand.

<sup>4</sup>Congdon (2005), p.36, actually finds the ratio of liquid assets to total assets to be reasonably stable and interprets this as stability of the *desired* ratio. Seemingly, this conflicts with my assumption of habit in the stock of money (as synonym for liquid assets) rather than habit in the ratio of money to bonds plus money. In the model considered here, however, the growth rate of the ratio  $M_t/(B_t + M_t)$  is proportional to the growth rate of real money if fiscal policy ensures that real debt grows at constant rate,  $(B_t/P_t + M_t/P_t)/(B_{t-1}/P_{t-1} + M_{t-1}/P_{t-1}) = k$ . More generally, by confining attention to the simpler case of habit in the stock of real money I implicitly assume that money is predominant in driving the ratio of liquid assets to total assets.

(3.2)

$B_t$  denotes end of period nominal government bonds. By assumption borrowing limits are finite and arbitrage opportunities across bond and money market are absent,  $i_t \geq i_t^m$ , which excludes exploding paths of consumption. Government bonds and money are the only assets traded on financial markets. Distributed ownership in firms and identical initial wealth across households ensure that households make identical choices and hold identical positions throughout.

### 3.2.2 Dynamic Money Demand

#### Habit in the Money Stock

Maximizing (3.1) subject to (3.2) by choosing the sequence  $\{C_t, N_t, M_t, B_t\}_{t=0}^{\infty}$  delivers, among other equilibrium conditions, the optimal demand for money,

$$E_t \left[ \frac{q_m(m_t - \phi m_{t-1}, \xi_t) - \phi \beta q_m(m_{t+1} - \phi m_t, \xi_{t+1})}{u_c(C_t - \eta C_{t-1}, \xi_t) - \eta \beta u_c(C_{t+1} - \eta C_t, \xi_{t+1})} \right] = \Delta_{it}.$$

Here  $m_t$  denotes real money balances. The left hand side is the discounted marginal utility of money in terms of the discounted marginal utility of consumption accounting for the fact that internal habit interlinks periods.<sup>5</sup> The marginal rate of substitution between real money and consumption then equals the price of money liquidity  $\Delta_{it} \equiv \frac{i_t - i_t^m}{1 + i_t} \geq 0$ . Linearizing money demand around a zero inflation steady state delivers

$$\hat{m}_t - \phi \hat{m}_{t-1} - \epsilon_t^m = \phi \beta E_t[\hat{m}_{t+1} - \phi \hat{m}_t - \epsilon_{t+1}^m] + (1 - \phi \beta)(1 - \phi) \left[ \frac{\eta_y}{(1 - \eta \beta)(1 - \eta)} \tilde{Y}_t - \eta_i \hat{\Delta}_{it} \right] \quad (3.3)$$

A variable with a hat on top denotes percentage deviation from steady state,  $\hat{m}_t = dm_t / \bar{m}$ , unless noted otherwise. A variable signified by a bar indicates the steady state value. Also, I exploit the equilibrium relationship  $Y_t = C_t$  and define  $\epsilon_t^m$  ( $g_t$ ) as the percentage change in money (consumption) required to offset changes in  $\xi_t$  on the marginal utility of money (consumption).<sup>6</sup> Moreover,  $\tilde{Y}_t$  denotes a lag polynomial in output using  $L$  to denote the lag operator,  $\tilde{Y}_t = E_t[(1 - \eta L)(1 - \eta \beta L^{-1})\hat{Y}_t - (1 - \eta \beta L^{-1})g_t]$ . This expression reflects habit formation in consumption. One may interpret  $\tilde{Y}_t$  and  $\hat{\Delta}_{it}$  as transaction and opportunity

<sup>5</sup>Notation  $q_m(\cdot)$  abbreviates  $\partial q(m_t - \phi m_{t-1}, \xi_t) / \partial(m_t - \phi m_{t-1})$ . Marginal utility of consumption is treated analogously. Notation  $q_m$  used below indicates that  $q_m(\cdot)$  is evaluated at steady state. For other equilibrium conditions and derivation of the budget constraint see Woodford (2003), chapter 3 and 5.

<sup>6</sup>In particular,  $\epsilon_t^m = -\frac{q_{m\xi}}{\bar{m}q_{mm}}\xi_t$  and  $g_t = -\frac{u_{c\xi}}{\bar{Y}u_{cc}}\xi_t$ . Also,  $\hat{\Delta}_{it} = d\Delta_{it} / (1 - \bar{\Delta}_i)$  is the absolute deviation from steady state for sufficiently small  $\bar{\Delta}_i$ .

cost motive of money demand, respectively. The two parameters

$$\eta_y = \frac{(1-\eta\beta)(1-\eta)\bar{Y}u_{cc}\bar{\Delta}_i}{(1-\phi\beta)(1-\phi)\bar{m}q_{mm}} > 0 \quad , \quad \eta_i = \frac{(\bar{\Delta}_i-1)(1-\eta\beta)u_c}{(1-\phi\beta)(1-\phi)\bar{m}q_{mm}} > 0$$

are the steady state income elasticity and the steady state interest rate semielasticity, respectively. The definitions imply that a permanent change in the interest rate differential leads to the same permanent percentage change in real money balances for each admissible value of  $\phi$ .<sup>7</sup>

Approximate short run elasticities  $\theta_y = \frac{(1-\phi\beta)(1-\phi)}{(1-\eta\beta)(1-\eta)}\eta_y$  and  $\theta_i = (1-\phi\beta)(1-\phi)\eta_i$  fall in absolute value when  $\phi$  increases. For illustration suppose that in equation (3.3) the interest rate differential falls while keeping consumption constant. Without habit in the money stock a substantial expansion of  $m_t$  ensures optimality. The expansion in  $m_t$  happens to be less pronounced once habit in the money stock is present because the household internalizes the fact that expanding  $m_t$  has adverse effects in the next period by increasing the marginal utility of  $m_{t+1}$ . Put differently, with habit formation benefits from money holdings increase in the current period while they are expected to fall in the next period if  $m_t$  extends. This expected loss makes  $m_t$  respond less sensitive to changes in the interest rate differential.

### Alternative Specifications

Dynamic money demand has alternatively been derived from the assumption that adjustment of money balances is subject to a penalty which is convex in money growth so that partial adjustment takes place in equilibrium. Two specifications that implement the partial adjustment mechanism are money adjustment costs and the assumption that adjusting money balances creates dis-utility. Goldfeld (1973) and Laidler (1990) appeal to the adjustment cost specification whereas Chari, Christiano, and Eichenbaum (1995), Christiano and Gust (1999), Nelson (2002), Nelson (2003) and Andrés, López-Salido, and Nelson (2008) apply the dis-utility specification. Goodfriend (1985) criticizes the partial adjustment mechanism on the grounds that in practice costs are fixed rather than contingent on the current volume of adjustment. Moreover, he notes that adjustment costs are small in practice whereas in theory costs are found to be substantial in order to match coefficients on lagged money close to unity in money demand.<sup>8</sup> In turn, Goodfriend (1985) argues that stochastic measurement

<sup>7</sup>This statement becomes more apparent if  $q(\cdot)$  is assumed to be of the power utility form. Then  $\bar{m}q_{mm} \propto [(1-\phi\beta)(1-\phi)]^{-1}$ .

<sup>8</sup>Papers that refer to the dis-utility specification typically specify adjustment costs in such a way that small costs are consistent with a large coefficient on lagged money in money demand (for instance, see

error in transaction and opportunity cost variables is consistent with the finding of a significant lagged money term in money demand regressions even though actual money demand is adjusted completely on a period by period basis. However, Taylor (1994) puts Goodfriend's hypothesis to a test and rejects the interpretation of stochastic mismeasurement for U.S. data on M1.<sup>9</sup>

### 3.2.3 Firms

Firms are indexed by  $j \in [0, 1]$ . Firm  $j$  produces quantity  $Y_t(j)$  with  $N_t(j)$  hours of labor and technology  $Y_t(j) = A_t f(N_t(j))$ . Productivity  $A_t > 0$  is an exogenous stochastic process with mean  $\bar{A} = 1$ . Technology  $f(\cdot)$  is increasing and concave. The firm operates on competitive factor markets but monopolistically competitive goods markets and faces household demand  $Y_t(j)/Y_t = (P_t(j)/P_t)^{-\theta}$ . As in Calvo (1983), the firm reoptimizes its price with probability  $\alpha$  in any given period. With probability  $(1 - \alpha)$  the firm's price is indexed to inflation in the previous period,  $P_t(j) = P_{t-1}(j)\pi_{t-1}^\kappa$ . Here  $\kappa$  denotes the degree of indexation. Such a price setting scheme makes reoptimized prices respond to expected marginal costs and implies a forward looking relationship between current inflation and marginal costs. In addition, past inflation influences current inflation because prices which are not reoptimized are indexed.

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Chari, Christiano, and Eichenbaum (1995), equation (30)). Unfortunately, the size of money adjustment costs is not identified in this specification up to first order so that the specification cannot be subjected to a test in linear models.

<sup>9</sup>Relative to the habit specification applied here, the adjustment cost specification modifies the lower bound on nominal interest rates. Suppose the household goes short in bonds  $B_t < 0$  to increase money holdings. If adjustment costs are sufficiently convex in money growth it is not optimal to engage in this arbitrage unboundedly because marginal benefits from going short in bonds are constant assuming the household is small whereas marginal adjustment costs increase in money growth. Thus it is not necessary to impose  $i_t \geq i_t^m$  in order to prevent unbounded consumption. Rather, situations  $i_t^m > i_t$  are well consistent with a finite path of consumption. By the same token, the relevant measure of opportunity costs is the interest rate differential  $i_t - i_t^m$  adjusted for marginal adjustment costs. Thus, adjustment costs make the marginal and the average return on money differ. Goodfriend (2000) proposes a carry tax on money with similar implications for the lower bound. Relative to a specification of money adjustment costs in, say, consumption units the dis-utility specification suppresses the fact that changes in the price of consumption affect the extent of money adjustment under optimal household behavior though this difference wipes out up to first order if dis-utility from money adjustment is assumed to vanish in steady state.

### 3.2.4 Equilibrium

The government balances its budget ensuring intertemporal solvency,  $T_t + B_t + M_t + \tau W_t N_t = (1 + i_{t-1})B_{t-1} + (1 + i_{t-1}^m)M_{t-1}$ , and controls both interest rates  $i_t$  and  $i_t^m$ . In the following the return on the monetary base is assumed to be fixed by monetary policy below the steady state real rate,  $i_t^m = \bar{i}^m < \bar{r}$ . The remaining bits of monetary policy are specified below and fiscal policy is left unspecified. In equilibrium goods markets clear,  $Y_t(j) = C_t(j)$ , as do labor markets,  $N_t = \int_0^1 N_t(j) dj$ .

Appendix C.5 summarizes the model once linearized around a zero inflation steady state and yet without a specification of monetary policy. It is straightforward to show existence and uniqueness of the steady state. The linear model is expressed in terms of the output gap  $x_t$  which is sticky price output  $\hat{Y}_t$  minus the natural level of output  $\hat{Y}_t^n$  that reveals under flexible prices. Habit in consumption implies that both past and expected output gaps matter for output gap dynamics. The aggregate supply relation links inflation and the output gap and is augmented by an adhoc disturbance  $u_t$  which generates a tradeoff between stabilizing inflation and the output gap.<sup>10</sup> The natural rate  $\hat{r}_t^n$  is the real interest rate consistent with the natural output level and is a function of preference and technology disturbances only. Evidently, no reference to money is necessary in this model to obtain equilibrium paths for inflation and the output gap as long as monetary policy does not introduce such a reference. For all equilibrium computations I use the solution method described in Sims (2002) and check local determinacy and uniqueness of equilibria numerically.

## 3.3 Calibration

A core parameter is the degree of habit in the money stock  $\phi$  which is set to 0.8. In general, there is a close correspondence between  $\phi$  and the coefficient on lagged money in estimated money demand functions so that this parameter should be compared to the evidence discussed in footnote 3.1.<sup>11</sup> The long-run income elasticity of money demand  $\eta_y$  is set to unity. Stracca (2003) provides an estimate of 0.75 using euro area data for M1 and Reynard (2007) reports a unit income elasticity using data for M2.

<sup>10</sup>Define  $\pi_t = P_t/P_{t-1}$ ,  $\hat{\pi}_t = d\pi_t$  and  $\hat{i}_t = d(1 + i_t)/(1 + \bar{i})$ . Also,  $a_t = \hat{A}_t$  and  $\bar{n}_t \equiv -v^{-1} \frac{h_{N_t}^{\bar{N}_t}}{h_N} \bar{\zeta}_t$ .

<sup>11</sup>If monetary policy either does not refer to money at all or responds to the quasi growth rate of money  $(1 - \phi L)\hat{m}_t$  persistence in money demand actually equals  $\phi$  after solving for expectations. For other policies the degree of persistence in money demand also depends on the eigenvalues of the entire system (B.1). However, the difference between  $\phi$  and the equilibrium elasticity between  $\hat{m}_t$  and  $\hat{m}_{t-1}$  seems usually fairly small in this case.



The long-run interest rate semielasticity  $\eta_i$  is equal to three. Reynard (2007) estimates semielasticities between 1.2 and 3.6 depending on the monetary aggregate and the sample period. For euro area data on M1 Andrés, López-Salido, and Nelson (2008) find an estimate of  $\eta_i$  consistent with three when imposing a unit income elasticity. Steady state velocity  $\bar{v}$  is set to its historical mean 4.47 using quarterly M1 velocity for euro area data from 1975:01 to 1999:04 in Fagan, Henry, and Mestre (2005) (FHM). The slope  $\mu$  of the aggregate supply relationship is equal to 0.097.<sup>12</sup> The steady state interest rate differential  $\bar{\Delta}_i$  is calibrated to 1.6 percent on an annual basis which is the average euro area differential between the three month nominal interest rate in the FHM data set and the own rate of M1 as computed in Stracca (2003). Labor taxes exactly offset the inefficiency generated by monopolistic competition.

Most other parameters are calibrated based on estimates in Onatski and Williams (2004) (OW) of the Smets and Wouters (2003) model which does not feature money explicitly. Onatski and Williams use quarterly euro area data from FHM. The degree of price indexation  $\kappa$  and habit formation in consumption  $\eta$  are set to 0.32 and 0.4, respectively, equal to estimates in OW. A subjective discount rate  $\beta$  of 0.99 is consistent with an annual real interest rate in euro area data of almost three percent. I set  $\sigma$  equal to 1.37 which is three times the estimate in OW. Rotemberg and Woodford (1997) argue that the intertemporal elasticity of substitution derived from output rather than consumption data is likely to be higher because investment is more interest sensitive than consumption. The higher value of  $\sigma$  also helps to obtain a weight in the loss function on interest rate variability more in line with values reported in the literature when taking into account the lower bound on nominal interest rates below (for instance, see Amato and Laubach (2004)).

All shock standard deviations are expressed relative to the standard deviation  $\sigma_a$  of the productivity disturbance. The  $g_t$  shock corresponds closest to OW's government spending shock and its standard deviation is set equal to  $\sigma_a$ . The standard deviation  $\sigma_n$  of the labor supply shock, here observationally equivalent to the productivity shock, is roughly seven times  $\sigma_a$ , and the standard deviation of the  $u_t$  disturbance which corresponds to OW's price markup shock is set to  $\frac{1}{2}\sigma_a$ . The standard deviation of the

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<sup>12</sup>To obtain this number I set the duration of price contracts to four quarters. This is at the upper bound of micro evidence in Nakamura and Steinsson (2007) but less extreme than a duration of roughly three and a half years estimated in Onatski and Williams (2004). The steady state markup is 20% and the labor supply elasticity with respect to real wages  $\nu$  is set to 0.2. Assuming a Cobb–Douglas technology with a labor coefficient of  $2/3$  implies  $\omega_w$  equal to 0.3 and  $\omega_p$  equal to 0.5. Here,  $\omega_w$  ( $\omega_p$ ) measures by how much higher output increases wages conditional on prices (prices conditional on wages). Both numbers are comparable to Giannoni and Woodford (2005).

monetary policy shock is taken to be one third of  $\sigma_a$  computed as the impact effect of the composite policy shock on the nominal interest rate in the policy rule estimated in OW. The standard deviation  $\sigma_m$  of the money demand disturbance is set equal to  $\sigma_a$ . All shocks are assumed white noise processes.

This calibration implies ratios  $std(\hat{\pi})/std(\hat{Y})$ ,  $std(\Delta\hat{m})/std(\hat{Y})$  and  $std(\hat{i})/std(\hat{Y})$  equal to 0.26, 0.40 and 0.14, respectively. The corresponding statistics in historical data are 0.45, 0.68 and 0.42.<sup>13</sup> Overall, output is too variable in the model such that model statistic consistently underestimate historical statistics. One reason for this is the higher value of  $\sigma$  assumed here which makes the output gap and thus output more variable. The interest rate statistic remains too low also because I omit the adhoc equity premium shock estimated to have non-trivial size in OW. Finally, the ratio  $std(\hat{\pi}_t^n)/std(\hat{Y}_t^n)$  is 1.31.

## 3.4 Optimal Policy

In this section I approximate household utility when habits persist with respect to money balances. Based on approximate utility I characterize optimal monetary policy for two boundary assumptions about the strength at which the monetary authority can commit itself to future action.

### 3.4.1 Welfare Loss

An Taylor approximation to utility (3.1) is conveniently split into three parts because utility is additively separable in consumption, money balances and labor. Appendix B.2 derives the second order approximation to the discounted sum of  $q(\cdot)$  whereas Woodford (2003), chapter 6, and Giannoni and Woodford (2005) provide details on approximating the remaining two parts. For the approximation I assume existence of a finite satiation level  $\bar{m}^s > 0$  in real money balances such that  $q_m((1 - \phi)\bar{m}^s) = 0$ . Also, the limiting value of  $q_{mm}$  is finite and negative when  $\bar{m}$  approaches  $\bar{m}^s$  from below.<sup>14</sup> Accordingly, at  $\bar{m} = \bar{m}^s$  the steady state interest rate differential  $\bar{\Delta}_i$  is zero

<sup>13</sup>Historical statistics are computed after taking logs and linearly de-trending real output and M1 denominated with the harmonized consumer price index. Inflation is the quarter to quarter change in the consumer price index and the three month nominal interest rate is on a quarterly basis. Data are from HFM for the sample period 1975:01 to 1999:04.

<sup>14</sup>These assumptions make  $q_{mm}$  discontinuous in the satiation point. Discontinuity of  $q_{mm}$  follows since  $q_m = 0$  for all values of real balances greater or equal to the satiation level,  $\bar{m} \geq \bar{m}^s$ . See Woodford (2003), chapter 6, assumption 6.1, for a similar assumption in the case of non-separable preferences and

and  $\bar{\Delta}_i$  can be treated as expansion parameter as long as the economy is sufficiently close to money satiation. Expected discounted household utility is proportional to

$$W = E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t L_t$$

and the period loss function is

$$L_t = \lambda_x(x_t - \delta x_{t-1} - \hat{x}^*)^2 + (\hat{\pi}_t - \kappa \hat{\pi}_{t-1})^2 + \lambda_m(\hat{m}_t - \phi \hat{m}_{t-1} - \epsilon_t^m - \hat{m}^*)^2 \quad (3.4)$$

omitting a residual that comprises terms independent of policy and terms of order higher than three. Beyond stabilizing the quasi differences of output gap and inflation the loss function puts weight on stabilizing the quasi difference of real money adjusted for money demand disturbances.

The target value at which stabilization should be achieved equals  $\hat{m}^* = (1 - \phi)\eta_i\bar{\Delta}_i$ . This value represents the inefficiency associated with the positive but small price for liquidity  $\bar{\Delta}_i$ . Absent shocks, extending average money balances above steady state by an amount  $\eta_i\bar{\Delta}_i > 0$  eliminates this inefficiency. The loss function also illustrates that it is crucial for the monetary authority to identify the correct money index  $(1 - \phi)L\hat{m}_t - \epsilon_t^m$  since money demand disturbances generate loss if not offset by the quasi difference of money.<sup>15</sup>

The weight attached to stabilizing the money index is

$$\lambda_m = \frac{\tilde{\lambda}_m}{(1 - \phi\beta)(1 - \phi)}.$$

Weights are defined as  $\tilde{\lambda}_m = \frac{\Xi_p}{\theta\bar{v}\eta_i}$  and  $\lambda_x = \delta_0 \frac{\Xi_p}{\theta}$  and are both independent of the degree of habit in the money stock. Additional parameters are explicit in appendix B.2.

Figure 3.1 plots weights  $\lambda_x$  and  $\lambda_m$  for different values of  $\phi$ . Whereas  $\lambda_x$  remains constant  $\lambda_m$  increases non-linearly in  $\phi$ . More weight is put on stabilizing the money index rather than the output gap if  $\phi$  is 0.7 or above. Calibrating  $\phi$  to 0.8 implies that stabilizing the money index is about three times as important as stabilizing the output gap. The monetary authority accepts more variation in the quasi difference of inflation and the output gap for keeping the money index stable when  $\phi$  is high because high  $\phi$  increases the preference of the household for a smooth path of real money balances. Notably,  $\lambda_m$  is about one tenth of  $\lambda_x$  if  $\phi$  equals zero.

for further discussion.

<sup>15</sup>For static money demand, the loss is often expressed in terms of the nominal interest rate rather than money balances. Here, such a substitution produces covariance terms in the loss function because money demand is dynamic.

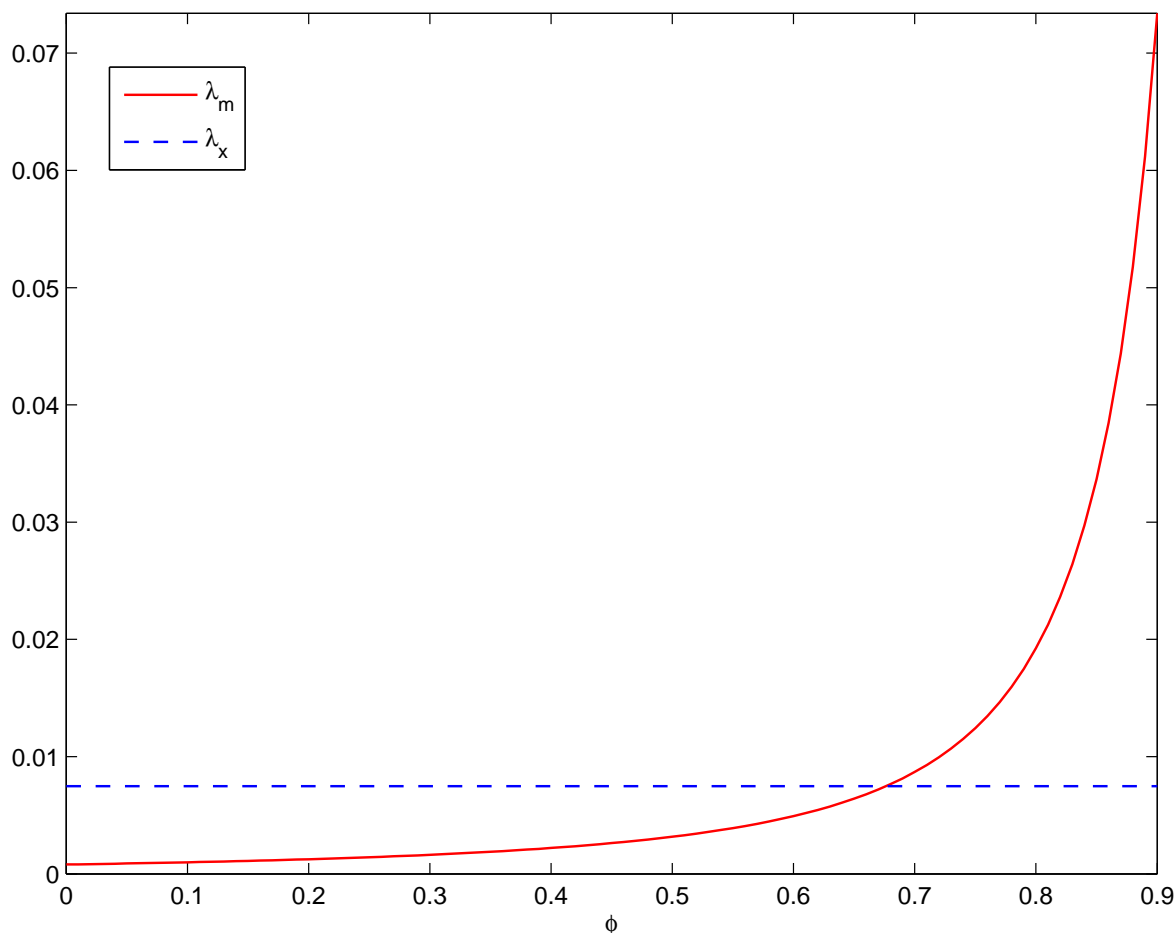


Figure 3.1: Weights  $\lambda_m$  and  $\lambda_x$  for different degrees of habit in the money stock.

### 3.4.2 Lower Bound

Extrapolating the case of static money demand, one may expect that imposing the lower bound  $\Delta_{it} \geq 0$  has effects similar to stabilizing money balances directly. If inflation is low concerns about the lower bound restrict variation in the interest rate differential. A differential which varies less thus would imply more stable money balances. To allow for this possibility I account for the lower bound along the lines of Amato and Laubach (2004), Woodford (2003) and Rotemberg and Woodford (1997).

In the linear quadratic framework here the basic idea is to replace the nonlinear constraint  $\Delta_{it} \geq 0$  by the requirement that the average interest rate differential remains at least  $k > 0$  standard deviations above the lower bound. If  $k$  is large enough violations of the lower bound occur infrequently. Adapting such a constraint to  $\hat{\Delta}_{it} + \bar{\Delta}_i$

equal to  $\Delta_{it}$  up to first order delivers

$$k \left\{ \mathbf{E} \left[ (\hat{\Delta}_{it} + \bar{\Delta}_i - \mathbf{E}[\hat{\Delta}_{it} + \bar{\Delta}_i])^2 \right] \right\}^{\frac{1}{2}} \leq \mathbf{E}[\hat{\Delta}_{it} + \bar{\Delta}_i] \quad (3.5)$$

referring to conditional discounted means to comply with discounting in the loss function,

$$\mathbf{E}[z_t] = E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t z_t .$$

Woodford (2003) shows that imposing a constraint such as (3.5) separately is equivalent to augmenting a loss function such as (3.4) with a term that penalizes squared deviations of the interest rate differential from a target value if the constraint is binding.<sup>16</sup> In this case the loss function can be expressed as

$$\mathcal{W} = E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t \quad , \quad \mathcal{L}_t = L_t + \lambda_i (\hat{\Delta}_{it} - \Delta_i^*)^2 \quad , \quad \lambda_i \geq 0 . \quad (3.6)$$

If the average interest rate differential equals the target value  $\Delta_i^* > \bar{\Delta}_i$  the new term is zero absent shocks. Long run monetary policy thus faces a tradeoff between addressing inefficiently low money balances and compliance with the lower bound. Whereas the first goal is consistent with an average interest rate differential that is below the one resulting in inefficient steady state the latter goal would require to push the average interest rate differential above its steady state value which then also goes hand in hand with higher average inflation.

Here I concentrate on losses due to incomplete stabilization of target variables. Applying the formula for the conditional variance  $\mathcal{W}$  is split additively into a component which depends only on deterministic aspects of the equilibrium paths of target variables and a component which depends exclusively on the equilibrium responses of target variables to shocks. The stabilization component is

$$\mathcal{W}^{stab} = \lambda_x \mathbf{V}[\Delta_\delta x_t] + \mathbf{V}[\Delta_\kappa \hat{\pi}_t] + \lambda_m \mathbf{V}[\Delta_\phi \hat{m}_t - \epsilon_t^m] + \lambda_i \mathbf{V}[\hat{\Delta}_{it}] \quad (3.7)$$

<sup>16</sup>A necessary condition to apply proposition 6.9 in Woodford (2003), chapter 6, section 4.2, is that the average interest rate differential is positive if the constraint is not binding. It can be shown that with  $\bar{r} > \bar{i}^m$  the optimal nominal average interest rate is  $i^{opt} = (1 - f)\bar{r} + f\bar{i}^m > \bar{i}^m$  which implies a positive average interest rate differential since  $0 < f < 1$  for the calibration in this paper and for a wide range of other calibrations. With  $\bar{r} > i^{opt}$  some deflation is optimal because  $\hat{m}^* > 0$  indicates a policy preference for increasing real money balances above inefficiently low steady state real money balances. One way to increase average money balances is to decrease the average nominal interest rate and, thereby, the price for liquidity. Decreasing  $i^{opt}$  below steady state also involves deflation which in turn is costly. The optimal resolution of this tradeoff is some deflation but maintains a positive price of liquidity on average.

using notation  $\mathbf{V}[z_t] = \mathbf{E}[z_t^2]$  and denoting the quasi difference with respect to  $\delta$  as  $\Delta_\delta$ . Thus, minimizing  $\mathcal{W}^{stab}$  is equivalent to minimizing  $\mathcal{W}$  with all target values  $\hat{x}^*, \hat{m}^*, \Delta_i^*$  set to zero.<sup>17</sup> In what follows  $k$  is set to 2.17 which is the mean relative to the standard deviation of the three month nominal interest rate from FHM. The standard deviation  $\sigma_a$  of the productivity shock is chosen in such a way that the lower bound is just not binding under the monetary policy rule  $\hat{i}_t = 0.95\hat{i}_{t-1} + 2\hat{\pi}_t + 0.3\Delta x_t + \zeta_{it}$  denoting with  $\zeta_{it}$  the monetary policy shock. This rule corresponds to the rule estimated in OW with response coefficients being rounded.

### 3.4.3 Discretion

I now solve the optimal policy problem assuming that the policy maker cannot commit to future action. Discretionary policy minimizes the loss function (3.6) subject to equilibrium conditions (B.1) taking private sector expectations as given. Appendix B.3 provides the lagrangian and optimality conditions.

#### A Special Case

Before turning to the general case, I consider a special case without habit persistence in consumption and absent price indexation ( $\eta = \kappa = 0$ ). Combining optimality conditions delivers

$$\lambda_i(\hat{\Delta}_{it} - \Delta_i^*) = \lambda_x \sigma(x_t - \hat{x}^*) + \mu \sigma \hat{\pi}_t + \lambda_m(\theta_i + \sigma \theta_y)((1 - \phi L)\hat{m}_t - \epsilon_t^m - \hat{m}^*) \quad (3.8)$$

using  $\hat{\Delta}_{it} = \hat{i}_t$ . The target criterion relates all four target variables to each other. The effective weight of a variable in the criterion is the variable's weight in the loss function corrected for its interest rate (differential) semielasticity. Therefore, even a large weight in the loss function is consistent with a small effective weight in the target criterion if the ability of the monetary authority to influence a target variable is small as measured by the interest rate semielasticity. A small effective weight is optimal because stabilizing a target variable with small semielasticity is "expensive" in terms of variation in other target variables.

<sup>17</sup>It is feasible to rewrite the stabilization component in terms of  $\mathbf{V}[\cdot]$  if the conditional expectation of target variables as of period zero is zero. Here this is true because attention is restricted to linear policies in a linear model assuming across policies that the initial state vector equals zero and that shocks have zero mean. In this case effects of positive and negative shocks average out in equilibrium and conditional expectations of target variables remain zero throughout. Moreover, if the lower bound is binding constraint (3.5) holds with equality and rewrites as  $k\mathbf{V}[\hat{\Delta}_{it}]^{1/2} - \bar{\Delta}_i = 0$  because conditional expectations are zero. With the lower bound binding I chose  $\lambda_i > 0$  numerically so that this condition holds true. Otherwise,  $\lambda_i = 0$ .

Interest rates affect the money index  $(1 - \phi L)\hat{m}_t - \epsilon_t^m$  via two channels. One channel enters directly through the opportunity cost motive in money demand and is represented by  $\theta_i$ . The second channel stems from interest rates affecting the output gap which in turn affects the money index via the transaction motive in money demand. This channel is represented by  $\sigma\theta_y$ . The effective weight of the money index rewrites as

$$\lambda_m(\theta_i + \sigma\theta_y) = \tilde{\lambda}_m(\eta_i + \sigma\eta_y) .$$

The crucial observation is that  $\tilde{\lambda}_m, \eta_i$  and  $\eta_y$  are all independent of  $\phi$ . Therefore, different degrees of habit in the money stock do not alter the effective weight attached to the money index (though habit does modify the definition of this index).

The independence occurs because habit in the money stock creates two counteracting effects which exactly offset each other. First, a large value of  $\phi$  implies a large weight  $\lambda_m$  because such a value of  $\phi$  makes the household averse against changes in real money balances relative to the habit stock. The household then accepts more variation in inflation and the output gap for keeping real money balances close to their habit level. Second, a large value of  $\phi$  implies a small overall interest rate semielasticity of money demand  $\theta_i + \sigma\theta_y$  because the marginal rate of substitution of money for consumption is small in this case. Reducing current period money balances increases current period marginal utility of money balances. But this reduction also decreases the habit level relevant to the next period thereby increasing marginal utility of money balances in that period. For internal habit this increase in next periods' marginal utility partly offsets the increase in current period marginal utility. As a result money demand becomes increasingly forward looking and less driven by transaction and opportunity cost motives as reflected by low short run elasticities.

### The General Case

Figure 3.2 shows impulse response functions after a productivity shock for the calibration in table 5.3. I first ignore the lower bound by setting  $\lambda_i = 0$ . When money demand is static (round markers) the nominal interest rate decreases to increase consumption, here identical to output, which in turn stabilizes the output gap. Also, high productivity decreases prices and pushes inflation below steady state.

For  $\lambda_m$  equal to zero a nominal interest rate equal to the natural real rate would achieve full stabilization of inflation and the output gap. Here  $\lambda_m > 0$  creates a trade-off between stabilizing inflation and the output gap versus stabilizing the money index because low opportunity costs and high output both increase money demand. Optimal discretionary policy thus does not lower the nominal rate all the way down to the

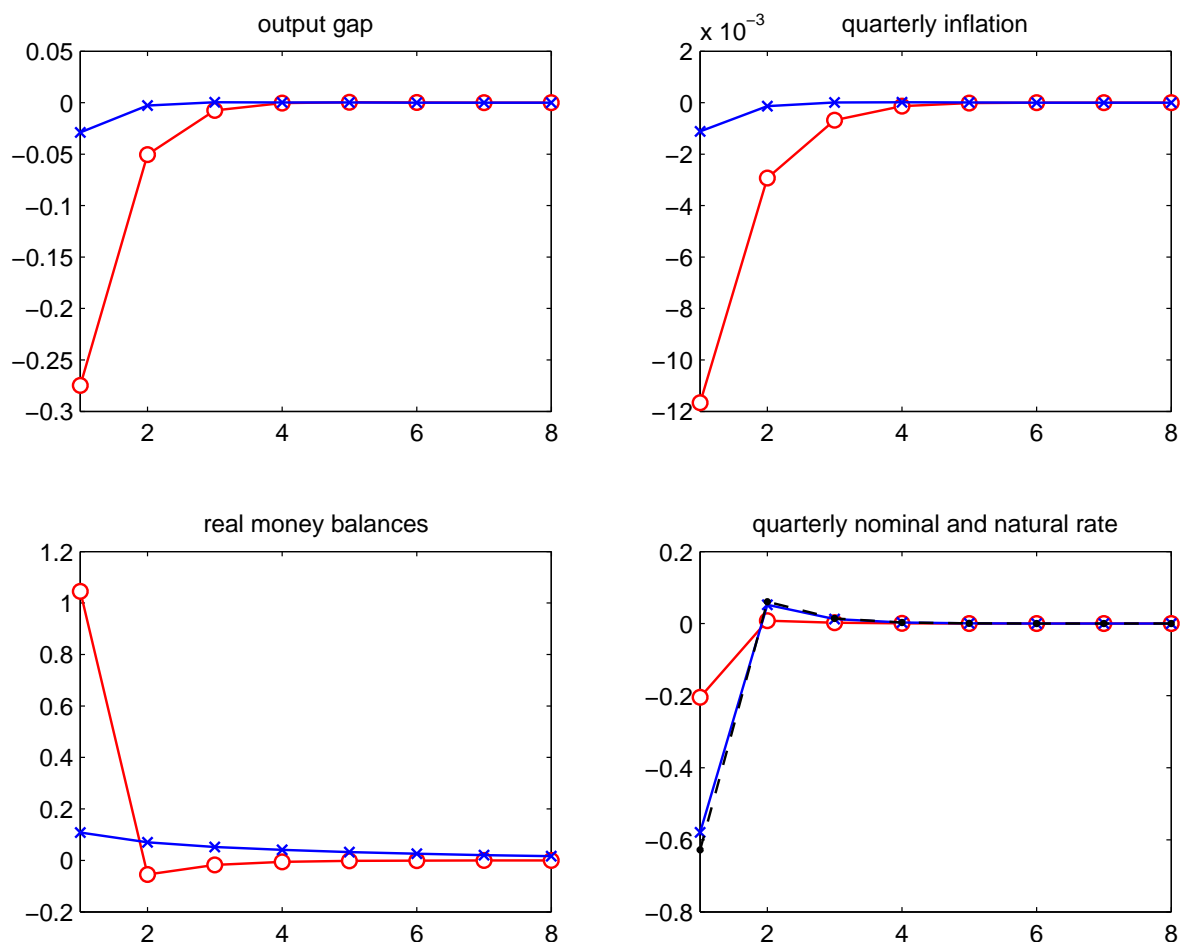


Figure 3.2: Impulse response function to a positive productivity shock  $a_t$  one standard deviation in size. Variables are in percentage deviation from steady state. Monetary policy acts under optimal discretion assuming that the lower bound does not bind,  $\lambda_i = 0$ . Round markers indicate responses under static money demand,  $\phi = 0$ . Crosses indicate responses under dynamic money demand,  $\phi = 0.8$ . Dots indicate the natural real rate.

natural rate. When money demand is dynamic (crosses) this tradeoff reduces visibly in that the nominal interest rate follows the natural real rate much more closely. The tradeoff becomes less severe because short run elasticities of money demand  $\theta_i$  and  $\theta_y$  are small if  $\phi$  is large. As a result, money demand is less affected by movements in nominal rates and output which leaves considerably more leeway for stabilizing inflation and the output gap. Thus, the striking observation is that all target variables are stabilized more successfully once money demand is dynamic.

How does imposing the lower bound modify this pattern? Figure 3.3 shows the



weight  $\lambda_i$  consistent with the lower bound for different values of  $\phi$  under optimal discretion. The lower bound becomes more of a concern for discretionary policy the more

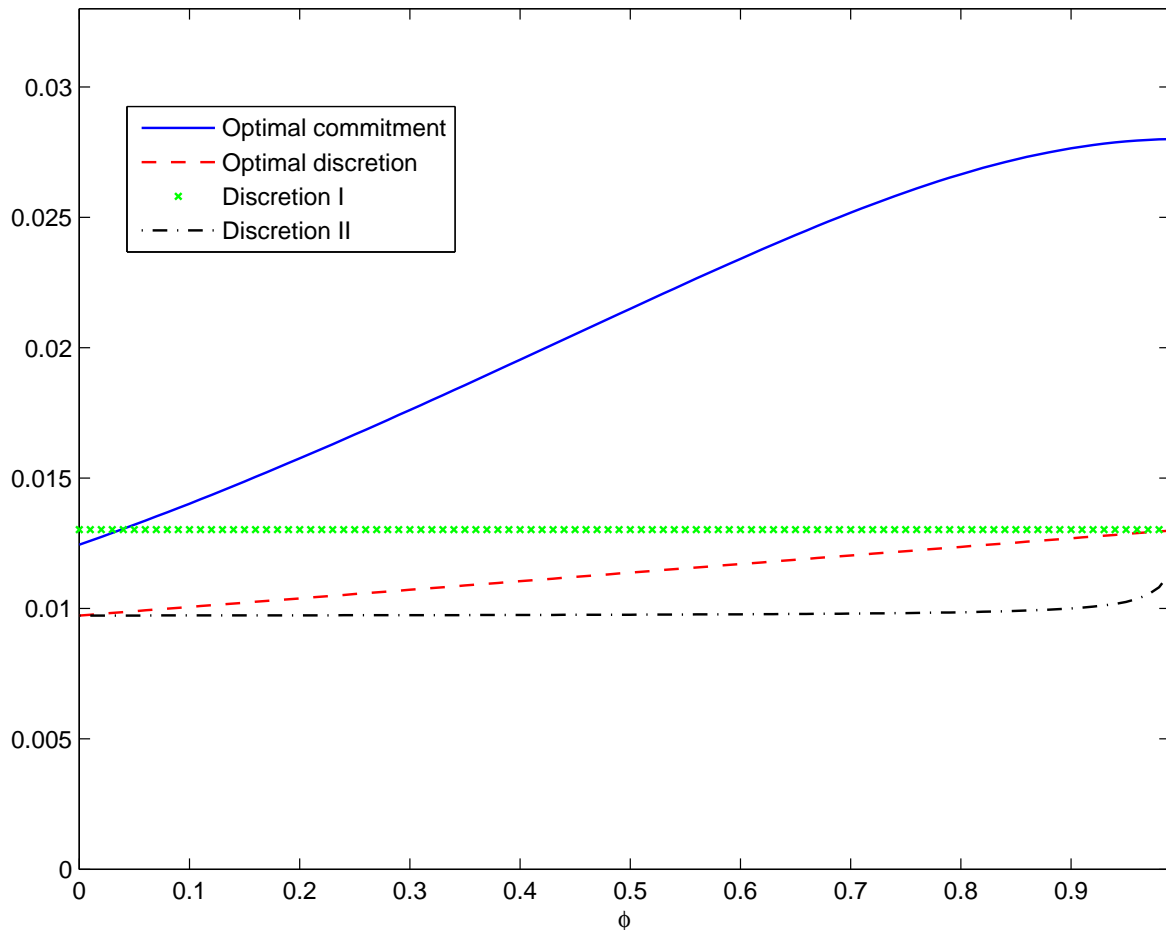


Figure 3.3: Weight  $\lambda_i$  for different degrees of habit in the money stock and across different policy regimes.

dynamic money demand is. Abstracting from disturbances other than to productivity concerns about the lower bound connect to the reduction of short run money demand elasticities. Once money demand elasticities are small optimal adjustment tends towards equating the nominal rate to the natural real rate because stabilizing inflation and the output gap creates less variation in the money index. The lower bound then matters because the natural rate is too volatile for the nominal rate to follow it.

Figure 3.4 shows adjustment to a productivity shock now imposing the lower bound. Stabilization of inflation and the output gap is less successful once accounting for the lower bound but marginal improvements in stabilizing inflation and the out-

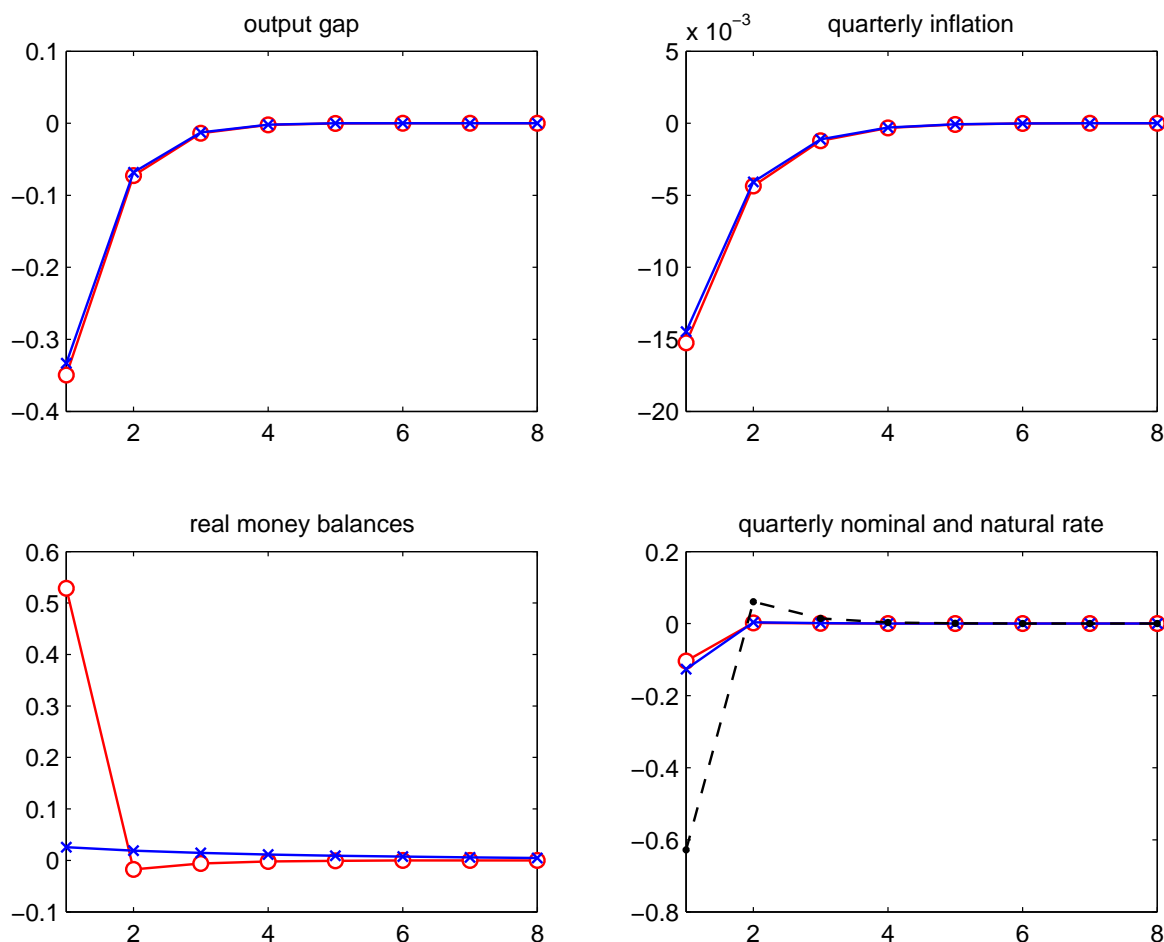


Figure 3.4: Impulse response function to a positive productivity shock  $a_t$  one standard deviation in size. Variables are in percentage deviation from steady state. Monetary policy acts under optimal discretion accounting for the lower bound,  $\lambda_t > 0$ . Round markers indicate responses under static money demand,  $\phi = 0$ . Crosses indicate responses under dynamic money demand,  $\phi = 0.8$ . Dots indicate the natural real rate.

put gap can still be achieved for dynamic money demand. Real money balances vary much less as a result of low money demand elasticities.<sup>18</sup>

Figure 3.5 shows adjustment to a money demand shock. Complete stabilization is achieved for both static and dynamic money demand. With static money demand the

<sup>18</sup>Similarly to the productivity shock the nominal rate responds more pronounced to the  $u_t$  disturbance once money demand is dynamic. Again, the reason is that decreased short run elasticities isolate money demand from movements in interest rates and output which also makes the lower bound matter more. Adjustment to the  $g_t$  disturbance is qualitatively identical to the one that follows a negative productivity shock.

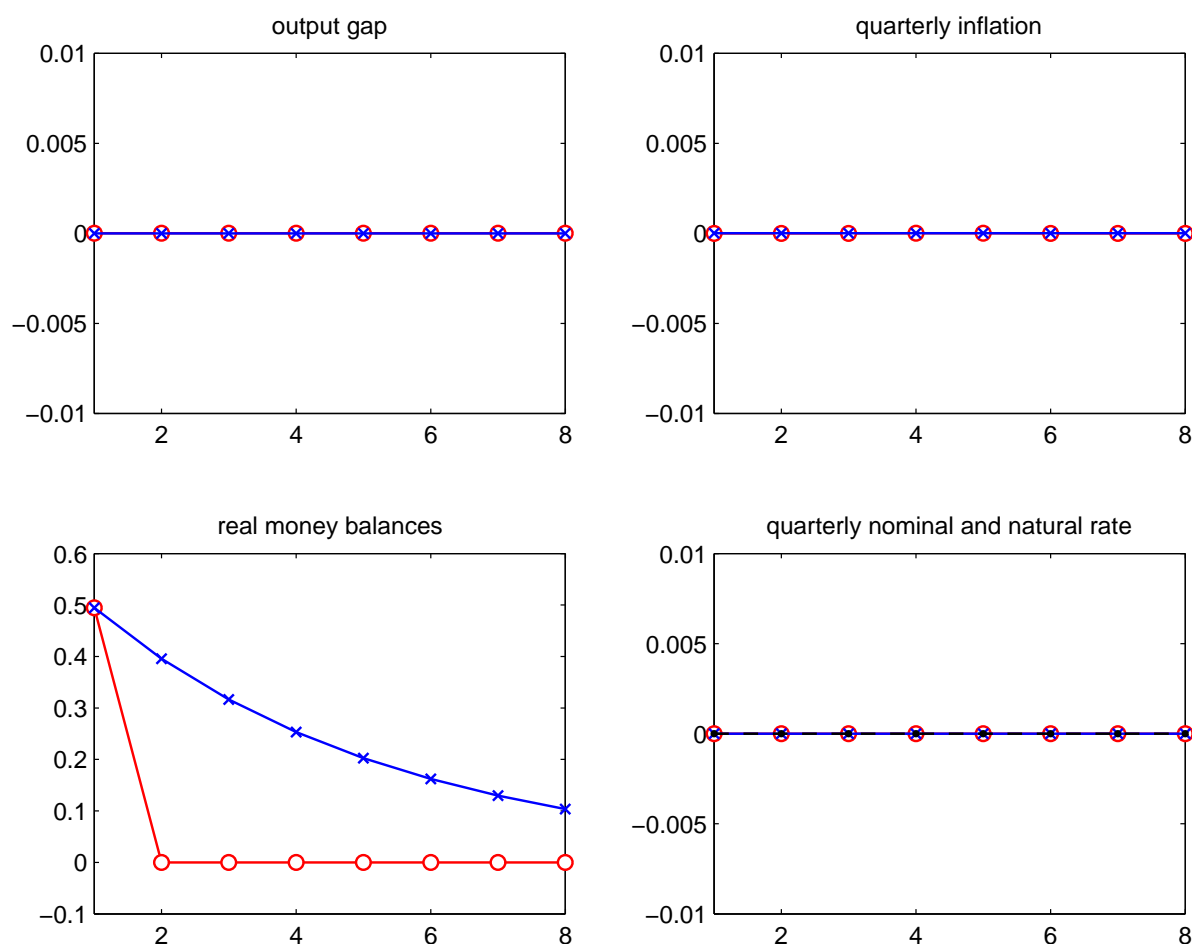


Figure 3.5: Impulse response function to a positive money demand shock  $\epsilon_t^m$  one standard deviation in size. Variables are in percentage deviation from steady state. Monetary policy acts under optimal discretion accounting for the lower bound,  $\lambda_i > 0$ . Round markers indicate responses under static money demand,  $\phi = 0$ . Crosses indicate responses under dynamic money demand,  $\phi = 0.8$ . Dots indicate the natural real rate.

monetary authority lets money demand fully accommodate the money demand shock in the same period. With dynamic money demand the shock leads to a long lasting deviation of real money balances from steady state. This deviation is consistent with complete stabilization because the monetary authority aims to stabilize money holdings relative to a fraction  $\phi$  of past holdings and the decay rate of the impulse response function is indeed equal to  $\phi$ . Money demand disturbances decouple from concerns about the lower bound because the natural rate is not affected by such disturbances.

### 3.4.4 Commitment

In this section I consider a monetary authority which credibly commits to future action and thus is in the position to manipulate private sector expectations. Appendix B.3 provides optimality conditions that characterize optimal policy in this case.

#### A Special Case

Again, consider a special case of no habit in consumption and absence of price indexation ( $\eta = \kappa = 0$ ). Optimality conditions combine to

$$\begin{aligned} & -\lambda_i(1 - \beta^{-1}L)(\hat{\Delta}_{it} - \hat{\Delta}_{it-1}) + \mu\sigma\hat{\pi}_t + \sigma\lambda_x(x_t - x_{t-1}) \\ & + \tilde{\lambda}_m(\eta_i + \sigma\eta_y - \eta_i\beta^{-1}L)(1 - L)(\hat{m}_t - \frac{\epsilon_t^m}{(1-\phi L)}) \\ & = -\mu\sigma\lambda_i\beta^{-1}(\hat{\Delta}_{it-1} - \Delta_i^*) + \mu\sigma\eta_i\tilde{\lambda}_m\beta^{-1}(\hat{m}_{t-1} - \frac{\epsilon_{t-1}^m}{(1-\phi L)} - \frac{\hat{m}^*}{1-\phi}). \end{aligned} \quad (3.9)$$

A difference of this target criterion relative to (3.8) is that the money index  $\hat{m}_t - (1 - \phi L)^{-1}\epsilon_t^m$  no longer involves the quasi difference of real money balances. Rather, the level of real money balances is adjusted for a sequence of current and past money demand disturbances with decay rate  $\phi$ .<sup>19</sup>

The criterion also differs in its reference to past states. This history dependence is intrinsic to optimally committed policy in forward looking models (Woodford (1999)). For the same reason as under discretion, however, effective weights attached to target variables or transformations thereof are independent of the degree of habit formation in the money stock: A strong preference to stabilize real money balances relative to the habit level is accompanied by low short run elasticities. Thus, the sole impact of  $\phi$  in (3.9) is to determine the appropriate weighting scheme for past money demand disturbances.

<sup>19</sup>To illustrate this difference contrast optimality conditions with respect to the quasi difference of real money under commitment and discretion, respectively,  $\lambda_m((1 - \phi L)\hat{m}_t - \epsilon_t^m - \hat{m}^*) = -(1 - \phi L)\psi_{mt}$  and  $\lambda_m((1 - \phi L)\hat{m}_t - \epsilon_t^m - \hat{m}^*) = -\psi_{mt}$  with  $\psi_{mt}$  denoting the lagrange multiplier on money demand. Under commitment history dependence implies that the marginal loss from increasing  $(1 - \phi L)\hat{m}_t$  is proportional to the shadow value of loosening the constraint. Thus, it is sufficient for optimality to ensure proportionality between  $\hat{m}_t$  and  $\psi_{mt}$  accounting for an infinite sequence of money demand disturbances. This outcome parallels findings for habit in consumption in a model without interest rate stabilization objective (Woodford (2003), chapter 8, section 2.3, and Giannoni and Woodford (2005)). Under discretion the lack of history dependence prevents proportionality between  $\hat{m}_t$  and  $\psi_{mt}$  from being optimal.

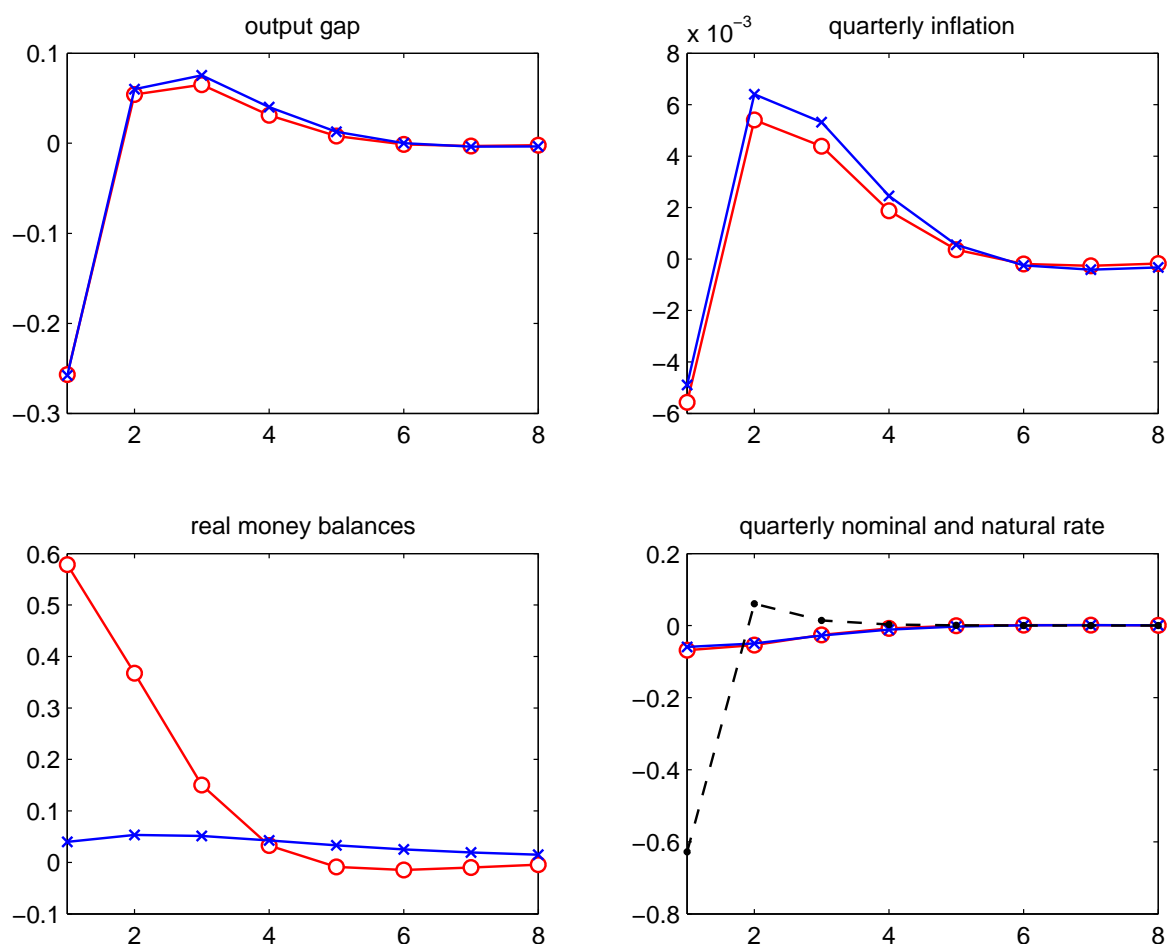


Figure 3.6: Impulse response function to a positive productivity shock  $a_t$  one standard deviation in size. Variables are in percentage deviation from steady state. Monetary policy acts under optimal commitment accounting for the lower bound,  $\lambda_i > 0$ . Round markers indicate responses under static money demand,  $\phi = 0$ . Crosses indicate responses under dynamic money demand,  $\phi = 0.8$ . Dots indicate the natural real rate.

### The General Case

Figure 3.6 shows adjustment to a productivity shock for committed monetary policy when imposing the lower bound. Owing to history dependent policy the nominal rate remains several periods below steady state to compensate for low inflation and low output gap. Contrary to discretion, stabilization of inflation and the output gap now is less complete when money demand is dynamic rather than static.<sup>20</sup> Less complete

<sup>20</sup>Under discretion  $\Delta_{\kappa}\hat{\pi}_t$  ( $\Delta_{\delta}x_t$ ) varies roughly 10% (9%) less under dynamic relative to static money demand after a productivity shock. Under commitment the same variable varies roughly 10% (4%)

stabilization derives from the lower bound being more of a concern for large values of  $\phi$  in line with the more pronounced increase of  $\lambda_i$  in figure 3.3.

Large values of  $\phi$  make money demand respond more to persistent deviations of interest rates (and output) from steady state because money demand is more forward looking in this case. This reinforces the tradeoff between stabilizing the quasi difference of inflation and the output gap versus stabilizing the money index. In fact, history dependence offsets some of the benefits that derive from low short run money demand elasticities under forward looking money demand.<sup>21</sup>

As for discretionary policy, shocks to money demand are completely stabilized. Indeed, adjustment paths are identical to the case of discretion in figure 3.5. A core assumption for complete stabilization to emerge is that solutions for inflation and the output gap can be obtained without any reference to  $\hat{m}_t$ . Additionally, it is crucial that the monetary authority identifies the correct money index in real time since otherwise money demand shocks spill over to the rest of the economy. Preventing such spill overs requires knowledge of the structural parameter  $\phi$  on the one hand and identification of money demand disturbance on the other. As both conditions seem rather strong future work should investigate losses associated with targeting transformations of money that do not correspond exactly to the money index identified here.

### 3.4.5 Discretion versus Commitment

Table 3.1 summarizes how components of the loss function change with dynamic rather than static money demand accounting for all shocks. Under both regimes, discretion and commitment, total losses fall when money demand turns dynamic. The reduction amounts to 1.36% under commitment and 0.97% for discretionary policy. Under discretion a substantial reduction in the variability of the money index is due to money demand being less driven by movements in opportunity costs and transaction volume. This implies that at the cost of more variability in  $\hat{\Delta}_{it}$  the quasi differences of output gap and inflation can be stabilized more successfully.

Under commitment again low short run money demand elasticities imply a substantial reduction of variability in the money index.<sup>22</sup> However, more variability is

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more measuring variation as the sum of the squared impulse response function.

<sup>21</sup>When it comes to the  $u_t$  disturbance dynamic money demand implies less stabilization of interest rates and  $\Delta_\delta x_t$  but slightly more stabilization of  $\Delta_\kappa \hat{\pi}_t$  relative to the case of static money demand.

<sup>22</sup>Under discretion the values of the statistic  $\mathbf{V}[\hat{m}_t]$  are 21.596 and 6.726 for low and high  $\phi$ , respectively. Under commitment corresponding statistics are 49.066 and 9.436 for low and high  $\phi$ , respectively. Thus, a substantial fraction of the reduction in losses also has to be devoted to changes in the definition

accepted for  $\Delta_\delta x_t$  reflecting that forward looking money demand reinforces tradeoffs among target variables. This offsets some of the benefits derived from the reduction in short run money demand elasticities.

Table 3.1: Discounted standard deviations and losses

Regime	$\phi$	$\lambda_i$	$\mathbf{V}[\Delta_\delta x_t]$	$\mathbf{V}[\Delta_\kappa \hat{\pi}_t]$	$\mathbf{V}[\Delta_\phi \hat{m}_t - \epsilon_t^m]$	$\mathbf{V}[\hat{\Delta}_{it}]$	$\mathcal{W}$
Discretion	0	.0097	2.361	0.584	19.175	0.912	0.615
Discretion	0.8	.0124	2.222	0.577	0.046	1.266	0.609
Commitment	0	.0123	9.263	0.348	46.646	0.340	0.434
Commitment	0.8	.0266	9.988	0.342	0.360	0.340	0.428

*Notes:* All statistics are based on the calibration in table 5.3. Statistics  $\mathbf{V}[\cdot]$  and  $\mathcal{W}$  are multiplied by  $10^5$ .

### 3.5 Revisiting the Relationship between Money Growth and Output Gap

It has been argued that money provides a useful index of aggregate demand because money and aggregate demand both co-vary with similar interest rates.<sup>23</sup> Indeed, Nelson (2002) shows that once money demand is augmented by a measure similar to the long term interest rate, which also shifts aggregate demand most effectively, money co-varies stronger with deviations of output from trend in an otherwise standard model. Similar to the specification of money demand in Nelson (2002), the money demand function here also involves a measure of a long term interest rate and thus allows to revisit the relationship between money growth and the output gap.

I demonstrate that the unconditional correlation between money growth and the output gap may indeed increase for intermediate values of  $\phi$  relative to the case of static money demand. However, I also show that this correlation fades out quickly for large values of  $\phi$  and entirely disappears when  $\phi$  approaches unity unless money receives extraordinarily large weight in monetary policy. For large values of  $\phi$  short run elasticities of money demand are negligible so that the structural reason for money growth to co-move with the output gap disappears.

Without habit persistence in the money stock money demand (3.3) makes real money net of money demand shocks exclusively driven by transaction and opportu-

of the money index.

<sup>23</sup>See Nelson (2003) and references therein, in particular footnote 6.

nity cost motive,  $\hat{m}_t - \epsilon_t^m = \eta_y \tilde{Y}_t - \eta_i \hat{\Delta}_{it}$ . With habit persistence in money, (3.3) can be iterated forward to obtain

$$\hat{m}_t - \phi \hat{m}_{t-1} - \epsilon_t^m = (1 - \phi\beta)(1 - \phi)E_t \sum_{s=0}^{\infty} (\phi\beta)^s [\eta_y \tilde{Y}_{t+s} - \eta_i \hat{\Delta}_{it+s}], \quad (3.10)$$

assuming  $\lim_{T \rightarrow \infty} E_t[(1 - \phi L)\hat{m}_T - \epsilon_T^m] = 0$ ,  $T > t$ . The money index  $(1 - \phi L)\hat{m}_t - \epsilon_t^m$  depends on a measure of discounted output and a discounted version of the long term interest rate differential.

Increasing  $\phi$  has two effects on the relationship between the money index and the measure of the long term differential. First, higher  $\phi$  puts more weight to short run interest rates farther in the future. One would expect this effect to emphasize co-variation between money growth and the output gap because the long term interest rate relevant for the output gap is not subject to discounting.<sup>24</sup> Second, increasing  $\phi$  decreases short run elasticities of money demand which makes the quasi difference of money co-move more with the exogenous component of money demand. Thus, increasing  $\phi$  improves the quality of the "signal" contained in the quasi difference of money due to the first effect but diminishes the strength of the "signal" due to the second effect. When  $\phi$  approaches unity the second effect dominates and money entirely decouples from any other endogenous variable.

Figure 3.7 provides correlations between money growth  $\Delta \hat{m}_t$  and the output gap  $\tilde{x}_t$  as a function of  $\phi$  and for several policy regimes. Regimes include optimal commitment, optimal discretion, discretion I, which ignores any reference to money setting  $\lambda_m = 0$ , and discretion II, which increases the weight for money with  $\phi$ ,  $\hat{\lambda}_m = \frac{\tilde{\lambda}_m}{(1-\phi)(1-\phi\beta)}$ . Whereas the right panel abstracts from the lower bound the left panel adds this constraint (figure 3.3 comprises weights  $\lambda_i$  for different values of  $\phi$ ).

The predominant observation is that correlations between  $\Delta \hat{m}_t$  and  $\tilde{x}_t$  converge to zero as  $\phi$  approaches unity. This is true unless policy creates a substantive link between the two variables, and discretion II has been constructed to illustrate such a case. The reference of policy to money is small for optimal commitment and discretion which makes these two cases essentially indistinguishable from discretion I for large values of  $\phi$ . The correlation between  $\Delta \hat{m}_t$  and  $\tilde{x}_t$  follows an inverted U-shape for optimal commitment when the lower bound constraint is in place. This pattern applies

<sup>24</sup>Iterating the Euler equation in (B.1) forward one obtains  $\tilde{x}_t = -\varphi^{-1}E_t \sum_{s=0}^{\infty} [\hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s}^n]$ . In addition to the difference with respect to discounting the two interest rate differentials also differ in their composition. Whereas money responds to the nominal interest rate on bonds adjusted for the nominal rate of return on money, the output gap responds to the expected real interest rate adjusted for the natural real rate. This is one reason why the difference between both measures does not only depend on  $\phi$  but also on details of the policy regime.



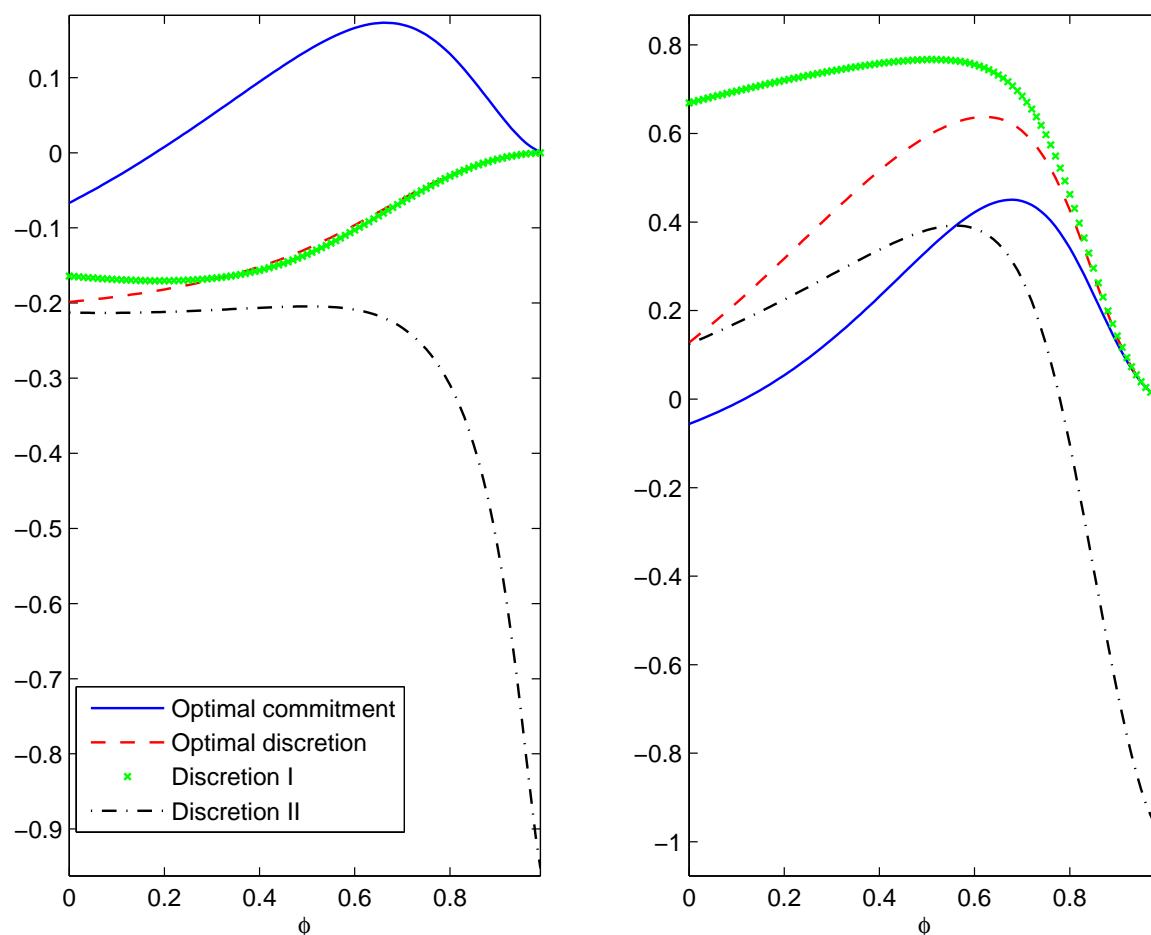


Figure 3.7: Contemporaneous correlations between a measure of the output gap  $\tilde{x}_t$  and money growth  $\Delta \hat{m}_t$  as a function of  $\phi$  for various policy regimes. In the left panel,  $\lambda_i > 0$  for each policy regime as indicated in figure 3.3. In the right panel,  $\lambda_i = 0$  throughout.

to all policy regimes once the lower bound is ignored in the right panel. Thus, the indicator properties of money growth crucially depend on the intrinsic persistence in money demand in the model presented here. Moreover, a substantial correlation between money growth and the output gap is likely injected by monetary policy itself if money demand heavily depends on its own lags.

It is evident from the figure that correlations can differ considerably across policy regimes. This cautions against statements about the indicator properties of money growth derived from historic correlations since such correlations may be subject to change as a consequence of operating monetary policy differently.<sup>25</sup> Woodford (1994)

<sup>25</sup>This is a variant of Lucas (1976) critique of econometric policy evaluation.

illustrates this and related fallacies that may arise if the only basis for assessing a variable's indicator property is measurement of the extent to which this variable associates with some other variable in historic data. Avoiding such pitfalls, he argues, requires analyzing candidate policy rules in the context of a structural model.

### 3.6 Conclusion

Estimated money demand is often intrinsically persistent beyond accounting for transaction volume and opportunity costs. In this paper I reconcile the typical New Keynesian model with this observation by means of habit formation in the money stock. It turns out that the weight for stabilizing a particular money index in the welfare based loss function increases in the degree of habit in money and is roughly three times the one attached to stabilizing the output gap for a reasonable calibration. However, the substantial weight in the loss function does not justify an equally substantial response to the money index under optimal policy because it goes hand in hand with low short run money demand elasticities. Stabilizing the money index then comes at large costs in terms of variation in other target variables. Nevertheless, optimal monetary policy improves its record relative to the case of static money demand because low short run elasticities reduce tradeoffs. A core assumption in the analysis is that monetary policy identifies the correct money index in real time. This assumption seems rather strong and future work should investigate losses associated with targeting transformations of money that do not correspond exactly to the money index identified here.

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# Chapter 4

## Productivity Catch Up in New EU Member States

New European Union member states experience real exchange rate appreciation and at the same time terms of trade that improve vis á vis the euro area. Whereas real appreciation is consistent with relatively fast growing production efficiency in the traded good sector improving terms of trade are not in the standard two country two sector real business cycle model. I augment the standard model by including endogenous product entry in the traded good sector and show that the augmented model is consistent with both observations simultaneously if productivity catch up reduces barriers to market entry of new firms. To obtain this result one has to account for the fact that observed import and export price indices have difficulties to track new products in time. Predictions of the standard model with respect to the real exchange rate and the terms of trade survive in the augmented model when productivity catch up concerns production efficiency.

### 4.1 Introduction

Over the past ten years or so appreciating real exchange rates and improving terms of trade coexist in several new European Union (EU) member states in Eastern Europe. The top panel of figure 4.1 shows real exchange rates defined as the price of a basket

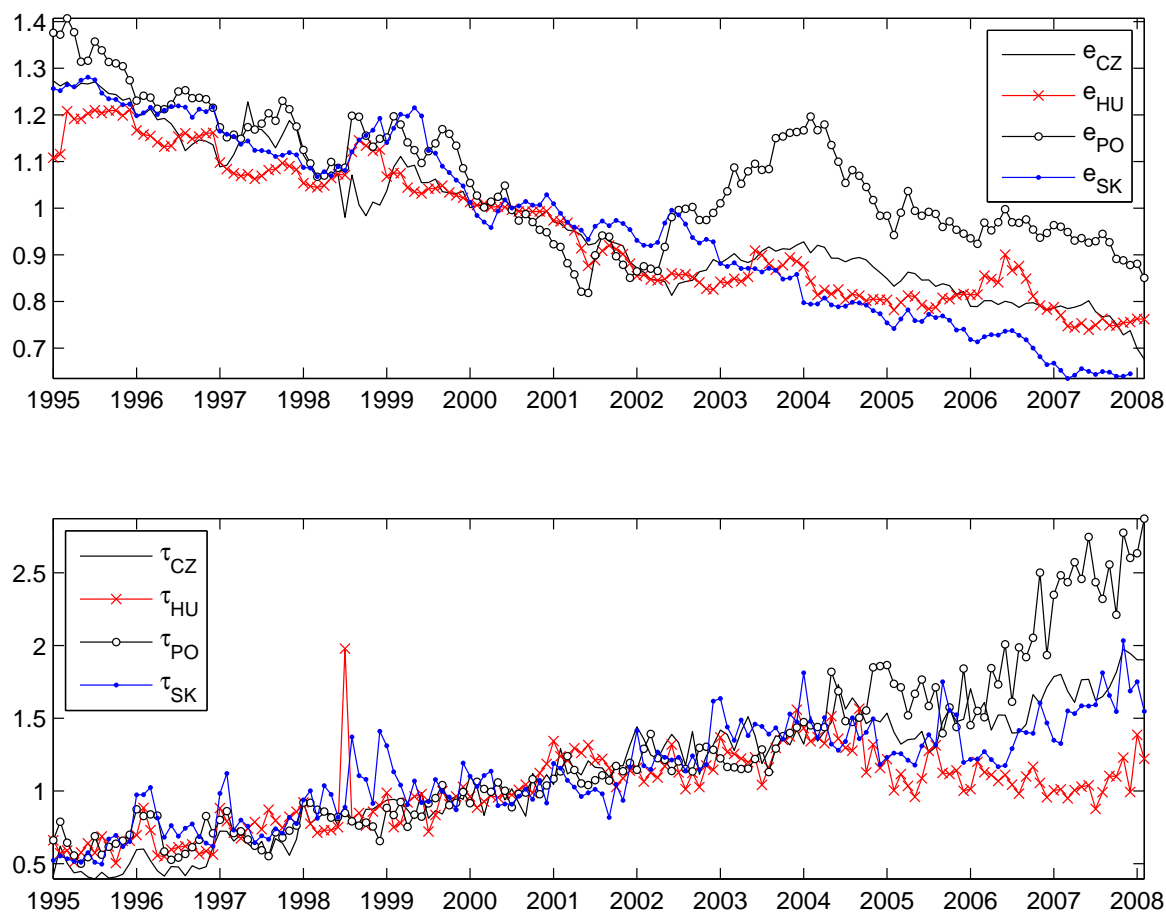


Figure 4.1: Top panel shows real exchange rates  $e = \mathcal{E}P^*/P$ . Here  $P$  is the consumer price index of a Visegrad 4 country,  $P^*$  is the consumer price index in the euro area and  $\mathcal{E}$  is a Visegrad 4 country's currency in terms of euro. Bottom panel shows Visegrad 4 country's terms of trade  $\tau = P_H/P_F$  vis á vis the euro area based on implicit unit export prices ( $P_H$ ) and unit import prices ( $P_F$ ) in common currency. All time series are normalized to the year 2000. Appendix C.1 describes data construction and sources.

of consumer goods in the euro area relative to the price of a basket of consumer goods in the Czech Republic (CZ), Hungary (HU), Poland (PO) or Slovakia (SK), respectively. For the same selection of countries the bottom panel shows terms of trade vis á vis the euro area measured as the unit export price relative to the unit import price. Whereas real exchange rates appreciate from 1995 until most recently terms of trade improve over the same period of time. Egert (2007) and Egert and Podpiera (2008) similarly report real appreciation and Fabrizio, Igan, and Mody (2007) similarly find improvements in the terms of trade of new EU member states vis á vis the euro area so that one may regard both observations as outstanding stylized facts in the data.



It is a common view that fast productivity growth in the traded relative to the non-traded good sector in new EU member states is a main driving force of real exchange rate appreciation. This common view is consistent with the standard two country two sector real business cycle (RBC) model in Stockman and Tesar (1995). According to that model relatively fast productivity growth implies wage growth in the traded good sector. When high wages spill over to the nontraded good sector, prices of non-tradables increase in line with Balassa (1964) and Samuelson (1964). As a result the real exchange rate appreciates. Indeed, Egert (2007) confirms that productivity differentials of new EU member states still are high according to euro area standards. However, it is a challenge for the standard model to simultaneously explain observed improvements in the terms of trade because relatively fast productivity growth in the traded good sector reduces export prices, which corresponds to deteriorating rather than improving terms of trade.

Here I propose a transmission mechanism of productivity growth which reconciles the standard two country two sector RBC model with both empirical facts. In particular, I extend the standard by endogenous product entry in the traded good sector so that the extended model exhibits two different margins of productivity in this sector. Intensive productivity growth enhances production efficiency of operating firms and thus coincides with the notion of productivity in the standard model. Extensive productivity growth reduces barriers to entry so that new firms which are associated with new products find it easier to enter the market. The core difference between the two productivity margins is their impact on marginal costs in the traded good sector. Intensive productivity growth reduces marginal costs which are proportional to wages net of intensive productivity. This is true despite the fact that wages grow because intensive productivity growth overcompensates wage growth. Terms of trade deteriorate because prices of traded products are a constant markup over marginal costs.

In contrast, extensive productivity growth reduces entry barriers and firm entry exerts upward pressure on wages because activities related to developing and establishing new products absorb labor. Wage growth increases marginal costs so that prices of both traded and nontraded products increase. As a result, the real exchange rate appreciates and terms of trade improve. Feenstra (1994) and Broda and Weinstein (2004) document an important measurement bias in import price indices due to a failure to track new products in time which relates to this result. The measure of the terms of trade which improves due to extensive productivity growth ignores the effect of product variety whereas a measure of the terms of trade which accounts for the effect of product variety contemporaneously deteriorates. The reason to adhere to

the first measure when comparing the model to data is that data on the terms of trade are derived from observed price indices which fail to track effects of product variety in time.

Debaere and Lee (2003) consistently obtain positive correlations between a country's terms of trade and its research and development induced productivity. Their interpretation of this finding is that fast growing countries can avoid deteriorating terms of trade by increasing product variety. Moreover, Feenstra and Kee (2004) find that across countries high productivity goes in hand with a large degree of product variety in exports. Both pieces of evidence support the link between extensive productivity and product variety relied on here. Kandogan (2006) decomposes the increase in highly disaggregated manufacturing exports of Central and Eastern European countries into the extensive margin (more products) and the intensive margin (more of established products). The sample of trading partners comprises the euro area as well as countries such as Japan, the United Kingdom and the U.S. for the period 1992 to 1999. Kandogan finds that the extensive margin is predominant in general and in particular so for CZ, PO, SK and HU. For instance, 56% of the increase in CZ exports is due to increased product variety. Corresponding figures for PO, SK and HU are 51%, 45% and 29%, respectively. This evidence suggests that modeling transition dynamics in new EU member states while not accounting for the extensive margin in trade omits a quantitatively important dimension.

Obviously, it is difficult to decide if a product is a quality upgraded version of an already existent product or rather an entirely new product so that product quality and product variety intertwine tightly. For practical purposes two reasons suggest that working with product variety is more desirable. First, modeling time variant and product specific quality quickly leads to an infinite dimensional state space similar to vintage capital models unless one resorts to simplifying assumptions. Second, measuring quality appears to be a challenging task which statistical authorities often handle by means of hedonic price regression (Ahnert and Kenny (2004)). The rapidly evolving literature on the new goods margin in open and closed economies, however, provides a variety of operative measures for the number of products.<sup>1</sup>

Beyond the extensive productivity margin the analysis here also offers a conventional demand side explanation for the observed patterns in international relative prices. Fostered government consumption involves appreciating real exchange rates and improving terms of trade. However, the demand side impulse has diametrical

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<sup>1</sup>Recent papers that study the new goods margin in international trade are Kehoe and Ruhl (2002), Broda and Weinstein (2004), Hummels and Klenow (2005), Debaere and Mostashari (2005) and Broda and Weinstein (2006), to name only a few.

implications for product variety so that it remains an interesting empirical matter to determine how much variation in the data should be attributed to this explanation.

In related work, Bruha and Podpiera (2007) set up a rich deterministic two country model to explain the trend appreciation in real exchange rates of new EU member states through investment into quality. They find that actual and measured real exchange rates may move in different directions because measurement bias in price indices drives a wedge between the two measures. Similarly, in a two country one sector model Dury and Oomen (2007) show that quality improvements lead to a depreciating real exchange rate. As in Bruha and Podpiera (2007) they find substantial differences between actual and measured real exchange rates. Both studies do not focus on explaining the terms of trade which is a main focus here. Fujiwara and HIRAKATA (2007) set up a two country model with firm entry and sticky prices and wages but restrict attention to a traded good sector and do not link their analysis to new EU member states. Corsetti, Martin, and Pesenti (2007) analyze the international transmission and welfare implications of extensive productivity growth in a two country one sector model. Corsetti, Martin, and Pesenti (2008) reconsider the classical transfer problem in a model where the set of exportables, importables and nontraded goods is endogenous.

## 4.2 Model

Let new EU member states (or parts thereof) represent the country labeled Home and let the euro area (or parts thereof) represent the country labeled Foreign. Foreign variables are marked by an asterisk throughout. Each country comprises a traded and a nontraded productive sector. In the traded good sector firm entry is endogenous and new firms assemble new products. In each country a representative firm combines domestic traded products and imported foreign traded products to an aggregate traded good. The firm combines the aggregate traded good with a nontraded good composed out of nontraded products to composite final output. In the country of production composite final output serves private consumption and is absorbed when new firms enter the traded good sector. Households can shift wealth across countries using one internationally traded bond. In each country a government conducts monetary and fiscal policy independent from the government in the other country. Governments consume domestically produced nontraded goods.

### 4.2.1 Firm Entry

Let  $h \in [0, H_t]$  index operating firms in the Home traded good sector denoting with  $H_t$  the endogenous upper bound of the firm interval which is interpreted as number of firms. There is a one to one match between products and firms so that firm  $h$  assembles product  $h$ . The value of firm  $h$  after period production equals expected discounted firm profits,

$$V_t(h) = E_t \sum_{s=t+1}^{\infty} [(1 - \delta)\beta]^{s-t} \left( \frac{\Lambda_s P_t}{\Lambda_t P_s} \right) D_s(h). \quad (4.1)$$

Here  $V_t(h)$  denotes nominal firm value in period  $t$ ,  $D_t(h)$  denotes nominal profits and  $P_t$  is the money price of composite final output. The term in round brackets jointly with  $\beta \in (0, 1)$  is the equilibrium stochastic discount factor of Home households so that  $\Lambda_t$  denotes households' marginal utility of consumption. Discounting from household perspective reflects that Home households are sole owners of Home firms and thus entitled to firm profits. No firm ever liquidates owing to bad macro shocks. However, at the micro level firms are subject to an exit causing shock which causes a fraction  $\delta \in (0, 1)$  of firms to exit at the end of each period. Accordingly, each firm faces exogenous exit probability  $\delta$  which reduces discounted firm value.

Entry of new firms is perfectly competitive because a large pool of entry candidates exists. When a new firm enters the market it needs the entry period to hire staff, to conduct research and development, to acquire and process information, to market and brand its product and to deal with administration and governmental regulation. These one time activities related to entry generate sunk costs  $F_E(t)$  in units of composite final output. Entry costs are specified as

$$F_E(t) = q (H_{Et} / \bar{H}_{Et})^{\phi_e} A_{Et}^{-\phi_E}, \quad q > 0, \quad \phi_e, \phi_E \geq 0.$$

A firm's costs increase with elasticity  $\phi_e$  whenever the number of new firms  $H_{Et}$  is above its deterministic growth path  $\bar{H}_{Et}$ . Exogenous firm exit ensures that  $H_{Et}$  remains positive throughout. This component of entry costs is similar to Corsetti, Martin, and Pesenti (2008) and captures the idea that a larger number of new firms makes it more demanding to differentiate a new product from the plentitude of other new products.<sup>2</sup> The second component of entry costs is extensive productivity  $A_{Et}$  which

<sup>2</sup>Entry costs in units of the composite final output comprise traded products assembled abroad so that entry absorbs international resources and benefits from efficiency gains realized abroad. For instance, Bilbiie, Ghironi, and Melitz (2007) and Corsetti, Martin, and Pesenti (2007) formulate entry costs in units of labor. When labor is immobile internationally, as is assumed here, this would restrict

evolves exogenously and reflects variation in the many dimensions of entry costs for which no explicit formulation is provided here.

The new firm starts production in the period after entry. Free entry ensures that expected discounted profits of a new firm equal entry costs  $F_E(t)$ . However, expected production prospects of the new firm are identical to the prospects of a firm already operating in the market after the operating firm has distributed current period profits to households because the new firm does not realize any profits in the entry period and there do not exist firm specific states. Hence, free entry implies

$$V_t(h) = P_t F_E(t) .$$

Combining the time to enter period with the exit shock the number of operating firms in period  $t$  evolves according to the recursive law of motion

$$H_t = (1 - \delta)[H_{t-1} + H_{E_{t-1}}] .$$

Firm entry in the Foreign traded good sector is symmetric. To fix notation let  $f \in [0, F_t^*]$  index operating firms in the Foreign traded good sector interpreting  $F_t^*$  as the number of operating firms in Foreign. The number of new Foreign firms is denoted by  $F_{E_t}^*$  and  $V_t^*(f)$  ( $D_t^*(f)$ ) indicates Foreign firm value (profit). Moreover,  $P_t^*$  stands for the money price of the Foreign composite final output in Foreign currency and parameters  $\beta, \delta, \phi_e, \phi_E$  and  $q$  are identical in Home and Foreign.

### 4.2.2 Households

Let  $j \in [0, 1]$  index Home households. Household  $j$  has expected discounted life time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t(j) - \frac{L_t(j)^{1+\varphi}}{1+\varphi} \right) , \quad \varphi > 0 \quad (4.2)$$

with  $\beta \in (0, 1)$  the subjective discount factor and period utility increasing in consumption  $C_t(j)$  and decreasing in labor  $L_t(j)$ . The household is subject to the budget con-

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entry to absorb national resources only. It is not easy to justify why one would consider traded goods such as, say, information technology of not being useful to complete the tasks required to set up a firm. Entry costs are sunk and so do not affect the liquidation value of a firm because the proceeds from entry activities are to a large extent either nonmarketable or depreciate quickly, such as research and development or the value of information. The entry setup applied here is similar to Bilbiie, Ghironi, and Melitz (2007).

straint

$$\begin{aligned}
P_t C_t(j) + B_{Ht}(j) + \mathcal{E}_t B_{Ft}(j) + \left[ \int_0^{H_t} V_t(h) dh + \int_{H_t}^{H_t + H_{Et}} V_t(h) dh \right] X_t(j) = \\
\left[ \int_0^{H_t} V_t(h) + D_t(h) dh \right] X_{t-1}(j) + R_{t-1} B_{Ht-1}(j) + \mathcal{E}_t B_{Ft-1}(j) R_{t-1}^* \Phi(t-1) \\
+ W_t L_t(j) + \int_0^1 D_t(n) dn - T_t. \tag{4.3}
\end{aligned}$$

Household  $j$  has consumption expenditure  $P_t C_t(j)$ , receives labor income  $W_t L_t(j)$  and profits  $\int_0^1 D_t(n) dn$  from owning profitable firms in the nontraded good sector and is subject to a lump sum tax  $T_t$  levied by the government. Residents in Home work exclusively in their own country and earn a nominal wage  $W_t$  on a competitive labor market. The household holds wealth in form of a nationally traded nominal bond  $B_{Ht}(j)$  in Home currency with nominal return  $R_t$  and an internationally traded nominal bond  $\mathcal{E}_t B_{Ft}(j)$  in Foreign currency converted in Home currency via the nominal exchange rate  $\mathcal{E}_t$ . The nominal exchange rate is defined as Home currency in terms of Foreign currency. When trading in Foreign bonds the household is subject to a premium  $\Phi(t) = \exp\{-\phi(\mathcal{E}_t B_{Ft}/(P_t Y_t))\}$  with  $\phi > 0$  and  $Y_t$  denoting composite final output. The premium reflects the costs the household faces when it engages in international financial transactions in Foreign currency. In particular, if Home lends to Foreign,  $\mathcal{E}_t B_{Ft} > 0$ , returns to lending are below the nominal return  $R_t^*$  in Foreign. If Home borrows from Foreign, Home's effective interest payment is above  $R_t^*$ . This setup is similar to Benigno (2001) and ensures a unique stationary evolution of the relative wealth position between Home and Foreign households.<sup>3</sup>

The household can hold wealth also in form of shares of a national mutual fund defined over firms in the traded good sector. In period  $t$  the household decides the fraction  $X_t(j)$  it wishes to hold of the mutual fund which is worth all firms that are expected to operate in the next period,  $\int_0^{H_t} V_t(h) dh + \int_{H_t}^{H_t + H_{Et}} V_t(h) dh$ , i.e. the value after production of currently operating firms plus the value of firms that entered in the present period. A respective investment in the previous period entitles the household to its share  $X_{t-1}(j)$  of the current period value of the mutual fund after dividends plus dividends from current period production,  $\int_0^{H_t} V_t(h) + D_t(h) dh$ . The household maximizes (4.2) subject to (4.3) choosing sequences  $\{C_t(j), L_t(j), B_{Ht}(j), B_{Ft}(j), X_t(j)\}_{t=0}^\infty$ .

<sup>3</sup>Actually, Benigno (2001) derives the cost interpretation pursued here explicitly by postulating intermediaries in the Foreign bond market. As alternative to debt elastic bond returns Bodenstein (2006) analyzes bond adjustment costs and a state dependent discount factor to achieve stationarity in a two country model. In the context of the rich deterministic growth path specified below the current setup turned out the easiest to implement.

Let  $j^* \in [0, 1]$  index Foreign households. Household  $j^*$  has expected discounted life time utility analogous to equation (4.2) with identical labor supply elasticity  $\varphi$ . The budget constraint of household  $j^*$  accounts for the fact that Home bonds are not traded internationally and that the Foreign household does not face costs to international financial transactions because it trades in its own currency. Accordingly,

$$\begin{aligned} P_t^* C_t^*(j^*) + B_{Ft}^*(j^*) + \left[ \int_0^{F_t^*} V_t^*(f) df + \int_{F_t^*}^{F_t^* + F_{Et}^*} V_t^*(f) df \right] X_t^*(j^*) = W_t^* L_t^*(j^*) \\ + \left[ \int_0^{F_t^*} V_t^*(f) df + \int_0^{F_t^*} D_t^*(f) df \right] X_{t-1}^*(j^*) + R_{t-1}^* B_{Ft-1}^*(j^*) + \int_0^1 D_t^*(n^*) dn^* - T_t^* . \end{aligned} \quad (4.4)$$

All variables have analogous definitions. Appendix C.2 provides the first order conditions that characterize optimal household behavior in Home and Foreign.

### 4.2.3 Production

Production is multistage and symmetric in both countries so that it suffices to describe Home production. First, intermediate firms assemble products in the traded and the nontraded good sector. Traded products are partly shipped abroad. Second, a representative firm combines traded products assembled in Home to a Home traded good and imported traded products assembled in Foreign to a Foreign traded good. It then combines the Home and Foreign traded good to the aggregate traded good. Similarly, nontraded products are combined to a composite nontraded good. Finally, the representative firm combines the aggregate traded good with the nontraded good to composite final output which is consumed by domestic households and absorbed by firm entry. Starting with the representative firm allows to obtain demand functions for the upstream production stages.

#### Representative Firm

The representative firm produces *composite final output*  $Y_t$  with technology

$$Y_t = (a_T^{a_T} a_N^{a_N})^{-1} Y_{Tt}^{a_T} Y_{Nt}^{a_N}, \quad a_N \in (0, 1), \quad a_N = 1 - a_T$$

combining the aggregate traded good  $Y_{Tt}$  and the nontraded good  $Y_{Nt}$  with unit elasticity of substitution. Cobb Douglas technology is the special case of CES technology that is consistent with a constant growth rate of  $Y_t$  if the aggregate traded good and the nontraded good grow at constant but different rates, which is the case along the

nonstochastic growth path considered below. Taking prices as given cost minimal production delivers the definition of the money price of composite final output

$$P_t = P_{Tt}^{a_T} P_{Nt}^{a_N}$$

and demand functions  $P_{Tt}Y_{Tt} = a_T P_t Y_t$  and  $P_{Nt}Y_{Nt} = a_N P_t Y_t$  for the aggregate traded and the nontraded good, respectively. The *aggregate traded good* is produced with technology

$$Y_{Tt} = \left[ a_H^\mu Y_{Ht}^{\mu-1} + a_F^\mu Y_{Ft}^{\mu-1} \right]^{\frac{\mu}{\mu-1}}, \quad \mu > 1, \quad a_H \in (0, 1), \quad a_H = 1 - a_F$$

which combines the Home traded good  $Y_{Ht}$  and the Foreign traded good  $Y_{Ft}$  with elasticity of substitution equal to  $\mu$ . Weights  $a_H$  and  $a_F$  govern the importance of each input in production. Cost minimal production delivers the definition of the price of the aggregate traded good and input demand functions,

$$P_{Tt} = \left[ a_H P_{Ht}^{1-\mu} + a_F P_{Ft}^{1-\mu} \right]^{\frac{1}{1-\mu}}, \quad \frac{Y_{Ht}}{Y_{Tt}} = a_H \left( \frac{P_{Ht}}{P_{Tt}} \right)^{-\mu}, \quad \frac{Y_{Ft}}{Y_{Tt}} = a_F \left( \frac{P_{Ft}}{P_{Tt}} \right)^{-\mu}. \quad (4.5)$$

The *Home traded good* is produced with technology

$$Y_{Ht} = H_t^{\rho - \frac{\nu}{\nu-1}} \left[ \int_0^{H_t} Y_t(h)^{\frac{\nu-1}{\nu}} dh \right]^{\frac{\nu}{\nu-1}}, \quad \nu > 1, \quad \rho > 0.$$

Here  $\nu$  denotes the across product elasticity of substitution and  $\rho$  governs the role of product variety in production. In line with Benassy (1996),  $\rho - 1$  represents the marginal output gain from spreading a given amount of production input over an array of products that includes one additional variety. Thus, as long as  $\rho > 1$  technology exhibits increasing returns to specialization, whereas  $\rho = 1$  makes technology completely insensitive to product variety and  $\rho < 1$  delivers technology with decreasing returns to specialization. A formally equivalent setup pursued in Bilbiie, Ghironi, and Melitz (2007) and Corsetti, Martin, and Pesenti (2008) interprets aggregation of intermediate products as home production. That is, instead of a representative firm doing product aggregation the household aggregates products itself before consumption. Here I prefer to make explicit the complete production structure so that firms rather than households provide the composite final output essential for firm entry. Formal results are not affected by the alternative interpretation. Cost minimal production taking product prices  $P_t(h)$  as given delivers

$$P_{Ht} = H_t^{-(\rho - \frac{\nu}{\nu-1})} \left[ \int_0^{H_t} P_t(h)^{1-\nu} dh \right]^{\frac{1}{1-\nu}}, \quad \frac{Y_t(h)}{Y_{Ht}} = H_t^{\rho(\nu-1)-\nu} \left( \frac{P_t(h)}{P_{Ht}} \right)^{-\nu}. \quad (4.6)$$



The *Foreign traded good* is produced with analog technology and identical parameters referring to the Foreign number of goods in the Foreign tradable sector. The *Home nontraded good* is produced with technology

$$Y_{Nt} = \left[ \int_0^1 Y_t(n)^{\frac{\nu_N-1}{\nu_N}} dn \right]^{\frac{\nu_N}{\nu_N-1}}, \quad \nu_N > 1. \quad (4.7)$$

There is no time variant extensive margin in the nontradable sector so that the definition of the price of the nontraded good and demand for nontraded products obtain from cost minimization as

$$P_{Nt} = \left[ \int_0^1 P_t(n)^{1-\nu_N} dn \right]^{\frac{1}{1-\nu_N}}, \quad \frac{Y_t(n)}{Y_{Nt}} = \left( \frac{P_t(n)}{P_{Nt}} \right)^{-\nu_N} \quad (4.8)$$

taking  $P_t(n)$  as given.

Production in Foreign is symmetric to the production in Home just described. In particular, parameters  $a_N, a_T, a_H$  and  $a_F$  are identical which is no technical necessity as the setup can be extended to allow for different weights in production indices. However, I stick to symmetry because otherwise expressions that describe the non-stochastic growth path below turn rather unhandy. One reason for this is that the law of one price no longer holds for traded goods (while it will continue to hold for traded products) which complicates derivation.

### Firms Assembling Traded Products

Home intermediate firms indexed by  $h$  maximize profits

$$D_t(h) = P_t(h)Y_t(h) + \mathcal{E}_t P_t^*(h)Y_t^*(h) - W_t L_t(h)$$

by choosing prices  $P_t(h)$  and  $P_t^*(h)$  for the Home and the Foreign market in local currency, respectively. Here  $L_t(h)$  denotes labor input of firm  $h$ . Firms take demand from the Home and the Foreign representative firm as given (equation (4.6) and the Foreign analog) and produce with linear technology  $Y_t(h) + Y_t^*(h) = A_{Ht} L_t(h)$  denoting with  $Y_t(h)$  ( $Y_t^*(h)$ ) the quantity produced for the Home (Foreign) market and with  $A_{Ht}$  intensive productivity in Home. Optimality requires prices that are set as a constant markup over marginal costs

$$P_t(h) = \frac{\nu}{\nu-1} \frac{W_t}{A_{Ht}}$$

with marginal costs being equal to efficiency wages. Optimality also implies the law of one price for Home traded products  $P_t(h) = \mathcal{E}_t P_t^*(h)$ . Profit maximization of Foreign

intermediate firms indexed by  $f$  is analog and implies that Foreign prices are set as a constant markup over marginal costs

$$P_t^*(f) = \frac{\nu}{\nu - 1} \frac{W_t^*}{A_{Ft}^*}$$

denoting with  $A_{Ft}$  intensive Foreign productivity in the tradable good sector. Moreover, the law of one price holds for Foreign products,  $P_t(f) = \mathcal{E}_t P_t^*(f)$ .

### Firms Assembling Nontraded Products

Home nontraded firms indexed by  $n \in [0, 1]$  maximize profits

$$D_t(n) = P_t(n)[Y_t(n) + \mathcal{G}_t(n)] - W_t L_t(n)$$

by choosing the Home currency price  $P_t(n)$ . Here  $L_t(n)$  denotes labor input of firm  $n$ . Firms take demand from the representative firm  $Y_t(n)$  and government demand  $\mathcal{G}_t(n)$  as given so that total demand is

$$Y_t(n) + \mathcal{G}_t(n) = \left( \frac{P_t(n)}{P_{Nt}} \right)^{-\nu_N} [Y_{Nt} + \mathcal{G}_{Nt}].$$

Technology is linear in labor,  $Y_t(n) + \mathcal{G}_t(n) = A_{Nt} L_t(n)$ , denoting with  $A_{Nt}$  productivity in the nontraded good sector. As before, optimality implies that prices are a constant markup over marginal costs,  $P_t(n) = \nu_N / (\nu_N - 1) W_t / A_{Nt}$ .

### 4.2.4 Clearing Conditions

I consider an equilibrium in which households  $j$  ( $j^*$ ), firms  $h$  ( $f$ ) and firms  $n$  ( $n^*$ ) are symmetric, respectively, households and firms optimize and markets clear. Here I describe market clearing in Home with the understanding that corresponding conditions apply to Foreign. Clearing of the market for composite final output requires that aggregate consumption and economy wide entry costs add up to composite final output,

$$Y_t = C_t + H_{Et} F_E(t)$$

using  $C_t = \int_0^1 C_t(j) dj$  to denote aggregate consumption. Labor market clearing requires that households' total labor supply equals overall labor demand in the non-traded and the traded good sector,

$$L_t = L_{Nt} + L_{Ht}$$

having  $L_t = \int_0^1 L_t(j) dj$ ,  $L_{Ht} = \int_0^{H_t} L_t(h) dh$  and  $L_{Nt} = \int_0^1 L_t(n) dn$ . Stock market clearing requires that the entire mutual fund is held by households so that  $X_t(j) = 1$  for all  $t$  and all  $j$ .

Define Home and Foreign trade balance as value of exports net of value of imports

$$TB_t = \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - P_{Ft} Y_{Ft}, \quad TB_t^* = P_{Ft} Y_{Ft} / \mathcal{E}_t - P_{Ht}^* Y_{Ht}^*,$$

respectively. Moreover, define Home and Foreign current account as net capital inflows

$$CA_t = \mathcal{E}_t (B_{Ft} - B_{Ft-1}), \quad CA_t^* = B_{Ft}^* - B_{Ft-1}^*$$

where aggregate Home and Foreign holdings of the Foreign bond are  $B_{Ft} = \int_0^1 B_{Ft}(j) dj$  and  $B_{Ft}^* = \int_0^1 B_{Ft}^*(j^*) dj^*$ , respectively. Integrating equation (4.3) over all households  $j$  delivers the Home economy wide budget constraint from which one derives the balance of payment condition in Home when combining it with aggregate firm profits, labor market clearing, stock market clearing, the Home trade balance and the Home current account,

$$CA_t = TB_t + \mathcal{E}_t B_{Ft-1} (R_{t-1}^* \Phi(t-1) - 1), \quad CA_t^* = TB_t^* + (R_{t-1}^* - 1) B_{Ft-1}^*.$$

Corresponding steps in Foreign deliver the Foreign balance of payment condition. Bond market clearing in Home requires  $\int_0^1 B_{Ht}(j) dj = 0$  since Home bonds are traded among symmetric households. Foreign bond market clearing obtains by combining Home and Foreign balance of payment conditions to

$$B_{Ft}^* + B_{Ft} = R_{t-1}^* [B_{Ft-1} \Phi(t-1) + B_{Ft-1}^*].$$

Absent the Home premium for international transactions,  $\Phi(t) = 1$  for all  $t$ , this condition reduces to  $B_{Ft}^* + B_{Ft} = 0$ .

Finally, to obtain a measure of real GDP consistent with the definition in the data define real GDP as  $GDP_t = (a_T^{a_T} a_N^{a_N})^{-1} (Y_{Ht} + Y_{Ht}^*)^{a_T} (Y_{Nt} + \mathcal{G}_{Nt})^{a_N}$  which is a composite exclusively comprising Home produced goods. The corresponding GDP deflator obtains as  $P_{GDP_t} = P_{Ht}^{a_T} P_{Nt}^{a_N}$ . Since  $P_{GDP_t}$  is the cost minimal price of  $GDP_t$  it can be shown that  $P_{GDP_t} GDP_t = P_{Ht} (Y_{Ht} + Y_{Ht}^*) + P_{Nt} (Y_{Nt} + \mathcal{G}_{Nt})$  which is consistent with defining nominal GDP as the sum of factor incomes in the Home economy by the economy wide budget constraint.

#### 4.2.5 Fiscal and Monetary Policy

The Home government combines nontraded products with technology analog to (4.7) so that the cost minimal price of total government consumption  $\mathcal{G}_{Nt}$  equals  $P_{Nt}$  and

government demand for nontraded products is  $\mathcal{G}_t(n) = (P_t(n)/P_{Nt})^{-v_N} \mathcal{G}_{Nt}$ . The government finances its consumption by lump sum taxes  $T_t$  and runs a balanced budget  $T_t = P_{Nt}\mathcal{G}_{Nt}$  in each period where the total amount of government consumption  $\mathcal{G}_{Nt}$  is taken as exogenous. Correspondingly, the Foreign government demands Foreign nontraded goods.

Home monetary policy follows a Taylor type rule which makes the nominal interest rate a function of inflation and of the deviation of composite final output from its growth path,  $R_t/R = (\pi_t/\pi)^{\alpha_\pi} (Y_t/\bar{Y}_t)^{\alpha_y}$  having  $\alpha_\pi > 1$  and  $\alpha_y \geq 0$ . Here  $\bar{Y}_t$  denotes Home composite final output along the nonstochastic growth path and  $R$  and  $\pi$  are the values of the nominal interest rate and inflation along this path. Foreign monetary policy follows the same rule but refers to Foreign variables so that all variables in the policy rule have an asterisk as superscript.

#### 4.2.6 Shocks

A novel feature of the model is that productivity processes in the traded good sector and productivity processes in the nontraded good sector are allowed to exhibit different growth rates along the nonstochastic growth path. This extension puts the model in the position to generate Balassa Samuelson type effects along the nonstochastic growth path so that the relative price of traded versus nontraded goods declines over long periods of time which is a property of the data. Let productivity processes evolve according to

$$\ln A_{Xt} = \lambda_X + \ln A_{Xt-1} + \ln z_{Xt} \quad , \quad \ln z_{Xt} = \psi_X \ln z_{Xt-1} + \epsilon_t^{AX} \quad , \quad \psi_X \in (0, 1)$$

where  $X$  stands for  $\{N, N^*, H, F^*, E, E^*\}$ . The nonstochastic growth path restricts growth rates across countries to  $\lambda_N = \lambda_{N^*}$ ,  $\lambda_H = \lambda_{F^*}$  and  $\lambda_E = \lambda_{E^*}$ . The unconditional mean  $E(z_{Xt})$  equals unity along the nonstochastic growth path and fundamental shocks  $\epsilon_t^{AX}$  are white noise. Government consumption follows

$$\ln \mathcal{G}_{Nt} = \lambda_N + \ln \mathcal{G}_{Nt-1} + \ln z_{Gt} \quad , \quad \ln z_{Gt} = (1 - \psi_G) \ln z_G + \psi_G \ln z_{Gt-1} + \epsilon_t^G$$

where the unconditional mean of  $E(z_{Gt})$  is allowed to take arbitrary values to simplify parametrization of the model,  $\epsilon_t^G$  again is white noise and  $\psi_G \in (0, 1)$ . All initial values are taken as given.

#### 4.2.7 Measurement

Observed price indices often fail to account for new products in time and Feenstra (1994) and Broda and Weinstein (2004) show that import and export price indices

exhibit a substantial upward bias due to this failure. Recently, Broda and Weinstein (2007) make similar observations for consumer price indices and Bils (2004) notes that mismeasurement in price indices converts into mismeasurement of quantity indices. Accordingly, among others Bilbiie, Ghironi, and Melitz (2007) and Corsetti, Martin, and Pesenti (2008) argue that a meaningful comparison of model predictions with observed data confronts the model's average price rather than expressions for prices which vary with the number of products to observed price indices.

In symmetric equilibrium prices of firms  $h$  equal the average price  $P_{ht} = P_t(h)$  so that definition (4.6) degenerates to  $P_{Ht} = H_t^{1-\rho} P_{ht}$ . Correspondingly, the Home currency price of the Foreign traded good degenerates to  $P_{Ft} = (F_t^*)^{1-\rho} P_{ft}$ . I define Home and Foreign traded good prices measured in Home as

$$P_{Ht}^m = H_t^{\rho-1} P_{Ht}, \quad P_{Ft}^m = (F_t^*)^{\rho-1} P_{Ft}.$$

Throughout measured variables are indicated with superscript  $m$ . The basic source of mismeasurement thus derives from  $P_{Ht}^m$  ( $P_{Ft}^m$ ) deviating from  $P_{Ht}$  ( $P_{Ft}$ ) which is the case if  $\rho \neq 1$  and  $H_t$  ( $F_t^*$ ) differs from unity over time. The fact that the actual price  $P_{Ht}$  declines when  $H_t$  increases assuming that  $\rho > 1$  and all else equal reflects that returns to specialization reduce the actual Home traded price when product variety increases. Measured prices do not reflect this variation along the extensive margin. With these definitions and in concordance with equation (4.5), the measured aggregate traded price equals<sup>4</sup>

$$P_{Tt}^m = \left[ a_H (P_{Ht}^m)^{1-\mu} + a_F (P_{Ft}^m)^{1-\mu} \right]^{\frac{1}{1-\mu}}.$$

so that measured prices of composite final output and of GDP are, respectively,

$$P_t^m = P_{Nt}^{a_N} (P_{Tt}^m)^{a_T}, \quad P_{GDPt}^m = P_{Nt}^{a_N} (P_{Ht}^m)^{a_T}.$$

Nontraded good prices are free of measurement bias because the extensive margin is constant over time in this sector. It is now straightforward to obtain conversion rates to convert actual into measured quantities,

$$Q_{Ht} = \frac{P_{Ht}}{P_{Ht}^m} = H_t^{1-\rho}, \quad Q_{Ft} = \frac{P_{Ft}}{P_{Ft}^m} = (F_t^*)^{1-\rho}, \quad Q_{Tt} = \frac{P_{Tt}}{P_{Tt}^m}, \quad Q_t = Q_{Tt}^{a_T}, \quad Q_{GDPt} = Q_{Ht}^{a_T}.$$

For instance, multiplying composite final output  $Y_t$  or a variable in the same units by  $Q_t$  converts this variable into units of  $Y_t^m$ . Corresponding conversion rates are easily obtained for the Foreign country.

<sup>4</sup>One may also object that measurement is difficult with respect to  $\mu$ . However, as long as  $\mu$  is constant over time this type of mismeasurement cancels out up to first order which is the order of approximation applied when solving for the model dynamics below. The reason is that there exists no substitution bias up to first order. Results in chapter 2 manifest this statement.

### 4.2.8 Real Exchange Rate and Terms of Trade

The real exchange rate is the Home currency price of the Foreign consumption composite in terms of the price for Home consumption composite,

$$e_t = \frac{\mathcal{E}_t P_t^*}{P_t} .$$

In light of the discussion in the previous section one obtains measured real exchange rates when replacing actual by measured prices,  $e_t^m = \mathcal{E}_t (P_t^m)^* / P_t^m$ . However, measurement bias in consumption prices does not affect measurement of the real exchange rate because the definition of the consumption basket and thereby of the consumption price is identical across countries. As a result measurement bias cancels out and  $e_t^m = e_t$ .

Terms of trade are the price of the Home traded good over the price of the Foreign traded good in Home currency. Correspondingly, measured terms of trade replace actual prices by measured prices so that

$$\tau_t = \frac{P_{Ht}}{P_{Ft}} , \quad \tau_t^m = \frac{P_{Ht}^m}{P_{Ft}^m} = \left( \frac{H_t}{F_t^*} \right)^{\rho-1} \tau_t .$$

Measurement bias affects the terms of trade as long as  $\rho \neq 1$ . In particular, if product variety in Home catches up to product variety in Foreign measured terms of trade remain below actual terms of trade during the catch up period as long as returns to specialization  $\rho > 1$  exist.

## 4.3 Model Solution

I solve for the model dynamics in three steps. Exogenous processes exhibit deterministic trends which endogenous variables inherit. Therefore, in a first step endogenous variables are transformed to remove deterministic trend components. Second, I solve for the nonstochastic steady state of the transformed model. Third, I approximate the transformed model linearly around this steady state and solve for the model dynamics accurate up to first order.

### 4.3.1 Transformation

Home and Foreign optimality conditions for households and firms plus clearing conditions allow to solve for the constant growth rate of each endogenous variable. Let

$\bar{A}_{Xt} = A_{X0} \exp(\lambda_X t)$  denote the deterministic part of exogenous productivity processes  $A_{Xt} = \bar{A}_{Xt} z_{Xt}$  so that  $\lambda_X$  is the deterministic growth rate. Being on a balanced growth path then coincides with  $z_{Xt} = 1$ . I assume a zero growth rate of total labor in Home and Foreign and require monetary policies to set identical nominal interest rates  $R = R^*$  along nonstochastic growth path. Level variables at the nonstochastic growth path are signified with a bar. I use notation  $G_X = \bar{X}_t / \bar{X}_{t-1}$  to denote the gross growth rate of a generic variable  $X_t$  and  $\pi_X = \bar{P}_{Xt} / \bar{P}_{Xt-1}$  for generic subscript  $X$  to denote respective inflation rates along the nonstochastic growth path.

Here I briefly describe some properties of the nonstochastic growth path. Along this path terms of trade and real and nominal exchange rates grow at zero rate. Net holdings of the internationally traded bond are zero,  $\bar{B}_{Ft} = \bar{B}_{Ft}^* = 0$ , so that the current account and the trade balance are zero,  $\bar{CA}_t = \bar{TB}_t = 0$ . Composite final output grows at rate

$$G_Y = G_{Y^*} = (1 + \lambda_H)^{\frac{a_T}{1-a_T(\rho-1)}} (1 + \lambda_E)^{\frac{a_T \phi_E (\rho-1)}{1-a_T(\rho-1)}} (1 + \lambda_N)^{\frac{a_N}{1-a_T(\rho-1)}}, \quad 1 \neq a_T(\rho-1)$$

which is a geometrically weighted average of intensive and extensive productivity growth in the traded good sector and of intensive productivity growth in the nontraded good sector.<sup>5</sup>

I obtain the growth rates  $G_{Y_N} = G_{Y_N^*} = 1 + \lambda_N$  for nontraded output. Traded quantities grow at rate

$$G_{Y_T} = G_{Y_H} = G_{Y_F} = G_{Y_T^*} = G_{Y_H^*} = G_{Y_F^*} = (1 + \lambda_H)(G_Y(1 + \lambda_E)^{\phi_E})^{(\rho-1)}.$$

For the number of firms it is true that  $G_H = G_{H_E} = G_{F_E^*} = G_{F^*} = G_Y(1 + \lambda_E)^{\phi_E}$  which implies that measured and actual terms of trade coincide along the nonstochastic growth path. The real wage grows at rate  $G_Y$  and labor hours in traded and nontraded sectors grow at zero rate. The real value of the average firm  $h$  actually shrinks to the extent that entry costs fall,  $G_{v_h} = G_{v_f^*} = (1 + \lambda_E)^{-\phi_E}$ . Inflation in the price of final composite output is  $\pi = \pi^* = \beta R / G_Y$ . Inflation of nontraded good prices is  $\pi_N = \pi_N^* = \pi G_Y / G_{Y_N}$  and inflation of traded good

<sup>5</sup>One obtains the scaling factor  $\bar{Y}_t$  of  $Y_t$  from  $G_Y$  as

$$\bar{Y}_t = \bar{A}_{Ht}^{\frac{a_T}{1-a_T(\rho-\alpha)}} \bar{A}_{Et}^{\frac{a_T \phi_E (\rho-\alpha)}{1-a_T(\rho-\alpha)}} \bar{A}_{Nt}^{\frac{a_N}{1-a_T(\rho-\alpha)}}.$$

Thus, along the nonstochastic growth path  $Y_t \propto \bar{Y}_t$  but not necessarily equal to. In order to obtain a stationary transformation of final composite output denominate final composite output as  $\tilde{Y}_t = Y_t / \bar{Y}_t$  where the wiggly indicates the stationary variable. Transforming all variables along these lines delivers the transformed model.

prices obeys  $\pi_T = \pi_H = \pi_F = \pi_{T^*} = \pi_{H^*} = \pi_{F^*} = \pi G_Y / G_{Y_T}$ . Accordingly, the Home relative price between traded and nontraded goods  $\bar{P}_{Tt} / \bar{P}_{Nt}$  declines at rate  $\pi_T / \pi_N = G_{Y_N} / G_{Y_T}$  for the parametrization pursued below. The Foreign relative price  $\bar{P}_{Tt}^* / \bar{P}_{Nt}^*$  behaves identically. Appendix C.3 provides transformed equilibrium conditions for the Home country where endogenous variables with wiggles indicate stationary variables.

### 4.3.2 Steady State and Dynamic Solution

A unique steady state exists for the transformed model. Appendix C.4 provides steady state solutions for selected Home quantities omitting the time subscript to indicate steady state values. I linearize Home and Foreign equilibrium conditions around this steady state and use the methods described in Sims (2002) to obtain the recursive law of motion for the linearized model which is shown numerically to have a unique and stationary solution. Appendix C.5 summarizes the linear model.

For numerical illustration of the model predictions I set elasticities of substitution in the tradable and nontradable sector  $\nu$  and  $\nu_N$  to 10 which implies a 11% steady state markup in each sector. The elasticity of substitution between Home and Foreign traded goods  $\mu$  is set to 2. Households stochastic discount factor  $\beta$  in Home and Foreign is set to 0.999 and the taste for variety parameter  $\rho$  is  $\nu / (\nu - 1)$  so that technology exhibits increasing returns to specialization in line with the standard Dixit Stiglitz index. The wage elasticity of labor supply  $1/\varphi$  is set to 1. The weight  $a_N$  in the composite final good  $Y_t$  ( $Y_t^*$ ) refers to goods produced domestically and is set to 0.4. The weight  $a_H$  in the aggregate traded good  $Y_{Tt}$  ( $Y_{Tt}^*$ ) refers to goods produced at Home and is also set to 0.4. The debt elasticity  $\phi$  of the Home nominal interest rate is set to 0.01. The exit probability of firms in the tradable sector  $\delta$  is set to 0.075 in line with evidence in Broda and Weinstein (2007). Absolute entry barriers  $q$  are normalized to unity and the steady state ratio  $\gamma_G$  of government over private consumption of the nontraded good is set to 1/3. The elasticity of entry costs with respect to new firms  $\phi_e$  is set to 0.1. Productivity growth in the tradable sector is set to 3 percent per year along both the intensive and the extensive margin which implies  $1 + \lambda_H = 1 + \lambda_E = 1.03^{1/4}$  interpreting a period in the model as one quarter. Productivity growth in the nontradable sector is 1 percent per year having  $1 + \lambda_N = 1.01^{1/4}$ . Both values imply an annual trend growth rate for  $\overline{GDP}_t^m$  of 2.6 percent in line with figures reported in Stockman and Tesar (1995) for a typical industrialized country. The nominal steady state interest rate  $R$  is set equal to 1.0125 which corresponds to 5 percent per annum. Jointly with  $\beta$  and  $G_Y$  this implies that aggregate long run inflation  $\pi$  is 2 percent per year. Shocks



$\hat{z}_{Ht}$ ,  $\hat{z}_{Et}$  and  $\hat{z}_{Gt}$  have autocorrelation coefficient 0.9 throughout. Shock standard deviations are normalized so that a shock of one standard deviation in size produces a one percent initial response in  $\widehat{GDP}_t^m$ . There is no need to attach numbers to parameters related to the monetary policy rule because once the transformed model is linearized around the steady state growth paths of real variables can be obtained independently from inflation, the nominal interest and nominal exchange rate growth.

Figure 4.2 shows the deterministic component of selected variables as implied by this parametrization. Evidently, composite final output increases along the nonstochastic growth path whereas the real exchange rate and terms of trade remain constant. The relative price of traded over nontraded goods in turn falls in line with Balassa Samuelson type effects.

## 4.4 Results

In this section I discuss dynamics of international relative prices induced by productivity catch up in the traded good sector. In particular, I discriminate between extensive productivity catch up (reduction in entry barriers) and intensive productivity catch up (increase in production efficiency). Dynamics induced by productivity catch up are contrasted with those following a demand side impulse which is advanced as alternative explanation for the patterns in international relative prices of new EU member states vis á vis the euro area.

Figure 4.3 shows dynamics induced by a catch up of extensive productivity  $\hat{z}_{Et}$  in Home as percentage deviation from the nonstochastic growth path. The catch up scenario is set up so that extensive productivity adjusts back to its hypothetical trend path. During the catch up period Home GDP grows, the real exchange rate  $\hat{e}_t$  appreciates and measured terms of trade  $\hat{\tau}_t^m$  improve. These model predictions are qualitatively consistent with the stylized facts in the data of new EU member states in figure 4.1. Interestingly, when export and import prices correctly account for product variety actual terms of trade  $\hat{\tau}_t$  remain above steady state during transition.

Figure 4.4 shows dynamics induced by a catch up scenario in intensive productivity  $\hat{z}_{Ht}$  in Home. For ease of comparison the extent of intensive productivity catch up is chosen so that measured GDP in Home deviates 1% from steady state in the initial period like in the case of extensive productivity catch up. As before, Home GDP grows and the real exchange rate appreciates. Contrary to extensive productivity catch up, however, measured and actual terms of trade now worsen during most of the transition. A catch up scenario in intensive productivity thus seems difficult to

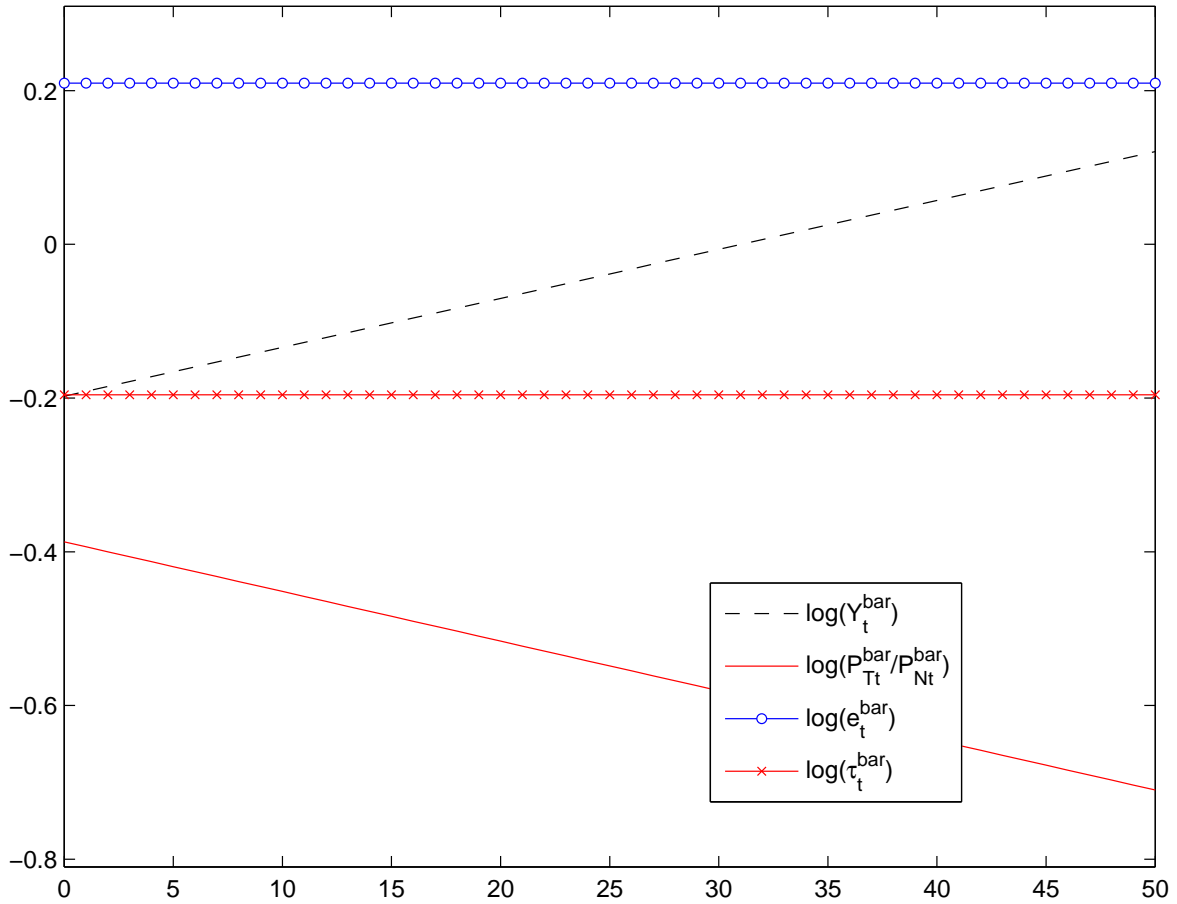


Figure 4.2: Deterministic components of selected variables. Deterministic components are computed as  $\log(\bar{Y}_t) = \log(\tilde{Y}) + \log(\bar{Y}_t)$ ,  $\log(\bar{e}_t) = \log(\bar{e})$ ,  $\log(\bar{\tau}_t) = \log(\bar{\tau})$  and  $\log(\frac{\bar{P}_{Tt}}{\bar{P}_{Nt}}) = \log(\frac{\bar{P}_T}{\bar{P}_N}) + (\rho - 1) \log(q) + [(\lambda_N - \lambda_H - (\rho - 1)\phi_E \lambda_E) \cdot t - (\rho - 1) \log(\bar{Y}_t)]$ .

reconcile with the observation that improvements rather than deteriorations in measured terms of trade are a predominant feature in new EU member states. Indeed, Egert (2007) concludes that the Balassa Samuelson effect is small at best for new EU member states.

Why do measured terms of trade improve during extensive productivity catch up, but deteriorate during intensive productivity catch up? The core difference between either scenario is that extensive productivity does not affect marginal costs directly whereas intensive productivity does so. This difference affects price dynamics of traded products because these prices are set as constant markup over marginal costs.

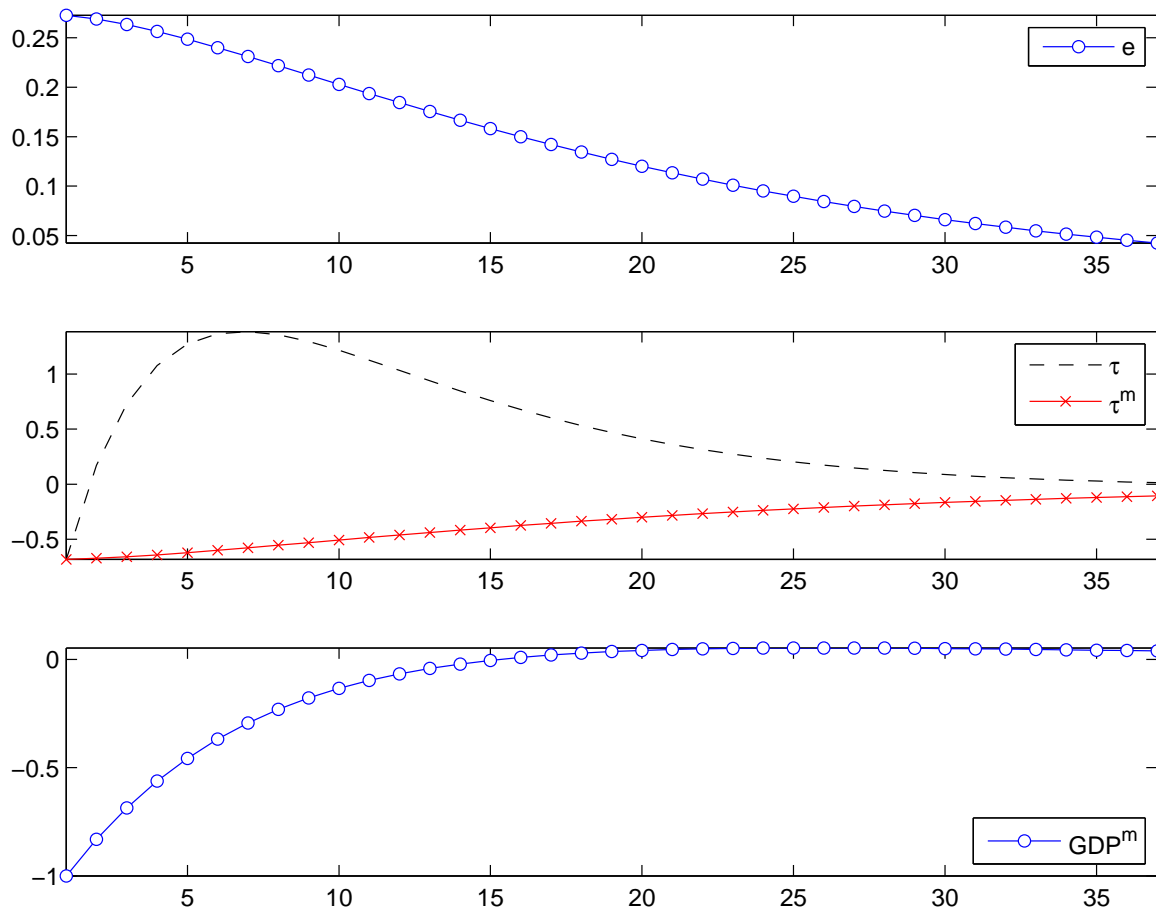


Figure 4.3: Impulse response function of endogenous variables as percentage deviation from non-stochastic growth path after negative  $\hat{z}_{Et}$  entry shock with standard deviation scaled so that measured Home GDP deviates 1% from steady state in the first period.

Figure 4.5 complements figure 4.3 by showing adjustment dynamics of more variables to the catch up in extensive productivity. Entry barriers above their nonstochastic growth path imply that few firms operate in the Home tradable sector so that product variety is poor. Labor is an abandoned factor because business activity related to firm entry is low. When entry barriers decrease during the catch up period labor demand increases due to firm entry. Accordingly, wages and thereby marginal costs  $\hat{w}_t - \hat{z}_{Ht}$  pick up, which also holds true for the relative price  $\hat{p}_{ht}$  proportional to marginal costs. Measured terms of trade improve because the Foreign price for traded products  $\hat{p}_{ht}$  in Home currency is less sensitive to Home productivity catch up. On the contrary, actual terms of trade which account contemporaneously for changes in relative pro-

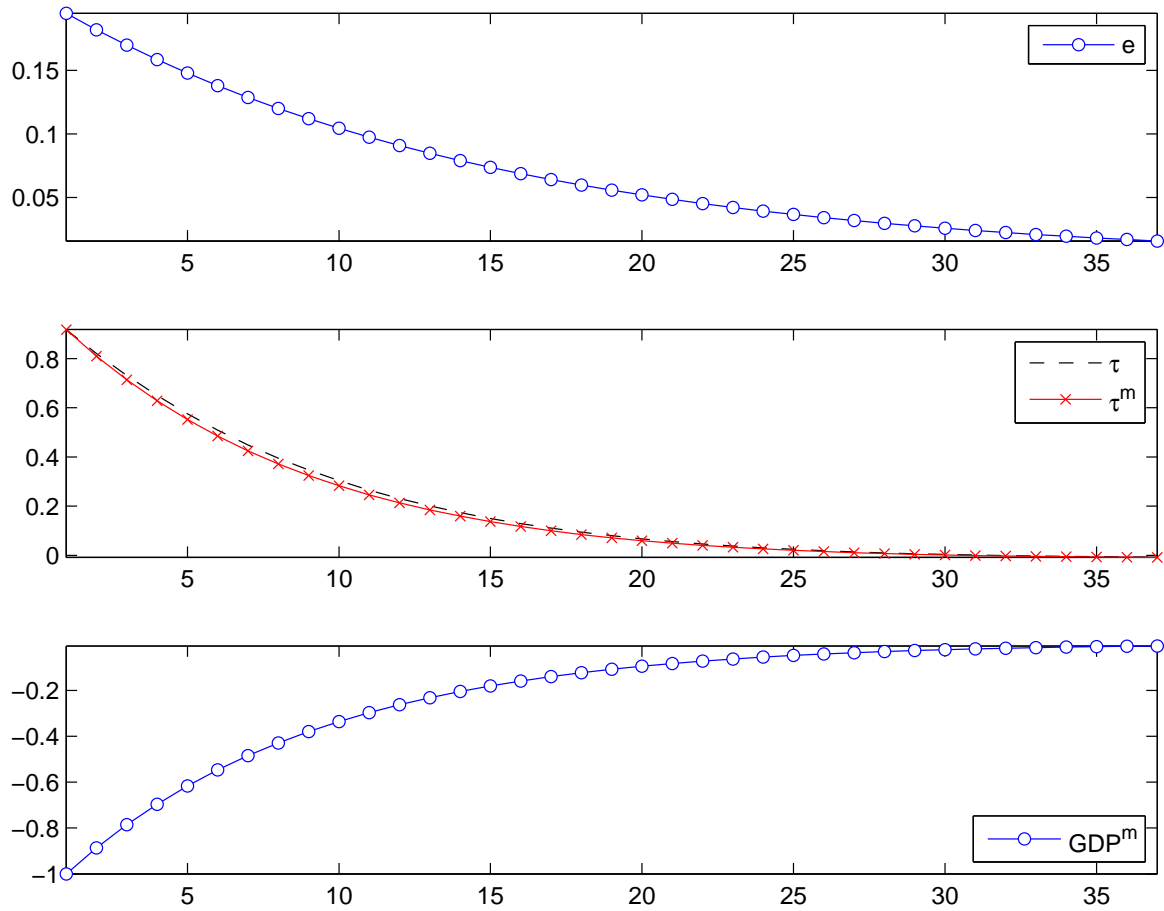


Figure 4.4: Impulse response functions of endogenous variables as percentage deviation from non-stochastic growth path after negative  $\hat{z}_{Ht}$  productivity shock with standard deviation scaled so that measured Home GDP deviates 1% from steady state in the first period.

duct variety deteriorate because Foreign product variety declines relative to Home product variety. The effect of product variety also shows up in the price  $\hat{p}_{Ht}$  of the Home traded good which is above steady state during transition. Low product variety is adverse for technology that features returns to specialization so that the good price declines when product variety starts to increase during the catch up. By the law of one price  $\hat{p}_{Ft}^*$  is high, too, and Foreign households absorb this negative spillover by replacing expensive imports with  $\hat{Y}_{Ft}^*$  and by reducing consumption. Fostered demand for  $\hat{Y}_{Ft}^*$  increases Foreign product variety slightly. Demand pressure on  $\hat{p}_{Ft}^*$  is offset by low wages because the reduction in consumption reduces labor demand. Prices  $\hat{p}_{Ft}^*$  and  $\hat{p}_{ft}^*$  remain extremely similar due to negligible dynamics in  $\hat{F}_t^*$ . By the law of

one price  $\hat{p}_{ft}^*$  is easily converted into  $\hat{p}_{ft}$  by adding the real exchange rate. Low initial product variety in Home implies that Home wages remain below Foreign wages so that  $\hat{p}_{ht}$  remains below  $\hat{p}_{ft}$  and measured terms of trade improve during transition.<sup>6</sup>

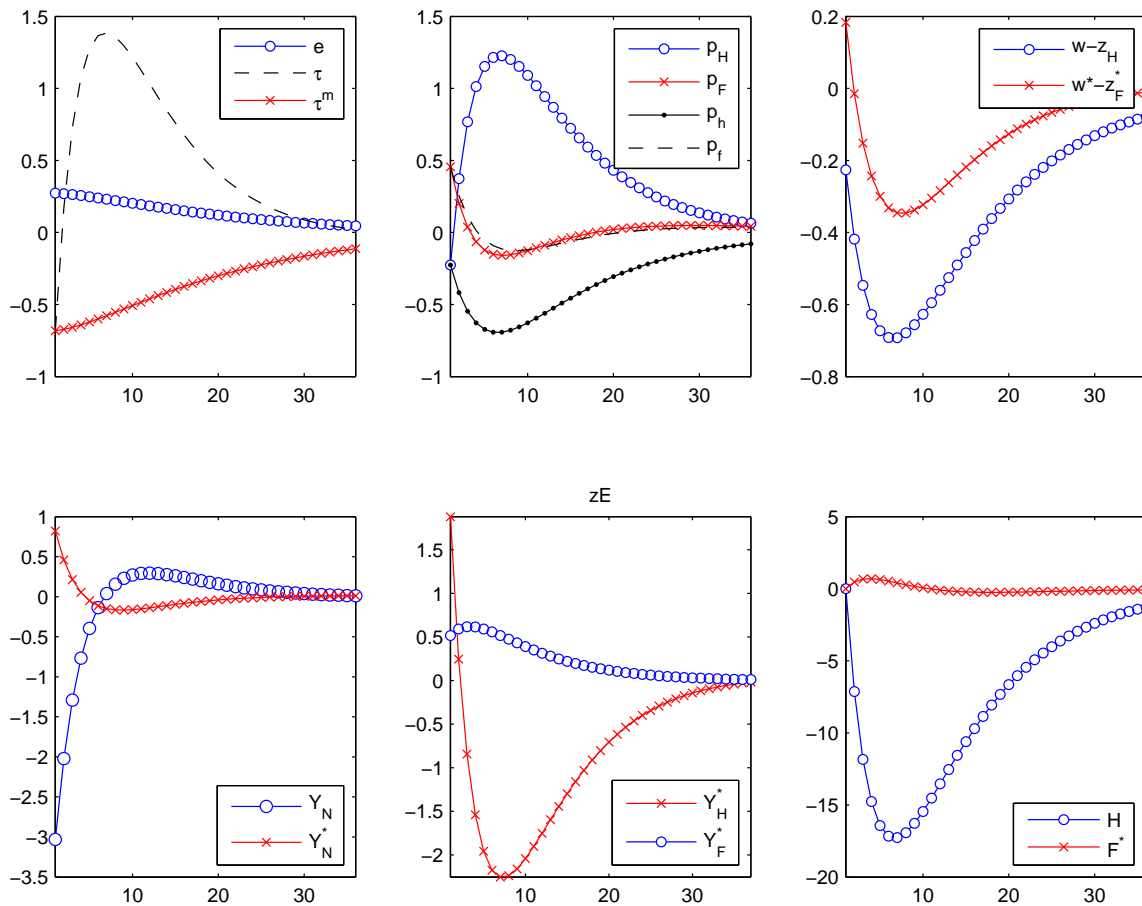


Figure 4.5: Impulse response functions of endogenous variables as percentage deviation from non-stochastic growth path after negative  $\hat{z}_{Et}$  entry shock with standard deviation scaled so that measured Home GDP deviates 1% from steady state in the first period.

Figure 4.6 complements figure 4.4 to contrast adjustment dynamics of extensive productivity catch up with those implied by intensive productivity catch up. Intensive productivity catch up makes marginal costs fall back to their nonstochastic growth path in the traded good sector because intensive productivity catch up reduces the

<sup>6</sup>Formulating entry barriers in units of labor also produces an appreciating real exchange rate and improving terms of trade. However, these price dynamics are accompanied by a decrease in composite final output because production in established firms falls due to firm entry.

effective wage. Accordingly, prices  $\hat{p}_{Ht}$  and  $\hat{p}_{ht}$  fall so that measured and actual terms of trade deteriorate. Intensive productivity catch up induces product variety to grow on its way back to steady state because advances in intensive productivity increase expected profits so that market entry becomes profitable to more firms. Nevertheless, discrepancies between actual and measured prices  $\hat{p}_{Ht}$  and  $\hat{p}_{ht}$  are less of a concern now because changes in intensive productivity affect product variety much less. As before, wage growth in Home is more pronounced than that in Foreign during the adjustment period and relative wages coincide with the relative price of Home versus Foreign nontraded goods absent changes in nontraded productivity. In turn, the relative price of nontraded goods equals the real exchange rate because the law of one price holds for traded goods. To this end, wage dynamics translate into real exchange rate appreciation.

Figure 4.7 shows adjustment dynamics induced by fostered growth in government consumption of the Home nontraded good. Dynamics of Home GDP, the real exchange rate and measured terms of trade are qualitatively indistinguishable from a catch up scenario in extensive productivity. At first sight, this seems to warrant the view that appreciating real exchange rates and improving terms of trade in new EU member states are demand driven phenomena. However, the demand side explanation has diametrically different implications concerning product variety. Whereas extensive productivity catch up extends product variety, fostered government consumption actually contracts product variety. The evidence in Kandogan (2006) that recent export growth in new EU member states is driven by the extensive margin to a large extent would thus make a case for extensive productivity catch up rather than the demand side explanation. Eventually, it remains an interesting empirical matter for future research to determine how much variation in the data should be attributed to each explanation.

## 4.5 Conclusion

In the data real exchange rates appreciate and terms of trade improve for new EU member states and vis á vis the euro area. This paper extends the standard two country two sector RBC model to account for both facts simultaneously by allowing for endogenous product entry and by accounting for frequently documented measurement bias in observed price indices which fail to track changes in product variety in time. Model predictions for a catch up scenario in which productivity reduces barriers to product entry are qualitatively consistent with observed data. Also, a demand side

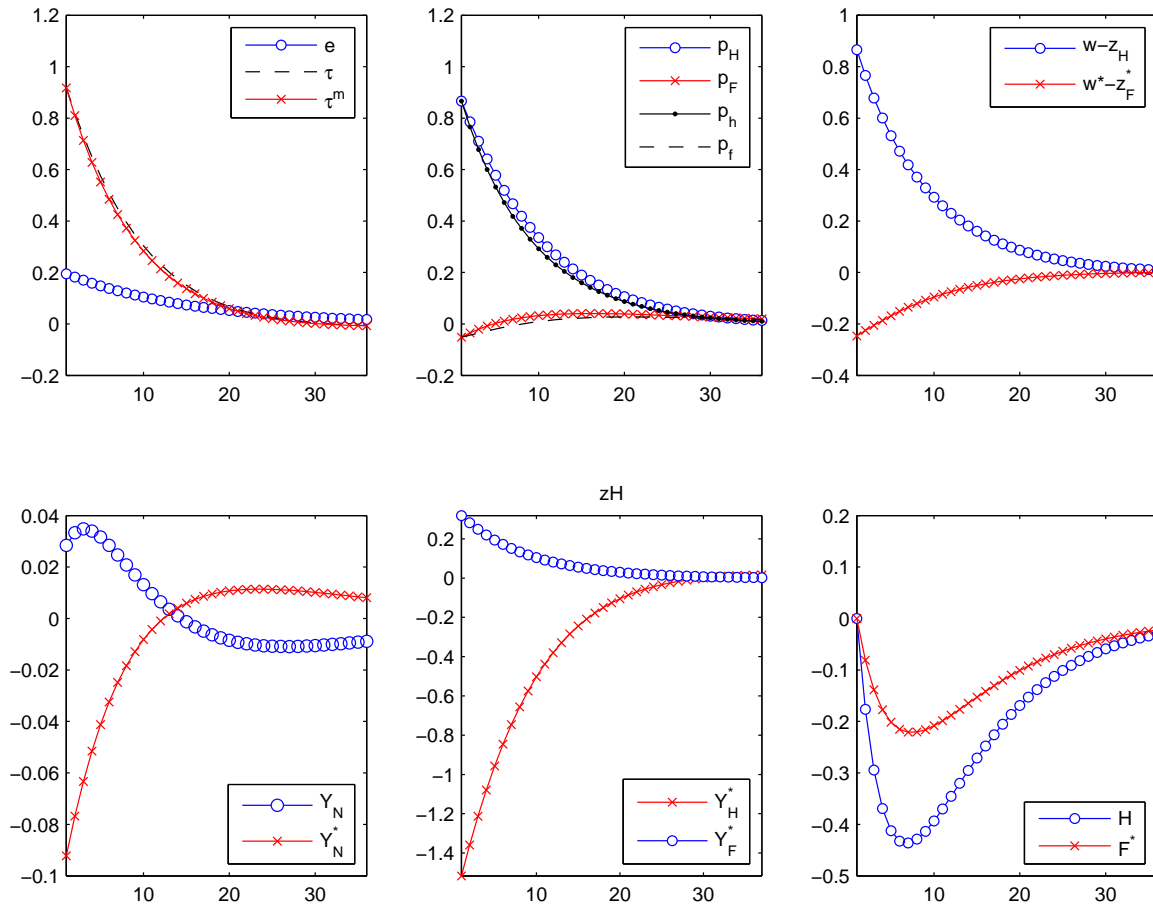


Figure 4.6: Impulse response functions of endogenous variables as percentage deviation from non-stochastic growth path after negative  $\hat{z}_{Ht}$  productivity shock with standard deviation scaled so that measured Home GDP deviates 1% from steady state in the first period.

impulse is shown to reproduce patterns in international relative prices found in the data, though it has different implications for product variety. The model lends itself to predict by how much and for how long inflation in new EU member states remains high when nominal exchange rates are fixed with entry into the European monetary union.

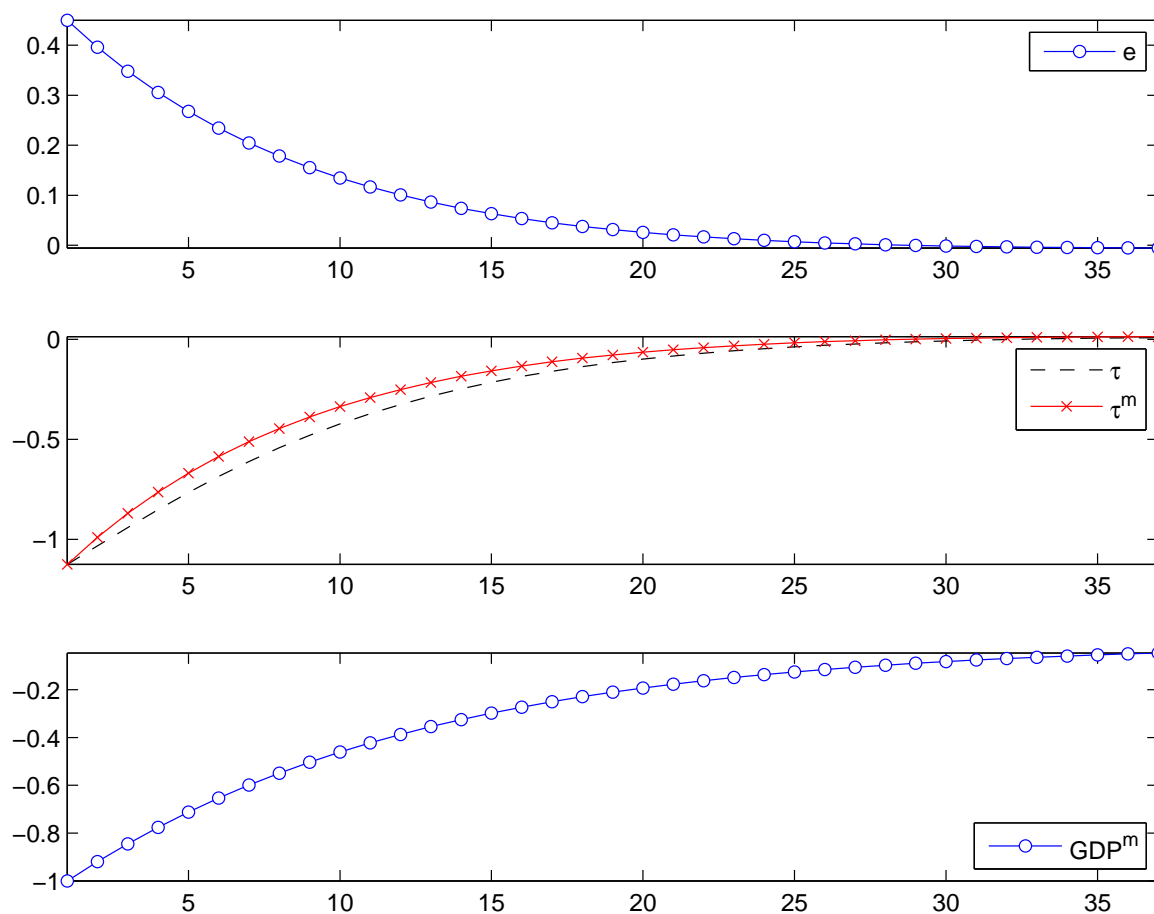


Figure 4.7: Impulse response functions of endogenous variables as percentage deviation from non-stochastic growth path after negative  $\hat{z}_{gt}$  government spending shock with standard deviation scaled so that measured Home GDP deviates 1% from steady state in the first period.

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## Chapter 5

# When, How Fast and by How Much do Trade Costs change in the Euro Area?

Empirical models set up to analyze the trade effect of the euro are often restrictive. We pursue a more general approach to estimate the When, How Fast and by How Much of adjustment in trade costs. Beyond the more general transition path our approach allows for sector specific impact of trade costs on sectoral trade while controlling for unobserved variation in trade costs at the sector level. We find gradual adjustment in trade costs between the years 2000 and 2003. Adjustment of individual sectors is extremely fast whereas aggregate adjustment spreads out because different sectors adjust at distinct times. Timing of the change in trade costs matters for conclusions about the size of this change. We provide independent evidence for the view that the euro has fostered euro area trade to the extent that we estimate rather than postulate the break point in the transition path of trade costs.

### 5.1 Introduction

The literature that assesses the impact of the euro on euro area trade has converged to a fairly uniform empirical model setup. The setup is based upon Gravity theory which links trade to trade costs, measures of inward and outward resistance and economic activity (Anderson and van Wincoop (2003), Anderson and van Wincoop (2004),

Baldwin (2006a)). Typically, Gravity theory is implemented as panel regression with a cross section that comprises countries both inside and outside the euro area. Trade costs contain a euro dummy which is zero before and unity after the introduction of the euro and the euro's trade effect is measured as the difference between the level shift for euro area insiders versus the one for euro area outsiders. The standard setup also accounts for the fact that the bulk of trade costs and the proxies for inward and outward resistance are either poorly measured or entirely unobserved by using time dummies as stand in for more accurate and explicit information. Frequently, time dummies are common to the entire cross section.

We attempt to generalize three substantive restrictions to which the conventional setup routinely refers. First, the euro dummy restricts the trade effect of the euro to materialize immediately and exhaustively with the introduction of the common currency. We replace the euro dummy by a smooth transition path and estimate timing, size and speed of adjustment of this path. The immediate and exhaustive level shift remains a special case in this specification. Smooth transition is flexible enough to capture anticipation effects, delayed and gradual adjustment and long run effects. Micco, Stein, and Ordonez (2003) and Flam and Nordström (2006b) report anticipation effects in euro area trade in the year before the euro introduction. DeNardis and Vicarelli (2007) argue that the euro may create trade effects in the medium and long run because home bias in preferences extends to former foreign markets in the course of time. Smooth transition has the potential to detect these effects because it does not force transition to complete within sample. Berger and Nitsch (2008) argue that several major events are candidates for changes in trade costs even though January 1999 has become the convention. The candidates for our sample are end of 1997 (the beginning of the third stage of the Economic and Monetary Union (EMU) is decided), January 1999 (factual start of third stage of EMU) and January 2002 (introduction of the euro as physical currency). To this end, a specification that is flexible with respect to the timing of the change in trade costs is a benefit.

Second, the standard setup restricts the level shift in trade costs to be identical among all euro area insiders and to be identical for euro area outsiders. Furthermore, the impact of proxies for trade costs others than the level shift is often restricted to be identical for the entire panel. We employ panel data that discriminate exports by partner country and trade sector. For example, the data comprise German exports to France in the Iron and Steel sector. Taking a mean group perspective (Pesaran and Smith (1995)) we allow each trade sector in each trade relationship to respond differently to changes in trade costs. Anderson and van Wincoop (2004) expect effects of switching from national to common currency to differ considerably across countries.

Suppose that one euro area country trades with countries both inside and outside euro area whereas another euro area country trades only within the euro area. If switching to a common currency reduces trade costs within the euro area the country with trade partners both inside and outside euro area experiences a change in relative trade costs whereas the other country does not. Changes in relative trade costs redirect trade which constitutes a national euro effect. Sector specific euro effects add to effects at the national level. Taglioni (2002) emphasizes that the extent of vertical differentiation, the magnitude of economies of scale, the degree of industrial concentration, the size of non-tariff barriers, the relative location of reference markets and competitors differ substantially across sectors. Moreover, sector specific exposure to exchange rate risk as a result of pricing strategies or a size distribution of firms that differ across sectors potentially implies asymmetric euro effects.

Third, the standard setup typically restricts unobserved or omitted trade costs and inward and outward resistance terms to be identical for all trade relationships and all trade sectors due to the use of common time dummies. We interpret this category of variables as latent states which we then estimate by means of the Kalman filter. This specification allows us to control for unobserved variation at the level of trade sectors and, at the same time, addresses the short (data) history of the euro because the specification is very parsimonious. Admittedly, we exchange parsimony against some computational complexity since state space modeling with latent variables requires numerical optimization. However, as Baldwin (2006a) points out the use of common time dummies is at odds with Gravity theory since inward and outward resistance terms are trade relationship specific. At the sector level Gravity theory predicts sector specific resistance terms (Anderson and van Wincoop (2004)). Tractability seems a major reason for imposing common time dummies because in the standard setup one set of time dummies for each trade relationship immediately exhausts all degrees of freedom.

Our main finding is that euro area trade increases relative to trade among euro area outsiders between the years 2000 and 2003 by 10 to 20 percent according to mean estimates. Contrary to the convention to specify a level shift in 1999 our results indicate that the transition period only starts about one year *after* the introduction of the euro. We obtain these conclusions about the When of the effect while leaving the timing of transition unrestricted. The timing of the change in euro area trade fits remarkably well with the third stage of EMU and the introduction of the euro as physical currency and provides independent evidence for the view that the common currency indeed reduces trade costs and, ultimately, is responsible for the increase in euro area trade. The fact that significant changes in trade between the euro area

and euro area outsiders occur during almost the same period of time adds to this interpretation. Our results neither support findings of anticipatory activity in other studies nor suggest that there is much scope left for long run effects to develop.

According to our estimates adjustment to the common currency at the aggregate level takes about three to four years whereas adjustment at the level of sectors is much faster. Aggregate adjustment is more spread out and gradual because different sectors adjust at different times. This conclusion holds true for trade within the euro area, trade among euro area outsiders and trade between the euro area and euro area outsiders. The high speed of adjustment at the sector level squares well with recent microfoundations of the euro's trade effect put forth in Baldwin and Taglioni (2005). These authors argue that adjustment of trade to the introduction of the euro is fast because reduction of exchange rate volatility induces a large number of small firms to enter export markets.

The extent by How Much trade within the euro area changes relative to trade among outsiders is somewhat larger though still consistent with earlier findings in the literature. However, mean estimates suggest that restricting the effect of trade costs to be identical for all sectors reduces the size of adjustment. We find strong evidence for sector specific coefficients when testing a general specification against a model which imposes effects of trade costs to be homogenous across sectors. This suggests that results based on the standard setup with coefficients common across trade sectors underestimates the extent by which trade costs adjust. For robustness we employ export and import data separately and even though import data seem less informative fairly similar results emerge qualitatively and quantitatively.

In the vast literature on the trade effect of the euro several studies exist which do not resort to all three restrictions which we propose to generalize here.<sup>1</sup> Flam and Nordström (2006a), Flam and Nordström (2006b), Micco, Stein, and Ordóñez (2003) and DeNardis and Vicarelli (2007) use repeated year dummies interacted with a euro dummy specific to the euro area to make inference on the timing of the effect. Baldwin and Taglioni (2006), Flam and Nordström (2006b), and Nitsch (2006) work with disaggregated sectoral data and estimate sector specific and sometimes also trade relationship specific coefficients.

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<sup>1</sup>Baldwin (2006b) and Baldwin (2006a) comprise exhaustive surveys of this literature.

## 5.2 Theory

Gravity theory relates equilibrium exports to the product of foreign expenditure and home production and to trade costs relative to resistance terms,

$$X_k^{ij} = E_k^j Y_k^i \left( \frac{\tau_k^{ij}}{\Pi_k^i P_k^j} \right)^{1-\sigma_k}. \quad (5.1)$$

Nominal equilibrium exports of reporter country  $i$  to partner country  $j$  in sector  $k$  are denoted  $X_k^{ij}$ , nominal expenditure in this sector is  $E_k^j$ , and nominal production is  $Y_k^i$ . The sectoral elasticity of substitution is  $\sigma_k > 1$ . The trade cost function  $\tau_k^{ij}$  summarizes all trade costs and is specified below. If bilateral trade costs  $\tau_k^{ij}$  are reduced say because a common currency decreases transaction costs exports from  $i$  to  $j$  increase. The variables  $P_k^j$  and  $\Pi_k^i$  represent inward and outward resistance terms, respectively. Both terms are weighted averages of bilateral trade costs relative to the welfare based price levels of the respective trading partner. Weights reflect the size of a sector. In particular, if importing country  $j$  faces high trade costs with respect to exporters other than  $i$  this increases inward resistance  $P_k^j$  and exports from  $i$  to  $j$  increase. Outward resistance  $\Pi_k^i$  reflects the notion that if from  $i$ 's perspective trade costs are higher for markets other than  $j$  more will be exported from  $i$  to  $j$ .

Neither resistance terms nor trade costs have easily to access empirical correspondents. For instance, Gravity consistent resistance terms require a stand on which foreign products compete with national products. Since we work with a small trade matrix we are likely to miss many important substitutes. Moreover, Baier and Bergstrand (2001), Feenstra (2003) and Anderson and van Wincoop (2004) point out that measured export and import price indices do not align with resistance terms because, for instance, such indices do not reflect home bias in consumer preferences. Adding to these difficulties we are likely to omit many important trade cost variables since detailed trade cost data is rare in general and even harder to obtain for a panel of trade flows at annual or even monthly frequency and at sector level.<sup>2</sup>

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<sup>2</sup>Anderson and van Wincoop (2004) survey trade costs and their availability. In the following we do not impose symmetry of sector specific trade costs because this assumption is particularly restrictive when working with sectoral trade data.

## 5.3 Empirical Model

### 5.3.1 Basic Setup

We interpret Gravity theory (5.1) as state space system which provides a conceptually straightforward account of unobserved trade costs and resistance terms. The log linear equation (5.1) jointly with a specification of measurable trade costs represents the observation equation of the state space system. Resistance terms and omitted trade cost variables are absorbed into the state equation. In this section we describe the most general panel model we take into consideration. When conducting specification tests below we describe the restrictions imposed to arrive at less general models. Let  $i$  denote reporters,  $j$  partners,  $k$  trade sectors and  $t$  time. Subsume reporter and partner indices under the trade relationship index  $s = ij, i \neq j$  and let  $S(K, T)$  denote the maximum number of trade relationships (sectors, observations). For each trade relationship  $s$  we estimate the following model:

$$y_{kt}^{(s)} = q_{it}\beta_{ik}^{(s)} + q_{jt}\beta_{jk}^{(s)} + (1 - \sigma^{(s)})[\ln(\tau_{kt}^{(s)}) + \lambda_{kt}^{(s)}] + u_{kt}^{(s)}, \quad (5.2)$$

$$\ln(\tau_t^{(s)}) = \theta_{0k}^{(s)} \left[ 1 + \exp\{-\theta_{1k}^{(s)}(\tilde{t} - \zeta_k^{(s)})\} \right]^{-1} + (Z_t^{(s)})' \boldsymbol{\gamma}_k^{(s)} + c^{(s)}, \quad (5.3)$$

$$\lambda_{kt}^{(s)} = \lambda_{kt-1}^{(s)} + v_{kt}^{(s)}, \quad (5.4)$$

$$u_{kt}^{(s)} \sim N(0, g_k^{(s)}) \quad , \quad v_{kt}^{(s)} \sim N(0, h_k^{(s)}) \quad , \quad E[u_{kt}^{(s)} v_{kr}^{(s)}] = 0 \quad \forall t, r. \quad (5.5)$$

Observation equation (5.2) specifies the log of sector  $k$  exports  $y_{kt}^{(s)}$  for trade relationship  $s$  conditional on scale variables  $q_{it}$  and  $q_{jt}$ , the log of measurable trade costs  $\tau_{kt}^{(s)}$  and the log of unobserved trade costs and resistance terms  $\lambda_{kt}^{(s)}$ . Equation (5.3) formalizes the log of measurable trade costs as a smooth transition path contained in square brackets plus measurable control variables  $Z_t^{(s)}$  and a constant  $c^{(s)}$ . Following Luukkonen, Saikkonen, and Teräsvirta (1988) we model smooth transition as a logistic distribution function where  $\theta_{0k}^{(s)}$  measures the size of transition while  $\theta_{1k}^{(s)} > 0$  governs the speed of transition. In order to immunize  $\theta_{1k}^{(s)}$  against the scale of the time index  $t$  the latter is standardized as  $\tilde{t} = t / (T \times \sqrt{0.08333})$  (Bauwens, Lubrano, and Richard (2000)). The transition path is centered around the coefficient  $\zeta_k^{(s)}$  which we estimate. Thus, we refrain from imposing a fixed break point in trade costs a priori and allow for gradual as well as immediate and exhaustive adjustment in trade costs. The state equation (5.4) specifies the evolution of sector specific unobserved variables



as a random walk process. Residuals of the observation equation  $u_{kt}^{(s)}$  and of the state equation  $v_{kt}^{(s)}$  are assumed uncorrelated at any lead or lag.

Control variables  $Z_t^{(s)}$  include real effective exchange rates of the reporting country  $reer_t^{(s)}$ , real bilateral exchange rates between both trading partners  $rex_t^{(s)}$  and real exchange rates of the reporting country relative to the U.S.  $rexus_t^{(s)}$ . Adding bilateral exchange rates among trading partners and with the U.S. separately gives a prominent role to variation in these two prices beyond their appearance in the real effective exchange rate. Appendix D.4 describes the construction of these variables. Flam and Nordström (2006b) and Baldwin (2006a) emphasize that exchange rates help to discriminate potential substitution effects due to changes in international prices from effects of introducing the common currency. When the euro devalued after its introduction products sold in euro became cheaper relative to products sold e.g. in U.S. dollar. This change in relative prices may have been redirecting part of euro area demand for foreign products back to the euro area. If such effects are not controlled for, the model may falsely attribute them to the introduction of the common currency. Also,  $Z_t^{(s)}$  includes a measure of exchange rate volatility  $vol_t^{(s)}$  to control for a potential link between exchange rate risk and trade. We describe construction of exchange rate volatility below. Finally, we add an index of energy prices  $en_t^{(s)}$  as a measurable proxy for transportation costs.

The dependent variable  $y_{kt}^{(s)}$  in (5.2) is likely to be nonstationary according to frequently inferred time series features.<sup>3</sup> In this case the empirical model may suffer from spurious findings in the sense that coefficient estimates of nonstationary right hand side variables fail consistency. Accordingly, balancing the regression model (5.2) – (5.5) requires at least one nonstationary variable on the right hand side. We regard scale variables  $q_{it}$  and  $q_{jt}$  and exchange rate measures as candidates to cointegrate with the dependent variable. Moreover, the latent state variable in (5.4) evolves nonstationary and is a further candidate for cointegration. To guard against spurious regressions we diagnose the stochastic features of model implied residuals  $u_{kt}^{(s)}$ . In case the latter residual processes are stationary, variables entering the observation equation are either stationary or nonstationary but cointegrated processes. Chang, Miller, and Park (2008) derive that the common (Q)ML interpretation of modeling stationary processes by means of the Kalman filter also applies for multivariate nonstationary

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<sup>3</sup>Corresponding unit root tests powerfully underscore the likelihood of stochastic trends in  $y_{kt}^{(s)}$ . In the light of the plentitude of time series entering the empirical models we refrain from providing detailed results on unit root testing. Doing so reveals that almost uniformly first differences of employed time series are stationary so that the highest order of stochastic trending is one.

processes sharing a common trend. As a consequence the validity of standard specification tests as e.g. likelihood ratio (LR) tests does not rely on the stationarity of  $y_{kt}^{(s)}$  or conditioning variables.

### 5.3.2 Functional Coefficients and Estimation

We collect all coefficients in the two vectors

$$\boldsymbol{\psi}_k^{(s)} = \left( \beta_{ik}^{(s)}, \beta_{jk}^{(s)}, \theta_{0k}^{(s)}, \theta_{1k}^{(s)}, \zeta_k^{(s)}, \boldsymbol{\gamma}_k^{(s)'}, h_k^{(s)}, g_k^{(s)} \right)' \quad \text{and} \quad \boldsymbol{\phi}^{(s)} = (\sigma^{(s)}, c^{(s)})'$$

where  $\boldsymbol{\psi}_k^{(s)}$  comprises sector specific coefficients and  $\boldsymbol{\phi}^{(s)}$  comprises coefficients not specific to sectors. To estimate sector specific coefficients  $\boldsymbol{\psi}_k^{(s)}$  we presume a parsimonious functional representation in which sector specific coefficients equal a common intercept term and slope coefficient multiplied by a sector specific scalar  $a_k^{(s)}$ ,

$$\boldsymbol{\psi}_k^{(s)} = (\mathbf{1} + \boldsymbol{\psi}_1^{(s)} a_k^{(s)}) \odot \boldsymbol{\psi}_0^{(s)}. \quad (5.6)$$

Here  $\mathbf{1}$  is a unit vector of appropriate dimension and  $\boldsymbol{\psi}_1^{(s)}$  and  $\boldsymbol{\psi}_0^{(s)}$  are vectors of unconditional coefficients. The operator ' $\odot$ ' signifies 'element-by-element' multiplication and the scalar  $a_k^{(s)}$  with  $\sum_k a_k^{(s)} = 1$  reflects the importance of sector  $k$  in reporting country  $i$ . Precisely, denote the relative average quantity traded in sector  $k$  as weight  $w_k^{(s)}$  and denote the rank associated with  $w_k^{(s)}$  conditional on  $s$  with  $\tilde{w}_k^{(s)}$ . The importance of sector  $k$  then is  $\tilde{a}_k^{(s)} = \tilde{w}_k^{(s)} / (\sum_k \tilde{w}_k^{(s)})$  and a corresponding mean zero weighting sequence is  $a_k^{(s)} = \tilde{a}_k^{(s)} - (1/K) \sum_k \tilde{a}_k^{(s)}$ .<sup>4</sup> Equation (5.2) restricts the elasticity of substitution  $\sigma^{(s)}$  to remain common for all sectors conditional on trade relationship  $s$ . Allowing for a linear functional relationship in the elasticity of substitution would produce a quadratic functional relationship in export elasticities with respect to e.g. control variables contained in  $Z_t^{(s)}$  because such elasticities comprise the product of  $\sigma^{(s)}$  with coefficients which already depend linearly on the functional variable  $a_k^{(s)}$ . We avoid such quadratic relationships by imposing  $\sigma_k^{(s)} = \sigma^{(s)}$ .

Ideally, sector specific coefficients should be flexible enough to reflect the many dimensions which make sectors transit differently and respond differently to changes

<sup>4</sup>Appendix D.1 describes computation of weights. As an alternative to the rank based weighting scheme we also experimented with original weights  $w_k^{(s)}$ . In this case functional estimates were dominated by a few very large sectors almost uniformly over all trade relationships because the distribution of weights is heavily skewed in that there are a few very large sectors but many small ones. This skewness is amplified by the fact that sector specific coefficients and thereby functional variables enter the model in a nonlinear fashion.

in control variables. To name only a few, sectors are likely to differ with respect to the tightness of competition, with respect to pricing strategies and strategic interaction or regarding the degree of product substitutability. Our functional specification pretends that all relevant dimensions are reasonably well represented by a sector's market size which obviously is not the case. However, besides it being an operative measure we nevertheless believe that market size is a useful functional variable in that it correlates with at least the more relevant dimensions.<sup>5</sup>

(Quasi) Maximum likelihood (QML) estimation of the empirical model deserves iterative optimization due to nonlinearities in model coefficients and the presence of the latent processes  $\lambda_{kt}^{(s)}$ . A few coefficients entering the state space model are estimated conditional on a restricted support. First, variance parameters are determined as exponentials of underlying parameters to ensure positive variation measures,

$$g_k^{(s)} = \exp(\underline{g_k^{(s)}}) \quad \text{and} \quad h^{(s)} = \exp(\underline{h_k^{(s)}}),$$

where the convention of underlining signifies that log likelihood optimization is done, for instance, with respect to  $\underline{g_k^{(s)}}$  rather than  $g_k^{(s)}$ . The second set of restrictions applies to coefficients of the smooth transition function. One observes that the term in squared brackets in (5.3) degenerates to a constant as  $\theta_{k1}^{(s)}$  tends to zero. The state space model might lack identification in this case because trade costs already comprise a constant. Accordingly, we restrict the support of  $\theta_{k1}^{(s)}$  to strictly positive values. Fastest 90% of transition is restricted to one month by imposing  $\theta_{k1}^{(s)}$  smaller than 232.89. Also, we impose bounds on the parameter space of the symmetry point  $\zeta_k^{(s)}$  to prevent the transition function from singling out the first twelve and the last twelve sample observations. Appendix D.1 provides details to obtain such bounds. We implement parameter restrictions with the cumulative Gaussian  $\Phi$ ,  $0 < \Phi < 1$ , as

$$\theta_{1k}^{(s)} = 232.89 \Phi(\underline{\theta_{1k}^{(s)}}) \quad \text{and} \quad \zeta_k^{(s)} = 0.30343 + 2.8826 \Phi(\underline{\zeta_k^{(s)}}).$$

Finally, we ensure  $\sigma^{(s)} > 1$  in line with economic theory. For optimizing over explicit or underlying coefficients obtaining  $(\boldsymbol{\psi}_k^{(s)'}, \boldsymbol{\phi}^{(s)'})'$  the *optmum* routine in GAUSS is used.

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<sup>5</sup>Alternative functional variables would be number of firms, profits, markups, exchange rate pass through or fixed costs of production. Besides data availability considerations relying on several functional variables would considerably boost the parameter space and render optimization a rather challenging task. Factor analysis is one means to reduce dimension and may turn out an interesting extension of the setup considered here.

## 5.4 Data

We use monthly bilateral export data from January 1995 to May 2006 (137 months). In EUROSTAT's COMEXT database export data is available in value (current euro) and volume (tons) and is disaggregated according to the HS two digit level. The HS classification provides a break down of aggregate trade into 99 trade sectors of which we consider  $K = 96$ .<sup>6</sup> Baldwin (2006a) discusses reasons why export and import data may differ in practice. Therefore, to check robustness of our results we explore our specification of Gravity for both export and import data which is also drawn from COMEXT. For estimation we convert export and import data into year 2000 euros. We rely on monthly data to collect as much information as possible around the hypothesized break point. At the two digit level trade data for some sectors is plagued by irregularly missing observations. We do not exclude such sectors from our analysis since Kalman filter recursions are easily modified to cope with irregularly missing observations. Appendix D.2 provides details on the employed Kalman filter. Hence, the empirical analysis does not suffer from imputed measures replacing missing observations and is not subject to sample selection bias as a consequence of excluding a potentially nonrandom fraction of trade sectors from the analysis.

Our trade matrix comprises Germany, France, Italy, the United Kingdoms, Sweden and Denmark so that we obtain  $S = 30$  trade relationships. Out of these 30 trade relationships six involve countries which both have adopted the euro (U2), six involve countries which both have *not* adopted the euro (O2), nine involve countries where the reporting country has adopted the euro but the partner country has national currency (OUT) and nine involve countries where partner country has adopted the euro but the reporter has not (IN). The first column of table 5.2 lists these trade relationships explicitly. We restrict attention to European Union (EU) member states to maintain as much homogeneity as possible along the country dimension. EU countries have been subjected to similar legislation and regulation in the wake of the European Single Market initiative after 1993.<sup>7</sup> The three EU countries with national currencies serve as reference group to which we compare effects in the three largest euro area countries.

Gravity theory suggests to use data on sector production and sector expenditure for scale variables. We use indices of industrial production as proxy for sector produc-

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<sup>6</sup>See <http://epp.eurostat.ec.europa.eu/newxtweb/setupdimselection.do> for COMEXT database. Sectors 77 and 98 do not contain any data for our sample. Also we drop sector 99 ('Other Products'). HS abbreviates Harmonized Commodity Description and Coding System. Baldwin and Taglioni (2006) address a wide range of possible misspecifications of the Gravity equation related to data construction.

<sup>7</sup>Berger and Nitsch (2008) discuss European integration and its interference to EMU trade.

tion and sector expenditure but allow for sector specific coefficients  $\beta_{ik}^{(s)}$  and  $\beta_{jk}^{(s)}$  to mitigate inferior data quality. Baldwin, Skudelny, and Taglioni (2005) estimate Gravity equations for two digit trade sectors and compare a specification with sector specific data on gross value added with a specification which uses aggregate GDP data as proxy for sectoral activity allowing for sector specific coefficients. They find that conclusions about the size of the change in trade costs are sensitive with respect to the proxy for sectoral activity but point to difficulties to obtain disaggregated data for gross value added. Flam and Nordström (2006b) report similar data problems with one digit annual data and estimate sector specific regressions with aggregate GDP data. In our case data availability is even more a constraint due to the monthly data frequency which motivates the choice of industrial production as proxy for sectoral activity. We measure nominal exchange rate volatility included in the set of control variables nonparametrically as

$$\left( vol_t^{(s)} \right)^2 = \frac{1}{D_t} \sum_{d=1}^{D_t} \left( \Delta \ln e_d^{(s)} - \frac{1}{D_t} \sum_{d=1}^{D_t} \Delta \ln e_d^{(s)} \right)^2. \quad (5.7)$$

Here  $e_d^{(s)}$  represents daily quotes of reporter  $i$ 's currency in terms of partner  $j$ 's currency and  $D_t$  is the number of days per month. A similar measure based on weekly data is proposed in Baldwin, Skudelny, and Taglioni (2005). Appendix D.4 provides further details on data construction and sources.

## 5.5 Model Selection and Diagnostic Checking

The fact that we jointly consider a set of 30 trade relationships for both imports and exports adds complexity to the provision of empirical results. In light of a plentitude of estimated empirical models, space considerations only allow a structured and condensed overview of particular model features. In this section we impose and test two particular restrictions on the smooth transition model outlined in section 5.3 with homogeneous coefficients  $\psi_1^{(s)} = 0$ : exclusion of unobservable components of trade costs ( $\lambda_{kt}^{(s)} = 0, \forall s, k, t$ ) and the euro dummy model ( $\theta_{1k}^{(s)} = 232.89, \zeta_k^{(s)} = 1.2137$ ). Then, the general homogenous smooth transition model is contrasted against a functional specification ( $\psi_1^{(s)} \neq 0$ ) which turns out preferable according to LR statistics. For the preferred model we illustrate the explanatory content of control variables  $\{vol_t^{(s)}, reer_t^{(s)}, rex_t^{(s)}, rexus_t^{(s)}, en_t^{(s)}\}$  ( $\gamma \neq 0$ ) and do extensive residual checking.

Results from specification testing and model diagnosis are documented in Table 5.2. By line the table displays results for particular trade relationships. For the

purpose of specification testing we mostly employ LR statistics to contrast alternative model specifications. Given that stochastic trends are likely to govern the dependent as well as explanatory variables in (5.2), a particular modeling issue is to guard against the potential of spurious regressions. For this purpose we test the null hypothesis of nonstationarity for estimated residuals of the observation equation. Residual based testing for unit roots is done by comparing standard ADF statistics with a 5% critical value of -4.74 (Fuller (1976)) which is relevant for testing residuals of static regressions involving 6 nonstationary variables. The lag order of the ADF regression is 3 throughout. Given that the number of potential nonstationary right hand side variables in the state space model excluding exogenous control variables is 3 (including  $\lambda_{kt}^{(s)}$ ) the critical value is likely conservative for the considered testing problem. Therefore and noting that estimated error sequences subjected to testing are not obtained from static cointegrating regressions it is clear that ADF tests provide more a descriptive view at overall model reliability. The analysis covers for each trade relationship dynamics for 96 industrial sectors. Therefore, Table 5.2 documents the empirical frequencies of rejections of the unit root null hypothesis rather than unit root statistics at sectoral level.<sup>8</sup> In a similar vein as outlined for unit root testing we also document diagnostic results to evaluate if model residuals feature serial correlation. Testing against serial correlation by means of a heteroskedasticity robust Wald statistic is detailed in Appendix D.3. We now discuss the specification issues raised above.

(i) Smooth transition

To assess the marginal contribution of the flexible smooth transition path in comparison with a conventional time dummy model, the homogenous model is alternatively estimated under restrictions that closely approximate the dummy variable model with shift ( $\theta_{1k}^{(s)} = 232.89$ ) occurring at the advent of the Euro in January 1999 ( $\zeta_k^{(s)} = 1.2137$ ). For numerous trade relations the smooth transition model is supported by LR statistics ( $LR_d$  in Table 5.2) that are significant at conventional significance levels. Out of 30  $LR_d$  statistics 10 and 12 (5 and 5) are significant at the 10% (5%) level when modelling imports and exports, respectively. Although LR tests inferring against the time dummy model are interpreted to follow an asymptotic  $\chi^2$  distribution it is worthwhile mentioning

<sup>8</sup>Tests on stationarity are only performed for sectors where the number of missing observations is at most 5. Consequently the frequencies of rejections of  $H_0$  documented in Table 5.2 refer to populations that depend on the trade relationship. In fact the number of diagnostic tests varies between 48 and 96 (53 and 96) for the analysis of import (export) relationships. The minimum numbers of 'complete' trade time series are obtained when modeling Italian imports from Sweden or Swedish exports to Italy.

that owing to a likely to restrictive alternative model, namely the homogenous state space specification, the true distribution of  $LR_d$  statistics is actually unknown. Consequently these specification tests should be treated with care and are rather a descriptive measure of model accuracy.

(ii) Latent variables

To describe the explanatory content of the latent processes,  $\{\lambda_{kt}^{(s)}\}_{t=1}^T$ , we estimate smooth transition regression models (5.2) and (5.3) excluding the state variable, and compare the resulting standard error estimate  $\sqrt{\hat{g}^{(s)}}$  with the corresponding quantity obtained from the state space model. In Table 5.2 'SR' is the ratio relating the latter quantities. Although these measures are purely descriptive they strongly underpin the explanatory content of the latent variables. Excluding sector specific latent variables involves a magnification of implied error variations by factors of 7.3 (imports of the United Kingdom from Denmark) up to 56.25 (French imports from Germany or French exports to Germany). At first sight these factors appear unreasonably large. Noting the potential nonstationarity of left hand side variables in (5.2) it is intuitive, however, to regard the latent sector specific (nonstationary) processes to potentially cointegrate with the trade variables. From this perspective a model excluding the latent states is likely to yield nonstationary residuals, so that inferential conclusions might be spurious. Accordingly, the marked reduction of variance estimates achieved by means of the state space representation becomes plausible.

(iii) Sector dependency

Most strikingly, the functional specification is uniformly and significantly supported when contrasting it against a homogeneous model. Introducing (only) 7 additional parameters smallest LR statistics ( $LR_f$ ) testing the functional against the homogeneous model (i.e.  $H_0 : \boldsymbol{\psi}_1^{(s)} = 0$  vs.  $H_1 : \boldsymbol{\psi}_1^{(s)} \neq 0$ ) are 635.7 and 734.6 for export and import characteristics, respectively. These statistics could be compared with critical values from a  $\chi^2(7)$  distribution, but, evidently all statistics are in favor of the functional model at any conventional significance level.

For the functional model first order residual correlation is detected for about one third of sector specific residual series.<sup>9</sup> In light of the documented evidence

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<sup>9</sup>Testing against joint autocorrelation at lag 1 to 12, the empirical evidence against serially uncorrelated model residuals is even stronger. For comparing the functional against the homogenous model specification (not reported) it is noteworthy that diagnostic model features also support the more general model class. Almost uniformly the frequency of significant autocorrelation test statistics is lower

in favor of serial residual correlation it is noteworthy that QML parameter estimates are inefficient but still unbiased in case of serially correlated error terms since lagged dependent variables are not included as explanatory variables. The efficiency aspect is, however, of minor importance as the interpretation of estimation results mostly relies on mean group estimators obtained as weighted averages of ML estimates (Pesaran and Smith (1995)). For the particular schemes employed to determine weighted mean group estimates see Appendix D.1.

For the special case of analyzing Danish imports from Germany we obtain that almost all (86 out of 93) sector specific residual sequences feature first order autocorrelation. We treat this diagnosis as a hint at potential model misspecification or computational obstacles and remove this particular trade relation from the sample when it comes to discussing the material implications of the estimated models in section 5.6.

(iv) Stochastic trends

Evidently, for both, modeling conditional import or export characteristics, the likelihood of stochastic trends featuring model implied residuals is rather limited. For most functional trade relationships almost all estimated sector specific residual sequences are found stationary. Thus, the conditional model is successful in filtering out common trends so that spurious regression results are unlikely. It is worthwhile to mention that the evidence against remaining stochastic trends is similarly weak when modeling trade dynamics by means of homogeneous conditional models. For space considerations we do not report model diagnostics for the state space model with cross sectionally homogeneous parameters.

(v) Exogenous control variables

Augmenting the functional model jointly with additional explanatory variables,  $vol_t^{(s)}$ ,  $reer_t^{(s)}$ ,  $rex_t^{(s)}$ ,  $rexus_t^{(s)}$ ,  $en_t^{(s)}$ , obtains LR statistics ( $LR_X$  in Table 5.2) that are mostly significant. With 5% significance 21 and 18 (out of 30) trade relationships are improved by including additional control variables. The marginal contributions of single variables to improved model accuracy are not discussed at this stage but rather when it comes to the economic discussion of the obtained (mean group) parameter estimates in the next section.<sup>10</sup>

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for the functional than it is for the homogenous model specification.

<sup>10</sup>We also assess the effect of each control variable separately on transition dynamics for the homogeneous and the functional state space model. These results (not provided here) indicate that in particular



## 5.6 Results

We discuss results of our preferred model which is the functional smooth transition state space model covering the full set of exogenous regressors. First, we inspect mean group coefficient estimates others than those determining transition. Then we turn to the characteristics of the transition path.

### 5.6.1 Coefficient Estimates

Table 5.3 reports estimated coefficients for our preferred specification for export and import data. The column labeled ALL provides weighted average coefficients where the weighted average accounts for all trade relationships. This set is disaggregated according to subsets U2, O2, IN and OUT of trade relationships in subsequent columns. Appendix D.1 provides details on the computation of mean group estimates and corresponding standard errors. Estimates of the elasticity of substitution  $\sigma$  are around 5 for both export and import data and across the various subsets of trade relationships.<sup>11</sup> Broda and Weinstein (2004) report elasticities of substitution around 4 using SITC three digit U.S. data for the period 1990–2001. Our estimates around 5 refer to two digit European data for a slightly different classification (HS instead of SITC) but overall appear to comply with estimates in Broda and Weinstein (2004). Considering the fact that our state space setup does not make use of data on international prices this is reassuring.

In line with theory industrial production in reporter ( $\beta_i$ ) and partner ( $\beta_j$ ) country significantly increases exports (imports) which holds over all trade relationships and for the majority of subsets. Though theory predicts a unit elasticity, estimates which differ from unity may signal ongoing change in the ratio of sectoral exports (imports) over industrial production. This explanation pairs well with the high estimation accuracy for some coefficients. Otherwise, Baldwin (2006a) argues that inferior data quality may be one factor pushing elasticities below unity. Indeed, our proxy of sectoral activity is identical for all sectors and even though industrial production certainly is a reasonably accurate measure for economic activity in some sectors (say, Manufacturing) it is likely to be less so for others (say, Food). Measurement errors may also underly the poor estimation precision which surrounds a few mean group estimates.

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the timing of transition as described below remains a very robust feature of our estimates.

<sup>11</sup>As side benefit our empirical specification of Gravity theory delivers estimates of the elasticity of substitution. Due to the use of time dummies this coefficient is commonly not uniquely identified in the standard setup.

Exchange rate volatility is usually considered an impediment to trade and in particular so for small firms without the financial stature to hedge exchange rate risk.<sup>12</sup> In our sample exchange rate volatility *vol* has a tendency to foster trade even though this is not a robustly significant feature of the estimates. Theories which draw on the option value of trade predict such a positive link between trade and exchange rate uncertainty. However, here it appears more likely that the positive tendency relates the trend decrease in exchange rate volatility due to the convergence process towards the third stage of EMU to a slight moderation in aggregate euro zone trade in the first half of our sample period.

A high effective exchange rate *reer* decreases exports with elasticity -0.2 when taken over all trade relationships. If domestic goods become more expensive relative to a weighted basket of foreign goods this reduces exports. However, the evidence over subsets is mixed so that in the aggregate this effect is not very precisely estimated. For imports the corresponding coefficient is larger in absolute value and more accurately estimated indicating that if domestic goods become more expensive relative to a weighted basket of foreign goods this fosters imports significantly (*reer* is the same regressor regardless of the dependent variable being exports or imports). Real bilateral exchange rates *rex* between reporter and partner country do not add significant information suggesting that real effective exchange rates *reer* already reflect bilateral variation to a sufficient degree. In contrast, real bilateral exchange rates with the U.S. *rexus* matter with a positive coefficient. This lends support to the substitution hypothesis discussed above: After 1999 the euro fell sharply so that products sold in euro became cheaper relative to products sold in U.S. dollar. As a consequence part of European demand for foreign products redirected to European products fostering exports and imports among European countries. Finally, the effect of energy prices is statistically significant over all trade relationships and for each subset individually. High energy prices as a measurable proxy for transportation costs accordingly reduce exports and imports.

### 5.6.2 Characteristics of Transition

To obtain estimates of the trade effect of the euro the literature relies almost exclusively on specifying an immediate and exhaustive level shift. Accordingly, there is

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<sup>12</sup>Baldwin and Taglioni (2005) argue that due to firm entry and exit into the sector of traded goods the true effect of exchange rate uncertainty on trade is non-linear. Indeed, Herwartz and Weber (2005) find evidence for non-linearities in the effect of exchange rate uncertainty on trade growth.

a predominant focus on a single dimension of euro transition which is size.<sup>13</sup> The smooth transition path estimated here allows to discriminate three dimensions of adjustment, namely size, speed of adjustment and timing. Each dimension is represented by one parameter,  $\theta_{k0}^{(s)}$ ,  $\theta_{k1}^{(s)}$  and  $\zeta_k^{(s)}$ , respectively.

Figure 5.1 shows the weighted average transition path for the subsets U2 and O2 (first column) and for the subsets IN and OUT (second column). The top row refers to import data whereas the bottom row refers to exports. Weighted average transition paths are averages over transition paths specific to each sector in each trade relationship based on estimates  $\theta_{k0}^{(s)}$ ,  $\theta_{k1}^{(s)}$  and  $\zeta_k^{(s)}$ . Corresponding weights are relative sector size  $w_k^{(s)}$  normalized so that they sum to unity for each subset. Probably the most remarkable feature of these plots is that transition takes off about one year *after* the introduction of the single currency in 1999. This timing pattern is robust across export and import data and, interestingly, also surfaces for trade flows that enter and leave the euro area as visible from the IN and OUT paths in the second column. Our results thus contrast the widespread convention to assume a break point in the year 1999.<sup>14</sup>

Evidently, our results are also not consistent with anticipatory activity as reported in Micco, Stein, and Ordonez (2003) and Flam and Nordström (2006b). One interpretation of this difference is that anticipation effects in fact are spurious findings due to unaccounted trade relationship or sector specific trade costs. Moreover, there is not much scope left for long run effects to develop according to our estimates. We obtain this conclusion about the 'When' of the change in trade costs while leaving timing coefficients  $\zeta_k^{(s)}$  unrestricted except from preventing them to single out the first few or last few sample observations. In our reading the timing of the change in euro area trade (costs) identified here, thus, provides independent evidence for the view that the common currency indeed reduces trade costs and, ultimately, is responsible for the increase in euro area trade because the period through which trade costs change fits remarkably well with the start of stage three of EMU and the introduction of the euro as physical currency.

Figure 5.1 also provides insight into the speed of adjustment. It takes about two to

<sup>13</sup>Some studies make inference on timing of the euro effect by interacting consecutive time dummies with a euro dummy. One potential flaw of such interaction terms is that they confuse omitted trade costs with the euro effect. This appears particularly likely when the specification relies on one set of time dummies common to all trade relationships and thus fails to control for trade relationship specific unobserved trade costs as suggested by theory.

<sup>14</sup>When comparing our approach to one that relies on a euro dummy one should take into account that most studies on the trade effect of the euro use annual data whereas we employ monthly data. It will nevertheless be difficult to reconcile the timing pattern identified here with a dummy specification even for annual data.

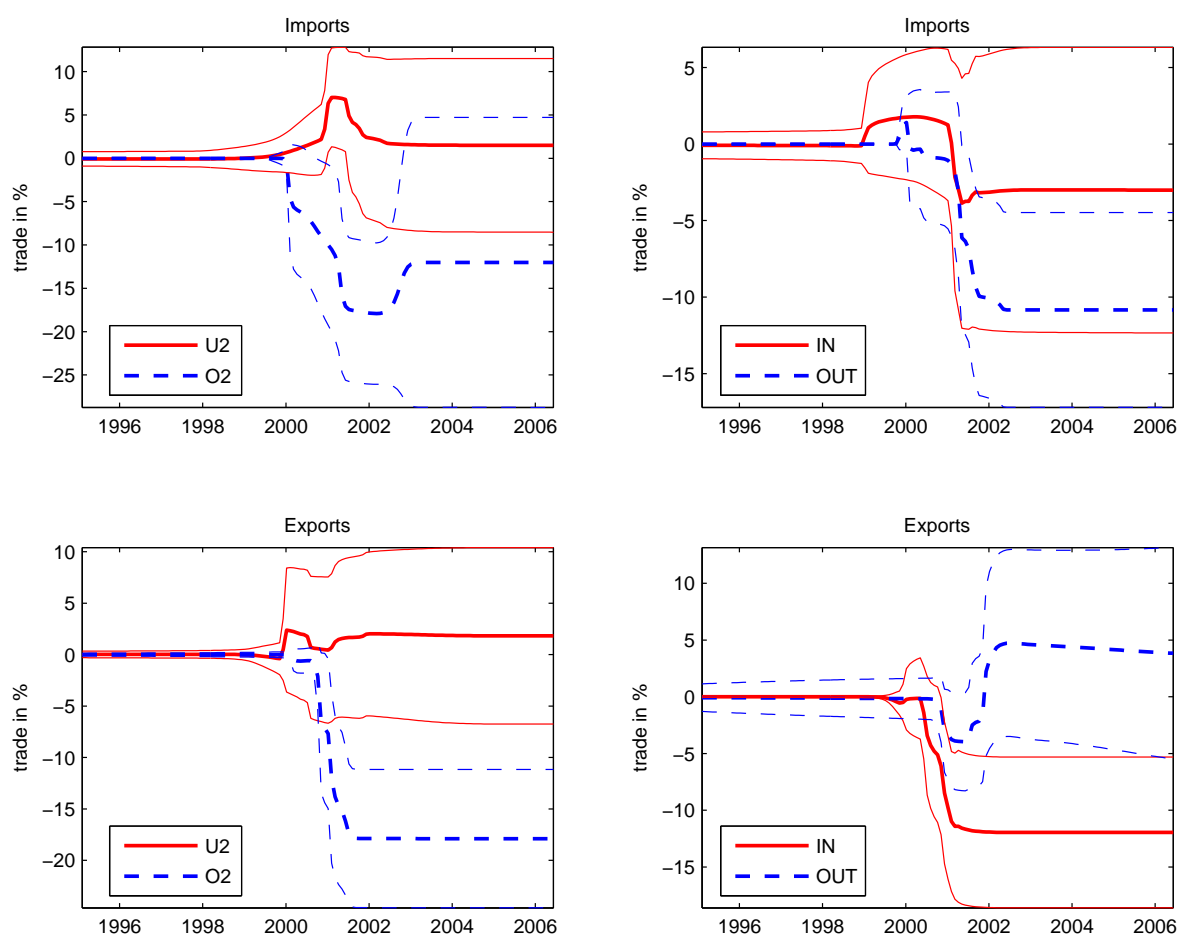


Figure 5.1: Weighted averages of sector specific transition paths of U2 and O2 countries (first column) and of IN and OUT countries (second column). All estimates are based on the functional smooth transition state space model. The first row refers to import data. The second row refers to export data. Confidence bands are based on a 10% significance level (see appendix D.1 for computation).

four years for trade to adjust to a new plateau which appears a fairly short transition period. This observation applies to trade within the euro area, trade among euro area outsiders and trade between the euro area and euro area outsiders. As it turns out adjustment of individual sectors is even faster. Aggregate adjustment then spreads out because different sectors adjust at distinct times. Table 5.3 reports estimates of  $\theta_1$  which indicate very fast adjustment throughout whereas mean estimates of  $\zeta$  jointly with their standard errors reflect the extent to which differences in timing exist. The high speed of adjustment at the sector level fits well with recent microfoundations of the euro's trade effect put forth by Baldwin and Taglioni (2005). These authors argue

that trade adjustment to the euro is fast because reduction of exchange rate volatility induces existing firms to export more and a large number of small firms to enter export markets.

Figure 5.1 also shows the size of adjustment which is the dimension of the transition path that compares most easily with related studies. At the end of our sample period mean effects are about 20 (13) percent for export data (imports) when taking the difference between adjustment of U2 versus O2 countries as is common practice. Most of the adjustment is a reduction of trade among O2 countries rather than an increase of trade among U2 countries. In table 5.1 we obtain essentially identical conclusions about long run effects that prevail once transition is fully completed. Baldwin (2006a) summarizes the literature on the euro's trade effect as suggesting a boost in trade of about 5 to 10 percent. Thus, our mean estimates are roughly twice as large compared to what has been reported so far even though confidence bands comprise effects of 5 to 10 percent in size. We obtain larger mean estimates for two reasons. First, suppose that mean paths in figure 5.1 indeed reflect the true adjustment. Minimizing the squared error between true adjustment and a euro dummy delivers smaller estimates because the euro dummy kicks in too early when its break point is the year 1999. To this end timing of the effect matters for conclusions about the size. Second, table 5.1 implies a mean difference of long run transition in the homogenous smooth transition state space model of about 10 (8) percent for exports (imports). Thus, it seems that restricting coefficients to be identical across sectors reduces the size of adjustment.

Finally, the second column of figure 5.1 shows adjustment of euro area trade with the United Kingdom, Sweden and Denmark. Exports of these countries into the euro area (IN) fall substantially after 2000 whereas euro area exports to these countries (OUT) slightly dip to reach a higher plateau thereafter. When it comes to import data IN indicates imports of the United Kingdoms, Sweden and Denmark from the euro area (the sets of countries subsumed under IN and OUT remain identical for export and import data). Even though mean estimates decrease at the end of 2001 nothing conclusive follows due to poor estimation precision. In turn, OUT indicates a significant decrease of euro area imports from the United Kingdoms, Sweden and Denmark between 2000 and 2003. Overall import and export data suggests that the euro area imports less from but exports more to European countries which have not adopted the euro. There exists a tight coincidence between U2/O2 transition dynamics and IN/OUT transition dynamics in that in both cases transition takes place during the 2000 to 2003 period. However, neither do we impose a restriction on timing coefficients for IN/OUT transition nor do we tie timing coefficients for IN/OUT to those for

Table 5.1: Long Run Adjustment as Change of Imports and Exports in Percent

	U2		O2		IN		OUT	
Imports	coef.	std.	coef.	std.	coef.	std.	coef.	std.
Func. Model	1.435	6.131	-12.01	10.14	-3.064	5.708	-10.84	3.857
Hom. Model	-2.958	5.786	-11.04	7.561	6.021	4.779	-7.684	4.713
Exports								
Func. Model	1.847	5.213	-17.90	4.077	-11.96	4.028	3.547	5.998
Hom. Model	0.686	4.683	-8.988	7.505	-13.66	2.945	-4.443	4.121

*Notes:* Reported statistics are  $\exp\{(1 - \sigma^{(s)})\theta_{k0}^{(s)}\} - 1$  which delivers percentage change in exports (imports) after completed transition. We then use weights  $w_k^{(s)}$  accordingly normalized to compute respective averages in the table. Weighted averages are based on the indicated subset of  $\theta_{k0}^{(s)}$  coefficients. 'coef.' abbreviates coefficient estimate and 'std' denotes the standard error. For computation of these statistic see appendix D.1.

U2/O2, which suggests that this timing coincidence is an important feature of the data. Transition estimates thus are consistent with the interpretation that EMU and the introduction of the euro creates detectable spillover effects for third European countries. Taken together with the observation that the euro area imports less from but exports more to third European countries one explanation consistent with our estimates is that the euro created stiffer competition among EMU exporters thereby depressing price markups. Accordingly, part of euro area trade with third countries is redirected back into the euro area whereas third countries import more from the euro area at the same time because euro area products become cheaper relative to products in the United Kingdoms, Sweden and Denmark.

## 5.7 Conclusion

Empirical models set up to analyze the trade effect of the euro appear restrictive along three dimensions. In this paper we pursue a more general approach which allows us to estimate the When, How Fast and by How Much of adjustment in trade costs. Beyond a more general notion of transition in trade costs this approach allows for a sector specific impact of trade costs on sectoral trade while controlling for unobserved variation in trade costs at the sector level. We find that adjustment in trade costs takes place between the years 2000 and 2003 which suggests that a euro dummy that kicks in with 1999 is to some degree misspecified. Adjustment of individual sec-

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tors is extremely fast whereas aggregate adjustment spreads out and is more gradual because different sectors adjust at different times. These findings support recent microfoundations of the euro's trade effect which predict the effect to happen quickly. In our reading, estimated transition paths provide independent evidence for the view that the euro has significantly increased euro area trade to the extent that we estimate rather than postulate the break point in the adjustment of trade costs.

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Table 5.2: LR Specification Tests and Model Diagnostics

$i, j$		Imports						Exports					
		LR <sub>d</sub>	SR	LR <sub>f</sub>	AR1	I(1)	LR <sub>X</sub>	LR <sub>d</sub>	SR	LR <sub>f</sub>	AR1	I(1)	LR <sub>X</sub>
GER,FRA	U2	5.11	5.1	7999.2	.33	.98	6.56	2.94	5.2	6240.5	.35	.98	17.7
GER,ITA	U2	6.45	4.5	11153.	.30	.99	12.8	3.74	4.8	6269.5	.31	.97	5.85
GER,UK	OUT	5.49	3.8	2868.1	.28	.96	7.31	4.35	4.1	4884.7	.31	.99	10.6
GER,SWE	OUT	4.14	3.2	2965.5	.17	.90	2.66	5.05	3.2	2260.9	.32	.93	9.04
GER,DK	OUT	2.32	3.4	4958.5	.31	.93	21.5	6.71	3.6	1668.5	.20	.98	10.5
FRA,GER	U2	8.44	7.5	5916.7	.25	.91	29.4	5.71	7.5	6939.3	.57	.90	33.6
FRA,ITA	U2	4.91	6.2	5279.1	.67	.83	47.6	2.87	6.0	4635.4	.39	.94	29.2
FRA,UK	OUT	2.67	5.3	5861.5	.25	.95	34.7	1.83	5.2	7747.5	.37	.95	52.7
FRA,SWE	OUT	1.60	3.5	2051.4	.25	.93	14.5	5.72	3.7	2553.2	.40	.90	16.8
FRA,DK	OUT	1.85	3.3	1991.8	.27	.91	25.7	3.61	3.6	3619.9	.34	.87	21.6
ITA,GER	U2	4.45	6.2	3028.8	.22	.92	30.2	4.95	6.7	6418.9	.46	.87	18.9
ITA,FRA	U2	3.79	5.6	3662.9	.41	.85	10.4	10.1	5.9	5658.9	.74	.80	20.6
ITA,UK	OUT	4.85	4.1	4023.7	.29	.90	50.7	2.63	4.2	4769.6	.37	.83	15.7
ITA,SWE	OUT	4.04	3.3	1790.1	.29	.92	14.3	1.70	3.3	1842.1	.53	.80	53.6
ITA,DK	OUT	10.3	3.1	734.6	.31	.92	16.7	3.25	3.4	2251.0	.41	.84	18.4
UK,GER	IN	5.05	4.0	2883.9	.21	.96	9.44	5.79	4.6	5122.7	.22	.97	25.9
UK,FRA	IN	3.47	4.0	3293.7	.20	.95	17.4	1.85	4.9	3382.7	.19	.98	8.53
UK,ITA	IN	3.69	3.5	3721.6	.19	.95	18.2	6.68	3.8	3337.3	.14	.94	10.6
UK,SWE	O2	0.83	3.3	1627.6	.22	.97	62.8	4.75	3.4	3029.2	.27	.93	3.02
UK,DK	O2	1.34	2.7	963.5	.27	.83	13.0	5.27	2.9	1280.7	.27	.90	7.98
SWE,GER	IN	1.45	4.5	2814.4	.30	.97	15.0	0.02	3.8	3125.5	.38	.91	19.6
SWE,FRA	IN	0.98	3.3	2145.4	.32	.99	11.1	0.90	3.6	1428.6	.35	.91	6.81
SWE,ITA	IN	1.84	3.1	2316.9	.44	.84	5.83	2.10	3.5	1585.3	.26	1.0	8.05
SWE,UK	O2	1.23	3.3	2262.3	.29	.94	50.8	0.11	3.4	1122.0	.40	.94	9.62
SWE,DK	O2	6.41	4.7	5520.2	.15	.98	46.5	2.57	4.1	3129.7	.26	.95	11.8
DK,GER	IN	4.13	4.0	4219.6	.93	.98	46.2	1.41	4.9	3630.2	.61	.94	36.9
DK,FRA	IN	3.39	3.1	2409.9	.33	.84	11.8	6.83	3.3	997.8	.21	.98	14.4
DK,ITA	IN	1.07	3.4	3193.7	.24	.97	3.60	1.24	3.2	635.7	.32	1.0	15.4
DK,UK	O2	0.56	3.1	2319.5	.22	.94	16.1	4.21	3.3	797.7	.30	.97	13.4
DK,SWE	O2	14.7	4.0	3194.4	.28	.97	10.9	19.7	4.9	5587.4	.20	1.0	5.94
crit	5%	5.99	-	14.07	.05	-	11.1	5.99	-	14.07	.05	-	11.1
	10%	4.60	-	12.02	-	-	9.24	4.60	-	12.02	-	-	9.24

Notes: LR<sub>d</sub> and LR<sub>f</sub> are LR statistics testing the smooth transition homogeneous model ( $\psi_1^{(s)} = 0$ ) against a time dummy homogeneous model and the functional smooth transition model, respectively. Conditional on the functional model LR<sub>X</sub> measures the explanatory content of five additional exogenous variables. 'SR' is the ratio of standard error estimates obtained when excluding the latent states  $\lambda_{kt}^{(s)} = 0$  over estimates from homogenous state space model. Estimated residuals of the observation equation (5.2) of the functional model excluding exogenous control variables ( $\psi_1^{(s)} \neq 0, \gamma = 0$ ) are diagnosed for stationarity ( $I(1)$ ) and first order serial correlation (AR1). Diagnostics (AR1,I(1)) are provided as frequencies of rejections of  $H_0$  over sectors. 'U2', 'O2', 'OUT' and 'IN' classify trade relationships.

Table 5.3: Mean Group Estimates

	ALL		U2		O2		IN		OUT	
	coef.	std.	coef.	std.	coef.	std.	coef.	std.	coef.	std.
Imports										
$\sigma$	5.016	0.072	4.949	0.121	5.458	0.082	5.074	0.099	5.176	0.067
$c$	7.161	1.260	8.386	2.583	8.412	2.323	4.497	2.195	7.236	2.937
$\beta_i$	0.957	0.169	1.067	0.267	0.901	0.510	1.184	0.351	0.419	0.359
$\beta_j$	0.215	0.130	0.080	0.319	0.205	0.448	0.336	0.173	0.408	0.204
$g$	0.499	0.042	0.376	0.074	0.736	0.067	0.613	0.047	0.624	0.061
$h$	0.048	0.007	0.036	0.004	0.058	0.006	0.046	0.002	0.043	0.003
$\theta_0$	0.042	0.037	0.000	0.017	0.038	0.023	0.010	0.015	0.031	0.010
$\theta_1$	203.3	12.91	199.2	30.37	182.6	33.23	220.0	15.46	231.4	4.056
$\zeta$	1.854	0.053	1.830	0.080	1.836	0.099	1.820	0.070	1.769	0.078
$vol$	0.038	0.021	0.084	0.051	0.084	0.030	-0.056	0.023	0.053	0.025
$reer$	0.488	0.165	0.347	0.369	-0.143	0.279	0.962	0.234	0.344	0.349
$rex$	0.006	0.161	-0.373	0.438	0.012	0.189	0.453	0.125	0.261	0.230
$rexus$	0.249	0.068	0.287	0.156	0.297	0.277	0.164	0.095	0.306	0.107
$en$	-0.051	0.014	-0.062	0.025	-0.052	0.021	-0.009	0.024	-0.068	0.030
Exports										
$\sigma$	5.345	0.077	5.494	0.168	5.402	0.230	5.314	0.095	5.090	0.113
$c$	8.812	1.195	11.571	2.995	10.617	1.749	5.528	1.304	5.982	1.531
$\beta_i$	0.871	0.126	0.914	0.244	0.306	0.308	1.043	0.239	0.821	0.237
$\beta_j$	0.637	0.157	0.490	0.248	-0.123	0.311	0.824	0.323	0.968	0.322
$g$	0.469	0.041	0.334	0.079	0.712	0.062	0.551	0.063	0.576	0.055
$h$	0.042	0.005	0.039	0.016	0.060	0.006	0.046	0.004	0.039	0.002
$\theta_0$	0.008	0.009	-0.001	0.013	0.046	0.012	0.031	0.010	-0.003	0.024
$\theta_1$	204.4	11.56	188.96	30.00	196.7	24.31	214.6	19.09	226.0	11.85
$\zeta$	1.772	0.047	1.682	0.109	1.776	0.047	1.718	0.044	1.973	0.078
$vol$	0.014	0.015	0.024	0.031	0.029	0.022	-0.039	0.030	0.033	0.024
$reer$	-0.208	0.128	-0.405	0.306	0.129	0.220	0.088	0.148	-0.186	0.233
$rex$	0.219	0.155	0.227	0.333	0.022	0.192	0.520	0.408	0.025	0.164
$rexus$	0.131	0.054	0.050	0.135	0.251	0.103	0.208	0.052	0.180	0.094
$en$	-0.030	0.012	-0.032	0.023	-0.031	0.029	-0.041	0.020	-0.017	0.027

Notes: Estimates are based on the functional smooth transition state space model. See appendix D.1 for computation of weighted average coefficients 'coef.' and corresponding standard errors 'std.'.



# Appendix A

## Appendix to Chapter 2

### A.1 Equilibrium Conditions

#### A.1.1 Household Optimality Conditions

Optimal household choices require

$$1 = \beta E_t \left[ \frac{u_c(C_{t+1}, \zeta_{t+1})}{u_c(C_t, \zeta_t)} \frac{(1+i_t)P_t}{P_{t+1}} \right] \quad (\text{A.1})$$

$$\frac{W_t}{P_t} = \frac{h_L(L_t, \zeta_t)}{(1-\tau)u_c(C_t, \zeta_t)} \quad (\text{A.2})$$

$$P_t C_t = W_t L_t + D_t \quad (\text{A.3})$$

where transversality conditions are assumed to hold (Woodford (2003), chapter 3, provides a detailed derivation) and the government budget constraint and bond market clearing has been used to simplify the budget constraint. Notation  $u_c(C_t, \zeta_t)$  abbreviates  $\partial u(C_t, \zeta_t) / \partial C_t$ .

#### A.1.2 Aggregate Supply Relationship

The optimal pricing condition to problem (2.8) is

$$0 = E_t \sum_{i=0}^{\infty} (\kappa\beta)^i \Omega_{t+i,t} Y_{t+i}(j) [(1-\theta) + \theta S_{t+i}(j) / P_t^*(j)] \quad (\text{A.4})$$

with marginal costs given by

$$\frac{S_t(j)}{P_t} = \frac{h_L(L_t, \zeta_t)}{(1-\tau)u_c(Y_t, \zeta_t)} \frac{1}{A_t f'(f^{-1}(Y_t(j)/A_t))} .$$

Combining marginal costs with optimal pricing once variables are expressed as percentage deviation from steady state and combining the result with the linearized recursive law of motion of the actual price level delivers

$$\hat{\pi}_t = \frac{(1-\kappa\beta)(1-\kappa)}{\kappa} (1 + \theta\omega_p)^{-1} \hat{s}_t + \beta E_t \hat{\pi}_{t+1} \quad (\text{A.5})$$

denoting with  $\hat{s}_t$  average real marginal costs. Parameters are defined analog to Woodford (2003) as

$$v = \frac{h_{LL}\bar{L}}{h_L}, \quad \chi = \frac{f}{\bar{L}f'}, \quad \omega_p = -\frac{\bar{Y}f''}{(f')^2}, \quad \omega_w = v\chi$$

and  $\omega = \omega_p + \omega_w$ . The natural output level which obtains under flexible prices is

$$\hat{Y}_t^n = \frac{v}{\omega + \sigma^{-1}} \bar{L}_t + \frac{1 + \omega}{\omega + \sigma^{-1}} a_t + \frac{\sigma^{-1}}{\omega + \sigma^{-1}} g_t$$

having  $\hat{A}_t = a_t$  and  $\bar{L}_t = -v^{-1}h_{L\zeta}/h_L\hat{\zeta}_t$ , so that real average marginal costs  $\hat{s}_t = (\omega + \sigma^{-1})x_t$  can be shown to be proportional to the output gap  $x_t = \hat{Y}_t - \hat{Y}_t^n$ . Thereby, equation (A.5) reformulates to equation (2.17) in the main text.

## A.2 Actual Price Level

Iterating backwards, the unit mass of operating firms is a sum over infinitely many entry cohorts each of size  $N_E = \delta/(1 - \delta)$  in the entry period,

$$1 = \sum_{s=t}^{-\infty} (1 - \delta)^{t-s+1} N_E.$$

Let the integer  $s \leq t$  be common to all goods that entered the market in this particular period and thus index entry cohorts. Under Calvo pricing and at date  $t$ , the price distribution of cohort  $s$  is truncated Poisson and summarized by

$$\Lambda_t(s) = (1 - \alpha) \sum_{k=t}^{s+1} \alpha^{t-k} (P_k^*)^{1-\theta} + \alpha^{t-s} (P_s^*)^{1-\theta}, \quad s < t. \quad (\text{A.6})$$

With  $s = t$  this reduces to  $\Lambda_t(t) = (P_t^*)^{1-\theta}$ . Under Calvo pricing and absent firm specific states other than prices, all reoptimizing firms chose identical price. Therefore, the actual price level (2.13) is equivalently expressed as

$$\begin{aligned} P_t^{1-\theta} &= \int_0^1 P_t(j)^{1-\theta} dj = \sum_{s=t}^{-\infty} (1 - \delta)^{t-s+1} N_E \Lambda_t(s) \\ &= (1 - \kappa) \sum_{i=0}^{\infty} \kappa^i (P_{t-i}^*)^{1-\theta} = (1 - \kappa) (P_t^*)^{1-\theta} + \kappa P_{t-1}^{1-\theta}. \end{aligned}$$

## A.3 Measurement Bias

### A.3.1 Substitution Bias

Linearize  $\pi_t^m$  in equation (2.2) as

$$\hat{\pi}_t^m = \int_{\mathcal{N}(t,\ell)} w_j^m \hat{\pi}_t(j) dj$$

exploiting steady state relationship  $\pi^m = \pi = \pi_j = 1$  and the fact that linearized measured weights integrate to zero by equation (2.3),

$$0 = \int_{\mathcal{N}(t,\ell)} w_j^m \hat{w}_{t-1,t}^m(j) dj .$$

Here  $w_j^m$  denotes the measured steady state weight homogenous among products  $j$  in symmetric steady state. Equivalently, linearize  $\pi_t^{eg}$  in equation (2.10) as

$$\hat{\pi}_t^{eg} = \int_{\mathcal{N}(t,\ell)} w_j^{eg} \hat{\pi}_t(j) dj$$

accounting for the fact that weights (2.11) integrate to zero once linearized. Thus,  $\hat{\mathcal{B}}_t^{sub} = \hat{\pi}_t^m - \hat{\pi}_t^{eg} = 0$  because  $w_j^m = w_j^{eg} = 1/(1-\delta)^\ell$  in symmetric steady state so that  $\hat{\pi}_t^m = \hat{\pi}_t^{eg}$ .

### A.3.2 New Product Bias

Finding a tractable expression for  $\mathcal{B}_t^{new} = \pi_t^{eg}/\pi_t$  requires to work out  $\pi_t^{eg}$  which is done based on equations (2.9). Depart from

$$(P_{t,t}^{eg})^{1-\theta} = \int_{\mathcal{N}(t,\ell)} P_t^{1-\theta}(j) dj .$$

The  $(1-\delta)^\ell$  products contained in  $\mathcal{N}(t,\ell)$  are composed out of an infinite number of entry cohorts  $s$  with each cohort having size  $N_E = \delta/(1-\delta)$  in its entry period,

$$(1-\delta)^\ell = \sum_{s=t-\ell}^{-\infty} (1-\delta)^{t-s+1} N_E .$$

The date  $t$  price distribution of those products which is summarized by  $P_{t,t}^{eg}$  is a sum over truncated Poisson distributions with zero weight on the prices of the  $\ell$  most recent entry cohorts. Using equation (A.6) in appendix A.2 which defines  $\Lambda_t(s)$  one obtains

$$(P_{t,t}^{eg})^{1-\theta} = \sum_{s=t-\ell}^{-\infty} (1-\delta)^{t-s+1} N_E \Lambda_t(s) .$$

For comparison, the date  $t - 1$  price distribution of the same products which is summarized by  $P_{t-1,t}^{eg}$  is a sum over truncated Poisson distributions with zero weight on the prices of the  $\ell$  most recent entry cohorts,

$$(P_{t-1,t}^{eg})^{1-\theta} = \sum_{s=t-\ell}^{-\infty} (1-\delta)^{t-s+1} N_E \Lambda_{t-1}(s).$$

Using equation (A.6) simplifies  $(P_{t,t}^{eg})^{1-\theta}$  to

$$\frac{(P_{t,t}^{eg})^{1-\theta}}{(1-\delta)^\ell} = (1-\alpha) \sum_{k=0}^{\ell-1} \alpha^k (P_{t-k}^*)^{1-\theta} + \alpha^\ell P_{t-\ell}^{1-\theta}$$

where the sum vanishes if  $\ell = 1$  and results on the actual price level have been applied. Analog transformation of  $(P_{t-1,t}^{eg})^{1-\theta}$  produces

$$\frac{(P_{t-1,t}^{eg})^{1-\theta}}{(1-\delta)^\ell} = (1-\alpha) \sum_{k=1}^{\ell-1} \alpha^{k-1} (P_{t-k}^*)^{1-\theta} + \alpha^{\ell-1} P_{t-\ell}^{1-\theta},$$

where the sum vanishes if  $\ell = 1$ . Linearizing  $(P_{t,t}^{eg})^{1-\theta}$  and  $(P_{t-1,t}^{eg})^{1-\theta}$  delivers

$$\hat{P}_{t,t}^{eg} = (1-\alpha) \sum_{k=0}^{\ell-1} \alpha^k \hat{P}_{t-k}^* + \alpha^\ell \hat{P}_{t-\ell}, \quad \hat{P}_{t-1,t}^{eg} = (1-\alpha) \sum_{k=1}^{\ell-1} \alpha^{k-1} \hat{P}_{t-k}^* + \alpha^{\ell-1} \hat{P}_{t-\ell},$$

exploiting the fact that  $\bar{P} = \bar{P}^*$ ,  $\bar{P}^{eg}/\bar{P}^* = (1-\delta)^{\ell/(1-\theta)}$  and  $(1-\alpha) \sum_{k=0}^{\ell-1} \alpha^k + \alpha^\ell = 1$ . Thus,  $\pi_t^{eg}$  linearizes as

$$\hat{\pi}_t^{eg} = \hat{P}_{t,t}^{eg} - \hat{P}_{t-1,t}^{eg} = (1-\alpha) \left[ \hat{P}_t^* - (1-\alpha) \sum_{k=1}^{\ell-1} \alpha^{k-1} \hat{P}_{t-k}^* - \alpha^{\ell-1} \hat{P}_{t-\ell} \right].$$

Linearizing  $\mathcal{B}_t^{new} = \pi_t^{eg}/\pi_t$  and using the linearized recursive law of motion of the actual price level delivers

$$\hat{\mathcal{B}}_t^{new} = \left( \frac{1-\alpha}{1-\alpha(1-\delta)} - 1 \right) \hat{\pi}_t + \frac{(1-\alpha)\delta}{1-\alpha(1-\delta)} \sum_{k=1}^{\ell-1} \alpha^k \hat{\pi}_{t-k},$$

which corresponds to the results in the proposition. Finally, linearizing equations (2.1) and (2.9) delivers  $\hat{P}_{t,t}^m = \hat{P}_{t,t}^{eg}$  and  $\hat{P}_{t-1,t}^m = \hat{P}_{t-1,t}^{eg}$  which warrants the discussion in the text.



### A.3.3 Properties of $a(L)$ and $b(L)$

Rewrite  $a(L)$  as

$$a(L) = \frac{1 - \alpha}{1 - \alpha(1 - \delta)} L^0 + \frac{(1 - \alpha)\delta}{1 - \alpha(1 - \delta)} \sum_{k=1}^{\ell-1} (\alpha L)^k \quad (\text{A.7})$$

$$= \frac{1 - \alpha}{1 - \alpha(1 - \delta)} \left( (1 - \delta)L^0 + \delta \frac{1 - (\alpha L)^\ell}{1 - \alpha L} \right) \quad (\text{A.8})$$

where it suffices to consider  $\ell > 1$ . Parameter intervals are  $\alpha \in [0, 1)$ ,  $\delta \in [0, 1)$  and  $1 < \ell \leq \infty$ . These intervals imply that all coefficients  $a_k$ ,  $k = 0, \dots, \ell - 1$  are nonnegative by equation (A.7). Representation (A.8) implies immediately  $a(1) < \infty$  so that  $a(L)$  is absolutely summable. Intervals for  $\alpha, \delta$  and  $\ell$  also imply that all coefficients of  $b(L)$ ,  $b_k = \frac{\alpha^{k+1}\delta}{1 - \alpha(1 - \delta)}$ ,  $k = 0, \dots, \ell - 1$  are nonnegative. Rewrite the polynomial  $b(L)$  as

$$b(L) = \frac{\delta}{1 - \alpha(1 - \delta)} \left( \frac{1 - (\alpha L)^{\ell+1}}{1 - \alpha L} - L^0 \right) \quad (\text{A.9})$$

from which one finds  $b(1) < \infty$  so that  $b(L)$  also is absolutely summable.



# Appendix B

## Appendix to Chapter 3

### B.1 Linear Model

$$\tilde{x}_t = E_t \tilde{x}_{t+1} - (1 - \eta\beta)\sigma(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n)$$

$$\tilde{x}_t = (x_t - \eta x_{t-1}) - \eta\beta E_t(x_{t+1} - \eta x_t)$$

$$\begin{aligned} \hat{\pi}_t - \kappa \hat{\pi}_{t-1} &= \mu[(x_t - \delta x_{t-1}) - \beta\delta E_t(x_{t+1} - \delta x_t)] \\ &\quad + \beta E_t(\hat{\pi}_{t+1} - \kappa \hat{\pi}_t) + u_t \end{aligned}$$

$$\begin{aligned} \hat{m}_t - \phi \hat{m}_{t-1} - \epsilon_t^m &= \phi\beta E_t[\hat{m}_{t+1} - \phi \hat{m}_t - \epsilon_{t+1}^m] \\ &\quad + (1 - \phi\beta)(1 - \phi) \left[ \frac{\eta_y}{(1 - \eta\beta)(1 - \eta)} \tilde{Y}_t - \eta_i \hat{\Delta}_{it} \right] \end{aligned} \tag{B.1}$$

$$E_t[\varphi(1 - \eta L)(1 - \eta\beta L^{-1}) + \omega] \hat{Y}_t^n = \varphi E_t(1 - \eta\beta L^{-1}) g_t + (1 + \omega) a_t + v \bar{n}_t$$

$$\hat{r}_t^n = -\varphi E_t[(1 - (1 + \eta\beta)L^{-1} + \eta\beta L^{-2})((1 - \eta L)\hat{Y}_t^n - g_t)]$$

$$x_t = \hat{Y}_t - \hat{Y}_t^n .$$

In line with Woodford (2003), chapter 3 and 5, and Giannoni and Woodford (2005) parameters are defined as

$$v = \frac{h_{NN}\bar{N}}{h_N}, \quad \chi = \frac{f}{\bar{N}f'}, \quad \omega_p = -\frac{\bar{Y}f''}{(\bar{f}')^2}, \quad \omega_w = v\chi, \quad \omega = \omega_p + \omega_w .$$

Moreover,  $\varphi = [(1 - \eta\beta)\sigma]^{-1}$  and parameters that make up the slope of the aggregate supply relationship are

$$\check{\chi} = \frac{\omega + \varphi(1 + \eta^2\beta)}{\beta\varphi}, \quad \vartheta = \frac{\beta}{2} \left( \check{\chi} + \sqrt{\check{\chi}^2 - 4\eta^2\beta^{-1}} \right) > 1, \quad \delta = \eta\vartheta^{-1}$$

$$\Xi_p = \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} (1 + \theta\omega_p)^{-1}, \quad \mu = \frac{\varphi\eta}{\delta} \Xi_p.$$

## B.2 Welfare Loss

This appendix presents the approximation to the discounted sum of  $q(\cdot)$ . The derivation is similar to Woodford (2003), chapter 6, section 4.1. Denote  $q_m$  as  $\partial q(\hat{m}_t - \phi\hat{m}_{t-1}, \zeta_t) / \partial(\hat{m}_t - \phi\hat{m}_{t-1})$  evaluated at steady state. Expand as

$$\begin{aligned} q(m_t - \phi m_{t-1}, \zeta_t) &= \bar{m}q_m[\hat{m}_t - \phi\hat{m}_{t-1} + \frac{1}{2}(\hat{m}_t^2 - \phi\hat{m}_{t-1}^2)] + \frac{\bar{m}^2 q_{mm}}{2}(\hat{m}_t^2 + \phi^2\hat{m}_{t-1}^2) \\ &\quad + \bar{m}q_{m\bar{\zeta}}\bar{\zeta}_t(\hat{m}_t - \phi\hat{m}_{t-1}) - \phi\bar{m}^2 q_{mmm}\hat{m}_t\hat{m}_{t-1} + tip + o(\|\zeta_t\|^3) \\ &= \frac{\bar{Y}q_m}{\bar{v}} \left\{ \hat{m}_t - \phi\hat{m}_{t-1} + \frac{1}{2}(1 - \sigma_m^{-1})\hat{m}_t^2 - \frac{\phi}{2}(1 + \phi\sigma_m^{-1})\hat{m}_{t-1}^2 \right. \\ &\quad \left. + \sigma_m^{-1}\epsilon_t^m(\hat{m}_t - \phi\hat{m}_{t-1}) + \phi\sigma_m^{-1}\hat{m}_t\hat{m}_{t-1} \right\} + tip + o(\|\zeta_t\|^3) \end{aligned}$$

using steady state velocity  $\bar{v} = \frac{\bar{Y}}{\bar{m}}$ ,  $d\bar{\zeta}_t = \zeta_t$  since  $\bar{\zeta} = 0$  and definitions  $\sigma_m^{-1} = -\frac{\bar{m}q_{mm}}{q_m}$  and  $\epsilon_t^m = -\frac{q_{m\bar{\zeta}}}{\bar{m}q_{mm}}\zeta_t$ . Define the interest rate cost of real balances as a fraction of  $\bar{Y}$  as

$$s_m = \frac{\bar{m}(1 - \phi\beta)q_m}{\bar{Y}(1 - \eta\beta)u_c} = \frac{\bar{m}\bar{\Delta}_i}{\bar{Y}} = \frac{\bar{\Delta}_i}{\bar{v}}$$

which is of order  $o(\|\bar{\Delta}_i\|)$  since the economy is close to satiation and inverse velocity  $\frac{\bar{m}}{\bar{Y}}$  is finite. Use  $q_m = \bar{\Delta}_i \frac{(1 - \eta\beta)u_c}{(1 - \phi\beta)}$  to obtain

$$\frac{q_m}{\bar{v}} = \frac{\bar{\Delta}_i}{\bar{v}} \frac{(1 - \eta\beta)u_c}{(1 - \phi\beta)} = s_m \frac{(1 - \eta\beta)u_c}{(1 - \phi\beta)}.$$

The ratio

$$\frac{s_m}{\sigma_m} = -\frac{\bar{m}(1 - \phi\beta)q_m}{\bar{Y}(1 - \eta\beta)u_c} \frac{\bar{m}q_{mm}}{q_m} = -\frac{\bar{m}}{\bar{Y}} \frac{\bar{m}(1 - \phi\beta)q_{mm}}{(1 - \eta\beta)u_c}$$

is well defined as a result of the finite limiting value of  $q_{mm}$ . Obtain

$$\begin{aligned} q(m_t - \phi m_{t-1}, \zeta_t) &= \frac{(1 - \eta\beta)\bar{Y}u_c}{(1 - \phi\beta)} \left\{ s_m(\hat{m}_t - \phi\hat{m}_{t-1}) - \frac{1}{2} \frac{s_m}{\sigma_m} \hat{m}_t^2 - \frac{1}{2} \phi^2 \frac{s_m}{\sigma_m} \hat{m}_{t-1}^2 \right. \\ &\quad \left. + \frac{s_m}{\sigma_m} (\hat{m}_t - \phi\hat{m}_{t-1})\epsilon_t^m + \phi \frac{s_m}{\sigma_m} \hat{m}_t\hat{m}_{t-1} \right\} + tip + \mathcal{O} \\ &= \frac{(1 - \eta\beta)\bar{Y}u_c}{(1 - \phi\beta)} \left\{ s_m(\hat{m}_t - \phi\hat{m}_{t-1}) - \frac{1}{2} \frac{s_m}{\sigma_m} (\hat{m}_t - \phi\hat{m}_{t-1} - \epsilon_t^m)^2 \right\} + tip + \mathcal{O} \end{aligned}$$

where in the first row terms of order three and higher are subsumed into the residual  $\mathcal{O}$  which now also accounts for  $\bar{\Delta}_i$  as expansion parameter. Rewrite the limiting interest rate elasticity with  $\bar{\Delta}_i \rightarrow 0$  as

$$\eta_i = \frac{(1-\eta\beta)u_c(\bar{\Delta}_i-1)}{(1-\phi\beta)(1-\phi)\bar{m}q_{mm}}$$

and obtain  $\frac{s_m}{\sigma_m} = ((1-\phi)\bar{v}\eta_i)^{-1}$ . Jointly with  $s_m = \frac{\bar{\Delta}_i}{\bar{v}}$  rewrite the expansion as deviation from target,

$$q(m_t - \phi m_{t-1}, \zeta_t) = -\frac{(1-\eta\beta)\bar{Y}u_c}{2} \frac{1}{(1-\phi\beta)(1-\phi)\bar{v}\eta_i} (\hat{m}_t - \phi\hat{m}_{t-1} - \epsilon_t^m - \hat{m}^*)^2 + tip + \mathcal{O}$$

denoting  $\hat{m}^* = (1-\phi)\eta_i\bar{\Delta}_i$ . The case treated in Woodford (2003) establishes with  $\phi = 0$ . Combining results here with those in Woodford (2003) delivers

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t U_t &= -\frac{(1-\eta\beta)\bar{Y}u_c}{2} \frac{\theta}{\Xi_p} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \delta_0 \frac{\Xi_p}{\theta} (x_t - \delta x_{t-1} - \hat{x}^*)^2 + (\hat{\pi}_t - \kappa \hat{\pi}_{t-1})^2 \right. \\ &\quad \left. + \frac{\Xi_p}{\theta(1-\phi\beta)(1-\phi)\bar{v}\eta_i} (\hat{m}_t - \phi\hat{m}_{t-1} - \epsilon_t^m - \hat{m}^*)^2 \right\} \end{aligned}$$

omitting terms of order higher than three and terms independent of policy. Here,  $U_t$  denotes the period utility function and additional coefficients are

$$\delta_0 = \vartheta\varphi, \quad \Phi_y = 1 - \frac{(1-\tau)(\theta-1)}{\theta}, \quad \hat{x}^* = \frac{\Phi_y}{\vartheta\varphi(1-\beta\delta)}.$$

## B.3 Optimal Policy

Optimal policy characterizes the path  $\{\hat{\pi}_t, x_t, \hat{i}_t, \hat{m}_t\}_{t=0}^{\infty}$  that minimizes the discounted loss subject to the equilibrium conditions (B.1) as of period  $t \geq 0$  and conditional on initial values for endogenous variables and the Lagrange multipliers equal to zero as well as paths of fundamental shocks  $\{g_t, a_t, u_t, \bar{n}_t, \epsilon_t^m\}_{t=0}^{\infty}$  and exogenous variables that are functions of those shocks  $\hat{r}_t^n, \hat{Y}_t^n, e_t^m$ . Attention here is restricted to local rational expectation bounded solutions so that relevant transversality conditions hold. The

corresponding Lagrangian is

$$\begin{aligned} \Theta = E_0 \sum_{t=0}^{\infty} \beta^t & \left\{ \frac{\lambda_x}{2} (\Delta_\delta x_t)^2 + \frac{1}{2} (\Delta_\kappa \hat{\pi}_t)^2 + \frac{\lambda_m}{2} (\Delta_\phi \hat{m}_t - \epsilon_t^m - \hat{m}^*)^2 + \frac{\lambda_i}{2} (\hat{\Delta}_{it} - \Delta_i^*)^2 \right. \\ & + \psi_{xt} \left[ a_0 x_t - \eta x_{t-1} - a_1 x_{t+1} + \eta \beta x_{t+2} + \varphi^{-1} \hat{i}_t - \varphi^{-1} \hat{\pi}_{t+1} - \varphi^{-1} \hat{r}_t^m \right] \\ & + \psi_{\pi t} \left[ (1 + \beta \kappa) \hat{\pi}_t - \kappa \hat{\pi}_{t-1} - \beta \hat{\pi}_{t+1} - b_0 x_t + \mu \delta x_{t-1} + \mu \delta \beta x_{t+1} - u_t \right] \\ & + \psi_{mt} \left[ \Delta_\phi \hat{m}_t - \epsilon_t^m - \phi \beta (\Delta_\phi \hat{m}_{t+1} - \epsilon_{t+1}^m) \right. \\ & \quad \left. - \theta_y [(1 + \eta^2 \beta) x_t - \eta x_{t-1} - \eta \beta x_{t+1} + \epsilon_t^m] + \theta_i \hat{\Delta}_{it} \right] \left. \right\} \end{aligned}$$

where with the law of iterated expectations  $E_0 \psi_{\pi t} E_t \hat{\pi}_{t+1} = E_0 \psi_{\pi t} \hat{\pi}_{t+1}$  and

$$a_0 = [1 + \eta(1 + \eta\beta)], \quad a_1 = [1 + \eta\beta(1 + \eta)], \quad b_0 = \mu(1 + \beta\delta^2).$$

**Discretion:** The system of optimality conditions is

$$\begin{aligned} x_t : \quad 0 &= \lambda_x (x_t - \delta x_{t-1} - \hat{x}^*) - \delta \beta \lambda_x (E_t x_{t+1} - \delta x_t - \hat{x}^*) \\ &+ [1 + \eta(1 + \eta\beta)] \psi_{xt} - \eta \beta E_t \psi_{xt+1} \\ &- \theta_y (1 + \eta^2 \beta) \psi_{mt} + \theta_y \eta \beta E_t \psi_{mt+1} \\ &- \mu(1 + \beta\delta^2) \psi_{\pi t} + \mu \delta \beta E_t \psi_{\pi t+1} \end{aligned}$$

$$\hat{\pi}_t : \quad 0 = (\hat{\pi}_t - \kappa \hat{\pi}_{t-1}) - \kappa \beta (E_t \hat{\pi}_{t+1} - \kappa \hat{\pi}_t) + (1 + \kappa \beta) \psi_{\pi t} - \kappa \beta E_t \psi_{\pi t+1}$$

$$\Delta_\phi \hat{m}_t : \quad 0 = \lambda_m (\Delta_\phi \hat{m}_t - \epsilon_t^m - \hat{m}^*) + \psi_{mt}$$

$$\hat{i}_t : \quad 0 = \lambda_i (\hat{\Delta}_{it} - \Delta_i^*) + \varphi^{-1} \psi_{xt} + \theta_i \psi_{mt}.$$

using  $\hat{\Delta}_{it} = \hat{i}_t - \hat{i}_t^m = \hat{i}_t$ .

**Optimal Commitment:** I restrict attention to period zero optimal commitment because responses to shocks do not differ from timeless optimal policy (see Woodford

(2003), chapter 7). The system of optimality conditions is

$$\begin{aligned}
 x_t : \quad 0 &= \lambda_x(x_t - \delta x_{t-1} - \hat{x}^*) - \delta\beta\lambda_x(E_t x_{t+1} - \delta x_t - \hat{x}^*) \\
 &\quad + [1 + \eta(1 + \eta\beta)]\psi_{xt} - [1 + \eta\beta(1 + \eta)]/\beta\psi_{xt-1} + \frac{\eta}{\beta}\psi_{xt-2} - \eta\beta E_t\psi_{xt+1} \\
 &\quad - \theta_y(1 + \eta^2\beta)\psi_{mt} + \theta_y\eta\psi_{mt-1} + \theta_y\eta\beta E_t\psi_{mt+1} \\
 &\quad - \mu(1 + \beta\delta^2)\psi_{\pi t} + \mu\delta\psi_{\pi t-1} + \mu\delta\beta E_t\psi_{\pi t+1}
 \end{aligned}$$

$$\begin{aligned}
 \hat{\pi}_t : \quad 0 &= (\hat{\pi}_t - \kappa\hat{\pi}_{t-1}) - \kappa\beta(E_t\hat{\pi}_{t+1} - \kappa\hat{\pi}_t) \\
 &\quad + (1 + \kappa\beta)\psi_{\pi t} - \psi_{\pi t-1} - \kappa\beta E_t\psi_{\pi t+1} - (\varphi\beta)^{-1}\psi_{xt-1}
 \end{aligned}$$

$$\Delta_\phi\hat{m}_t : \quad 0 = \lambda_m(\Delta_\phi\hat{m}_t - \epsilon_t^m - \hat{m}^*) + \psi_{mt} - \phi\psi_{mt-1}$$

$$\hat{i}_t : \quad 0 = \lambda_i(\hat{\Delta}_{it} - \Delta_i^*) + \varphi^{-1}\psi_{xt} + \theta_i\psi_{mt} .$$





# Appendix C

## Appendix to Chapter 4

### C.1 Data

Consumer prices are from IFS of the IMF, except the series for the euro area which is from the ECB Statistical Data Warehouse. Consumer prices are not seasonally adjusted. Value (current euro) and quantity (tons) trade data are from Comext database of Eurostat (series are total exports and total imports, source is EU27 Trade since 1995 by SITC).<sup>1</sup> Trade data is not seasonally adjusted. The reporter is the euro zone (EZ) and partner countries are CZ, HU, PO and SK, respectively. For each partner country I compute implicit unit prices by dividing value by quantity data. I then assume that EZ exports to the partner country equal partner country's imports from EZ to obtain import and export unit prices of the partner country. Unit price indices are by no means innocent measures of export and import prices because value and quantity data are in some cases adjusted separately (see section 4.7 in EC (2006)). Also, Baldwin (2006a) discusses the source of differences between imports of country A to country B and exports of country B to country A.

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<sup>1</sup>See <http://epp.eurostat.ec.europa.eu/newxtweb/> .

## C.2 Household Optimality Conditions

Optimal household behavior in Home is characterized by

$$\begin{aligned}\Lambda_t(j) &= C_t(j)^{-1} \\ 1 &= \beta R_t^* \Phi(t) E_t \frac{\Lambda_{t+1}(j)}{\Lambda_t(j)} \frac{1}{\pi_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \\ 1 &= \beta R_t E_t \frac{\Lambda_{t+1}(j)}{\Lambda_t(j)} \frac{1}{\pi_{t+1}} \\ \frac{V_{ht}}{P_t} &= (1 - \delta) \beta E_t \frac{\Lambda_{t+1}(j)}{\Lambda_t(j)} \frac{V_{ht+1} + D_{ht+1}}{P_{t+1}} \\ \frac{W_t}{P_t} &= L_t(j)^\varphi C_t(j).\end{aligned}$$

where symmetry of all  $h$  firms in equilibrium  $V_{ht} = V_t(h)$  is taken into account. Iterating the Euler condition for shares forward delivers equation (4.1) in the main text assuming that the discounted expected firm value converges to zero asymptotically. Optimal household behavior in Foreign is characterized by

$$\begin{aligned}\Lambda_t^*(j^*) &= C_t^*(j^*)^{-1} \\ 1 &= \beta R_t^* E_t \frac{\Lambda_{t+1}^*(j^*)}{\Lambda_t^*(j^*)} \frac{1}{\pi_{t+1}^*} \\ \frac{V_{ft}^*}{P_t^*} &= (1 - \delta) \beta E_t \frac{\Lambda_{t+1}^*(j^*)}{\Lambda_t^*(j^*)} \frac{V_{ft+1}^* + D_{ft+1}^*}{P_{t+1}^*} \\ \frac{W_t^*}{P_t^*} &= L_t^*(j^*)^\varphi C_t^*(j^*).\end{aligned}$$

where symmetry of all  $f$  firms in equilibrium  $V_{ft}^* = V_t^*(f)$  is taken into account. Furthermore, household optimality requires that flow budget constraints and respective transversality conditions hold.

### C.3 Transformed Home Economy

Let  $\kappa \equiv (1 - \delta)(1 + \lambda_E)/G_Y$ . Household optimality conditions are

$$\begin{aligned} 1 &= \beta/G_Y R_t^* \Phi(t) E_t \frac{\tilde{C}_t}{\tilde{C}_{t+1}} \frac{1}{\pi_{t+1}^*} \frac{e_{t+1}}{e_t} \\ 1 &= \beta/G_Y R_t E_t \frac{\tilde{C}_t}{\tilde{C}_{t+1}} \frac{1}{\pi_{t+1}} \\ \tilde{v}_{ht} &= \kappa \beta E_t \frac{\tilde{C}_t}{\tilde{C}_{t+1}} [\tilde{v}_{ht+1} + \tilde{d}_{ht+1}] \\ \tilde{w}_t &= L_t^q \tilde{C}_t \end{aligned}$$

Let  $b_{Ft} = B_{Ft}/P_t^*$  and  $\tilde{b}_{Ft} = b_{Ft}/\tilde{Y}_t$  to have  $\Phi(t) = \exp(-\phi(e_t \tilde{b}_{Ft})/\tilde{Y}_t)$ . Price setting implies

$$\tilde{p}_{ht} = \frac{\nu}{\nu - 1} \frac{\tilde{w}_t}{z_{Ht}}, \quad \tilde{p}_{ft}^* = \frac{\nu}{\nu - 1} \frac{\tilde{w}_t^*}{z_{Ft}^*}, \quad \tilde{p}_{Nt} = \frac{\nu_N}{\nu_N - 1} \frac{\tilde{w}_t}{z_{Nt}}, \quad \tilde{p}_{ft} = e_t \tilde{p}_{ft}^*, \quad \tilde{p}_{ht} = e_t \tilde{p}_{ht}^*$$

denoting with a small character the relative price  $p_{Xt} = P_{Xt}/P_t$  for generic subscript  $X$  and using  $A_{N0} = \mathcal{G}_{N0}$ . Firm entry, firm value and aggregate firm profits in the traded good sector are

$$\begin{aligned} \tilde{H}_t &= \kappa \tilde{H}_{t-1} + \kappa \tilde{H}_{Et-1} \\ \tilde{v}_{ht} &= q \tilde{H}_{Et}^{\phi_e} z_{Et}^{-\phi_E} \\ \tilde{v}_{ht} &= \sum_{s=t+1}^{\infty} [\kappa \beta]^{s-t} \left( \frac{\tilde{C}_s}{\tilde{C}_t} \right)^{-1} \tilde{d}_{hs} \\ \tilde{H}_t \tilde{d}_{ht} &= \tilde{p}_{Ht} \tilde{Y}_{Ht} + e_t \tilde{p}_{Ht}^* \tilde{Y}_{Ht}^* - \tilde{w}_t L_{Ht}. \end{aligned}$$

Clearing conditions and production functions transform according to

$$\begin{aligned} L_t &= L_{Nt} + L_{Ht} \\ \tilde{Y}_t &= \tilde{C}_t + \tilde{H}_{Et} \tilde{v}_{ht} \\ z_{Ht} L_{Ht} &= \tilde{H}_t^{-(\rho-1)} (\tilde{Y}_{Ht} + \tilde{Y}_{Ht}^*) \\ z_{Nt} L_{Nt} &= \tilde{Y}_{Nt} + z_{gt} \\ \tilde{t}b_t &= e_t \tilde{p}_{Ht}^* \tilde{Y}_{Ht}^* - \tilde{p}_{Ft} \tilde{Y}_{Ft} \\ \tilde{c}a_t &= e_t \tilde{b}_{Ft} - (G_Y \pi_t^*)^{-1} e_t \tilde{b}_{Ft-1} \\ \tilde{c}a_t &= \tilde{t}b_t + [R_{t-1}^* \Phi(t-1) - 1] (G_Y \pi_t^*)^{-1} e_t \tilde{b}_{Ft-1} \end{aligned}$$

where  $tb_t = TB_t/P_t$ ,  $ca_t = CA_t/P_t$  and  $\tilde{t}b_t = tb_t/\tilde{Y}_t$ ,  $\tilde{c}a_t = ca_t/\tilde{Y}_t$ . Home production indices, prices and demand functions related to the representative firm are

$$\begin{aligned}\tilde{Y}_t &= (a_T^{a_T} a_N^{a_N})^{-1} \tilde{Y}_{Tt}^{a_T} \tilde{Y}_{Nt}^{a_N}, \quad 1 = \tilde{p}_{Tt}^{a_T} \tilde{p}_{Nt}^{a_N}, \quad \tilde{p}_{Tt} \tilde{Y}_{Tt} = a_T \tilde{Y}_t, \quad \tilde{p}_{Nt} \tilde{Y}_{Nt} = a_N \tilde{Y}_t \\ \tilde{Y}_{Tt} &= \left[ a_H^\mu \tilde{Y}_{Ht}^{\mu-1} + a_F^\mu \tilde{Y}_{Ft}^{\mu-1} \right]^{\frac{\mu}{\mu-1}}, \quad \tilde{p}_{Tt} = \left[ a_H \tilde{p}_{Ht}^{1-\mu} + a_F \tilde{p}_{Ft}^{1-\mu} \right]^{\frac{1}{1-\mu}} \\ \tilde{Y}_{Ht} &= a_H \left( \frac{\tilde{p}_{Ht}}{\tilde{p}_{Tt}} \right)^{-\mu} \tilde{Y}_{Tt}, \quad \tilde{Y}_{Ft} = a_F \left( \frac{\tilde{p}_{Ft}}{\tilde{p}_{Tt}} \right)^{-\mu} \tilde{Y}_{Tt} \\ \tilde{Y}_{Ht} &= \tilde{H}_t^\rho \tilde{Y}_{ht}, \quad \tilde{Y}_{Ft} = (\tilde{F}_t^*)^\rho \tilde{Y}_{ft}, \quad \tilde{p}_{Ht} = \tilde{H}_t^{1-\rho} \tilde{p}_{ht}, \quad \tilde{p}_{Ft} = (\tilde{F}_t^*)^{1-\rho} \tilde{p}_{ft}.\end{aligned}$$

## C.4 Steady State Home Economy

Abbreviate  $\mu_v = \frac{v}{v-1}$  and  $\mu_N = \frac{v_N}{v_N-1}$ . Assume that government consumption is proportional to  $\tilde{Y}_N$  in steady state  $z_g = \gamma_g \tilde{Y}_N$  with  $\gamma_g \geq 0$ . Let  $\theta = a_F/a_H \tilde{\tau}^{\mu-1}$ . The following steady state quantities obtain as long as the conditions  $1 \neq a_T(\rho-1)$  and  $\mu \neq \frac{-a_T(\rho-1)}{1-a_T(\rho-1)}$  are fulfilled.

$$\begin{aligned}\tilde{e} &= \left( \frac{a_H}{a_F} \right)^{\frac{-1}{\mu-a_T(\mu-1)(\rho-1)}} \\ \tilde{\tau} &= \left( \frac{a_H}{a_F} \right)^{\frac{1-a_T(\rho-1)}{\mu-a_T(\mu-1)(\rho-1)}} \\ \tilde{L} &= \left[ \Gamma_{CY}^{-1} \left( \frac{a_N(1+\gamma_g)}{\mu_N} + \frac{a_T}{\mu_v} \right) \right]^{1/(1+\varphi)} \\ \tilde{L}_N &= \frac{a_N(1+\gamma_g)}{\mu_N \Gamma_{CY}} \tilde{L}^{-\varphi} \\ \tilde{L}_H &= \frac{a_T}{\mu_v \Gamma_{CY}} \tilde{L}^{-\varphi} \\ \tilde{Y}_N(1+\gamma_g) &= \tilde{L}_N \\ \tilde{H} &= \left[ \frac{\Gamma_{HY}}{(a_T^{a_T} a_N^{a_N})} \tilde{L}_H^{a_T} \tilde{Y}_N^{a_N} (a_H + a_F \tilde{\tau}^{\mu-1})^{\frac{a_T}{\mu-1}} \right]^{\frac{1}{1-a_T(\rho-1)}} \\ \tilde{Y} &= \frac{q}{\Gamma_{HY}} \tilde{H} \\ \frac{\tilde{p}_N}{\tilde{p}_T} &= \frac{a_N \tilde{Y}_T}{a_T \tilde{Y}_N} = \frac{\mu_N}{\mu_v} [a_H(1+\theta)]^{\frac{-1}{1-\mu}} \tilde{H}^{\rho-1}\end{aligned}$$

Parameters are  $\Gamma_{HY} = \frac{a_T \kappa \beta (1-1/\mu_v)}{(1-\kappa\beta)}$  and  $\Gamma_{CY} = 1 - \frac{1-\kappa}{\kappa} \Gamma_{HY}$ .

## C.5 Linear Model

A hat on top of a variable indicates percentage deviation from steady state. Different from this convention define  $\hat{t}b_t = (d\tilde{t}b_t/\tilde{Y})$ ,  $\hat{c}a_t = (d\tilde{c}a_t/\tilde{Y})$ ,  $\hat{b}_{Ft} = (d\tilde{b}_{Ft}/\tilde{Y})$ . Also, let  $\hat{R}_t = \hat{r}_t + E_t\hat{\pi}_{t+1}$  and  $\hat{R}_t^* = \hat{r}_t^* + E_t\hat{\pi}_{t+1}^*$ . These two equations plus the two monetary policy rules can be used to obtain paths of  $\hat{\pi}_t, \hat{\pi}_t^*, \hat{R}_t, \hat{R}_t^*$ . Nominal exchange rate growth is obtained by  $\Delta\hat{e}_t = \Delta\hat{\mathcal{E}}_t + \hat{\pi}_t^* - \hat{\pi}_t$ .

### Home Country

$$\begin{aligned}
\hat{C}_t &= E_t\hat{C}_{t+1} - \hat{r}_t \\
\hat{v}_{ht} &= \beta\kappa E_t\hat{v}_{ht+1} + (1 - \beta\kappa)E_t\hat{d}_{ht+1} - \hat{r}_t \\
\hat{w}_t &= \varphi\hat{L}_t + \hat{C}_t \\
\hat{H}_t &= \kappa\hat{H}_{t-1} + (1 - \kappa)\hat{H}_{Et-1} \\
\hat{v}_{ht} &= \phi_e\hat{H}_{Et} - \phi_E\hat{z}_{Et} \\
(1 - \frac{1}{\mu_v})[\hat{H}_t + \hat{d}_{ht}] &= \hat{p}_{Ht} + \frac{1}{1+\theta}\hat{Y}_{Ht} + \frac{\theta}{1+\theta}\hat{Y}_{Ht}^* - \frac{1}{\mu_v}[\hat{w}_t + \hat{L}_{Ht}] \\
\hat{Y}_t &= \Gamma_{CY}\hat{C}_t + (1 - \Gamma_{CY})[\hat{H}_{Et} + \hat{v}_{ht}] \\
(\frac{a_N(1+\gamma_G)}{\mu_N} + \frac{a_T}{\mu_v})\hat{L}_t &= \frac{a_N(1+\gamma_G)}{\mu_N}\hat{L}_{Nt} + \frac{a_T}{\mu_v}\hat{L}_{Ht} \\
\hat{z}_{Ht} + \hat{L}_{Ht} &= (1 - \rho)\hat{H}_t + \frac{1}{1+\theta}\hat{Y}_{Ht} + \frac{\theta}{1+\theta}\hat{Y}_{Ht}^* \\
\hat{z}_{Nt} + \hat{L}_{Nt} &= \frac{1}{1+\gamma_G}\hat{Y}_{Nt} + \frac{\gamma_G}{1+\gamma_G}\hat{z}_{Gt} \\
\hat{p}_{Ht} &= (1 - \rho)\hat{H}_t + \hat{w}_t - \hat{z}_{Ht} \\
\hat{p}_{Nt} &= \hat{w}_t - \hat{z}_{Nt} \\
\hat{p}_{Nt} + \hat{Y}_{Nt} &= \hat{Y}_t \\
\hat{Y}_{Ht} - \hat{Y}_{Ft} &= \mu\hat{p}_{Ft} - \mu\hat{p}_{Ht} \\
\hat{Y}_t &= \frac{a_T}{1+\theta}\hat{Y}_{Ht} + \frac{a_T\theta}{1+\theta}\hat{Y}_{Ft} + a_N\hat{Y}_{Nt} \\
0 &= \frac{a_T}{1+\theta}\hat{p}_{Ht} + \frac{a_T\theta}{1+\theta}\hat{p}_{Ft} + a_N\hat{p}_{Nt} \\
\hat{P}_{GDPt} &= a_T\hat{P}_{Ht} + a_N\hat{P}_{Nt} \\
\widehat{GDP}_t &= \frac{a_T}{1+\theta}\hat{Y}_{Ht} + \frac{a_T\theta}{1+\theta}\hat{Y}_{Ht}^* + \frac{a_N}{1+\gamma_G}\hat{Y}_{Nt} + \frac{a_N\gamma_G}{1+\gamma_G}\hat{\mathcal{G}}_{Nt} \\
\hat{P}_{GDPt}^m &= -\hat{Q}_{GDPt} + \hat{P}_{GDPt} \\
\widehat{GDP}_t^m &= \hat{Q}_{GDPt} + \widehat{GDP}_t \\
\hat{Q}_{GDPt} &= a_T(1 - \rho)\hat{H}_t.
\end{aligned}$$

## Foreign Country

$$\begin{aligned}
\hat{C}_t^* &= E_t \hat{C}_{t+1}^* - \hat{r}_t^* \\
\hat{\vartheta}_{ft}^* &= \beta \kappa E_t \hat{\vartheta}_{ft+1}^* + (1 - \beta \kappa) E_t \hat{d}_{ft+1}^* - \hat{r}_t^* \\
\hat{w}_t^* &= \varphi \hat{L}_t^* + \hat{C}_t^* \\
\hat{F}_t^* &= \kappa \hat{F}_{t-1}^* + (1 - \kappa) \hat{F}_{Et-1}^* \\
\hat{\vartheta}_{ft}^* &= \phi_e \hat{F}_{Et}^* - \phi_E \hat{Z}_{Et}^* \\
(1 - \frac{1}{\mu_v}) [\hat{F}_t^* + \hat{d}_{ft}^*] &= \hat{p}_{Ft}^* + \frac{\theta}{1+\theta} \hat{Y}_{Ft}^* + \frac{1}{1+\theta} \hat{Y}_{Ft}^* - \frac{1}{\mu_v} [\hat{w}_t^* + \hat{L}_{Ft}^*] \\
\hat{Y}_t^* &= \Gamma_{CY} \hat{C}_t^* + (1 - \Gamma_{CY}) [\hat{F}_{Et}^* + \hat{\vartheta}_{ft}^*] \\
\left( \frac{a_N(1+\gamma_G^*)}{\mu_N} + \frac{a_T}{\mu_v} \right) \hat{L}_t^* &= \frac{a_N(1+\gamma_G^*)}{\mu_N} \hat{L}_{Nt}^* + \frac{a_T}{\mu_v} \hat{L}_{Ft}^* \\
\hat{Z}_{Ft}^* + \hat{L}_{Ft}^* &= (1 - \rho) \hat{F}_t^* + \frac{\theta}{1+\theta} \hat{Y}_{Ft}^* + \frac{1}{1+\theta} \hat{Y}_{Ft}^* \\
\hat{Z}_{Nt}^* + \hat{L}_{Nt}^* &= \frac{1}{1+\gamma_G^*} \hat{Y}_{Nt}^* + \frac{\gamma_G^*}{1+\gamma_G^*} \hat{Z}_{Gt}^* \\
\hat{p}_{Ft}^* &= (1 - \rho) \hat{F}_t^* + \hat{w}_t^* - \hat{Z}_{Ft}^* \\
\hat{p}_{Nt}^* &= \hat{w}_t^* - \hat{Z}_{Nt}^* \\
\hat{p}_{Nt}^* + \hat{Y}_{Nt}^* &= \hat{Y}_t^* \\
\hat{Y}_{Ht}^* - \hat{Y}_{Ft}^* &= \mu \hat{p}_{Ft}^* - \mu \hat{p}_{Ht}^* \\
\hat{Y}_t^* &= \frac{a_T}{1+\theta} \hat{Y}_{Ht}^* + \frac{a_T \theta}{1+\theta} \hat{Y}_{Ft}^* + a_N \hat{Y}_{Nt}^* \\
0 &= \frac{a_T}{1+\theta} \hat{p}_{Ht}^* + \frac{a_T \theta}{1+\theta} \hat{p}_{Ft}^* + a_N \hat{p}_{Nt}^* \\
\hat{P}_{GDPt}^* &= a_T \hat{P}_{Ft}^* + a_N \hat{P}_{Nt}^* \\
\widehat{GDP}_t^* &= \frac{a_T}{1+\theta} \hat{Y}_{Ft}^* + \frac{a_T \theta}{1+\theta} \hat{Y}_{Ft}^* + \frac{a_N}{1+\gamma_G^*} \hat{Y}_{Nt}^* + \frac{a_N \gamma_G^*}{1+\gamma_G^*} \hat{G}_{Nt}^* \\
(\hat{P}_{GDPt}^m)^* &= -\hat{Q}_{GDPt}^* + \hat{P}_{GDPt}^* \\
(\widehat{GDP}_t^m)^* &= \hat{Q}_{GDPt}^* + \widehat{GDP}_t^* \\
\hat{Q}_{GDPt}^* &= a_T (1 - \rho) \hat{F}_t^* .
\end{aligned}$$

## Linkages

$$\hat{r}_t - \hat{r}_t^* = E_t \hat{e}_{t+1} - \hat{e}_t - e\phi \hat{b}_{Ft}$$

$$\hat{p}_{Ht} = \hat{e}_t + \hat{p}_{Ht}^*$$

$$\hat{p}_{Ft} = \hat{e}_t + \hat{p}_{Ft}^*$$

$$\frac{1+\theta}{a_T\theta} \hat{t}b_t = \hat{\tau}_t + \hat{Y}_{Ht}^* - \hat{Y}_{Ft}$$

$$\hat{c}a_t = \hat{t}b_t + e \frac{R-1}{G_Y\pi} \hat{b}_{Ft-1}$$

$$\hat{c}a_t = e \hat{b}_{Ft} - \frac{e}{G_Y\pi} \hat{b}_{Ft-1}$$

$$\hat{\tau}_t = \hat{p}_{Ht} - \hat{p}_{Ft}$$

$$\hat{\tau}_t^m = (\rho - 1)[\hat{H}_t - \hat{F}_t^*] + \hat{\tau}_t$$





# Appendix D

## Appendix to Chapter 5

### D.1 Weighted Moments and Transition

Mean group estimation in dynamic panel models is considered by Pesaran and Smith (1995). In case of panel heterogeneity the mean group estimator measures marginal impacts for an average cross section member. In the spirit of Phillips and Moon (1999) mean group estimation also guards against spurious inferential conclusions that might be attributed to single equation regressions with nonstationary data. As a possible statistical quantity characterizing the parameter heterogeneity one may consider the standard deviation of mean estimates.

**Construction of Weights:** Weights  $w_k^{(s)}$  represent average relative imports (exports) in a given sector  $k$  and conditional on trade relationship  $s$  where the average is taken over the period 2001:06 to 2006:05,

$$w_k^{(s)} = \frac{\bar{y}_k^{(s)}}{\sum_s \bar{Y}^{(s)}} \quad , \quad \sum_s \sum_k w_k^{(s)} = 1 \quad ,$$

where  $\bar{Y}^{(s)} = \sum_k \bar{y}_k^{(s)}$  with  $k = 1, \dots, K$  and  $s = 1, \dots, S$ .

**Coefficients and Standard Errors:** Let  $sub$  denote a subset of the 30 trade relationships we consider, i.e. either ALL, U2, O2, IN or OUT where ALL denotes the full set of trade relationships. Let  $S$  denote the number of trade relationships in  $sub$ . Then, the weighted mean group estimator and a corresponding standard error are, respectively,

$$\beta^{(sub)} = \sum_{s \in sub} \sum_k \frac{w_k^{(s)}}{\sum_{s \in sub} \sum_k w_k^{(s)}} \beta_k^{(s)} \quad \text{and} \quad \omega_{\beta}^{(sub)} = \left( \frac{1}{S} \sum_{s \in sub} \sum_k \frac{w_k^{(s)}}{\sum_{s \in sub} \sum_k w_k^{(s)}} (\beta_k^{(s)} - \bar{\beta})^2 \right)^{\frac{1}{2}} .$$

**Transition:** Smooth transition is a logistic cumulative distribution function (omit  $s$  superscript and  $k$  subscript),  $[1 + \exp\{-\theta_1(t_\kappa/(T\sqrt{0.083}) - \zeta)\}]^{-1} = \kappa$  where  $\kappa$  denotes percent of completed transition at date  $t_\kappa$ . Solve for  $t_\kappa$ ,

$$t_\kappa = T\sqrt{0.083} \left( \zeta - \frac{1}{\theta_1} \ln\left(\frac{1}{\kappa} - 1\right) \right) .$$

The number of months needed to complete medium  $(1 - 2\alpha)\%$  transition is the difference  $t_{1-\alpha} - t_\alpha = 2T\sqrt{0.083}/\theta_1 \ln((1 - \alpha)/\alpha)$ . With  $\alpha = .05$ ,  $T = 137$  and  $\theta_1 = 232.89$  the fastest 90% of transition happen in one month. The symmetry point of the transition function obtains with  $\kappa = 0.5$  as  $t_{0.5} = T\sqrt{0.083} \zeta$ . Converting the lower and upper bound of  $\zeta \in [0.30343, 3.186]$  into month delivers [12,126]. To approximate a dummy that kicks in at 1999:01 fix the symmetry point  $\zeta = 1.2137$  and set the transition speed to  $\theta_1 = 232.89$ .

## D.2 Kalman Recursions

Given the parameters of the state-space model in (5.2) to (5.5),  $\boldsymbol{\psi}_k^{(s)'}$ ,  $\boldsymbol{\phi}^{(s)'}$ , the Kalman filter provides sequentially linear projections for the dynamic system. The likelihood of the model is computed stepwise. In the following reported estimates will have a second index reflecting the time point up to which data for the computations are collected. Such an extra index easily allows to discriminate between forecasts and updates. The analyst is assumed to have some guess concerning the initial states of the system (denoted  $\lambda_{k,0|0}^{(s)}$ ) and their variances ( $P_{k,0|0}^{(s)}$ ). The Kalman recursions for regression models with missing observations are given by the following steps (Jones (1985)):

1. Computation of a one step ahead forecast for the state and the associated variance:

$$\begin{aligned} \lambda_{k,t|t-1}^{(s)} &= \lambda_{k,t-1|t-1}^{(s)} \\ P_{k,t|t-1}^{(s)} &= P_{k,t-1|t-1}^{(s)} + h_k^{(s)}. \end{aligned}$$

2. The forecast of the state and observable explanatory variables are used to obtain a prediction for the dependent variable:

$$y_{k,t|t-1}^{(s)} = q_{it}^{(s)} \beta_{ik}^{(s)} + q_{jt}^{(s)} \beta_{jk}^{(s)} + (1 - \sigma^{(s)}) \left( \ln(\tau_{k,t}^{(s)}) + \lambda_{k,t|t-1}^{(s)} \right). \quad (\text{D.1})$$

3. Comparing  $y_{kt}^{(s)}$  and  $y_{k,t|t-1}^{(s)}$  is feasible in case that  $y_{kt}^{(s)}$  is observed. Then, the prediction error  $u_{kt}^{(s)}$  with variance  $W_{kt}^{(s)}$  is obtained as:

$$\begin{aligned} w_{kt}^{(s)} &= y_{kt}^{(s)} - y_{k,t|t-1}^{(s)} \\ W_{kt}^{(s)} &= (1 - \sigma^{(s)})^2 P_{k,t|t-1}^{(s)} + g_k^{(s)}. \end{aligned}$$

4. The latter quantities contribute to the models' log likelihood with

$$l_{kt}^{(s)} = -0.5 \ln(2\pi) - 0.5 \left( u_{kt}^{(s)} \right)^2 / W_{kt}^{(s)} - 0.5 \ln W_{kt}^{(s)}. \quad (\text{D.2})$$

5. The innovation  $u_{kt}$  and its variance are used to update the current estimate of the state vector:

$$\begin{aligned} \lambda_{k,t|t}^{(s)} &= \lambda_{k,t|t-1}^{(s)} + P_{k,t|t-1}^{(s)} (1 - \sigma^{(s)}) u_{kt}^{(s)} / W_{kt}^{(s)} \\ P_{k,t|t}^{(s)} &= P_{k,t|t-1}^{(s)} + (1 - \sigma^{(s)})^2 \left( P_{k,t|t-1}^{(s)} \right)^2 / W_{kt}^{(s)}. \end{aligned}$$

Note that the log likelihood function integrates over all time and sector specific estimates  $l_{kt}$  given in (D.2), i.e.

$$l = l(\boldsymbol{\psi}_k^{st}, \boldsymbol{\phi}^{st})' = \sum_k \sum_t l_{kt}.$$

In case a particular observation on  $y_{kt}^{(s)}$  is missing, steps 3. and 4. are left out and the updating in step 5. becomes

$$\begin{aligned} \lambda_{k,t|t}^{(s)} &= \lambda_{k,t|t-1}^{(s)} \\ P_{k,t|t}^{(s)} &= P_{k,t|t-1}^{(s)}. \end{aligned}$$

## D.3 Serial Correlation Tests

Serial correlation might easily be diagnosed by means of Portmanteau type test statistics exploiting the autocorrelation coefficients of the estimated model residuals  $\hat{u}_{kt}^{(s)}$ . To obtain an indication of serially correlated error terms which is robust under heteroskedasticity, however, we rather use the following auxiliary regression:

$$\hat{u}_{kt}^{(s)} = c + \kappa_1 \hat{u}_{kt-1}^{(s)} + \dots + \kappa_h \hat{u}_{kt-h}^{(s)} + v_t,$$

where  $c$  is an intercept term and  $v_t$  a white noise disturbance. We test the null hypothesis  $H_0 : \kappa_1 = \kappa_2 = \dots = \kappa_h = 0$  by means of a Wald-test

$$\omega_h = \hat{\kappa}' (\text{Cov}[\hat{\kappa}])^{-1} \hat{\kappa} \xrightarrow{d} \chi^2(h). \quad (\text{D.3})$$

To implement the statistic in (D.3) we use the heteroskedasticity consistent covariance estimator for the estimated parameter vector  $\hat{\kappa} = (\hat{\kappa}_1, \hat{\kappa}_2, \dots, \hat{\kappa}_h)'$  (White (1980)). With respect to the choice of the lag order  $h$  we consider tests on serial correlation at lag 1 and joint correlation at lags 1 to 12. The latter choices appear reasonable noting that monthly data enter our analyses.

## D.4 Data Appendix

We seasonally adjusted trade data by means of seasonal dummies. Merging data in value and in volume allows to express exports and imports in constant prices of 2000. To do so, we compute implicit unit price deflators and use the average of the 12 price observations in 2000 to re-value volumes. Monthly industrial production data comes from International Financial Statistics (IFS) of the IMF. Monthly exchange rates are market rates from IFS. Daily exchange rate data used to compute exchange rate volatility comes from the FED historical database.<sup>1</sup> The IFS indicators of real effective exchange rates based on unit labor costs in manufacturing represent the product of the index of the ratio of the relevant indicator (in national currency) for the country listed to a weighted geometric average of the corresponding indicators for 20 other industrial countries. Bilateral real exchange rates are computed as  $rex = eP_{par}/P_{rep}$  where  $e$  is reporter's currency in terms of partner's currency and  $P_{rep}$  ( $P_{par}$ ) denotes reporter's (partner's) producer price index. For *rexus* the partner country is the U.S. Producer price indices and the energy price index are drawn from IFS. All indices are normalized to a base year 2000. We take natural logs of all series unless otherwise noted.

<sup>1</sup><http://www.federalreserve.gov/releases/h10/Hist/default1999.htm>

# Deutsche Zusammenfassung

Die Arbeit besteht aus vier Essays, die theoretische und empirische Beiträge in den Bereichen der monetären Ökonomie und der ökonomischen Integration liefern. Der erste Essay untersucht die geldpolitischen Folgen von Messfehlern in der Inflationsrate. Es wird ein Konjunkturmodell mit Markteintritt von Gütern entwickelt, welches neben einer Stabilisierungsfunktion für die Zentralbank einen Messfehler in der Inflationsrate nachbildet. Der Messfehler entsteht durch neue Güter, die bei der Berechnung des Konsumentenpreisindex anfangs unberücksichtigt bleiben. Das Modell zeigt, dass zwischen einem solchen Messfehler und dem Konjunkturzyklus ein systematischer Zusammenhang besteht. Dieser Zusammenhang impliziert, dass gemessene Inflation weniger variabel, dafür aber stärker persistent über den Konjunkturzyklus ist. Wenn der Messfehler von der Zentralbank nicht aufgefangen wird, führt er zu einer unzureichenden Stabilisierung der Inflationsrate. Der zweite Essay zeigt eine Divergenz zwischen stilisierten Fakten und einem häufig verwendeten monetären Modellrahmen auf. Während die stilisierten Fakten eindeutig auf eine dynamische Geldnachfragefunktion hindeuten, bleibt die Standardformulierung der Geldnachfrage rein statisch. Ich erarbeite eine Erweiterung des Standardmodells, die eine dynamische Geldnachfrage liefert. Im Rahmen des erweiterten Modells untersuche ich optimale Geldpolitik. Es zeigt sich, dass die Geldmenge zwar aus wohlfahrtstheoretischer Sicht bedeutend ist, aus geldpolitischer Sicht aber nicht. Dieses Ergebnis steht im Zusammenhang mit der anhaltenden Debatte über die geldpolitische Strategie der Europäischen Zentralbank. Der dritte Essay widmet sich dem Angleichungsprozess der neuen Mitgliedsstaaten der Europäischen Union. Diese Länder erfahren in den letzten Jahren eine Aufwertung ihres realen Wechselkurses bei gleichzeitiger Verbesserung der außenwirtschaftlichen Handelsbedingungen. Während das Zwei-Länder-Zwei-Güter-Modell realer Konjunkturzyklen beide Fakten zusammen nicht erklären kann, zeigt meine Analyse, dass eine Erweiterung des Modells hinsichtlich endogener Gütervielfalt dieses Problem löst. Damit scheint eine wichtige Triebkraft des Angleichungsprozesses eine Form von Produktivitätswachstum zu sein, welche

die Güervielfalt erhöht, gleichzeitig aber wenig Einfluss auf die Produktionsmenge ausübt. Der vierte Essay beinhaltet eine empirische Analyse des Effektes der gemeinsamen Europäischen Währung auf den Handel. Empirische Arbeiten mit dem Ziel, den Handelseffekt des Euro zu quantifizieren, verwenden oft einen ähnlichen und sehr speziellen Modelrahmen. Gemeinsam mit Helmut Herwartz verfolgen wir einen allgemeineren Ansatz, der es erlaubt, den Handelseffekt in die drei Dimensionen des 'Wann, Wie schnell, und Wie stark' zu zerlegen. Wir finden einen graduellen Anstieg der europäischen Handelsaktivität zwischen den Jahren 2000 und 2003 und zeigen, dass Annahmen bezüglich dem Zeitpunkt des Effektes entscheidend sind für Schlussfolgerungen bezüglich der Stärke des Effektes.