

# Equivalence Scales: Identification and Estimation

A Cross-Sectional Analysis of German Data

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## Abstract

In many applications, in social policy and development policy as well as poverty and inequality analysis, households of different demographic composition have to be compared with respect to income and well-being. The needs and the welfare of a household depend on household income, on the number of household members, and on other demographic characteristics. To compare the income of two households, the effect of the number of persons and of other characteristics has to be accounted for. This can be done with a demographic index, a so called equivalence scale.

The aim of this work is the development of the necessary theory and the estimation of economic equivalence scales which are based on ordinal utility theory. This is not only a problem of applying the right econometric procedures to the data, since economic equivalence scales also suffer from a fundamental identification problem, that must be solved first.

I propose that economic equivalence scales can be identified, if interpersonal comparability of identical reference units (individuals or couples) is assumed and a model of household behaviour is developed that allows to deduce the effect of a change in household demography on the reference unit. This effect can work through redistribution between household members and through a change in virtual prices due to scale effects from joint consumption of household public goods. Three basic models are addressed in this work, each of which achieves the separation and identification of scale effects and redistribution in different ways. To facilitate the analysis, the assessment of childless single and couple households is separated from the assessment of couples with and without children.

For families, separation of the scale effects and redistribution remains incomplete. Two models are analyzed: the Rothbarth model (Chapter 3) focuses on redistribution, but does not account for scale effects, while in the Barten-Gorman model (Chapter 4) redistribution is not independent of scale-effects related changes in virtual prices.

The household model is best developed in the collective framework for childless households (Chapter 5), where actually *two* distinct reference units (a man and a woman) living in one household affect each other via a change in virtual prices and via redistribution. This model also allows for the analysis of distributional effects inside the household, their effects on equivalence scales for men and women, and the distributional differences between married and cohabiting couples.

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