Introduction

Because of the close relations of complex analysis to mathematical physics the theory of boundary value problems in complex analysis is very rich. Since the basic initial considerations of B. Riemann and D. Hilbert a deep theory has been developed mainly in the former Soviet Union by F.D. Gakhov, N.I. Muskhelishvili, I.N. Vekua and their students. The investigations are devoted mainly to analytic functions, particular equations as e.g. the generalized Beltrami equation and to elliptic partial differential equations with analytic coefficients. Two boundary value problem are in the center of interest, the Riemann problem of linear conjugancy and the Riemann-Hilbert boundary value problem. The literature on these problems is enormous and still growing nowadays because of their many connections to other branches of mathematics and to applied problems and due to their structural concepts.

Subjects of this thesis are basic boundary value problems in complex analysis. They are the Schwarz, the Dirichlet, the Neumann and the Robin boundary value problems. The Schwarz problem is the simplest form of the Riemann-Hilbert problem. The Dirichlet problem is related to the Riemann jump problem. The Neumann problem is connected to the Dirichlet problem and the Robin condition is a combination of Dirichlet and Neumann conditions. Recently these boundary value problems have been investigated in regular domains and in particular on the unit disc $\mathbb{D}$ of the complex plane $\mathbb{C}$ for complex model equations also of higher order, see [9], [10], [11]. The aim of these investigations is to develop a unique theory for strongly and non-strongly elliptic equations of arbitrary order. Particular non-regular domains are half planes.

In this thesis the upper half plane $\mathbb{H}$ is consisted with the listed four basic boundary value problems studied as well for the homogeneous as for the inhomogeneous Cauchy-Riemann equation. Some second order equations as the Bitsadze and the Poisson equations are considered. Moreover, the inhomogeneous polyanalytic and the inhomogeneous polyharmonic equation are treated where higher order Cauchy-

Pompeiu representations are given and a higher order Dirichlet problem is solved for the polyharmonic equation.

While the Schwarz problem is uniquely solvable the Dirichlet problem is not in general. As turns out the Dirichet problem is unconditionally solvable only in the case of the second order Poisson equation.

As a basic aid for the Dirichlet problem a polyharmonic Green function is developed for the upper half plane $\mathbb{H}$, a counter part for the one known for the unit dick $\mathbb{D}$. It not only serves to solve the respective Dirichlet problem, but also leads to some orthogonal decompositions of functions by modifying the respective Cauchy-Pompeiu representation formulas of higher order.

The only basic tools for this work are the Gauss divergence theorem for regular domains in the complex plane, the properties of the Pompeiu operator and the Poisson kernel function.