Chapter 4

Price Effects and Equivalence Scales

In Chapter 3 the Rothbarth method for the estimation of equivalence scales was explored. Its advantage is that it is easily estimated and the assumptions on the underlying household model and on demographic separability in particular are clear and testable. There are, however, some serious drawbacks: the method does not account for possible substitution effects that affect the demand for the identifying adult good when separability is violated. Some goods become relatively more expensive to consume than others when children are present, because of different economies of scale in joint consumption and because of the additional needs of children for various groups of goods. The price of a package holiday, for example, may rise sharply when parents have to pay up for their children, and clothing will become more expensive when parents' clothes have to be matched by corresponding children's wear. On the other hand, the effective price of a cigarette or a drink is not affected at all, as long as the children are under a certain age. These changes in the relative cost of goods affect the perceived prices and could cause substitution effects, leading to a biased estimate when the Rothbarth method is applied. In addition, adult goods are sometimes not observed or difficult to distinguish from other goods.

The Barten (1964)[1] and Gorman (1976) methods of estimating equivalence scales overcome some of these problems by allowing for substitution between different groups of goods. In both cases demographic effects are included into a complete demand system via good specific scale factors that embody the needs of additional household members. In the Gorman method, scale

[1] Barten originally intended the method as a means of estimating a complete demand system without information on prices if equivalence scales are known *a priori*. Muellbauer (1977) points to the usefulness of the method for estimating equivalence scales if a complete demand system can be estimated. Pollak and Wales (1980) refer to the method as demographic scaling.

factors are combined with overheads that pick up the fixed costs of children. The scale factors affect the effective price of each good and generate possible substitution effects that can be integrated into the estimation process.

In order to employ the Barten and Gorman methods, a complete demand system must be estimated. This is possible only if the reaction of household demands to price changes can be observed. Price changes can either occur in time series data or in regional data. Using time series data is difficult in the case of Germany, because the income and expenditure survey is carried out only every five years and commodity groupings have changed between survey periods.[2] Regional price differences cannot be used either, because the German Statistical Survey does not collect comparable regional price indexes.[3] Due to these limitations, up to now only the rather restrictive extended linear expenditure system has been used to estimate Barten equivalence scales from German data. In this chapter, a new method will be presented which allows for the estimation of Barten equivalence scales and a complete quadratic expenditure system (QES) from one cross section only.[4]

Identification of the demand system is achieved by exploiting the curvature of quadratic Engel curves of the QES. Because derivatives of the Engel curves are used for identification, the parameters of the model are only identified if Engel curves exhibit sufficient curvature – a linear model would not be identified. Empirical investigations confirm that the curvature of the estimated German expenditure functions is not sufficient to achieve a good enough estimate of all demand system parameters. Still, satisfactory results can be achieved from a single cross section, if Rothbarth-like constraints are imposed on the scale factors, i.e. some scale factors are fixed based on a priori considerations. For example, scale factors for pure adult goods can be fixed at a value of one: a couple with children faces the same scaled price for this good as a childless couple. The demand system is estimated using a full information maximum likelihood approach. In contrast to traditional Rothbarth scales, this procedure has the advantage, that information on several adult goods can be combined in the estimation and substitution effects are accounted for.

The plan of the chapter is as follows: First the Barten and Gorman methods and the respective literature will be reviewed. Then the quadratic expenditure system will be explored. In the appendix I will prove that the parameters can be recovered using a limited information approach, and the full information approach will be reviewed. Values of scale factors that can

[2]See the discussion on the details of the data set in the Appendix. It is possible that this situation will improve with planned changes in the data collection mechanism of the EVS.

[3]See Linz and Eckert (2004) for a discussion of regional cost of living in Germany.

[4]Kohn and Missong (2003) use a QES to estimate equivalence scales from German data, but they are not successful in estimating Barten scales. A translating model is estimated instead (see below).

be possibly fixed will be determined in the empirical section. Different specifications of the full information approach will be compared: Two Barten models and a Gorman model with different degrees of additional restrictions, Rothbarth style constraints on clothing and on the adult goods tobacco and alcohol. To facilitate following the line of reasoning, a flow chart of the chapter's structure is given in Appendix 4.E to this chapter.

4.1 Household Composition and Demographic Scaling

When households of different composition are assessed within one demand system, demands can be affected by composition in two ways: First, household members have different needs. Children for example have different needs from adults. They probably need less food and different clothing, no alcohol or cigarettes, and they might drink more milk than adults. Second, economies of scale from the joint consumption of household public goods or in household production can generate more effective individual consumption from a given amount of a good than a division of the amount by the number of heads in the household might suggest. E.g., a couple gets twice as much individual consumption out of a TV set than a person living alone (provided they share the same preferences about which program to watch), and a family can share the fixed cost of most household appliances. There are of course limits to sharing. A family of five will probably need a larger refrigerator than a person living alone, but the cost of an appropriately sized fridge will still be less than those of five separate ones.

The Barten Model: Demographic Scaling

In the Barten Model it is assumed that observed quantities for a household of type s can be converted into the equivalent consumption of a reference household of type r by scaling them according to a vector of goods specific scale factors:

$$\boldsymbol{m}(s) = \{m_1^s, ..., m_n^s\}. \tag{4.1}$$

The reference household is usually either a single adult or a childless couple. The m_i^s are determined relative to the values of the reference household for which all m_i^r are normalized to one. Reference equivalent consumption in a household of type s is

$$q_i^r = \frac{q_i^s}{m_i^s}, \tag{4.2}$$

where q_i^s is observed household consumption and m_i^s is the respective scale factor. q_i^r is the equivalent consumption of a reference household. Suppose the reference household is a single adult, household s is a childless couple, and $m_{apple}^s = 2$. Then, if the couple is observed eating two apples this is equivalent to the consumption of one apple by a single adult. Thus, q_i^r is the effective individual consumption of all adults in the household if the reference household is a single adult. When couples with children are compared with childless couples as reference, q_i^r is the joint consumption of both parents together. Recalling the example of the TV set above, the scale factor relative to a single adult for the TV set would be equal to one, while the factor for the refrigerator of a family of five would be less than five but more than one, because there are some economies of scale, but a refrigerator is not entirely public due to congestion. For the remainder of the chapter, the reference household is always a childless couple.

Scale factors reflect economies of scale as well as different needs. Assume the reference household r is a childless couple and the compared household s is a couple with two children. In this constellation a scale factor m_i^s of 1.6 can have two meanings: for clothes a factor of 1.6 implies that there are no savings from sharing or buying things together, but the children use fewer or cheaper clothes than the parents (unequal needs, no economies of scale). Then again a scale factor of 1.6 for housing might mean that the children need as much space as the parents (children need a lot of space), but they can share kitchen and bathroom (same needs, economies of scale). These two effects cannot be separated from each other except if all compared households have identical individuals, i.e. households contain only adults. In this case scale factors reflect economies of scale only.[5]

The utility function of household type s can be written in terms of the utility function of the reference household u^* by inserting scaled quantities:

$$u^s(\boldsymbol{q}^s) = u^*(q_1^s/m_1^s, ..., q_n^s/m_n^s) \ . \tag{4.3}$$

The utility function u^* is assumed to be equal for all households. It is assumed that preferences of a couple do not change when they have children. Parents have the same utility from consuming quantities q_i^s with their children as from consuming scaled quantities q_i^s/m_i^s when they were living alone.

Marshallian demands $g_i(\mu, p_1, ..., p_n)$ and expenditure functions $x_i(\mu, p_1, ..., p_n)$ for the reference household r (with $m_i = 1\,\forall i$) can be inferred by maximizing the utility function u^* subject to the budget constraint:

$$\max u^*(q_1, ..., q_n) \text{ s.t. } \mu = \sum_{i=1}^{n} q_i p_i \ , \tag{4.4}$$

where μ is total expenditure.

Accordingly, maximizing the scaled utility function u^s (4.3) leads to the

[5] See Nelson (1988) for an application.

Marshallian demand functions for household type s. Using 4.3 one can write:

$$\max u^*(q_1^s/m_1^s, ..., q_n^s/m_n^s) \text{ s.t. } \mu = \sum_{i=1}^{n} \frac{q_i^s}{m_i^s} p_i m_i^s \ . \tag{4.5}$$

Noting the similarity between scaled quantities $\frac{q_i}{m_i^s}$ and scaled prices $p_i^s = p_i m_i^s$ in equation 4.5 and quantities q_i and prices p_i in equation 4.4, the solution for g_i can be used to find the demands g_i^s:

$$\begin{aligned} \frac{g_i^s}{m_i^s} &= g_i(\mu, p_1^s, ..., p_n^s) \\ \text{or} \quad g_i^s &= m_i^s g_i(\mu, p_i^s, ..., p_n^s) \ . \end{aligned} \tag{4.6}$$

Multiplication with the price p_i gives the expenditure function of household type s:

$$\begin{aligned} x_i^s &= p_i m_i^s g_i(\mu, p_1^s, ..., p_n^s) \\ &= p_i^s g_i(\mu, p_1^s, ..., p_n^s) \\ &= x_i(\mu, p_1 m_1^s, ..., p_n m_n^s) \ . \end{aligned} \tag{4.7}$$

Note that normalization of the scale factors for the reference household leads to the equalities $q_i^r = g_i(\mu, p_1, ..., p_n)$ and $x_i^r = x_i(\mu, p_1, ..., p_n)$.

The Gorman Method: Demographic Scaling and Translating

Demographic scaling explains observed changes in demands exclusively by substitution effects. This can be a problem if only small quantities of a good are consumed by the reference household, while the compared household type consumes a lot of it. This applies mostly to pure children's goods, e.g. baby food. In the extreme case, if the reference household is a childless couple that is not consuming any baby food at all, it is possible to explain a young parent's demand for baby food by a multiplicative scaling factor only if the childless couple's demand is the result of a corner solution of the utility maximization program. Even if the demand of the reference couple is not zero but small compared to that of the parents, scaling factors can become very large, leading to an extreme change in scaled prices and excessive substitution.

This problem can be solved partly by selecting wider commodity groups that encompass adult and children's goods, e.g. all basic foods. This can be a sensible solution if it is assumed that parents provide their children with food, clothing, housing etc. in the same way as they care for themselves, no matter if the children consume the same commodities or special children's versions of these commodities.

Gorman (1976) suggests a different solution: children have minimum needs that have to be satisfied: a minimum amount of food, clothing, toys, etc.

For each commodity, these needs incur a subsistence level of fixed costs or overheads β_i^s which depends on household composition and is independent of the household's income. The household utility function becomes:

$$u^s = u(q_1^s - \beta_1^s, q_2^s - \beta_2^s, ..., q_n^s - \beta_n^s) \tag{4.8}$$

The subsistence level reduces the discretionary income of the household and Marshallian demands change to:

$$g_i^s = \beta_i^s + g_i\left(\mu - \sum_{j=1}^{n} p_j\beta_j^s, p_1, .., p_n\right) \tag{4.9}$$

Pollak and Wales (1980) call this method demographic translating.

The Gorman method is a combination of translating and scaling. With translating and scaling, the reference equivalent quantities are:

$$q_i^r = q_i^s/m_i^s - \beta_i^s \ , \tag{4.10}$$

and the utility function and demands can be written as

$$u^s = u(q_1^s/m_1^s - \beta_1^s, q_2^s/m_2^s - \beta_2^s, ..., q_n^s/m_n^s - \beta_n^s) \qquad \text{and} \tag{4.11}$$

$$g_i^s = \beta_i^s + m_i^s g_i(\mu - \sum_{j=1}^{n} p_j\beta_j^s, m_1^s p_1, .., m_n^s p_n), \tag{4.12}$$

respectively.

Now define scaled overheads:

$$\tilde{\beta}_i^s = \beta_i^s/m_i^s \ . \tag{4.13}$$

Using scaled overheads and scaled prices ($p_i^s = m_i^s p_i$), the expenditure function can be written as:

$$x_i^s = p_i^s\left(\tilde{\beta}_i^s + g_i(\mu - \sum_{j=1}^{n} p_j^s\tilde{\beta}_j^s, p_1^s, .., p_n^s)\right) . \tag{4.14}$$

Substitution of 4.12 into 4.11 gives the indirect utility function

$$V^s = V^*\left(\mu - \sum_{i=1}^{n} p_i^s\tilde{\beta}_i^s, p_1^s, .., p_n^s\right), \tag{4.15}$$

where $V^*(.)$ is the indirect utility function of the reference household.

Translating and scaling are nested in this model with all $\tilde{\beta}_i^s = 0$ for scaling and all $m_i^s = 1$ for the pure translating case.

Scaling, Translating and Equivalence Scales

For the determination of a cost of living index or an equivalence scale it is useful to calculate the cost function by substituting the Marshallian demand functions into the utility function and inversion. The cost function of household s follows from the cost function of the reference household c^* through scaling and translating:

$$\begin{aligned} c^s(u, \boldsymbol{p}) &= c^*(u, m_1^s p_1, ..., m_n^s p_n) + \sum_{j=1}^{n} p_j m_j^s \tilde{\beta}_j^s \\ &= c^*(u, \boldsymbol{p}^s) + \boldsymbol{p}^{s\prime} \tilde{\beta}^s \end{aligned} \tag{4.16}$$

where $\boldsymbol{p}$, $\boldsymbol{p}^s$ and $\tilde{\beta}^s$ are the vectors of prices, scaled prices, and adjusted overheads, respectively.

A general equivalence scale $m_r^s(u_0)$ of household type s with respect to the reference household type r at any particular utility level u_0 can be written in terms of the cost functions:

$$m_r^s(u_0) = \frac{c^*(u_0, \boldsymbol{p}^s) + \boldsymbol{p}^{s\prime} \tilde{\beta}^s}{c^*(u_0, \boldsymbol{p})}. \tag{4.17}$$

It is convenient to express the equivalence scale not in terms of utility level u_0 but in terms of income of the reference household μ^r and prices. Substitution of the indirect utility function 4.15 gives then

$$m_r^s(\mu^r) = \frac{c^*\left(V^*\left(\mu^r, \boldsymbol{p}\right), \boldsymbol{p}^s\right) + \boldsymbol{p}^{s\prime} \tilde{\beta}^s}{\mu^r}. \tag{4.18}$$

This equivalence scale will generally change with the level of the reference income μ^r. Again, scaling and translating are nested within this general formula with the above-mentioned constraints: $\tilde{\beta}^s = \mathbf{0}$ for scaling and $\boldsymbol{p}^s = \boldsymbol{p}$ for translating.

For the Barten method the cost function becomes:

$$c^s(u, \boldsymbol{p}) = c^*(u, \boldsymbol{p}^s), \tag{4.19}$$

and the equivalence scale is:

$$m_r^s(\mu^r) = \frac{c^*\left(V^*\left(\mu^r, \boldsymbol{p}\right), \boldsymbol{p}^s\right)}{\mu^r}. \tag{4.20}$$

4.2 A Review of the Literature

The method of demographic scaling was first suggested by Barten (1964). His idea was to use the demographic price effects to estimate price elasticities, when only cross section data without direct price variation are available. This was generalized by Muellbauer (1974) to the estimation of all parameters of a demand system. However, Muellbauer shows in the same paper that, if applied to data without price variation, the Barten model suffers from a general identification problem, which does not permit parameters other than the income responses to be completely identified from cross-sectional data alone.[6]

If data with price variation either from several cross sections or from regional price indices are available, the estimation of Barten equivalence scales poses no problem in principle. However, because of the difficulty of the estimation procedure and the demands on the data, there exists only a limited number of studies which apply the method.

Using data from five cross sections (1968–1973), Muellbauer (1977) estimates Barten scales in a Working-Leser demand system with 10 commodity groups. In a test he rejects the Barten hypothesis against an unrestricted model. Three explanations for rejection are suggested apart from the Barten hypothesis being actually wrong: 1) the functional form for Barten prices[7] is too restrictive. 2) The demand system is too restrictive. 3) Systematic differences in time series and cross section behaviour: Muellbauer finds that direct price elasticities are significantly lower than pseudo price elasticities derived from demographic change. This could mean that price responses that are caused by Barten price effects are fundamentally different from those that are experienced over time. Households are adapted to long run price changes that are caused by household composition, while in time series data short run changes are more prominent, to which households react only slowly.

Barnes and Gillingham (1984) estimate a QES including demographic effects into the model via scaling, translating and a combination of both. Using microdata and time as well as regional price variations for the US, they reject all three possibilities of including demographics against an unpooled model with a disaggregation into singles, couples and families with children. Equivalence scales are not estimated.

Nelson (1988) estimates a demographically scaled QES for households with adults only. The aim of the exercise is to estimate economies of scales for food, shelter, household furnishings/operations, clothing and transportation. Equivalence scales are not reported. Again, scaling is rejected against a dis-

[6] See the next section for a detailed presentation of Muellbauer's argument.

[7] The functional form of the goods specific scales was $m_{ih} = 1 + \delta_i a_h$, where a_h is the number of children in the household. All children have the same weight, there are no economies of scale in child costs.

aggregation into households with one, two and three-plus members. Data are taken from two cross sections with regional price variation.

Pollak and Wales (1980, 1981) study the inclusion of demographic variables in demand systems. Naturally, the same methods that are used to estimate equivalence scales can be applied here. Given their criticism on the validity of economic equivalence scales (Pollak and Wales, 1979), it comes as no surprise that they are not interested in their estimation. Nevertheless, both papers are relevant here, because they test the performance of different methods against each other. In the 1980 paper, Pollak and Wales use a Barten procedure, which they call demographic scaling, on the QES and a generalized translog (GTL) demand system. They also use demographic translating, which applies fixed cost to each additional family member, and find scaling generally superior to translating. In the 1981 paper, five specifications were tested against each other and against an unpooled specification: scaling, translating, the Gorman procedure, the "reverse Gorman" procedure, where translating and scaling are interchanged[8] and a Prais-Houthakker[9] procedure. In contrast to all of the above-mentioned authors, Pollak and Wales restrict their survey to households of couples with children present. For these, they find that the Barten, the Gorman and the Prais-Houthakker methods are all valid representations of demographic effects; only translating was rejected against the unpooled specification. In pairwise tests, scaling (the Barten method) performed better than both Gorman methods and translating. The modified Prais-Houthakker performed best of all, but this method is of limited use here, because it does not always yield a theoretically plausible demand system, nor does it allow for the identification of equivalence scales – and Pollak and Wales were not concerned with equivalence scales estimation in the first place. In the estimation they use aggregated time series data. Some caution is in place when their results are applied to micro data.

Jorgenson and Slesnick (1987) use the Barten approach combined with a translog indirect utility function to estimate equivalence scales. Admittedly, they are interested in equivalence scales only in so far as they can be used for the aggregation of consumer data, but they stretch the approach to its limits, when they estimate not only the cost of children but the cost of other characteristics as well.

Estimation of Barten equivalence scales for Germany encounters some difficulties, because the German income and expenditure survey (EVS) is undertaken only once every five years, making the estimation of demand systems from several cross sections rather difficult. The extended linear expenditure system (ELES, Lluch, 1973) can be estimated from only one cross section and has been used by various studies on German data (Scheffter, 1991; Merz and

[8]The reverse Gorman procedure is distinct from the Gorman procedure only for specific functional forms of the m_i^s and $\tilde{\beta}_i^s$ parameters.

[9]See page 16.

Faik, 1995; Faik, 1995; Stryck, 1997; Missong and Stryck, 1998). However, the ELES is very restrictive: it has only linear Engel curves and its identification relies on a special assumption on the saving behaviour of households.[10]

Kakwani and Son (2005) also estimate Barten equivalence scales from data without price variation, for Australia. They show that knowing the compensated demand elasticity with respect to household composition for only one good is sufficient to estimate a Barten equivalence scale. Alternatively, if the size of the elasticity is unknown, but it is known that the elasticities for two goods are identical, the equivalence scale can be estimated as well. The approach differs from the one followed in this chapter in two ways. First it is in the demand system used; Kakwani and Son use a Working-Leser demand system. This is not a pure formality, because the QES used here allows for the recovery of additional information from the non-linearity of its demand curves. Second, they do not fix a scale factor, but what they call the index of economies of scale. In contrast to the scale factor, this index includes the compensated price reactions of the household. In the paper, expenditures on health care are assumed to be purely private. If this is a valid assumption certainly depends on the health care system of the country in question.

4.3 Identification of the Barten Model

Barten equivalence scales can be estimated in a complete demand system, if data with sufficient price variation are available. In the present work micro-data from only one cross section of the German EVS were used, and scales are identified without the direct observation of price reactions.[11] If successful, this approach has additional merits: in the estimation of Barten equivalence scales it is assumed that the responses to changes in the direct goods prices are the same as the responses to a shift in scaled prices caused by a change in household composition. Both reactions can be different, e.g. because households need time to adapt to changes in goods prices, and therefore observed price reactions are not complete, while households have a long time to adapt to the permanent changes in scaled prices caused by a new household member.

Identification When Scale Factors are Known

Muellbauer (1974) shows that, if applied to one cross section without price variation, the Barten model suffers from a general identification problem, which does not permit parameters other than the income responses to be completely identified. The argument goes as follows:

[10] For a detailed discussion of the ELES, see Section 2.2.5, p. 25.

[11] Two cross sections (1993 and 98) were available, but relative price variation between years was smaller than the error introduced by a change in the system of how commodity groups are organized. Therefore a standard estimation procedure with price variation was not feasible.

According to equation 4.6, the Marshallian demand equations for a household with the vector of demographic variables $\boldsymbol{s}$ can be written in the form:

$$q_i(\boldsymbol{s}, \mu, p_1, ..., p_n) = m_i^s g_i(\mu, p_1^s, ..., p_n^s), \quad i = 1, ..., n \tag{4.21}$$

A change in household composition parameter s will affect demand of good i directly through the change in the scale factor m_i^s and indirectly through the changes in the scaled prices $p_i^s = p_i m_i^s$:

$$\frac{\partial q_i}{\partial s} = \frac{\partial m_i}{\partial s} \cdot g_i + m_i \sum_{j=1}^{n} \frac{\partial g_i}{\partial p_j^s} \frac{\partial p_j^s}{\partial s} \tag{4.22}$$

where the index s has been dropped from the m_i.

Substituting

$$\frac{\partial p_i^s}{\partial s} = p_i \frac{\partial m_i}{\partial s} \quad \text{and} \quad \frac{\partial g_i}{\partial p_j^s} = \frac{1}{m_j} \frac{\partial g_i}{\partial p_j}$$

and writing in elasticity form gives the directly observable uncompensated household type elasticity of demand:

$$\left.\frac{s}{q_i} \frac{\partial q_i}{\partial s}\right|_{\bar{\mu}} = \frac{s}{m_i} \frac{\partial m_i}{\partial s} + \sum_{j=1}^{n} \left[\frac{p_j}{g_i} \frac{\partial g_i}{\partial p_j}\right] \left[\frac{s}{m_j} \frac{\partial m_j}{\partial s}\right] \tag{4.23}$$

which can be rewritten as:

$$\phi_i = \gamma_i + \sum_j \varepsilon_{ij} \gamma_j \ , \tag{4.24}$$

where

$$\phi_i = \frac{s}{q_i} \frac{\partial q_i}{\partial s} \ ,$$
$$\gamma_i = \frac{s}{m_i} \frac{\partial m_i}{\partial s} \ \text{and}$$
$$\varepsilon_{ij} = \frac{p_j}{g_i} \frac{\partial g_i}{\partial p_j}$$

are the uncompensated elasticity of demand with respect to household composition, the elasticity of the scale factors with respect to household composition and the uncompensated price elasticity of demand, respectively.

$$\eta_i = \frac{\mu}{g_i} \frac{\partial g_i}{\partial \mu}$$

is the income elasticity of the demand for good i.

Substitution of the Slutsky equation $\varepsilon_{ij} = \tilde{\varepsilon}_{ij} - w_j \eta_i$ and rewriting gives:

$$\phi_i = [\gamma_i - \sum_j w_j \eta_i \gamma_j] + \sum_j \tilde{\varepsilon}_{ij} \gamma_j \tag{4.25}$$

which can be written in matrix form:

$$\boldsymbol{\phi} = [\boldsymbol{I} - \boldsymbol{\eta} \boldsymbol{w}'] \gamma + \tilde{\boldsymbol{E}} \gamma \tag{4.26}$$

From this equation, $\boldsymbol{\gamma}$ can be identified if $\tilde{\boldsymbol{E}}$ is known and different from zero. But with only one cross section, $\tilde{\boldsymbol{E}}$ is not known and the scale factors cannot be identified. Conversely, $\tilde{\boldsymbol{E}}$ can be identified, if $\boldsymbol{\gamma}$ is known a priori, but this is not the case either. For an approximate solution, one could set all compensated price elasticities equal to zero, but with $\tilde{\boldsymbol{E}} = \mathbf{O}$, $\boldsymbol{\gamma}$ cannot be determined, because the matrix $[\boldsymbol{I} - \boldsymbol{\eta} \boldsymbol{w}']$ is singular.

$\tilde{\boldsymbol{E}}$ and $\boldsymbol{\gamma}$ can be estimated in a complete demand system, if data with sufficient price variation are available. However, with only one cross section and no information on prices available, a different route to the estimation of Barten equivalence scales must be found.

Identification when a Compensated Demand Elasticity is Known

Kakwani and Son (2005) show that identification of Barten scales from only one cross section is possible if the compensated demand elasticity with respect to household composition for one good is known. Consider the derivative of the cost function $c = c(u, p^s)$ (equation 4.19) with respect to demography:

$$\begin{aligned} \frac{\partial c}{\partial s} &= \sum_{i=1}^{n} \frac{\partial c}{\partial p_i^*} \frac{\partial p_i^*}{\partial s} \\ &= \sum_{i=1}^{n} q_i^* p_i \frac{\partial m_i}{\partial s} \end{aligned} \tag{4.27}$$

4.27 can be written in terms of elasticities as the elasticity of total cost with respect to household composition. This elasticity is closely related to the differences in the value of the equivalence scales of compared household types:

$$\phi^* = \frac{s}{c} \frac{\partial c}{\partial s} = \sum_{i=1}^{n} w_i \gamma_i \tag{4.28}$$

The Hicksian demand equation is:

$$q_i^*(u, s) = m_i h_i(u, p_1^s, ..., p_n^s) \tag{4.29}$$

and its derivative with respect to household demography is:

$$\begin{aligned}\frac{\partial q_i^*}{\partial s} &= \frac{\partial m_i}{\partial s} h_i + \sum_{j=1}^{n} m_i \frac{\partial h_i}{\partial p_j^s} \frac{\partial p_j^s}{\partial m_j} \frac{\partial m_j}{\partial s} \\ &= \frac{\partial m_i}{\partial s} h_i + \sum_{j=1}^{n} \frac{m_i}{m_j} \frac{\partial h_i}{\partial p_j} p_j \frac{\partial m_j}{\partial s}\end{aligned} \tag{4.30}$$

Analogous to the Marshallian demands for the uncompensated elasticities, this gives the compensated demand elasticity with respect to household composition:

$$\phi_i^* = \gamma_i + \sum_{j=1}^{n} \tilde{\varepsilon}_{ij} \gamma_j \tag{4.31}$$

Plug the Slutsky equation in the form $\tilde{\varepsilon}_{ij} = \varepsilon_{ij} + \varepsilon_i w_j$ into the compensated demand equation:

$$\phi_i^* = \gamma_i + \sum_{j=1}^{n} \varepsilon_{ij} \gamma_j + \sum_{j=1}^{n} \eta_i w_j \gamma_j \tag{4.32}$$

Substitution of Marshallian demand (4.25) and the household composition elasticity of the cost function (4.28) gives:

$$\phi_i^* = \phi_i + \eta_i \phi^* \tag{4.33}$$

This result implies that identification of the elasticity of the cost function with respect to demography and therefore equivalence scales is possible if either the compensated change of demand for one good with respect to demography is known or if compensated demands for two goods are assumed to be identical. The problem is that compensated demands cannot be known in advance even for a perfectly public or private good: in the case of a perfectly private good, the compensated demand elasticity with respect to household size would be smaller than one, in the case of a perfectly public good, the compensated demand elasticity would be higher than one, due to substitution effects.

In the case of a private or public good with a very low substitution elasticity, the bias that is introduced when the elasticity is assumed to be zero is small and with this assumption identification is possible. Kakwani and Son (2005) use health expenditures in an application of the approach and assume that the substitution elasticity for the good is zero. Apart from the fact that the assumption is not testable in the model, this approach is not always feasible. If health costs are covered by health insurance or a public health care system, they may not be observable. And when they are observable, they are often not income elastic. A look at equation 4.33 shows that in this case the change of the cost function with respect to demography is not identified.

Only the scale factor elasticities can safely be assumed to take a value of one for a perfectly private good and zero for a public good. But these are not identified by the method of Kakwani and Son.

A Proposed Alternative Approach to Identification

I suggest an alternative approach. When all compensated price elasticities are zero, Equation 4.26 can be rewritten as

$$\boldsymbol{\phi} = [\boldsymbol{I} - \boldsymbol{\eta}\boldsymbol{w}']\boldsymbol{\gamma}. \tag{4.34}$$

As argued before, $\boldsymbol{\gamma}$ cannot be identified from this equation, because $\boldsymbol{I} - \boldsymbol{\eta}\boldsymbol{w}'$ is singular. However, if at least one γ_k is known in advance, all other γ_i can be identified. γ_k could be fixed a priori, either because it is an adult good or a public good ($\gamma_k = 0$ in both cases), or because it is an assignable and private good, when γ_i can be determined by the relation of adult and child expenditures on the good.

A demand system, where all compensated price elasticities are zero would be very restrictive. However, at a subsistence expenditure level, there is no possibility for substitution and price elasticities are zero at this point. Relative changes in scaled prices ($\boldsymbol{\gamma}$) can be determined at the subsistence level and then be used to determine price reactions at higher incomes.

The quadratic expenditure system defines a subsistence level and is restrictive enough to allow identification of the complete demand system from a very limited amount of price variation. Thus, using the QES, equivalence scales can be estimated by fixing one or more scale factors instead of fixing the compensated demographic demand elasticity as in the approach suggested by Kakwani and Son (2005). The subsistence level itself can be determined simultaneously with all other parameters of the demand system. The QES is described in the next section.

4.4 The Quadratic Expenditure System

The quadratic expenditure system (QES) (Pollak and Wales, 1978; Howe et al., 1979) is far less restrictive than the linear expenditure system (LES) or the extended linear expenditure system (ELES). Unlike data demanding flexible forms it is very parsimonious in its parameters, so that in theory it can be estimated from data with price variation from as few as only two cross sections. Unfortunately, the QES demand equations are highly non-linear in their parameters. Non-linear estimation procedures such as full information maximum likelihood (FIML) are required for estimation. The robustness and precision of the results directly depend on the degree of price variation in the data. Hence, few empirical studies of the QES actually use only two cross sections for estimation (e.g. Kohn and Missong, 2003), while most studies draw on more than just two cross sections (Pollak and Wales, 1980) or exploit data on regional price variation (Barnes and Gillingham, 1984).

The demands in the linear expenditure system (LES) are linear functions of total expenditure μ, where the a_i are the marginal budget shares, and the b_i can be interpreted as necessary or minimum consumption of each commodity:

$$x_i = p_i q_i = p_i b_i + a_i \left(\mu - \sum p_j b_j\right) \tag{4.35}$$

The LES can be characterized by the following indirect utility function from which the demand equations (4.35) can be derived by Roy's identity ($q_i(\mu, \boldsymbol{p}) = -\frac{\partial V/\partial p_i}{\partial V/\partial \mu}$):

$$V^* = -\frac{\prod p_j^{a_j}}{\mu - \sum p_j b_j} \tag{4.36}$$

However, empirical evidence shows that the marginal budget share is not constant for most goods (i.e. Engel curves are not linear)[12]. The introduction of a quadratic term is a natural extension to the LES leading to a more accurate representation of observed demand behaviour. The Quadratic Expenditure System (QES) can be derived by generalizing the LES indirect utility function according to

$$V^*(\mu, \boldsymbol{p}) = -\frac{\prod p_j^{a_j}}{\mu - \sum p_j b_j} - \frac{\psi(\boldsymbol{p})}{\prod p_j^{a_j}} \tag{4.37}$$

where $\psi(\boldsymbol{p})$ is required to be a linear homogeneous function of prices. The LES is nested within the QES by $\phi(\boldsymbol{p}) = 0$. For empirical purposes it is convenient to set $\psi(\boldsymbol{p})$ equal to $\sum_{j=1}^{n} p_j c_j$.[13] Nesting of the LES for this specification implies that all $c_j = 0$. Application of Roy's identity and multiplication with p_i yields the expenditure functions:

$$x_i = p_i b_i + a_i \left(\mu - \sum_{j=1}^{n} p_j b_j\right) + \left(p_i c_i - a_i \sum_{j=1}^{n} p_j c_j\right) \prod_{j=1}^{n} p_j^{-2a_j} \left(\mu - \sum_{j=1}^{n} p_j b_j\right)^2 . \tag{4.38}$$

[12] See for example Banks et al. (1997), Blundell et al. (1993), Lewbel (1991) and – for German data – Missong (2004).

[13] See Pollak and Wales (1992). This is not the only possible demand system quadratic in expenditures, see van Daal and Merkies (1989). The functions $\prod p_j^{a_j}$ and $\sum p_j b_j$ are also special cases and can be replaced by any function that is linearly homogeneous in prices.

The QES parameters are not as conveniently interpreted as in the LES. The sum of the b_i, $\sum p_j b_j$, can still be seen as some kind of subsistence level of expenditures, but the parameters a_i and c_i lack any direct economic interpretation. Therefore, it is economically more meaningful to summarize the model in terms of price and income elasticities at different income levels.

Demographic effects can be included into the quadratic demand system via demographic translating and via demographic scaling as described in Section 4.1. With demographic scaling, demographic effects enter the demand system as prices, and the expenditure equations change in the following way:

$$\begin{aligned} x_i^s = & m_i^s p_i b_i + a_i \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right) \\ & + \left(m_i^s p_i c_i - a_i \sum_{j=1}^n m_j^s p_j c_j \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right)^2 , \end{aligned} \tag{4.39}$$

where x_i^s is the expenditure of a household of type s on good i. The scaling parameters for the reference household are normalized to one (or any other convenient value), while the m_i^s for any other household type depend on demography (and on the normalization for the reference household).

Demographic translating can be included easily into the QES, because the demand system already contains translation parameters: the b_i. Replace each b_i by b_i^s that depends on demography with

$$b_i^s = b_i + \tilde{\beta}_i^s , \tag{4.40}$$

where $\tilde{\beta}_i^s$ is the scaled overhead of household type s. If convenient, scaled overheads $\tilde{\beta}_i^r$ can be normalized to zero for the reference household and $b_i^r = b_i$.

With scaling and translating (the Gorman method) the expenditure equations become:

$$\begin{aligned} x_i^s = & m_i^s p_i b_i^s + a_i \left(\mu - \sum_{j=1}^n m_j^s p_j b_j^s \right) \\ & + \left(m_i^s p_i c_i - a_i \sum_{j=1}^n m_j^s p_j c_j \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j^s \right)^2 . \end{aligned} \tag{4.41}$$

Inversion of the indirect utility function (4.37) gives the cost function, which can be used to calculate equivalence scales and cost of living indexes:

$$c^*(V, \boldsymbol{p}) = \mu = \sum_{j=1}^n p_j b_j - \frac{\prod_{j=1}^n p_j^{2a_j}}{V \prod_{j=1}^n p_j^{a_j} + \sum_{j=1}^n p_j c_j} \tag{4.42}$$

Using the cost function (4.42), the indirect utility function (4.37), Equation 4.18, and substituting b_i^s and b_i^r, the overall equivalence scale of household type s relative to household type r can be calculated at the reference income level μ^r:

$$
\begin{aligned}
m_r^s(\mu^r) &= \frac{c^*(V^*(\mu^r, \boldsymbol{p}^r), \boldsymbol{p}^s))}{\mu^r} \\
&= \frac{1}{\mu^r}\left[\sum_{j=1}^{n} m_j^s p_j b_j^s - \frac{\prod_{j=1}^n (m_j^s p_j)^{2a_j}}{V^r \prod_{j=1}^n (m_j^s p_j)^{a_j} + \sum_{j=1}^n m_j^s p_j c_j}\right]
\end{aligned}
\tag{4.43}
$$

with

$$
V^r = -\frac{\prod_{j=1}^n (m_j^r p_j)^{a_j}}{\mu^r - \sum_{j=1}^n m_j^r p_j b_j^r} - \frac{\sum_{j=1}^n m_j^r p_j c_j}{\prod_{j=1}^n (m_j^r p_j)^{a_j}} \ . \tag{4.44}
$$

For the standard reference household, all $m_i^r = 1$ and all $b_i^r = b_i$, and Equation 4.44 simplifies to:

$$
V^r = -\frac{\prod_{j=1}^n p_j^{a_j}}{\mu^r - \sum_{j=1}^n p_j b_j} - \frac{\sum_{j=1}^n p_j c_j}{\prod_{j=1}^n p_j^{a_j}} \ . \tag{4.45}
$$

4.5 Identification of the Barten Model in a QES

In Appendix 4.A to this chapter it is shown that – due to the quadratic term in the demand equations – all parameters of the QES can be identified from only one cross section, if demographics enter the demand system via scaling and at least two different household types are present. The estimate relies, however, on the curvature of the demand functions as defined by the functional form of the QES. In contrast, the LES, which is nested within the QES but lacks a quadratic term in its Engel curves, is not identified from just one cross section without additional information.

Experimental results show that not all parameters of the QES are well identified in practical application, because demands for some goods tend to be too linear, and the likelihood function is therefore too flat. To reach an estimate that does not rely on the non-linearities of the expenditure equations, it is necessary to determine some parameters of the demand system a priori based on additional information. If enough parameters are known in advance so that the LES is fully identified, then the QES is also identified even if the quadratic terms in all expenditure equations were statistically not significant.

But how many parameters must be known to identify the LES from a single cross section with Barten scaling? Consider two household types r and s. For the reference household r, scale factors are normalized to one. With scaling, the expenditure equations of the LES (4.35) for household types r

and s change to:

$$\begin{aligned} x_i^r &= p_i b_i + a_i(\mu - \sum_{j=1}^{n} p_j b_j) \\ x_i^s &= p_i m_i^s b_i + a_i(\mu - \sum_{j=1}^{n} m_j^s p_j b_j) \end{aligned} \tag{4.46}$$

Expenditure equations for both household types can be integrated into the following linear expenditure equation, which can be estimated using linear regression methods:

$$x_i = \theta_{1i}^r R + \theta_{1i}^s S + \theta_{2i}\mu \qquad \text{for} \quad i \in \{1, \ldots, n\} \;, \tag{4.47}$$

where R is a dummy for the household being of type r and S is a dummy for the household being of type s. The parameters of the LES are related to the thetas in Equation 4.47 in the following way:

$$\theta_{1i}^r = b_i - a_i \sum_{j=1}^{n} b_j \tag{4.48}$$

$$\theta_{1i}^s = m_i^s b_i - a_i \sum_{j=1}^{n} m_j^s b_j \tag{4.49}$$

$$\theta_{2i} = a_i \tag{4.50}$$

$\hat{a}_i$ is directly determined by Equation 4.50. The b_i and m_i^s are, however, not identified, because there are $2n$ parameters, but only $2(n-1)$ independent equations.[14] To identify the demand system, at least two parameters – either m_i^s or b_i – have to be known in advance. There is no straightforward way of fixing the b_i parameters[15], but it is possible to determine some of the m_i^s parameters from additional information contained in the data and some a priori considerations.

If at least two m_i^s are fixed, the LES is fully identified. Therefore, the QES is identified as well, even if it degenerates to the LES. With the additional information that is contained in the curvature of the Engel curves, the model can even be estimated when only one m_i^s is fixed a priori.

It is also possible to estimate a Gorman model if at least two overheads (β_i^s) for families with children are known in advance and the nonlinearities of the QES are used. This is shown in appendix 4.B to this chapter. The

[14] Due to the adding up restriction, not all thetas are independent of each other, because $\sum_{i=1}^{n} \theta_{1i}^r = 0$, $\sum_{i=1}^{n} \theta_{1i}^s = 0$ and $\sum_{i=1}^{n} \theta_{2i} = 1$.

[15] The ELES is a special case, where the b_i for saving is fixed at a value of zero. This eliminates the corresponding m_i^s as well, and the model is just identified.

different possibilities will be explored in the empirical application in section 4.7.

4.6 Fixing a Scale Factor

Before fixing any scale factor, some thoughts must be given to possible preference effects. Taste changes can play an important role in the Barten model. They can affect the demand for vices (tobacco and alcohol), expenditures on vacation, food, housing, even transportation. Not always is the effect of taste changes relevant. For example if a family with two children rents a house for vacation and gets there by car, while the couple used to fly to Majorca before the children came, then this is an intra-group substitution driven by the change in the relative price of renting an apartment and going there by car where there are high economies of scale, versus a flight trip, the price of which might double when the number of heads doubles. This does not mean that the couple's taste for vacation has changed; it is merely a substitution effect. The effect of preference changes is more important when a priori assumptions on scale factors are used to identify the model: if the size of scale factors is not independent of preference changes, an adjustment of a priori judgements might be necessary.

The Barten Model and Taste Changes

In the Barten model, preferences for couples with children are inferred from the preferences of childless couples. To determine the household technology that includes the economies of scale of living together and the needs of children, it is assumed that adult preferences do not change when people become parents. Actually, preference changes and household technology are lumped together in the good-specific scale factors. It would be interesting to separate taste changes from household technology.

Instead of assuming that tastes do not change, preference changes can be parametrized, and the model can be estimated together with the taste changes.[16] Then preference changes can be disentangled from household technology using data for couples with children.

Assume taste changes are represented by good-specific factors l_i^s, and true scale factors $\tilde{m}_i$ represent the pure effect of the change in household technology – in contrast to the usual scale factors m_i that do not separate taste changes from household technology. Then the demand for a family with children would become:

$$x_i^s = l_i^s \tilde{m}_i^s p_i g_i(\mu;\ l_1^s \tilde{m}_1^s p_1, \ldots, l_n^s \tilde{m}_n^s p_n)\ . \tag{4.51}$$

[16] Browning et al. (2004) apply this idea in the context of a collective model of equivalence scales.

This is the base equation of the estimated demand system. l_i^s and m_i^s cannot be separated from each other, only their product $m_i^s = l_i^s \tilde{m}_i^s$ can be estimated.[17] However, true scale factors for some goods are known, or can be evaluated from other sources. E.g. for adult goods the true scale factor $\tilde{m}_i^s$ is equal to one and any change in scale factors m_i^s can only be attributed to taste changes. If there are preference changes in the consumption of a specific good, they must be taken into account when scale factors m_i^s are fixed. This will be discussed for each commodity group in the following paragraphs.

Which Scale Factors can be Fixed?

There are two types of goods which are candidates for fixing a scale factor: perfectly public and perfectly private goods. The factor for a good that can be assumed to be perfectly public can be fixed at a value of one: The personal consumption of the public good of each household member is equal to the total purchased quantity, irrespective of the number of heads in the household. As it is very difficult to find a good that can be attributed as perfectly public, in the following discussion only private goods are considered.

For a perfectly private good, however, a scale factor cannot be fixed at the number of persons in the household (possibly with a normalization[18]), if not all members of the household are identical. This procedure does not take into account the different needs of children. To be able to fix a factor for goods that are perfectly private, the consumption of these goods must be *assignable* to the persons who consume them. The best candidates are goods that can be assigned to adults and children (e.g. clothing), and adult goods (e.g. tobacco and alcohol), which are assignable to adults only. This is analogous to the Rothbarth method. The method of fixing a scale factor for an assignable good is shown for the more general Gorman technology with a childless couple as a reference.

Take a good i of which household s consumes the quantity q_i^s and of which parents are observed to consume the quantity q_i^a. With (unscaled) overheads β_i^s, scale factors m_i^s and scaled prices $p_i^s = m_i^s p_i$, write the Hicksian demand

[17] Taste changes of this type could be identified if it were assumed that (1) having children has an effect on preferences, but not the number of children, and that (2) the true scale factors are a function that is linear in the number of children. Then true scale factors become: $\tilde{m}_i^s = 1 + n^c \tilde{m}_i^c$, where n^c is the number of children and $\tilde{m}_i^c$ are the cost of each additional child. If l_i^c is the preference change effect of having children, the combined scale factors are: $m_i^s = l_i^c(1 + n^c \tilde{m}_i^c)$.

If at least families with one and two children are observed, $\tilde{m}_i^c$ and l_i^c are identified and taste changes and household technology can be separated.

[18] To normalize the factor for the reference household to one, the scale factor must be divided by the number of persons in the reference household: the scale factor for a private good in a three adult household would be 1.5, given a two adult reference household.

equation for the Gorman model:

$$q_i^s = m_i^s h_i(u, p_1^s, ..., p_n^s) + \beta_i^s \tag{4.52}$$

The demand for the parents' quantity q_i^a is exactly the same as the demand of a childless reference household r at scaled prices $p_1^s, ..., p_n^s$:

$$q_i^a = q_i^r = h_i(u, p_1^s, ..., p_n^s) \tag{4.53}$$

Substitution gives:

$$q_i^s = m_i^s q_i^a + \beta_i^s \tag{4.54}$$

Two cases can be distinguished: assignable goods and pure adult goods. For assignable goods, the parameters m_i^s and β_i^s can be determined by linear regression, because q_i^s and q_i^a are known. Pure adult goods are consumed only by parents. Therefore, q_i^s is equal to q_i^a. The need of children for the good is zero: there is no child-related overhead ($\beta_i^s = 0$), and the need of parents is equal to that of a childless couple ($m_i^s = 1$).

Assignable Goods: Clothing

For clothing it is possible to determine scaling and translating parameters directly if expenditures for clothing of adults and children are observed separately. Clothing for adults and children cannot be treated as two separate categories, as this would necessitate very high values of the scale factor for clothing explain expenditures on children's clothes, provided childless couples buy at least some children's clothes at all.[19] High values will lead to extreme price changes in this category and therefore to extreme substitution effects. It is also improbable that adult and children's clothing is separable. Under the assumption of a benevolent dictator, parents want their children to be as well-dressed as themselves. This links clothing expenditures for children to clothing expenditures for parents.

In the EVS 93 data, clothing for children and clothing for parents cannot be separated completely, because there are different categories for clothing for adults and for children, as well as categories that are not directly assignable to one of the two groups.[20] Two possibilities are explored to estimate scale fac-

[19] Even childless couples are observed buying some children's clothes. Assume for example, that a childless couple buys DM 10 worth of children's clothes and an equivalent couple with one child buys children's clothes for DM 100. Disregarding any substitution effects, the scale factor would be $m_i^s = 10$. Because of the high scale factor, the childless couple would buy less children's clothing at scaled prices, so that the corrected scale factor would be even higher. According to the Gorman method, the difference in expenditures could also be interpreted as an overhead of $\beta_i^s =$ DM 90, but this would require that marginal expenditures on children's clothing are equal between childless couples and parents.

[20] For the EVS 93, only *sports wear, hosiery, headgear, gloves, accessories for clothes and shoes, other shoes, outside changes and repairs of clothes and shoes*, and *rent for*

tors for clothing. One is to use assignable expenditures on adult clothing $\tilde{q}^a_{cloth}$ as a proxy for total expenditures on adult clothing q^a_{cloth} and predict q^a_{cloth} from a regression on a sample of childless households. A second possibility is to assume that the relation between the assignable and the total amount is the same for children's clothes and for adult clothes: $\tilde{q}^c_{cloth}/q^c_{cloth} = \tilde{q}^a_{cloth}/q^a_{cloth}$. With $q_{cloth} = q^c_{cloth} + q^a_{cloth}$ it follows that:

$$q^a_{cloth} = q_{cloth} \frac{\tilde{q}^a_{cloth}}{\tilde{q}^a_{cloth} + \tilde{q}^c_{cloth}} \tag{4.55}$$

This assumption is far easier to handle because no auxiliary regressions are necessary. It can be tested by comparing its results with those of the first method.

Equation 4.54 is estimated with linear regression. Table 4.1 shows the results of both methods. The first method exhibits higher standard errors, because it is a two-step method: first total expenditures on adult clothing are predicted and then scale factors are estimated from predicted values. The prediction error has to be accounted for, leading to higher standard errors.

Household type		AA	AAC	AACC	AACCC
Method 1	m^s_{cloth}	1.06	1.30	1.47	1.50
		(0.027)	(0.045)	(0.041)	(0.069)
	β^s_{cloth}	−118.5	153.1	162.9	624.8
		(123.5)	(174.8)	(159.1)	(230.5)
Method 2	m^s_{cloth}	0.996	1.15	1.31	1.33
		(0.003)	(0.015)	(0.014)	(0.029)
	β^s_{cloth}	104.8	546.8	724.4	1092.5
		(15.0)	(57.1)	(50.4)	(94.5)

Table 4.1: *Estimated scaling and translating parameters for clothing and different household types, for two estimation techniques. Parameters are not normalized for the reference household. Method 1: Total expenditures on adult clothing are predicted from a sample of childless households. Method 2: Observed expenditures are scaled up. Standard error in parentheses.*

Results from both methods differ significantly: Method 1 shows significant translation parameters only for a family of five and higher scale factors, while method 2 generates significant translation parameters for all household types and lower scale factors. Barten scaling alone is rejected for all household types only with method 2. However, with limited data, it is difficult to

clothes and accessories are not directly assignable. Unfortunately some assignable items are not given as annual expenses, but only as monthly expenses. They are not comparable with the other numbers and cannot be used. These are: *other outer garments for men, women, boys and girls*, and *underwear for men, women, children and babies.*

estimate a demand system with a Gorman household technology. The choice is to impose further restrictions on the demand system by fixing more scale factors and translation parameters, or to limit the way how demography enters the demand system. To compare both possibilities, Gorman and Barten equivalence scales are estimated. This requires the estimation of Barten scale factors (Table 4.2). Here, differences between both methods are very small and significant only for household type AAC.

Household type		AA	AAC	AACC	AACCC
Method 1	m^s_{cloth}	1.04	1.34	1.52	1.69
		(0.013)	(0.019)	(0.019)	(0.028)
Method 2	m^s_{cloth}	1.02	1.29	1.49	1.65
		(0.001)	(0.005)	(0.005)	(0.011)

Table 4.2: *Restricted estimated Barten scale factors for clothing and different household types. Parameters are not normalized for the reference household. Two estimation techniques. Method 1: predicted adult clothing Method 2: scaled adult clothing. Standard error in parentheses.*

Because method 2 generates significant overheads, its results are used as restriction for the Gorman model, while results of method 1 are used for the Barten model. Normalized parameters are given in Table 4.5, together with other fixed factors.

Adult Goods: Alcohol and Tobacco

In principle, scale factors for tobacco and alcohol, which are pure adult goods, can be fixed at a value of one. However, of all categories of goods, the taste for these adult goods is probably affected most by having children. Therefore, fixing scale factors for these goods can be particularly problematic.

The preference for alcohol seems to be less affected by the presence of children. As shown in Table 4.3, the percentage of households with expenditures on alcohol does not decrease significantly with the number of children in the household. A logit regression of buying alcohol on ages of partners, net income and the squares of these variables as well as on the number of children, children present, children over the age of twelve present and children under the age of two present, shows that only the income variables and the presence of children under the age of two has any statistically significant effect on the probability of buying alcohol. The presence of infants reduces the number of households buying alcohol by 7 percentage points, but for families with the youngest child two years or older, no effect is seen. These results are consistent with children not having any lasting effect on their parents' general taste for alcohol.

Household type	AA	AAC	AACC	AACCC
Percentage of households buying alcohol	89.1%	86.6%	87.8%	84.6%
Average expenditures on alcohol (DM/year)				
a) of all households	1042	854	856	826
b) of those reporting alcohol purchases	1170	987	976	977

Table 4.3: *Incidence and amount of alcohol consumption among different family types: percentage of households that were observed buying tobacco during a one month detailed recording period, and annualized expenditures on alcohol.*

Despite the percentage of alcohol buying households being the same, total expenditures on alcohol (Table 4.3) are strongly reduced when children are present. The question is if this is a preference change – parents drink less than their childless peers – or an income effect. A strong influence of the number of children on alcohol expenditures would point to an income effect: every additional child reduces the equivalent income of the parents, and therefore alcohol consumption. An stronger influence of the presence of children on alcohol consumption would rather point to a preference effect. However, when regressing total alcohol expenditures on the same regressors as above, the number of children and the presence of children have a significant joint negative effect. Unfortunately, the model fit is not significantly worse if either of the two variables is left out.[21] If alcohol is affected by a preference effect cannot be decided. To extract a possible preference effect, it might be a reasonable approach to fix the scale factor for alcohol at the same level for all households with children, but let it vary with respect to the childless reference household.

Table 4.4 gives an overview of the number of households that have recorded some tobacco expenditures during the one month detailed recording period of the survey. The higher incidence of smoking households among couples with one child compared to childless couples is due to the higher average age of childless couples in the selected data (the median childless couple is 14 years older than the median couple with one child). When controlled for ages of both partners, net household income and their squares, each child reduces the number of smoking households by roughly 5 percentage points. Parents seem to suspend smoking while they have young babies: taking families with children aged 2–12 as a benchmark, the youngest child being younger than two years of age reduces smoking by a further 8 percentage points. Having

[21] Statistically, the two variables are jointly significant on the 1% level with an F-value of 7.87 against a critical value of 4.61. A comparison of the full model with a model where the number of children is left out is not significant with an F-value of 3.19, a comparison with a model without the children dummy is not significant with an F-value of 3.17, both against a critical value of 6.64 on the 1% level.

Household type	AA	AAC	AACC	AACCC
Observed smokers	47.9%	51.0%	46.3%	41.0%
relative to AA	1.00	1.06	0.97	0.86
Mean expenditures on tobacco (DM/year)				
a) of all households	583	602	506	432
relative to all AA	1.00	1.03	0.87	0.74
b) of smoking households	1217	1181	1094	1055
relative to smoking AA	1.00	0.97	0.90	0.87

Table 4.4: *Incidence of smoking among different family types: percentage of households that reported buying tobacco during a one month detailed recording period and annualized mean expenditures on tobacco of smoking households.*

children over the age of twelve increases smoking incidence again by about 4 percentage points.

How are these results to be interpreted in terms of fixing a scale factor for smoking? If an addicted person could only decide either to smoke an amount that satisfies the addiction or to give up smoking completely, but not to continue smoking but smoke less, then the lower incidence of smoking in larger households could be interpreted as an income effect, that is increasing with the number of children. The not fully compensated additional costs of children would lead to lower smoking expenditures, where some smokers would quit because of a reduced income and others would continue to smoke. If this were the case, it were entirely sensible to fix scales for smoking at a level of one.

However, tobacco expenditures of smoking households depend negatively on income: tobacco is an inferior good. Having children as well as the number of children also have a *negative* effect on tobacco expenditures.[22] This cannot be an income effect, because the deduction of the cost of children reduces parents' effective income, which would lead to an *increase* of tobacco consumption. In terms of the Barten model, this cannot be a substitution effect either, because as an adult good, tobacco becomes relatively cheaper and parents would substitute towards tobacco, not away from it. This observation implies, that the lower incidence of smoking is a direct effect of a change in preferences.

The numbers on the percentage of tobacco buying households among families with children are a clear rejection of the assumption that only the first child has an influence on preferences. Indeed, each additional child increases the probability for the parents to quit smoking. As a consequence and un-

[22] Tobacco expenditures of smoking households were regressed on: age of man, age of woman, net income, age of man squared, age of woman squared, net income squared, number of children, and dummies for children present, children under two years present and children over age 12 present.

like the scale factor for alcohol, the scale factor for tobacco cannot be fixed between families with children.

If possible, tobacco should not be used as an assignable good in the estimation process. Preference effects are too strong to give a sensible assessment of the value at which its scale factor should be fixed. If a scale factor has to be fixed at all, it is more plausible to see the lower number of smoking households as a preference effect, and then to model lower expenditures on smoking by these households as an income and substitution effect. Under this assumption scale factors can be fixed at the incidence of observed smoking relative to the reference household (Table 4.4, line 2).

Summary: Scale Factors used in Estimations

As a summary, all values of scale factors that are used in the estimation process are collected in Table 4.5. Scale factors for clothing are fixed according to parameter estimates by method 2 for the Barten model and by method 1 for the Gorman model.[23] It is assumed that no preference effects act on clothing demands. Preference effects on tobacco consumption are strong. Therefore, if necessary, scale factors for tobacco are fixed at the relative percentage of households that were observed buying tobacco for both the Barten and the Gorman model. Overheads for tobacco are set to zero in the Gorman model. There might be some preference effect on alcohol consumption, but the size of the effect could not be determined using a priori information. If necessary, scale factors for alcohol are fixed to have the same value for all households with children. This value, $\hat{m}^{\bar{s}}_{alc}$, is estimated together with all other model parameters. Overheads for alcohol are zero in the Gorman model.

Model	line #	Good		AA	AAC	AACC	AACCC
Barten	1	Clothing	m_i^s	1.00	1.29	1.47	1.63
	2	Tobacco	m_i^s	1.00	1.06	0.97	0.86
	3	Alcohol	m_i^s	1.00	$\hat{m}^{\bar{s}}_{alc}$	$\hat{m}^{\bar{s}}_{alc}$	$\hat{m}^{\bar{s}}_{alc}$
Gorman	4	Clothing	m_i^s	1.00	1.16	1.31	1.34
	5		β_i^s	0	442	620	988
	6	Tobacco	m_i^s	1.00	1.06	0.97	0.86
	7		β_i^s	0	0	0	0
	8	Alcohol	m_i^s	1.00	$\hat{m}^{\bar{s}}_{alc}$	$\hat{m}^{\bar{s}}_{alc}$	$\hat{m}^{\bar{s}}_{alc}$
	9		β_i^s	0	0	0	0

Table 4.5: *Normalized scale factors and overheads used in the estimation of the Barten and Gorman model*

[23]Scale factors for the reference household are normalized to one, overheads to zero. Tables 4.1 and 4.2 show scales larger than one for childless couples. Normalization leads to the differences between the values in those tables and in Table 4.5.

4.7 Estimation

The model is estimated for ten different commodity groups: *food, clothing, housing, home & furniture, transportation, recreation, personal care, vacation, tobacco* and *alcohol*.[24] Four different household types are considered: childless couples (AA), and couples with one to three children under the age of 16. (AAC, AACC and AACCC). Due to computational limitations, only data from West German households are used with the age of the present adults between 30 and 60. Table 4.6 gives an overview over the number of cases in each group, and the income and expenditure range.

Household type	AA	AAC	AACC	AACCC
# of cases	3593	1971	2971	1350
Net household income				
Minimum	3788	19230	12450	24330
Median	75310	72760	77080	83260
Maximum	387600	409500	373100	399700
Total expenditures (μ)				
Minimum	12360	13330	13660	20120
5^{th} percentile	26240	29050	31730	33960
Median	47740	48480	51400	54760
95^{th} percentile	88870	86780	90690	91600
Maximum	231000	198000	211700	181700

Table 4.6: *Case numbers of household types, net household income and total expenditures on the modelled basket of goods. EVS 1993.*

In a preliminary step, parameters of quadratic Engel curves for the ten categories and four household types are estimated. A Hausman test[25] indicates endogeneity of total expenditure because of bulk purchases. Therefore, total expenditure is instrumented by net household income, net household income squared, age of wife and age of wife squared.[26] The parameters are determined from separate two stage least squares regressions in share form (Table 4.7), which have the following stochastic specification:

$$w_{it} = \theta_{1i}/\mu_t + \theta_{2i} + \theta_{3i}\mu_t + u_{it}, \qquad (4.56)$$

where μ_t is total expenditures, w_{it} is the income share of category i, and u_{it} is an error term that is assumed to be approximately normally distributed.

[24] For a more detailed description of the commodity groups see the Appendix, p. 197.

[25] (Hausman, 1978) and (Wooldridge, 2002, p.119)

[26] See Deaton (1985) or Blundell (1986) for a discussion of instruments for household expenditures.

An F-test was carried out to test the inclusion of the quadratic term into the model, F-values are reported in Table 4.7. The quadratic term is indeed highly significant for most Engel curves, most notably for *housing*, *transportation* and *tobacco*, where it is significant for all household types. It is significant for some household types for most other goods. Exceptions are *recreation* and *personal care* with no significant quadratic terms and *clothing* with a significant quadratic term only for one household type (AACC). In addition, the slopes for most goods are significantly different from each other between household types. Therefore it is promising to proceed with the estimation of the quadratic expenditure system in the described manner.

In a detailed test of parametric Engel curves on data of the EVS 93, Missong (2004) shows, that for most goods, quadratic and log-quadratic curves compare much better with non-parametric curves than linear and loglinear specifications. The only exceptions are *food* and *care*, which includes health care and personal care. This lends additional support to the quadratic demand system specification.

The Engel curves and the respective expenditure shares for the 5–95 percentile range of expenditures are shown in figures 4.1 and 4.2. The income share for those commodity groups of which children's needs are highest increase with the number of children. These groups are *food*, *housing* and *recreation*, which also includes toys, child care and educational expenditures. The share for *clothing* increases only slightly. The share of the adult goods *tobacco* and *alcohol* is reduced, as expected. Striking is the strong reduction in vacation expenditures. The shift of the Engel curves for the *vacation* category to the right is similar in size to that of *alcohol*. But while children do not consume alcohol, they generate considerable cost when the family is travelling. Thus one would expect vacation expenditures to shift less than those for alcohol. The higher than expected shift can be explained by intra-group substitution of family vacation for the type of travelling people without children do. This would also explain why the first child has a stronger shifting effect than subsequent children: the switch to family vacation happens with the first child. The cost of family vacation increases only slightly with a second or third child. Therefore, the share of vacation expenditures continues to fall with the number of children, reflecting the effect of a lower equivalent income. Note also the u-shaped form of the Engel-curves for tobacco. They are shifted with an increasing number of children not only to the right, but also downwards, due to a lower incidence of smokers in families with more than one child (see Table 4.4).

Household type		Food	Clothing	Housing	Home & furniture	Transport	Recreation	Personal care	Vacation	Tobacco	Alcohol
		Commodity Group									
AA	θ^r_{1i}	3176.1	−917.2	1269.6	−875.2	−1420.3	−865.9	208.6	−2331.1	1255.4	500.1
		[6.04]***	[−2.58]**	[2.01]*	[−1.30]	[−3.86]***	[−1.90]	[1.33]	[−4.14]***	[6.48]***	[2.83]**
	θ^r_{2i}	0.094	0.098	0.345	0.062	0.192	0.091	0.016	0.138	−0.033	−0.003
		[3.82]***	[5.90]***	[11.68]***	[1.97]*	[11.12]***	[4.24]***	[2.25]*	[5.25]***	[−3.69]***	[−0.38]
	θ^r_{3i}	$6.38 \cdot 10^{-7}$	$1.30 \cdot 10^{-7}$	$-2.06 \cdot 10^{-6}$	$1.05 \cdot 10^{-6}$	$-1.05 \cdot 10^{-6}$	$3.56 \cdot 10^{-7}$	$1.38 \cdot 10^{-7}$	$2.07 \cdot 10^{-7}$	$3.56 \cdot 10^{-7}$	$2.35 \cdot 10^{-7}$
		[2.51]*	[0.76]	[−6.75]***	[3.23]**	[−5.89]***	[1.61]	[1.83]	[0.76]	[3.80]***	[2.75]**
	F	49.54***	2.56	44.81***	32.46***	33.57***	22.10***	26.12***	1.11	82.79***	11.44***
AAC	θ^s_{1i}	3264.6	−340.4	1860.0	−3678.5	−2103.8	−622.4	−38.4	−77.8	1776.9	−40.2
		[3.27]**	[−0.49]	[1.53]	[−2.80]**	[−2.95]**	[−0.65]	[−0.12]	[−0.08]	[5.03]***	[−0.15]
	θ^s_{2i}	0.103	0.087	0.314	0.168	0.236	0.078	0.033	0.019	−0.052	0.016
		[2.33]*	[2.85]**	[5.86]***	[2.88]**	[7.48]***	[1.84]	[2.38]*	[0.47]	[−3.33]***	[1.36]
	θ^s_{3i}	$5.86 \cdot 10^{-7}$	$2.04 \cdot 10^{-7}$	$-1.57 \cdot 10^{-6}$	$5.65 \cdot 10^{-8}$	$-1.55 \cdot 10^{-6}$	$6.95 \cdot 10^{-7}$	$-4.58 \cdot 10^{-8}$	$1.09 \cdot 10^{-6}$	$5.12 \cdot 10^{-7}$	$2.42 \cdot 10^{-8}$
		[1.34]	[0.67]	[−2.95]**	[0.10]	[−4.96]***	[1.65]	[−0.33]	[2.69]**	[3.30]***	[0.21]
	F	32.86***	4.32*	17.32***	2.80	122.04***	4.25*	3.85*	26.22***	87.44***	0.39
AACC	θ^s_{1i}	4817.6	1093.2	−2129.1	573.6	−1344.9	−3330.1	2.7	−1562.8	1465.5	414.3
		[5.72]***	[1.83]	[−1.93]	[0.54]	[−2.12]*	[−4.01]***	[0.01]	[−2.16]*	[5.28]***	[1.69]
	θ^s_{2i}	0.063	0.023	0.476	−0.013	0.188	0.198	0.029	0.078	−0.039	−0.003
		[1.86]	[0.97]	[10.70]***	[−0.31]	[7.36]***	[5.91]***	[2.81]**	[2.67]**	[−3.46]***	[−0.34]
	θ^s_{3i}	$8.41 \cdot 10^{-7}$	$8.51 \cdot 10^{-7}$	$-2.80 \cdot 10^{-6}$	$1.63 \cdot 10^{-6}$	$-1.00 \cdot 10^{-6}$	$-4.10 \cdot 10^{-7}$	$-4.76 \cdot 10^{-8}$	$3.99 \cdot 10^{-7}$	$3.53 \cdot 10^{-7}$	$1.90 \cdot 10^{-7}$
		[2.69]**	[3.83]***	[−6.85]***	[4.13]***	[−4.25]***	[−1.33]	[−0.50]	[1.48]	[3.43]***	[2.08]*
	F	103.33***	79.42***	112.48***	46.58***	150.15***	14.27***	4.48*	14.63***	66.59***	15.94***
AACCC	θ^s_{1i}	2819.4	−1206.1	1290.2	−3839.0	−790.4	−1033.3	66.2	648.5	1948.8	95.8
		[2.18]*	[−1.26]	[0.78]	[−2.47]*	[−0.84]	[−0.82]	[0.18]	[0.61]	[4.79]***	[0.26]
	θ^s_{2i}	0.152	0.112	0.352	0.164	0.172	0.092	0.022	−0.018	−0.057	0.009
		[3.00]**	[3.02]**	[5.43]***	[2.70]**	[4.70]***	[1.87]	[1.54]	[−0.42]	[−3.62]***	[0.65]
	θ^s_{3i}	$1.34 \cdot 10^{-7}$	$-2.15 \cdot 10^{-8}$	$-1.55 \cdot 10^{-6}$	$-7.88 \cdot 10^{-8}$	$-8.89 \cdot 10^{-7}$	$6.72 \cdot 10^{-7}$	$1.57 \cdot 10^{-8}$	$1.17 \cdot 10^{-6}$	$4.98 \cdot 10^{-7}$	$5.45 \cdot 10^{-8}$
		[0.29]	[−0.06]	[−2.67]**	[−0.14]	[−2.71]**	[1.52]	[0.12]	[3.13]**	[3.50]***	[0.42]
	F	5.14*	0.21	6.90**	2.22	1.24	5.51*	1.39	15.64***	67.83***	0.25

Table 4.7: *Parameters for quadratic Engel curves for ten goods. West German households, EVS 1993. t-values in square brackets. F values are given for the significance of a model including the quadratic term θ^s_{3i} versus a linear model. Significance levels: * = 5%, ** = 1%, *** = 0.1%.*

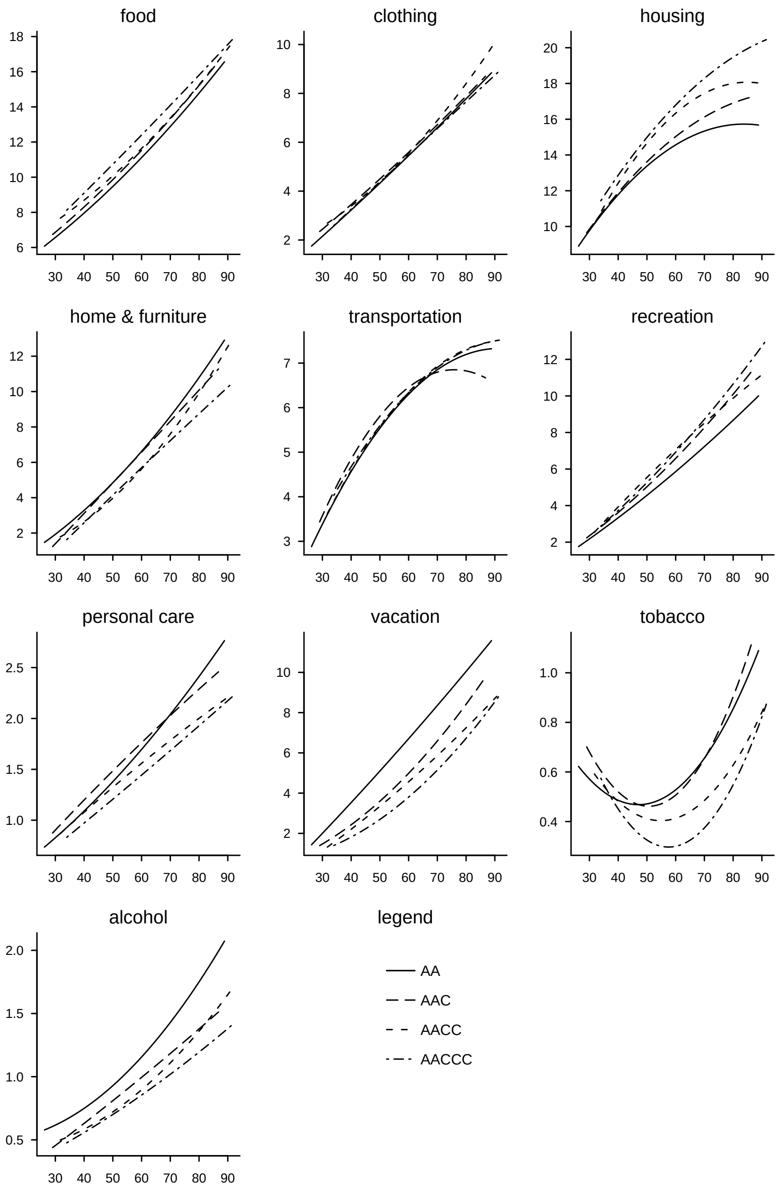

Figure 4.1: *Quadratic Engel curves for all expenditure categories based on Table 4.7. 5th–95th percentile of expenditure range.x-axis shows total expenditures, y-axis shows expenditures on respective good, both in 1000 DM.*

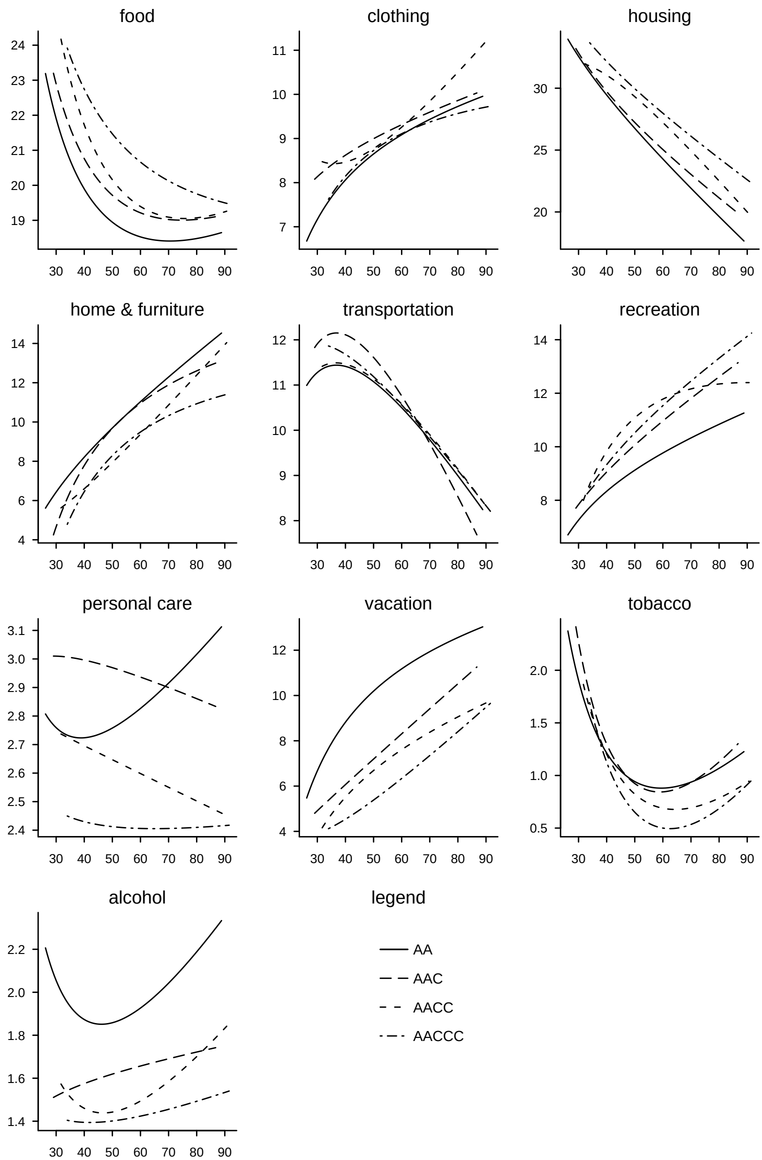

Figure 4.2: *The respective expenditure shares of the quadratic Engel curves-based on Table 4.7. 5th–95th percentile of expenditure range. x-axis shows total expenditures in 1000 DM, y-axis shows expenditure share of respective good in percentage points.*

Estimated Models

Results for three different models are reported: Two Barten models (*BaC* and *BaCTA*) and one Gorman model (*GoCTA*).[27] Model *BaC* uses a restriction on the scale factor for clothing taken from Table 4.5 (line 1). Model *BaCTA* uses the same restriction on scale factors for clothing, plus additional restrictions on alcohol and tobacco (Table 4.5, lines 1–3). The main purpose of model *BaCTA* is to evaluate the sensitivity of the proposed procedure to more restrictive assumptions. It is also a baseline for model *GoCTA*, which uses the Gorman method with an appropriate restriction on scale factors and overheads for clothing and restrictions on tobacco and alcohol that are analogous to the restrictions in model *BaCTA*, with no overheads (Table 4.5, lines 4–9). It would be preferable not to impose any restrictions on scale factors for tobacco, but without some assumption on a third commodity group, the estimation procedure for the Gorman model does not converge.[28]

Gorman and Barten models with a restriction on tobacco similar to that for alcohol (same scale factors for all families with children) were also tried, but these led to extreme values of uncompensated price elasticities for most goods. Results are not reported.

Equivalence Scales

The central result of this chapter and a useful summary statistic is the overall equivalence scale m_r^s (shown in Table 4.8 for the median income of the reference household). All three models show similar values for the equivalence scales at the median income level. Scales for a household with three children are slightly higher for the Gorman model, but the difference is not significant. Standard errors for the more constrained Barten model *BaC* are naturally lower than for the Gorman model, but they are also lower than those of the even more restricted Barten model *BaCTA* with its additional restrictions on tobacco and alcohol scale factors.

Even though Equivalence scales are almost identical between models at the median income level, differences are larger at other incomes, and at lower incomes in particular. Scales for models *BaCTA* and *GoCTA* are falling over the relevant income range between DM 20,000 and DM 120,000,[29] but to different degrees. Scales for model *BaC* are almost constant over the given range, with the maximum around the median income and a decrease of no

[27] The first two letters of the model names indicate the type of model, Gorman (*Go*) or Barten (*Ba*). The other letters indicate the employed restrictions on scale factors and overheads for clothing (C), tobacco (T) and alcohol (A).

[28] This is a numerical problem. As shown in Appendix 4.B, only two restrictions are necessary for identification

[29] This range corresponds approximately to the 1$^{\text{st}}$ and 99$^{\text{th}}$ percentile of total expenditures of the reference household.

Household type	Barten Models *BaC*	*BaCTA*	Gorman Model *GoCTA*
AA	1.00	1.00	1.00
AAC	1.13	1.12	1.14
	(0.018)	(0.022)	(0.028)
AACC	1.22	1.20	1.25
	(0.019)	(0.024)	(0.032)
AACCC	1.40	1.37	1.44
	(0.028)	(0.031)	(0.057)

Table 4.8: *Equivalence scales for different household types and three models, evaluated at the median income level of the reference household. Reference household is a childless couple. Standard errors in parentheses are estimated with delta method.*

more than two percentage points towards the upper end of the income range. The decrease is stronger for model *BaCTA* with up to 4 percentage points. The reduction of equivalence scales with rising income is most pronounced in the Gorman model (*GoCTA*). This model can reflect changes of equivalence scales with income better than the other two models, because of the added flexibility of the fixed cost term. Again, scales are falling for all household types: 3 percentage points for a one child family, 4 points for a two children family, but almost 13 points for families with three children. Differences between scales at different income levels are even higher in model *GoCTA*, when the income range is extended downwards, while they do not change for the two Barten models.[30] A comparison of scales for models *BaC* and *GoCTA* is shown in Figure 4.3.

At the median income, scales are quite low, with a single child costing about 13% of a couple, i.e. 26% of an adult. These results are in line with other estimates for Germany (Table 2.1), but scales are much lower than the Rothbarth scales that were estimated in chapter 3 using private adult goods. The difference can be attributed to the substitution effects that are accounted for in the Barten model, but ignored in the Rothbarth model. It is interesting that the non-child floor space scales that have been calculated with the Rothbarth model are of a similar magnitude as the estimates of all three models.[31] This could imply, that substitution goes very far in the Barten and Gorman model, since it was argued that the floor space scale indicates a lower bound to the equivalence scale.

[30] For *GoCTA*, differences increase to 4, 6 and 19 percentage points between households equivalent to the DM 15,000 income level and households at the DM 120,000 income level for household types AAC, AACC and AACCC, respectively.

[31] Child cost are more restricted in the Rothbarth model, with particular low economies of scale in having children imposed. Therefore scales are lower for households with three children relative to the models presented here.

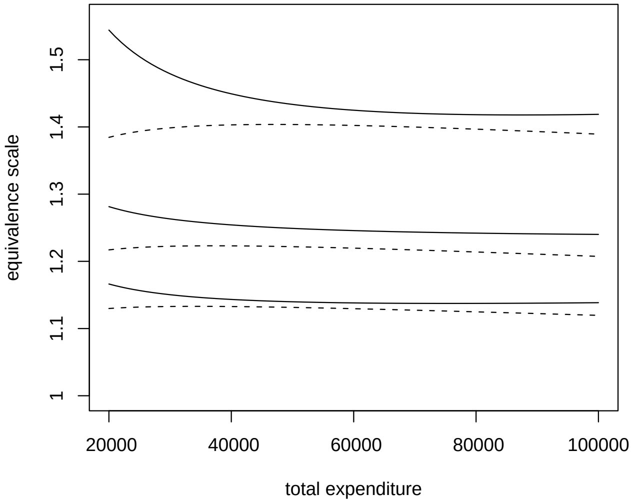

Figure 4.3: *Income dependence of equivalence scales for families with one, two and three children, model* GoCTA *(solid lines) and model* BaC *(dashed lines).*

In contrast to other works, scale factors for children were not pooled, i.e. scale factors were estimated separately for every family type without any assumption about the relationship between scale factors for family types, be it a constant cost for each child with $m_i = 1 + k\delta_i$, where k is the number of children and δ_i is the additional cost per child per commodity group as in Muellbauer (1977) or Merz and Faik (1995), or a constant elasticity of child cost as in Plug et al. (1997) and Schwarze (2003). Therefore the method allows for an investigation on possible economies or diseconomies of scale in the monetary cost of children.[32]

It is an interesting question if the second child is less “expensive” than the first one and if the third child is even cheaper in a purely material sense. Clothes and toys for children can be handed down, children can share a room and the cost of fuel for the car barely depends on the number of children that are sitting in it. On the other hand, the average apartment might be made for a family of four and a small car sits two children in the back as comfortably as one, but not three. These effects are visible in the results.

Table 4.9 shows the cost of the second and third child relative to the first for all three estimated models. All models show the same pattern: the cost of the second child is somewhat lower than that of the first, while the cost

[32]There are certainly also strong economies of scale in the time that has to be spent with children, for their education and for house work. This is, however, beyond the scope of this investigation.

	BaC	*BaCTA*	*GoCTA*
first child	1.00	1.00	1.00
second child	0.69	0.67	0.78
	(0.21)	(0.24)	(0.33)
third child	1.38	1.51	1.33
	(0.26)	(0.35)	(0.44)

Table 4.9: *Cost of second and third child relative to the first. Three models, standard errors in parentheses estimated with delta method.*

of the third child is somewhat higher.[33] In both Barten models (*BaC* and *BaCTA*), the relative costs for the second and third child are different from one at a 90 percent confidence level – the cost of the second child is lower and the cost of the third is higher. The pattern is similar for the Gorman model *GoCTA*, but not as pronounced and not statistically significant. When added together, three children are approximately as expensive as three single children, while two children are less expensive than two single children.

Scale factors for all major commodity groups – *food, housing, home and furniture, transportation and recreation* – increase more for the third child than for the second. Only clothing shows slight economies of scale even for the third child. In the detailed description of the results of model *BaC*, the particular effects will be discussed.

Model *BaC*: Fixed Scales for Clothing

Model *BaC* is taken as the reference model. Three issues are of interest in the analysis of the results: first, a closer look at the good-specific scale factors shows which goods are "child intensive", and is informative about the overall plausibility of the scales. Second, the overall equivalence scales indicate economies of scale in having a second child, but diseconomies of scale in having a third child: the second child is less expensive than the first child, while the third is more expensive. This finding is reproduced in all models and warrants a closer investigation with respect to which goods generate these diseconomies of scale. Third, the calculation of compensated quantities and a comparison with goods specific scale factors gives a better impression of the actual strength of substitution effects.

Table 4.10 shows scale factors for all ten commodity groups and all family types. The differences of the good-specific scale factors can be attributed mainly to differences in children's needs and to differences in economies of scale from joint household consumption. Both effects are visible in the two

[33] The estimation of standard errors with the delta method takes account for the fact that estimates of relative child cost for the third child are not independent of the estimate for the second. This is reflected in higher standard errors.

	AA	AAC	AACC	AACCC
Food	1.00	1.11	1.22	1.39
		(0.016)	(0.017)	(0.024)
Clothing	1.00	1.29	1.47	1.63
Housing	1.00	1.12	1.24	1.44
		(0.019)	(0.019)	(0.028)
Home & furniture	1.00	1.09	1.12	1.32
		(0.054)	(0.057)	(0.088)
Transportation	1.00	1.16	1.20	1.35
		(0.024)	(0.023)	(0.033)
Recreation	1.00	1.45	1.75	2.16
		(0.059)	(0.070)	(0.108)
Personal care	1.00	1.15	1.06	1.07
		(0.026)	(0.027)	(0.039)
Vacation	1.00	0.86	0.83	0.94
		(0.054)	(0.059)	(0.087)
Tobacco	1.00	1.02	0.90	0.81
		(0.036)	(0.036)	(0.054)
Alcohol	1.00	0.85	0.87	0.91
		(0.041)	(0.038)	(0.057)

Table 4.10: *Good-specific scale factors for a Barten model with ten goods and fixed factors for clothing (Model* BaC*). West German households, EVS 1993. Standard errors are given in parentheses.*

groups with the highest factors: clothing and recreation. For clothing there is no joint consumption between parents and children, which leads to very high scales. But children's needs are lower than those of the parents, probably because children's clothes are cheaper than adult clothes. Economies of scale are also observable: Clothes can be handed down to younger siblings, thus decreasing the extra clothing cost of an additional brother or sister.

The factors for recreation are by far the highest of all, because this commodity group contains many child specific goods and services for education, leisure activities and entertainment, like expenses on school and child care, books, toys and so on. According to the scaling estimates, the first child costs 90% of an adult[34] in this category. In the recreation group there is some joint consumption, e.g. of consumer electronics, but the needs of children in this group are very high. There could be some sharing between children, as the indicated costs are lower for the second and third child. Other scales for food, housing, home & furniture and transportation are smaller, because there is strong joint consumption as well as lower needs of children compared to adults.

[34] A child costs 45% of a couple, or 0.45/0.5 of half a couple, which is 90% of one adult.

The scale factor for personal care is slightly decreasing with the number of children. This can be interpreted as children having fixed costs for personal care. One has to buy these things only once. Another interpretation would be a preference effect: if women with more children spend less on personal care (which is mainly a women's good), they would partly offset their children's share in the cost of this commodity. This argument will reappear in the discussion of the cost of the third child in this section.

The scale factor for tobacco is 1.02 for families with one child and strongly decreasing in the number of children. This is in line with the discussion of the tobacco scale factor in section 4.6. Factors are slightly higher than those given in Table 4.4, row 4. This means that even though tobacco as an adult good became relatively cheaper, it is substituted *away from.* This will be discussed below.

The scale factor for alcohol is lower than one for families with children, and increasing in the number of children, but the increase is not significant. Again, this is in line with the discussion of the scale factor for alcohol in section 4.6: there is some preference effect, but it does not depend on the number of children. The effect of the number of children on alcohol consumption that is observed when income is controlled for is a pure income effect that disappears, once incomes are equalized.

Scale Factors: The Cost of the Third Child

The discussion of the overall equivalence scale revealed that in all models the third child is more expensive than the first, while the second is less expensive, although differences were significant only for the Barten models. With the third child being more expensive than the second, one would expect that some commodity groups still show additional economies of scale for the third child, e.g. clothing, but the savings in these groups are outweighed by additional expenses in other groups that show strong diseconomies of scale, e.g. housing or transportation. Therefore, some scale factors should increase more for the third child, but not all of them. However, if there is a higher increase in scale factors for *all* commodity groups, this would be an indicator that the equivalence scale for families with three children is biased upwards, because a bias of the equivalence scale will cause a bias of the same relative size in all scale factors (and vice versa).

Indeed, scale factors for clothing exhibit clear economies of scale in children's clothes. The additional clothing cost of the third child is lower than that of the second which again is lower than that of the first child. This is a plausible result, because clothes can be handed down from one child to the next. Savings are decreasing with the number of children because there is a limit to how often clothes can be handed down to a brother or a sister. For *food* and *housing*, the second child costs approximately the same as the first child, for *home & furniture*, for *transportation* and for *recreation*, the second child costs considerably less than the first. For all five categories the third child is significantly more expensive than the second.

Higher cost of the third child might be plausible for *housing*: the average apartment is too small for a large family, such that a family of five is more likely than a smaller family to live in self-owned housing, which is usually a house and therefore more expensive than an apartment. When corrected for equalized income, each additional child increases the probability that a family is living in self owned quarters: the first child increases the probability by 1.6%, the second by an additional 6.6% and the third by a further 8.4%. A similar argument could be brought forward for *home & furniture*: furnishing and caring for a house is more expensive than for a flat. However, standard errors for this category are too high for the higher cost of the third child to be statistically significant.

For *transportation* it can be assumed that a second child can easily be taken in any family car with barely any additional cost. This is not true for a third child. Many small cars seat only four persons; luggage space is a similar issue. So, for the same amount of transportation a family of five on average needs a larger car than a family of four or three. This can be tested via expenditures on the annual tax on motor vehicles, because the tax is proportional to the displacement of the motor, which can be taken as a proxy for the size of the car. When controlled for net income, expenditures and the number of cars in the household, adding a third child increases these significantly by about DM 33 (Table 4.11). This is equivalent to an increase in displacement of about 0.3 litres, implying that – at the same income level – families with three children have a significantly larger car than families with two or one children. This difference is increased to DM 40, when the regression is controlled for equalized income and expenditures. A similar increase from the second over the first child cannot be observed. The uncompensated increase is not significant at a value of DM 12 and the equalized increase is significant on the 95% level with a value of DM 16.

There is, however, no equally convincing argument for the disproportionally high increase in the cost of *food*, where the second child is almost (90%) as expensive as the first, while the third child is over 50% more expensive. It could be that the increased demands of the care for three children lead to time substitution from cooking from scratch to the cooking of more expensive prefabricated meals. This is not testable without more detailed food consumption data.[35]

In summary, the evidence is mixed. There is a plausible story for all commodity groups for the higher cost of the third child. It is interesting that the costs of the third child are higher for almost all commodity groups, with the only exception being clothing and tobacco. The argument of increased cost

[35]Such data have been collected for the 1993 EVS, but they were not available to the author.

	income & consumption	
	uncompensated	equivalent
(Intercept)	75.92***	68.44***
Net income	−0.38***	−0.37***
Private consumption	1.48***	1.61***
Number of cars	185.82***	185.29***
Number of children ≥ 1	−13.90	−6.66
Number of children ≥ 2	12.11	16.28*
Number of children ≥ 3	32.57***	40.18***
R^2	0.1646	0.1644

Table 4.11: *Relationship between expenditures on the tax on motor vehicles and the number of children. EVS 1993. For a compact presentation, standard errors are suppressed. Significance levels: * = 5%, ** = 1%, *** = 0.1%.*

would have found better support, had there been at least some commodity groups with clear additional economies of scale or at least no diseconomies, e.g. food or recreation. The cost of the third child might indeed be higher than that of the second, but the evidence is not very strong. The higher cost of the third child could also stem from an underestimation of the cost of the second child. Here an underestimation leads to a double bias: the estimated cost of the second child are too low and those of the third too high. Nevertheless, the higher cost of the third child is present in all models and significant in *BaC* and *BaCTA*. A bias of the estimated differences can only be attributed to errors in the model specification.

Substitution Effects

It is interesting to understand how substitution effects affect the demand for different categories of goods. In section 4.8 uncompensated and compensated elasticities are reported. Here, a different, more direct approach is set out. The compensated demands for all categories are calculated and compared with the demands of the reference household and the scale factors.

A household with two children has a scale factor for *recreation* of 1.75. But this household does not consume 1.75 times as much recreation as a childless couple with an equivalent income. At median income, this household consumes only 1.50 times as much as the equivalent childless couple, because the household substitutes away from recreation which has now become relatively more expensive. Table 4.12 shows the relation between the expenditures of households with children at the median expenditure level of their household type relative to the expenditures of a reference household with equivalent

	Relative expenditures at equivalent income levels			Relative difference to scale factors		
	AAC	ACC	AACCC	AAC	AACC	AACCC
Food	1.12	1.22	1.40	0.5%	0.3%	0.6%
Clothing	1.23	1.37	1.54	−5.1%	−6.8%	−5.4%
Housing	1.13	1.24	1.44	0.4%	−0.2%	−0.4%
Home & furniture	1.11	1.17	1.36	2.1%	4.4%	3.8%
Transportation	1.16	1.21	1.37	−0.8%	1.0%	1.9%
Recreation	1.30	1.50	1.80	−10.5%	−14.5%	−16.6%
Personal care	1.15	1.11	1.16	−0.3%	4.0%	8.4%
Vacation	0.98	1.00	1.15	13.7%	20.4%	21.9%
Tobacco	0.96	0.71	0.47	−6.2%	−20.2%	−41.2%
Alcohol	0.92	0.95	1.02	7.2%	8.9%	12.3%

Table 4.12: *Expenditures of families with children relative to the reference childless couple at expenditure levels equivalent to the median of the reference household and difference between direct factors and estimated scale factors as a percentage value. Model* BaC.

income (i.e. with the same utility level):

$$\tilde{m}_i^s = \frac{q_i^s(u_0)}{q_i^r(u_0)} = \frac{m_i^s h_i(u, p_1^s, \ldots, p_n^s)}{h_i(u, p_1, \ldots, p_n)}$$

As children have an effect on prices, households substitute away from those goods that experience a price increase that is higher than the average and substitute towards those goods that experience a price increase lower than the average. The equivalence scale is then the true price index reflecting the summary of these changes. Indeed, a comparison of the equivalence scales in Table 4.8 with scale factors in Table 4.10 predicts exactly those goods towards which households substitute ($\tilde{m}_i^s > m_i^s$) and those from which they substitute away ($\tilde{m}_i^s < m_i^s$).

The only exception is tobacco, which has some of the lowest scale factors of all commodity groups, which even fall with an increasing number of children. Therefore one would expect substitution towards tobacco, but the opposite is observed. This results from a irregular positive compensated price elasticity for tobacco, which is the only commodity that does not meet the Slutsky regularity conditions.[36] The issue will be further discussed in section 4.8.

Table 4.12 also shows the difference between relative actual expenditures m_i^* and scale factors m_i as a percentage. This is the percentage that an

[36]The conditions are that compensated price responses are symmetric (this is imposed by the QES) and form a negative semidefinite matrix. A necessary condition for the matrix to be negative semidefinite is that all diagonal elements (i.e. all own price responses) have to be negative.

adult in a household with children effectively consumes more or less than an adult in the reference household. For example an adult in a couple with two children effectively consumes 6.8% less *clothing* but 4.4% more *home & furniture*. These numbers have to be interpreted with care. For a couple with two children, the value of m_i^* for *vacation* is exactly 1.00 – a couple with two children at the median income has exactly the same vacation expenditures as an equivalent childless couple – but the adult equivalent consumption is shown to have increased by 20.4%. This is because the scale factor is less than one: the vacation needs of a family (in money terms) are lower than those of a childless couple, leading to an increase of effective vacation consumption for the parents, even though they spend the same. The same pattern is found for alcohol expenditures. Of course, this could be interpreted as a preference shift effect. If prices would not change, parents would consume less alcohol and less vacation and be just as happy, but because prices shift they do consume more of these goods. This would be the interpretation of the numbers in terms of the Barten model.

Substitution effects are rather small for the larger commodity groups. Nevertheless the effect on the general equivalence scales is not negligible. If the possibility of substitution is neglected, the costs of the first child increase by more than a quarter, while the effect on the costs of the second and third child are somewhat lower. This is in line with the Rothbarth results in chapter 3. It remains an open question whether the additional precision that is achieved by using demographic scaling is worth the cost in computational effort and loss of robustness.

Model *BaCTA*: Fixed Scales for Clothing, Tobacco and Alcohol

In model *BaCTA*, scale factors for clothing were fixed as in model *BaC*. Scale factors for smoking were fixed according to the relative numbers of tobacco-buying households, and scale factors for alcohol were held equal for all households with children, thus allowing for an effect of having children on alcohol demand, but not for an effect of any additional child after the first (Table 4.5, lines 1–3).

Scale factors for tobacco and alcohol in this model are not very different from the estimated scale factors of model *BaC*. Consequently, other estimates shown in Table 4.13 do not differ significantly either. This similarity is a confirmation that small changes in the restrictions have only small effects on estimation outcomes.

Model *BaCTA* can be compared against the more general model *BaC* using a likelihood ratio test: The test statistic takes a value of 10.50 which has to be compared against a critical value of 15.09 for five restrictions[37] at a

[37] The five restrictions are: the three fixed values for the tobacco scale factors (three

	A	AAC	AACC	AACCC
Food	1.00	1.11	1.21	1.38
		(0.017)	(0.018)	(0.025)
Clothing	1.00	1.29	1.47	1.63
Housing	1.00	1.11	1.22	1.43
		(0.021)	(0.021)	(0.029)
Home & furniture	1.00	1.05	1.06	1.25
		(0.066)	(0.071)	(0.102)
Transportation	1.00	1.16	1.18	1.33
		(0.026)	(0.026)	(0.036)
Recreation	1.00	1.49	1.81	2.24
		(0.076)	(0.097)	(0.150)
Personal care	1.00	1.15	1.05	1.04
		(0.027)	(0.030)	(0.043)
Vacation	1.00	0.79	0.72	0.81
		(0.071)	(0.082)	(0.108)
Tobacco	1.00	1.06	0.97	0.86
Alcohol	1.00	0.85	0.85	0.85
		(0.036)	(0.036)	(0.036)

Table 4.13: *Good-specific scale factors for a model with ten goods and fixed factors for clothing, tobacco and alcohol (Model* BaCTA*). West German households. Standard errors are given in parentheses.*

1% significance level. Model *BaCTA* cannot be rejected against model *BaC*. Therefore, additional restrictions on alcohol and tobacco are a reasonable starting point for the estimation of a Gorman model.

The Gorman Model: Model *GoCTA*

Good-specific scale factors and overheads for model *GoCTA* are reported in Table 4.14. To compare the scale factors and overheads of the Gorman model with the scale factors of the Barten models, virtual scale factors are defined as the relative quantities of the actual and reference equivalent consumption of a given household, analogous to Equation 4.2:

$$q_i^r = \frac{q_i^s}{m_i^{s*}} , \tag{4.57}$$

where q_i^s is the actual consumption of household type s and q_i^r is the reference equivalent quantity. Substitution of q_i^r by Equation 4.10 ($q_i^r = q_i^s/m_i^s - \beta_i^s$) and rearrangement gives an equation for the virtual scale factor that depends

restrictions) and the equality of all three alcohol scale factors, which is equivalent to two further restrictions.

on the quantity consumed by household type s:

$$m_i^{s*} = \frac{q_i^s}{q_i^r} = \frac{q_i^s}{q_i^s/m_i^s - \beta_i^s} \ . \tag{4.58}$$

In the Barten model, virtual scale factors and Barten scale factors are identical. Virtual scale factors for expenditures that are equivalent to the 5th, 50th and 95th percentile incomes are shown in Table 4.15, opposite page.

Individual overheads and scale factors are not well determined in the model, nor are the sums of overheads. Estimates of overheads and scale factors are not independent of each other. Standard errors for virtual scale factors are somewhat lower, because they are a function of both overheads and scale factors where errors partly cancel out. Nevertheless, results have to be interpreted with care, because standard errors are high and only virtual scale factors for the larger commodity groups *food*, *housing* and *vacation* are

	Scale factors m_i^s			Overheads β_i^s		
	AAC	AACC	AACCC	AAC	AACC	AACCC
Food	1.20	1.23	1.27	−417	124	1127
	(0.111)	(0.083)	(0.225)	(694)	(513)	(1306)
Clothing	1.16	1.31	1.34	442	620	988
Housing	1.29	1.57	1.83	−1081	−2015	−1799
	(0.135)	(0.186)	(0.294)	(921)	(1130)	(1712)
Home & furniture	1.04	1.23	1.31	143	−70	382
	(0.133)	(0.128)	(0.250)	(229)	(218)	(342)
Transportation	0.75	0.77	0.71	1101	1054	1661
	(0.196)	(0.230)	(0.301)	(302)	(340)	(465)
Recreation	1.38	1.46	2.12	471	1014	1276
	(0.155)	(0.136)	(0.349)	(224)	(223)	(382)
Personal care	0.67	0.68	0.39	396	273	460
	(0.255)	(0.309)	(0.440)	(148)	(174)	(243)
Vacation	1.09	1.03	1.15	−575	−437	−292
	(0.096)	(0.109)	(0.177)	(181)	(161)	(229)
Tobacco	1.06	0.97	0.86	0	0	0
Alcohol	0.87	0.87	0.87	0	0	0
	(0.048)	(0.048)	(0.048)			
Sum of overheads				479	563	3802
				(1517)	(1343)	(2275)

Table 4.14: *Good-specific scale factors and overheads for a model with ten goods, fixed factors for clothing and tobacco, equal factors for households with children for alcohol (Model* GoCTA*). West German households. Standard errors are given in parentheses.*

	AAC			AACC			AACCC		
Percentile	5%	50%	95%	5%	50%	95%	5%	50%	95%
Food	1.13	1.15	1.17	1.25	1.24	1.24	1.49	1.42	1.37
	(0.031)	(0.024)	(0.051)	(0.025)	(0.026)	(0.043)	(0.039)	(0.086)	(0.129)
Clothing	1.49	1.29	1.22	1.89	1.53	1.41	2.29	1.69	1.49
	(0.015)	(0.004)	(0.002)	(0.030)	(0.008)	(0.003)	(0.063)	(0.016)	(0.004)
Housing	1.12	1.18	1.22	1.19	1.31	1.41	1.43	1.56	1.66
	(0.041)	(0.037)	(0.070)	(0.067)	(0.035)	(0.079)	(0.130)	(0.052)	(0.117)
Home & furniture	1.12	1.07	1.06	1.18	1.21	1.22	1.61	1.42	1.36
	(0.097)	(0.105)	(0.118)	(0.098)	(0.090)	(0.108)	(0.231)	(0.215)	(0.227)
Transportation	0.97	0.87	0.83	0.99	0.89	0.85	0.96	0.85	0.82
	(0.259)	(0.229)	(0.217)	(0.292)	(0.262)	(0.252)	(0.466)	(0.388)	(0.365)
Recreation	1.89	1.55	1.45	2.80	1.88	1.64	7.66	3.14	2.46
	(0.267)	(0.154)	(0.149)	(0.451)	(0.171)	(0.140)	(5.819)	(0.713)	(0.422)
Personal care	0.89	0.81	0.77	0.83	0.77	0.75	0.46	0.44	0.43
	(0.347)	(0.310)	(0.290)	(0.350)	(0.333)	(0.323)	(0.589)	(0.532)	(0.521)
Vacation	0.65	0.95	1.04	0.74	0.94	1.00	0.94	1.08	1.13
	(0.099)	(0.059)	(0.082)	(0.083)	(0.079)	(0.097)	(0.119)	(0.130)	(0.159)
Tobacco	1.06	1.06	1.06	0.97	0.97	0.97	0.86	0.86	0.86
Alcohol	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
	(0.048)	(0.048)	(0.048)	(0.048)	(0.048)	(0.048)	(0.048)	(0.048)	(0.048)

Table 4.15: *Good-specific virtual scale factors for model* GoCTA. *Standard errors are given in parentheses.*

well determined. Note that, even though overheads and scale factors were fixed for *clothing*, virtual scale factors show a (small) positive standard error for this commodity, because of the error of the quantity q_i^s in Equation 4.58. For *tobacco* and *alcohol*, overheads are zero and the virtual scale factor does not depend on expenditures.

At the median income level, virtual scale factors of model *GoCTA* are similar to those of the Barten models, with the notable exception of *vacation*. Both *Transportation* and *personal care* also show point estimates of virtual scale factors that are quite different from the results of the Barten models, but estimates for these commodities are too imprecise for the difference to be significant.

Virtual scale factors for some commodity groups vary strongly over the income range. *Housing* and *vacation* show a significant increase of virtual scale factors with rising income. Poorer households seem to cut back on these expenditures. Factors for *clothing* and *recreation* are falling in income. These two commodity groups contain many specific children's goods and services, that have a higher weight on poorer households: While other commodity groups merely reflect scaled parents' expenditures, scaling alone is not sufficient here. This gives support to the criticism of the Barten method, that scaling alone cannot explain the expenditures of families.

In summary, the increased effort necessary and the wider error margins have to be weighted against the value of the improved representation of family expenditure patterns in the Gorman model. The choice of model depends on the focus of interest. The Barten model gives a good representation of the average household, where fixed cost have a low weight compared to scaled parents' expenditures, but if the aim is to estimate equivalence scales for poor households, then the fixed cost that are found in the Gorman model have a high weight and cannot be ignored.

4.8 The Expenditure System: Parameters and Elasticities

Minimum Expenditure Levels

Another useful summary statistic for an evaluation of the estimated expenditure system – apart from equivalence scales – are the sums of subsistence expenditures $\sum_{j=1}^{n} m_j^s p_j b_j$ (for the Barten model) and $\sum_{j=1}^{n} m_j^s p_j \tilde{b}_j^s$ (for the Gorman model), shown in Table 4.16.

Model *GoCTA* shows much lower subsistence levels[38] than both Barten models. Despite the high standard errors for scale factors and overheads

[38]The subsistence level in the expenditure system should not be confused with the poverty line which is based on social considerations. The poverty line should be higher than the subsistence levels estimated here.

Household type	Barten Models *BaC*	*BaCTA*	Gorman Model *GoCTA*
AA	28278	26036	14342
	(1838)	(1777)	(828)
AAC	32039	29259	17013
	(2186)	(2150)	(1081)
AACC	34571	31400	18692
	(2400)	(2349)	(1198)
AACCC	39524	35821	23296
	(2753)	(2657)	(1514)

Table 4.16: *Subsistence levels for four household types and three models. Standard errors in parentheses have been determined by the delta method.*

discussed above, the subsistence level is indeed better determined in the Gorman model than in the Barten models. This reflects the better ability of the Gorman model to accommodate differences in needs between households at different income levels.

One problem of the QES is that it has no economic interpretation for incomes below the subsistence level. Does this pose a problem here? This certainly depends on the percentages of households whose expenditure level is below the subsistence level, which are given in Table 4.17. Two numbers are shown in each cell: The first number is based on instrumented total expenditures that have been estimated together with the model using net income and other variables as instruments. This reduces the variance of total expenditures and therefore the number of households with very low expenditures. The second number is based on actual expenditures.

Household type	*BaC*	*BaCTA*	*GoCTA*
AA	1.1 / 7.3	0.4 / 4.9	0.0 / 0.1
AAC	1.6 / 9.4	0.3 / 5.4	0.0 / 0.2
AACC	1.6 / 9.1	0.4 / 4.6	0.0 / 0.1
AACCC	3.9 / 14.5	1.2 / 7.4	0.0 / 0.5

Table 4.17: *Percentage of households whose total expenditures are below the subsistence level for: instrumented expenditures / actual expenditures.*

The number of households whose instrumented expenditures are below the subsistence level is indeed low in all models. It is even zero for the Gorman model, where subsistence levels are close to the observed minimum expenditures and only a fraction of a percent of households exhibit actual expenditures below the subsistence level. The higher subsistence level of the Barten models is problematic if equivalence scales for poor households are the object of interest, because the estimated expenditure system has no economic interpretation for these households. If equivalence scales at the subsistence level are to be estimated, then the Gorman model must be preferred, while the Barten model can be used for some "average" equivalence scale at the mean income level.

Price and Income Elasticities

Since parameters of the QES other than the subsistence levels lack an obvious economic interpretation, the overall effects are best represented by the calculation of income and price elasticities. The income elasticity η_i^s of good i for household type s is:

$$\begin{aligned}\eta_i^s =& \frac{\partial q_i}{\partial \mu} \cdot \frac{\mu}{q_i} = \frac{\partial x_i}{\partial \mu} \cdot \frac{\mu}{x_i} \\ =& \frac{\mu}{x_i} \left[a_i + 2 \left(p_i m_i^s c_i - a_i \sum_{j=1}^n m_j^s p_j c_j \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right) \right],\end{aligned}$$

where m_i^s is the scale factor for the household.

The uncompensated own price elasticity for good i is:

$$\begin{aligned}\varepsilon_{ii}^s =& \frac{\partial q_i}{\partial p_i} \cdot \frac{p_i}{q_i} = \frac{\partial x_i}{\partial p_i} \cdot \frac{p_i}{x_i} - 1 \\ =& -1 + \frac{p_i}{x_i} \Bigg[(1 - a_i) m_i^s b_i - \\ & 2 m_i^s b_i \left(p_i m_i^s c_i - a_i \sum_{j=1}^n m_j^s p_j c_j \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right) + \\ & \left((1 - 3a_i) m_i^s c_i + 2 \frac{a_i^2}{p_i} \sum_{j=1}^n m_j^s p_j c_j \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right)^2 \Bigg]\end{aligned}$$

Finally the uncompensated cross price elasticities of good i with respect to price k are:

$$\begin{aligned}\varepsilon_{ik}^s =& \frac{\partial q_i}{\partial p_k} \cdot \frac{p_k}{q_i} = \frac{\partial x_i}{\partial p_k} \cdot \frac{p_k}{x_i} \\ =& \frac{p_k}{x_i} \Bigg[-a_i m_k^s b_k - \\ & 2 m_k^s b_k \left(p_i m_i^s c_i - a_i \sum_{j=1}^n m_j^s p_j c_j \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right) + \\ & \left(-a_i m_k^s c_k - 2 \frac{a_k}{p_k} \left(p_i m_i^s c_i - a_i \sum_{j=1}^n m_j^s p_j c_j \right) \right) \prod_{j=1}^n (m_j^s p_j)^{-2a_j} \left(\mu - \sum_{j=1}^n m_j^s p_j b_j \right)^2 \Bigg]\end{aligned}$$

with $i \neq k$

According to the Slutsky equation, it follows that the compensated price elasticity is:

$$\begin{aligned}\tilde{\varepsilon}_{ik}^{s} &= \frac{\partial h_i^s(V, p_1 \ldots p_n)}{\partial p_k} \cdot \frac{p_k}{h_i^s(V, p_1 \ldots p_n)} \\ &= \varepsilon_{ik}^{s} + \eta_i^s w_k^s\end{aligned}$$

Good	AA	AAC	AACC	AACCC
Food	−0.190	−0.165	−0.159	−0.136
Clothing	−0.362	−0.307	−0.291	−0.266
Housing	−0.224	−0.197	−0.188	−0.162
Home & furniture	−0.389	−0.370	−0.373	−0.335
Transportation	−0.278	−0.239	−0.241	−0.213
Recreation	−0.410	−0.330	−0.301	−0.258
Personal care	−0.232	−0.197	−0.216	−0.207
Vacation	−0.377	−0.433	−0.454	−0.433
Tobacco	0.453	0.386	0.489	0.523
Alcohol	−0.206	−0.227	−0.230	−0.217

Table 4.18: *Model* BaC*: Estimated compensated price elasticities at the median expenditure level for different household types.*

Good	AA	AAC	AACC	AACCC
Food	−0.195	−0.164	−0.152	−0.142
Clothing	−0.314	−0.253	−0.234	−0.228
Shelter	−0.175	−0.151	−0.140	−0.129
Health	−0.276	−0.285	−0.261	−0.256
Mobility	−0.233	−0.180	−0.176	−0.171
Education	−0.333	−0.262	−0.233	−0.221
Others	−0.266	−0.260	−0.253	−0.253

Table 4.19: *Estimated compensated price elasticities at the median expenditure level for different household types. (Kohn and Missong, 2003)*

Compensated price elasticities for model *BaC* are shown in Table 4.18. They were estimated at the median of the respective income distribution of each household type. Except for the elasticities for tobacco they are all negative. However, the positive compensated price elasticity for tobacco is sufficient to violate the theoretical condition of negative semidefiniteness of the Slutsky matrix of the entire demand system. This points again to the difficulties posed by fitting a relatively simple model to fit families of differing compositions when strong preference effects are mixed with price effects. That the problem can be localized to tobacco expenditure can be seen when the model is fitted to non-smoking households (i.e. tobacco expenditure equals zero). All the remaining estimated compensated price elasticities are positive so the demand system satisfies this important necessary theoretical condition for the demand system. The estimated equivalence scales change only marginally. The same applies when the Barten model is estimated with the restriction of negative compensated own price elasticities imposed. See Appendix 4.D for calculations.

For comparison with model *BaC*, in Table 4.19 results from Kohn and Missong (2003) are shown. The results were estimated in a quadratic expenditure system with data from two years (1988 and 1993). Demographics entered the model as translation parameters. It should be noted that commodity groups are not directly comparable, in Kohn and Missong's work, food also contains alcohol and tobacco, mobility also contains the purchase of new and used cars and expenditures on transportation services for travel. Education corresponds to recreation, but does still contain "other expenditures during travel". Clothing and shelter are identical to the categories clothing and housing.

As it turns out, the estimated elasticities for both models are of a similar magnitude. The elasticities for the *food* category are almost identical, while elasticities for other comparable categories are slightly higher in model *BaC*. The differences can be explained by two reasons: The different incorporation of demographic effects in both models (scaling versus translating) and a stronger reaction of households to long term demographic price changes which have been used to estimate model *BaC* than to short term changes in the prices of goods (which have been used by Kohn and Missong).

It could be argued, that the observation of similar estimates is not surprising, because elasticities were both estimated with the QES, which is not a flexible functional form[39], and therefore has income and own-price elasticities that are not entirely independent of each other. However, the specification of Kohn and Missong's model with demographic translating ensures that no information from Barten scaling is used that could outweigh direct price effects. In addition, the results from the Gorman model, model *BaCTA* and results from unreported preliminary models show that price elasticities in the QES can vary over a wide range, depending on the specification and the additional informantion used. Therefore, it is reassuring to note that resulting elasticities are approximately of the same size as those that were estimated with a model that used actual price variation instead of a demographically induced price variation.

Income elasticities are well identified from cross section data. Therefore, the influence of the specifications is rather small. Table 4.20 shows the estimated income elasticities at the median expenditure level. Elasticities are

[39]See Pollak and Wales (1992) for a discussion of flexible functional forms.

Good	AA	AAC	AACC	AACCC
Food	0.643	0.637	0.630	0.618
Clothing	1.299	1.286	1.267	1.279
Housing	0.817	0.838	0.830	0.824
Home & furniture	1.357	1.450	1.483	1.532
Transportation	0.839	0.849	0.873	0.891
Recreation	1.364	1.358	1.315	1.284
Personal care	0.691	0.677	0.734	0.791
Vacation	1.839	1.953	2.004	2.124
Tobacco	−0.675	−0.836	−1.144	−1.559
Alcohol	0.915	0.928	0.940	0.952

Table 4.20: *Model* BaC*: Estimated income elasticities at the median expenditure level for different household types.*

very similar among household types. This is partly owed to the restrictive nature of the Barten scaling approach, where demography can enter only through prices. Therefore, income elasticities can only vary in so far as prices influence them and as the median income of the household types is not equivalent. Again, tobacco stands out as the only inferior good and as the only good where the absolute values of elasticities increase strongly with household size.

Table 4.21 shows estimated compensated price elasticities at the median income of the respective household types for model *BaCTA*. Elasticities are on average about 23% higher than in model *BaC*, but the general pattern of differences between commodity groups is the same, with the exception of tobacco and alcohol. In comparison, compensated price elasticities for tobacco are higher relative to those of other goods and elasticities for alcohol are lower.

Good	AA	AAC	AACC	AACCC
Food	−0.227	−0.201	−0.195	−0.173
Clothing	−0.429	−0.371	−0.355	−0.335
Housing	−0.266	−0.240	−0.232	−0.206
Home & furniture	−0.462	−0.452	−0.460	−0.427
Transportation	−0.338	−0.296	−0.303	−0.276
Recreation	−0.484	−0.397	−0.366	−0.324
Personal care	−0.279	−0.242	−0.268	−0.265
Vacation	−0.436	−0.514	−0.546	−0.538
Tobacco	0.612	0.508	0.635	0.754
Alcohol	−0.232	−0.259	−0.268	−0.268

Table 4.21: *Model* BaCTA*: Estimated compensated price elasticities at the median expenditure level for different household types.*

This is the effect of the additional restrictions on these goods.

Table 4.22 shows compensated price elasticities for the Gorman model *GoCTA*. Model *GoCTA* shows even higher compensated price elasticities than model *BaCTA*, with elasticities on average being about 65% higher than in model *BaC*, at the median income level.

Good	AA	AAC	AACC	ACCC
Food	−0.275	−0.268	−0.266	−0.243
Clothing	−0.496	−0.471	−0.461	−0.448
Housing	−0.307	−0.306	−0.302	−0.275
Home & furniture	−0.548	−0.583	−0.614	−0.580
Transportation	−0.464	−0.442	−0.462	−0.433
Recreation	−0.554	−0.500	−0.469	−0.426
Personal care	−0.421	−0.398	−0.453	−0.460
Vacation	−0.403	−0.545	−0.591	−0.599
Tobacco	0.830	0.750	1.010	1.282
Alcohol	−0.252	−0.306	−0.327	−0.341

Table 4.22: *Model* GoCTA*: Estimated compensated price elasticities at the median expenditure level for different household types.*

4.9 Conclusion

In this chapter Barten and Gorman equivalence scales were estimated in a quadratic expenditure system from a single cross section. The identification problem of equivalence scales is solved in these models by assuming a certain structure of the household decision process, where parents complement their own consumption with adequate consumption quantities for their children according to the children's (exogenous) needs. Children's needs are a fixed proportion of parents' needs in the Barten model, while the Gorman model also allows for an additional fixed cost term for children. The need to complement parents' consumption leads to a scaling of prices.

It is a special feature of the methods used in this work, that an estimation of scaled (Barten) prices is possible *without* using direct short run price elasticities. The standard procedure to estimate Barten-Gorman equivalence scales is to use directly observed price elasticities to identify the changes in scaled prices caused by a change in the number of persons in a household. However, the reactions to a long run change in scaled prices that is caused by an additional household member might be different from short run price reactions. Hence, the limitation to data from a single cross section can be seen as an advantage, not only because smaller data requirements facilitate the application but also for theoretical reasons.

Identification of the demand system was possible, because of a close relationship between income and price elasticities and their derivatives in the QES. To improve estimates and to rely less on the second order information contained in the derivatives, some scale factors were fixed to act as price changes. To fix scaled prices, an analysis of possible constraints was carried out, and scaled prices for different household types were calculated for clothing, tobacco and alcohol.

The Rothbarth method discussed in Chapter 3 was shown to suffer from a potential bias, especially at middle and high incomes, due to neglected substitution effects. The Barten and Gorman models can account in part for these substitution effects and are therefore likely to give a better estimate of the studied equivalence scale.

In the empirical analysis, however, it was found that the Barten model seems to be too restrictive to reflect the characteristics of households at the lower end of the income range where fixed cost play a bigger role. The model can be used for the median income household, where all Barten and Gorman models lead to similar equivalence scale estimates. The Gorman model is well suited for the application to households of all income levels, but it is more difficult to estimate and has higher data requirements. Therefore, the Rothbarth model should be considered as an alternative for the estimation of equivalence scales for poor households.

4.A Estimation of the QES from one cross section with demographic scaling and limited information

The good-specific scale factors act on demand similarly to a change in prices. Therefore, if good-specific scale factors are known, and if demographic scaling alone is the appropriate way to correct for demographic effects, a complete demand system can be derived from a single cross section, provided scale factors vary among goods. However, generally scale factors are not directly observable and can be estimated only jointly with the parameters of the demand system. Even if the demand system is known a priori, the scale factors cannot be identified from data without price variation, in the absence of further restrictions (see section 4.3).

In this section I will show, that scale factors and all parameters of the QES can still be determined using a single cross section. A limited information approach is employed that builds on the approach followed by Kohn and Missong (2003) and first suggested by Ding and Hadri (1996). Both use a limited information approach and linear estimation techniques to determine the parameters of a QES from data with price variation from two cross sections. Instead, I use only one cross section and price effects that are determined by the variation in household composition between two different household types.[40]

In the demand equations for the reference household r and for the compared household s prices are normalized to one, because only data without price variation are used. For simplicity of notation, the m_i^r are normalized to one, and omitted:

$$x_i^r = b_i + a_i\left(\mu - \sum_{j=1}^n b_j\right) + \left(c_i - a_i\sum_{j=1}^n c_j\right)\left(\mu - \sum_{j=1}^n b_j\right)^2 \tag{4.59}$$

$$\begin{aligned} x_i^s = {} & m_i^s b_i + a_i\left(\mu - \sum_{j=1}^n m_j^s b_j\right) \\ & + \left(m_i^s c_i - a_i\sum_{j=1}^n m_j^s c_j\right)\prod_{j=1}^n \left(m_j^s\right)^{-2a_j}\left(\mu - \sum_{j=1}^n m_j^s b_j\right)^2 \end{aligned} \tag{4.60}$$

It is assumed that both households r and s have the same reference utility functions. Utility is only affected by the unknown scaling factors m_i^s that

[40] It should be noted, that I use the curvature of the demand equations for the estimation of the system. The model is not identified from only one cross secion for the LES. Therefore the model is only poorly identified. (compare Wooldridge, 2002, p.234) The QES will be identified though, if one of the scale factors can be fixed. We will explore this possibility in detail later.

depend on the household type.

It is straight-forward to estimate the complete demand system from n unrestricted Engel curves for the reference household r and for the compared household s:

$$\begin{aligned} x_i^r &= \theta_{1i}^r + \theta_{2i}^r \mu + \theta_{3i}^r \mu^2 \qquad \text{for} \quad i \in \{1, \ldots, n\} \\ x_i^s &= \theta_{1i}^s + \theta_{2i}^s \mu + \theta_{3i}^s \mu^2 \qquad \text{for} \quad i \in \{1, \ldots, n\} \ , \end{aligned} \tag{4.61}$$

where the thetas are related to the parameters of the demand system as follows:

$$\theta_{1i}^r = b_i - a_i \sum_{j=1}^n b_j + \theta_{3i}^r \left(\sum_{j=1}^n b_j \right)^2 \tag{4.62}$$

$$\theta_{2i}^r = a_i - 2\theta_{3i}^r \sum_{j=1}^n b_j \tag{4.63}$$

$$\theta_{3i}^r = c_i - a_i \sum_{j=1}^n c_j \tag{4.64}$$

$$\theta_{1i}^s = m_i^s b_i - a_i \sum_{j=1}^n m_j^s b_j + \theta_{3i}^s \left(\sum_{j=1}^n m_j^s b_j \right)^2 \tag{4.65}$$

$$\theta_{2i}^s = a_i - 2\theta_{3i}^s \sum_{j=1}^n m_j^s b_j \tag{4.66}$$

$$\theta_{3i}^s = \left(m_i^s c_i - a_i \sum_{j=1}^n m_j^s c_j \right) \prod_{j=1}^n \left(m_j^s \right)^{-2a_j} \tag{4.67}$$

Not all information that is contained in the n Engel curves for household s can be used. One has to choose the curves for two goods k and l as the identifying equations. For all other goods only the intercept terms are used. The complete information of the Engel curves for the reference household is used. As with limited information approaches in general, the result is not independent of which information is left out, i.e. it is not independent of the choice of k and l. The size of this effect will be discussed in the empirical part of the chapter.

Writing the equations for the θ_{2i}^r and θ_{2i}^s for both identifying Engel curves

$i = \{k, l\}$ gives the following equation system:

$$\begin{aligned} \theta^r_{2k} &= a_k - 2\theta^r_{3k} \sum_{j=1}^{n} b_j \\ \theta^r_{2l} &= a_l - 2\theta^r_{3l} \sum_{j=1}^{n} b_j \\ \theta^s_{2k} &= a_k - 2\theta^s_{3k} \sum_{j=1}^{n} m^s_j b_j \\ \theta^s_{2l} &= a_l - 2\theta^s_{3l} \sum_{j=1}^{n} m^s_j b_j \quad , \end{aligned} \tag{4.68}$$

which can be solved for a_k, a_l, $\sum_{j=1}^n b_j$ and $\sum_{j=1}^n m^s_j b_j$.[41] The missing a_i can be estimated from the other θ^r_{2i} and $\widehat{\sum_{j=1}^n b_j}$:

$$a_i = \theta^r_{2i} + 2\theta^r_{3i} \widehat{\sum_{j=1}^{n} b_j} \tag{4.69}$$

The b_i can be estimated from the θ^r_{1i}, $\hat{a}_i$ and $\widehat{\sum_{j=1}^n b_j}$.

$$b_i = \theta^r_{1i} + \theta^r_{2i} \widehat{\sum_{j=1}^{n} b_j} + \theta^r_{3i} \left(\widehat{\sum_{j=1}^{n} b_j} \right)^2 \tag{4.70}$$

Using the constancy of the b_i, the m^s_i can then be calculated from the intercept terms of household type s's Engel curves θ^s_{1i}:

$$m^s_i = \frac{1}{\hat{b}_i} \left(\theta^s_{1i} + \hat{a}_i \widehat{\sum_{j=1}^{n} m^s_j b_j} - \theta^s_{3i} \left(\widehat{\sum_{j=1}^{n} m^s_j b_j} \right)^2 \right) \tag{4.71}$$

Estimation of the c_i is straight forward. Again, c_l and c_k are assumed to be constant over household types. Replace a_i and m_i by their estimates $\hat{a}_i$ and $\hat{m}_i$. Then the following equation system can be solved for c_k, c_l, $\sum_{j=1}^n c_j$

[41] Note that the relation $\sum_{j=1}^n b_j / \sum_{j=1}^n m^s_j b_j$ is already the estimate of the general equivalence scale at the subsistence level.

and $\sum_{j=1}^{n} m_j^s c_j$:

$$\begin{aligned} \theta_{3k}^r &= c_k - \hat{a}_k \sum_{j=1}^{n} c_j \\ \theta_{3l}^r &= c_l - \hat{a}_l \sum_{j=1}^{n} c_j \\ \theta_{3k}^s \prod_{j=1}^{n} \hat{m_j^s}^{2\hat{a}_j} &= \hat{m_k^s} c_k - \hat{a}_k \sum_{j=1}^{n} m_j^s c_j \\ \theta_{3l}^s \prod_{j=1}^{n} \hat{m_j^s}^{2\hat{a}_j} &= \hat{m_l^s} c_l - \hat{a}_l \sum_{j=1}^{n} m_j^s c_j \end{aligned} \tag{4.72}$$

All other c_i can be derived from the equations for θ_{3i}^r (4.64), by replacing a_i with $\hat{a}_i$ and $\sum_{j=1}^{n} c_j$ by $\widehat{\sum_{j=1}^{n} c_j}$:

$$c_i = \theta_{3i}^r + \hat{a}_i \widehat{\sum c_j} \tag{4.73}$$

Interpretation of the Identification Procedure

The identification of the a_k, a_l, $\sum_{j=1}^{n} b_j$ and $\sum_{j=1}^{n} m_j^s b_j$ (equation system 4.68) can be interpreted in terms of the derivatives of the Engel curves of good k and l. Write the derivatives of Equation (4.61) with respect to μ:

$$\frac{\mathrm{d}x_i^s}{\mathrm{d}\mu} = \theta_{2i}^s + 2\theta_{3i}^s \mu, \quad s = \{r, s\}, i = \{k, l\}. \tag{4.74}$$

Now substitute Equation (4.63) for θ_{2i}^r and (4.66) for θ_{2i}^s, respectively:

$$\begin{aligned} \frac{\mathrm{d}x_i^r}{\mathrm{d}\mu} &= a_k - 2\theta_{3i}^r \sum_{j=1}^{n} b_j + 2\theta_{3i}^r \mu \\ \frac{\mathrm{d}x_i^s}{\mathrm{d}\mu} &= a_k - 2\theta_{3i}^s \sum_{j=1}^{n} m_j^s b_j + 2\theta_{3i}^s \mu, \quad i = \{k, l\}. \end{aligned} \tag{4.75}$$

a_k and a_l can be eliminated by equalizing $\mathrm{d}x_k^r/\mathrm{d}\mu$ and $\mathrm{d}x_k^s/\mathrm{d}\mu$, and $\mathrm{d}x_l^r/\mathrm{d}\mu$ and $\mathrm{d}x_l^s/\mathrm{d}\mu$, respectively. The remaining two equations can be solved for $\sum_{j=1}^{n} b_j$ and $\sum_{j=1}^{n} m_j^s b_j$. This implies that the subsistence levels are exactly those two income levels where the slopes of the respective pairs of Engel curves are equal, i.e. both Engel curves for good k have the same slope at an income level of $\sum_{j=1}^{n} b_j$ for the reference household and $\sum_{j=1}^{n} m_j^s b_j$ for the compared household, and both Engel curves for good l have an identical slope at the same income levels.

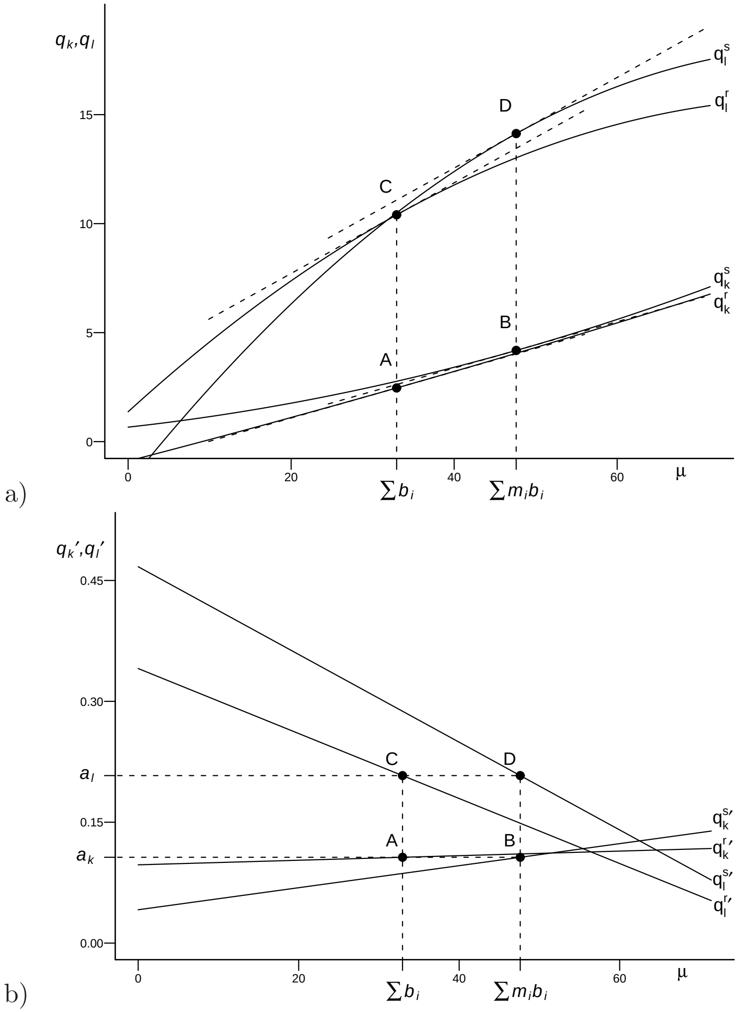

Figure 4.4: *Identification of the minimum expenditure parameters and the* a_k*,* a_l*. a) Engel curves, b) the respective derivatives.* μ *is given in 1000 DM.*

This is shown in Figure 4.4.[42] Fig. 4.4 a) shows the Engel curves of goods k and l for both household types while b) shows their derivatives. The Engel curves for good k have the same slope in points A and B, while the Engel curves for good l have the same slope in points C and D, where A and C are at the same income level $\sum_{j=1}^{n} b_j$ and B and D are at the same income level

[42] In Figure 4.4, the results from the estimation in section 4.7 are used with clothing and housing as goods k and l.

$\sum_{j=1}^{n} m_j^s b_j$. The Engel curves also have equal slopes at other income pairs, but there is only one distinguished pair of incomes where both pairs of Engel curves have the same slope at the same time.

This also explains, why the linear expenditure system cannot be identified in the same way: in the LES the slopes of the Engel curves are constant and equal at all incomes, therefore there is no distinguished pair of incomes, any pair of incomes has these properties.

The result also has an economic interpretation: the income elasticities are closely connected to the price elasticities. Any substitution would lead to a change in the income elasticity. At the subsistence level, however, there is no possibility of substitution left, and therefore the income elasticities of all goods is the same in the compared household as in the reference household.

Problems and Tests

Not all available information has been used to calculate the parameters, namely the demand equations for the compared household s except for equations k and l. Also the additional information that could be gained by a joint estimation of the model with several different household types cannot be used.

The additional information is not all lost, as it can be used to test the model in different ways. For $i \notin \{k, l\}$, the a_i, c_i and m_i can be estimated from the equations for θ_{1i}^s, θ_{2i}^s and θ_{3i}^s (4.65 – 4.67) and compared with the estimates which were described above. It is also possible to compare estimates of the model for different choices of the identifying equations k and l. The second test is applied in the estimation of the model with the limited information method below.

Estimation

It turns out that the result depends strongly on the choice of the identifying equations. The limited information approach has been applied to a comparison of household types AA and AACC, using the results for the Engel curve parameters from Table 4.7 and two different pairs of identifying equations, namely clothing and housing and food and housing.

For a first assessment, the sum of the minimum expenditure levels is printed in Table 4.23. The result strongly depends on the choice of identifying equations. Even for the model with the lowest error terms (clothing/housing), errors are very high, and the minimum expenditure levels are not well determined. The difference between the results is also remarkable. At least the numbers from both models are statistically not different – because of high standard errors.

	clothing/housing	food/housing
AA	35714	210309
	(30102)	(4155245)
AACC	49565	177911
	(20522)	(3047117)

Table 4.23: *Subsistence levels for household types AA and AACC, limited information approach and two pairs of identifying equations. Standard errors in parentheses*

Results for scale factors are even more discouraging than those of Table 4.23: statistically none of the scale factors is significantly different from one. Therefore, no further results are reported.

In principle, identification of a demographically scaled QES from only one cross section is possible. However, the limited information approach ignores too much information to attain a satisfactory accuracy of the estimated parameters. Also, it limits the inclusion of additional restrictions, such as restrictions on the range or the value of certain scale factors. Therefore, the estimation should not be done with a limited information approach, but with a standard full information maximum likelihood estimation (see appendix 4.C).

4.B Identification of the QES with a Gorman Model

The identification of the Gorman model and the QES can be achieved in a similar way. In addition to the scale factors, also scales overheads b_i^s have to be identified. These can easily be included into the demand system, because the QES already contains overheads in the form of the translation parameters b_i. These can be replaced by household-type specific overheads $\tilde{b}_i^s$ by adding scaled overheads:

$$\tilde{b}_i^s = b_i + b_i^s \ . \tag{4.76}$$

Scaled overheads for the reference household are normalized to zero, so that $\tilde{b}_i^r = b_i$.

Rewrite equation system 4.68 with household-type specific overheads:

$$\begin{array}{ll} \theta_{2k}^r = a_k - 2\theta_{3k}^r \sum_{j=1}^n \tilde{b}_j^r & \theta_{2k}^s = a_k - 2\theta_{3k}^s \sum_{j=1}^n m_j^s \tilde{b}_j^s \\ \theta_{2l}^r \ = a_l - 2\theta_{3l}^r \sum_{j=1}^n \tilde{b}_j^r & \theta_{2l}^s \ = a_l - 2\theta_{3l}^s \sum_{j=1}^n m_j^s \tilde{b}_j^s \end{array} , \tag{4.77}$$

which can be solved for a_k, a_l, $\sum_{j=1}^n \tilde{b}_j^r$ and $\sum_{j=1}^n m_j^s \tilde{b}_j^s$. The missing a_i and the $\tilde{b}_i^r$ can be estimated as in Equations 4.69 and 4.70. However, in contrast to the pure Barten system, the m_i^s are not determined at this stage, because the $\tilde{b}_i$ are not constant between household types: $\tilde{b}_i^s \neq \tilde{b}_i^r$. Only the product

$m_i^s \tilde{b}_i^s$ is identified according to:

$$m_i^s \tilde{b}_i^s = \theta_{1i}^s + \hat{a}_i \widehat{\sum_{j=1}^{n} m_j^s \tilde{b}_j^s} - \theta_{3i}^s \left(\widehat{\sum_{j=1}^{n} m_j^s \tilde{b}_j^s} \right)^2 . \tag{4.78}$$

$$\begin{aligned} \theta_{3i}^r &= c_i - \hat{a}_i \sum_{j=1}^{n} c_j \\ \theta_{3i}^s &= \left(m_i^s c_i - \hat{a}_i \sum_{j=1}^{n} m_j^s c_j \right) \prod_{j=1}^{n} \left(m_j^s \right)^{-2\hat{a}_j}, \quad i \in \{1, \ldots, n-1\} \end{aligned} \tag{4.79}$$

This is an equation system with $2(n-1)$ independent equations and $2n$ unknowns: the m_i^s and the c_i. Therefore, 2 of the unknowns have to be determined independently for the system to be solvable. Because they can be interpreted in terms of a household technology, the m_i^s are suited for fixing, as suggested for clothing, tobacco and alcohol in section 4.6. Alternatively, the overhead b_i^s can be fixed, allowing for the determination of the respective m_i^s from $\tilde{b}_i^s = \hat{b}_i + b_i^s$ and Equation 4.78. The equation system is also non-linear, because of the term $\prod_{j=1}^{n} \left(m_j^s \right)^{-2\hat{a}_j}$, but an iterative solution is possible.

In summary, the QES can also be estimated jointly with a Gorman model from a single cross section, if restrictions on at least two m_i^s or b_i^s are applied. Given the unsatisfactory results of the limited information approach for the Barten model, only a full information maximum likelihood estimation was applied to this model. Results are reported in section 4.7.

4.C Full Information Maximum Likelihood Estimation of the QES

In the previous section it was shown that it is possible to recover all parameters of the QES and the good-specific scales with a limited information approach from a single cross section. The result, however, was somewhat disappointing because of very high asymptotic standard errors. To improve the situation, this section will explore a full information maximum likelihood estimation of the model. The description in this section follows Pollak and Wales (1992, ch. 5).

The demand system is estimated in share form. The stochastic specification of the demand system is

$$w_{it} = \omega_{it}(\boldsymbol{\beta}, \boldsymbol{s}_t) + u_{it} \quad , \tag{4.80}$$

where $\boldsymbol{\beta}$ is the vector of all parameters, $\boldsymbol{s}_t$ is the set of explanatory variables and u_t is a random disturbance. There are n goods. Define the n-vector of disturbances $\tilde{\boldsymbol{u}}_t' = (u_{1t}, \ldots, u_{nt})$. $\tilde{u}$ is multivariate normal distributed with mean $\mathbf{0}$ and the covariance matrix $\tilde{\boldsymbol{\Omega}}$, which is assumed to be the same for all observations and disturbances are uncorrelated across observations: $E(\tilde{\boldsymbol{u}}_t \tilde{\boldsymbol{u}}_s') = 0, \forall s \neq t$.

The assumption of a constant covariance matrix is the reason for the specification in share form. If the disturbances are added to demand equations in expenditure form, the assumption of a constant covariance matrix is less plausible. It implies that the covariance is the same regardless of the expenditure on a good. In a cross section, as used here, total expenditures vary widely and with them good-specific expenditures. It is not persuasive, that expenditures for one good have the same variance for a high level of total expenditure as for a low level.[43]

According to the adding up restriction, budget shares add up to unity:

$$\sum_{i=1}^{n} w_{it} = \sum_{i=1}^{n} \omega_{it}(\boldsymbol{\beta}, \boldsymbol{s}_t) = 1 \quad . \tag{4.81}$$

Therefore the error terms add to zero ($\sum_{i=1}^{n} u_{it} = 0$) for each t and the covariance matrix $\tilde{\Omega}$ becomes singular.

It is assumed that $\tilde{\boldsymbol{u}}$ has a multivariate normal distribution with mean 0 and covariance matrix $\tilde{\boldsymbol{\Omega}}$ for all t. Due to the singularity of $\tilde{\boldsymbol{\Omega}}$, the density of $\tilde{\boldsymbol{u}}$ may be expressed in terms of the density of any $n-1$ of the goods. So one equation is dropped. Barten (1969) proves that the parameter estimate is irrespective of which equation is dropped. For convenience the n^{th} equation

[43] For an extended discussion of the topic see Pollak and Wales (1992, p.130).

is dropped. Then the vector of disturbances becomes $\boldsymbol{u}_t' = (u_{1t}, \ldots, u_{(n-1)t})$ with the covariance matrix $E(\boldsymbol{u}_t \boldsymbol{u}_t') = \boldsymbol{\Omega}$.

Under these assumptions, the density distribution for $\boldsymbol{u}_t$ is given by:

$$f(\boldsymbol{u}_t) = (2\pi)^{\frac{n-1}{2}} |\boldsymbol{\Omega}|^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}\boldsymbol{u}_t'\boldsymbol{\Omega}^{-1}\boldsymbol{u}_t\right) \tag{4.82}$$

The logarithm of the likelihood function for a sample of T independent observations is given by:

$$L(\boldsymbol{\beta}, \boldsymbol{\Omega}) = -\tfrac{n-1}{2} T \log 2\pi - \tfrac{T}{2} \log |\boldsymbol{\Omega}| - \tfrac{1}{2} \sum_{t=1}^{T} \boldsymbol{u}_t' \boldsymbol{\Omega}^{-1} \boldsymbol{u}_t \tag{4.83}$$

It is convenient to concentrate the likelihood function with respect to $\boldsymbol{\Omega}$:

$$L^*(\boldsymbol{\beta}) = -\tfrac{n-1}{2} T (\log 2\pi + 1) - \tfrac{T}{2} \log |\boldsymbol{S}| \tag{4.84}$$

where $\boldsymbol{S}$ is a $(n-1) \times (n-1)$ matrix with the ij^{th} element given by

$$s_{ij} = \frac{1}{T} \sum_{t=1}^{T} u_{it} u_{jt}, \quad i, j = 1, \ldots, n-1. \tag{4.85}$$

$\boldsymbol{S}$ is the sample covariance matrix of the residuals for the first $n-1$ goods. Maximization of the likelihood function is equivalent to minimizing $\boldsymbol{S}$. Maximization is carried out using a Newton like method and the software package R.

To estimate the QES, for household t of type s_t and with income μ_t, demand equations are written in share form and an error term is added:

$$w_{it} = \frac{p_i m_i^{s_t} b_i}{\mu_t} + \frac{a_i}{\mu_t} \left(\mu_t - \sum_{j=1}^{n} m_j^{s_t} p_j b_j \right) + \\ \left(\frac{p_i m_i^{s_t} c_i}{\mu_t} - \frac{a_i}{\mu_t} \sum_{j=1}^{n} m_j^{s_t} p_j c_j \right) \prod_{j=1}^{n} (m_j^s p_j)^{-2a_j} \left(\mu_t - \sum_{j=1}^{n} m_j^{s_t} p_j b_j \right)^2 + u_{it} \tag{4.86}$$

The n^{th} equation is dropped and the system is estimated using the described maximum likelihood procedure.

4.D Additional Models

All estimated models (*BaC*, *BaCTA* and *GoCTA*) lead to demand systems that are ill-behaved in the theoretical sense of being inconsistent with the Slutsky condition (i.e. having a positive compensated own-price elasticity for tobacco). This is caused by the fact that families with children reduce tobacco consumption despite the reduced relative effective price of tobacco. To show that the problem is limited to tobacco consumption, two more Barten models were estimated with additional restrictions. In model *BaC-nosmoke*, the sample was restricted to non-smoking households. In this model, the number of households is approximately halved to 1871, 966, 1596 and 797 households of types AA, AAC, AACC and AACCC, respectively. The second model (*BaC-restricted*), was estimated with all households included in the sample and the restriction imposed that all compensated own price elasticities be negative. In both models, the restrictions on parameters or the selection of households have no significant influence on estimates, as shown below.

First of all, equivalence scale estimates do not differ significantly between models *BaC-nosmoke*, *BaC-restricted* and the *BaC* reference model, as shown in Table 4.24. Standard errors are higher for model *BaC-nosmoke* because of the smaller number of households in the non-smoking sample.

Household type	AA	AAC	AACC	AACCC
Model *BaC-nosmoke*	1.00	1.13	1.24	1.44
		(0.027)	(0.031)	(0.043)
Model *BaC-restricted*	1.00	1.13	1.22	1.40
		(0.019)	(0.021)	(0.028)
Model *BaC*	1.00	1.13	1.22	1.40
		(0.018)	(0.019)	(0.028)

Table 4.24: *Equivalence scales for models* BaC-nosmoke *and* BaC-restricted*, evaluated at the median income level of the reference household. Values for model* BaC *are given as comparison. Reference household is a childless couple. Standard errors in parentheses are estimated with delta method.*

Naturally, no parameters for tobacco were estimated in model *BaC-nosmoke*, while the outcomes for all model parameters other than those for tobacco are quite similar to model *BaC*; values of the good-specific scale factors are shown in Table 4.25, on the opposite page. Compensated price elasticities are also of approximately the same size, as shown in Table 4.26. Only the values for alcohol are about 40% higher than previous estimates. All in all, model *BaC-nosmoke* confirms the results obtained before.

	AA	AAC	AACC	AACCC
Food	1.00	1.10	1.23	1.42
		(0.023)	(0.023)	(0.032)
Clothing	1.00	1.29	1.47	1.63
Housing	1.00	1.10	1.26	1.46
		(0.030)	(0.028)	(0.043)
Home & furniture	1.00	1.09	1.03	1.30
		(0.078)	(0.102)	(0.129)
Transportation	1.00	1.16	1.18	1.37
		(0.032)	(0.037)	(0.050)
Recreation	1.00	1.47	1.85	2.29
		(0.101)	(0.148)	(0.228)
Personal care	1.00	1.20	1.11	1.08
		(0.039)	(0.042)	(0.066)
Vacation	1.00	0.89	0.91	0.98
		(0.093)	(0.106)	(0.155)
Tobacco	—	—	—	—
Alcohol	1.00	0.77	0.77	0.88
		(0.081)	(0.086)	(0.109)

Table 4.25: *Good-specific scale factors for a Barten model with ten goods and fixed factors for clothing (Model* BaC-nosmoke*). West German households, EVS 1993. Standard errors are given in parentheses.*

Good	AA	AAC	AACC	AACCC
Food	−0.197	−0.177	−0.164	−0.141
Clothing	−0.336	−0.289	−0.270	−0.251
Housing	−0.258	−0.238	−0.219	−0.193
Home & furniture	−0.399	−0.386	−0.403	−0.352
Transportation	−0.312	−0.277	−0.276	−0.242
Recreation	−0.429	−0.351	−0.310	−0.268
Personal care	−0.251	−0.215	−0.229	−0.228
Vacation	−0.382	−0.439	−0.448	−0.446
Tobacco	—	—	—	—
Alcohol	−0.285	−0.341	−0.344	−0.310

Table 4.26: *Model* BaC-nosmoke*: Estimated compensated price elasticities at the median expenditure level for different household types.*

The requirement that all compensated own price elasticities be negative in model *BaC-restricted* is a complex and nonlinear restriction on the parameters and was applied using a barrier function.[44] The constraint affects allmost exclusively the parameters for tobacco consumption. Especially the demand system parameters (a, b and c) change significantly, while the tobacco scale-factors are only slightly reduced with values of 1.01, 0.84 and 0.77 for household types AAC, AACC and AACCC, respectively. All other model parameters are subject only to insignificant changes with the values of most parameters varying by less than one percent. Except for tobacco all own price and income elasticities remain almost unchanged (again less than one percent change). The compensated own price elasticity for tobacco is now just below zero, and the income elasticity at the median income is still negative, but less so with values of -0.447, -0.288, -0.265 and -0.189 for household types AA, AAC, AACC and AACCC, respectively.

The values of estimated model parameters and derived parameters such as elasticities and equivalence scales other than those for tobacco are not affected by the restriction. The model fit, however, is reduced, and significantly so: The likelihood ratio test statistic between the unrestricted model *BaC* and the restricted model *BaC-restricted* has a value of 64.43. Both models are nested, but the restriction does not act directly on any single parameter. The restriction influences mainly the three tobacco parameters. The critical value for three restrictions on the 1% confidence level is only 11.34. The restricted model is rejected against the unrestricted model.

The positive compensated own price elasticity of tobacco is therefore not the result of a weak identification, but it is caused by preference changes among parents or by negative external effects of tobacco consumption that are not well represented in the Barten model. Maybe health effects on children should be included into the effective price of smoking, but this will not be possible without a generalization of the Barten framework that can account for such indirect effects.

[44] A logarithmic barrier is added to enforce the constraint during optimization. The barrier function is chosen so that the objective function should decrease at each iteration that steps over the barrier. See Lange (2001), p. 185ff.

4.E Flow Chart of Chapter 4

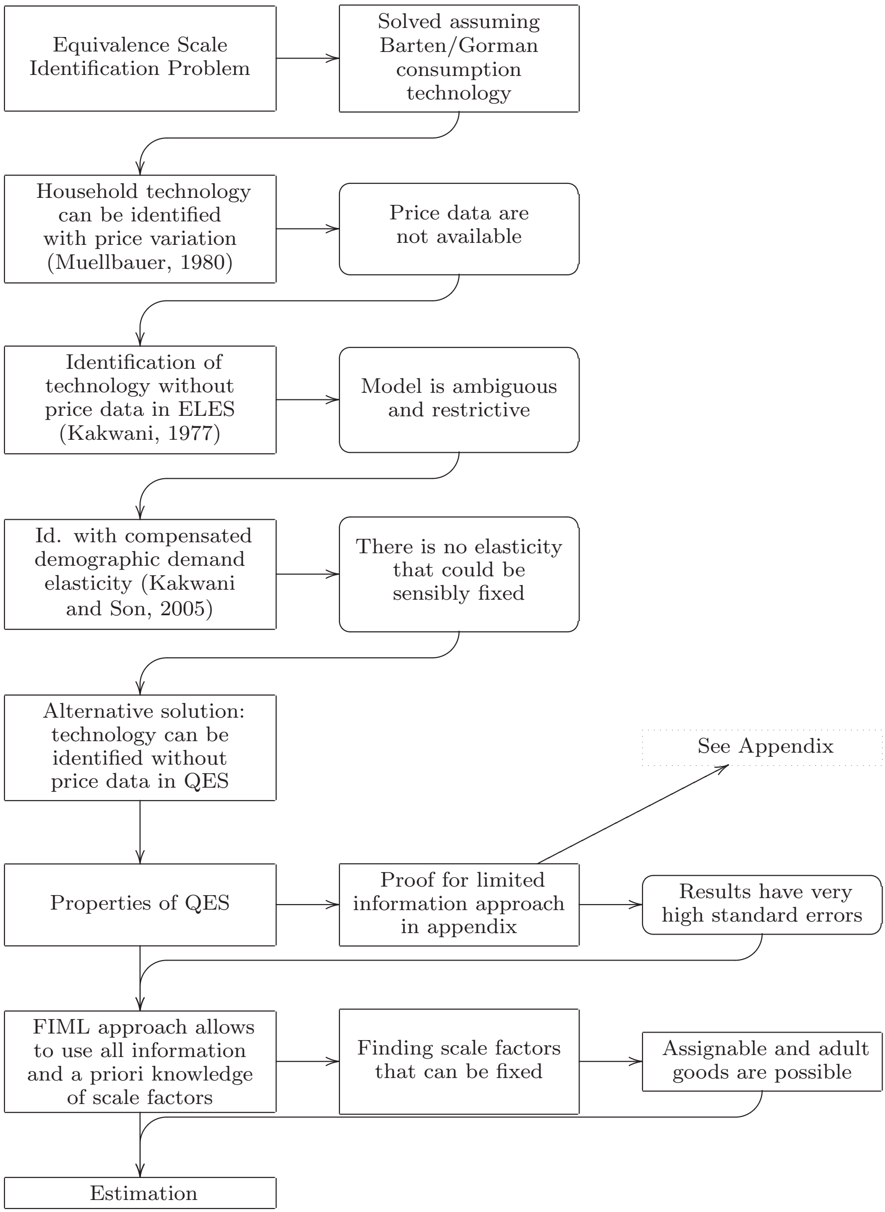

Figure 4.5: *Flow chart chapter 4*