Conclusion

In this thesis we have presented research on the problem of attaching labels to features in graphs, networks, diagrams, or cartographic maps. We have approached the problem from two sides. On the one hand we have developed a general framework for maximizing the number of features that receive a label by extending the classical constraint-satisfaction framework to maximum variable-subset constraint satisfaction. The features and label candidates of a label-placement problem are called variables and values, respectively, in the context of constraint satisfaction. The fact that two candidates intersect and thus should not both appear in a solution is expressed by symmetric binary constraints. Our new framework is of interest not only for label-placement problems, but for any overconstraint system where it is possible to drop some of the variables in order to find an assignment for the remaining variables that satisfies all constraints. We have proposed an algorithm, EI-1, that achieves edge-irreducibility, a new form of local consistency similar to arc-consistency in classical constraint satisfaction. EI-1 considers pairs of variables and removes their redundant values, i.e. values whose removal does not reduce the size of an optimal solution. We have also given an efficient algorithm, EI-1*, that combines EI-1 with a simple heuristic and proved to be very effective for labeling point sets.

It would be interesting to extend EI-1 to problems where constraints are not necessarily symmetric or binary, and to take priorities into account. The rules that EI-1 checks in order to achieve edge-irreducibility for all pairs of variables can easily be generalized to any constant number r of variables. However, it is not clear whether the generalized rules achieve r-irreducibility and whether there are efficient and practical algorithms that can apply these rules for a constant $r > 2$ exhaustively.

Our hope is that EI-1* or other efficient algorithms based on higher degrees of irreducibility will substitute simulated annealing and other iterative methods of gradient descent for the wide variety of problems that fit into the framework of maximum variable-subset constraint satisfaction.

In this thesis we have also investigated special cases of the label-placement problem. For labeling points with squares or with circles we have shown that the NP-hardness for picking labels from finite candidate sets carries over to infinite candidate sets, more precisely to the case where we require that the boundary of a label contains the point to be labeled. For rectangles of fixed

height we have given a corresponding positive result, namely a polynomial-time approximation scheme (PTAS) for label-number maximization. Since this scheme uses stabbing lines of equal spacing, it is not clear how to extend our result to arbitrary rectangles. Even for choosing a non-intersecting subset of maximum cardinality from n (fixed-position) rectangles only a factor-$O(\log n)$ approximation algorithm is known [AvKS98].

We have shown that one cannot expect to find a PTAS for maximizing the size of uniform circular labels. Although we proposed an efficient approximation algorithm for this problem, there is still a large gap between the approximation factor of about 1/20 that we have shown for our algorithm and the factor 1/2 that we conjecture to be best possible.

While the hardness results and approximation schemes above are predominantly of theoretical interest, the algorithm EI-1* for features with a constant number of label candidates and the greedy algorithm for sliding rectangular labels, our method for labeling polygonal chains, and our toolbox for designing flexible geometric algorithms can and should be applied in practice.

The point-labeling algorithm Rules, which is described in Section 3.2, is actually used by the city authorities of Munich for labeling ground-water drill holes. We also cooperated with a company that wants to label tourist shops in Berlin. This company plans to offer the following on-line service[2]. A user can select a subset of the shops according to what articles (s)he is interested in. After pressing the submit button, the corresponding shops in the current map area are labeled with their names. More information on a shop can be obtained by clicking on its label.

[2] `http://von-kunst-bis-krimskrams.de/`