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Abstract

A train timetable is a crucial component of railway transportation systems as it directly impacts the system's performance and the customer satisfaction. Various approaches can be found in the literature that deal with timetable generation. However, the approaches proposed in the literature differ significantly in terms of the use case for which they are intended. Differences in objective function, timetable periodicity, and solution methods have led to a confusing number of works on this topic. Therefore, this paper presents a compact literature review of approaches to train timetable generation. The reviewed papers are briefly summarized and categorized by objective function and periodicity. Special emphasis is given to approaches that have been applied to real-world railway data.

1 The Railway Planning Process

Railway planning consists of a series of steps that rail operators follow to plan, operate and maintain railway systems. The planning process is a highly complex task that involves various stakeholders and requires a thorough understanding of all aspects of a railway transportation system. According to Bussieck et al. (1997), railway operators typically structure this process hierarchically in order to effectively manage coordination and collaboration between all stakeholders, as the different steps of the process build on one another. The process begins with planning the infrastructure network, including stations and tracks. Next, lines connecting the stations and their frequencies are defined, followed by determining a detailed schedule for trains and assigning rolling stock.

Mathematical optimization approaches are often used to model and solve the underlying problems in each step, ensuring that the optimal solution is found. However, as pointed out by Dauzère-Pérès et al. (2015), it is typically not possible to solve all steps in one integrated model due to the high complexity of large railway networks. Therefore, the individual models are usually solved sequentially, which reduces complexity.

According to Huisman et al. (2005), railway planning problems can be classified by the planning horizon (strategic, tactical, and operational) or by the physical location of the problems. In this paper, the classification is done by planning horizon, as also discussed in Lusby et al. (2011) and shown in figure 1. The planning process presented by Lusby et al. (2011) is divided into three levels: the strategic level, which deals with long term decision such as infrastructure planning and line planing within the network. The tactical level, which focuses on resource allocation, including timetable generation and scheduling rolling stock and crews. The operational level, which deals with daily problems that need to be handled in
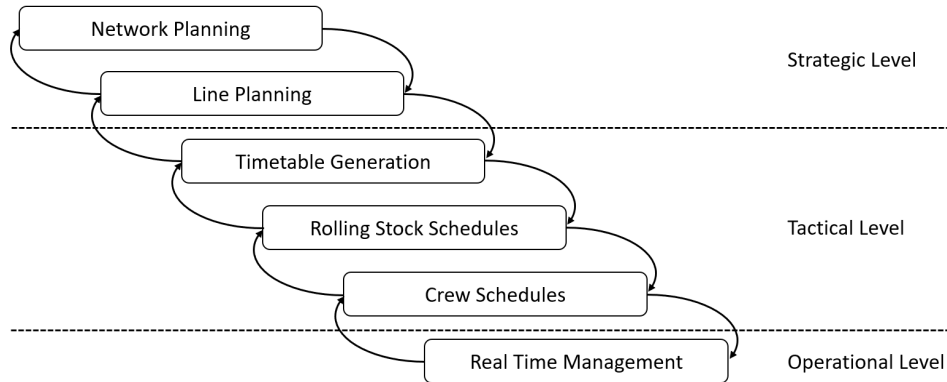
Figure 1: The Railway Planning Process (Lusby et al. (2011))

real-time, such as disruption management. As figure 1 illustrates, each step builds upon the results of the previous step, requiring the process to be followed hierarchically. However, it is also possible that results from one step may require adjustments in the previous step, creating a two-way hierarchy. For more detailed information on the entire process, see Lusby et al. (2011).

Each of the problems described in the planning process can be solved using heuristic or optimal solution approaches. Various approaches with different focuses are presented and discussed in the literature. In particular, the Train Timetabling Problem (TTP) is widely discussed, since there are many different use cases and objectives for this problem, and a large number of possible methods to solve it. This work aims to provide a comprehensive overview of existing approaches for generating train timetables, with a focus on methods applied to real-world train data. The reviewed papers are categorized based on their objective function.

2 The Train Timetabling Problem

The Train Timetabling Problem is a complex optimization problem that involves determining a schedule for trains that operates efficiently and effectively while taking into account various constraints. The problem involves determining the departure and arrival times of trains at each station, as well as the assignment of trains to tracks, while considering factors such as track capacity, rolling stock and crew availability, and travel time between stations (Arenas (2014)). The combination of these factors makes the TTP a highly complex problem that is challenging to solve and, as proven by Caprara et al. (2006), it is NP-hard. However, the TTP is a crucial component of railway planning as it directly impacts the transportation system's performance and customer satisfaction and therefore receives much attention in the literature.

A good timetable typically needs to satisfy three main objectives, which may sometimes be in conflict with each other (Lusby et al. (2011)). One goal is to maximize profits for the railway operator by reducing operating costs. Another objective is to maximize the timetable's quality by increasing passenger satisfaction, which can be done by pro-

viding more direct connections, reducing waiting times, and shortening train travel times. Timetable robustness is also an important objective, which aims to decrease the emergence of secondary delays and delay propagation (D'Ariano et al (2007)). A typical way to increase the robustness of a timetable is to include additional time slots, called buffer times, into the timetable which can be used to mitigate delays (Cacchiani and Toth (2018)).

The periodicity of a timetable must be chosen based on the specific use case. Timetables can be either periodic or aperiodic. A periodic timetable is one in which trains operate on a regular, recurring schedule, with the same times and frequencies of trains at each station. An example of this would be a train that departs from a station every hour on the hour. An aperiodic timetable is one in which trains do not operate on a regular, recurring schedule. For instance, a train that departs from a station at random times without a set pattern would be considered to have an aperiodic timetable. The main difference between the two is that in periodic timetable the schedule repeats in a specific period, while in aperiodic timetable the schedule does not repeat in any specific period. An aperiodic timetable is more flexible and adaptable to changing demand and other factors, but it is harder to memorize and therefore less preferred by customers (Goerigk (2014)). However, it is more flexible and less expensive to maintain as it can be adapted to low demand situations outside of peak hours. It is commonly used in passenger rail transport (Lusby et al. (2011)).

A specific solution approach for the periodic timetabling problem is the Periodic Event Scheduling Problem (PESP), which was first introduced by Serafini and Ukovich (1989). PESP is a mathematical problem that arises in the context of scheduling events that occur on a regular, recurring basis. It is frequently used in scheduling public transportation, such as trains or buses, where trains or buses operate on a regular, recurring schedule. The problem involves determining the optimal schedule for a set of events subject to a set of constraints, such as resource availability (e.g., tracks or buses) and the need to minimize the overall schedule cost. Formally, it is a problem of allocating resources to tasks over a period of time, where the tasks recur with a specific period and may have different deadlines, durations, and priorities. The objective is to minimize total cost, which can be defined as the sum of resource usage costs, tardiness penalties, or other metrics. PESP is often solved using mathematical optimization techniques, such as mixed-integer programming, to find the optimal schedule that satisfies all constraints while minimizing costs. Many approaches have been proposed to solve the PESP, but there is still room for improvement in terms of performance and scalability.

In the following section, the reviewed literature is categorized according to which of the three objectives it aims for. Special emphasis is also given on the timetable periodicity for which the solution approach is intended and the data set used to apply the approach.

3 Literature Review of Solution Approaches for Train Timetabling

3.1 Approaches to Maximize the Profit of the Railway Operator

The following timetabling approaches aim to increase the profit of the railway operator. However, they achieve this in different ways. For example, by reducing the number of trains for a timetable or by using infrastructure capacity efficiently. Others simply try to find a feasible timetable in an efficient way to save costs and effort in creating it. The approaches also differ in terms of their applicability as they are designed for either periodic or non-periodic timetables.

Periodic Timetables

One of the most important models for creating periodic timetables, is the Periodic Event Scheduling Problem (PESP). The PESP was first introduced by Serafini and Ukovich (1989). Their paper proposes a mathematical model for scheduling periodic activities, including a model for scheduling periodic events with specific time constraints. They also show that the problem is NP-complete. In their paper, a general framework is proposed to deal with a large class of scheduling problems in a periodic environment, including periodic event scheduling and resource management. Specific applications such as a periodic version of the Job-Shop Scheduling problem and scheduling vehicles are also considered. Their work has lead to a variety of timetabling approaches which use adapted or improved variants of the PESP.

One PESP based approach is presented by Odijk (1996). The authors discuss a method for creating train schedules, which involves adding periodic time window constraints to the PESP. These constraints allow for arrival and departure times to be related to each other based on a clock, rather than a linear time axis. The paper introduces a new algorithm that uses constraint generation to solve the problem, and provides an example of how it works in a real-life scenario. The results show that the algorithm is effective for problem instances of moderate size.

In another work, Zimmermann and Lindner (2003) focus on creating cost-optimal periodic train schedules. They analyze existing approaches for solving the PESP and improve and extend those algorithms in order to achieve a better performance for real-world instances.

The paper from Kroon and Peeters (2003) describes an extension of an existing PESP based mathematical model for railway timetabling, which includes the ability to handle variable trip times for trains. Unlike existing models, their approach allows for small deviations from fixed trip times, making it useful in cases where a feasible solution does not exist in the original model.

Besides PESP based approaches, other works suggest models based on integer programming. For example, in Caprara et al. (2002) the authors present an approach to determine a periodic timetable for important lines, called corridors, in railway networks. A corridor is a single track between two major stations with different intermediate stations in between. The idea is, to first determine the timetable for the few important lines in a network and then, the timetable for the remaining lines is relatively easy to find. The authors define an ideal timetable for every train, which represents the most desired timetable for each train. Those ideal timetables need to be adapted to satisfy all constraints in the network. The objective is to maximize the profit by minimizing the difference from each train's ideal timetable. The problem is formulated using a directed multigraph and solved using integer linear programming and lagrange relaxation. The approach is applied to real world instances from the Italian railway company.

The previous paper is extended in Caprara et al. (2006) by adding additional constraints for example manual block signaling, station capacities or maintenance operations. Compared to the previous approach, this approach can be modified to be applied in real time management. The model is again applied to data from the Italian railway company.

Another integer programming approach is discussed in Borndörfer et al. (2005). The authors propose an approach for conducting an auction of railway slots. The auction design is based on an iterative combinatorial auction, but with certain restrictions on the types of slot bundles that can be bid on. The authors present an integer programming method for

determining the winners of the auction. They present computational results from simulations of the auction in a specific area of the German railway network, which suggest that the auction can lead to more efficient use of railway capacity.

In Borndörfer and Schlechte (2008) the authors propose a new integer programming formulation for the train timetabling problem. Their approach uses additional variables called configuration variables, and shows that the relaxed linear programming version of the formulation can be solved efficiently. They also demonstrate the approach on a specific area of the German railway network.

The previous paper is extended in Borndörfer and Schlechte (2007) and the authors compare two types of integer programming formulations for the train timetabling problem: one using packing constraints to model block conflicts and another using additional configuration variables. They show that both formulations can be solved efficiently.

In a more recent paper from Liu and Han (1017), another approach based on integer programming and time expanded graphs is discussed for train scheduling on a railway line. The goal is to minimize the total dwell time and deviation of departure time from the earliest departure time at the origin station while keeping the running time fixed. The authors propose an integer linear programming model and a branch-and-price algorithm to solve the problem. They also conduct a case study on a specific section of a Chinese railway line and found that considering different headway times can lead to more efficient and feasible train schedules.

Non-Periodic Timetables

A Column Generation algorithm for solving the train timetabling problem for both periodic and non-periodic schedules on a single track is proposed in Cacchiani et al. (2008). The algorithm is based on solving the relaxed linear programming version of an integer linear programming formulation, in which each variable corresponds to a full timetable for a train, rather than previous approaches that used variables corresponding to specific train departures and arrivals at specific times. The proposed methods are applied to real world data from the Italian railway company and are able to solve small instances optimally.

An approach specifically developed for non-periodic timetables is for example presented in Brännlund et al. (1998). The authors developed an integer program to create a profit-maximizing train timetable. They used a Lagrangian relaxation approach to separate the problem into one dynamic program for each physical train. The approach has been tested on a real data provided by the Swedish National Railway Administration, which includes scheduling 18 passenger trains and 8 freight trains on a single track. The computation time for this example is relatively fast and the resulting timetable is close to optimal.

Jiang et al. (2017) in their study, proposed a method for optimizing non-periodic train scheduling by allowing for additional stops or skipping stops on a double-track line, and considering deceleration and acceleration times. They improved an existing heuristic method to account for these new factors and tested it on a real-world instance. Results showed that the proposed method can effectively improve the current timetable.

3.2 Approaches to Maximize the Quality of the Timetable

The following approaches aim to increase the timetable quality. Therefore, the objective functions mostly focus on minimizing passenger waiting times or minimizing passenger and/or train travel times. As before, the approaches can either be applied for periodic or

non-periodic timetables.

Periodic Timetables

Based on the PESP, Liebchen (2003) proposes a new method for formulating instances of the cyclic timetabling problem in public transportation companies. It introduces the concept of integral cycle bases, which are a more general class of directed cycle bases that enable the modeling of cyclic timetabling problems. The paper also presents algorithms for constructing integral cycle bases of small width, which results in notable reductions of running times for cyclic timetabling. The method is applied to data from long distance train in Germany and the Berlin subway.

In Liebchen (2008), the author discusses the use of discrete optimization in the computation of the timetable for the Berlin subway network. The optimized timetable offers improvements for both passenger waiting times and the operating efficiency of the subway operator.

In Zhang et al. (2019), the authors describe how an existing model for the PESP is transformed into a multi-commodity network flow model with two coupled schedule networks and side track capacity constraints. The goal is to solve a cyclic train timetabling problem, which aims to synchronize limited operational resources towards a master periodic schedule of transport services. The paper demonstrates the effectiveness of the proposed reformulation using real-world examples from a Chinese high-speed railway corridor.

Other approaches from Nachtigall and Voget (1996) or Arenas (2014) use genetic algorithms to find timetables. In Nachtigall and Voget (1996), the authors discuss a method for creating periodic train timetables that minimize waiting times for passengers. They suggest using a combination of manual timetabling techniques and genetic algorithms for networks that are too large for deterministic optimization methods. They introduce a new algorithm, called local improvement procedure, which combines elements of a greedy algorithm, local optimization, and genetic algorithm. They test this approach on real-world data and find that it is effective in solving the problem at hand.

Arenas (2014) present a constraint-based model and a genetic algorithm to generate feasible periodic train timetables, and they also present case studies that were solved using the algorithm. The article describes the application of the presented approach on two case studies, showing the use of the algorithm on subsets of the Dutch and French rail networks. Their solution is efficient and flexible and can be easily modified to add, remove or modify constraints to satisfy other criteria such as reducing waiting time for passengers.

A number of other approaches use linear programming models. Vansteenwegen and Van Oudheusden (2006) are discussing a new approach to formulating instances of the cyclic timetabling problem in public transportation companies, which aims to reduce waiting times for passengers. The approach involves calculating ideal buffer times for each transfer and using them in a linear program that minimizes a generalized waiting cost function. The approach is shown to produce good results in a small part of the Belgian railway network.

The paper of Heydar et al. (2013) is about investigating the capacity of a single track rail line that follows a cyclic timetable, using the work of Bergmann (1975) as a starting point. The authors restate the problem using improved notation, modify the mathematical model by adding a second objective and removing unnecessary variables, and perform the first numerical analysis of this problem using randomly generated instances. The objective is to minimize the length of the dispatching cycle, and the total dwell time of the local trains at all stations combined.

Corman et al. (2017) discuss the problem of determining real-time measures to reduce disturbances in railway systems. They integrate train scheduling models that include all conditions relevant to efficient operations from the viewpoint of operations managers and delay management models that focus on the impact of rescheduling decisions on the quality of service for passengers. The authors present a mixed integer linear program that is able to be solved optimally by a commercial solver for small instances of the dutch railway network. The paper also describes four heuristic procedures that are developed to find good solutions for large and complex instances when the commercial solver fails to find a feasible solution.

The work of Zhou and Zhong (2007) is about solving the train timetabling problem by minimizing the total train travel time. It presents a method that uses a branching scheme to eliminate train conflicts by adding precedence relations between trains, and solves the resulting subproblems as longest path problems. The paper also presents a Lagrangian relaxation based lower bound rule and beam search heuristic algorithm to provide initial tighter upper bounds. The case study demonstrates that the beam search algorithm offers superior performance with limited computing efforts.

Non-Periodic Timetables
A hybrid approach for timetabling using a simulated annealing heuristic is proposed in Robenek et al (2017). The authors combine the regularity of cyclic timetables with the flexibility of non-cyclic ones and evaluate the results based on passenger satisfaction. The performance of the timetable was assessed on the Israeli railway network.

A genetic algorithm is presented in Xu et al. (2014). The authors also developed an improved simulation model for scheduling trains, called the GA-ITAS method. The method aims to find an optimal balanced schedule with the least delay-ratio by considering the impacts of train velocity. The research uses several indices to evaluate the performance of the generated schedule and numerical experiments were implemented on a Chinese rail line with 17 stations.

The paper of Yuhua et al. (2022) discusses a study on the integration of train timetabling, train service connection, rolling stock assignment, and passenger boarding choices in urban rail transportation. The study aims to minimize total passenger waiting time and operating cost while taking into consideration time-based origin-destination dependent passenger demand and different types of rolling stock. The study proposes a mixed-integer linear programming model for the integrated problem and uses an iterative programming approach for solving the problem. The approach is tested on data from a Chinese railway line, and the results show that it is efficient and effective and compared to other optimization approaches it has advantages both on computation time and solution quality.

3.3 Approaches to Maximize the Robustness of the Timetable

The approaches presented in this chapter aim to maximize timetable robustness in order to prevent delay propagation throughout the network. The papers are again categorized into methods for periodic timetables and methods for non-periodic timetables.

Periodic Timetables
Based on the PESP, Kroon et al. (2007) develop a stochastic optimization model to improve periodic timetables by allocating buffer times. The model uses a combination of a

timetabling model and a simulation model, and aims to minimize the total average delay. The results of the study indicate that by re-allocating buffer times, a significant reduction in average delay can be achieved. The study was conducted on a Dutch railway corridor, and the average delay was reduced by about 30%. A similar approach is presented in Kroon et al. (2008). The approach in this paper results in timetable that is maximal robust against stochastic disturbances.

Another approach to find robust timetables for the PESP is addressed in Goerigk (2014). The authors present a recovery approach based on integer programming and a bicriteria local search algorithm for larger instances. They analyze the trade-off between costs and robustness using German railway data and the results suggest, that both objectives can be balanced.

Other works maximize timetable robustness using Branch and Bound algorithms. D'Ariano et al (2007) study the problem of generating a new conflict-free timetable for the real-time management, when the initial timetable is disrupted. The problem is modeled as a job shop scheduling problem and a branch and bound algorithm is used to efficiently find feasible solutions. The algorithm was tested on an area of the Dutch railway network and was able to find near-optimal solutions quickly and efficiently.

Also Branch and Bound based is the heuristic of Maróti and Gábor (2017). It solves the stochastic program proposed by Kroon et al. (2008) in a much shorter computation time using specific node and variable selection rules. However, the faster computational time results in a considerable optimality gap.

In Liebchen et al. (2010) the authors propose a new technique for computing delay-resistant periodic timetables by combining approaches from timetabling and delay management. Their example of real-world data from Germany shows that the approach can significantly reduce passenger delays while only slightly increasing passenger travel times.

In Bešinović et al. (2016), an integrated approach is presented to first determine a feasible periodic timetable and then improve the robustness of this timetable. The trade-off between minimal travel times and maximal robustness is optimized using an integer linear programming formulation. The framework is tested in a part of the Dutch railway network and it shows that it is able to generate a robust timetable. The framework can also be applied for evaluating the robustness of existing timetables.

Non-Periodic Timetables

The paper of Cacchiani et al. (2020) discusses a scheduling problem for trains, where the goal is to determine the departure and arrival times at stations, as well as the stopping patterns for trains, while taking into account uncertain passenger demand. The paper proposes mixed integer linear programming models to derive robust solutions for scheduling, which are based on the technique of Light Robustness. This technique handles uncertainty by inserting a protection against increased passenger demand.

In Fischetti et al. (2009), the authors propose different methods to improve the robustness of a given non-periodic timetable using linear and stochastic programming. The effectiveness of these methods is tested on real-world test cases from the Italian railway company, and the results show that the proposed techniques provide robust solutions in short computation time.

A bicriteria optimization approach for non-periodic robust timetabling is presented in Schöbel and Kratz (2009). The article discusses the trade-off between achieving the best possible solution and a more robust solution by including the robustness of a timetable as

an additional objective function.

The paper of Cachiani et al. (2012) proposes a framework that allows the user to chose a solution for a non-periodic timetable based on the trade-off between robustness and efficiency. To determine the timetable, a lagrangian heuristic is applied with adjustable parameters for the importance of the robustness. The results on an Italian railway corridor suggest that the framework generates high quality solutions in short computation time.

3.4 Categorization of the Reviewed Literature

The preceding summary of the reviewed papers has shown that there are four main distinguishing characteristics that can be used to categorize the papers: The objective function, the underlying solution method, the cycle of periodicity, and the data basis. The objective functions can be very different, as there a different possible objectives for different use cases. However, it is possible to assign them all to one of the three categories presented: maximizing profit, maximizing timetable quality or maximizing robustness. The underlying solution methods can also vary considerably, especially since some approaches are based on exact methods and others on heuristic approaches. The most commonly used approaches in the literature are based on Linear (Integer) Programming in general, PESP, Column Generation, Genetic Algorithms or Branch and Bound. Regarding the periodicity, most of the papers focus on periodic timetables. However, some papers also focus an aperiodic or on hybrid timetables. Nearly all the approaches discussed have been applied to real-world data from various railway operators. However, some authors use only one or a few lines while others use different sized parts of the entire railway network.

In the following table 1, all reviewed papers are sorted by the year of publication. The table also shows the objective function and the main solution method. The Cycle column represents the periodicity of the timetable for which the approach can be applied. The last two columns show the data base used to evaluate the approach and whether the approach is applicable for entire train networks or for train lines.

PAPER	OBJECTIVE	METHOD	CYCLE	DATA	INSTANCE
Serafini and Ukovich (1989)	Minimize number of trains	PESP	Periodic	n.a.	n.a.
Odijk (1996)	Feasible timetable	PESP with time window constraints	Periodic	Dutch railway	Single station with 3 lines
Nachtigall and Voget (1996)	Minimal waiting time for passengers changing trains	Genetic algorithm	Periodic	German railway	Network
Brännlund et al. (1998)	Maximize profit	Integer programming and lagrange relaxation	Non-periodic	Swedish railway	Single track
Caprara et al. (2002)	Minimize the difference from each train's ideal timetable	Integer programming and lagrange relaxation	Periodic	Italian railway	A single one-way track

Zimmermann and Lindner (2003)	Minimize total cost	Several algorithms based on PESP	Periodic	German railway	Network
Liebchen (2003)	Minimize passenger waiting time and the number of vehicles to operate the timetable	Integral cycle bases based on PESP	Periodic	German railway and Berlin subway	Different pairs of single lines
Kroon and Peeters (2003)	Feasible timetable	Extension of PESP	Periodic	Dutch railway	Network
Borndörfer et al. (2005)	Maximize the network proceeds	Auction approach based on integer programming	Periodic	German railway	Network
Caprara et al. (2006)	Minimize the difference from each train's ideal timetable	Lagrangian heuristic	Periodic	Italian railway	Different single tracks
Vansteenwegen and Van Oudheusden (2006)	Improve passenger service	Linear programming	Periodic	Belgian railway	Network
Borndörfer and Schlechte (2007)	Maximize total number of trains	Integer programming and LP-Relaxation	Periodic	German railway	Network
D'Ariano et al (2007)	Minimize the maximum secondary delay for all trains	Branch and Bound	Periodic	Dutch railway	Network
Zhou and Zhong (2007)	Minimize total train travel time	Branch and Bound	Periodic	Chinese railway	Single track
Kroon et al. (2007)	Minimize average train delay	Stochastic optimization variant of PESP	Periodic	Dutch railway	Single line and network
Kroon et al. (2008)	Maximize robustness	PESP based stochastic optimization model	Periodic	Dutch railway	Network

Cacchiani et al. (2008)	Minimize the difference from each train's ideal timetable	Column Generation	Hybrid	Italian railway	Different corridors
Borndörfer and Schlechte (2008)	Maximize the network proceeds	Integer programming and LP-Relaxation	Periodic	German railway	Network
Liebchen (2008)	Reduce passenger waiting times	PESP	Periodic	Berlin subway	Network
Schöbel and Kratz (2009)	Robust optimization	Bicriteria optimization	Non-periodic	n.a.	Network
Fischetti et al. (2009)	Improve robustness	Linear and stochastic programming	Non-periodic	Italian railway	Single line
Liebchen et al. (2010)	Increase delay resistance	Branch and Prize	Periodic	German railway	Network
Cachiani et al. (2012)	Improve robustness and efficiency	Lagrange optimization	Non-periodic	Italian railway	Corridor
Heydar et al. (2013)	Minimize dwell time	Mixed integer programming	Periodic	Created instances	Single track
Xu et al. (2014)	Balanced timetable with low delay-ratio	Genetic algorithm	Non-periodic	Chinese railway	Single line
Arenas (2014)	Minimize constraint violations	Genetic algorithm	Periodic	Dutch and French railway	Network
Goerigk (2014)	Increase robustness	Bicriteria algorithm based on PESP	Periodic	German railway	Network
Bešinović et al. (2016)	Maximize robustness	Integer programming and monte carlo simulation	Periodic	Dutch railway	Network
Maróti and Gábor (2017)	Increase robustness	Branch and Bound	Periodic	Dutch railway	Network
Jiang et al. (2017)	Increase the number of scheduled trains	Lagrangian based heuristic	Non-periodic	Chinese railway	Single line

Corman et al. (2017)	Minimize total passenger travel time	Mixed integer programming and heuristics	Periodic	Dutch railway	Network
Liu and Han (1017)	Minimizes the weighting sum of total dwell time and deviation of the earliest departure time	Integer programming and time expanded graph	Periodic	Chinese railway	Double track line
Robenek et al (2017)	Maximize passenger satisfaction	Simulated annealing	Hybrid	Israeli railway	Network
Zhang et al. (2019)	Minimize journey time	Integer programming and time space network based on PESP	Periodic	Chinese railway	Double track corridor
Cacchiani et al. (2020)	Improve robustness	Mixed integer program	Hybrid	Chinese railway	Single line
Yuhua et al. (2022)	Minimize waiting time for passengers and costs for operators	Mixed integer program	Non-periodic	Chinese railway	Single line

Table 1: Overview of the Reviewed Literature on Train Timetable Generation

4 Conclusion

This paper presents a compact literature review on approaches for train timetable generation. The different papers are grouped into three categories based on their objective function. The categories are profit maximization, quality maximization and robustness maximization. In total, 36 papers are reviewed and briefly summarized. All papers are compared in a tabular overview according to their objective function, the main solution method, and the periodicity of the timetable for which the approach can be applied. Special emphasis is given on papers that apply and evaluate their approach to real-world data. Therefore, the data basis and instance used in each paper is highlighted as well.

The results show that there are a variety of objective functions and solution methods for the TTP. Nevertheless, it is still possible to classify the different objectives into one of the three main categories identified. The presented tabular overview provides a comprehensive comparison of the main contributions in the literature on generating train timetables. In summary, this paper gives an introduction to the topic and the current state of research.

References

Arenas, D., Chevirer, R., Hanafi, S., & Rodriguez, J. (2014). Solving the periodic timetabling problem using a genetic algorithm. arXiv. Retrieved from https://arxiv.org/abs/1411.6998 doi: 10.48550/ARXIV.1411.6998

Bešinović, N., Goverde, R. M., Quaglietta, E., & Roberti, R. (2016). An integrated micro–macro approach to robust railway timetabling. Transportation Research Part B: Methodological, 87, 14-32. Retrieved from https://www.sciencedirect.com/science/article/pii/S0191261516000291 doi: https://doi.org/10.1016/j.trb.2016.02.004

Bergmann, D. R. (1975). Integer programming formulation for deriving minimum dispatch intervals on a guideway accommodating through and local public transportation services. Transportation Planning and Technology, 3 (1), 27-30. Retrieved from https://doi.org/10.1080/03081067508717088 doi: 10.1080/03081067508717088

Borndörfer, R., Grötschel, M., Lukac, S., Mitusch, K., Schlechte, T., Schultz, S., & Tanner, A. (2006). An auction approach to railway slot allocation. Competition and Regulation in Network Industries, 7 , 163-197.

Borndörfer, R., & Schlechte, T. (2007). Models for railway track allocation. In C. Liebchen, R. Ahuja, & J. Mesa (Eds.), Atmos 2007 - 7th workshop on algorithmic approaches for transportation modeling, optimization, and systems (Vol. 07001).

Borndörfer, R., & Schlechte, T. (2008). Solving railway track allocation problems. In J. Kalcsics & S. Nickel (Eds.), Operations research proceedings 2007 (pp. 117–122). Berlin, Heidelberg: Springer Berlin Heidelberg.

Brännlund, U., Lindberg, P. O., Nõu, A., & Nilsson, J.-E. (1998). Railway timetabling using lagrangian relaxation. Transportation Science, 32 , 358-369. doi: 10.1287/trsc.32.4.358

Bussieck, M. R., Winter, T., & Zimmermann, U. T. (1997). Discrete optimization in public rail transport. Mathematical Programming, 79 , 415-444.

Cacchiani, V., Caprara, A., & Fischetti, M. (2012). A lagrangian heuristic for robustness, with an application to train timetabling. Transportation Science, 46 (1), 124–133. doi: https://doi.org/10.1287/trsc.1110.0378

Cacchiani, V., Caprara, A., & Toth, P. (2008, 06). A column generation approach to train timetabling on a corridor. 4OR, 6, 125-142. doi: 10.1007/s10288-007-0037-5

Cacchiani, V., Qi, J., & Yang, L. (2020). Robust optimization models for integrated train stop planning and timetabling with passenger demand uncertainty. Transportation Research Part B: Methodological, 136, 1-29. doi: https://doi.org/10.1016/j.trb.2020.03.009

Cacchiani, V., & Toth, P. (2018). Robust train timetabling. In R. Borndörfer, T. Klug, L. Lamorgese, C. Mannino, M. Reuther, & T. Schlechte (Eds.), Handbook of optimization in the railway industry (pp. 93–115). Cham: Springer International Publishing. doi: 10.1007/978-3-319-72153-8_5

Caprara, A., Fischetti, M., & Toth, P. (2002). Modeling and solving the train timetabling problem. Operations Research, 50 , 851-861. doi: 10.1287/opre.50.5.851.362

Caprara, A., Monaci, M., Toth, P., & Guida, P. L. (2006). A lagrangian heuristic algorithm for a real-world train timetabling problem. Discrete Applied Mathematics, 154 (5), 738-753. doi: https://doi.org/10.1016/j.dam.2005.05.026

Corman, F., D'Ariano, A., Marra, A. D., Pacciarelli, D., & Samà, M. (2017). Integrating train scheduling and delay management in real-time railway traffic control. Transportation Research Part E: Logistics and Transportation Review, 105, 213-239. doi:

https://doi.org/10.1016/j.tre.2016.04.007

D'Ariano, A., Pacciarelli, D., & Pranzo, M. (2007). A branch and bound algorithm for scheduling trains in a railway network. European Journal of Operational Research, 183 (2), 643-657. doi: https://doi.org/10.1016/j.ejor.2006.10.034

Dauzère-Pérès, S., De Almeida, D., Guyon, O., & Benhizia, F. (2015). A lagrangian heuristic framework for a real-life integrated planning problem of railway transportation resources. Transportation Research Part B: Methodological, 74, 138-150. doi: https://doi.org/10.1016/j.trb.2015.01.008

Dollevoet, T., Corman, F., D'Ariano, A., & Huisman, D. (2014). An iterative optimization framework for delay management and train scheduling. Flexible Services and Manufacturing Journal, 26, 490-515. doi: https://doi.org/10.1007/s10696-013-9187-2

Fischetti, M., Salvagnin, D., & Zanette, A. (2009). Fast approaches to improve the robustness of a railway timetable. Transportation Science, 43 (3), 321-335. doi: https://doi.org/10.1287/trsc.1090.0264

Goerigk, M. (2015). Exact and heuristic approaches to the robust periodic event scheduling problem. Public Transport, 7, 101-119. doi: https://doi.org/10.1007/s12469-014-0100-5

Heydar, M., Petering, M. E., & Bergmann, D. R. (2013). Mixed integer programming for minimizing the period of a cyclic railway timetable for a single track with two train types. Computers & Industrial Engineering, 66 (1), 171-185. doi: https://doi.org/10.1016/j.cie.2013.06.003

Huisman, D., Kroon, L., Lentink, R., & Vromans, M. (2005). Operations research in passenger railway transportation. Statistica Neerlandica, 59, 467-497. doi: 10.1111/j.1467-9574.2005.00303.x

Jiang, F., Cacchiani, V., & Toth, P. (2017). Train timetabling by skip-stop planning in highly congested lines. Transportation Research Part B: Methodological, 104, 149-174. doi: https://doi.org/10.1016/j.trb.2017.06.018

Kroon, L., Maróti, G., Helmrich, M. R., Vromans, M., & Dekker, R. (2008). Stochastic improvement of cyclic railway timetables. Transportation Research Part B: Methodological, 42 (6), 553-570. doi: https://doi.org/10.1016/j.trb.2007.11.002

Kroon, L., & Peeters, L. (2003). A variable trip time model for cyclic railway timetabling. Transportation Science, 37, 198-212. doi: 10.1287/trsc.37.2.198.15247

Kroon, L. G., Dekker, R., & Vromans, M. J. C. M. (2007). Cyclic railway timetabling: A stochastic optimization approach. In F. Geraets, L. Kroon, A. Schoebel, D. Wagner, & C. D. Zaroliagis (Eds.), Algorithmic methods for railway optimization (pp. 41–66). Berlin, Heidelberg: Springer Berlin Heidelberg.

Liebchen, C. (2003). Finding short integral cycle bases for cyclic timetabling. In G. Di Battista & U. Zwick (Eds.), Algorithms - esa 2003 (pp. 715–726). Berlin, Heidelberg: Springer Berlin Heidelberg.

Liebchen, C. (2008). The first optimized railway timetable in practice. Transportation Science, 42 (4), 420–435. doi: 10.1287/trsc.1080.0240

Liebchen, C., Schachtebeck, M., Schöbel, A., Stiller, S., & Prigge, A. (2010). Computing delay resistant railway timetables. Computers & Operations Research, 37 (5), 857-868. doi: https://doi.org/10.1016/j.cor.2009.03.022

Liu, P., & Han, B. (2017). Optimizing the train timetable with consideration of different kinds of headway time. Journal of Algorithms & Computational Technology, 11 (2), 148-162. doi: 10.1177/1748301816689685

Lusby, R., Larsen, J., Ehrgott, M., & Ryan, D. (2011, 10). Railway track allocation: Models

and methods. OR Spectrum, 33 , 843-883. doi: 10.1007/s00291-009-0189-0

Maróti, G. (2017). A branch-and-bound approach for robust railway timetabling. Public Transport, 9 (1), 73-94. doi: 10.1007/s12469-016-0143-x

Nachtigall, K., & Voget, S. (1996). A genetic algorithm approach to periodic railway synchronization. Computers & Operations Research, 23 (5), 453-463. doi: https://doi.org/10.1016/0305-0548(95)00032-1

Odijk, M. A. (1996). A constraint generation algorithm for the construction of periodic railway timetables. Transportation Research Part B: Methodological, 30 (6), 455-464. doi: https://doi.org/10.1016/0191-2615(96)00005-7

Robenek, T., Sharif Azadeh, S., Maknoon, Y., & Bierlaire, M. (2017). Hybrid cyclicity: Combining the benefits of cyclic and non-cyclic timetables. Transportation Research Part C: Emerging Technologies, 75 , 228-253. doi: https://doi.org/10.1016/j.trc.2016.12.015

Schöbel, A., & Kratz, A. (2009). A bicriteria approach for robust timetabling. In R. K. Ahuja, R. H. Möhring, & C. D. Zaroliagis (Eds.), Robust and online large-scale optimization: Models and techniques for transportation systems (pp. 119–144). Berlin, Heidelberg: Springer Berlin Heidelberg. doi: https://doi.org/10.1007/978-3-642-05465-5_5

Serafini, P., & Ukovich, W. (1989). A mathematical model for periodic scheduling problems. SIAM Journal on Discrete Mathematics, 2 (4), 550-581. doi: https://doi.org/10.1137/0402049

Vansteenwegen, P., & Oudheusden, D. V. (2006). Developing railway timetables which guarantee a better service. European Journal of Operational Research, 173 (1), 337-350. doi: https://doi.org/10.1016/j.ejor.2004.12.013

Xu, X., Li, K., Yang, L., & Ye, J. (2014). Balanced train timetabling on a single-line railway with optimized velocity. Applied Mathematical Modelling, 38 (3), 894-909. doi: https://doi.org/10.1016/j.apm.2013.07.023

Yuhua, Y., Samà, M., Dario, P., & Shaoquan, N. (2022). Train timetabling with passenger data and heterogeneous rolling stocks circulation on urban rail transit line. Soft Computing. doi: 10.1007/s00500-022-07057-0

Zhang, Y., Peng, Q., Yao, Y., Zhang, X., & Zhou, X. (2019). Solving cyclic train timetabling problem through model reformulation: Extended time-space network construct and alternating direction method of multipliers methods. Transportation Research Part B: Methodological, 128 , 344-379. doi: https://doi.org/10.1016/j.trb.2019.08.001

Zhou, X. S., & Zhong, M. (2007). Single-track train timetabling with guaranteed optimality: Branch-and-bound algorithms with enhanced lower bounds. Transportation Research Part B: Methodological, 41 , 320-341. doi: https://doi.org/10.1016/j.trb.2006.05.003

Zimmermann, U. T., & Lindner, T. (2003). Train schedule optimization in public rail transport. In W. Jäger & H.-J. Krebs (Eds.), Mathematics — key technology for the future: Joint projects between universities and industry (pp. 703–716). Berlin, Heidelberg: Springer Berlin Heidelberg. doi: 10.1007/978-3-642-55753-8_53