Chapter 8

Appendix

8.1 nj-Symbols

The Wigner 3j-Symbols

The coupling of two angular momenta j_1 and j_2 to the total angular momentum j can be written as

$$j = j_1 + j_2 \quad (m = m_1 + m_2). \tag{8.1}$$

The state of the overall system can be represented either by the product $|j_1, m_1 > \cdot |j_2, m_2 >$ or by $|j, m >$.

The linear relation between the two representations is given by

$$|j, m >= \sum_{m_1, m_2} C(j_1, j_2, m_1, m_2|j, m)| \; |j_1, m_1 > |j_2, m_2 > . \tag{8.2}$$

The expansion coefficients $C(j_1, j_2, m_1, m_2|j, m)$ are the so called Clebsch-Gordan coefficients (vector addition coefficients). Often the nj-symbols are used since they possess higher symmetries. The 3j-symbols are related to the Clebsch-Gordan coefficients by the following equation:

$$\begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{pmatrix} = (-1)^{j_1 - j_2 - m} \frac{1}{\sqrt{2j+1}} C(j_1, j_2, m_1, m_2|j, -m). \tag{8.3}$$

with $|j_1 - j_2| \leq j \leq j_1 + j_2$ and $m_1 + m_2 + m = 0$.

The Clebsch-Gordan coefficient for the maximum j, m value can easily be given, since for $j = j_1 + j_2$ and $m = m_1 + m_2$ there is only one term in the sum of equation 8.3. Because of the wavefunction normalization, it must hold that

$$C(j_1, j_2, j_1, j_2|j_1 + j_2, j_1 + j_2) = 1. \tag{8.4}$$

The other Clebsch-Gordan coefficients can be obtained by using the raising and lowering operators j_+ and j_-. These are tabulated, e.g. in Rotenberg *et al.* [RBMJ59].

Special cases

$$\begin{pmatrix} j & j & 0 \\ m & -m & 0 \end{pmatrix} = (-1)^{j-m} \frac{1}{\sqrt{2j+1}}. \tag{8.5}$$

If $j_1 + j_2 + j_3$ is odd then

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ 0 & 0 & 0 \end{pmatrix} = 0. \tag{8.6}$$

If $j_1 + j_2 + j_3$ is even then

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ 0 & 0 & 0 \end{pmatrix} = (-1)^p \sqrt{\frac{(2p-2j_1)!(2p-2j_2)!(2p-2j_3)!}{2p+1} \times \frac{p!}{(p-j_1)!(p-j_2)!(p-j_3)!}} \tag{8.7}$$

with $2p = j_1 + j_2 + j_3$.

The 6j-Symbols

These occur in the coupling of three angular momentum vectors $j_1 + j_2 + j_3 = j$. Coupling is possible according to two schemes:

$$j_1 + j_2 = j_{12} \;\; and \;\; j_{12} + j_3 = j, \tag{8.8}$$

or

$$j_2 + j_3 = j_{23} \;\; and \;\; j_{23} + j_1 = j. \tag{8.9}$$

The wavefunctions in these two schemes are related as follows:

$$|(j_1 j_2) j_{12}, j_3, jm >= (-1)^{j_1+j_2+j_3+j} \sum_{j_{23}} \sqrt{(2j_{12}+1)(2j_{23}+1)} \times \begin{Bmatrix} j_1 & j_2 & j_{12} \\ j_3 & j & j_{23} \end{Bmatrix} |j_1, (j_2 j_3) j_{23}, jm > . \tag{8.10}$$

Using the definition of the $3j$-symbols given above, the $6j$ symbol can be determined as follows:

$$\begin{Bmatrix} a & b & e \\ d & c & f \end{Bmatrix} = \sum_{g,h} (-1)^{a+b+c+d+e+f+h_1+h_2+h_3} \begin{pmatrix} a & b & e \\ g_1 & g_2 & g_3 \end{pmatrix} \begin{pmatrix} a & c & f \\ -g_1 & h_2 & h_3 \end{pmatrix} \times \begin{pmatrix} d & b & f \\ -h_1 & -g_2 & h_3 \end{pmatrix} \begin{pmatrix} d & c & e \\ h_1 & -h_2 & -g_3 \end{pmatrix}. \tag{8.11}$$

For further descriptions see Refs. [SW92] and [SP86], and references therein.

8.2 Nuclear constants relevant for this work

The nuclear constants which were used to determine the magnetic hyperfine fields and to determine the temperature in the cryostat at the sample position are listed in tables 8.1 and 8.2.

isotope	nuclear angul. momentum I	nuclear moment in μ_N	half-life time
^{198}Au	2	0.5934(4)	2.696 d
^{60}Co	5	3.799(8)	5.271 y
^{54}Mn	3	3.2819(13)	312 d

Table 8.1: *Nuclear moments and half-lives of ^{198}Au, ^{60}Co and of the nuclear thermometer ^{54}Mn.*

isotope	γ energy in keV I	U_2A_2	U_4A_4
^{198}Au	412	-0.4751	-0.3709
^{60}Co	1173 1332	-0.4206	-0.2428
^{54}Mn	835	-0.4949	-0.4467

Table 8.2: *Nuclear parameters related to the γ-transitions for the present systems, where U_kA_k is the product of the deorientation parameters U_k and the angular distribution coefficients A_k of order k (see also [SP86]). For the present case, only 2nd and 4th order terms are relevant.*