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Abstract

High nonresponse rates have become a rule in survey sampling. In panel surveys there occur additional sample losses due to panel attrition, which are thought to worsen the bias resulting from initial nonresponse. However, under certain conditions an initial wave non-response bias may vanish in later panel waves. We study such a "Fade away" of an initial nonresponse bias in the context of regression analysis. By using a time series approach for the covariate and the error terms we derive the bias of cross-sectional OLS-estimates of the slope coefficient. In the case of no subsequent attrition and only serial correlation an initial bias converges to zero. If the nonresponse affects permanent components the initial bias will decrease to a limit which is determined by the size of the permanent components.

Attrition is discussed here in a worst case scenario, where there is a steady selective drift into the same direction as in the initial panel wave. It is shown that the fade away effect dampens the attrition effect to a large extent depending on the temporal stability of the covariate and the dependent variable. The attrition effect may by further reduced by a

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weighted regression analysis, where the weights are estimated attrition probabilities on the basis of the lagged dependent variable.

The results are discussed with respect to surveys with unsure selection procedures which are used in a longitudinal fashion, like access panels.

Key words: Regression Analysis, Nonresponse Bias, Panel Attrition, Inverse Probability Weighting.

1 Introduction

High nonresponse rates have become the rule in survey sampling. They not only reduce case numbers but they have the potential to bias standard statistical analysis. In panel surveys there may occur additional sample losses due to drop-out of respondents, called panel attrition.

Often panel attrition is thought to worsen the bias resulting from initial nonresponse. However, this may not hold. On the contrary, under certain conditions an initial wave nonresponse bias may vanish in later panel waves. Rendtel (2013) coined the term "Fade away" for the vanishing of initial nonresponse bias in later panel waves. Rendtel and Alho (2022) explain a fade away effect in a Markov chain setting for a distribution on a finite state space. Here we establish a similar statement for regression analysis. The nonresponse is defined here by an explicit impact of the dependent variable of the regression model on the probability of participation. This nonresponse is non-ignorable in the sense of Rubin (1976).

Empirically, panel attrition often depends on field work related causes, like change of address, move from the parental home or divorce, see Behr et al. (2005), Iacovou and Lynn (2017) and the overview of Watson and Wooden (2008). These causes are frequently unrelated to the analysis variable. For our analysis we assume a worst case scenario, where attrition continues to depend on the value of the actual dependent variable, thus creating a steady selective drift into the same direction as the initial nonresponse.

We will use here a time series approach for the covariate and the error term. In this setting we analyse the bias of cross-sectional OLS estimates of the regression slope coefficient. For a linear response model we are able to derive an analytic expression for the resulting bias and its decline in later panel waves. This formula treats the case with no attrition. The more realistic case of a logit response model and the presence of attrition is treated by means of a simulation study. The factors of this environment are: 6 temporal scenarios, 3 scenarios for initial nonresponse and 3 scenarios for attrition. We will investigate the development of the bias over the first 10 waves of a panel. Our results state that it is worthwhile to check the temporal stability of the covariate values and the regression error terms. This is a new aspect in survey nonresponse analysis.

Weighting approaches are widely used to compensate for nonresponse, see, for example, Särndal and Lundström (2005). This is mostly done for cross-sectional surveys. Cornesse et al. (2020) conclude, that "weighting does not sufficiently reduce bias in nonprobability sample surveys". However, this statement refers to cross-sectional surveys and the estimation of population totals and means. Here we discuss a special solution for panel surveys where we can use the lagged dependent variable as a proxy for the dependent variable in the actual wave. This offers the opportunity to estimate a proxy for the attrition probability which can be used in a weighted regression analysis.

Our results offer a new view on samples with a high nonresponse or with uncontrolled sampling or self-sampling which result from web-surveys, see, for example, Couper (2000) or Bethlehem (2010). If such samples are used over longer time periods, for example via access panels, their results may become more reliable after some time. In the presence of a substantial fade away effect the results from such longitudinal surveys become more and more reliable at later points in time. Usually the bias studies of nonprobability surveys are done in a mere cross-sectional fashion, see, for example, Cornesse et al. (2020). Hence our results will open a new floor for the use of non-probability samples in a longitudinal fashion, which are discussed in the conclusions.

In the next section we describe the regression time series model and derive our bias result for the linear response model. Then we formulate the simulation setting in Section 3. We study the bias of cross-sectional estimates of the slope coefficient until wave 10 in Section 4. Here we use the logit response model and assume no attrition. Section 5 introduces attrition. In Section 6 we investigate the use of the lagged dependent variable for a weighted regression analysis. We summarize our findings and give recommendations in Section 7.

2 The nonresponse bias in regression analysis

In this section we derive a bias result for cross-sectional estimates of the slope coefficient in a simple regression setting. For this purpose we use a time-series approach for the dependent variable Y and the covariate X. For nonresponse at the start of the panel we use a linear response model with Y as the explanatory variable. Here we assume no attrition in later panel waves.

2.1 The cross-sectional estimator

For an individual i at time t with covariate value X_{it}, the response is of the form

$$Y_{it} = \delta_t + \beta X_{it} + \epsilon_{it}, \tag{1}$$

where the error terms are independent over units with $E[\epsilon_{it}] = 0$ and $Var(\epsilon_{it}) = \sigma^2$. The objective of the analysis is either to estimate the finite population effect β of the covariate on the response, the intercept δ_t, or both. We will concentrate on the first one.

In order to estimate the slope coefficient β by the cross-sectional sample of wave t with sample size $n(t)$ we compute the OLS-estimator, at time t, from the observations $Y_{it} = y_{it}$, as

$$\hat{\beta}_t = \sum_{i=1}^{n(t)} (x_{it} - \bar{x}_t)(y_{it} - \bar{y}_t) \Big/ \sum_{i=1}^{n(t)} (x_{it} - \bar{x}_t)^2, \tag{2}$$

where the averages $\bar{x}_t$ and $\bar{y}_t$ have been computed from the observed sample in wave t.

The corresponding estimates for the interceps means are

$$\hat{\delta}_t = \bar{y}_t - \hat{\beta}_t \bar{x}_t. \tag{3}$$

2.2 Effect of initial nonresponse under a dynamic time series model

We will assume that the covariate values are realizations of random variables X_{it}. They, and the error terms ϵ_{it} have a time-independent component,

$$X_{it} = \mu_i + \xi_{it}, \quad \epsilon_{it} = \nu_i + \eta_{it}, \tag{4}$$

where all μ, ξ, ν and η-terms are independent of each other, with $E[\mu_i] = E[\xi_{it}] = E[\nu_i] = E[\eta_{it}] = 0$, and $Var(\mu_i) = \kappa, Var(\xi_{it}) = 1 - \kappa, Var(\nu_i) = \gamma\sigma^2, Var(\eta_{it}) = (1-\gamma)\sigma^2$, where $0 \leq \kappa \leq 1$ and $0 \leq \gamma \leq 1$.

The intended conclusions regarding the explanatory variables are that $E[X_{it}] = 0$ and $Var(X_{it}) = 1$. The error term has correspondingly $E[\epsilon_{it}] = 0$ and $Var(\epsilon_{it}) = \sigma^2$.

In addition, the explanatory variables and the error terms have dynamic, autoregressive components,

$$\xi_{it} = \rho\xi_{i,t-1} + \zeta_{it}, \quad \eta_{it} = \phi\eta_{i,t-1} + \upsilon_{it}, \quad t = 1, 2, ..., \tag{5}$$

where $\mid \rho \mid < 1, \mid \phi \mid < 1, E[\zeta_{it}] = E[\upsilon_{it}] = 0, Var(\zeta_{it}) = (1-\kappa)(1-\rho^2)$, and $Var(\upsilon_{it}) = (1-\gamma)(1-\phi^2)\sigma^2$. The independent sequences ζ_{it} and υ_{it}, are independent of all other random variables, and of each other. An implication of the autoregressive structure is that values at $t > 0$ can be written, e.g., as $\xi_{it} = \rho^t\xi_{i0} + \tilde{\zeta}_{it}$, where the second term is independent of the first; for $t = 0$ we have $\tilde{\zeta}_{i0} \equiv 0$.

If, at time $t = 0$, an individual decides to respond, we define $R_i = 1$, otherwise $R_i = 0$. We first restrict our attention to the case in which *no further attrition occurs*, but *allow response probability to depend on* Y_{i0} (i.e., missingness is *not at random*),

$$P(R_i = 1 \mid Y_{i0}) = a + bY_{i0}, \tag{6}$$

where $0 < a < 1$ and $\mid b \mid$ is small enough so that the probability remains in interval $[0, 1]$. The marginal probability of responding is then $P(R_i = 1) = a + b\delta_0$.

Define $\tilde{\beta}_t$ as the large sample limit of $\hat{\beta}_t$, but based on the respondents. In Appendix A we show in detail that under mild conditions we have the limit

$$\tilde{\beta}_t = \beta \times (1 - \frac{b^2\sigma^2(\kappa + \rho^t(1-\kappa))(\gamma + \phi^t(1-\gamma))}{(a + b\delta_0)^2 - b^2\beta^2(\kappa + \rho^t(1-\kappa))^2}) \tag{7}$$

for $t = 0, 1, \ldots$, or the *estimate is attenuated towards zero.*

Since $(\kappa + \rho^t(1-\kappa))(\gamma + \phi^t(1-\gamma)) \leq 1$ for all $t > 0$, the bias is always *smaller for* $t > 0$ than at $t = 0$. If $\kappa = 0$ or $\gamma = 0$, then the bias goes to zero, as $t \to +\infty$. In other words, the *initial bias will fade away.*

Interestingly, for the fade away it is sufficient that $\kappa = 0$, although it is related to the explanatory variable, not to the residual, and nonresponse determined by x-values only, would be ignorable. This would be the same as randomizing individual effects across the values of explanatory variables.

Finally, equation 7 shows that the bias in the average of y-values in (3) will fade away, provided that both $\kappa = 0$ *and* $\gamma = 0$ hold.

The bias approximation in equation 7 reveals different scenarios:

- **No bias**
  - If $\sigma^2 = 0$, the probability of participating depends only on X, then there is no bias in the OLS estimates. This is the Missing at Random (MAR) case.
  - if $b = 0$, the probability of response is constant over units. This is the Missing Completely at Random (MCAR) case.
  - If $\beta = 0$ there is no bias present in OLS estimates of β.
  - In case of independent covariate values ($\kappa = \rho = 0$) the bias will vanish within one time period.
  - In case of independent values of the residual term ($\gamma = \phi = 0$) the bias will vanish within one time period.
- **Complete Fade-away of bias**

- There are either no permanent components of $X_{i,t} \quad (\kappa = 0)$ or $\epsilon_{i,t} \quad (\gamma = 0)$. The speed will depend upon the AR-factors ρ for the covariate and ϕ for the error terms.

- **Partial Fade-away of bias**

  - A fraction of the initial bias will remain permanent, if there are both permanent components for the covariate and the error term. The fraction will be $\kappa\gamma$ of the initial bias.

  - The speed of the convergence will depend upon the autoregressive factors ρ for the covariate and ϕ for the error terms.

This result leads to some questions:

1. The above bias formula is an asymptotic result which is based on a linear response model. However, this approach does not cover the standard logit model which is nonlinear in the y-values. Therefore we will investigate by a simulation study whether the implications of the above bias formula still hold under a logit response setting.

2. The practical relevance of the above result depends on the speed of the fade-away effect, which depends on many parameters in the model setting. This suggests the investigation of realistic parameter settings in the simulation runs. We assume here also a set of alternative response scenarios.

3. The proof of the analytic bias formula did not use the normal distribution of the x-covariate. We needed only zero expectation and symmetry with respect zero. Thus we also check the simulation results against deviations from the standard normality assumption for the x-covariate which will be inappropriate in many realistic instances.

4. We have not developed analytical formulas for the case of attrition. The impact of attrition on the regression results shall be demonstrated in the framework of a simulation study. Here we should also investigate a few alternative scenarios.

3 Design of the simulation study

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To generate the data from the model, we set the starting values of the regression model parameters to $\delta_t = 0$ for the intercept and $\beta_t = 1$ for the slope. We then generate a synthetic data set of size $N = 20,000$ units from the model, which is replicated over 100 Monte Carlo repetitions. This sample size is higher than in empirical applications. However, our focus is here on a high precision of the bias estimates. Further, we assume that the distribution of the covariate $X_{i,t} = \mu_i + \xi_{i,t}$ and the error term $\epsilon_{i,t} = \nu_i + \eta_{i,t}$ are normally distributed. The covariate $X_{i,t}$ consists of two components: the permanent component μ_i and the transient component $\xi_{i,t}$, both the components are uncorrelated with each other having expectations 0 and variances $\sigma^2_\mu = \kappa$ and $\sigma^2_{\xi_t} = (1-\kappa)$, respectively:

$$\mu \sim \ N(0, \sigma^2_\mu) \quad \text{and} \quad \xi_t \sim N(0, \sigma^2_{\xi_t}),$$

Likewise, the error term $\epsilon_{i,t}$ is decomposed into two uncorrelated components: the permanent component ν_i and the transient component $\eta_{i,t}$ having expectations 0, and variances $\sigma^2_\nu = \gamma\sigma^2$ and $\sigma^2_{\eta_t} = (1-\gamma)\sigma^2$, respectively, where σ^2 is the total variance of the error term:

$$\nu \sim \ N(0, \sigma^2_\nu) \quad \text{and} \quad \eta_t \sim N(0, \sigma^2_{\eta_t}),$$

Further, it is assumed that the error terms $\zeta_{i,t}$ and υ_{it} of the auto-regressive models are independent and identically normally distributed with expectations zero, and variances $\sigma^2_\zeta = (1-\kappa)(1-\rho^2)$ and $\sigma^2_\upsilon = (1-\gamma)(1-\phi^2)\sigma^2$, respectively.

$$\zeta_t \sim \ N(0, \sigma^2_\zeta) \quad \text{and} \quad \upsilon_t \sim \ N(0, \sigma^2_\upsilon),$$

To check the size of the non-response bias of the regression estimators and its fade-away effect in later panel waves, we consider different model stabilities of the covariate and error term. A total of six Scenarios $A - F$ is considered. In the first three scenarios we assume equal stabilities of the covariate and residual components, while there are some mixed cases (unequal stabilities) in the last three scenarios.

- Scenario A: Low stability $\kappa = \gamma = \rho = \phi = 0.30$,

- Scenario B: Medium stability $\kappa = \gamma = \rho = \phi = 0.50$,
- Scenario C: High stability $\kappa = \gamma = \rho = \phi = 0.70$,
- Scenario D: No permanent components $\kappa = \gamma = 0$, Equal serial correlation for the error terms $\phi = 0.5$ and the x-values $\rho = 0.5$
- Scenario E: No serial correlation for the x-values and the error terms $\rho = \phi = 0$. Equal permanent components for the error terms $\gamma = 0.5$ and the x-values $\kappa = 0.5$.
- Scenario F: This scenario is motivated by the result for the linear response, where the bias becomes zero if the x-values are independent over time. Thus $\kappa = \rho = 0$ while we set $\gamma = \phi = 0.5$ for the residual terms.

We then simulate the initial nonresponse variable $R_{i,t=1}$ at wave 1[1]. In our simulations we use logit response model:

$$P(R_{i,t} = 1|Y_{i,t}) = \frac{\exp(a + bY_{i,t})}{1 + \exp(a + bY_{i,t})} \quad (8)$$

We choose $a = 1.0$ to scale the initial response rate to a value in the interval $(0.6, 0.7)$ which is thought to be a realistic range for the response rate. The value of b determines the size of the bias. The resulting response probability depends on the value of Y. The variance σ^2 of y is equal to the sum of the variances of variance of X and ϵ. In our simulation setting we have $\sigma = \sqrt{1+1}$. Here we scaled the b-value by multiples of σ.Thus we have $b = \tilde{b}/\sigma$. For $\tilde{b} = 1.0$, 1.5 and 2.0 we obtain $b = 0.707$, 1.06 and 1.41. These values define the Nonresponse Scenarios N_1, N_2 and N_3.

Table 1 displays the resulting response probabilities for the Nonresponse Scenarios N_1, N_2 and N_3. The selective power of the nonresponse scenarios is quite pronounced, For example, changing from $y = -\sigma$ to $y = \sigma$ increases in Scenario N_2 the response probabilities from 0.465 to 0.887. The coefficients of the logit model can be used to interpret the effect of the explaining covariate, here Y, on the odds ratio P(Response)/P(non-Response). For $Y = 0$ the odds ratio for response is given by 2.73. For $y = -\sigma = -\sqrt{2}$ the odds ratio is decreased by the factor

[1]In the subsequent sections we index time by waves. Thus we start at time zero with the first panel wave.

Scenario	b	Effect on odds ratio	Response Probability $y = -2 \times \sigma$	$y = -\sigma$	$y = \sigma$	$y = 2 \times \sigma$
N_1	0.71	0.37	0.398	0.572	0.845	0.912
N_2	1.06	0.22	0.246	0.465	0.887	0.957
N_3	1.41	0.13	0.138	0.398	0.918	0.978

Table 1: Response probabilities for different y-values and different values of b. Y-values at different multiples of the marginal standard deviation σ of Y.

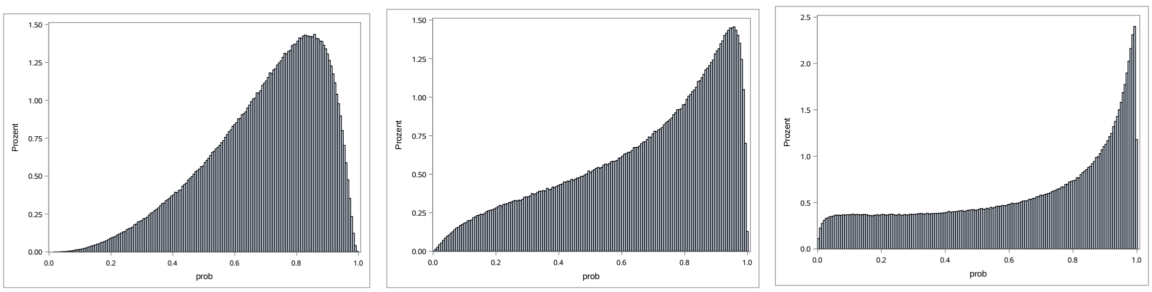

Figure 1: Comparison of the distribution of response probabilities for the Nonresponse Scenarios N_1 **(Left)** with mean response probability 0.698, N_2 **(Middle)** with mean response probability 0.671 and N_3 **(Right)** with mean response probability 0.648 .

$\exp(-b \times \sqrt{2})$. The resulting effects of nonresponse on the odds ratios are displayed in column 3 of Table 1. These effects on the odds ratio are substantial.

For these three nonresponse scenarios we obtain different pattern of response probabilities in the initial sample after response, see Figure 1. With increasing values of b the average response rate drops from 0.698 to 0.648. But also the shape of the distribution of response probabilities varies. Figure 1 demonstrates that the distribution becomes more and more left-skewed, which reflects the fact that persons with low response probabilities are discarded form the sample. The distribution for Scenario N_1 corresponds good to the empirical example given in in Figure 1 of Alho et al. (1993) for participation in an US Post Enumeration Survey.

We estimate the slope coefficient β in each Monte Carlo replication r, $(r = 1, \ldots, R)$, where R stands for the number of Monte Carlo replications. Let $\hat{\beta}_{tr}$ be the regression estimator of β in wave t in the r^{th} simulation run, then the average based on all replications is $\hat{\beta}_t = \frac{1}{R} \sum_{r=1}^{R} \hat{\beta}_{tr}$. Finally, we compute the relative bias in percent: $RB_t = 100 \times (\hat{\beta}_t - \beta)/\beta$. We also display the estimation results on the basis of the so-called full-sample, which is the sample before initial nonresponse. This serves as a control that the initial sample size is large enough to display stable results.

Each replicate of the 100 simulation runs delivers an estimate of the bias. From the replications we compute the 5-percent and the 95-percent points of the 100 replications divided by $\sqrt{100} = 10$, which may be taken as a confidence interval for the true bias. The original confidence intervals of the single simulation runs are quite broad in the range of 5 percentage points. They become larger in later panel waves due to the decreasing case numbers of the observations which enter the estimate of b.

4 Initial nonresponse without attrition

In this section we display the fade-away effect under the logit response model. In Appendix B we compare simulation results with the analytical formula results if we approximate the logit function by a linear approximation. It turns out, however, that the linear approximation of the logit function is poor in realistic situations and therefore the results obtained under the assumption of a linear response model are not exact. In general the bias under the logit model is smaller than the bias under the linear approximation.

Figure 2 displays a fade-away effect under the Scenarios A to F for the logit response model and the nonresponse Scenario N_2. Note that the response model for Scenarios A to F is identical. Therefore the initial bias for a fixed response scenario is the same for Scenarios A to F. Yet the consequences of initial nonresponse depend intrinsically on the temporal changes of the distribution of the $Y_{i,t}$, resulting in different temporal developments of the initial bias. Figure 2 displays clearly the basic features of the fade-away effect: the level where the bias stabilizes is fixed by the proportion of the permanent components while the speed of the fade away effect is determined by the size of the serial correlation. The smaller the correlation is the faster is the fade away of the initial bias. For Scenario A the permanent bias level is already reached at wave 3, while it takes until wave 6 in Scenario B. In Scenario C we have to wait until wave 9 until the permanent bias is reached. The levels of the permanent bias are quite different depending on the fraction of permanent y-values. If this fraction is as low as 0.3 as in Scenario A, the initial bias of 13.2 percent is reduced 1.1 percent, while it is 3.0 percent in Scenario B with a fraction of 0.5 permanent components. Finally, in Scenario C with a fraction of 0.7 of permanent components the bias stabilizes at 6.3 percent.

The last three Scenarios D and E disentangle the serial and the permanent effects, while Scenario F creates independent x-values and the residual values have permanent components and serial correlation. In the case of Scenario F the bias formula of the linear response model predicts an immediate vanishing of the initial bias in Wave 2. This holds also under the logit response model. Under Scenario D with no permanent components the bias converges rapidly to zero. In Scenario E with no serial correlation the bias level of Scenario B is already reached in Wave 2. The level of the bias is approached under Scenario B four waves later, after the bias due to the serial components has faded-away.

Now we will study the impact of the response mechanisms N_1, N_2 and N_3 on the fade-away effect. We do this only for the Scenarios A, B and C as these Scenarios exhibit longer declines of the initial bias. Here we are interested in the decline of the bias relative to the bias at the initial wave. Therefore we compute a normed bias which is the ratio of the bias in the current wave divided by the bias at the initial wave. This facilitates the comparison of different nonresponse scenarios and gives a measure of the speed of the fade-away effect. To be precise we obtain the following biases (in percent points): 7.44 for Scenario N_1, 13.20 for Scenario N_2 and 18.71 for Scenario N_3.

Figure 3 displays the speed and the level of the fade-away effect. For low temporal stability the final bias reduction is already reached in wave 4. Here the original size of the bias is reduced by a factor 0.09. However, with higher temporal stability the reduction factor is still considerable: 0.22 under Scenario B und 0.5 under Scenario C. Figure 3 clearly demonstrates that the reduction of the initial bias is mainly a matter of the temporal stability of the x and y-variables but not of the response parameter b in the Logit model. There seems to be a slight advantage for models with a highly selective nonresponse pattern as they show a faster decline of the initial bias.

Next we will check the stability of the fade-away effect under departures of the x-variable $X_{i,t=0}$ from the normal distribution at the start of the panel. Remember that $X_{i,t=0}$ is the sum of two independent variables $X_{i,t=0} = \mu_i + \xi_{i,t=0}$ having expectations 0 and variances $\sigma_\mu^2 = \kappa$ and $\sigma_{\xi_t}^2 = (1 - \kappa)$, respectively. Instead of a normal distribution we assume for each component a symmetric, discrete distribution on the points $-\sigma_\mu, +\sigma_\mu$ resp. $-\sigma_\xi, +\sigma_\xi$. For these x-values we use

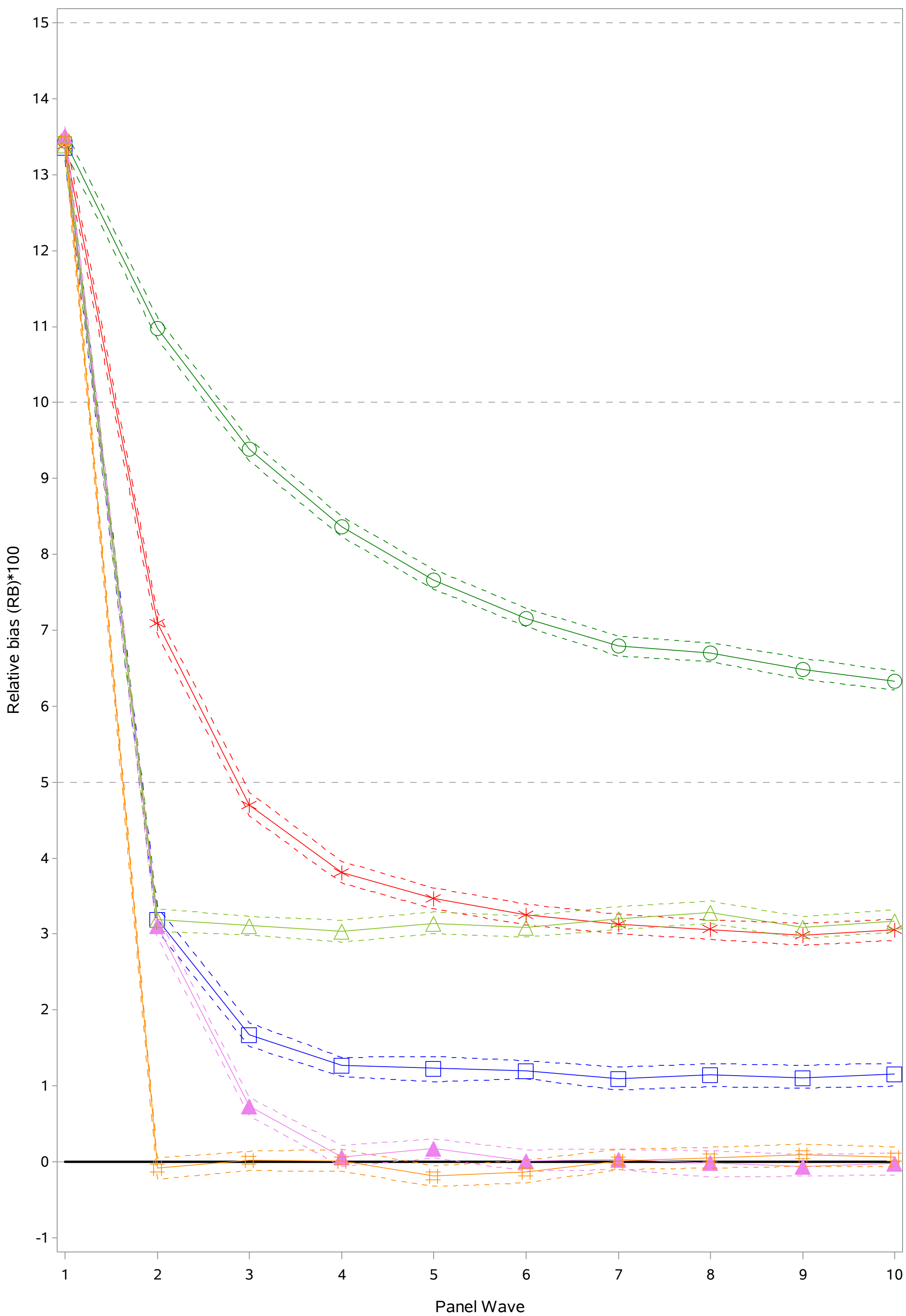

Figure 2: Comparison of the simulated relative bias of cross-sectional estimates of the slope coefficient β under Scenarios A to F under Nonresponse Scenario N_2. Legend: Square=A, Star=B, Circle=C, Trianglefilled=D, Triangle=E, Hash=F. Confidence limits by dashed lines.

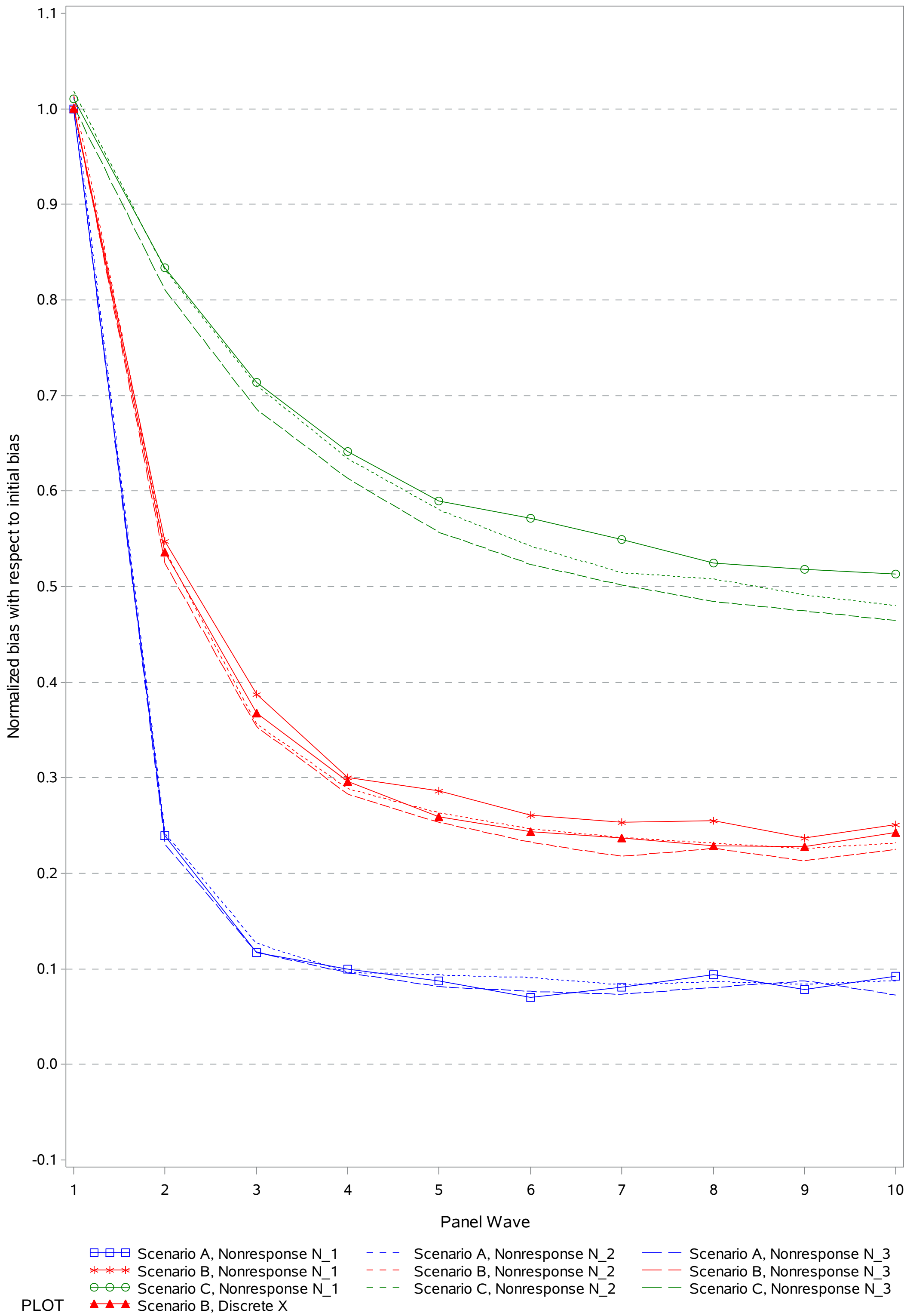

Figure 3: Comparison of the reduction of the initial bias (normed bias) for Scenarios A,B and C under the Nonresponse Scenarios N_1, N_2 and N_3 and for discrete X-values at $t = 0$.

the medium stability Scenario B with the medium nonresponse Scenario N_2. The resulting bias for β is 14.03 percent points which is similar to the bias under the normal distribution, which is 13.38 percent points. The decline of this bias is displayed in Figure 3 by the line with the filled triangles. There is virtually no difference to the fade-away behaviour under the normal distribution.

5 Attrition after initial nonresponse

In this section we extent our simulation design to cover also panel attrition. Panel attrition in wave t is defined here as nonresponse, indicated by $R_{i,t} = 0$, with response in the preceding panel wave $t-1$. We assume that the drop-out by attrition is permanent, i.e. units do not return to the sample in later panel waves. Similar to initial nonresponse we assume a direct dependence of attrition on the dependent variable $Y_{i,t}$.

$$P(R_{i,t} = 1|Y_{i,t}, R_{i,t-1} = 1) = \frac{\exp(a^* + b^* Y_{i,t})}{1 + \exp(a^* + b^* Y_{i,t})} \quad (9)$$

Note, that attrition acts here during the entire panel into the same direction as the initial nonresponse. This is a worst case scenario. As mentioned in the introductions attrition is often linked to field work, for example residential mobility which may be uncorrelated to the dependent variable Y. Also, in empirical attrition analysis which is based on automatic selection procedures of all variables from the previous panel wave, there appear no stable attrition variables, see, for example the attrition analysis of the German Socio-Economic panel (SOEP) in Kroh et al. (2017 Table 4.3)). Thus a steady selective shift into the same direction, which we assume here, is not supported by empirical findings.

The parameters a^* and b^* have to be chosen to give realistic attrition rates in the range of 10 percent. With higher attrition rates the panel will come to an end simply by running out of sample size. For $a^* = 2$ we are in the vicinity of a response probability of 0.90. Like in the case of initial nonresponse we scale the value of b^* with respect to y-values at $\pm\sigma$, where $\sigma = \sqrt{2}$ is the marginal variance of Y. The odds ratio for response at $y = 0$ is given by $\exp(2)/(1 + \exp(2)) = 0.881$. Column 3 of Table 2 displays the effect on the odds ratio for $y = -b^* \times \sigma = -b^* \times \sqrt{2}$ for different

Scenario	b^*	Effect on odds ratio	Response Probability $y = -2 \times \sigma$	$y = -\sigma$	$y = \sigma$	$y = 2 \times \sigma$
ATT_1	0.353	0.60	0.786	0.838	0.913	0.937
ATT_2	0.530	0.47	0.718	0.813	0.926	0.955
ATT_3	0.707	0.37	0.642	0.784	0.937	0.968

Table 2: Response probabilities for different y-values and different values of b. Y-values at different multiples of the marginal standard deviation σ of Y.

values of b^*, which define the attrition scenarios ATT_1, ATT_2 and ATT_3. The effect of attrition on the odds ratio scale is substantial, however somewhat smaller compared to the initial response scenarios of Table 1.

Despite the low attrition rate of about 10 percent the cumulative effect on the sample size of the persisting sample can be substantial. Figure 4 compares the decline of sample sizes due to sample attrition for the Scenarios A, B, C and F. Under initial response scheme N_2 has lowered the sample size from 20000 to 13410. Under Scenario ATT_2 the cumulative effect of attrition from wave 2 to wave 10 is substantial: from sample size 13410 down to sample sizes of about 4000 to 5000, which means a loss of 2/3 of the sample size at the start of the panel. Surprisingly, the strongest losses of case numbers appear in Scenarios A and F where the bias will be shown to be smallest among all scenarios, while in Scenario C with the largest bias the loss of cases is the smallest. This indicates that the pure decline of case numbers is not a valid indicator of an attrition bias.

Figure 5 compares the bias of the Scenarios A, B, C and F under attrition scheme ATT_2. Here Scenario F no longer results in a zero bias as in the case of no attrition. Instead, the attrition results in each wave of a bias of about 2.5 percent. As the bias would immediately disappear in the subsequent wave we conclude that attrition results in each wave in a new bias of about 2.5 percent. This demonstrates that the attrition parameters are well scaled to a steady trend with respect to the resulting bias. Under attrition schema ATT_1 the drift is about 1.1 percent, while it is about 4.0 percent under attrition scheme ATT_3.

For each Scenario A,B and C and Nonresponse Scheme N_2 Figure 5 displays three biases: the bias without attrition (solid line) and the bias under attrition (dotted line). The third line (dashed line) results from a weighting correction which will be discussed below. From Figure 5 we

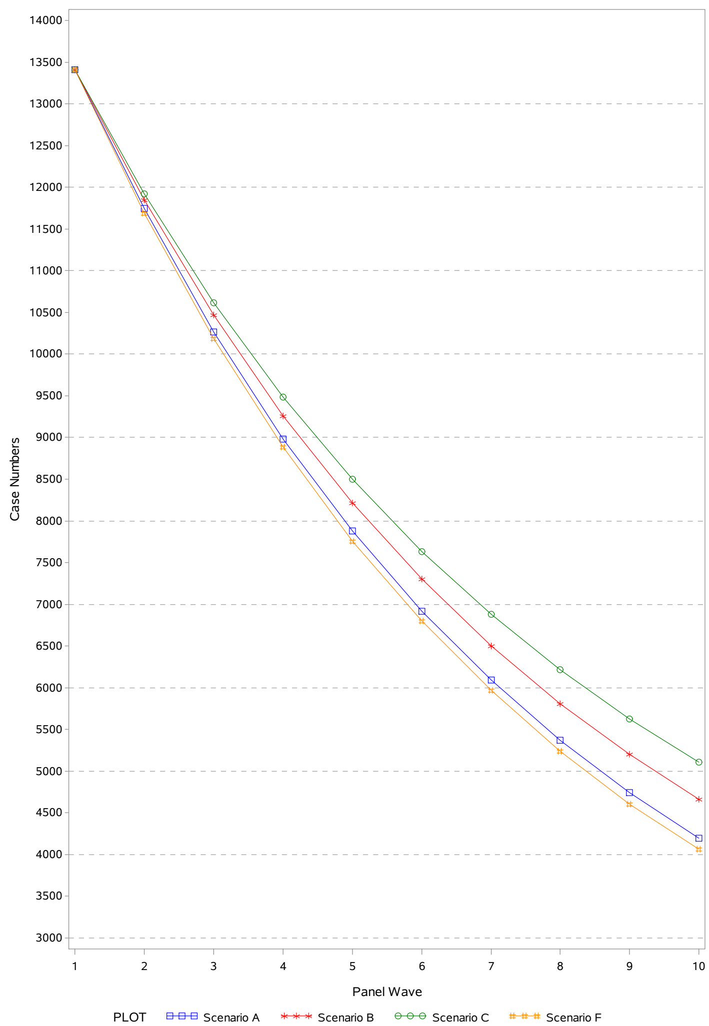

Figure 4: Comparison of case numbers in response samples under attrition after initial nonresponse for Scenarios A to F.

conclude that the impact of attrition increases with increasing temporal stability of the x- and the y-variable. At wave 10 the effect of attrition on the bias results in an increase of about 4 percentage points (Scenario A), 6 percentage points (Scenario B) and 8 percentage points (Scenario C). These differences are considerably smaller than the cumulative effect of $9 \times 2.5 = 22.5$ separate increases. Therefore the fade-away effect is also present during attrition. However, after a decrease in waves 2 and 3 the bias increases again due to attrition. With increasing temporal stability the attrition effect becomes more pronounced. In Scenario C the original initial bias is exceeded in Wave 7. We report here, that for the attrition Scenario ATT_3 and stability Scenario C the discrepancy between the cumulated drift ($9 \times 4 = 36$ percentage points) and the increase of the bias (11.4 percentage points) becomes even much stronger. On the contrary, for the low attrition Scenario ATT_1 the difference between the cumulated drift ($9 \times 1.1 = 9.9$ percentage points) and the increase of the bias (3.5 percentage points) results in the lowest fade away effect.

6 Bias correction by inverse probability weighting

A standard strategy to reduce a bias due to nonresponse is to use a weighted analysis, where the units with $R_{i,t} = 1$ are weighted by their inverse response probability $w_{i,t} = R_{i,t}/P(R_{i,t} = 1)$. The formal argument behind this strategy is $E_{R_{i,t}}(w_{i,t}) = \frac{E(R_{i,t})}{P(R_{i,t}=1)} = 1$ Here the expectation is with respect to the distribution of $R_{i,t}$ with all other variables being fixed.

This reasoning is in the framework of design-based inference where the analysis is done with respect to the randomisation of the selection procedure, see, for example, Särndal et al. (1992, Chapter 2). In our analysis the selection procedure is described by response and attrition events. The design-based approach is used to estimate shares, ratios and totals in a fixed population. However, this approach is also used in a regression setting, see Kott (1991). One motivation to incur design-weights in regression analysis is the intention to estimate a regression relationship at population level, see Bethlehem (2010) and Särndal and Lundström (2005) for a discussion. In our simulation framework the population[2] is given by the sample before initial nonresponse

[2]If the data come from a real survey one would like to make inferences about the population where the survey is taken from. In this case one has to use the selection probabilities of the survey design multiplied with the response probabilities.

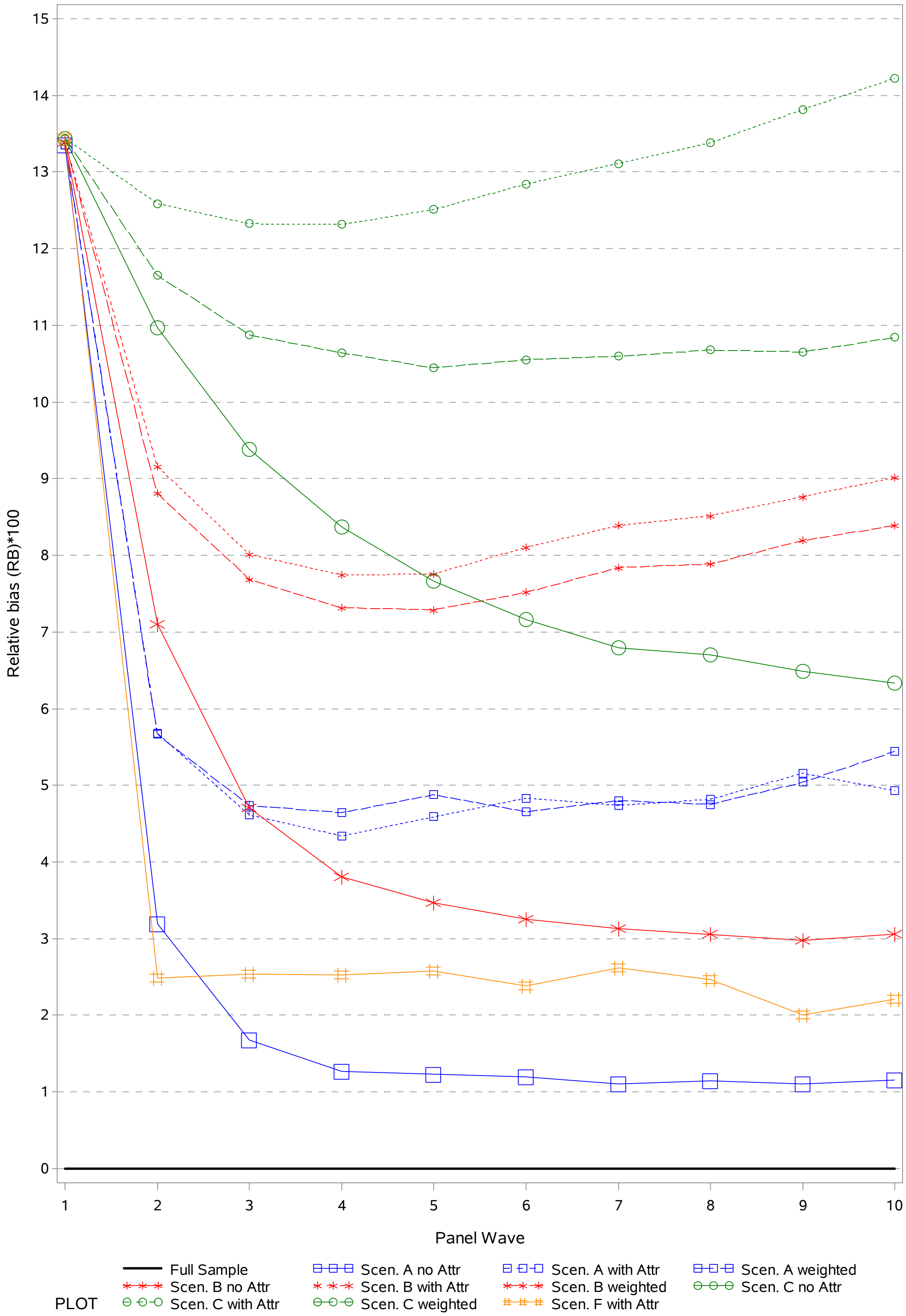

Figure 5: Relative bias under attrition for Scenario A, B, C and F . Legend: Square=A, Star=B, Circle=C, Hash=F

and attrition. Here the weighted $X'X$-matrix and $X'Y$ vector estimate their population values. Another motivation for a weighted regression is the protection of the analysis against uncontrolled stratification which is not incorporated in the regression model by control variables (Dumouchel and Duncan 1983).

In our case, the attrition process is with respect to the dependent variable $Y_{i,t}$. So, there is no exact way to control this selection by this covariate. However, we can use the known values of $Y_{i,t-1}$ as a proxy for $Y_{i,t}$-values. The approximation becomes better with increasing temporal stability of the y-values. In the setting of the simulation Scenarios A,B and C we expect a good performance of the weighted regression in Scenario C.

The weights are computed in a sequential fashion. The initial weights $w_{i,t=1}$ for the first wave are set to 1. For each subsequent wave t we estimate a logit response model for participation in wave t given participation in wave $t-1$ conditional on the value of $Y_{i,t-1}$. This gives the estimated values $\hat{P}(R_{i,t} = 1|R_{i,t-1}, Y_{i,t-1})$. These values are used to compute

$$w_{i,t} = w_{i,t-1} \times \frac{R_{i,t}}{\hat{P}(R_{i,t} = 1|R_{i,t-1}, Y_{i,t-1})} \tag{10}$$

The values of $w_{i,t}$ are used for the weighted regression analysis of the sample of wave t.

The results of this weighting strategy are displayed in Figure 5 for attrition Scenario ATT_2 by the dashed lines. As expected the best reduction of the attrition bias appears in Scenario C with the highest temporal stability. Here the increase of the bias from wave 4 onwards is almost stopped by the use of the weights. The reduction of the bias component which is due to attrition is substantial: from 8 percentage points to 4.5 percentage points. For Scenario B the reduction of the attrition bias is visible, but much smaller. For Scenario A there is no such reduction effect. However, in this scenario the attrition is not strong enough to result in an increasing bias.

To sum up: despite a loss of 2/3 of the sample size and a steady drift towards an additional bias of about two percentage points the initial bias does not increase if we apply a weighting strategy by the lagged dependent variable.

7 Summary and Conclusions

We demonstrated a fade away effect in regression analysis: For non-ignorable nonresponse cross-sectional OLS-estimates of the slope coefficient tend to produce less biased estimates in later panel waves. The size of this effect depends on the temporal stability of the covariate and the regression errors.

We presented an analytical result for the case of a linear response model and a time series setting for the x- and the y-variable. In this setting we prove that initial bias of the OLS-estimate for the slope coefficient is reduced over time. Our result proposes a monotonically decreasing attenuation bias. We could also prove an interesting finding: despite nonignorable initial nonresponse it is sufficient to control the distribution of the covariate to remove the bias in later panel waves: if the covariate values are serially independent the bias will be completely removed in one wave. These findings hold not only under a linear response model but also under the logit response models in our simulation setting.

In our simulation runs the reduction of the initial bias was substantial: without attrition it ranges to 91 percent (low stability) to 50 percent (high stability). The rate of reduction is almost independent from the initial nonresponse nor does it depend on the distribution of the x-covariate. It is simply a matter of percentage of the fixed components in the time series model. The serial components govern the speed of the fade-away effect. For moderate serial effects the final bias is reached within 4 to 6 panel waves.

In our simulations we studied a worst case scenario of a permanent selective drift by attrition into the same direction as the initial nonresponse. The losses due to attrition amount 2/3 of the sample size at the start of the panel. Despite these losses in sample size and the selective drift the resulting bias which is due to attrition is much less than the expected cumulative drift of $9 \times 2.5 = 22.5$ percentage points: it ranges from 4 (low stability) to 8 (high stability) percentage points. Contrary to initial nonresponse, the size of the selective drift plays an important rule of the temporal stability of the bias. Thus it can happen that under a substantive drift by attrition and stable x- and y-variables the nonresponse bias is increased beyond the level of the initial bias. However, such a situation can be prevented by a weighting strategy where the lagged dependent variable is used to predict attrition in the actual wave. The predicted attrition probabilities can be used in a weighted regression analysis. The approach is efficient for stable y-variates, where the attrition effect is largest. There the increase of the bias due to the attrition effect is reduced from 7.8 (without weighting) to 4.5 (with weighting) percentage points. In all cases the bias stayed below the level at the start of the panel. Despite a substantial loss in case numbers and a steady selective drift due to attrition the bias can be controlled by the fade away effect combined by a simple weighting strategy in the regression analysis.

These results can be extended to more complex situations. For example, the sample may be subdivided into groups. All groups are supposed to have the same slope coefficient β but group-specific intercepts. The well-known *Analysis of Covariance (ANCOVA)* estimator of β is a mixture of group-specific OLS-estimates of β. Hence, if these group-specific estimators show a fade-away behaviour, then the ANCOVA estimator will do so too. There is an important difference between the group-specific and the unspecific OLS-estimator with respect to the fade-away effect. Here the unspecific OLS-estimator incurs residual terms with a much higher permanent component than the group-specific estimator. The permanent components are due to the group-specific impact of the group on the dependent variable. Therefore we can expect a higher fade-away effect for the ANCOVA estimator than for the unspecific OLS-estimator.

The above result may be used to cover two cases. The first case refers to stratified probability samples at the start of a panel survey. If the user can identify the strata then he can use the ANCOVA estimator for strata and use the survey design weights to combine the separate strata estimations. The second case refers to non-probability samples, especially if they are sampled from access panels. Often these samples are controlled by some quota variables. For example, the quotas refer to totals from a large population surveys like gender, age-group and education level. As the selection into the access panel did happen some periods before its use we may regard it as a longitudinal sample where a fade away effect is present. Here we may assume that the residual fluctuations after control of the quota variables have a low serial correlation if the quota variables have a high impact on the dependent variable. The stated fade away effect together with the possibility to control attrition via the lagged dependent variable support the use of nonprobability samples in a longitudinal context, as proposed in the introduction.

Our results may be used for practical recommendations. In order to judge the potential size of the fadev away effect one should estimate the temporal stability of the covariates and the error term. In case of small and moderate fixed components one may be willing to accept a fast decline of an initial non-response bias. In case of attrition one should check the stability of the dependent variable. If it is high then we recommend to use weights which compensate for attrition via a logit model with the lagged dependent variable as explaining covariate. If the sample is controlled by some stratification or quota variables the stability should be checked within the strata or quota cells and the ANCOVA estimator should be used.

An alternative to unweighted or weighted regression analyses – as discussed above – is the use of nonignorable nonresponse models, see, for example Little and Rubin (2002, Chapter 11). These models and their results rely heavily on untestable distributional assumptions. The message of the fade away effect in regression analysis is: under appropriate conditions it is not necessary to switch to the use of risky nonignorable nonresponse models. If there is a fast temporal change of the covariates or the error terms unweighted regression results become more reliable in later points of time, while the results of nonignorable nonresponse models may even increase a potential nonresponse bias, if the model is wrong.

We have treated here the use of cross-sectional estimators for the estimation of a regression model. With longitudinal data one might look for the use of panel estimators which analyse data from several panel waves simultaneously. Here especially the Within-estimator (Greene 2011), which bases on departures from the individual mean, seems to be a promising candidate which is robust against selective effects related to individual permanent components, see Khan (2020) for details.

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A The derivation of the bias formula

To simplify, assume that the third moments vanish, or $E[X_{it}^3] = E[\mu_i^3] = E[\xi_{it}^3] = E[\epsilon_{it}^3] = E[\nu_i^3] = E[\eta_{it}^3] = E[\zeta_{it}^3] = E[\upsilon_{it}^3] = 0$. This is true if the distributions are symmetric around zero.

The true regression coefficient is $\beta = Cov(X_{it}, Y_{it})/Var(X_{it})$. for all i. The large sample limit of the estimator obtained from respondents at time t is

$$\tilde{\beta}_t = Cov(X_{it}, Y_{kit} \mid R_i = 1)/Var(X_{it} \mid R_i = 1). \tag{11}$$

To compute this, we need the four conditional moments $E[X_{it} \mid R_i = 1], E[Y_{it} \mid R_i = 1], E[X_{it}^2 \mid R_i = 1]$, and $E[X_{it}Y_{it} \mid R_i = 1]$.

Take first $t = 0$. By conditioning and then taking repeated expectations, we have $E[X_{i0} \mid R_i = 1] = E[R_i X_{i0}]/P(R_i = 1) = E[E[R_i X_{i0} \mid X_{i0}]]/P(R_i = 1) = E[(a + b(\delta_o + \beta X_{i0}))X_{i0}]/P(R_i = 1)$, so that

$$E[X_{i0} \mid R_i = 1] = \beta b/(a + b\delta_0), \tag{12}$$

because $E[X_{i0}] = 0$, $Var(X_{i0}) = 1$, and $P(R_i = 1) = E[P(R_i = 1 \mid X_{i0})] = E[a + b(\delta_{io} + \beta X_{i0})] = a + b\delta_0$. Under the assumption that the third moment of X_{it} is zero, we have

$$E[X_{i0}^2 \mid R_i = 1] = E[(a + b(\delta_0 + \beta X_{i0}))X_{i0}^2]/(a + b\delta_0) = 1. \tag{13}$$

Thus, we have the variance

$$Var(X_{i0} \mid R_i = 1) = 1 - \beta^2 b^2/(a + b\delta_0)^2. \tag{14}$$

Conditioning on Y_{i0}, and noting that $E[Y_{i0}] = \delta_0$ and $Var(Y_{i0}) = \beta^2 + \sigma^2$, we have that

$$E[Y_{i0} \mid R_i = 1] = E[(a + bY_{i0})Y_{i0})]/(a + b\delta_0) = (a\delta_0 + b(\delta_0^2 + \beta^2 + \sigma^2))/(a + b\delta_0). \tag{15}$$

Then, starting from $E[X_{i0}Y_{i0} \mid R_i = 1] = E[R_i X_{i0} Y_{i0}]/P(R_i = 1)$, one can condition on both X_{i0} and Y_{i0}, to get that $E[R_i X_{i0} Y_{i0}] = E[(a + bY_{i0})X_{i0}Y_{i0}])$. Noting that $E[X_{i0}Y_{i0}] = \beta$ and that

$E[X_{i0}Y_{i0}^2] = E[X_{i0}(\delta_0 + \beta X_{i0} + \epsilon_{i0})^2] = 2\delta_0\beta$, we obtain the result

$$E[X_{i0}Y_{ki0} \mid R_i = 1] = E[(a + bY_{i0})X_{i0}Y_{i0}]/(a + b\delta_0) = \beta(a + 2b\delta_0)/(a + b\delta_0). \tag{16}$$

Hence, the large sample limit of the regression coefficient becomes, after some algebra,

$$\tilde{\beta}_0 = \beta \times (1 - \frac{b^2\sigma^2}{(a + b\delta_0)^2 - b^2\beta^2}). \tag{17}$$

Since (14) is positive, the denominator in (17) must be positive. Thus, there is *attenuation towards* 0.

Consider the case $t > 0$. Assume that there is no attrition. Conditioning on both X_{i0} and X_{it}, we have $E[X_{it} \mid R_i = 1] = E[R_iX_{it}]/P(R_i = 1) = E[(a + b(\delta_0 + \beta X_{i0}))X_{rt}]/(a + b\delta_0)$. Taking into account that $X_{i0} = \mu_i + \xi_{i0}$, and $X_{it} = \mu_i + \rho^t\xi_{i0} + \tilde{\zeta}_{it}$ (as noted below (5)), we get that

$$E[X_{it} \mid R_i = 1] = \beta b(\kappa + \rho^t(1 - \kappa))/(a + b\delta_0). \tag{18}$$

Similarly, by conditioning we get that $E[X_{it}^2 \mid R_i = 1] = E[(a + b(\delta_{i0} + \beta X_{ki0}))X_{kit}^2]/(a + b\delta_{i0})$. Since the first and third moments are assumed to vanish, we have that $E[X_{i0}X_{it}^2] = 0$. Using $E[X_{it}^2] = 1$ we get that

$$E[X_{it}^2 \mid R_i = 1] = (a + b\delta_0)/(a + b\delta_0) = 1. \tag{19}$$

Therefore, the variance is

$$Var(X_{it} \mid R_i) = 1 - \beta^2b^2(\kappa + \rho^t(1 - \kappa))^2/(a + b\delta_0)^2. \tag{20}$$

As a check, note that if $t = 0$, this expression agrees with (14).

Now consider terms involving Y_{it}. First, condition on both Y_{i0} and Y_{it} to get that $E[Y_{it} \mid R_i = 1] = E[(a + bY_{i0})Y_{it}]/(a + b\delta_0)$. Since, $Y_{i0} = \delta_0 + \beta(\mu_i + \xi_{i0}) + \nu_i + \eta_{i0}$, and $Y_{it} = \delta_t + \beta(\mu_i + \rho^t\xi_{i0} +$

$\tilde{\zeta}_{it}) + \nu_i + \phi^t \eta_{i0} + \tilde{\upsilon}_{it}$, it follows that

$$E[Y_{it} \mid R_i = 1] = ((a + b\delta_0)\delta_t + b\beta^2(\kappa + \rho^t(1-\kappa)) + b(\gamma + \phi^t(1-\gamma))\sigma^2)/(a + b\delta_0). \tag{21}$$

Similarly, by conditioning on X_{it}, Y_{it} and Y_{i0}, we get that $E[X_{it}Y_{it} \mid R_i = 1] = E[(a+bY_{i0})X_{it}Y_{it}]/(a+b\delta_0)$. Note first that $E[X_{it}Y_{it}] = \beta$. Here, we note that $Y_{i0} = \delta_0 + \beta X_{i0} + \epsilon_{i0}$, and $Y_{it} = \delta_t + \beta X_{it} + \epsilon_{it}$. Recalling that $E[X_{i0}X_{it}^2] = 0$, we can show that $E[Y_{i0}X_{it}Y_{it}] = \delta_0\beta + \delta_t\beta E[X_{i0}X_{it}]$, whence

$$E[X_{it}Y_{it} \mid R_i = 1] = (\beta(a + b\delta_0) + \beta b\delta_t(\kappa + \rho^t(1-\kappa)))/(a + b\delta_0). \tag{22}$$

These lengthy expressions reduce to the relative tidy result in (7). As a check, take $t = 0$ to get formula (17).

B A comparison of simulation results and the bias formula

Equation (7) is an asymptotic result which uses the linearity of the response equation and the restriction of the support of $P(R_i = 1|Y_{i,1}) = a + bY_{i,1}$ to the interval $(0, 1)$. In this appendix we investigate the validity of the formula if these restrictions are not met exactly.

The range restriction can be only fulfilled if the support of the random components of $X_{i,1}$ and $\epsilon_{i,1}$ is restricted. We did use here a Beta(1,1) distribution, which is a rectangular distribution on $(0, 1)$, which is transformed to a distribution with zero mean and variance 1. These transformed rectangular distributions are then multiplied by $\sqrt{\kappa}$ for the realization of the time independent component μ_i of $X_{i,t}$. Similarly, we multiply by $\sqrt{\gamma}$ for the realisation of ν_i, the time independent realisation of $\epsilon_{i,t}$, see equation (4). The time dependent components $\zeta_{i,t}$ of $X_{i,t}$ are realized by multiplying the transformed distributions by $\sqrt{(1-\kappa)(1-\rho^2)}$. And finally, for the time dependent components $\upsilon_{i,t}$ of $\epsilon_{i,t}$ we use the multiplication factor $\sigma\sqrt{(1-\gamma)(1-\phi^2)}$, see equation(5). For $\delta_0 = 0$ and $\beta = 1$ we obtain then ranges for valid values of a and b in the response equation (6). For $a = 0.5$, which defines the average response rate, the maximum value of b resulting in valid probabilities is given by $b = 0.095$.

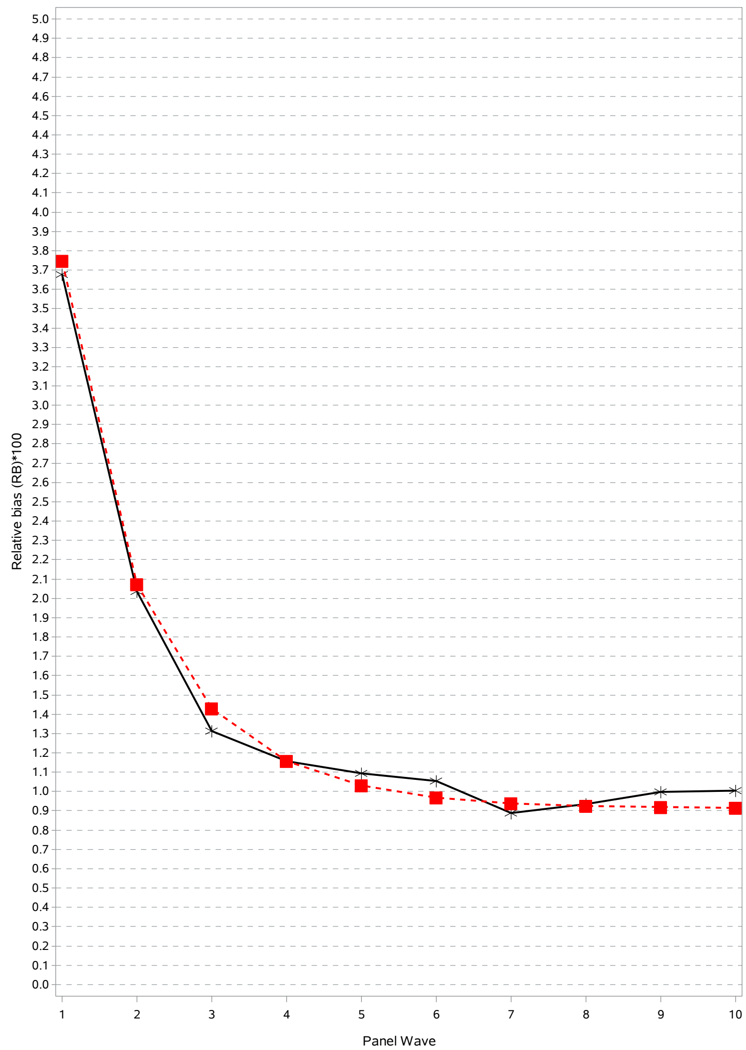

Figure 6: Comparison of the simulated bias (Solid line) and the bias by formula (Dotted line) of cross-sectional estimates of the slope coefficient β under Scenario B under a linear response model.

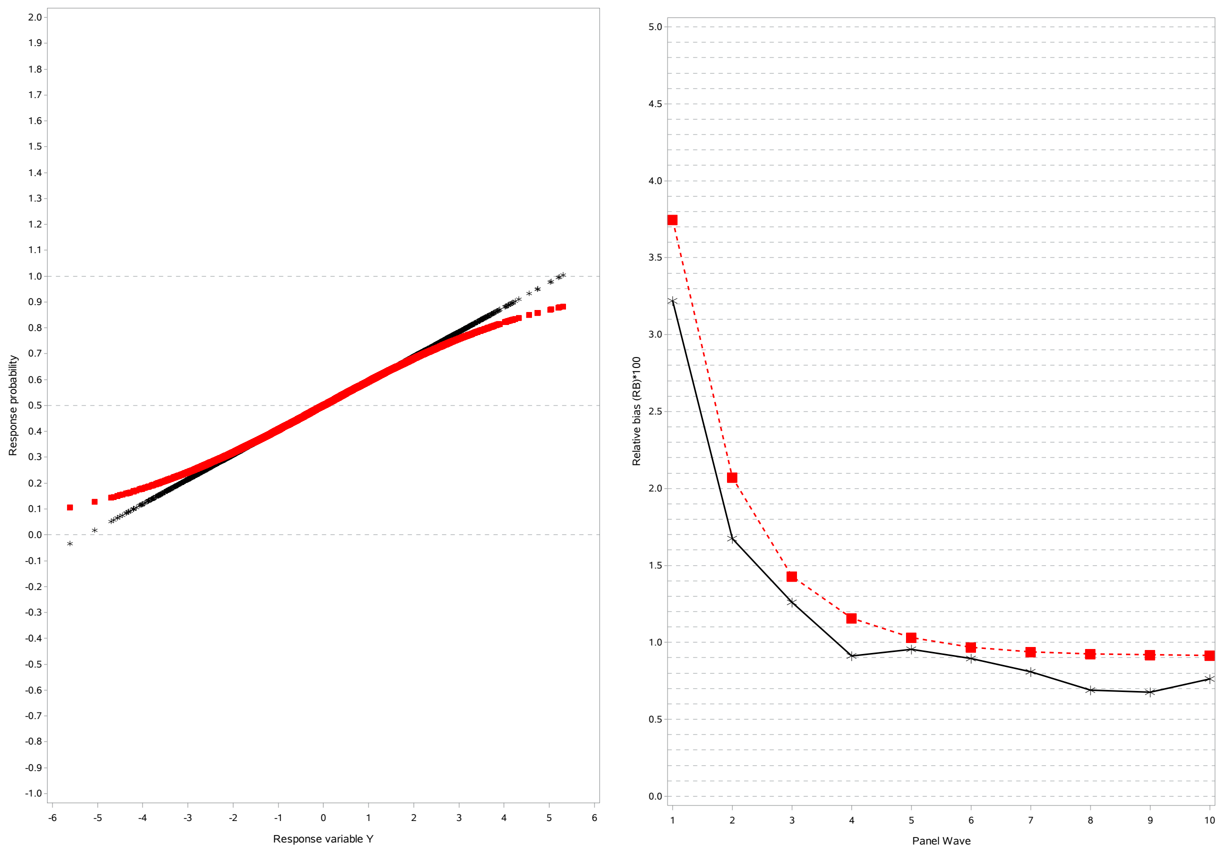

Figure 7: Comparison of the simulated and the bias by formula of cross-sectional estimates of the slope coefficient β with minor departures from the linear response model. **Left**: Comparison of linear (Stars) and Logit (Squares) response probabilities. **Right**: Simulation values (Solid line) and formula values (Dotted line).

Figure 6 shows a good coincidence of the formula results with the simulation results if the support of the x- and the ϵ-variable is controlled, such that the resulting response probabilities are within the interval $(0, 1)$. The departure is not larger than 0.1 percentage points. Here the response probabilities range from 5 percent for $y = -4.6$ to 95 percent for $y = 4.6$.

Now we switch to deviations from the finite support assumption. Thus we assume a normal distribution of the above variance components and the logit response model of equation (8). In order to use the linear response model we use a linear approximation of the logit function. We use an evaluation at a point where the logit function is 0.5. If the logit parameters are $\hat{a}$ and $\hat{b}$ then the linear approximation is given by $P(R = 1|y) = 0.5 + \frac{1}{4}\hat{b}(y - \hat{a}/\hat{b})$ for the linear response model. This gives the corresponding values $a = 0.5 - \hat{a}/4$ and $b = \hat{b}/4$. Therefore the values $\hat{a} = 0$ and $\hat{b} = 0.095 \times 4 = 0.38$ for the logit response model result in $a = 0.5$ and $b = 0.095$ like in Figure 6 .

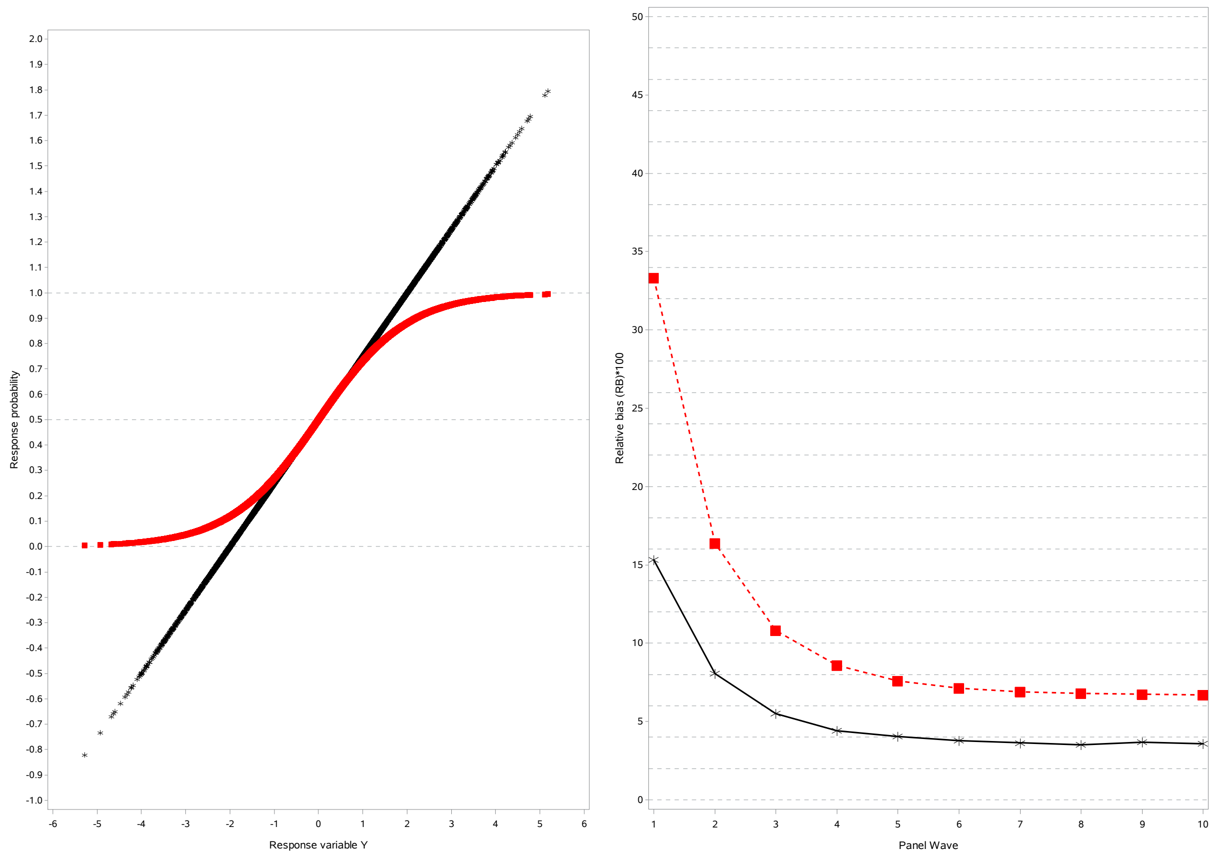

Figure 8: Comparison of the simulated and the bias by formula of cross-sectional estimates of the slope coefficient β with large departures from the linear response model. **Left**: Comparison of linear (Stars) and Logit (Squares) response probabilities. **Right**: Simulation values (Solid line) and formula values (Dotted line).

For these values we get of share of values out of the range of the interval $(0, 1)$ of only 0.02 percent. Figure 7 (Left panel) compares the response probabilities for the linear approximation and the logit case. The right panel compares the bias values from the bias formula for the linear model with the simulated values under the logit model. Despite the small deviations with respect to the percentage of observations with non-conforming probabilities and departures from linearity there is a noticeable overestimation of the bias by Equation 7.

If we increase the size of $\hat{b}$ to $\hat{b} = 1$ we will get a more pronounced initial bias. Now the percentage of observations where the linear response approximation is outside the interval $(0, 1)$ is 15.7 percent. Figure 8 (Left panel) compares the response probabilities for the linear case and the logit case. Here we observe major departures of the logit probabilities from the linear case. The right panel of Figure 8 displays a substantial overestimation by the bias under the linear model.

Because of this overestimation we did not use the bias formula for the logit cases of our simulations.

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