Essays in Macroeconomics

DISSERTATION

submitted for the degree of

Doctor Rerum Politicarum (Dr. rer. pol.)

Flora Budianto

Freie Universität Berlin
School of Business and Economics

Referees:

Prof. Dr. Mathias Trabandt
Freie Universität Berlin

Prof. Lutz Weinke, Ph. D.
Humboldt–Universität zu Berlin

Date of defense: July 5, 2019 May 2019

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is entirely the outcome of my own work.

Flora Budianto

Acknowledgements

Writing this thesis has been an exciting, creative and – sometimes – challenging journey. I would like to thank all those who joined and supported me along my way. First and foremost, I would like to thank Mathias Trabandt. I am indebted to Mathias for his unfailing support, encouragement and guidance, as well as for being a kind and dedicated advisor and a source of inspiration and motivation. During my time at Freie Universität Berlin, I have grown as a person and economist and I have learned so much that goes beyond the pages of this thesis.

I would also like to thank Lutz Weinke and Sebastian Schmidt. My second advisor Lutz Weinke was my teacher from the early days, laying the groundwork for this thesis. I am very grateful to Lutz for his continuous support over the years and, especially, for his commitment, help and enthusiasm during my job market and the last phase of the dissertation. Sebastian has brought me to the advanced methods in contemporary macroeconomics which became an essential part of this thesis. I am very grateful to Sebastian for his invaluable advice and support, as well as for many inspiring discussions and helpful comments. I am especially thankful to Sebastian for his support during my job market.

In addition, I benefited from discussions with many researchers who crossed my path at Freie Universität Berlin, Humboldt-Universität zu Berlin, European Central Bank and at conferences and seminars. I would particularly like to thank Drago Bergholt, Martin Bodenstein, R. Anton Braun, Lawrence Christiano, Günter Coenen, Fiorella De Fiore, Michael Ehrmann, Martin Eichenbaum, Philipp Engler, Francesco Furlanetto, Jordi Galí, Frank Heinemann, Ivan Jaccard, Aikaterini Karadimitropoulou, Jesper Lindé, Helmut Lütkepohl, Junior Maih, Cristina Manea, Gernot Müller, Dieter Nautz, Johannes Pfeifer, Stephanie Schmitt-Grohé and Lars Winkelmann.

Part of the research was financially supported by the Research Training Group 1659 "Interdependencies in the Regulation of Markets". Moreover, I am grateful to the European Central Bank (ECB) for their hospitality during my research stay in 2017. I would like to thank my mentors at the ECB Ivan Jaccard and Sebastian Schmidt. Thank you also to Mathias Trabandt for supporting my temporary leave from Freie Universität Berlin.

I am very grateful to my colleagues and friends for important support, inspiring discussions, practical help, as well as for sharing many valuable moments together. I would particularly like to thank Hannah Liepmann, Pablo Anaya, Daniel Nachtigall, Catalina Martinez Hernandez, Gregor Böhl, Grzegorz Dlugoszek, Simon Jukartis, Emanuel Gasteiger, Wenjuan Chen, Christian Höynck, Adriana Grasso, Jorgo Georgiadis, Simon Voigts, Caterina Forti Grazzini, Martín Harding, Sandra Pasch, Stefan Gebauer, Daniel Bierbaumer, Thore Kockerols, Sven Hartjenstein, Falk Mazelis and Maximilian Muhn. I am especially thankful to Hannah for sharing our ups and downs over the years, to Pablo for always having my back, and to Gregor for being a calming voice in turbulent times and especially during my job market.

Furthermore, I would like to thank those who worked with me at the Chair of Macroeconomics at Freie Universität Berlin. Stefan Hasenclever, Nicki Kämpf, Fabian Seyrich and Giorgio Varanini have always provided excellent, important and helpful research assistance. I am very grateful to Anemeri Bennemann Frohberg not only for practical help but above all for guiding my way over the years with her positive energy and encouragement.

Last but not least, I would like to thank my family who has always been a constant source of comfort and encouragement for me. Looking past, I was blessed with your unconditional support, love and faith throughout this journey. Looking forward, I will be very happy to share what comes with you.

Abstract

This dissertation consists of three essays which investigate the *economic implications of monetary and fiscal policy*, with an emphasis on times of severe recessions. In the first essay, I study the implications of a higher central bank's inflation target as a policy tool to reduce the risk of hitting the zero lower bound (ZLB) on nominal interest rates. As a novel and surprising result, I find that higher inflation targets can, in fact, increase the risk of hitting the ZLB and produce deep recessions more often. Using a standard New Keynesian model, I show that a higher inflation target changes the price-setting behavior of firms in a substantial way. Specifically, firms become more forward-looking, inflation is more volatile and, thereby, the nominal interest rate fluctuates more. I show that even with more "room-to-manoeuvre" for the nominal interest rate due to a higher inflation target, the higher volatility of the nominal interest rate implies that the economy ends up – on net – more often at the ZLB. In the second essay, I analyze the implications of a temporary cut of the value-added tax rate (VAT) in deep recessions with a binding ZLB on nominal interest rates. Standard New Keynesian models predict that a temporary VAT cut has adverse effects at the ZLB. However, I show that a temporary VAT cut has highly expansionary effects in normal times and also in deep recessions once I account for a more realistic consumption basket, including non-durable and durable goods. The reason is that purchases of durable goods are highly intertemporally substitutable - consumers will stock up on storable goods when prices are currently low. Most interestingly, I find that the boom in demand for durables affects the demand for non-durables: consumption of durables and non-durables increase in response to the temporary VAT cut. In both essays, I use global solution techniques to correctly account for the effects from uncertainty on the solution of the model. I propose an algorithm that is considerably faster and more efficient compared to the standard implementation of the method in the literature. In the third essay, I describe the algorithm and discuss the performance of my approach based on an application for a New Keynesian model with a ZLB constraint on the nominal interest rate. The main innovation of the algorithm is to show that the model can be represented as a system of equations over the pre-defined grid. This way, it is possible to determine the solution of the model in one computation step instead of solving for each grid point individually.

Keywords: zero lower bound, inflation targets, durable goods, consumption taxes, monetary policy, fiscal policy.

Zusammenfassung

Diese Dissertation besteht aus drei Aufsätzen, welche die *ökonomischen Implikationen von Geld- und Fiskalpolitik* mit einem Fokus auf schwere Rezessionen untersuchen. Der erste Aufsatz analysiert die Implikationen einer Anhebung des Inflationsziels einer Zentralbank als Politikinstrument für die Senkung des Risikos die Nullzinsgrenze zu erreichen. Als ein bislang unbeachtetes und unerwartetes Ergebnis zeigt meine Studie auf, dass ein höheres Inflationsziel das Risiko einer Absenkung des Zinses zur Nullgrenze, in der Tat, erhöhen und häufiger zu tiefen Rezessionen führen kann. Auf Basis eines klassischen neukeynesianischen Modells kann ich darlegen, dass ein höheres Inflationsziel das Preissetzungsverhalten von Firmen substantiell ändert. Firmen agieren vorwärtgerichteter, Inflation wird volatiler und der Nominalzins fluktuiert folglich stärker. In meiner Studie zeige ich, dass ein höheres Inflationsziel zwar den Handlungsspielraum für Zinsänderungen erweitert, die höhere Inflationsvolatilität aber dazu führt, dass die Nullzinsgrenze – im Endeffekt – häufiger erreicht wird. Der zweite Aufsatz untersucht die Implikationen einer temporären Absenkung der Mehrwertsteuer in tiefen Rezessionen mit bindender Nullzinsgrenze. Klassische neukeynesianische Modelle sagen voraus, dass eine temporäre Mehrwertsteuersenkung adverse Effekte an der Nullzinsgrenze hat. In meiner Studie kann ich allerdings zeigen, dass eine temporäre Mehrwertsteuersenkung sowohl in normalen Zeiten als auch in tiefen Rezessionen expansiv wirkt sofern ein realistischerer Konsumkorb berücksichtigt wird, welcher kurzlebige Verbrauchsgüter und langlebige Gebrauchsgüter beinhaltet. Der Grund ist, dass Anschaffungen von langlebigen Gebrauchsgütern intertemporal stark substituierbar sind – Konsumenten erweitern ihren Bestand an Gebrauchsgütern in Zeiten niedriger Preise. Interessanterweise kann ich auch zeigen, dass der Nachfrageboom für langlebige Gebrauchsgüter die Nachfrage nach kurzlebigen Verbrauchsgütern beeinflusst: der Konsum von langlebigen und kurzlebigen Gütern steigt in Folge einer temporären Mehrwertsteuersenkung. In beiden Aufsätzen verwende ich globale Lösungstechniken, um die Effekte von Unsicherheit auf die Lösung des Modells in korrekter Weise zu berücksichtigen. Ich schlage einen Algorithmus vor, der wesentlich schneller und effizienter ist im Vergleich zur Standardimplementierung der Methode in der Literatur. Im dritten Aufsatz beschreibe ich den Algorithmus und diskutiere die Leistungsfähigkeit meines Ansatzes basierend auf einer Anwendung für ein neukeynesianisches Modell unter der Restriktion der Nullzinsgrenze. Die wesentliche Innovation meines Algorithmus geht darauf zurück zu zeigen, dass das Modell als Gleichungssystem auf einem vordefinierten Grid dargestellt werden kann. Auf diese Weise ist es möglich die Lösung des Modells in einem einzigen Auswertungsschritt zu bestimmten anstelle das Modell für jeden Gridpunkt individuell zu lösen.

Schlagwörter: Nullzinsgrenze, Inflationsziel, langlebige Gebrauchsgüter, Konsumsteuern, Geldpolitik, Fiskalpolitik.

Table of contents

List of figures

List of tables

Chapter 1

Introduction

The primary tool that central banks have to fight recessions is to reduce interest rates so as to encourage borrowing and spending and stabilize the economy. However, during the Great Recession many advanced economies experienced that there is a natural limit to how low nominal interest rates can go: it is known as the zero lower bound (ZLB) or effective lower bound. To date, the ZLB on nominal interest rates is still a major concern for economists and policy markers as it appears to change the economic environment in many ways.

In a recent policy debate, prominent economists have proposed to raise the central bank's inflation target to increase long-run nominal interest rates and provide more "room-to-manoeuvre" for monetary policy to reduce nominal interest rates in recessions before reaching the ZLB (e.g. Blanchard et al. (2010) or Ball (2013)). The discussion whether or not to raise the inflation target was further fueled by the recent decline in the natural interest rate documented for major advanced economies. The natural interest rate is defined as the short-term real interest rate consistent with the economy operating at its full potential. A declining natural rate of interest poses real challenges for a central bank. Given that inflation and inflation expectations are stable, a lower natural interest rate is associated with lower nominal interest rates. Consequently, the ZLB would be reached more frequently, limiting the central bank's ability to stabilize the economy in recessions. In Chapter 2 titled "Inflation Targets and the Zero Lower Bound", I study the implications of a higher inflation target as a policy tool to reduce the risk of reaching the ZLB on nominal interest rates. However, as a novel and surprising result, I find that higher inflation targets can, in fact, increase the risk of reaching the ZLB and produce deep recessions more often. Using a standard New Keynesian model, I show that a higher inflation target changes the price-setting behavior of firms in a substantial way. Specifically, firms become more forward-looking at higher target rates of inflation. This reflects the increased relevance of future expected demand for today's price-setting decisions because firms anticipate that inflation will erode their real profits over time. When economic shocks are persistent, as is commonly considered in the literature to gauge the persistent effects of the financial crisis and the Great Recession, inflation becomes more volatile and, thereby, the nominal interest rate fluctuates more. I show that even with more "room-to-manoeuvre" for the nominal interest rate due to a higher inflation target, the higher volatility of the nominal interest rate implies that the economy ends up - on net - more often at the ZLB. Overall, my main findings adds a new angle to the policy debate over whether the inflation target should be raised. In fact, my results caution against raising the inflation target while neglecting the consequences for the behavior of households and firms.

Recent research suggests that fiscal policy has very different effects when nominal interest rates have reached the ZLB (e.g. Eggertsson (2010)). One particular finding is that fiscal policy that targets the supply side can have negative macroeconomic effects if the policy leads to deflationary pressures at the ZLB. If the central bank cannot adjust the nominal interest rate, deflationary pressures translate into an increase of the real interest rate, contracting demand. Chapter 3 titled "Are Consumption Tax Cuts Expansionary in a Liquidity Trap?" investigates the macroeconomic effects of a temporary cut of the value-added tax rate (VAT). This study is motivated by a fiscal experiment in the United Kingdom (UK) from 2008–2009. In an effort to stimulate private demand in the onset of the global financial crisis, the UK government lowered the standard VAT rate from 17.5% to 15% between December 2008 and December 2009. The VAT is a general consumption tax that is collected incrementally along the supply chain. Retailers pay the VAT on their sales but can reclaim the VAT paid on goods and services purchased for use in earlier stages of production. Ultimately, the tax burden will be paid by the final consumer. Consumers typically pay the final sales price which is inclusive of the VAT. The particular set-up of the VAT system can lead to deflationary pressures in response to a tax cut. A lower VAT rate will only affect final consumers if retailers are able to adjust their final sales price inclusive of the VAT. If a VAT cut is not immediately passed on to final prices, consumers will expect a period of falling prices following the tax cut.

First, I study the implications of a temporary reduction in the VAT rate in a standard New Keynesian model and show that this policy is expansionary in normal times but contractionary when nominal interest rates are constrained by the ZLB. A potential problem is that the standard New Keynesian model only accounts for consumption of non-durable goods. However, in countries that levy VAT, consumer durables typically represent roughly 40% of total consumption expenditures on goods and services that are subject to the VAT. Second, I revisit the analysis in a New Keynesian model that allows for non-durable goods and durable goods consumption. Using the extended model, I find that a temporary VAT cut is expansionary in normal times *and* in deep recessions when the ZLB is reached. The reason is that purchases of durable goods are highly intertemporally substitutable. Consequently, consumers will stock up on storable goods in response to the VAT cut. However, most interestingly, I find that the boom for durable goods affects the demand of non-durables. Hence, consumption of both durable goods *and* non-durable goods increase. A temporary VAT cut in a recession would increase the demand for durable goods and dampen the fall in employment and wages. Higher aggregate wages prevent marginal cost of firms and prices to deteriorate. I find that this effect is sufficiently strong to reduce deflationary pressures and dampen the rise in the real interest rate. My results show that a simple New Keynesian model neglects an important transmission channel to study the effects of consumption tax policies. In fact, the analysis points to the possibility that fiscal policy that seeks to stimulate consumption via the supply side can be expansionary even in a low interest rate environment.

The ZLB constraint on nominal interest rates can be interpreted as an occasionally binding constraint in macroeconomic models. Solving models with occasionally binding constraints poses a non–trivial challenge to contemporary macroeconomic analysis. The major problem with models that feature non-linear elements such as the ZLB constraint is that they typically have no closed-form solution and, hence, require alternative solution strategies. For the analysis in Chapter 2 and 3, I use global solution techniques to solve the New Keynesian models subject to the ZLB constraint on the nominal interest rate. The global solution method is based on an approximation of the model's solution on a finite grid. An important advantage of this approach is that it is able to accurately take into account the effects from future shock uncertainty for the solution of the model. A growing body of work shows that future shock uncertainty can have non-negligible implications for the decision functions of households and firms (e.g. Fernández-Villaverde et al. (2015), Gust et al. (2017), Nakata (2016) or Richter and Throckmorton (2017)). However, the main drawback of grid–based solution methods is typically the high computational burden. I propose an algorithm that is considerably faster and more efficient compared to standard implementation of the method in the literature.

Chapter 4 titled "An Efficient Algorithm to Solve DSGE Models with Occasionally Binding Constraints" describes the algorithm and discusses the performance of my approach based on an application for a New Keynesian model with a ZLB constraint on the nominal interest rate. The model features habit formation and price and wage rigidities. The algorithm is based on a policy function iteration method with time iteration. I use a collocation method to interpolate the value of the policy function between grid points and extrapolate it outside the grid, based on Miranda and Fackler (2002) or Schmidt (2013). The main innovation of my algorithm is to show that the model can be represented as a system of equations over the pre-defined grid. This way, I can solve for the complete unknown policy function in one computation step instead of solving for the policy function on each grid point individually. The solution method is fast even without parallel computing. I provide detailed results of accuracy tests and solution times for alternative specifications of the New Keynesian model. Overall, I can show that the algorithm can solve the model with many state variables fast and accurately.

Chapter 2

Inflation Targets and the Zero Lower Bound

Does a higher inflation target help to reduce the risk of hitting the zero lower bound (ZLB) on nominal interest rates? Recently, higher inflation targets for central banks have been proposed to allow for more "room-to-manoeuvre" in deep recessions. Advocates of this proposal suggest an inflation target of 4% to reduce the risk of hitting the ZLB. I show that the opposite may happen: a 4% inflation target can, in fact, increase the risk of hitting the ZLB compared to a 2% inflation target. Using a standard New Keynesian model, a higher inflation target changes the price-setting behavior of firms in a substantial way. Specifically, firms become more forward-looking, inflation is more volatile and, thereby, the nominal interest rate fluctuates more. I show that even with more "room-to-manoeuvre" for the nominal interest rate due to a higher inflation target, the higher volatility of the nominal interest rate implies that the economy ends up – on net – more often at the ZLB.

2.1 Introduction

Prominent economists have advocated to raise the central bank's inflation target to increase long-run nominal interest rates and provide more "room-to-manoeuvre" for monetary policy to reduce its policy rate in recessions. For example, Blanchard et al. (2010) and Ball (2013) propose a target inflation rate of 4%, arguing that the benefits to the economy could be substantial while the costs of 4% inflation are likely to be rather small. Others have voiced concern over the potential risks associated with a higher inflation target. In his 2010 Jackson Hole speech, Ben Bernanke observed the following:

> "Inflation expectations appear reasonably well-anchored, and both inflation expectations and actual inflation remain within a range consistent with price stability. In

> this context, raising the inflation objective would likely entail much greater costs than benefits. Inflation would be higher and probably more volatile under such a policy, undermining confidence and the ability of firms and households to make longer-term plans, while squandering the Fed's hard-won inflation credibility."

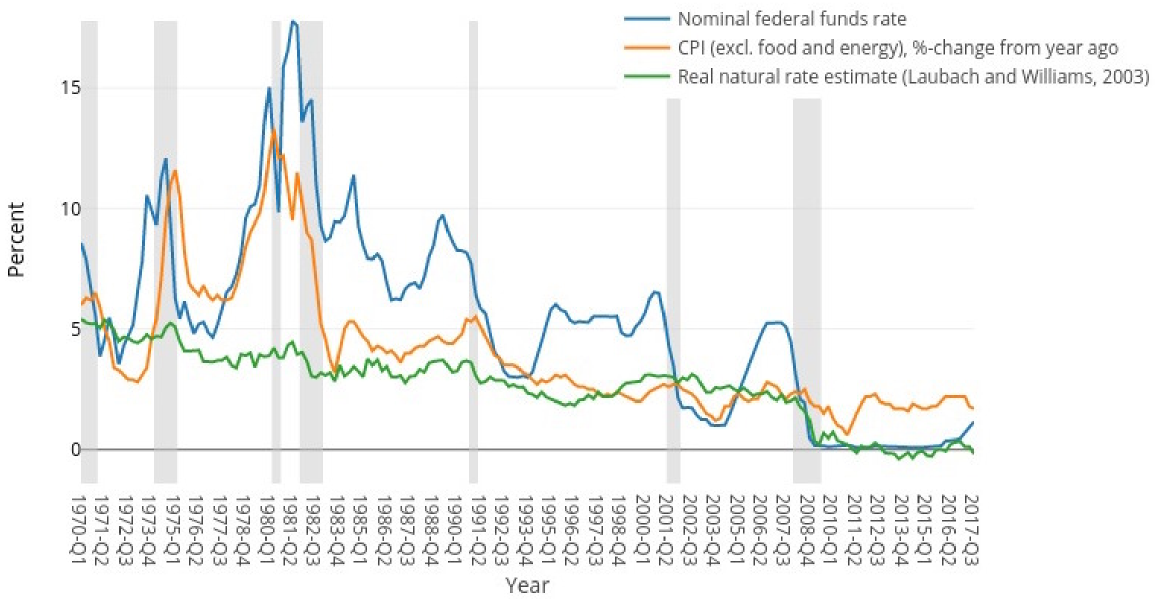
The discussion about whether or not to raise the inflation target was fueled by the recent decline in the natural real interest rate (e.g. Del Negro et al. (2017), Hamilton et al. (2015), Gagnon et al. (2016), and Eggertsson et al. (2017)). Figure 2.1 shows the evolution of the nominal effective Federal Funds Rate, inflation and an estimate of the real natural interest rate for the United States. Given that inflation has been fairly stable in the past 20 years, lower long-run real interest rates lead to lower average nominal interest rates. As a consequence, the ZLB would be reached more frequently, hampering the ability of monetary policy to stabilize the economy. Proponents of higher inflation targets argue that higher long-run nominal interest rates will provide more leeway for monetary policy.

In light of this debate, the central question I seek to answer in this Chapter is whether higher target rates of inflation *necessarily* reduce the probability of hitting the ZLB. To this end, I investigate the implications of higher inflation targets for the behavior of households and firms and the consequences for monetary policy - especially, when the economy is hit by large adverse shocks.

In macroeconomic models with nominal rigidities, positive trend inflation changes the price-setting behavior of firms (see e.g. Kiley (2007), Ascari and Ropele (2009) or Ascari and Sbordone (2014)). Firms become more forward-looking at higher target rates of inflation because they anticipate that inflation will erode their real profits over time. When economic shocks are persistent, as is commonly considered in the literature to gauge the persistent effects of the financial crisis and the Great Recession, inflation becomes more volatile. So far, the literature has not considered the implications of higher inflation volatility caused by higher inflation targets in deep recessions with a binding ZLB.

Intuitively, higher inflation targets generate the following trade-off. On the one hand, higher steady state inflation increases long-run nominal interest rates, allowing for larger decreases of nominal interest rates before the ZLB starts to bind. On the other hand, when higher inflation targets are a source of increased inflation volatility, larger cuts in the nominal interest rate are required to stabilize the economy in the wake of adverse macroeconomic developments.

Fig. 2.1 The Nominal FED Funds Rate, Consumer Price Index (CPI) and the Real Natural Rate

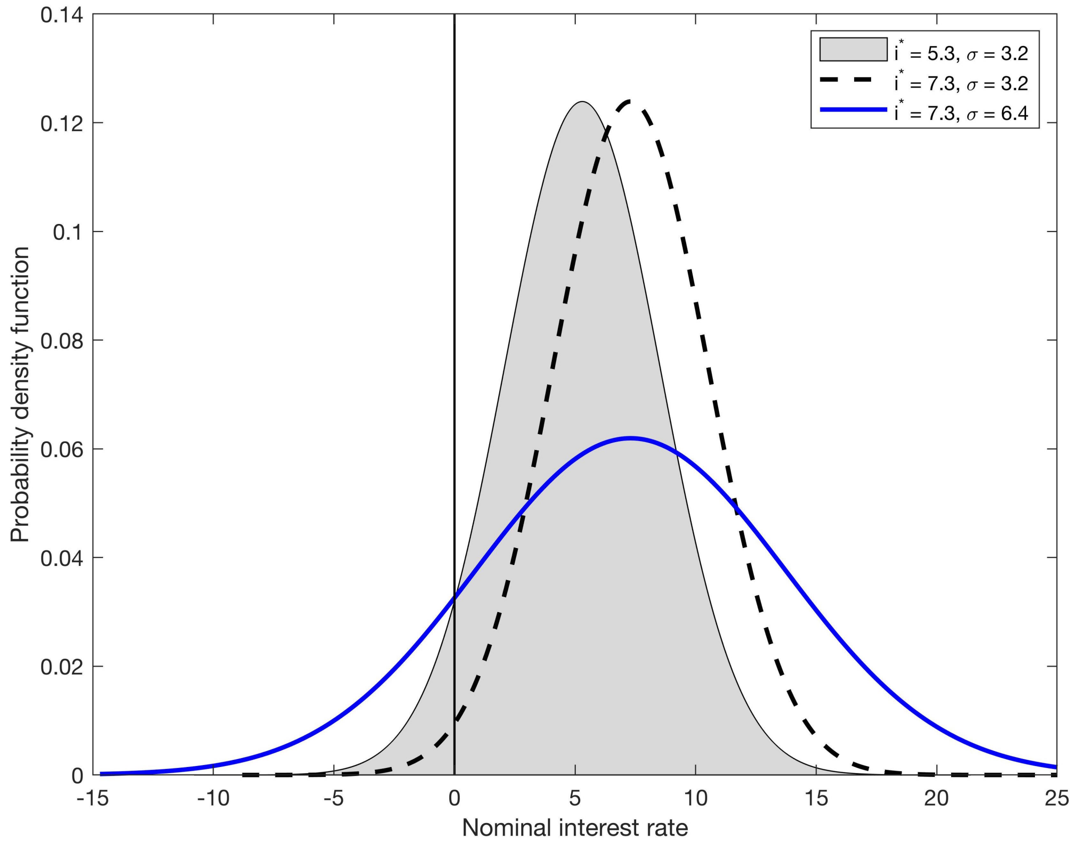


Source: FRED

These considerations can be illustrated in a simple experiment. Figure 2.2 shows illustrative probability density functions for the short-term nominal interest rate. In the benchmark case, the mean of the distribution equals the average of the federal funds rate from 1970 to 2017 (gray shaded curve). The variance, say σ^2, is chosen such that the probability of hitting the ZLB is 5% as in Reifschneider and Williams (2000). The vertical line indicates the nominal interest rate at zero percent. All realizations to the left of the vertical line represent instances in which the ZLB is binding. Suppose an increase in the inflation target from 2% to 4% moves the mean of the distribution by 2 percentage points but the variance is kept unchanged (black dashed line). It is straightforward to see that ZLB episodes become less likely in this case. However, if a higher inflation target changes the mean and also the variance of the density – as perhaps a result of the change in the behavior of households and firm – the distribution becomes more dispersed (blue solid line). Specifically, I assume that the standard deviation of the nominal interest rate doubles. In Figure 2.2, the level increase in the mean of the nominal interest rate is dominated by the increase in the volatility of the distribution. All told, the ZLB binds more often.

In this Chapter, I examine the implications of higher inflation targets in a standard New Keynesian model. The model features Calvo (1983) price setting frictions. Following Ascari

Fig. 2.2 Probability Density Functions for the Nominal Interest Rate



and Sbordone (2014), the model abstracts from price indexation. In the model, a higher inflation target is a source of larger fluctuations of inflation in response to economic shocks. Based on the model, I provide an example in which larger inflation volatility has a first-order effect on the probability of hitting the ZLB. In this case, raising the inflation target would be associated with *more* frequent rather than less frequent episodes at the ZLB. Hence, adopting a higher inflation target is ineffective in reducing the probability of hitting the ZLB and aggravates the effects of large adverse shocks on the economy.

I develop and discuss the results of this Chapter analytically and numerically. The analytical part is useful to understand the intuition. For tractability, I follow Eggertsson (2010) or Eggertsson and Woodford (2003) in assuming a two-state Markov chain process for the economic shocks. This way, it is possible to represent the equilibrium conditions by aggregate supply (AS) and aggregate demand (AD) schedules. While a positive inflation target does not affect the aggregate demand side directly, it can change the slope of the AS curve. Specifically, higher rates of steady state inflation imply a steeper AS curve, triggering larger fluctuations of inflation in response to variations in demand.

The intuition for this result is as follows. Price-setters anticipate that they may be stuck with today's price for some period of time while their real profits erode over time due to future inflation. The higher steady state inflation, the higher the degree of forward-lookingness for today's price decisions, reflecting the increased relevance of future expected demand. Assume that the economy is hit by a large and persistent recessionary demand shock such that firms expect future demand to be low. In this case, as a higher inflation target makes firms more forward-looking, firms tend to cut prices more aggressively in order to maximize the net present value of real profits. All else equal, the expectation of a persisting contraction implies a stronger period of disinflation the higher the target rate of inflation.

The numerical section reports the results based on stochastic simulations of the model. In this section, I assume an AR(1) process for the economic shocks and revisit the frequency and average duration of ZLB episodes for the benchmark case at 2% and an alternative target of 4%. The model is calibrated to generate a ZLB probability of around 5% for the baseline scenario at 2% (Reifschneider and Williams (2000)). I find that the likelihood of ZLB events increases to 7.2% when the target rate is 4%. Moreover, the duration of a ZLB episode is 2.5 quarters longer at 4% inflation compared to the baseline case. The quantitative results are sensitive to the persistence of the shocks, the degree of price rigidity and the degree of inflation indexation. I provide an extensive sensitivity analysis in a separate section.

For the stochastic simulations, I solve the model using a global solution technique to account for future shock uncertainty affecting the decision rules of households and firms. The solution approach is based on a collocation method (see e.g Judd (1998), Miranda and Fackler (2002) and Schmidt (2013)). Based on this method, I develop an algorithm that exploits the fact that the model equations can be represented as a system of difference equations over a pre-defined grid. The major advantage of this approach is a significant speed improvement compared to the standard implementation of the algorithm for this global solution method.

All told, my main findings shed new light on the ongoing debate about inflation targets and the ZLB. Based on the conjecture in the Jackson Hole speech by Ben Bernanke (2010), I investigate the implications of higher inflation targets for the behavior of firms and households. My results caution against proposals to raise the inflation target as a policy tool to reduce the risk of ZLB episodes. My analysis highlights the importance to keep in mind that a higher inflation target moves the mean and, more importantly, the variance of the distribution of the nominal interest rate. I show that because of a change in the firms' price-setting behavior, the effect on the variance dominates the effect on the mean of the nominal interest rate.

The remainder of the Chapter is organized as follows. Section 2.2 provides a review of related literature. Section 2.3 develops the New Keynesian that I use in my analysis. Section 2.4 presents the analytical analysis, and Section 2.5 reports the simulation results from the numerical analysis. Section 2.6 provides a sensitivity analysis. Finally, Section 2.7 concludes.

2.2 Related Literature

A substantial body of work has analyzed the potential for the ZLB to impede macroeconomic performance. Starting in the early 1990s, Summers (1991) raised concerns that, as nominal interest rates cannot fall below zero, monetary policy faces a trade-off between achieving low average inflation and macroeconomic stability given that the latter occasionally requires negative real interest rates to offset contractionary disturbances. Among others, Orphanides and Solow (1990), Reifschneider and Williams (2000), Coenen et al. (2004), Williams (2009), and Kiley and Roberts (2017) consider and quantify the effects of the ZLB in structural and semi-structural models.

Reifschneider and Williams (2000) and Kiley and Roberts (2017), in particular, study the consequences of higher steady state nominal interest rates for the likelihood of hitting the ZLB. Based on stochastic simulations of the FRB/US model using historical shocks, Reifschneider and Williams (2000) analyze how the steady state distributions of macroeconomic variables vary when policymakers adopt different inflation targets. Their findings suggest that the zero lower bound binds around 10% of the time at an inflation target of 0%. Raising the inflation target rate to 2% would reduce the ZLB risk to 5%. At an inflation target of 4%, the federal funds rate would be at ZLB only 1% of the time.

Similarly, the results in Kiley and Roberts (2017) also lend support to the idea that higher inflation targets may be beneficial. These authors find that the ZLB binds more frequently, as much as 40% of the time. The ZLB constraint makes it harder for monetary policy to achieve its 2% inflation objective. As a result, Kiley and Roberts (2017) suggest that policymakers seek an inflation target near 3%.

The complexity of the FRB/US model makes it difficult to trace out the underlying key mechanisms that give rise to the findings in Reifschneider and Williams (2000) and Kiley and Roberts (2017). One important difference, however, is the absence of price indexation in my model compared to previous work. Price indexation reduces the degree of forward-lookingness of price-setting firms as prices are set mechanically. Consequently, with full indexation, inflation volatility is unaffected at positive rates of steady state inflation. In my analysis, I choose to abstract from price indexation in the model as there is little to no empirical micro-evidence that firms adjust their prices mechanically. I dwell deeper on this issue in Section 2.6. In addition, Kiley and Roberts (2017) solve the model under the assumption of certainty equivalence (perfect foresight) whereas I allow for shock uncertainty to affect the model solution using global solution methods.

Several papers have studied the optimal inflation target in light of the ZLB likelihood and the economic costs associated with higher inflation. Ngo (2018) finds that the probability of hitting the ZLB probability falls monotonically in the inflation target when the central maximizes household welfare. In difference to Ngo (2018), I abstract from price indexation and consider persistent demand shocks. Both features are crucial for my perhaps surprising result that a higher inflation can increase the risk of hitting the ZLB. He finds that the optimal target rate is greater than 2% when the unconditional probability of hitting the ZLB is larger than 2.5%. Schmitt-Grohé and Uribe (2010) find that the optimal inflation rate is around 0%. However, they assume that the central bank is able to commit to policy plans such that the ZLB risk is zero. Coibion et al. (2012) report that the optimal inflation rate is around 1.5%. I conduct a positive analysis of the implications of higher inflation targets. I show that a higher inflation target can be associated with substantial welfare losses as it increases the probability of deep recessions with a binding ZLB.

A possible concern of higher target rates of inflation is the higher risk of unanchoring of inflation expectations (see e.g. Bundesbank (2018)). An extensive literature studies the role of trend inflation for equilibrium determinacy (e.g. Kiley (2007), Ascari and Ropele (2009), Ascari and Sbordone (2014) or Arias et al. (2017)). The general consensus is that higher steady state inflation shrinks the determinacy region, requiring monetary policy to act more aggressively in response to economic disturbances.

2.3 The Model

This section presents the basic New Keynesian DSGE model with positive trend inflation as in Ascari and Sbordone (2014). In the baseline model, I abstract from price indexation. The role of price indexation will be discussed in the sensitivity analysis in Section 2.6.

Households. The representative household maximizes expected lifetime utility of consumption, C_t, and labor, N_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left\{ \ln C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right\} \tag{2.1}$$

subject to the period-by-period budget constraint:

$$P_t C_t + B_t \leq (1+i_{t-1}) B_{t-1} + P_t w_t N_t + \Gamma_t. \tag{2.2}$$

Here, P_t is the price of the final good, B_t represents holdings of a one-period bond with nominal return i_t, w_t is the real wage, and Γ_t denotes firms' profits that are distributed to households. ζ_t is an exogenous discount factor shock. The household optimization problem yields the following first-order conditions:

$$w_t = N_t^{\varphi} C_t \tag{2.3}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \delta_{t+1} \right] \tag{2.4}$$

where $1+\pi_t \equiv P_t/P_{t-1}$ denotes the (gross) inflation rate and $\delta_{t+1} \equiv \frac{\zeta_{t+1}}{\zeta_t}$.

Final good firms. A final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{j,t}$ indexed by $j \in (0,1)$ and a production function, $Y_t = \left(\int_0^1 Y_{j,t}^{(\varepsilon-1)/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}$, with $\varepsilon > 1$. Taking prices as given, the representative final good producer chooses quantities of intermediate goods $Y_{j,t}$ to maximize profits, resulting in the following demand function:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{2.5}$$

where $P_{j,t}$ is the price for intermediate good j. The aggregate price index is $P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj\right)^{1/(1-\varepsilon)}$.

Intermediate goods firms. Intermediate inputs $Y_{j,t}$ are produced by a continuum of monopolistically competitive firms indexed by $j \in (0,1)$ using the linear production technology $Y_{j,t} = N_{j,t}$. Intermediate good producers face price setting frictions as in Calvo (1983). In each period, a firm can reoptimize its nominal price, defined as $P_{j,t}^*$, with fixed probability $1-\theta$. With probability θ, the firm keeps its price unchanged. Firms do not index their price to either past inflation or trend inflation. The firm chooses its optimal re-set price P_t^* to maximize profits

$$E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} - w_{t+i} \right) Y_{j,t+i} \tag{2.6}$$

subject to the demand schedule in Eq. (5). $\Lambda_{t,t+i} \equiv \beta \frac{C_t}{C_{t+i}}$ is the stochastic discount factor, evaluating future profits in utility units since the household is the owner of the firm. Note that in the model, the real wage equals real marginal cost of the firm. Finally, the aggregate price level evolves as:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \tag{2.7}$$

The solution to the profit maximization problem can be written as:

$$\frac{P_{j,t}^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon} w_{t+i} Y_{t+i}}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon - 1} Y_{t+i}}. \tag{2.8}$$

Eq. (2.8) can be used to show that trend inflation changes the price-setting behavior of firms (see for example Kiley (2007), Ascari and Ropele (2009) or Ascari and Sbordone (2014)). Note that $\frac{P_{t+i}}{P_t} = (1+\pi_{t+i}) \times \cdots \times (1+\pi_{t+1}))$. Thus, firms use future expected inflation rates to discount future marginal cost that is, $\left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon}$ and $\left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon-1}$ in the nominator and denominator of Eq. (2.8), respectively. The higher future expected inflation, the larger the weight on future expected marginal cost. Hence, firms effectively become more forward-looking, placing more weight on future rather than on current economic conditions. I will discuss the implications of this feature of the model in further detail in the next sections.

Price dispersion. Price dispersion is an important characteristic of models with staggered price setting. For intermediate good firm j, it holds that $Y_t \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} = N_{j,t}$. Integrating over j and rearranging yields:

$$Y_t = \frac{1}{s_t} N_t \tag{2.9}$$

with

$$s_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} dj. \tag{2.10}$$

The variable s_t measures the relative price dispersion across intermediate firms. s_t is bounded from below by unity (e.g. Yun (1996), Christiano et al. (2011) or Schmitt-Grohé and Uribe (2007)) and can be interpreted as a resource cost: the higher is the dispersion in relative prices, the more labor input is needed to produce a given level of output. It is straightforward to see that s_t evolves as:

$$s_t = (1-\theta)\, \frac{P^{*}_{j,t}}{P_t}^{-\varepsilon} + \theta\, \pi_t^{\varepsilon}\, s_{t-1}. \tag{2.11}$$

Market clearing. The market clearing conditions in the goods market and the labor market are:

$$Y_t = C_t \tag{2.12}$$

$$N_t = \int_0^1 N_{j,t}\, dj. \tag{2.13}$$

Monetary policy. The central bank sets the short-term nominal interest rate according to a Taylor rule subject to the ZLB:

$$1 + i = \max\left\{1, (1+\bar{i}) \left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\overline{\overline{Y}}}\right)^{\phi_Y}\right\} \tag{2.14}$$

where ϕ_π and ϕ_Y denote the response coefficients to inflation and output, respectively. $\bar{i}$ represents the steady state nominal interest rate, which is given by $\bar{i} \equiv \frac{1+\overline{\pi}}{\beta} - 1$. $\overline{Y}$ is steady state output.

2.3.1 The Log-Linearized Model

This section provides the log-linear approximations of the equilibrium conditions around a steady state with a positive inflation target, i.e. $\overline{\pi} > 0$. In Section 2.6, I will show results based on the non-linear model.

Imposing the resource constraint $Y_t = C_t$, the Euler Equation is given by:

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) - E_t\widehat{\delta}_{t+1} \tag{2.15}$$

where $\widehat{\pi}_t$ denotes the log-deviation of gross inflation and $\widehat{i}_t$ the log-deviation of the nominal interest rate from its steady state. Similarly, for the monetary policy rule, we have:

$$\widehat{i}_t = \max\left\{-\bar{i}, \phi_\pi\widehat{\pi}_t + \phi_y\widehat{Y}_t\right\} \tag{2.16}$$

where $\bar{i}$ is the steady state nominal interest rate.

The New Keynesian Phillips curve is derived by log-linearizing a recursive formulation of Eq. (2.8) and combining it with the log-linearized expressions of Eqs. (2.7), (2.9) and (2.11). The New Keynesian Phillips curve consists of two equations that describe the dynamics of inflation, $\widehat{\pi}_t$, and the evolution of the present discounted value of marginal cost, $\widehat{\psi}_t$:

$$\widehat{\pi}_t = \beta\alpha(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})[(1+\varphi)\widehat{Y}_t + \varphi\widehat{s}_t] + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} \tag{2.17}$$

$$\widehat{\psi}_t = (1-\beta\,\theta\,\overline{\pi}^{\varepsilon})\left((1+\varphi)\widehat{Y}_t + \varphi\widehat{s}_t\right) + \beta\,\theta\,\overline{\pi}^{\varepsilon}E_t\left(\widehat{\psi}_{t+1} + \varepsilon\,\widehat{\pi}_{t+1}\right) \tag{2.18}$$

with the following expressions:

$$\alpha(\overline{\pi}) = 1 + \varepsilon\,\overline{\pi}\,(1-\theta\,(1+\overline{\pi})^{\varepsilon-1}) > 0 \tag{2.19}$$

$$\kappa(\overline{\pi}) = \frac{(1-\theta\beta\,(1+\overline{\pi})^{\varepsilon})(1-\theta\,(1+\overline{\pi})^{\varepsilon-1})}{\theta\,(1+\overline{\pi})^{\varepsilon-1}} > 0 \tag{2.20}$$

$$\eta(\overline{\pi}) = \beta\,\overline{\pi}\,(1-\theta\,(1+\overline{\pi})^{\varepsilon-1}) > 0 \tag{2.21}$$

Finally, the evolution of price dispersion s_t is derived from log-linearizing Eq. (2.11):

$$\widehat{s}_t = \theta\,(1+\overline{\pi})^{\varepsilon}\,\widehat{s}_{t-1} + \left[\frac{\varepsilon\,\theta\,(1+\overline{\pi})^{\varepsilon-1}}{1-\theta\,(1+\overline{\pi})^{\varepsilon-1}}\,\overline{\pi}\right]\widehat{\pi}_t. \tag{2.22}$$

Note that for $\overline{\pi} = 0$, it holds that $\eta(\overline{\pi}) = 0$, $\alpha(\overline{\pi}) = 1$ and $\widehat{s}_t \approx 0$. In this case, the New Keynesian Philipps curve reduces to:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\theta\beta)(1-\theta)}{\theta}(1+\varphi)\widehat{Y}_t. \tag{2.23}$$

2.4 Analytical Results

This section presents the key analytical results. For simplicity and analytical tractability, I assume that $\varphi = 0$ in this section, i.e. linear labor disutility. This assumption simplifies the specification of the New Keynesian Phillips curve.[1] In the numerical analysis of Section 2.5, I allow for the more general case of $\varphi > 0$.

A recursive formulation of the New Keynesian Phillips curve. Iterating the New Keynesian Phillips curve in Eq. (2.17) forward yields:

$$\widehat{\pi}_t = \sum_{j=0}^{\infty} \beta^j \alpha(\overline{\pi})^j E_t \left[\kappa(\overline{\pi})(1+\varphi)\widehat{Y}_{t+j} + \eta(\overline{\pi})\beta\,\widehat{\psi}_{t+1+j}\right] \tag{2.24}$$

In this analysis, I focus on low to moderate values for the inflation target, $\overline{\pi}$. In particular, assume that $1 \leq 1+\overline{\pi} < (\theta)^{-\frac{1}{\varepsilon-1}}$, $1 \leq 1+\overline{\pi} < (\theta\beta)^{-\frac{1}{\varepsilon}}$.[2] Furthermore, assume $\beta\alpha(\overline{\pi}) < 1$ such that the infinite sum in (2.24) converges to a finite number. Then, it holds that $\alpha(\overline{\pi}) \geq 1$ implying that the weight of future expected demand and marginal cost in determining current inflation, $\widehat{\pi}_t$, increases progressively in j.

More generally, a higher trend inflation alters the firms' price-setting behavior: an increase in $\overline{\pi}$ *decreases* the weight on current demand (i.e. $\partial\kappa(\overline{\pi})/\partial\overline{\pi} < 0$) and *increases* the weights on future expected demand and marginal cost (i.e. $\partial\alpha(\overline{\pi})/\partial\overline{\pi} > 0$, and $\partial\eta(\overline{\pi})/\partial\overline{\pi} > 0$).[3] Hence, the contemporaneous relationship between inflation and output weakens and inflation becomes less sensitive to variations in current demand. Instead, firms become effectively

[1] When labor disutility is linear, the labor supply equation (2.3) is independent of hours worked and also of the measure of relative price dispersion, s_t. Consequently, the variable s_t, which is a predetermined variable, does not affect the dynamics of inflation and output. It only determines the evolution of labor.

[2] These conditions ensure that $\alpha(\overline{\pi}) > 0$, $\kappa(\overline{\pi}) > 0$ and $\eta(\overline{\pi}) > 0$.

[3] For a formal proof, see Ascari and Ropele (2009).

more forward-looking, placing more weight on future than on present economic activity.

The exogenous process. In this section, I assume that the exogenous process $\widehat{\delta}_t$ follows a two-state Markov chain as in Eggertsson and Woodford (2003) and Eggertsson (2010). In the short run the discount factor shock is $\widehat{\delta}_t = \widehat{\delta}_S$ and reverts back to steady state, $\widehat{\delta}_t = 0$, with probability $1 - \mu$ in each period. This approach allows to derive the main results analytically and represent the equilibrium conditions of the model as Aggregate Supply (AS) and Aggregate Demand (AD) schedules. Let T denote the period in which $\widehat{\delta}_t$ has reverted to the steady state. Hence, the time $t \geq T$ denotes the long run while $t < T$ is the short run. I focus on a long-run equilibrium at positive interest rates (i.e. $\bar{i} = \frac{1+\bar{\pi}}{\beta} - 1 > 0$) and $\widehat{Y}_t = \widehat{\pi}_t = \widehat{s}_t = \widehat{\psi}_t = 0$.[4]

First, consider a large contractionary discount factor shock, $\widehat{\delta}_S$, such that the ZLB is binding:

$$\widehat{\delta}_S > \widehat{\delta}^*(\overline{\pi}), \tag{2.25}$$

where $\widehat{\delta}^*(\overline{\pi}) = [\phi_\pi\, \omega_\pi(\overline{\pi}) + \phi_Y\, \omega_Y(\overline{\pi})]^{-1}\, \bar{i}$ and the analytical expressions for $\omega_\pi(\overline{\pi})$ and $\omega_Y(\overline{\pi})$ are shown in Table 1. Then, there exists a locally unique bounded equilibrium at zero interest rate in the short run such that $\widehat{i}_t = -\bar{i}$ and $\widehat{\pi}_t = \widehat{\pi}_S^{ZLB}$, $\widehat{Y}_t = \widehat{Y}_S^{ZLB}$ and $\widehat{\psi}_t = \widehat{\psi}_S^{ZLB}$.[5]

Second, suppose the condition in Eq. (2.25) is not satisfied so that the ZLB is not reached. Denote this case as "normal times" (NT). Then, there exists a locally unique bounded equilibrium with $\widehat{i}_t > -\bar{i}$ and $\widehat{\pi}_t = \widehat{\pi}_S^{NT}$, $\widehat{Y}_t = \widehat{Y}_S^{NT}$ and $\widehat{\psi}_t = \widehat{\psi}_S^{NT}$. Appendix A.0.2 provides the conditions for determinacy of the rational-expectations equilibrium.

The short-run AS. For periods $t < T$, the two-state Markov chain concept implies that $E_t\widehat{\pi}_{t+1} = E_t\widehat{Y}_{t+1} = 0$ with probability $1 - \mu$ or alternatively $E_t\widehat{\pi}_{t+1} = \widehat{\pi}_t$ and $E_t\widehat{Y}_{t+1} = \widehat{Y}_t$ with probability μ. Combining the New Keynesian Phillips curve in (2.17) and the present discounted value of marginal cost in (2.18) yields the short-run AS curve:

[4] Appendix A.0.2 provides a proof showing that there exists a locally unique bounded equilibrium such that $\bar{i} = \frac{1+\bar{\pi}}{\beta} - 1 > 0$ and $\widehat{Y}_t = \widehat{\pi}_t = \widehat{\psi}_t = 0$ in $t \geq T$. Here, I implicitly assume a long-run monetary policy regime which excludes that the zero lower bound is binding as a steady state outcome (see for example Eggertsson (2010), Eggertsson and Woodford (2003)).

[5] Generally, my analysis focuses only on zero lower bound events that are driven by real disturbances and does not consider liquidity traps resulting from self-fulfilling expectations as for example in Mertens and Ravn (2010).

Table 2.1 Summary of Parameters and Analytical Expressions

Parameter	Description	Parameter	Description
$\tilde{\alpha}(\overline{\pi})$	$\alpha(\overline{\pi})+\eta(\overline{\pi})\frac{\theta\,\overline{\pi}^{\varepsilon}\,\varepsilon\,\mu}{1-\beta\,\theta\,\overline{\pi}^{\varepsilon}\,\mu}$	$\omega_{\pi}(\overline{\pi})$	$\frac{\tilde{K}(\overline{\pi})}{1-\mu+\phi_Y+\tilde{K}(\overline{\pi})\,(\phi_{\pi}-\mu)}$
$\tilde{\kappa}(\overline{\pi})$	$\kappa(\overline{\pi})+\eta(\overline{\pi})\,\mu\,\frac{1-\theta\,\overline{\pi}^{\varepsilon}\,\varepsilon}{1-\beta\,\theta\,\overline{\pi}^{\varepsilon}\,\mu}$	$\omega'_{\pi}(\overline{\pi})$	$\frac{\tilde{K}(\overline{\pi})}{1-\mu-\tilde{K}(\overline{\pi})\,\mu}$
$\tilde{K}(\overline{\pi})$	$\frac{\tilde{\kappa}(\overline{\pi})}{1-\beta\,\tilde{\alpha}(\overline{\pi})\,\mu}$	$\omega_Y(\overline{\pi})$	$\frac{1}{1-\mu+\phi_Y+\tilde{K}(\overline{\pi})\,(\phi_{\pi}-\mu)}$
		$\omega'_Y(\overline{\pi})$	$\frac{1}{1-\mu-\tilde{K}(\overline{\pi})\,\mu}$

$$\widehat{\pi}_t = \beta\,\tilde{\alpha}(\overline{\pi})\,\mu\widehat{\pi}_t + \tilde{\kappa}(\overline{\pi})\,\widehat{Y}_t \tag{2.26}$$

where the expressions for $\tilde{\kappa}(\overline{\pi})$ and $\tilde{\alpha}(\overline{\pi})$ are provided in Table 1. Define the slope of the AS curve as $\tilde{K}(\overline{\pi}) \equiv \tilde{\kappa}(\overline{\pi})/(1-\beta\,\tilde{\alpha}(\overline{\pi})\,\mu)$. The slope of the AS curve depends on the relative weights attached to future and present terms. I can show that the AS curve becomes steeper when the inflation target increases if the following sufficient condition is satisfied.

Proposition 1. *A sufficient condition for a steeper AS curve if the inflation target rate is increased (i.e.* $\frac{\partial\tilde{K}(\overline{\pi})}{\partial\overline{\pi}} > 0$*) is:*

$$\beta\,\alpha(\overline{\pi})\,\mu \geq \frac{-\frac{\partial\kappa(\overline{\pi})/\partial\overline{\pi}}{\kappa(\overline{\pi})}}{-\frac{\partial\kappa(\overline{\pi})/\partial\overline{\pi}}{\kappa(\overline{\pi})}+\frac{\partial\alpha(\overline{\pi})/\partial\overline{\pi}}{\alpha(\overline{\pi})}} > 0. \tag{2.27}$$

The proof can be found in Appendix A.0.3. Proposition 1 is a key result: a steeper AS curve is associated with larger changes in inflation in response to variations in demand in the short run. Intuitively, the inequality in Proposition 1 states that the effective discounting on future expected output must be larger than the relative elasticity of the slope of the New Keynesian Phillips curve, $\kappa(\overline{\pi})$, with respect to an increase in the inflation target $\overline{\pi}$. This condition ensures that the weight on future expected output is large enough to compensate for the weaker response to current output.

To gain more intuition, consider first the special case in which $\mu = 0$. The discount factor shock $\widehat{\delta}_S$ reverts to the steady state with probability one in the next period. Here, Proposition

1 would be clearly violated and the slope of the AS equation becomes *flatter*. The reason is the following: forward-looking firms anticipate that the economy will return to its long-run equilibrium in the next period and are reluctant to change their (relative) price because with some probability they will be unable to readjust the price while inflation erodes their real profits over time. Consequently, Proposition 1 is satisfied only if $\mu > 0$ is sufficiently large. The expectation of a persistent contraction in demand can lead to a stronger change in inflation when $\overline{\pi}$ is higher because firms cut prices more aggressively in order to maximize the net present value of real profits.

The short-run AD. Combining the New IS curve in (2.15) and monetary policy in (2.16) yields the Aggregate Demand curve. For $t < T$, the Aggregate Demand curve in normal times and at the ZLB are:

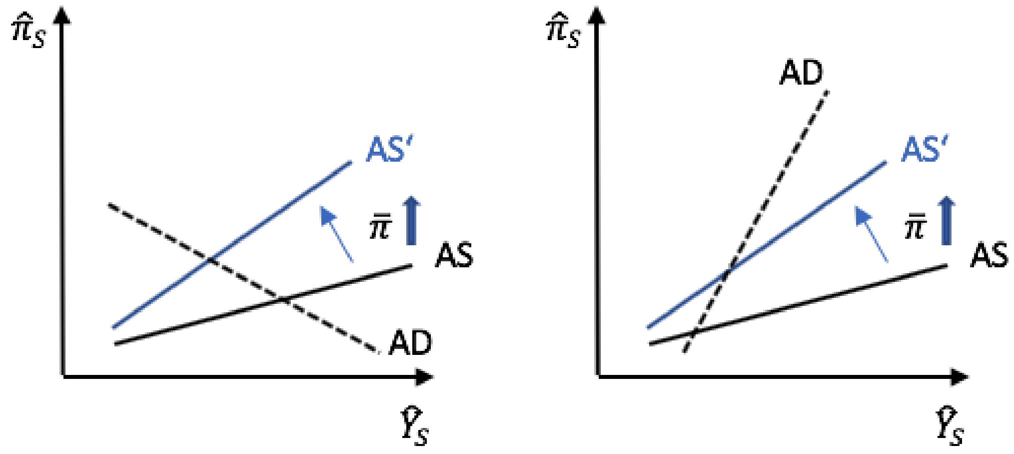
AD: Normal Times
$$\widehat{Y}_S^{NT} = \mu \widehat{Y}_S^{NT} - \left[\phi_\pi \widehat{\pi}_S^{NT} + \phi_Y \widehat{Y}_S^{NT} - \mu \widehat{\pi}_S^{NT}\right] - \mu \widehat{\delta}_S \tag{2.28}$$

AD: ZLB
$$\widehat{Y}_S^{ZLB} = \mu \widehat{Y}_S^{ZLB} - \left[-\bar{i} - \mu \widehat{\pi}_S^{ZLB}\right] - \mu \widehat{\delta}_S \tag{2.29}$$

At the ZLB, higher inflation expectations decrease the real interest rate, reducing the households' desire to save and making current spending relatively cheaper. Hence, the slope of the AD curve becomes positive at the ZLB.

The AS-AD schedules. Figure 2.3 illustrates the short-run AS and AD schedules graphically. Note that a higher inflation target does not affect the aggregate demand side but only the aggregate supply side as it affects the firms' price-setting behavior. All else equal, a higher inflation target increases the slope of the AS curve if $\widehat{\delta}_S$ is sufficiently persistent while the AD curve remains unaffected. Hence, higher inflation targets lead to a larger response in inflation both in normal times and at the ZLB.

Fig. 2.3 Aggregate Supply and Aggregate Demand Schedules



(a) Normal Times (b) At ZLB

Inflation target and macroeconomic volatility. The intersection of the AS and AD curves represent the short-run equilibrium of the model. The solutions in normal times and at the ZLB are:

Case 1: Normal Times $$\widehat{\pi}_S^{NT} = -\omega_\pi(\overline{\pi})\,\mu\,\widehat{\delta}_S \tag{2.30}$$

Case 2: ZLB $$\widehat{\pi}_S^{ZLB} = -\omega'_\pi(\overline{\pi})\left[\overline{i} - \mu\,\widehat{\delta}_S\right] \tag{2.31}$$

where $\omega_\pi(\overline{\pi})$ and $\omega'_\pi(\overline{\pi})$ are defined in Table 1. Based on the solutions, I can show that a higher inflation target amplifies the impact of the shock on inflation. This effect is stronger when the ZLB is binding because the adjustment of the nominal interest rate is too weak to match the drop in inflation, further deteriorating output and prices.

Proposition 2. *Given that* $\frac{\partial \tilde{K}(\overline{\pi})}{\partial \overline{\pi}} > 0$, *an increase in* $\overline{\pi}$ *leads to a stronger inflation response to the shock* $\widehat{\delta}_S$*:*

$$\partial\omega_\pi(\overline{\pi})/\partial\overline{\pi} > 0 \quad and \quad \partial\omega'_\pi(\overline{\pi})/\partial\overline{\pi} > 0 \tag{2.32}$$

Generally, inflation responds stronger at the ZLB than in normal times, that is, $\partial\omega'_\pi(\overline{\pi})/\partial\overline{\pi} > \partial\omega_\pi(\overline{\pi})/\partial\overline{\pi} > 0$.

The proof can be found in Appendix A.0.5.. Similarly, the solution for output is:

Case 1: Normal Times $$\widehat{Y}_S^{NT} = -\omega_Y(\overline{\pi})\,\mu\,\widehat{\delta}_S \tag{2.33}$$

Case 2: ZLB $$\widehat{Y}_S^{ZLB} = -\omega_Y'(\overline{\pi})\,\left[\bar{i} - \mu\,\widehat{\delta}_S\right] \tag{2.34}$$

Proposition 3. *Given that $\frac{\partial \tilde{K}(\overline{\pi})}{\partial \overline{\pi}} > 0$, an increase in $\overline{\pi}$ leads to a weaker response of output to the shock $\widehat{\delta}_S$ in normal times but a stronger response at the ZLB:*

$$\partial\,\omega_Y(\overline{\pi})/\partial\overline{\pi} < 0 \quad \textit{and} \quad \partial\,\omega_Y'(\overline{\pi})/\partial\overline{\pi} > 0 \tag{2.35}$$

In normal times, higher fluctuations in inflation lead to larger adjustments of the nominal and real interest rate, dampening the effects of the shock on output. This is not possible at the ZLB: lower inflation leads to a higher real interest rate causing a larger contraction in demand.

Inflation volatility and ZLB probability. A steeper AS curve can lead to a higher probability of reaching the ZLB. Suppose the shock $\widehat{\delta}_S$ is drawn from a normal distribution such as $\widehat{\delta}_S \sim \mathcal{N}(0,\sigma^2)$ where σ is the unconditional standard deviation of the shock. As in (2.25), define $\widehat{\delta}^*(\overline{\pi})$ as the minimum shock that needs to materialize such that the ZLB binds. Furthermore, denote the probability to observe the shock $\widehat{\delta}_S = \widehat{\delta}^*(\overline{\pi})$ for target rate $\overline{\pi}$ as p. Hence, the lower $\widehat{\delta}^*(\overline{\pi})$, the likelier it becomes to reach the ZLB.

A higher inflation target can reduce the threshold $\widehat{\delta}^*(\overline{\pi})$. Suppose $\overline{\pi}' > \overline{\pi}$. It is likelier to reach the ZLB at $\overline{\pi}'$ iff $\widehat{\delta}^*(\overline{\pi}') < \widehat{\delta}^*(\overline{\pi})$ which is satisfied if

$$\frac{\bar{i}'}{\bar{i}} < \frac{\phi_\pi\,\omega_\pi(\overline{\pi}') + \phi_Y\,\omega_Y(\overline{\pi}')}{\phi_\pi\,\omega_\pi(\overline{\pi}) + \phi_Y\,\omega_Y(\overline{\pi})} \tag{2.36}$$

where $\bar{i}'$ is the steady state nominal interest rate at inflation target $\overline{\pi}'$. To understand how (2.36) is related to the slope of the AS curve, $\tilde{K}(\overline{\pi})$, consider the following example. Let $\phi_\pi = 1.5$ and $\phi_Y = 0.5$ and normalize the slope of the AS curve at inflation target rate $\overline{\pi}$ to 1. In this case, the likelihood to be at the ZLB for $\overline{\pi}'$ increases only if:

$$\tilde{K}(\overline{\pi}') > \frac{\frac{\bar{i}'}{\bar{i}} - 0.5}{1.5 - \frac{\bar{i}'}{\bar{i}}}. \tag{2.37}$$

Imagine that the higher inflation target, $\overline{\pi}'$, is associated with an increase in the steady state nominal interest rate from 4% to 5% such that $\frac{\bar{i}'}{\bar{i}} = 1.25$. Then, the zero bound regime becomes likelier if the slope of the AS curve for $\overline{\pi}'$ is three times higher than before, i.e. $\tilde{K}(\overline{\pi}') > 3$.

2.5 Numerical Results

This section reports the results from numerical simulations of the model for alternative inflation targets. There are two important differences compared to Section 2.4. First, the assumption of linear labor disutility is removed, allowing the dispersion of relative prices to affect inflation and output dynamics (i.e. $\varphi > 0$). Second, I assume that $\widehat{\delta}_t$ follows an AR(1) process given by:

$$\widehat{\delta}_t = \rho\, \widehat{\delta}_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2). \tag{2.38}$$

2.5.1 Solution Method and Calibration

The log-linearized model is solved using a global solution method. More specifically, I use a collocation method to obtain approximations of the unknown policy functions as in Judd (1998), Miranda and Fackler (2002) or Schmidt (2013). This method allows to take the uncertainty arriving from the stochastic nature of the discount factor shock correctly into account in that firms' and households' expectations represent probability distributions over future economic outcomes. Numerous papers have studied the effects of shock uncertainty for the solution of the linearized and nonlinear New Keynesian model. Examples are Lindé and Trabandt (2018*b*), Adam and Billi (2006), Adam and Billi (2007), Fernández-Villaverde et al. (2015), Gust et al. (2017), Nakata (2016) and Richter and Throckmorton (2017). These authors show that accounting for future shock uncertainty may or may not have non-negligible implications for equilibrium dynamics.

I discretize the continuous stochastic process for $\widehat{\delta}_t$ using the Rouwenhorst (1995) method. The Rouwenhorst (1995) method approximates the continuous process through a finite-state Markov chain that mimics closely the underlying process. Based on the collocation method by Miranda and Fackler (2002) or Schmidt (2013), I develop an algorithm that exploits the fact that the log-linearized model can be represented on a pre-defined grid as a linear system of difference equations. Then, the algorithm solves for the policy function approximations iteratively. In each iteration, the procedure identifies the collocation nodes at which the ZLB constraint is binding and updates the solution of the model accordingly. The details of the application of the algorithm are described in Appendix A.0.6. Chapter 4 discusses the algorithm in further detail.

The model is parameterized at quarterly frequency. The parameter values are reported in Table 2.2. The discount factor β is set to 0.995 to match a steady state nominal interest rate of 4% annually. The elasticity of substitution between differentiated goods, ε, is set to 6, implying a steady-state price markup of 20 percent in line with Rotemberg and Woodford (1997). The price contract parameter θ is set to 0.84 so that the slope of the Phillips curve, $\kappa(\overline{\pi})$, given β and ε, equals 0.022 at a 2% inflation target. Moreover, based on the values for β and ε, the weight on expected inflation in the Phillips curve, $\alpha(\overline{\pi})$, is 1.004 and the coefficient on the present discounted value for marginal cost, $\eta(\overline{\pi})$, is 0.07/100 at a 2% inflation target.

It may be possible that firms adjust prices more frequently when the inflation target is higher[6]. If this is the case, price-setting may – in fact– become *less* forward-looking, dampening the amplification effects of higher trend inflation on inflation dynamics. Ultimately, the sensitivity of the frequency of price adjustment to variations in the inflation target remains an empirical question. However, the evidence on the relationship between higher trend inflation and the frequency of price adjustment is mixed. For example, Fernández-Villaverde and Rubio-Ramírez (2008) and Cogley et al. (2010) study estimated versions of the New Keynesian model for the pre- and the post-Volcker periods. These authors find that the estimate of the Calvo probability has remained quite stable over time. Based on micro-data from Argentina, Alvarez et al. (2011) report that the frequency of price changes is fairly unresponsive for inflation rates below 10%. Nakamura et al. (2018) report a substantial increase in the frequency of price adjustment for the US in the early 1980s when inflation increased to 16%. However, the frequency of price adjustment is found to be stable for low inflation periods. The focus of my analysis lies on low to moderate rates for the inflation

[6] For example, Bakhshi et al. (2007) study a model with state-dependent pricing as in Dotsey et al. (1999). In this economy, the New Keynesian Phillips curve is a function of steady state inflation but is also a function of the steady state distribution of price vintages and the number of price vintages. As trend inflation rises, firms adjust their prices more frequently and the number of vintages declines.

Table 2.2 Benchmark Parameterization

Parameter	Value	Description
β	0.995	Discount factor (quarterly)
φ	1	(Inverse) labor supply elasticity
ε	6	Substitution elasticity of inputs
θ	0.84	Prob. of not resetting price
ϕ_π	1.5	Taylor rule: Inflation response coefficient
ϕ_Y	0.125	Taylor rule: Output response coefficient
ρ	0.9	AR(1)-coefficient discount factor shock
σ	0.125/100	Standard deviation of shock innovation
		5% ZLB probability at $\overline{\pi} = 2\%$ annually
$\alpha(\overline{\pi})$	1.0042	Inflation weight in Phillips curve at $\overline{\pi} = 2\%$
$\kappa(\overline{\pi})$	0.022	Slope of Phillips curve at $\overline{\pi} = 2\%$
$\gamma(\overline{\pi})$	0.07/100	Marginal cost coefficient in Phillips curve at $\overline{\pi} = 2\%$

target. It therefore seems reasonable to assume that the Calvo parameter is independent of the level of $\overline{\pi}$.

For monetary policy, I use the standard Taylor (1993) rule parameters $\phi_\pi = 1.5$ and $\phi_Y = 0.125$. Finally, in the baseline calibration, the AR(1)-coefficient of the discount factor shock, ρ, is set to 0.9 to match the persistent effects of the financial crisis and the Great Recession. The unconditional standard deviation of the discount factor shock is calibrated to $\sigma = 0.125/100$ so that the probability of being at the ZLB is around 5% at an inflation target rate of 2% (Reifschneider and Williams (2000)).

2.5.2 Numerical Experiment

The main analysis in this section is based on computations of moments from simulations of the model. In computing these simulations, I generate 6000 samples of 200 periods (i.e. 50 years) initialized at the model's non-stochastic steady state following Kiley and Roberts (2017). The shocks are drawn from the unconditional distribution with standard deviation σ. Similar to Kiley and Roberts (2017), Reifschneider and Roberts (2006) and Reifschneider (2016), the stochastic simulations allow for back-to-back recessions. The economy has not necessarily fully recovered from a recessionary period before additional adverse shocks are realized. All else equal, the simulations are performed for an economy with inflation targets ranging from 0% to 4% annually.

Table 2.3 reports the results of the stochastic simulations of the New Keynesian model. The numbers in brackets are the standard deviations of selected model variables based on a simulation of the New Keynesian model that ignores the ZLB constraint. In this case, the nominal interest is allowed to fall below zero. In the baseline scenario ($\overline{\pi} = 2\%$), the probability of reaching the ZLB is 5.14%. At an inflation target of 3%, the ZLB probability declines relative to the baseline case. The ZLB is expected to bind only 2.16% of the time. The standard deviation of inflation and the nominal interest rate are slightly higher compared to $\overline{\pi} = 2\%$. However, this channel is not sufficiently strong at $\overline{\pi} = 3\%$. Here, a higher inflation target renders monetary policy more scope to stabilize the economy, dampening the fluctuations in output.

Table 2.3 Simulation Results

	Zero Lower Bound		Standard Deviation (in %)			
$\overline{\pi}$	Probability	Duration	$\widehat{\pi}$	$\widehat{y}$	$\widehat{i} - E\widehat{\pi}$	$\widehat{i}$
0%	22.86%	4.60	0.60 (0.26)	1.87 (0.43)	0.48 (0.25)	0.29 (0.44)
1%	10.10%	3.15	0.32 (0.26)	0.67 (0.40)	0.27 (0.25)	0.43 (0.45)
2%	5.14%	2.51	0.31 (0.28)	0.48 (0.35)	0.26 (0.25)	0.46 (0.47)
3%	2.16%	2.19	0.32 (0.31)	0.29 (0.25)	0.25 (0.26)	0.49 (0.50)
4%	7.21%	5.07	0.56 (0.50)	0.90 (0.75)	0.31 (0.28)	0.67 (0.70)

Note: Mean duration at ZLB in quarters. The model with $\overline{\pi} = 1\%$ and $\overline{\pi} = 0$ are solved and simulated using $\sigma = 0.11/100$. $\widehat{\pi}$: inflation, $\widehat{y}$: output, $\widehat{i} - E\widehat{\pi}$: real interest rate, $\widehat{i}$: nominal interest rate. Numbers in brackets are standard deviations from a model which ignores ZLB constraint.

The volatility effect dominates at a 4% inflation target rate. Macroeconomic performance deteriorates significantly relative to the baseline case with inflation and output volatility being substantially higher. The additional margin for monetary policy at $\overline{\pi} = 4\%$ becomes effectively irrelevant in the wake of large recessions: monetary policy responds to larger fluctuations of inflation by larger cuts in the nominal interest rate. In an economy with $\overline{\pi} = 4\%$, the ZLB is expected to bind around 7.21% of the time. The expected average duration at the ZLB increases by 2.5 quarters relative to the baseline case (5.07 quarters vs. 2.51 quarters). Output volatility increases substantially because it becomes more difficult for monetary policy to stabilize the economy in response to adverse shocks.

The probability of reaching the ZLB is substantially higher for inflation targets 0% and 1% relative to the benchmark case. Higher fluctuations in inflation, output and the real interest rate can be mostly attributed by the inability of monetary policy to stabilize the economy when the ZLB is reached. The simulated standard deviations of inflation, output and the real interest rate differ, in parts, significantly from their counterparts in an economy in which the ZLB is not a binding constraint.

Fig. 2.4 Simulated Probability Density Functions: $\overline{\pi} = 2\%$ vs. $\overline{\pi} = 4\%$

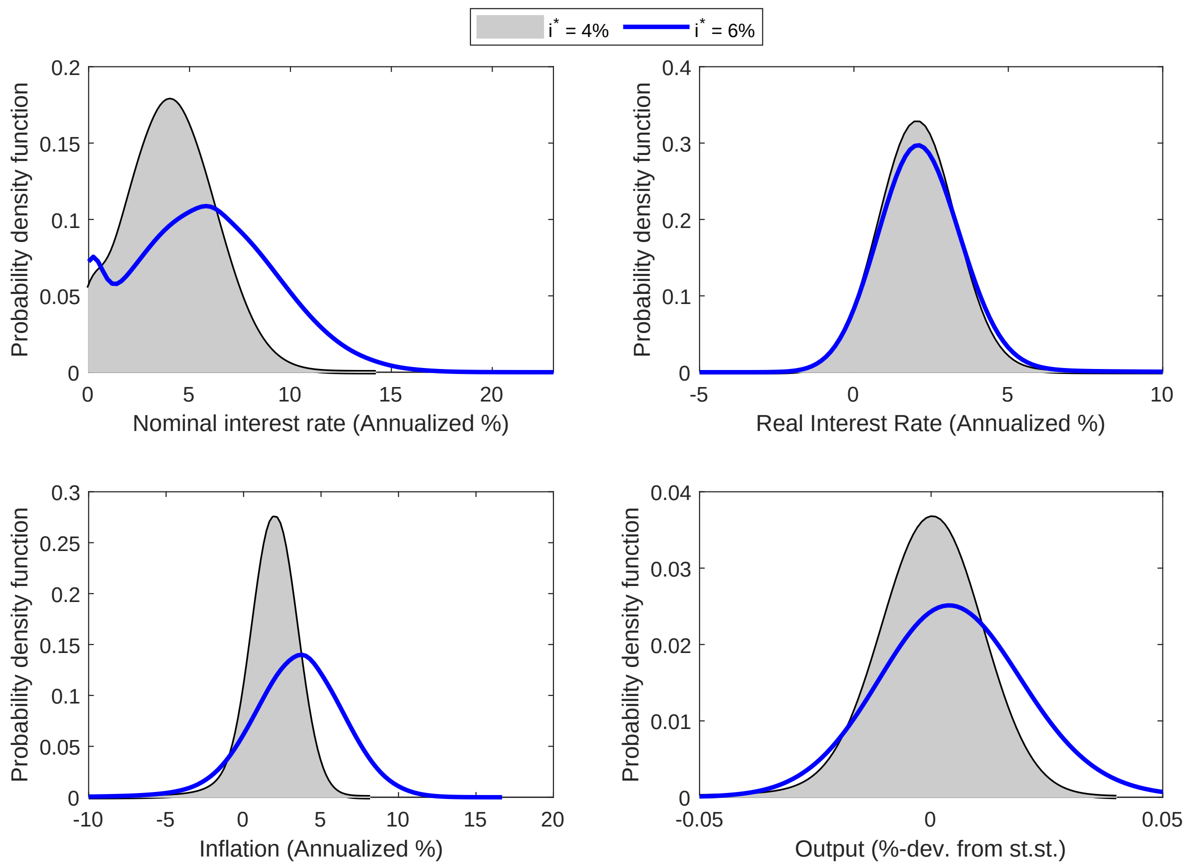
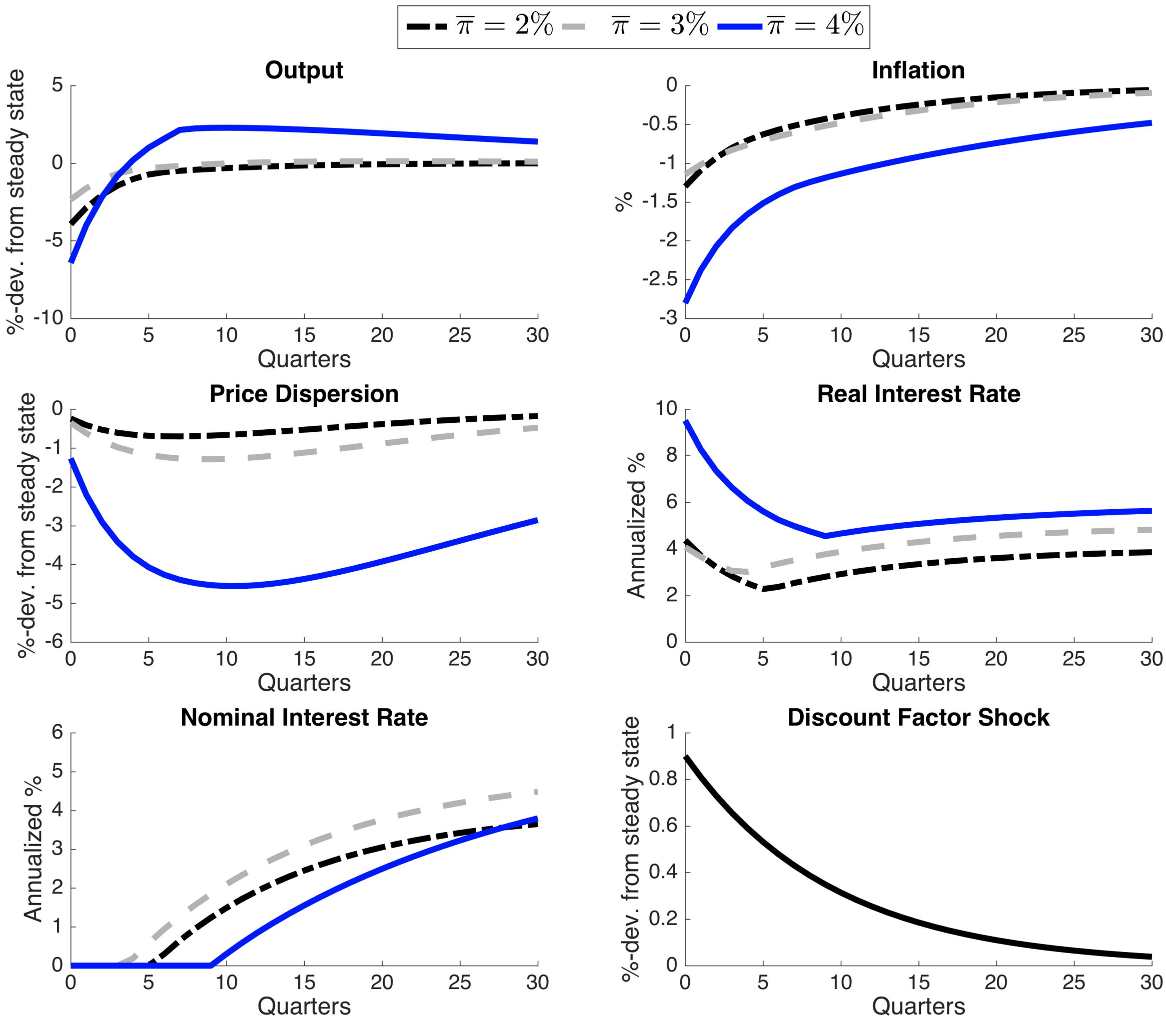


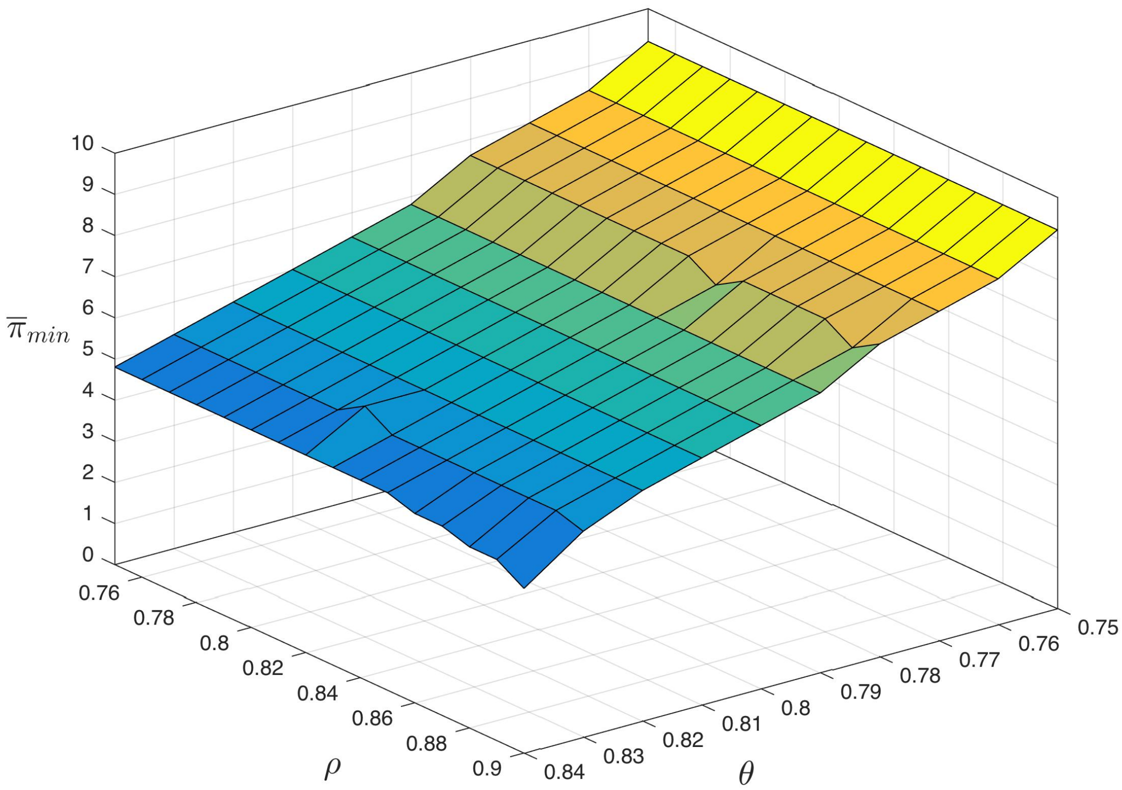
Figure 2.4 plots the probability density functions for the nominal interest rate, the real interest rate, inflation and output from simulations of the New Keynesian model with $\overline{\pi} = 2\%$ (gray shaded area) and $\overline{\pi} = 4\%$ (blue solid line), respectively. The distributions for the nominal interest rate are truncated at the ZLB. In general, the distributions of all variables are more dispersed in an economy with $\overline{\pi} = 4\%$. The shape of the distributions for the real interest rate is fairly similar in both scenarios. However, there are slightly more positive realizations of the real interest rate at $\overline{\pi} = 4\%$ as a result of the higher probability of being at the ZLB.

Fig. 2.5 Impulse Responses to Positive Discount Factor Shock



To further illustrate the macroeconomic implications of higher inflation targets, I simulate the model for a large adverse discount factor shock that generates an expected 4-quarter liquidity trap in the benchmark scenario (i.e. $\overline{\pi} = 2\%$). Figure 2.5 compares the impulse responses for the benchmark case for a target at 2% (black dash-dotted line), at 3% (gray dashed line) and 4% (blue solid line), respectively. At $\overline{\pi} = 4\%$, the fall in inflation is substantial and leads to a larger cut of the nominal interest rate as compared to the benchmark case. At the ZLB, the real interest rate increases as expected inflation declines, rendering current spending more costly and, hence, contracting demand. The large contraction leads to a longer period at the ZLB: at $\overline{\pi} = 4\%$, the nominal interest rate is zero for 8 quarters.

Fig. 2.6 5% ZLB Probability: Minimum Inflation Target



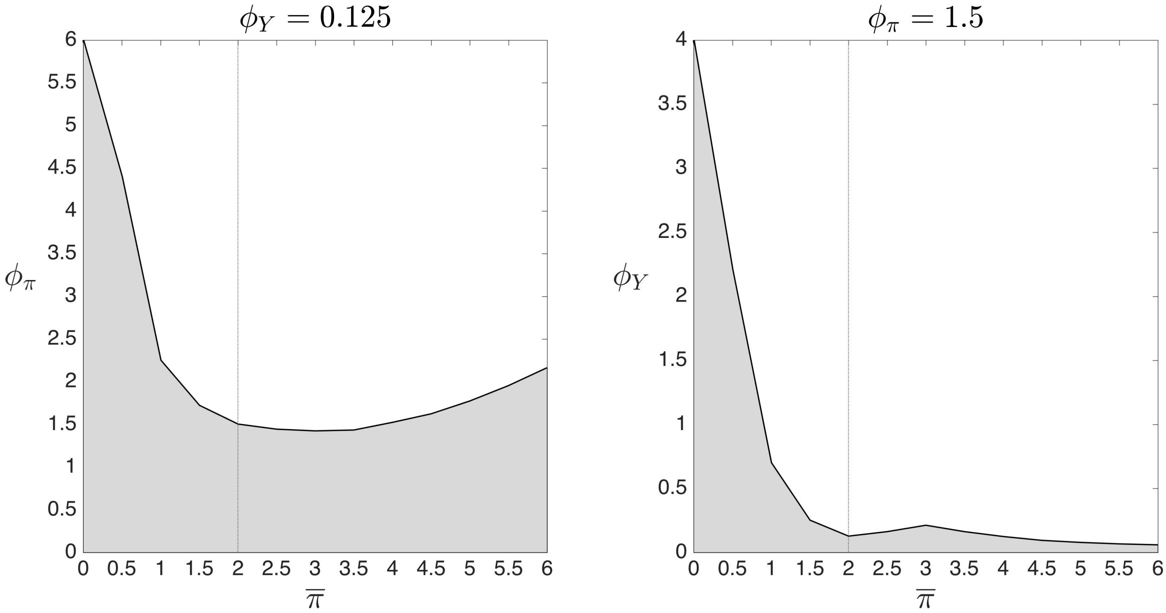
Note: min. $\overline{\pi}$: minimum inflation target, θ : Calvo parameter, ρ: AR(1)-coefficient of discount factor shock

The previous results are sensitive to the degree of shock persistence and the degree of price rigidities. Firms are more forward-looking if prices are more rigid and the shock is more persistent. Figure 2.6 shows the minimum inflation target rate that is associated with a (marginally) higher probability of reaching the ZLB relative to the benchmark case of $\overline{\pi} = 2\%$ for different combinations of the shock persistence ρ and Calvo parameter θ.[7] Figure 2.6 shows ρ on a range between 0.75 and 0.9 and θ between 0.75 and 0.84. $\overline{\pi}_{min}$ is 9.2% for

[7] All else equal, I simulated the New Keynesian model for different values of ρ, θ and $\overline{\pi}$. All other parameter values are shown in Table 2.2.

$\rho = 0.75$ and $\theta = 0.75$. This means that $\overline{\pi} = 9.2\%$ is the lowest target rate that would be associated with a higher probability of hitting the ZLB relative to $\overline{\pi} = 2\%$. Interestingly, $\overline{\pi}_{min}$ declines with higher θ for given ρ but remains nearly constant as ρ increases for given θ. Hence, the degree of price rigidities, θ, plays a key role in determining $\overline{\pi}_{min}$.

Fig. 2.7 5% ZLB Probability: Minimum Value for Taylor Coefficients



So far, I have assumed that a higher inflation target does not change how monetary policy responds to fluctuations in inflation and output. However, a higher inflation target can increase the risk that inflation expectations become unanchored. As Ascari and Sbordone (2014) and Ascari and Ropele (2009) and others argue, higher trend inflation shrinks the determinacy region requiring a stronger policy response to inflation and a weaker response to output to ensure determinacy. The previous analysis has shown that different monetary policy strategies are desirable not only due to a higher risk of unanchored inflation expectations. A stronger policy response to inflation could lower the risk of reaching the ZLB for higher inflation target rates. I simulated the New Keynesian model again for different combinations of the inflation target and the Taylor coefficients on inflation, ϕ_π, and output, ϕ_Y. Figure 2.7 reports the lowest value for ϕ_π and ϕ_Y that leads to a probability at the ZLB similar to the one at $\overline{\pi} = 2\%$, i.e. around 5%, for different levels of the inflation target. The left panel shows the minimum value of the Taylor coefficient on inflation ϕ_π while holding ϕ_Y constant at 0.125. The minimum ϕ_π increases for target rates higher than 4% relative to the baseline case (gray vertical line at $\phi_\pi = 1.5$). Hence, it would require a stronger response to deviations of inflation from its target to avoid the ZLB. Between $\overline{\pi} = 2\%$ and 4%, the minimum value of the Taylor coefficient on inflation is lower relative to the baseline. In this case, monetary policy can allow for a weaker response to deviations of inflation from its target to achieve a 5% probability of reaching the ZLB. The right panel in Figure 2.7 shows the minimum value of the Taylor coefficient on output ϕ_Y while ϕ_π is constant at 1.5. Here, ϕ_Y declines relative to the benchmark for $\overline{\pi} > 3\%$. An interpretation of this results is that inflation stabilization is more important than output stabilization in order to reduce the ZLB probability.

2.6 Sensitivity Analysis

In this section, I examine the robustness of the main findings in the previous sections. In particular, I study the implications of higher inflation targets in the non-linear New Keynesian model and examine the role of price indexation. Furthermore, I test the sensitivity of the main findings against an alternative solution method and, finally, consider additional shocks for the stochastic simulations.

Solution to the Non-Linear Model. The analysis so far is based on the semi-loglinear New Keynesian model. All equilibrium conditions were log-linearized around the steady state with positive inflation target, except for the interest rate rule. The implicit assumption is that the linearized solution is sufficiently accurate. However, recent research by Lindé and Trabandt (2018*a*,*b*) and Boneva et al. (2016) find that the results of the linearized model can substantially differ from the solution of the non-linear model.

Boneva et al. (2016) study fiscal multipliers at the ZLB in a tractable non-linear New Keynesian model with zero inflation in steady state. Their findings complement earlier work by Eggertsson (2010), Christiano et al. (2011) or Woodford (2011) in that they suggest that supply side policies are effective in stabilizing the economy even in a low interest rate environment. Boneva et al. (2016) argue that the reason for the conflicting results is the solution method. Boneva et al. (2016) use the non-linear New Keynesian model and show that the dispersion of relative price have non-negligible effects in the non-linear model. In the linearized version of the model, the dispersion of relative prices is approximately zero.

Lindé and Trabandt (2018*a*,*b*) analyze the implications of non-linearities in a New Keynesian model that allows for non-linearities in the price-setting behavior of firms as in Kimball

(1995). Under Kimball (1995) preferences, each producer faces a price elasticity of demand that is increasing in its own relative price, while the desired price mark-up over marginal cost is decreasing in the relative price. Thus, a firm will temper any price increases as this would endogenously reduce its desired mark-up. This effect is absent in a linearized model. Lindé and Trabandt (2018*a*) compare the effects of government spending in a linearized and a non-linear model. They find that fiscal spending multipliers in the non-linear model are significantly lower than in the linearized model at constant nominal interest rates. In the non-linear model inflationary pressures in response to the government spending shock are muted under the Kimball (1995) specification, dampening the drop in the real interest rate. Building on their previous work, Lindé and Trabandt (2018*b*) show that non-linearities in price and wage-setting can explain the small decline in inflation in the Great Recession.

I consider a model with a Dixit-Stiglitz (1977) aggregator and a constant price elasticity of demand. Hence, the non-linearities in firms' price-setting behavior that are present under Kimball (1995) are not relevant for my analysis. However, there might be other effects that are associated with the non-linear model. I redo the stochastic simulations from the previous section for the non-linear New Keynesian model. Table 2.4 summarizes the simulation results.[8] The main finding is robust: an inflation target of 4% is associated with a higher probability of hitting the ZLB relative to the benchmark case of 2%. However, the differences are quantitatively smaller in the non-linear model. Macroeconomic variables are less volatile which reduces both the incidence of the ZLB and the average duration at the ZLB. An interpretation of this results is that the linearized model overstates the degree of forward-lookingness of firms.

[8]The unconditional standard deviation of the discount factor shock, σ is chosen such that the ZLB is binding for 5% of the time for $\overline{\pi} = 2\%$.

Table 2.4 Simulation Results for the Non-Linear Model

	Zero Lower Bound		Standard Deviation (in %)		
$\overline{\pi}$	Probability	Duration	$\widehat{\pi}$	$\widehat{y}$	$\widehat{i}$
2%	5.12%	1.61	0.21	0.30	0.31
3%	1.05%	1.05	0.23	0.16	0.29
4%	5.61%	2.03	0.34	0.45	0.35

Note: Mean duration at ZLB in quarters. $\widehat{\pi}$: inflation, $\widehat{y}$: output, $\widehat{i}$: nominal interest rate.

Price Indexation. For the main analysis, I considered a New Keynesian model that abstracts from price indexation. To match the persistence of inflation in the data, prices are often indexed to steady-state inflation or to past inflation rates in the literature (see for example Yun (1996), Christiano et al. (2005), and Smets and Wouters (2007)). Price indexation mitigates the amplification of inflation in response to economic shocks. Moreover, price indexation can completely neutralize the effects of trend inflation if firms index their prices fully to trend inflation (e.g. Ascari and Ropele (2009) or Ascari and Sbordone (2014)). Here, the steady state allocation coincides exactly with the allocation under flexible prices.

However, the assumption of price indexation is hardly justified on empirical and theoretical grounds. First, studies based on micro-data on prices do not show that prices change all the time, a property that price indexation would imply (Ascari and Sbordone (2014)). Second, as Cogley and Sbordone (2008) or Ascari and Sbordone (2014) have shown, price indexation is not needed to match the inflation persistence in the data when trend inflation has been correctly taken into account in the New Keynesian Phillips curve. Cogley and Sbordone (2008) and Ascari and Sbordone (2014) estimate a hybrid version of the New Keynesian Phillips curve under positive trend inflation and find no evidence for a significant coefficient on the backward-looking term.

To analyze the role of price indexation in generating the previous results, I assume the following version of the maximization problem of a price-setting firm j:

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\left(\frac{P^*_{j,t}}{P_{t+i}} \Omega_{t,t+i-1} \right) - w_{t+i} \right) Y_{j,t+i} \tag{2.39}$$

subject to

$$Y_{j,t+1} = \left(\frac{P_{j,t}}{P_{t+1}} \Omega^{\nu}_{t,t+i-1} \right)^{-\varepsilon} Y_{t+1} \tag{2.40}$$

where

$$\Omega_{t,t+i-1} = \overline{\pi}^{(1-\omega)} \left(\frac{P_{t+i-1}}{P_{t-1}} \right)^{\omega}. \tag{2.41}$$

$\Omega_{t,t+j-1}$ represents a general price indexation rule. The parameter $0 \leq \nu \leq 1$ measures the overall degree of indexation, while the parameter $0 \leq \omega \leq 1$ denotes the degree of indexation to the inflation target and/or past realized inflation. Suppose that either $\omega = 0$ (indexation to the inflation target only) or $\omega = 1$ (indexation to past realized inflation only). Moreover, I set ν to $\nu = 0.139$, $\nu = 0.5$ or $\nu = 1$. $\nu = 0.139$ corresponds to the estimate in Smets and Wouters (2003) for indexation to past inflation. Finally, I choose the unconditional standard deviation of the discount factor shock such that the ZLB probability is roughly 5% at $\overline{\pi} = 2\%$. All other parameters are shown in Table 4.1.

Table 2.5 reports the simulation results. Price indexation dampens fluctuations in inflation. The main findings are not robust under a price indexation rule for the baseline parameterization as shown in Table 2.2. Under the benchmark parameterization, a 4% inflation target is associated with a lower probability of reaching the ZLB relative to $\overline{\pi} = 2\%$ for all specifications of ν and ω. Under mild price indexation to steady state inflation ($\nu = 0.139$), the previous result can be restored if the persistence of the discount factor shock is sufficiently high. Reaching the ZLB becomes likelier at $\overline{\pi} = 4\%$ if $\rho = 0.92$.

The Role of Uncertainty. The main analysis is based a solution of the New Keynesian model that allows for future shock uncertainty. To assess the sensitivity of the solution method for the main results, I redo the analysis using a perfect foresight approach to solve the model. This method ignores any potential effects of the variance of the discount factor shock on the decision rules of firms and households.

Table 2.5 Simulation Results for the Log-Linearized Model with Price Indexation

	ZLB Probability			
	Indexation to Inflation Target ($\omega = 0$)		Indexation to Past Inflation ($\omega = 1$)	
	$\rho = 0.90$	$\rho = 0.92$	$\rho = 0.90$	$\rho = 0.92$
$\overline{\pi}$	$\nu = 0.139$			
2%	5.1%	5.2%	5.1%	5.2%
4%	4.6%	6.3%	0%	0.5%
$\overline{\pi}$	$\nu = 0.5$			
2%	5.3%	5.3%	5.0%	5.0%
4%	2.3%	4.9%	0%	0%
$\overline{\pi}$	$\nu = 1$			
2%	5.9%	5.1%	5.1%	5.1%
4%	1.6%	3.8%	0%	0%

Table 2.6 compares the simulation results in the previous section with the deterministic solution. In both cases, the discount factor shock is drawn from a normal distribution with mean zero and standard variance $\sigma = 0.125/100$. Shock uncertainty generally affects the solution of the model, elevating the effects of adverse shocks and the ZLB risk. The frequency of ZLB events for the deterministic model is lower by roughly 2 percentage points for target rates 2% and 4%, respectively. The results for an inflation target of 3% differ only minimally between the stochastic and the deterministic approach: as ZLB events are generally rare, shock uncertainty basically does not impact the model solution.

Overall, my main findings do not hinge on the specific solution method. The quantitative differences between both approaches reflect the impact of future shock uncertainty on the decision rules of households and firms.

Table 2.6 Simulation Results: Stochastic Model vs. Deterministic Model

	Stochastic Model		Deterministic Model	
	ZLB		ZLB	
$\overline{\pi}$	Probability	Duration	Probability	Duration
2%	5.14%	2.51	3.2%	2.09
3%	2.16%	2.19	1%	1.50
4%	7.21%	5.07	5.5%	3.88

Note: Mean duration at ZLB in quarters.

Effects of Additional Shocks. In addition to the discount factor shock, I consider a price markup shock which affects inflation dynamics via the New Keynesian Phillips curve. In this case, the Phillips curve is given by:

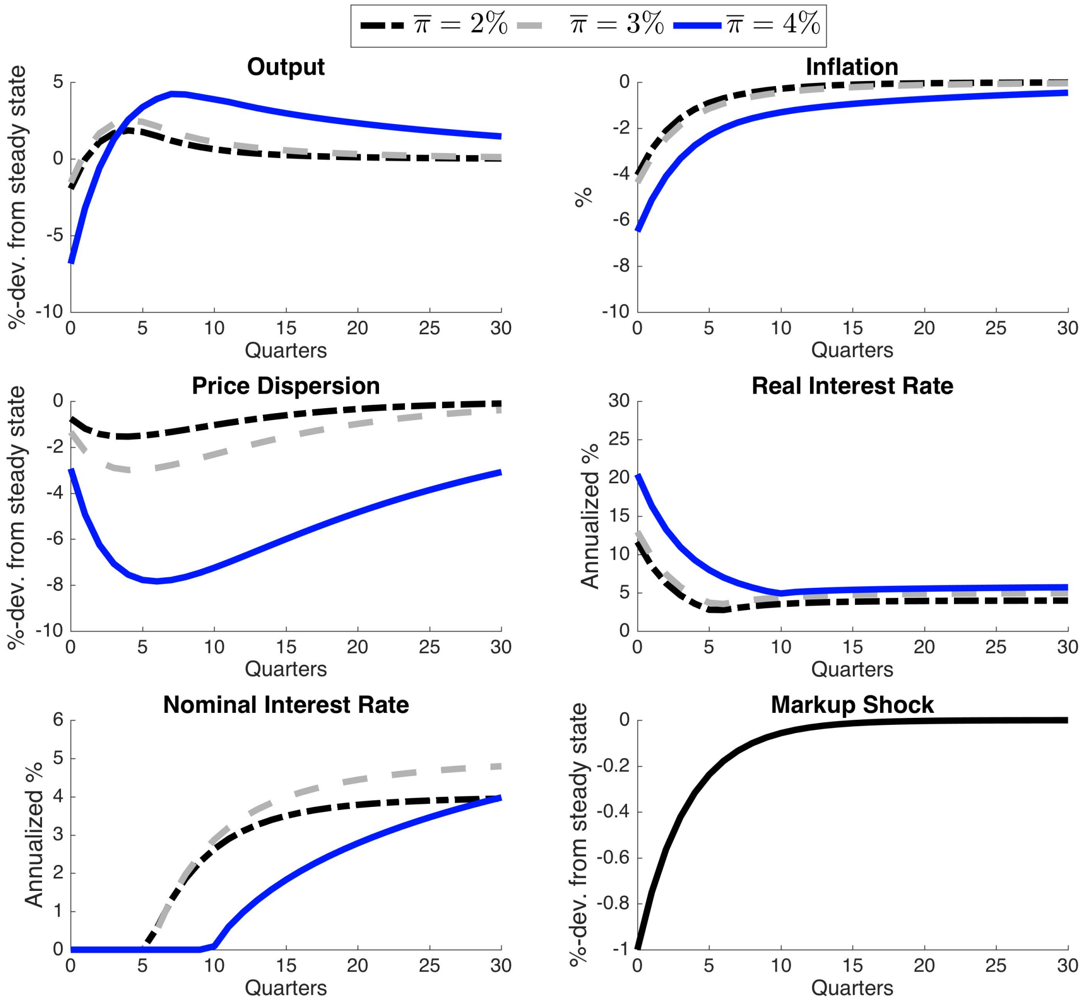
$$\widehat{\pi}_t = \beta\alpha(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})[(1+\varphi)\widehat{Y}_t + \varphi\widehat{s}_t] + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} + \widehat{\gamma}_t \tag{2.42}$$

where $\widehat{\zeta}_t$ is an exogenous shock following an AR(1) process

$$\widehat{\gamma}_t = \rho_M\widehat{\gamma}_{t-1} + \varepsilon_M \quad \text{with } \varepsilon_M \sim \mathcal{N}(0, \sigma_M^2). \tag{2.43}$$

Assume that $\rho_M = 0.75$. All other parameters are shown in Table 4.1 in the previous section. Figure 2.8 shows the impulse responses to a large negative markup shock for an economy with inflation target 2% (dash-dotted black line), 3% (dashed gray line) and 4% (blue solid line), respectively. The negative markup shock causes a decline in inflation. To match the large decline in inflation, the nominal interest rate is reduced to its lower bound. As a consequence, the real interest rate increases and thus output falls. The ZLB lasts for 5 quarters in the baseline case $\overline{\pi} = 2\%$. A 4$ inflation target amplifies inflation dynamics, further deteriorating economic activity. For $\overline{\pi} = 4\%$, the ZLB lasts for 10 quarters. Note that in response to smaller markup shocks that do not imply a binding ZLB inflation and output would move in opposite directions.

Fig. 2.8 Impulse Responses to Negative Markup Shock



I solve the model allowing for random disturbances of the discount factor shock and a price markup shock. I assume that both shocks are uncorrelated. For simplicity, assume that $\sigma = \sigma_M$ which is chosen such that the observed probability of being at the ZLB is about 5%. Table 2.7 summarizes the simulation results for alternative values of the AR(1)-coefficient for the price markup shock, ρ_M. A higher inflation target leads to a higher probability of hitting the ZLB only if the cost push shock is sufficiently persistent. A 4% inflation target is associated with a higher risk of hitting the ZLB relative to a 2% target for $\rho_M \geq 0.75$.

Table 2.7 Simulation Results: Discount Factor Shock and Markup Shock

	Zero Lower Bound		Standard Deviation (in %)		
	Probability	Duration	$\widehat{\pi}$	$\widehat{y}$	$\widehat{i}$
$\overline{\pi}$	$\rho_M = 0.5$				
2%	5.12%	2.01	0.25	0.43	0.25
4%	2.51%	1.85	0.31	0.48	0.29
$\overline{\pi}$	$\rho_M = 0.75$				
2%	5.11%	2.66	0.34	0.51	0.35
4%	8.70%	5.50	0.55	1.22	0.44

Note: Mean duration at ZLB in quarters. ρ_C: AR(1) coefficient for cost-push shock. $\widehat{\pi}$: inflation, $\widehat{y}$: output, $\widehat{i}$: nominal interest rate.

2.7 Conclusion

This paper studies the implications of inflation targets for the behavior of firms and households in light of the current debate to raise the inflation target as a policy tool to reduce the incidence of the ZLB. I argue that the level of inflation itself can be a source of macroeconomic volatility, especially in deep recessions with binding ZLB. This generates a non-trivial economic trade-off as higher fluctuations in inflation require stronger adjustment in the nominal interest rate, limiting the effective safety margin for monetary policy in recessions. My paper presents a case in which inflation volatility has a first-order effect. Based on stochastic simulations of the New Keynesian model, I find that higher inflation targets are associated with a higher probability of hitting the ZLB. In this example, an increase in the inflation target rate from 2% to 4% would increase the probability of hitting the ZLB from around 5% to 7.21%.

The main focus of this paper is to investigate the role of higher inflation targets as a driving factor for inflation volatility. In this context, the choice of a simple New Keynesian framework seems reasonable to present the results in a tractable way. However, a direction for future research would be to examine to which extent the key findings in this paper are robust to extensions of the model. One dimension involves the estimation of the parameters of the model. In particular, it is straightforward to see that crucial parameters are the persistence of the shocks, the average duration of a firm's price, as well as the degree of indexation. For the latter, macroeconomic evidence commonly suggests that the slope of the Phillips curve is rather low, corresponding to a high duration between price changes (e.g. Smets and Wouters (2003, 2007) or Altig et al. (2011)). Other considerations concern the relevance of the mechanism that are described in this paper for larger models incorporating more nominal and real frictions (see e.g. Smets and Wouters (2003, 2007) or Christiano et al. (2005)). In this context, it would be interesting to study the role of wage rigidities for inflation dynamics. On the one hand, wage stickiness as modelled for example in Erceg et al. (2000) can reinforce price stickiness and, hence, forward-looking behavior of firms. This generates an additional source of higher inflation volatility when inflation target are raised. On the other hand, wage rigidities add persistence to inflation, counterbalancing the effects of higher inflation targets on inflation volatility.

While this paper focuses on the implications of the level of the inflation target for the ZLB risk, it generally abstracts from other issues that relate to the implementation of a higher inflation target. One concern is how to manage the transition to a higher target rate. It would involve effective communication of the permanent nature of the monetary regime shift as well as the conduct of policies to move up inflation and inflation expectations. The recent experience in Japan shows how difficult the effective implementation of an inflation target could be. As part of a large-scale policy package known as Abenomics, the Bank of Japan introduced an official inflation target of 2% in 2013. Currently, while inflation expectations have generally increased, inflation is still below target, suggesting that private agents are still doubtful whether the adopted inflation target is indeed credible (e.g De Michelis and Iacoviello (2016)).

Overall, the main finding in this paper adds a new angle to the debate over whether the inflation target should be raised. It highlights the role of the level of the inflation target itself for inflation volatility, effectively determining the likelihood of reaching the ZLB on nominal interest rates. As a consequence, the results caution against raising the target rate of inflation while ignoring how this would change the behavior of economic agents. Finally, they call for rethinking and reevaluating the benefits and risks involved in changing a central bank's inflation target.

Chapter 3

Are Consumption Tax Cuts Expansionary in a Liquidity Trap?

Do temporary value-added tax (VAT) cuts stimulate aggregate consumption? I show that the canonical New Keynesian model predicts that they are expansionary in normal times but contractionary in deep recessions (i.e. when an effective lower bound on nominal interest rates is binding). A potential issue is that standard models only account for consumption of non-durable goods. However, in countries that levy VAT, consumer durables typically represent roughly 40% of total consumption expenditures on goods and services that are subject to the VAT. I allow for consumption of durable goods in the New Keynesian model and now find that the previous results are completely overturned: temporary consumption tax cuts have large positive macroeconomic effects both in normal times and in a liquidity trap. The reason is that purchases of durable goods are highly intertemporally substitutable - consumers will stock up on storable goods when prices are currently low. But most interestingly, I observe that the boom in the durable goods sector spills over to the non-durable goods sector. The VAT cut becomes expansionary for both non-durable and durable goods consumption. The findings of this paper suggest that it is important to distinguish between different types of consumption goods to study the aggregate effects of consumption tax changes.

3.1 Introduction

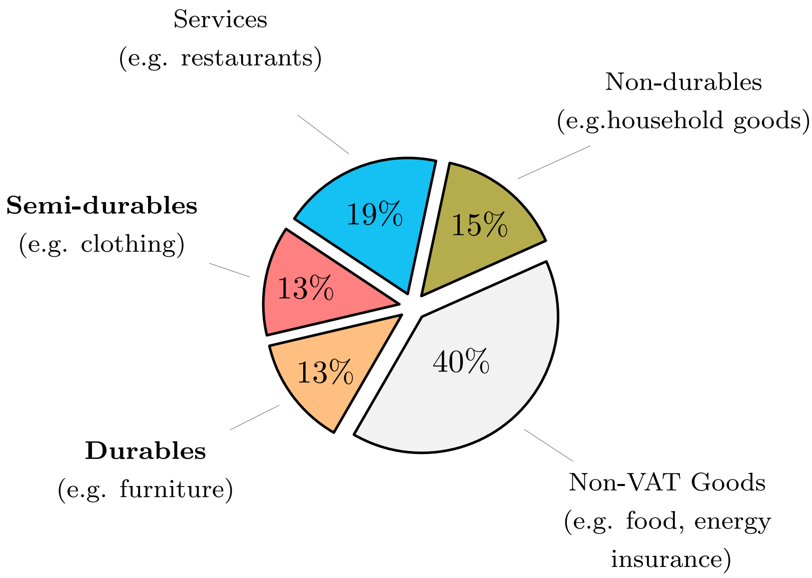
Recent research suggests that fiscal policy has very different effects when nominal interest rates have reached a zero lower bound (ZLB). For example, Eggertsson (2010) finds that employment and output decrease in response to a cut in the labor tax rate, a property he refers to as the "paradox of toil". In a standard New Keynesian model, he shows that such a tax cut would generate deflationary pressures by reducing the marginal costs of firms. If the central bank cannot accommodate by lowering nominal interest rates, real interest rates increase and, thereby, contract demand.

Using a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework, I analyze the effects of a novel fiscal policy: a temporary cut in the value-added tax (VAT). Around 150 countries levy VAT, including all countries of the European Union. The VAT is a general consumption tax that is collected incrementally along the supply chain. Retailers pay the VAT on their sales but can receive reimbursements for the previous VAT they paid at earlier stages of production. Hence, the taxation is effectively based on the value-added at every point of sale along the supply chain. Ultimately, the tax burden will be paid by the final consumer. Consumers typically pay the final sales price which is *inclusive* of the VAT. The VAT system is distinctly different from a sales tax. For example, the United States and Canada levy a sales tax. With a sales tax, the consumption tax is only collected once at the final point of purchase by a consumer. Moreover, the sales tax will be added directly on top of the final price when a good or service is purchased. The particular set-up of the VAT system may give rise to deflationary pressures in response to a tax cut. A lower VAT rate will only affect final consumers if retailers are able to adjust their final sales price inclusive of the VAT. If a VAT cut is not immediately passed on to final prices, consumers will expect a period of falling prices following the implementation of the lower tax rate.

This study is generally motivated by a fiscal experiment in the United Kingdom (UK). In an effort to boost private consumption expenditure in the onset of the global financial crisis, the UK government lowered the standard VAT rate by 2.5 percentage points (from 17.5% to 15%) from December 2008 to December 2009. The tax change was entirely unanticipated by retailers and consumers and implemented only one week after it was announced. The VAT cut affected the major share of total household expenditure. Figure 3.1 shows the composition of an average private consumption basket in the UK in 2008.[1] While 40% of total expenditures are on goods and services that either attract zero VAT (e.g. food) or the reduced VAT (e.g. energy), the remaining 60% of consumer spending is subject to the standard VAT rate. Within the latter group, semi-durable and durable goods together contribute to 43% of consumer expenditure.

[1] The composition of the representative consumption baskets in other European countries is very similar (see Figure B.1 in Appendix B.).

Fig. 3.1 Representative Consumption Basket of Goods and Services for UK in 2008



Source: Eurostat, own calculations

I study the following questions: (i) What are the macroeconomic effects of a temporary VAT cut in normal times and in deep recessions (i.e. when the ZLB on nominal interest rates is reached)? And (ii) is it important to distinguish between different types of consumption goods that are subject to the VAT (i.e. perishable goods (non-durables and services) and long-lived goods (semi-durables and durables)? In the standard New Keynesian model, households consume a bundle of non-durable goods. Using the standard New Keynesian model, I find that a VAT cut is expansionary in normal times *but* contractionary when nominal rates are at the ZLB, consistent with the conjecture in Eggertsson (2010). However, the novel contribution of this study is that this result no longer holds if I account for an empirically more realistic household consumption basket. I extend the standard New Keynesian model to allow for non-durable and durable goods consumption. Semi-durables and durables are consumer goods with extended product life yielding utility over time. Using the extended model, I find that a temporary VAT cut is expansionary both in normal times *and* in deep recessions at the ZLB for nominal interest rates. I show that in response to the VAT cut, demand for durable goods increases, but – most interestingly – also demand for non-durable goods increases.

The reason is the following: purchases of durable goods are highly intertemporally substitutable and consumers stock up on durables when prices are currently low after the VAT cut. In a New Keynesian model with non-durable and durable consumption, Barsky et al. (2007) show that the shadow value of long-lived goods is rather unresponsive to temporary economic disturbances because it reflects utility from expected service flows over the lifetime of the durable goods. The fact that the shadow value is nearly constant implies that consumers are indifferent on the timing of durable goods purchases. When their relative price is low – for example, when the VAT rate is reduced– consumers increase spending on consumer durables. But more interestingly, I find that demand for non-durables increase in response to a VAT cut even in deep recessions with a binding ZLB on nominal interest rates. The higher demand for durable goods dampens the fall in employment and wages in the economy in a recession. Higher aggregate wages prevent marginal cost of firms and prices to deteriorate. I find that this effect is sufficiently strong to reduce deflationary pressure and dampen the rise in the real interest rate. The results show that a simple New Keynesian model neglects an important transmission channel to study the effects of consumption tax policies. In fact, the analysis points to the possibility that fiscal policy that seeks to stimulate consumption via the supply side can be expansionary even in a low interest rate environment.

Fig. 3.2 Summary of Results

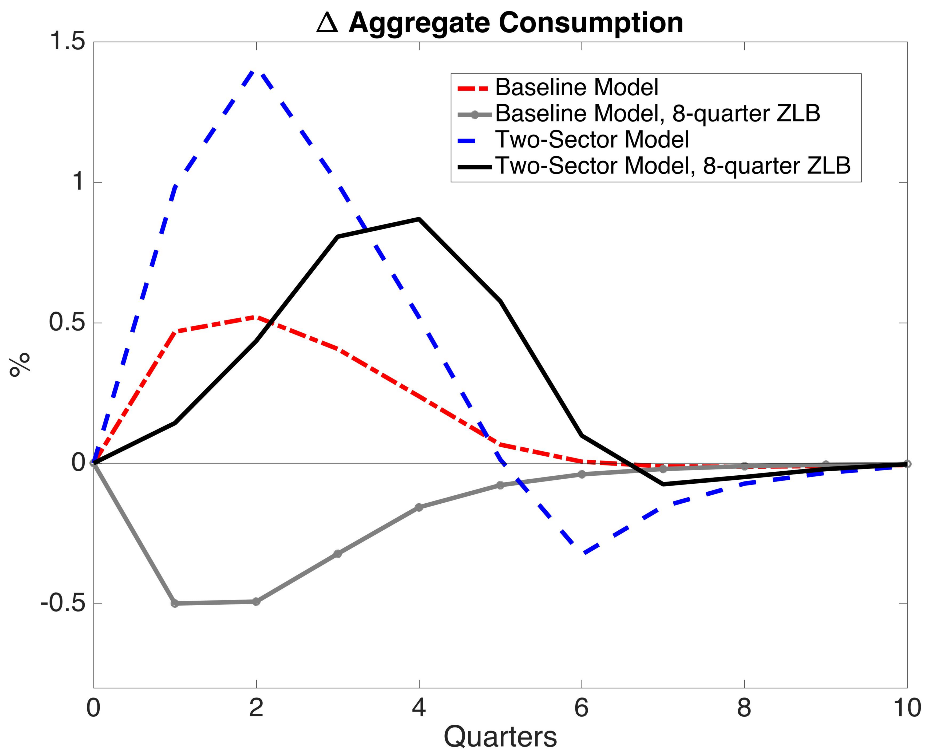


Figure 3.2 illustrates the main results. The *baseline model* denotes the standard New Keynesian model. It features price and wage rigidities. The extended model is a *two-sector model* in which one sector produces non-durable consumption goods and the other produces

durable consumption goods. Workers are assumed to be perfectly mobile between the sectors. Price and wage rigidities are present in both sectors. Now, I consider the following scenario: assume that the models are subject to a large transitory adverse shock and monetary policy responds by lowering the nominal interest rate to zero for an expected duration of 8 quarters. In addition, a VAT cut may or may not be implemented. If it is implemented, the VAT will be reduced in the first period when the ZLB on nominal interest rates is binding and will remain lower for 4 quarters consistent with the UK experience. The tax cut is entirely unexpected by households and firms. Figure 3.2 shows the differences of impulse responses for aggregate consumption between a scenario with a VAT cut and a scenario without a VAT cut for the different New Keynesian models. A VAT cut lowers aggregate consumption in the standard New Keynesian model (gray line with circles) but stimulates demand in the New Keynesian model with non-durable and durable goods consumption (black solid line). Furthermore, I assume an alternative scenario in which the model is subject to a small adverse shock and monetary policy is perfectly accommodative. In this case, the standard New Keynesian model (red dash-dotted line) and the extended model (blue dashed line) predict that the VAT cut is expansionary.

The results are sensitive to the choice of two key parameters: the share of durable goods in the overall consumption basket and the relative degree of price rigidity in the durable goods sector and the non-durable goods sector. The share of durable goods is calibrated to match its contribution to the representative consumption basket for the UK in 2008. The degree of price rigidities determines to which extent the VAT cut will be passed on to consumer prices. Ultimately, the degree of tax pass-through to consumer prices is an empirical question. Several papers have estimated the pass-through of VAT changes to consumer prices. For example, Bernedek et al. (2015) and Carare and Danninger (2008) consider a series of VAT changes in European countries and find that the pass-through on aggregate prices is less than full on impact. For the UK in 2008/2009, Crossley et al. (2014) document that prices did not immediately adjust by the size of the tax cut. Using UK data, I estimate the degree of the pass-through from the temporary VAT cut in 2008/2009 on consumer final prices. The evidence suggests that consumer prices of semi-durable and durable goods declined significantly on impact after the implementation, while the pass-through was limited for non-durable goods and services. Full pass-through of the lower VAT rate on prices, however, can be rejected in both cases.

I solve all models using a global solution technique to account for future shock uncertainty affecting the decision rules of households and firms. The solution approach is based on a collocation method that solves for the decision functions of households and firms on a pre-defined grid (see e.g Judd (1998), Miranda and Fackler (2002) and Schmidt (2013)). One major drawback of grid-based solution methods is the computational burden. Computation times increase with the size of the model and the number of endogenous state variables and exogenous processes. Based on this method, I develop an algorithm that exploits the fact that the model equations can be represented as a system of difference equations over a pre-defined grid. The main advantage of this approach is a significant speed improvement compared to the standard implementation of the algorithm for this global solution method. The algorithm is particularly useful for the two-sector model which features a non-negligible number of state variables.

Throughout the analysis, I assume that tax changes do not impose any direct income effects for consumers, i.e. Ricardian equivalence holds in all models. In principle, it cannot be ruled out that income effects matter for households' spending decisions. This might, for example, be the case if consumers face borrowing constraints. I abstract from such considerations to focus mainly on understanding the role of price dynamics for aggregate demand. My findings show that price dynamics in a multi-sector model can be fundamentally different and, hence, change the implications of fiscal policy that targets the supply side in a low interest rate environment.

All told, my main findings shed new light on the discussion about the effects of supply side shocks in a liquidity trap. A VAT cut can have expansionary macroeconomic effects in normal times *and* in severe recessions with a binding ZLB on nominal interest rates if the model accounts for a more realistic consumption basket. Moreover, a temporarily lower VAT rate can be an appealing policy option in practice for three main reasons. First, the policy can be applied in many countries. Second, it affects a large share of total private spending and, hence, can be an effective approach to stimulate demand. And third, the implementation can be very fast. In the UK, the lower VAT rate was set in place only one week after it was announced. For example, tax rebates can take months to become effective.

The remainder of this Chapter is organized as follows. Section 3.3 describes the baseline New Keynesian model and the two-sector New Keynesian model. Section 3.4 summarizes the calibration of the models and explains the solution method. Section 3.5 discusses the results of the simulations and the tax multipliers. Section 3.6 reports relevant robustness checks. Finally, section 3.7 concludes.

3.2 Related Literature

A number of papers point out that durable goods consumption have important implications for monetary and fiscal policy (e.g. Erceg and Levin (2006), Barsky et al. (2015), Basu and Di Leo (2017)). The mechanisms that are stressed in this Chapter are closest to Boehm (2019). He studies the effects of fiscal policy in a two-sector New Keynesian model and finds that the size of government spending multiplier depends on the composition of goods that are purchased by the government. More specifically, he finds that the fiscal mutiplier tends to be smaller if the government buys durables or investment goods. The crowding-out effect for these goods is particularly strong because private demand is highly price elastic. Hence, a policy that increases the relative price of durable goods has contractionary effects in this model.

My study is also related to the literature that analyzes the relationship between consumption and inflation expectations. Using a New Keynesian model, Correia et al. (2013) suggest to raise consumption taxes to induce positive inflation expectations at the ZLB. They show that by engineering an increasing path for consumption taxes over time, it would be possible to generate negative real interest rates and stimulate demand in a liquidity trap. Their analysis differs from mine in one key dimension. Correia et al. (2013) study a sales tax that allows for instantaneous pass-through on consumer final prices. Consequently, their optimal tax policy might have completely different implications if the consumption tax was a value-added tax.

D'Acunto et al. (2017), Bachmann et al. (2015) and Ichiue and Nishiguchi (2015) empirically test the hypothesis that higher (expected) consumer prices have expansionary effects at constant nominal interest rates. While Bachmann et al. (2015) do not find any effects for consumer behavior, D'Acunto et al. (2017) and Ichiue and Nishiguchi (2015) document that people tend to increase durable goods purchases when they expect consumer prices to increase in the future. This result is in line with my results. The demand for new durables is particularly high (i) in the first period of the VAT cut when relative price of durables is lowest and (ii) in the last period of the VAT cut when households are expecting final prices to increase again once the tax rate has reverted to its old level.

3.3 The Models

This section describes the model economies. The baseline model is a standard New Keynesian model with wage rigidities as in Erceg et al. (2000). The second model is a two-sector New Keynesian model that allows for non-durable and durable goods consumption similar to Barsky et al. (2007). In addition, I introduce a *value-added tax (VAT)* on all consumption goods. The tax is collected from all sellers in each stage of the supply chain and is a percentage of the sale price. In this model economy, the supply chain consists of a final good producer and intermediate good producers. The VAT is distinctly different from a general sales tax. In particular, the sales tax is typically imposed on the customers directly at the final stage as a percentage on top of the sale price.

3.3.1 The Baseline New Keynesian Model

The model is formulated in discrete time with an infinite horizon. The economy is populated by households, a final good producer, a continuum of intermediate good producers and a monetary policy authority.

Final good production. A final good, $Y_{C,t}$, is produced by competitive and identical firms using the technology

$$Y_{C,t} = \left[\int_0^1 Y_{C,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{with } \varepsilon > 1. \tag{3.1}$$

The representative firm chooses input goods, $Y_{C,t}(j)$, $j \in [0,1]$, to maximize her profits. Let τ_t be the VAT rate in period t. The firm sells its good at a price inclusive of the VAT, $P_{C,t}$ but pays the fraction $\frac{\tau_t}{1+\tau_t} P_{C,t}$ of the final price as a tax.[2]

The representative final good producer purchases intermediate good j at final price inclusive of VAT, $P_{C,t}(j)$. However, the final good producer is allowed to recover the tax from purchasing intermediate goods. It will be compensated with the amount $\frac{\tau_t}{1+\tau_t} P_{C,t}(j)$ that has been collected from the intermediate good producer.

[2]Given the final price $P_{C,t}$, the producer receives the net-of-tax price $P_{C,t}^{net} = P_{C,t} - P_{C,t} \frac{\tau_t}{1+\tau_t} = \frac{1}{1+\tau_t} P_{C,t}$. Hence, the VAT rate can be interpreted as a tax on the net-price with $P_{C,t} = (1+\tau_t) P_{C,t}^{net}$. To further illustrate the idea of the VAT, imagine that τ_t is 10 % and final consumer price inclusive tax is $P_{C,t} = 1.1$. The VAT share of the sale price is $\frac{\tau_t}{1+\tau_t} \approx 0.091$. This implies that the retailer's tax liabilities are $P_{C,t} \frac{\tau_t}{1+\tau_t} = 1.1 \times 0.091$. Her effective net-of-tax price is : $P_{C,t}^{net} = P_{C,t} - P_{C,t} \frac{\tau_t}{1+\tau_t} = 1$.

The representative final good firm maximizes profits

$$P_{C,t}Y_{C,t} - \int_0^1 P_{C,t}(j)Y_{C,t}(j)dj - \underbrace{\left(P_{C,t}\frac{\tau_t}{1+\tau_t}Y_{C,t} - \int_0^1 P_{C,t}(j)\frac{\tau_t}{1+\tau_t}Y_{C,t}(j)\,dj\right)}_{\text{VAT liabilities}} \tag{3.2}$$

subject to the production function in Eq. (3.1). Rewriting Eq. (3.2) shows that taxation is based on the value-added, $\frac{1}{1+\tau_t}\left(P_{C,t}Y_{C,t} - \int_0^1 P_{C,t}(j)Y_{C,t}(j)\,dj\right)$.

The firm's demand for input j and the price index, $P_{C,t}$, are:

$$Y_{C,t}(j) = \left(\frac{P_{C,t}(j)}{P_{C,t}}\right)^{-\varepsilon} Y_{C,t}, \tag{3.3}$$

$$P_{C,t} = \left(\int_0^1 P_{C,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}. \tag{3.4}$$

Note that for the representative final good producer, the VAT only scales the profits but does not affect the optimal choice of input $Y_{C,t}$. This holds as long as final good producers are perfectly competitive.

Intermediate good production. Intermediate goods are produced by a unit continuum of differentiated firms, each of which produces a single variety j. Each intermediate good firm has a constant returns to scale production function $Y_{C,t}(j) = N_{C,t}(j)$ using labor $N_t(j)$ as the only input factor. Retailers have monopolistic power in the product market and are competitive in the factor market, i.e. labor market.

Every retailer sets her price inclusive VAT, $P_{C,t}(j)$, subject to the demand in Eq. (3.3), facing price frictions as in Calvo (1983):

$$P_{C,t}(j) = \begin{cases} P_{C,t-1}(j) & \text{with probability } \theta_C \\ P^*_{C,t}(j) & \text{with probability } 1-\theta_C \end{cases} \tag{3.5}$$

where $P^*_{C,t}(j)$ is the firm's optimal re-set price. Given VAT rate τ_t, a firm that is allowed to re-optimize its sale price in period t maximizes:

$$E_t \sum_{s=0}^{\infty} (\theta_C \beta)^s \Lambda_{t+s} \left[P^*_{C,t}(j) - \underbrace{P^*_{C,t}(j) \frac{\tau_{t+s}}{1+\tau_{t+s}}}_{\text{VAT liabilities}} - P_{C,t}\, mc_{C,t+s} \right] Y_{C,t+s}(j) \tag{3.6}$$

subject to Eq. (3.3). Λ_{t+s} is the firm's stochastic discount factor and $mc_{C,t}$ is real marginal cost which is defined as:

$$mc_{C,t} = \frac{W_t}{P_{C,t}}. \tag{3.7}$$

where W_t is the nominal wage rate. The optimal relative re-set price satisfies:

$$\frac{P^*_{C,t}}{P_{C,t}} = \frac{\sum_{s=0}^{\infty} (\theta_C \beta)^s \frac{\varepsilon}{\varepsilon-1} mc_{C,t+s} \left(\Pi^C_{t,t+s}\right)^{\varepsilon} Y_{C,t+s}}{\sum_{s=0}^{\infty} (\theta_C \beta)^s \Lambda_{t+s} \frac{1}{1+\tau_{t+s}} \left(\Pi^C_{t,t+s}\right)^{\varepsilon-1} Y_{C,t+s}} \tag{3.8}$$

where $\Pi^C_{t,t+s}$ denotes the *cumulative* gross inflation rate over s periods:

$$\Pi^C_{t,t+s} = \begin{cases} 1 & \text{for } s=0 \\ \left(\frac{P_{C,t+1}}{P_{C,t}}\right) \times \ldots \times \left(\frac{P_{C,t+s}}{P_{C,t+s-1}}\right) & \text{for } s = 1,2,\ldots \end{cases} \tag{3.9}$$

Suppose that there is an increase in the tax rate τ_t. This affects the optimal re-set price of the profit-maximizing producers as follows: producers that are able to adjust their sale price would pass on the higher tax rate to the buyer by increasing $P_{C,t}(j)$. All other firms will adjust their price in the following periods. Suppose instead that it is credibly communicated in period t that the VAT rate is raised in the following period. Price-setting is forward-looking, reflecting the relevance of future expected marginal cost and price dynamics for today's optimal price. This implies that future expected changes in the VAT rate will be fully internalized in today's price-setting decision.

Log-linearizing the price-setting condition around a zero inflation steady state yields the New Keynesian Phillips curve:

$$\widehat{\Pi}_{C,t} = \beta E_t \widehat{\Pi}_{C,t+1} + \kappa_C \left(\widehat{mc}_{C,t} + \frac{\overline{\tau}}{1+\overline{\tau}} \widehat{\tau}_t \right), \text{ with } \kappa_C \equiv \frac{(1-\theta_C)(1-\beta\theta_C)}{\theta_C} \tag{3.10}$$

where $\widehat{\Pi}_{C,t}$ are log-deviations of (gross) inflation from steady state. The " $\widehat{\ }$ " denote log-deviation from the steady state and " $\bar{\ }$ " denotes the steady state value. In the presence of price setting frictions, there exists an imperfect pass-through of the VAT on aggregate inflation.

Specifically, a change in the tax rate $\widehat{\tau}_t$ cannot be passed on instantly to final sale prices. The degree of immediate pass-through depends on the degree of price rigidity, reflected by the slope of the Phillips curve κ_C.

Households. There is a continuum of households indexed by $i \in [0,1]$. Households are subject to wage-setting frictions as in Erceg et al. (2000). More specifically, assume that there exists a continuum of monopolistically competitive households, each of which supplies a differentiated labor service $N(i)$ to the production sector. A representative labor aggregator combines households' labor services in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index, $N_{C,t}$, is given by:[3]

$$N_{C,t} = \left[\int_0^1 N_t(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{3.11}$$

where ε_w represents the elasticity of substitution between labor types. Let $W_t(i)$ denote the nominal wage for labor of type i. Wages are set by workers of each type and taken as given by all firms. Cost minimization yields the demand schedule for each labor type i:

$$N_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t \tag{3.12}$$

where W_t is the aggregate wage index defined as

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\varepsilon_w} di \right]^{\frac{1}{1-\varepsilon_w}}. \tag{3.13}$$

The life-time utility of the individual household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log \left(C_t(i) - h_C C_{t-1} - \nu_{C,t} \right) - \frac{N_t(i)^{1+\phi}}{1+\phi} \right\} \tag{3.14}$$

where $C_t(i)$ is consumption of a non-storable consumption good and C_t is aggregate consumption. The parameter $h_C \in [0,1]$ disciplines the degree of external habit formation. $\nu_{C,t}$ is a consumption preference shock as in Erceg and Lindé (2014) or Blanchard et al. (2016).

[3] As in Erceg et al. (2000), I assume that the representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands.

The household has the following period t budget constraint:

$$P_{C,t}C_t(i) + B_t(i) \leq (1+i_{t-1})B_{t-1}(i) + W_t(i)N_t(i) + \Gamma_t - T_t \quad (3.15)$$

where $P_{C,t}$ denotes the price of the consumption good, $W_t(i)$ is individual nominal wage rate and $B_t(i)$ is a risk-free nominal government bond, paying interest rate i_t in the following period. The household pays lump-sum taxes net of transfers T_t and receives a share Γ_t of the profits of all intermediate firms. Each household maximizes expected lifetime utility with respect to consumption, labor supply and holdings of bonds subject to its budget constraint (3.15) and the labor demand function (3.12). Combining the first-order conditions for consumption and bond holdings yields the Euler equation:[4]

$$1 = \beta E_t \left\{ \frac{1+i_t}{1+\pi_{C,t+1}} \frac{C_t - h_C C_{t-1} - \nu_{C,t}}{C_{t+1} - h_C C_t - \nu_{C,t+1}} \right\} \quad (3.16)$$

where $\pi_{C,t}$ denotes the inflation rate.

Households set nominal wages as staggered contracts. In particular, assume that a constant fraction $(1-\theta_w)$ of households can renegotiate its wage in each period. The remainder θ_w stays with the past wage $W_{t-1}(i)$. The first-order condition associated with the choice of the optimal wage $W_t^*(i)$ is given by

$$E_t \sum_{k=0}^{\infty} (\beta\,\theta_w)^k \left\{ \frac{1}{C_{t+k} - hC_{t+k-1} - \nu_{C,t}} N_{t+k}(i) \left(\frac{W_t^*(i)}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{t+k} \right) \right\} = 0 \quad (3.17)$$

where $MRS_t \equiv N_t(i)^{\phi} \left(C_t - hC_{t-1} - \nu_{C,t}\right)$.

Define the real wage as $w_t = W_t/P_{C,t}$. Log-linearizing the optimal wage condition above around the steady state yields the wage inflation equation:

$$\widehat{\Pi}_t^w = \beta E_t \widehat{\Pi}_{t+1}^w + \kappa_w \widehat{\mu}_t \quad (3.18)$$

where $\widehat{\Pi}_t^W = \widehat{w}_t - \widehat{w}_{t-1}$ and $\widehat{\mu}_t = \widehat{MRS}_t - \widehat{w}_t$ and $\kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\phi)}$.

[4] Erceg et al. (2000) show that if there exist state contingent claims that insure households against idiosyncratic wage risk, and if preferences are separable in consumption and leisure, households will be identical in their choice of consumption and bond-holdings. They will only differ in their wage and in their labor supply.

Monetary and Fiscal Policy. Monetary policy follows a Taylor rule subject to a zero lower bound constraint on the nominal interest rate i_t:

$$1+i_t = \max\left\{1, (1+\bar{i})\left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_C}\right)^{\gamma_\pi}\right\} \quad \text{with}\, \gamma_\pi > 1 \tag{3.19}$$

where $\bar{i}$ is the steady state nominal interest rate and $\overline{\pi}_C$ is the inflation target.

The government can temporarily change the value-added tax rate. These changes are modeled as a shock to the tax rate τ_t :

$$\tau_t = \overline{\tau}\, exp\left(\varepsilon_t^\tau\right) \quad \text{with}\ \varepsilon_t^\tau \sim \mathscr{N}(0, \sigma_\tau^2)\ i.i.d. \tag{3.20}$$

where $\overline{\tau}$ is the steady state tax rate. The government collects value-added tax and balances its budget in every period:[5]

$$T_t + (1+i_{t-1})\, B_{t-1} = \tau_t P_{C,t} Y_{C,t} + B_t. \tag{3.21}$$

Market clearing. I assume market clearing in the final goods market, $Y_t = C_t$, and in the labor market. The labor market clearing condition is given by

$$N_{C,t} = \int_0^1 N_{C,t}(j)\, dj. \tag{3.22}$$

For intermediate good firm j, it holds that $\left[P_{C,t}(j)/P_{C,t}\right]^{-\varepsilon} Y_{C,t} = N_{C,t}(j)$. Integrating over j and rearranging yields:

$$Y_{C,t} = \frac{N_{C,t}}{p_{C,t}} \tag{3.23}$$

with

$$p_{C,t} = \int_0^1 \left(\frac{P_{C,t}(j)}{P_t}\right)^{-\varepsilon} dj \tag{3.24}$$

$p_{C,t} \geq 1$ can be interpreted as a measure of output loss due to price dispersion (Yun (1996)).

[5] The balanced budget constraint is not restrictive here as Ricardian equivalence holds. Alternatively, it can be assumed that bonds $B_t = 0$ while $T_t = \tau_t P_{C,t} Y_{C,t}$.

3.3.2 A New Keynesian Model with Durable Goods

In this section, I extend the baseline New Keynesian model by allowing for durable goods consumption as in Barsky et al. (2007). Durable goods are storable and can be used repeatedly.

Firms and Price Setting. The production of non-durable goods and durable goods takes place in two distinct sectors. Sector C produces a non-durable consumption good, while sector X produces a durable consumption good. Assume that labor can move freely between the two industries and the nominal wage is determined on a joint labor market. The industry structure of the durable goods sector is identical to the production sector for non-durables described above. Similarly, intermediate good firms in the durable goods sector face price setting rigidities.

Denote X_t to be newly produced durable goods. The New Keynesian Phillips curve for sector $S \in \{C, X\}$ is:

$$\widehat{\Pi}_{S,t} = \beta E_t \widehat{\Pi}_{S,t+1} + \kappa_S \left(\widehat{mc}_{S,t} + \frac{\overline{\tau}}{1+\overline{\tau}} \widehat{\tau}_t \right) \tag{3.25}$$

where $\kappa_S = \frac{(1-\theta_S)(1-\beta\theta_S)}{\theta_S}$. Note that $mc_{S,t} = MC_{S,t}/P_{S,t}$ is real marginal cost in terms of the sector-specific price index $P_{S,t}$.

Households. Households derive utility from non-durables and from service flows of durable goods. Let $D_t(i)$ be the stock of durables of household i. The household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\alpha) \log \left(C_t(i) - h_C C_{t-1} - \nu_{C,t} \right) + \alpha \log \left(D_t(i) - h_D D_{t-1} - \nu_{D,t} \right) - \frac{N_t(i)^{1+\phi}}{1+\phi} \right\} \tag{3.26}$$

subject to the nominal budget constraint

$$P_{C,t} C_t(i) + P_{X,t} X_t(i) + B_t(i) \leq (1 + i_{t-1}) B_{t-1}(i) + W_t(i) N_t(i) + \Gamma_t - T_t \tag{3.27}$$

and the accumulation equation for durables

$$D_t(i) = X_t(i) + (1-\delta) D_{t-1}(i). \tag{3.28}$$

α is the share of durables in the households overall consumption basket and δ is the depreciation rate on the existing stock of durable goods. $\nu_{C,t}$ and $\nu_{D,t}$ are consumption preference shocks for non-durable goods and durable goods consumption, respectively. $X_t(i)$ is new investment of durable goods.

Let λ_t be the Lagrange multiplier on the budget constraint in Eq. (3.27) and η_t be the Lagrange multiplier on the accumulation equation of durable goods in Eq. (3.28). Note that η_t can also be interpreted as the shadow value for newly purchased durables. Optimal behavior of the household requires:

$$\frac{P_{X,t}}{P_{C,t}} = \frac{\eta_t}{\lambda_t} \tag{3.29}$$

$$\eta_t = U_{D,t} + \beta(1-\delta)E_t\eta_{t+1} \tag{3.30}$$

$$\lambda_t = U_{C,t} \tag{3.31}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{1+i_t}{1+\pi_{C,t+1}} \right\} \tag{3.32}$$

with

$$U_{C,t} = (1-\alpha)\left(C_t(i) - h_C C_{t-1} - \nu_{C,t}\right)^{-1} \tag{3.33}$$

$$U_{D,t} = \alpha\left(D_t(i) - h_D D_{t-1} - \nu_{D,t}\right)^{-1} \tag{3.34}$$

Eq. (3.29) characterizes the demand function for new durables as a function of the relative price $P_{X,t}/P_{C,t}$. Eq. (3.30) relates the shadow value of additional durables to the discounted flow utility from durable goods consumption. Finally, combining Eq. (3.31) and (3.32) gives the standard Euler equation for non-durable consumption goods.

The wage setting problem follows Erceg et al. (2000) and is identical to the solution in the benchmark model. On a general note, the implications of the two-sector model as discussed in Barsky et al. (2007) is at odds with VAR evidence. Following a monetary policy shock, the model would generate negative co-movement across sector output. Several remedies have been proposed in the literature (e.g. Monacelli (2009), Sterk (2010), Carlstrom and Fuerst (2010)). One of those is to incorporate wage rigidities (see for example Carlstrom and Fuerst (2010) or Cenesiz and Guimaraes (2018)). I follow the literature in using wage rigidities to avoid a co-movement puzzle in a two-sector model.

Monetary and Fiscal Policy. The central bank follows a Taylor-type policy rule that targets weighted inflation in the durable and non-durable goods sector:

$$1+i_t = \max\left\{1, (1+\bar{i})\left(\left(\frac{1+\pi_{X,t}}{1+\overline{\pi}_X}\right)^{1-\alpha}\left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_C}\right)^{\alpha}\right)^{\gamma_\pi}\right\} \quad \text{with}\, \gamma_\pi > 1 \tag{3.35}$$

Again, the shock to the VAT rate is:

$$\tau_t = \overline{\tau}\, exp\left(\varepsilon_t^\tau\right) \quad \text{with } \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2)\ i.i.d. \tag{3.36}$$

As before, the government balances it budget in every period:

$$T_t + (1+i_{t-1})\, B_{t-1} = \tau_t \left(P_{C,t}Y_{C,t} + P_{X,t}Y_{X,t}\right) + B_t. \tag{3.37}$$

where $Y_{X,t}$ and $Y_{C,t}$ are output in the durable and the non-durable goods sector, respectively.

Market Clearing. I assume that the labor market clearing condition in every sector S is given by

$$N_{S,t} = \int_0^1 N_{S,t}(j)\, dj \text{ with } S \in \{C, X\}. \tag{3.38}$$

Furthermore, I assume that the aggregate labor market is cleared:

$$N_t = N_{C,t} + N_{X,t}. \tag{3.39}$$

Finally, assume that the markets for non-durable consumption goods and durable consumption goods clear:

$$C_t = \frac{N_{C,t}}{p_{C,t}}. \tag{3.40}$$

and

$$X_t = \frac{N_{X,t}}{p_{X,t}}. \tag{3.41}$$

with

$$p_{S,t} = \int_0^1 \left(\frac{P_{S,t}(j)}{P_t}\right)^{-\varepsilon} dj \;\text{ with } S \in \{C, X\} \tag{3.42}$$

where the ratio $p_{S,t} \geq 1$ with $S \in \{C, X\}$ denotes Yun's (1999) price dispersion measure for the non-durable and the durable goods sector, respectively.

3.3.3 Key Properties of the Model

This section discusses two key properties of the two-sector New Keynesian model.

Demand elasticity for new durables. The inverse demand function for new purchases of durables can be derived using the household's optimality conditions in Eqs. (3.29), (3.30) and (3.34). Solving forward gives:

$$\frac{P_{X,t}}{P_{C,t}} = \lambda_t^{-1} E_t \sum_{k=0}^{\infty} [\beta(1-\delta)]^k \alpha \left(D_{t+k} - h_D D_{t+k-1} - \nu_{D,t+k}\right)^{-1} \tag{3.43}$$

given the stock of durables

$$D_t = X_t + (1-\delta)D_{t-1}. \tag{3.44}$$

As stated in Boehm (2019), a demand elasticity for durable goods is difficult to obtain analytically due to the dynamic structure of the model. Instead, I simulate a 1% decrease in the price for durables ceteris paribus and calculate the exact demand elasticities for different values of the depreciation rate δ using $\beta = 0.9975$ and $\alpha = 0.4$. The parameter α is chosen in such that the share of durable goods consumption in the overall consumption basket of the household corresponds to around 40%. For the UK in 2008, the share of expenditure on durable goods among all goods and services that are subject to the standard VAT rate is roughly 40%. For simplicity, I assume for now that $h_D = 0$ and $\nu_{D,t} = 0$.

Table 3.1 Demand Elasticity for Durable Goods

Rate of Depreciation	Demand Elasticity
$\delta = 1$ (Non-durable)	1
$\delta = 0.7$ (1.3 quarters service life)	2.03
$\delta = 0.5$ (2 quarters service life)	3.9
$\delta = 0.1$ (2.5 years service life)	91.7
$\delta = 0.01$ (25 years service life)	5025.7

Table 3.1 reports the price elasticities of demand for new durable goods for different depreciation rates δ. For a durable good with 2 quarters of service life, a 1 % drop in the price is associated with an increase of spending by 3.9 %. Similarly, demand would go up by 91.7 % for a good that lasts for 2.5 years. The price elasticities are computed given an exogenous change in the price for durables *all else equal*. However, in general equilibrium, demand for

new durable goods will be ultimately determined by the (effective) relative price $\frac{P_{X,t}}{P_{C,t}}\lambda_t$.

The Euler equation for the stock of durables. Combining the household's optimality conditions in Eqs. (3.32) and (3.29) yields the Euler equation for durables:

$$\eta_t = \beta E_t \left\{ \eta_{t+1} \frac{1+i_t}{1+\pi_{X,t+1}} \right\}. \tag{3.45}$$

Eq. (3.45) shows that the household seeks to smooth consumption of durable goods over time. However, there is a distinct difference between consumption smoothing of durables and non-durables. The marginal utility from durables, η_t, is derived from the stock of durables, D_t, which is inherently less volatile than the flow. In the limiting case when durable goods are never used up ($\delta \to 0$), Barsky et al. (2007) show that the shadow value of the durable good η_t is quasi-constant for the following reasons. First, durables with low depreciation rates have high stock-to-flow ratios. The steady-state stock-flow ratio is $1/\delta$ in this model. A high stock-to-flow ratio implies that even large changes in the production of durables have only small effects on the total stock of durables. Barsky et al. (2007) show that the latter implies that the aggregate stock of durables is approximately constant $D_t \approx \overline{D}$. Second, if β is close to 1, η_t largely depends on marginal utility flows in the distant future. In a stationary environment with temporary shocks, η_t will be dominated by future utility flows in steady-state.[6]

To sum up, spending on new durables is subject to a high demand elasticity while at the same time the degree of consumption smoothing is naturally high for the existing stock of durables. Since the marginal utility from a *stock* of durables is naturally smooth, consumers are willing to tolerate larger fluctuations in *new purchases* of durables. Hence, consumers will find it optimal to buy durables when the (relative) price is temporarily low and use up the stock of durables when the (relative) price is high.

[6]Note that if $\eta_t \approx \overline{\eta}$, the Euler equation for durables implies that there is a one-to-one relationship between the nominal interest rate and expected inflation in the durable goods sector. A first-order approximation of the the durable Euler equation around the steady state approximately gives

$$\widehat{i}_t \approx E_t \widehat{\pi}_{X,t+1}. \tag{3.46}$$

This relationship shows that there is a pure Fisher effect for the prices of durables (Barsky et al. (2015)). This is the result of an extreme form of consumption smoothing. In every period, the consumer derives utility from the same, constant stock of durables.

3.4 Calibration and Solution Method

3.4.1 Calibration

I calibrate the New Keynesian model at quarterly frequency. The following parameter values apply to both models: the household's discount factor β is set to 0.9975 and the steady state (net) inflation rate to zero, so that $\overline{\pi}_C = \overline{\pi}_X = 0$. This implies an annual steady state interest rate of 1% in line with Christiano et al. (2011). The inverse Frisch elasticity ϕ is set to unity, which is consistent with Barsky et al. (2007), Kimball and Shapiro (2008) and Hall (2009). The elasticity of substitution between differentiated goods, ε_C, is set to 6. This implies a steady-state price markup of 20 percent in line with Rotemberg and Woodford (1997). Furthermore, I assume that $\varepsilon_W = 2$ and $\theta_W = 0.45$ which corresponds to a wage contract duration of roughly 2 quarters consistent with the findings in Galí (2011). The habit persistence parameters are $h_C = h_D = 0.5$. Macro-based estimates of habits persistence range from 0.55 in Smets and Wouters (2003), 0.76 in Altig et al. (2011) to 0.98 as reported by Bouakez et al. (2005). Micro-based estimates are smaller, e.g. Ravina (2007) documents values from 0.29 to 0.5. For monetary policy, I use the standard Taylor (1993) rule parameter $\gamma_\pi = 1.5$.

Furthermore, α is set to 0.4 to correspond to around 40 % of total expenditures for semi-durable and durable goods subject to the standard VAT rate. I choose $\delta = 0.5$ as a compromise between durable and semi-durable goods.

Finally, I define the preference shock for non-durable goods consumption as $\widehat{\mu}_{C,t} = E_t \widehat{v}_{C,t+1} - \widehat{v}_{C,t}$. $\widehat{\mu}_{C,t}$ and $\widehat{v}_{D,t}$ follow an AR(1)-process:

$$\widehat{\mu}_{C,t} = \rho_C \widehat{\mu}_{C,t-1} + \varepsilon_{C,t} \quad \text{with} \quad \varepsilon_{C,t} \sim \mathcal{N}(0, \sigma_C^2) \; i.i.d. \tag{3.47}$$

$$\widehat{v}_{D,t} = \rho_D \widehat{v}_{D,t-1} + \varepsilon_{D,t} \quad \text{with} \quad \varepsilon_{D,t} \sim \mathcal{N}(0, \sigma_D^2) \; i.i.d. \tag{3.48}$$

The AR(1) coefficients are set to $\rho_C = \rho_D = 0.8$ to gauge the effects of the global financial crisis and the Great Recession. Finally, I set the unconditional standard deviations to $\sigma_C = \sigma_D = 0.025/100$.

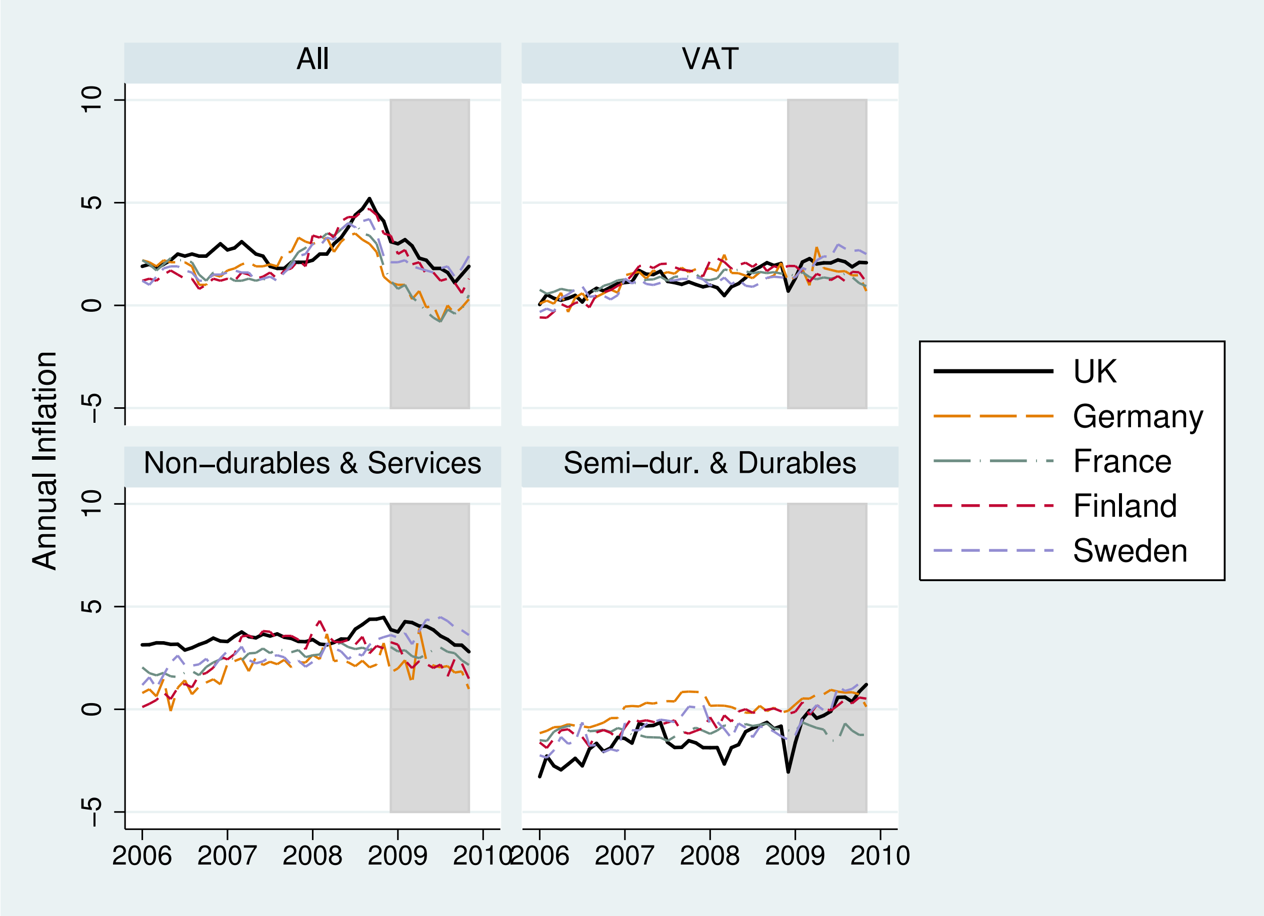
Evidence on Imperfect VAT Pass-through. To calibrate the degree of price rigidities in the two sectors, θ_C and θ_X, I draw on evidence from the temporary VAT cut in the UK in 2008/2009. The UK government lowered its standard VAT rate by 2.5 p.p. (from 17.5% to 15%). The VAT on goods subject to the reduced rate of 5% (mainly food and energy) and on alcohol and tobacco remained unchanged.

A full pass-through of the VAT cut in the UK would have implied a fall in consumer prices by 2.1 percentage points.[7] Between November 2008 and December 2008, annual CPI inflation in the UK fell by 1 percentage point. A number of papers have estimated the pass-through of the temporary tax shock on aggregate inflation. Pike and Turner (2009) find that around 38% of the tax cut was passed on to consumer prices. However, their estimate may be biased because they assume that any reduction in prices for VAT goods between December 2008 and 2009 was due to the cut. Crossley et al. (2014) estimate the effects of the tax cut on prices and spending using inflation dynamics of non-VAT goods to identify the counterfactual. They cannot reject full pass-through in the first month of implementation. Over the entire period, however, their estimate is insignificant suggesting that prices were reversed again before the tax cut ended. In an alternative approach, Crossley et al. (2014) estimate a counterfactual path for prices in the UK using data on prices in other OECD countries. This specification rejects full pass-through in the first month at 10% significance level. Again, the results suggest that at least part of the VAT cut was reversed early.

Following Crossley et al. (2014), I use the evolution of prices and purchases in 9 other European countries to estimate the counterfactual. A counterfactual analysis is necessary because many other factors might have affected prices in December 2008. One reason could be the drop in food and energy prices in late 2008. I consider the following countries: Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Portugal and Sweden. Figure 3.3 shows the Consumer Price Index (CPI) between 2006 and 2010 for different sub-categories and countries. The index of consumer prices for "VAT goods and services" is a weighted index of prices for all goods and services at the standard VAT rate. Price dynamics of non-durables and services that accrue the standard tax rate are summarized by the category "Non-durables and Services". Likewise, the CPI for semi-durable and durable consumption goods at standard VAT rate are shown under "Semi-durables and Durables". All indices are computed as annual

[7] Suppose P^{net} is the net-of-tax price. Then, the rate of change in the sale price under full pass-through is $\frac{1.15\times P^{net}-1.175\times P^{net}}{1.175\times P^{net}} = -0.021$.

Fig. 3.3 Harmonized Index of Consumer Prices (Annual Rate of Change) by Consumption Category for Selected Countries



rate of change. It is the change in the index of a certain month compared with the index of the same month in the previous year expressed in percentappe.

The shaded area indicates the temporary VAT cut in the UK. Between November and December 2008, inflation for "VAT Goods and Services" in the UK fell by 1.5 percentage points. Almost half of the drop was reversed in February 2009 when inflation in VAT goods rose again by 0.65 percentage points. Most of the fall in inflation of VAT goods came from prices of semi-durable and durable goods. For these goods, prices changes by about 2.5 percentage points. In contrast, inflation for non-durables and services changed minimally over the same period.

Despite some variation across countries prior to the UK VAT cut, the evolution of inflation in the European countries displays by and large a similar trend before December 2008 and after January 2009. Moreover, the policy impact of the VAT cut is evident, especially for semi-durables and durables.

Detailed information on the data is provided in Appendix B. The data is used on a monthly basis and covers the period from January 2002 to December 2010. I estimate the pass-through of the VAT cut on inflation using the following difference-in-difference specification separately for each good category k, where k are either non-durables/services or semi-durables/durables:

$$Inflation_{i,t}^{a,k} = \beta_0 + \beta_1 UK_i + \beta_2 UK_i * Cut_t^m + \beta_3 Cut_t^m \tag{3.49}$$

$$+ \beta_4 totalsales_{i,t-1} + \beta_5 totalsales_{i,t-1}^k + \beta_6 Trend_{i,t} + \varepsilon_{i,t} \tag{3.50}$$

where $Inflation_{i,t}^{a,k}$ is the Consumer Price Index. $Trend_{i,t}$ is a (one-sided) HP-filtered trend. $totalsales_{i,t-1}^k$ are retail sales or turnover in good group k.[8] $totalsales_{i,t-1}$ are aggregate retail sales of the previous period.

The superscript m denotes the time span which is considered in the specification. Hence, $Cut_t^{dec08-jan09}$ is a dummy set to 1 for all countries in December 2008 and January 2009. Suppose $m = dec08$. Then, β_2 is an estimate of inflation pass-through from the VAT cut unique to the UK in December 2008. Intuitively, the coefficient picks up changes in inflation in UK in the first month of the VAT cut after differencing out any price changes that were common in December 2008 for all countries.

All regression results are reported in Table 3.2. The estimates of β_2 are negative in the first month of implementation indicating that prices fell in December 2008 for all good categories. Zero pass-through on inflation can be rejected at 1% significance level. Interestingly, the estimated pass-through coefficients for non-durables/services and semi-durables/durables are insignificant from February onwards suggesting that price changes were undone very early.

I use the estimation results to calculate the average degree of inflation pass-through implied by the estimates in Table 3.2. Figure 3.4 shows the results. Overall, full pass-through of the tax shock to consumer prices in the first month of implementation can be rejected for all VAT goods and services. The degree of pass-through averages around 25% for non-durables and services and 75% for semi-durables and durables in December 2008. Note that the standard errors for the estimates for semi-durables and durables are higher compared to the standard errors in the other regression. Yet, the lower bound of the 95% interval would imply a pass-through of 65% and remains higher than the lower bound of the same interval for non-durable goods and services.

[8] Sales are included with a one period lag to avoid simultaneity issues.

Table 3.2 Regression Results

Annual inflation (p.p.)	(1) Non-durables and Services	(2) Semi-durables and Durables
$UK{*}Cut^{dec08}$	-0.418***	-1.583***
	(0.156)	(0.232)
$UK{*}Cut^{dec08-jan09}$	-0.418 **	-1.16**
	(0.178)	(0.573)
$UK{*}Cut^{dec08-feb09}$	-0.208	-0.78
	(0.169)	(0.628)
$UK{*}Cut^{dec08-mar09}$	-0.0008	-0.328
	(0.154)	(0.585)
$UK{*}Cut^{dec08-apr09}$	0.034	-0.198
	(0.165)	(0.527)
$UK{*}Cut^{dec08-may09}$	-0.073	-0.198
	(0.145)	(0.527)
$UK{*}Cut^{dec08-jun09}$	-0.093	-0.033
	(0.126)	(0.422)
100 % Pass-Through corresponds to	-2.128	-2.128
No. of observations	1043	1043
(Avg.) Adj. R^2	0.51	0.71

(i) Robust standard errors in parentheses

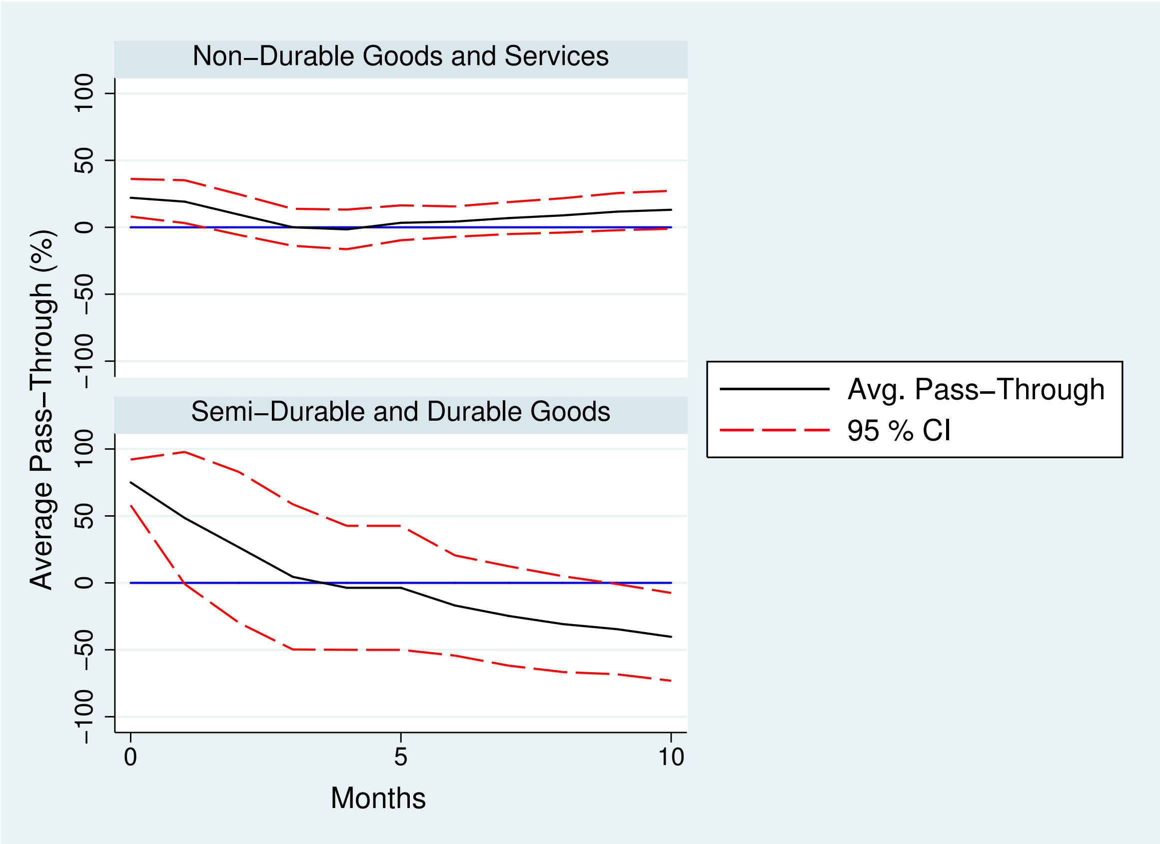
(ii) * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The Calvo parameters θ_C and θ_X are chosen to reflect the general findings of the estimation. Consequently, I set θ_C to 0.75 implying that retailers are able to change their sale prices with a probability of 25%. θ_X is 0.35 such that firms face a probability of 65% to adjust their price in every period.[9] Bils and Klenow (2004) document that prices of durable goods are generally more flexible than those of non-durable goods. However, Nakamura and Steinsson (2008) do not find any systematic evidence of larger flexibility of durable prices, and estimate an average frequency of adjustment of about four quarters regardless of durability. To investigate

[9] The choice of θ_X reflects the lower bound of the estimated immediate pass-through on prices of semi-durables and durables in response to the tax shock.

the sensitivity of the results to the parameter choice, I will consider different values for θ_D in the next section. Table 3.3 reports the baseline calibration for the model parameters.

Fig. 3.4 Estimated Price Pass-Through of VAT cut



3.4.2 Solution Method

For both New Keynesian models, I log-linearize all model equations around a deterministic steady state, except for the max-operator in the Taylor rule. All log-linearized equilibrium equations are provided in Appendix B. The models are solved using a global solution method. In particular, the solution method approximates the value of the decision functions of the agents under rational expectations at a finite number of grid points based on a collocation method (i.e. Judd (1998), Miranda and Fackler (2002), Schmidt (2013), Schmidt (2017)). Further details are provided in Chapter 4.

The baseline New Keynesian model has 4 state variables (i.e. 2 endogenous state variables (non-durable goods consumption and the real wage) and 2 exogenous processes (preference shock and VAT shock)). The two-sector model has 6 state variables in total (i.e. 4 endogenous

Table 3.3 Calibration of the Baseline Model and the Two-Sector Model

Parameter	Value	Description
β	0.9975	Discount factor (quarterly)
α	0.4	Share of durables in utility
δ	0.5	Depreciation rate of durables
ϕ	1	Inverse labor supply elasticity
h_C	0.5	Habit formation parameter: Non-durables
h_D	0.5	Habit formation parameter: Durables
ε_C	6	Elasticity of substitution btw. varieties: ND sector
ε_D	6	Elasticity of substitution btw. varieties: D sector
ε_w	2	Elasticity of substitution btw. labor input
θ_C	0.75	Share of non-price adjusting firms (ND sector)
θ_X	0.35	Share of non-price adjusting firms (D sector)
θ_w	0.45	Share of non-wage adjusting workers
γ_π	1.5	Monet. policy: Inflation coefficient
$\overline{\tau}$	0.175	Steady State VAT rate
$\tilde{\tau}$	0.15	Temporary VAT rate
ρ_C	0.8	AR(1) parameter (preference shock, ND)
ρ_D	0.8	AR(1) parameter (preference shock, D)
σ_C	0.025/100	Standard deviation of shocks to $\widehat{\mu}_C$
σ_D	0.025/100	Standard deviation of shocks to $\widehat{v}_D$

state variables (non-durable and durable goods consumption, the real wage and the relative price between durables and non-durables) and 2 exogenous state variables (preference shock and VAT shock)). The special characteristic of the VAT shock is its deterministic nature. Given that the VAT cut is perfectly credible, households and firms are able to perfectly foresee the path of the tax rate. Suppose the VAT cut lasts for 4 quarters. The model will have a distinctly different solution for each of the 4 periods of the VAT cut. I propose an iterative scheme to solve for the solution of the model in each period. My approach solves backward for the decision functions starting in the last period of the VAT cut in $t = 4$. Given the solution in $t = 4$, it is possible to find the decision functions in period $t = 3$. This scheme will be repeated for periods $t = 2$ and $t = 1$. The details of the solution method are described in Appendix B.

3.5 Results

In this section, I present the main findings of the analysis and discuss the implications of a temporary VAT cut in the baseline model and in the two-sector New Keynesian model.

3.5.1 Constructing a Liquidity Trap and a Temporary VAT Cut

For the baseline scenario, I assume that the economy is hit by a large adverse preference shock similar to Erceg and Lindé (2014), Christiano et al. (2011) so the the ZLB on the nominal interest rate is binding for 8 quarters. More specifically, I assume in the baseline New Keynesian model that the economy is subject to a temporary, positive preference shock $\widehat{\mu}_{C,t}$. In the two-sector model, the economy is hit by a similar positive preference shock that affects both non-durable goods consumption and durable goods consumption equally.

Then, I add the following tax experiment to the baseline scenario: assume that the value-added tax is cut by 2.5 percentage points (from 17.5% to 15%) for 4 periods and returns to its previous level again. The policy is implemented in the same period when ZLB starts to bind.[10] Households and firms do not anticipate the tax shock but are able to foresee the path of the VAT rate in the following periods. Figure 3.5 shows the evolution of the tax rate.

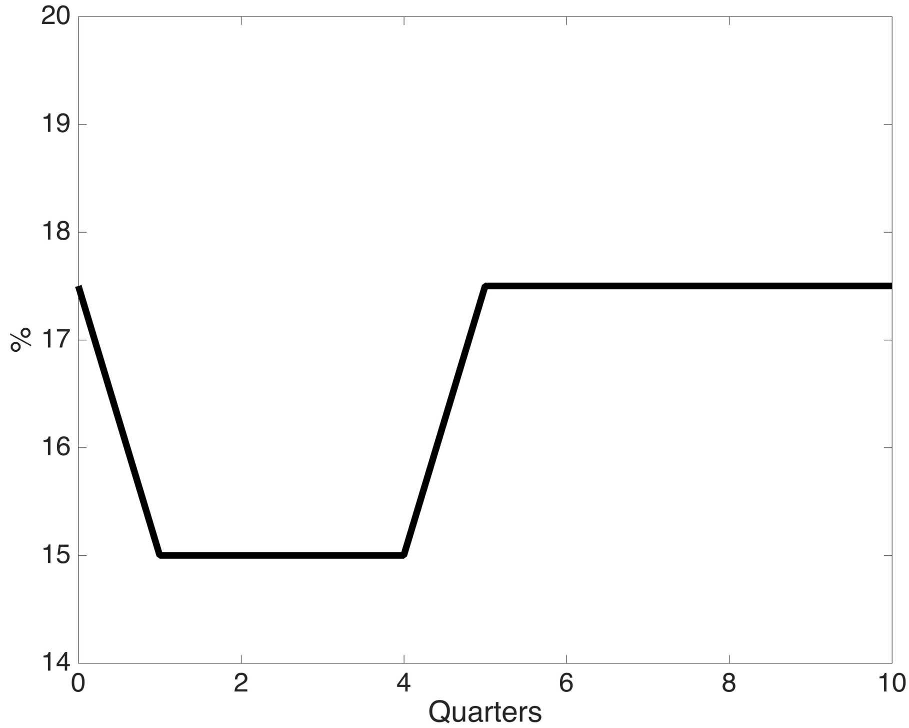
3.5.2 Simulation Results

Baseline New Keynesian Model: Unconstrained Monetary policy. Suppose for now that the ZLB constraint on the nominal interest rate is ignored. Figure 3.6 shows the impulse responses to an adverse preference shock in the baseline New Keynesian model with a temporary VAT cut (black solid line) and without a temporary VAT cut (blue dashed line). The tax shock dampens the fall in aggregate consumption and output.

A temporary VAT cut affects consumer prices in the following way: under sticky prices, a fraction of retailers are unable to adjust their sale price on impact in response to the VAT cut. Since firms are profit maximizers, consumers will expect those firms to adjust prices later and, hence, anticipate a period of falling prices. Monetary policy matches deflationary pressure

[10]In March 2009, the Bank of England lowered the base interest rate (Official Bank Rate) for secured overnight lending from 1% to 0.5%. It had remained at this level until it was further dropped in August 2016 to 0.25%. The UK temporary VAT cut came into effect when the base rate was not yet close enough to a lower bound. Hence, the scenario that is considered in this Chapter studies the general implications of a temporary VAT cut in a New Keynesian model but does not necessarily recount the historical facts.

Fig. 3.5 A Temporary VAT cut

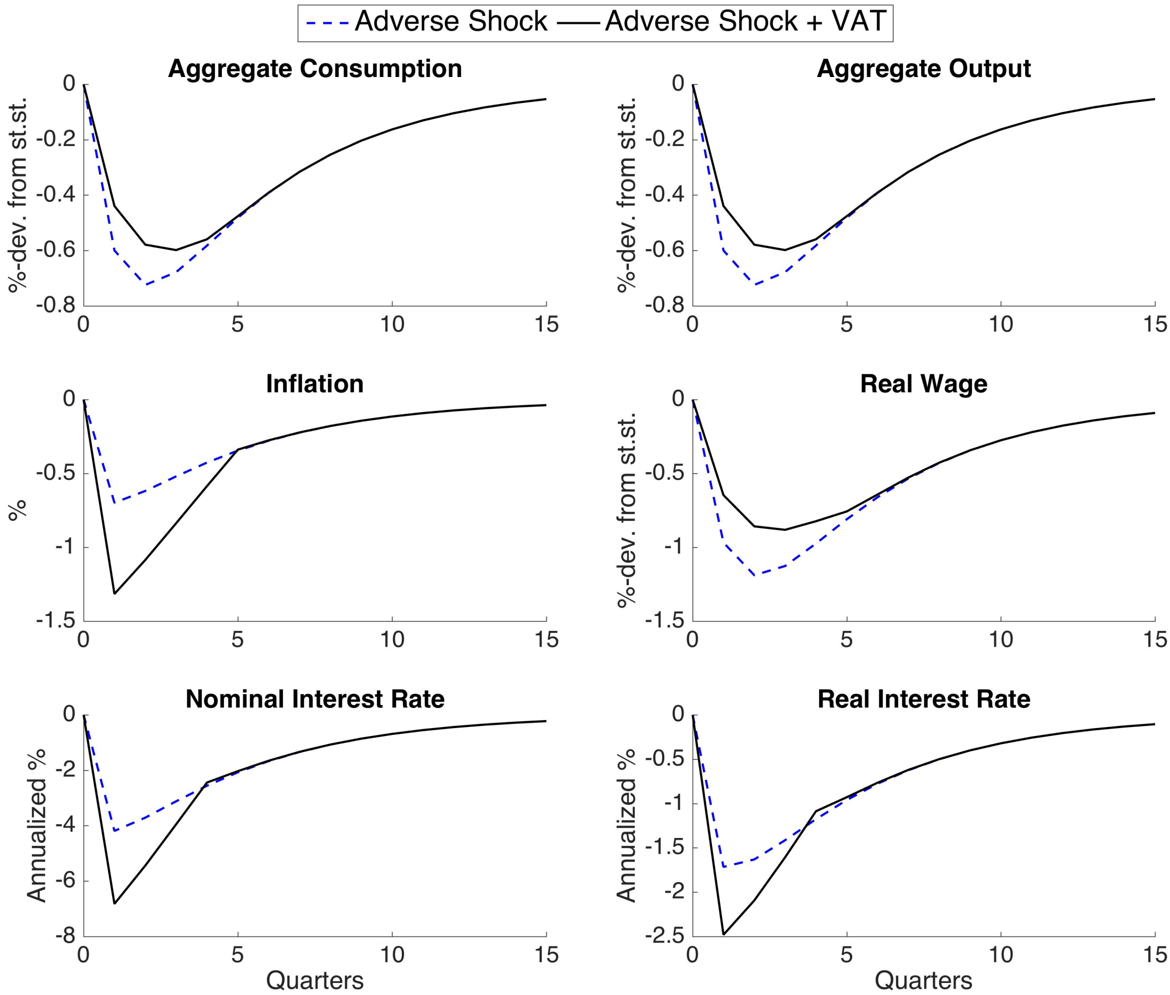


by lowering the nominal interest rate and, hence, reduces the real interest rate so that current demand expands.

Baseline New Keynesian Model: Monetary policy with ZLB constraint. Figure 3.7 shows the impulse responses for an expected 8-quarter liquidity trap. Now, a temporary VAT cut deepens the recession at the ZLB causing a larger drop in consumption and output (black solid line). If the central bank is unable to accommodate to deflationary pressure, the real interest rate increase, further discouraging private spending.

In 2008, the UK government has stressed two major working channels of the VAT cut (Treasury (2008)). First, consumers will enjoy the same basket of goods at lower prices and may spend the extra savings (*income effect*). Second, temporarily lower prices encourage consumers to forward the purchase of goods either for immediate consumption or hoarding (*intertemporal substitution*). Both channels do not work in the standard New Keynesian model with imperfect price pass-through of the value-added tax. The positive *income effect* stemming from cheaper prices is more than offset by the large drop in income in equilibrium that is necessary to reduce the households' temporary desire to save when the real interest rate

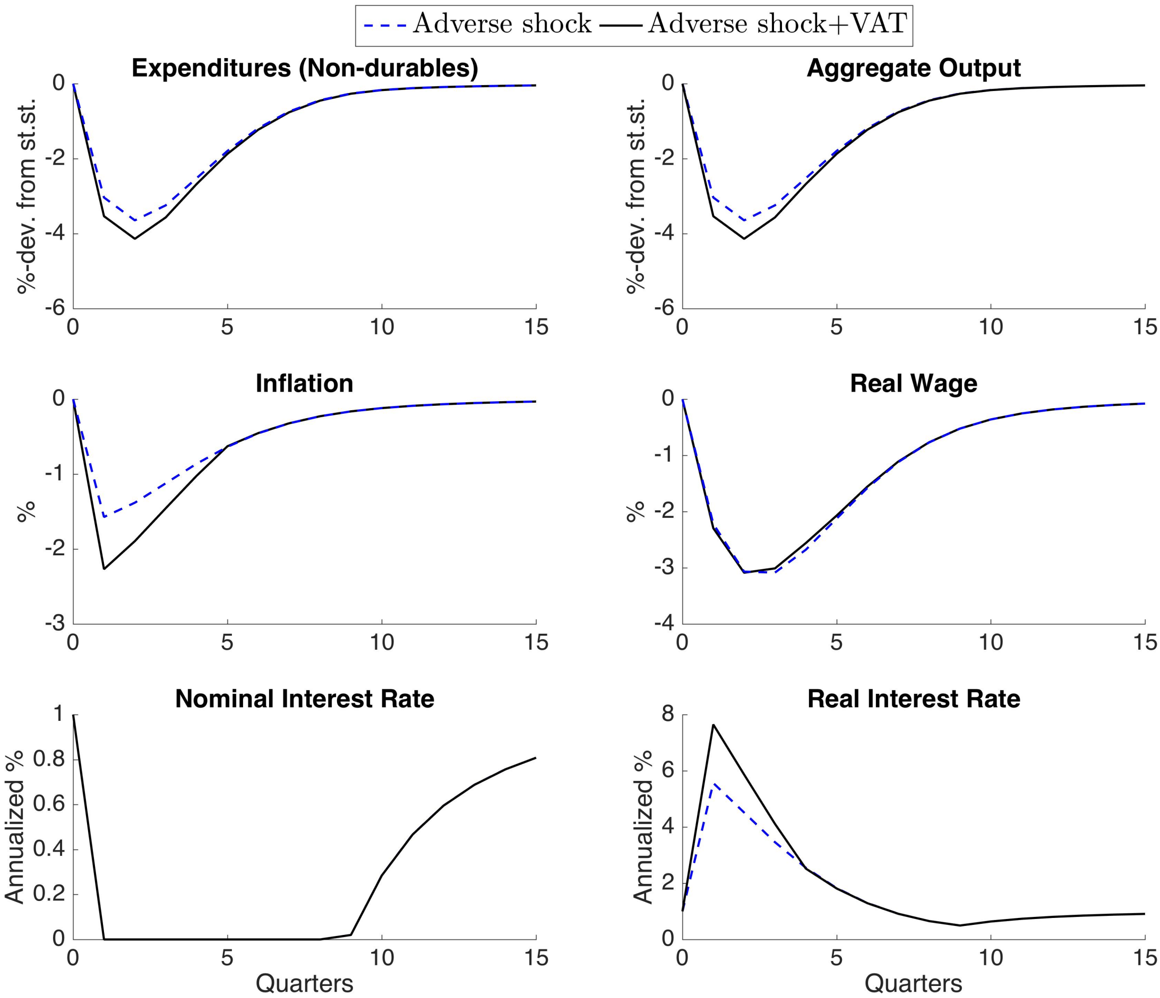
Fig. 3.6 Impulse Responses: Unconstrained Monetary Policy in Baseline Model



rises. Furthermore, in their attempt to smooth marginal utility of consumption, households delay purchases rather than forward them.

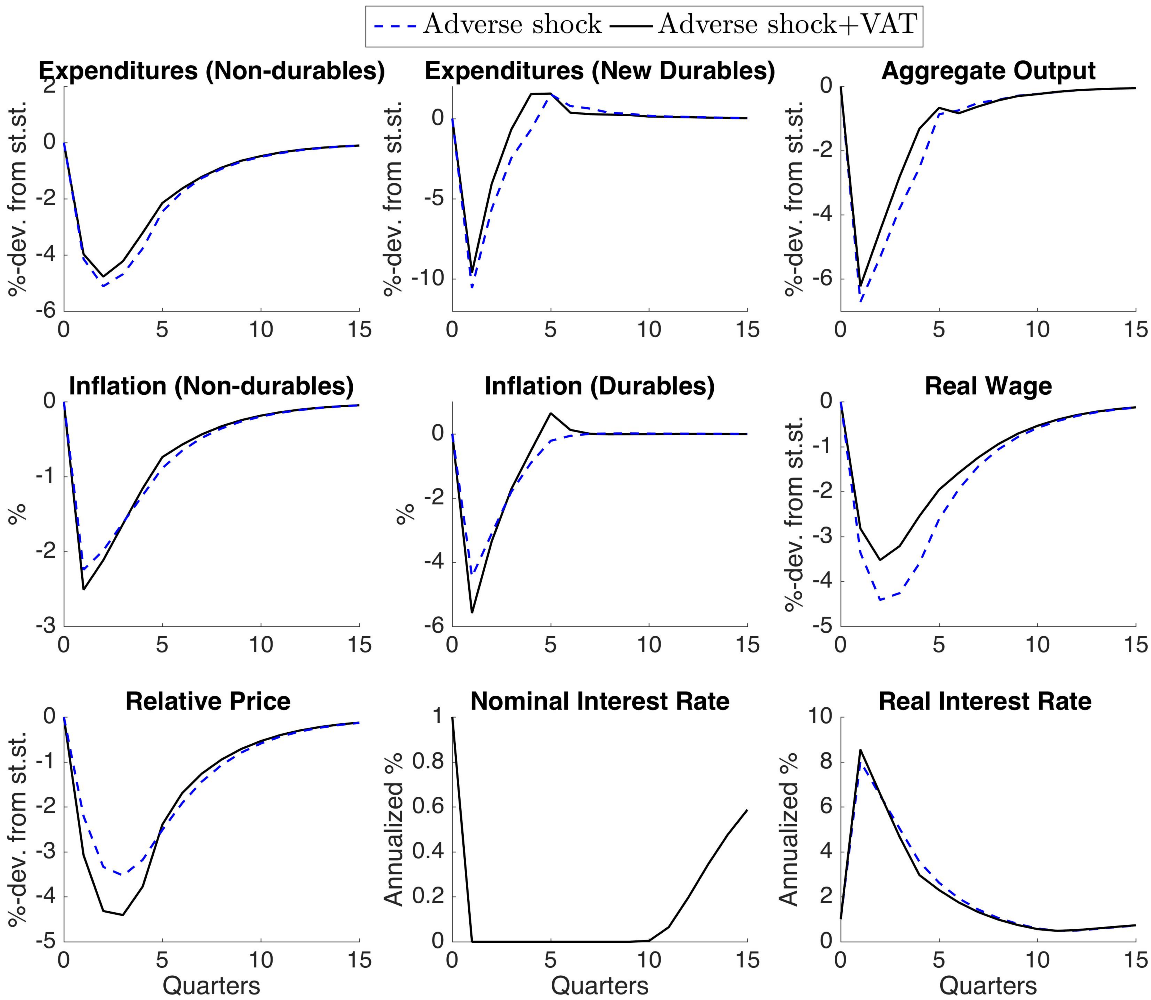
The Two-Sector New Keynesian Model. Figure 3.8 shows the impulse responses to a large adverse shock that leads to a binding ZLB on the nominal interest rate for an expected duration of 8 quarters in the two-sector New Keynesian model. The (gross) real interest rate is defined in terms of inflation of non-durable goods, i.e. $1+r_t \equiv E_t \frac{1+i_t}{1+\pi_{C,t+1}}$. In the two-sector model, a VAT cut dampens the recession and has expansionary effects on private spending for non-durable and durable goods (black solid line). Aggregate output is higher by roughly 0.3 percentage points on impact. For non-durable goods, the difference is roughly 0.08 percentage points. The largest difference can be observed for durable goods. Spending on new durables differ by nearly 0.75 percentage points in the two scenarios.

Fig. 3.7 Impulse Responses: 8-Quarter Liquidity Trap in Baseline Model



Given that prices for durable goods are more flexible, the relative price $\frac{P_{X,t}}{P_{C,t}}$ falls on impact in response to a VAT cut and shifts spending to new durable goods. Spending on new durables increases substantially because the price elasticity of demand is high. The interesting observation in the two-sector model, however, is that demand for non-durable goods also increases. The reason is the following: the boom in the durable goods sector dampens the fall in labor hours and, hence, in the real wage in both sectors assuming a frictionless labor market. This attenuates the fall in marginal costs and prices in the durable *and* non-durable goods sector. The latter dampens the increase in the current real interest rate at the ZLB and further reduces the expected future real interest rate. Expecting lower real interest rates in the future, households start to expand current consumption of non-durable goods. This observation points to the role of price dynamics in general equilibrium for multi-sector models. The expansion in the durable goods sector has implications for inflation dynamics in both sectors.

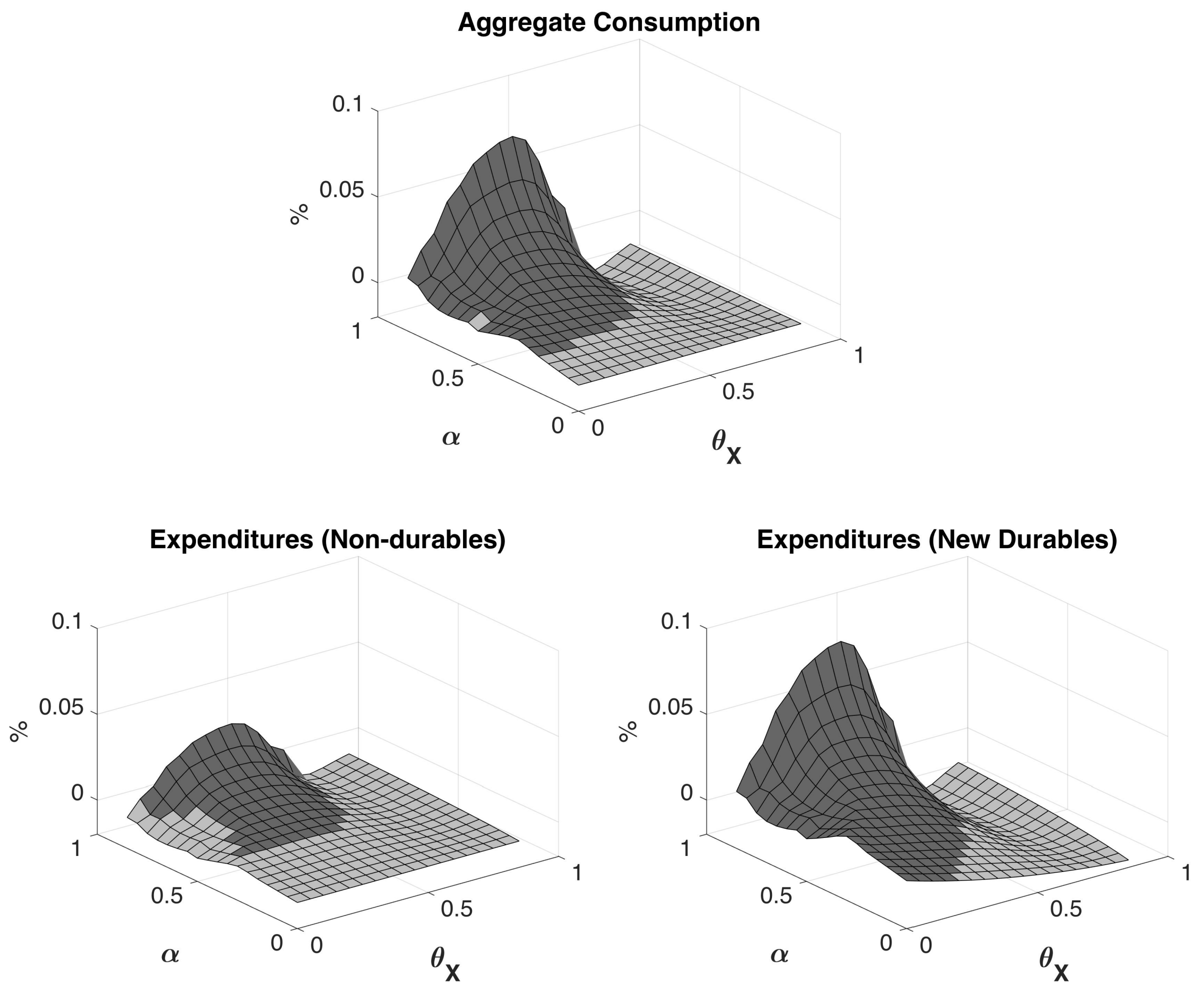
Fig. 3.8 Impulse Responses: 8-Quarter Liquidity Trap in Two-Sector Model



Note: the relative price is expressed as price of durable goods relative to the price of non-durables. The real interest rate is expressed in terms of inflation of non-durable goods.

Note that demand peaks one quarter before the VAT cut ends. The anticipated increase in the VAT rate in period 4 generates positive inflation expectations. Higher inflation expectations at the ZLB reduces the real interest rate and, hence, increase consumers' incentives to increase current spending. Consistent with the prediction of the UK government, a temporarily lower VAT rate induces an intertemporal substitution effect for goods that can be hoarded and consumed at a later point in time. The expansion stops after the tax cut and consumers start enjoying service flows from the accumulated stock of durables. In fact, demand for new durables remains lower (black solid line) compared to the case without VAT change (blue dashed line) after the VAT cut has ended.

Fig. 3.9 Simulation Results in Two-Sector Model: The Relative Importance of α and θ_X



Notes: the plots report the impact tax multipliers for aggregate consumption, expenditures on non-durable goods and durable goods.

The results in the two-sector model are sensitive to the choice of two key parameters: α, the share of durable goods, and θ_X, the degree of price rigidities in the durable goods sector. Figure 3.9 shows the difference between the simulations with VAT cut and without VAT cut in the period when the adverse preference shock hits the economy for different combinations of α and θ_X. The dark-shaded areas represent instances when the difference is positive, i.e. the VAT cut is expansionary. The light-shaded areas show negative difference, i.e. the VAT cut is contractionary. The VAT cut is on aggregate expansionary only if the share of durables α is sufficiently large and the Calvo parameter θ_X small, i.e. prices of durables sufficiently flexible. α is a key parameter for generating positive spill-over effects from the durable to the non-durable goods sector. For low values of α, the size of the durable goods sector is too small to change aggregate wage and inflation dynamics. The ratio of the Calvo parameters θ_C and θ_X determines the extent to which the relative price for durables can fall in response to the VAT cut. Given the baseline calibration $\theta_C = 0.75$ and $\alpha = 0.4$, the highest value for θ_X to ensure that the tax policy is expansionary for consumption of non-durables and durables is $\theta_X = 0.47$.

3.5.3 Tax Multipliers

This section reports the tax multipliers. Let $t + T_n$ be the exit date of the liquidity trap. It will be endogenously determined by the first period in which the nominal interest rate exceeds the zero lower bound (Erceg and Lindé (2014)) with

$$T_n = \min_j \left(i_{t+j} > 0 \right). \tag{3.51}$$

To generate liquidity traps of the same lengths, I adjust the size of the preference shocks in the models accordingly.[11]

The tax multiplier on aggregate output Y_t is defined as:

$$m_t = \frac{\Delta Y_t}{-\Delta(\tau_t \times Y_t)} \tag{3.52}$$

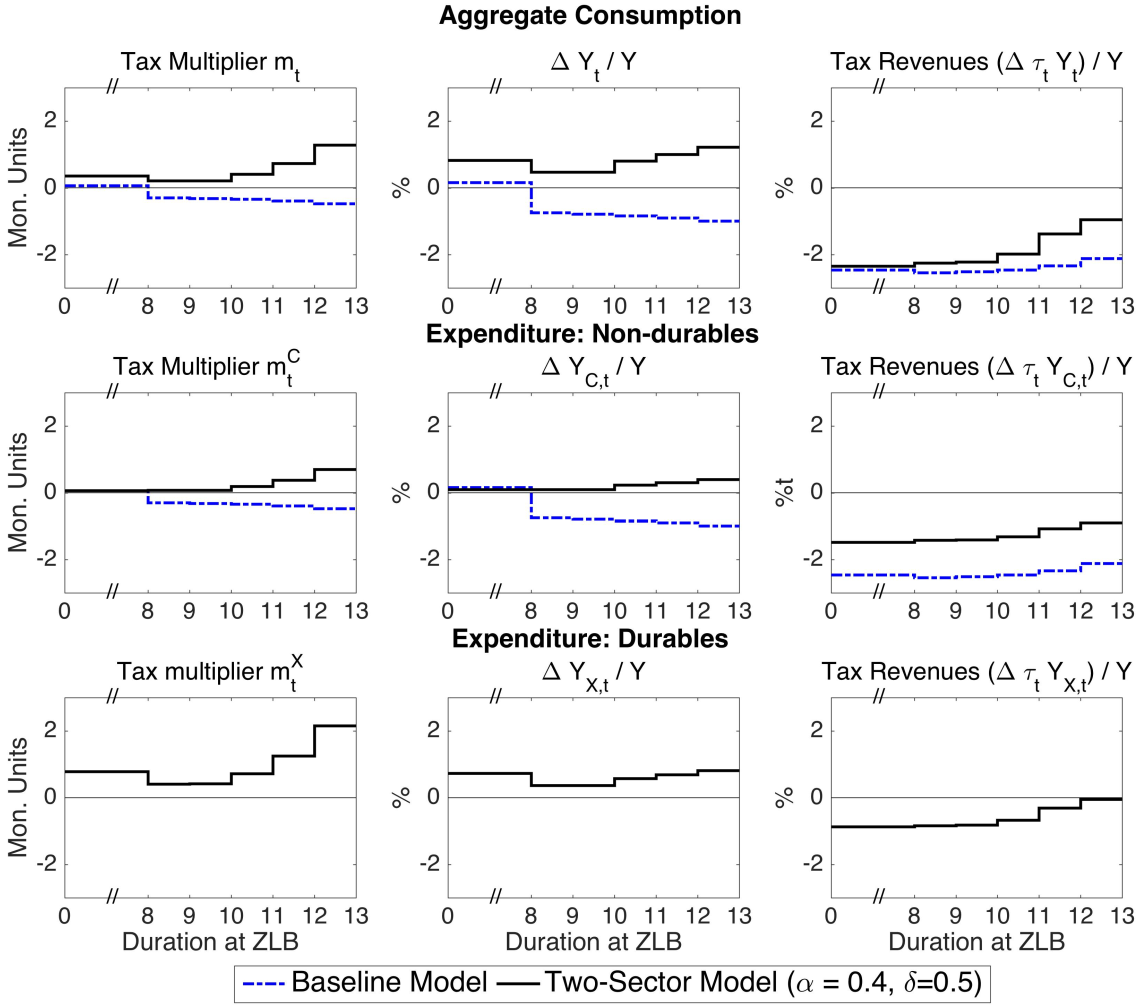
where the Δ-operator represents the difference between the scenario with and without the tax cut. $\Delta(\tau_t \times Y_t)$ is the difference in tax revenues. Hence, $m_t > 0$ ($m_t < 0$) is the gain (loss) in output for any monetary unit of tax revenue that is forgone. A multiplier above 1 would imply that the change in output outweighs the loss in tax revenues. Finally, m_t^S with $S \in \{C, X\}$ is the tax multiplier for private spending on non-durable and durable goods, respectively:

$$m_t^S = \frac{\Delta Y_t^S}{-\Delta(\tau_t \times Y_t)}. \tag{3.53}$$

[11] Alternatively, we can fix the size of the shocks and assume that monetary policy is *passive* over a horizon T where T is the duration of the liquidity trap (see e.g. Kiley (2016)). There is a drawback to this approach for the presented model framework. The central bank implicitly promises to keep a certain interest rate for T periods. In fact, if the recessionary shock is rather small, a *passive* policy would keep the interest rate lower for longer than under *active* monetary policy when nominal rates are free to adjust to changes in inflation. This has expansionary macroeconomic effects if agents are rational and forward-looking. It is hard to control for these effects when I calculate the tax multipliers.

A VAT cut from 17.5% to 15% for 4 quarters. Figure 3.10 shows the average impact tax multipliers in the two models. Duration 0 stands for the scenario in which monetary policy is ignored. The analysis is restricted to liquidity traps which are expected to last longer than 8 quarters. For smaller preference shocks, the expansionary effects of the 4-quarter VAT cut can be large enough to change the expected duration at the ZLB.

Fig. 3.10 Average Tax Multipliers: Baseline Model vs. Two-Sector Model



The tax multipliers are positive in all models when the ZLB is ignored. However, the multiplier for aggregate consumption in the two-sector model is notably larger (0.35) than in the baseline model (0.16). This observation goes back to the boom in the durable goods sector. The tax multiplier for newly purchased durables is roughly 0.78. The difference between the multipliers for non-durables is negligible (0.065 in the basic model vs. 0.063 in the two-sector model).

The tax multipliers in the baseline model are negative in a liquidity trap and further decline with the duration at the ZLB (-0.31 for a 8-quarter vs. -0.52 for a 12-quarter liquidity trap). Suppose for simplicity that $h_C = 0$. Then, solving the Euler equation forward yields

$$\widehat{C}_t = E_t \left\{ \widehat{C}_{t+T_n+1} - \sum_{i=0}^{T_n} \left(\log\beta - E_t \widehat{\pi}_{C,t+i+1} + \widehat{\mu}_{C,t+i} \right) \right\} \tag{3.54}$$

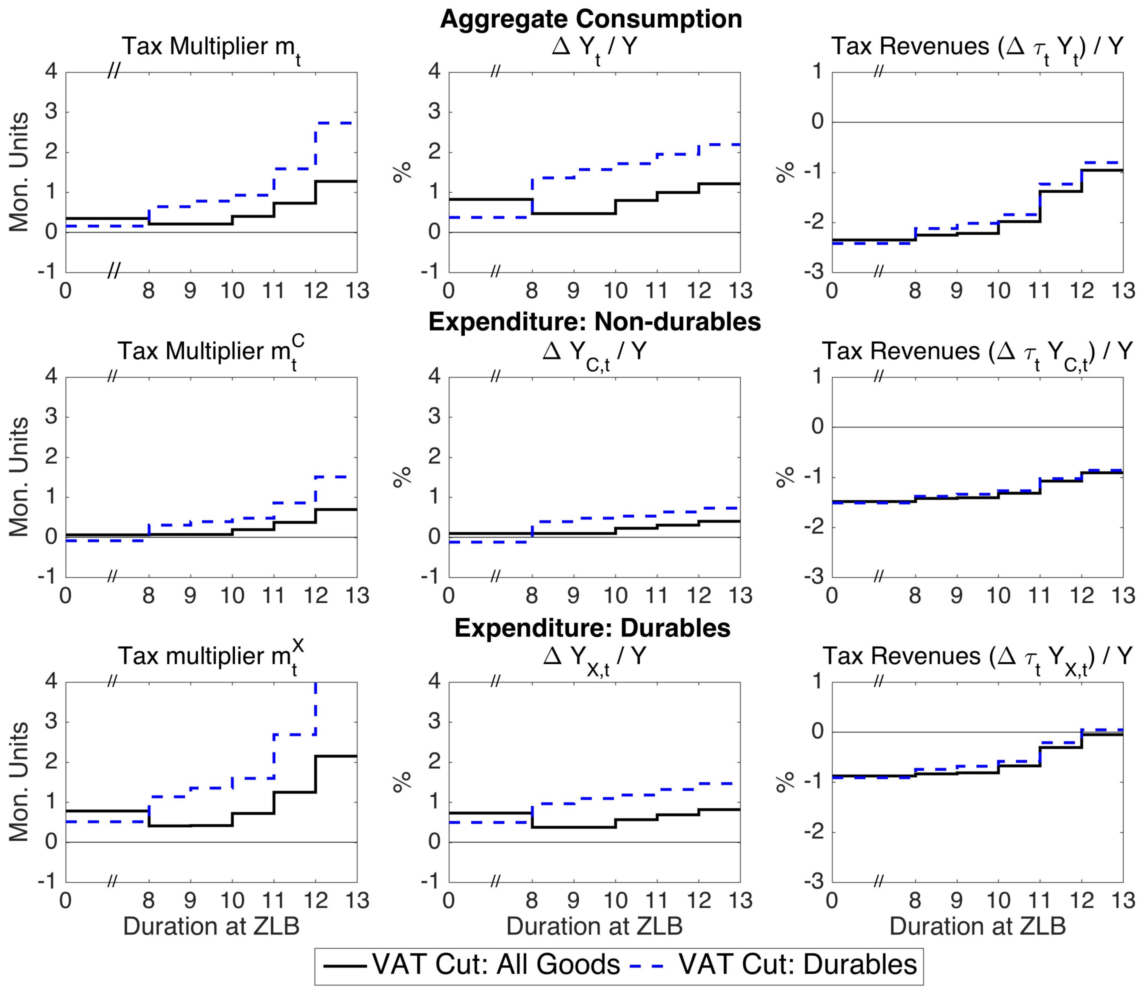
where T_n is the ZLB duration. Current consumption, $\widehat{C}_t$, depends on four terms. First, it depends positively on expected consumption when the economy exits the liquidity trap, $E_t\widehat{C}_{t+T_n+1}$. Second, it depends negatively on the sum of the lower bound of the (log-linearized) gross nominal interest rate, $\log\beta$, over the liquidity trap and, third, on the cumulative expected inflation from 0 to T_n, $E_t\widehat{\pi}_{C,t+i+1}$. And finally, consumption depends negatively on preference shocks, $\widehat{\mu}_{C,t+i}$. Imagine, monetary policy could completely stabilize the economy once it exits the liquidity trap, so that $E_t\widehat{C}_{t+T_n+1} = E_t\widehat{\pi}_{t+T_n+1} = 0$. Then, the difference in current consumption in a scenario with and without VAT cut depends only on the cumulative expected inflation until T_n:

$$\widehat{C}_t^{VAT} - \widehat{C}_t = E_t \left\{ \sum_{i=0}^{T_n} \left(\widehat{\pi}_{C,t+T_n+1}^{VAT} - \widehat{\pi}_{C,t+T_n+1} \right) \right\}. \tag{3.55}$$

The VAT cut has contractionary effects and further amplifies the fall in prices. Hence, the gap between expected inflation in Eq. (3.55) can be non-zero even when the temporary VAT shock is over. Furthermore, the sum in Eq. (3.55) decreases in T_n.

In the two-sector model, tax multipliers are strictly positive and further increase with the duration at the ZLB. Consequently, the expression in Eq. (3.55) is positive and the sum increases in T_n. At the same time, the shadow value of non-durables, λ_t, falls when non-durable goods consumption increases. This further reduces the effective relative price for durables, $\frac{P_{X,t}}{P_{C,t}}\lambda_t$, amplifying the boom in the durable goods sector. Intuitively, the loss in utility that the consumer suffers if he sacrifices the last unit of non-durables in order to purchase durables instead, becomes smaller as λ_t falls. Interestingly, tax multipliers reach values larger than 1 in long liquidity traps. This result comes from the combination of two effects. First, the expansionary effect of a VAT cut on consumption on impact rises with T_n. Second, the loss in tax revenue shrinks as the difference in $\widehat{C}_t^{VAT} - \widehat{C}_t$ becomes larger. For even longer liquidity traps, it could be possible that the VAT cut is self-financing.

Fig. 3.11 Average Tax Multipliers: Targeted VAT Cut

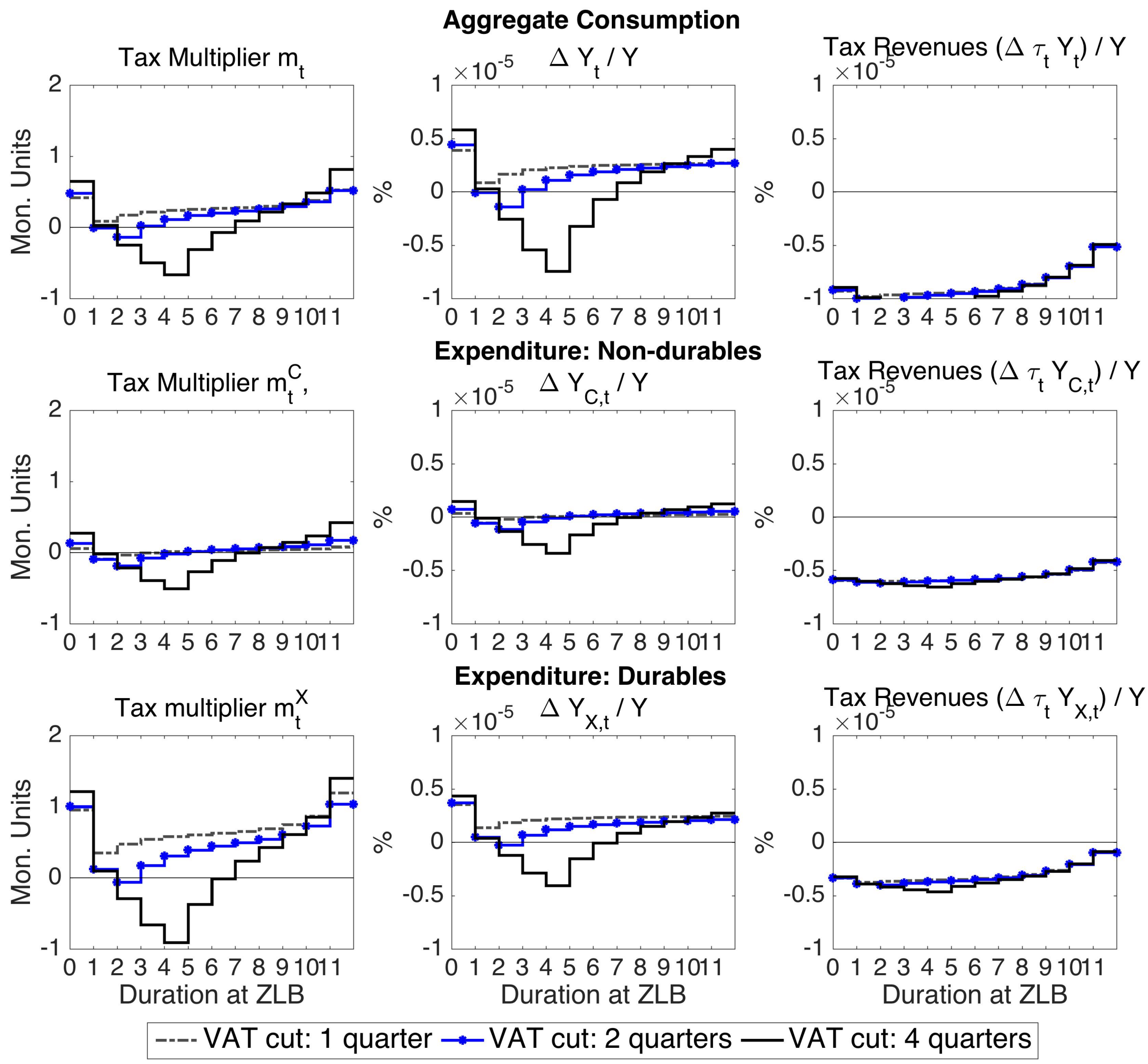


Tax multipliers are higher if if the temporary VAT cut is restricted to durable goods only. This way, any deflationary pressure on prices of non-durable goods could be completely eliminated. Figure 3.11 shows the tax multipliers for the targeted VAT policy. The tax shock is highly effective in a liquidity trap and less expensive to finance.

Marginal Tax Multipliers. In this section, I consider VAT shocks that are sufficiently small to leave the liquidity trap duration unaffected. Figure 3.12 shows marginal tax multipliers illustrating the impact of a minor change in the fiscal instrument. In general, the size of the tax multipliers depends on the duration of the VAT shock. A longer tax cut is associated with smaller or even negative multipliers if the ZLB duration is rather short. For long liquidity traps, however, the fiscal instrument can be very effective.

As the tax shock is very small, it will affect the relative price of durable goods and non-durable goods only minimally. Hence, the dynamics of the effective relative price $\frac{P_{X,t}}{P_{C,t}}\lambda_t$ will be mainly influenced by the shadow value of non-durable goods λ_t. Additionally, if the duration of the VAT cut exceeds the length of the liquidity trap, monetary policy is able to respond to changes in inflation attributed to the reversion of the tax rate to its previous level. Consequently, the fall in the real interest rate will be muted in period 4 discouraging spending on non-durable goods and new durable goods.

Fig. 3.12 Marginal Tax Multipliers: Two-Sector Model



3.6 Sensitivity Analysis

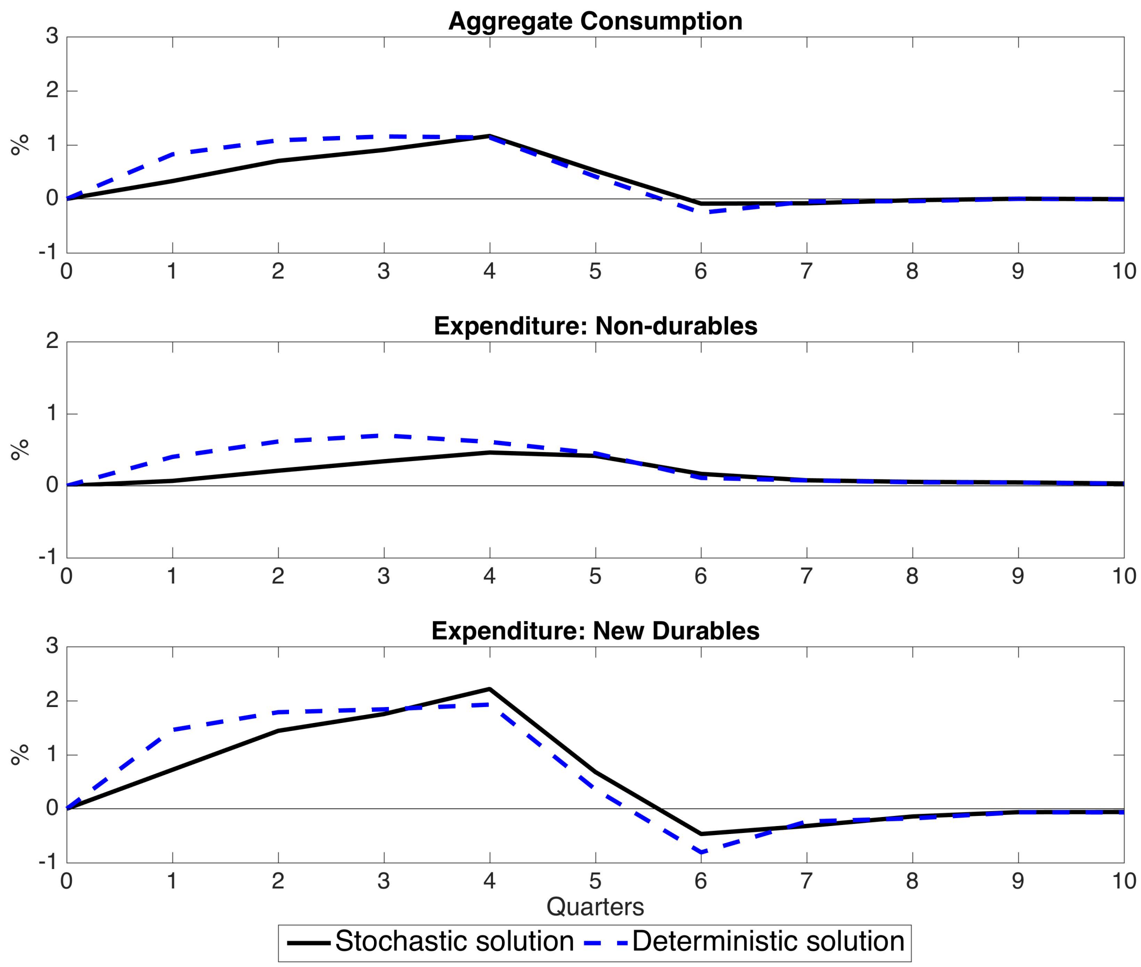
In this section, I examine the robustness of the main findings in the previous sections. In particular, I test the sensitivity of the main findings against an alternative solution method and study the implications of habit formation for durable and non-durables goods. Finally, I discuss the implications of a temporary sales tax cut.

Stochastic solution vs. deterministic solution. Using a global solution method allows future shock uncertainty to affect the decision rules of the households and firms. Numerous papers have pointed out that the model solution under shock uncertainty can substantially differ from the deterministic solution, e.g. when certainty equivalence is assumed (e.g. Lindé and Trabandt (2018*b*), Adam and Billi (2006), Adam and Billi (2007), Fernández-Villaverde et al. (2015), Gust et al. (2017), Nakata (2016) and Richter and Throckmorton (2017)).

I redo the previous exercises based on the New Keynesian models that are solved under certainty equivalence. As before, I simulate the different models for a scenario in which the economy is subject to a large contractionary preference shock that generates an expected 8-quarter liquidity trap. Likewise, I assume that the VAT cut is implemented in the same period when the preference shock hits the economy and lasts for 4 quarters. Figure 3.13 shows the difference of the impulse responses between the scenario with VAT cut and without VAT cut in the two-sector model. The effectiveness of the tax cut is smaller on impact in the stochastic model (solid black line) relative to the deterministic model (blue dashed line). The effects are reversed after the tax cut has ended in period 4. An explanation for this result is that future expected changes in the real interest rate due to the reversal of the tax rate are less relevant in determining current consumption decisions under uncertainty.

The role of habit formation. This section explores the role of external habit for non-durable goods and durable goods in the two-sector model. Similarly to the approach taken above, the impulse responses in Figure 3.14 are obtained as the difference between the simulation with tax shocks and the simulation with preference shock only. The contractionary preference shocks generate an expected 8-quarter liquidity trap. The VAT shock reduces the tax rate by 2.5 percentage points and lasts for 4 quarters. Figure 3.14 shows the comparison of impulse responses for alternative specifications of the two-sector model. In particular,

Fig. 3.13 Two-Sector Model: Stochastic vs. Deterministic Solution

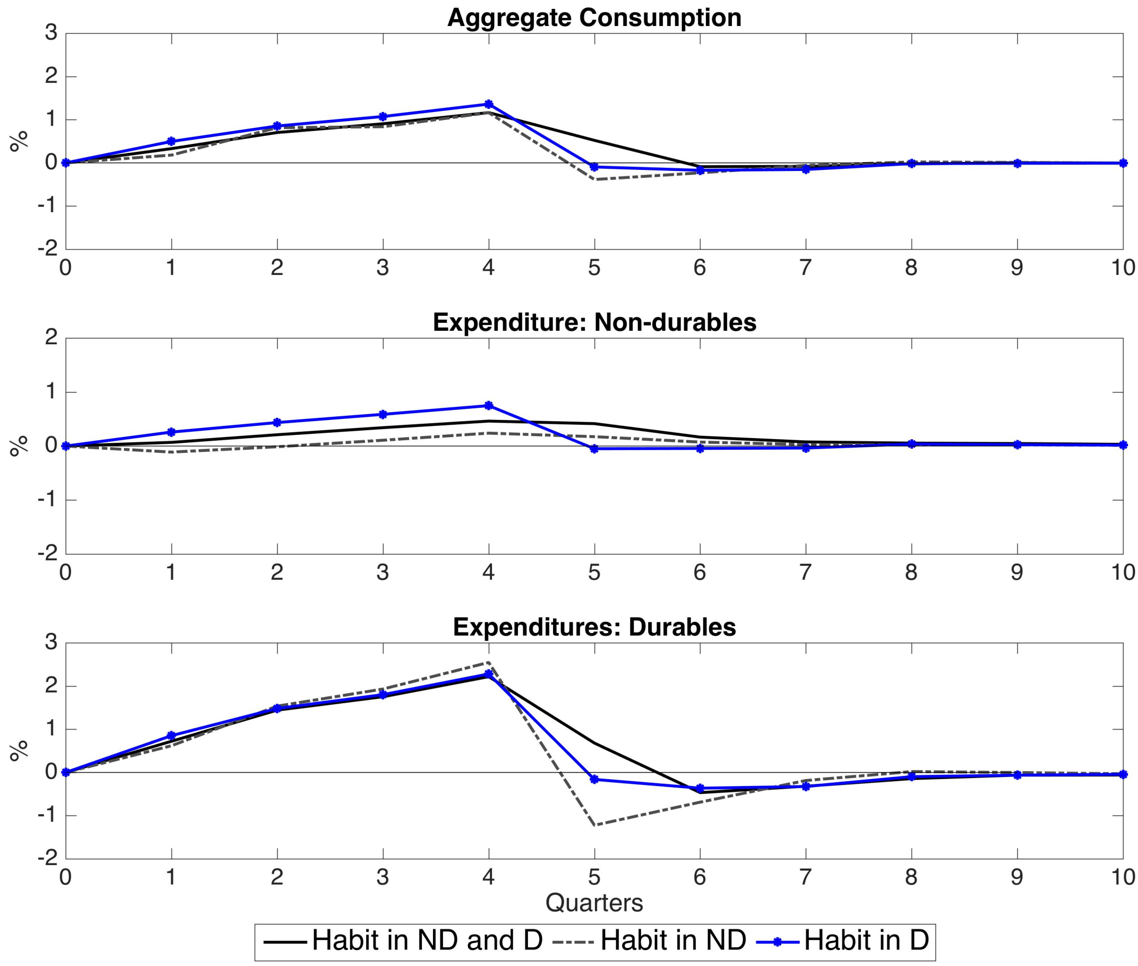


Note: the plots show the difference of the impulse responses between a scenario with VAT cut and without VAT cut.

the black solid lines show the responses of aggregate consumption, non-durable and durable goods consumption for the model discussed in the previous section. The black dash-dotted line is the two-sector model with $h_C = 0.5$ and $h_D = 0$ while the blue lines with dots corresponds to the model with $h_C = 0$ and $h_D = 0.5$. The impulse responses for spending on new durable goods vary only marginally for alternative model specifications in the first 4 quarters after the shocks. Hence, the differences in the response of aggregate consumption can be mainly attributed to the dynamics of non-durable goods consumption.

The tax cut is most effective in the first 4 quarters for the third case, i.e. $h_C = 0$ and $h_D = 0.5$. In contrast, tax cut is contractionary for non-durable good consumption on impact if $h_C = 0.5$ and $h_D = 0$. Essentially, habit for non-durable goods dampens the response in spending and, hence, reduces the spill-over effects from the durable goods sector.

Fig. 3.14 Two-Sector Model: Habit Formation in Consumption of Non-durables (ND) and Durables (D)



Note: the plots show the difference of the impulse responses between a scenario with VAT cut and without VAT cut.

VAT vs. Sales Tax. While all countries in the European Union levy a value-added tax[12], the consumption tax in the United States or in Canada is a sales tax. The sales tax is collected by the retailer at the final sale to the end-consumer.

[12]Other countries with a VAT system include Australia, Singapore and New Zealand.

Suppose, a sales tax is charged on newly purchased non-durables and durables. A sales tax would affect the budget constraint in the following way:

$$P_{c,t}(1+\tau_t^S)C_t(i)+P_{x,t}(1+\tau_t^S)X_t(i)+B_t(i)\leq(1+i_{t-1})B_{t-1}(i)+W_t(i)N_t(i)+\Gamma_t-T_t \quad (3.56)$$

where τ_t^S is the sales tax rate. The household's first order conditions are:

$$\frac{P_{X,t}}{P_{C,t}}(1+\tau_t^S)=\frac{\eta_t}{\lambda_t} \quad (3.57)$$

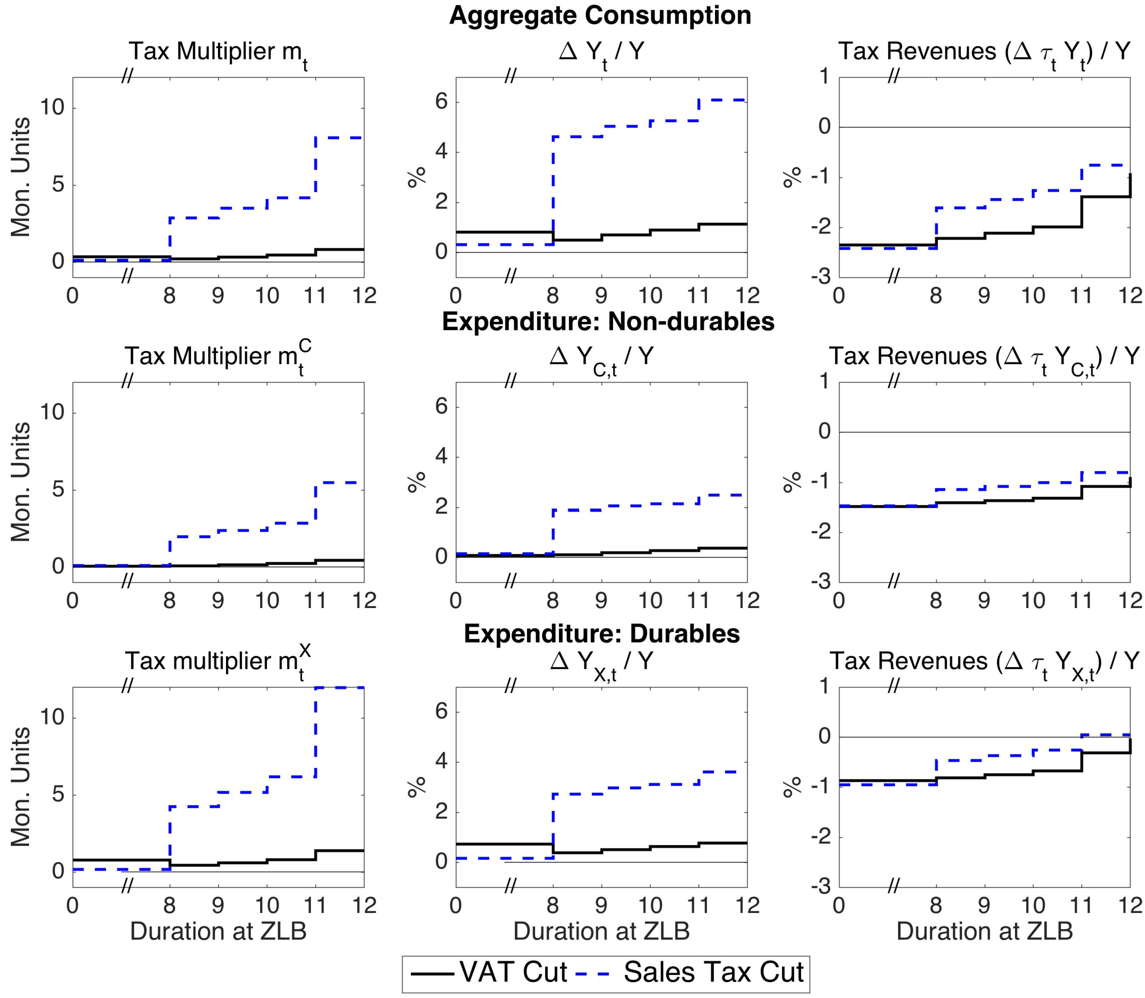
$$\eta_t=U_{D,t}+\beta(1-\delta)E_t\eta_{t+1} \quad (3.58)$$

$$U_{C,t}=\beta E_t\left\{U_{C,t+1}\frac{1+i_t}{1+\pi_{C,t+1}}\frac{1+\tau_t^S}{1+\tau_{t+1}^S}\right\} \quad (3.59)$$

An unexpected, temporary drop in the tax rate will affect the effective relative price of durables instantaneously and make durables cheaper (see Eq. (3.57)). Furthermore, consumers anticipate a positive change in $\frac{1+\tau_{t+1}^S}{1+\tau_t^S}$ in the last period of the VAT cut. Higher $\frac{1+\tau_{t+1}^S}{1+\tau_t^S}$ reduces the expected real interest rates at the ZLB. Under complete markets, consumers are extremely forward looking and react to distant changes in the real interest rate (McKay et al. (2016)). The plots on the right hand side in Figure 3.15 show the tax multipliers for a 4-quarter temporary cut in the tax rate for a sales tax and value-added tax. A temporary tax cut in the sales tax has significantly larger expansionary effects than a VAT cut.[13]

[13] Sales tax multipliers for a 12-quarter liquidity trap would exceed the value 100 and are too large to report here. The reason is that for longer liquidity traps, the change in the tax revenue becomes very close to zero or even positive.

Fig. 3.15 Two-Sector Model: VAT vs. Sales Tax



3.7 Conclusion

Motivated by a temporary VAT cut from 2008 to 2009 in the United Kingdom, this paper studies a similar tax experiment using a New Keynesian framework. In a standard New Keynesian model, I find that a transitory cut in the VAT rate is expansionary in normal times but contractionary when the ZLB on nominal interest rates is binding. Lowering the tax rate at the ZLB generates deflationary pressure on consumer prices. Hence, the real interest rate increases and contracts demand.

However, the baseline New Keynesian model does not consider long-lived consumption goods that account for roughly 40% of all goods and services subject to the VAT in the UK. I revisit the analysis using a two-sector New Keynesian model which allows for durable goods consumption. In this model, a VAT cut is highly expansionary even in deep recessions with a binding ZLB. But most interestingly, I find that consumers increase spending on durable goods but only on non-durable goods. The reason is the following: the temporary fall in prices stimulates demand for durable goods. Assuming a frictionless labor market, the boom in the durable goods sector affects aggregate wage dynamics and reduces deflationary pressure in both sectors. Hence, expected real interest rates fall and reduce the households' desire to save. These observations hinge on the choice of two key parameters: the share of the durable goods in the households' consumption basket and the relative degree of price rigidity in the durable goods sector and the non-durable goods sector. A boom in the sector for durable goods can affect aggregate wage dynamics only if the consumption share for these goods is sufficiently large. Likewise, consumers only increase spending on new durables if they become relatively cheaper.

This analysis focuses on consumption behavior based on the intertemporal substitution channel in response to tax shocks. Ricardian equivalence holds in the models so that tax changes have no direct income effects. Although the primary focus of this study is to understand how price dynamics for durable and non-durable goods together affect intertemporal consumption decisions, it would still be a promising avenue to explore similar tax experiments in a more elaborate setting which abandons the Ricardian equivalence assumption.

Overall, the main findings in this Chapter point out that the standard New Keynesian model, which only features consumption of non-durable goods, ignores important transmission channels that determine aggregate inflation dynamics. The two-sector model allows to study the relative importance of price dynamics for durable goods and non-durable goods in determining aggregate dynamics. My results shed new light on the role of supply-side policies as a means to stimulate the economy both in normal times and in severe recessions.

Chapter 4

An Efficient Algorithm to Solve DSGE Models with Occasionally Binding Constraints

I propose an algorithm to solve Dynamic Stochastic Equilibrium Models (DSGE) with occasionally binding constraint efficiently and fast. The algorithm is based on a policy function iteration method with time iteration. I use a collocation method to approximate off-grid values of the policy function. However, the main advantage of my algorithm compared to standard implementation of these methods is the fact that the model can be represented as a system of equations over the pre-defined grid. This way, I can solve for the complete policy function in one computation step instead of solving for the policy function on each grid point individually. The solution method is fast even without parallel computing. I apply the method to a New Keynesian model with a zero lower bound constraint (ZLB) on the nominal interest rate. The model features habit formation, price and wage rigidities. I show that the algorithm can solve the model with many state variables fast and accurately.

4.1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are an important tool for macroeconomic analysis. During much of the recent period, the prospects of interest rates at the zero lower bound (ZLB), potential changes in monetary and fiscal policy regimes or the importance of heterogeneity of firms and households for macroeconomic outcome, have become crucial elements in contemporary macroeconomics. Much work has been devoted to model these scenarios and incorporate non-linearities in DSGE models. However, these models often fail to have closed-form solutions and require alternative solution strategies. A major problem for the solution and estimation of nonlinear models is the heavy computational burden. Solving this class of models often requires iterative procedures where the computational expense grows rapidly with the dimensionality of the model and number of the state variables.

In this chapter, I propose an algorithm to solve DSGE models with occasionally binding constraints efficiently and fast. The special focus of the analysis is on a model with a ZLB constraint on the nominal interest rate. The solution method fully takes into account the stochastic nature of the model. The proposed approach contributes to the class of policy function iteration methods with time iteration. However, the major advantage of the algorithm comes from using the fact that the model can be represented as a system of equations over the pre-defined grid. Using a collocation method to interpolate and extrapolate the value of the policy functions, it is possible to solve for the entire unknown policy function in one computation step.

Another strategy to reduce the computational time is to take advantage of parallel computing (e.g. Richter et al. (2014), Gust et al. (2017) or Fernández-Villaverde and Zarruk Valencia (2018)). Multi-core processing allows to run computations simultaneously. Richter et al. (2014) document the relative speed gains from parallelization. Running their solution method on each core of the 6-core processor (3.47GHz each) simultaneously can reduce the computational time by a factor of 24 for their model as compared to using a single core.

My solution method uses parallel computing in parts. Hence, parallelization will provide additional speed gains but they do not increase one-to-one with the number of cores or workers. This clearly generates a trade-off: on processors with many cores, policy function iteration methods that rely on parallel computing can be very fast whereas the speed improvements of my algorithm would be bounded. However, basic computers typically come with 2- or 4-cores. Thus, my algorithm may be particularly interesting for the application on processors with a low number of cores and, hence, for a larger community of researchers.

The general idea of the proposed method is to find the values of the policy functions for a defined grid set on the state space. I use a collocation method to approximate the policy functions at points that lie between the grid nodes or outside of the grid. The collocation method provides a very efficient interpolation and extrapolation scheme to approximate the policy function over the entire state space. I use spline functions to interpolate or extrapolate the values of the policy function. The key innovation of the algorithms goes back to the fact that the values of the spline functions are constant when solving for the policy function. Hence, it is possible to construct a matrix which is defined over the entire grid and contains the values of the spline functions *before* the algorithm starts. I refer to this matrix as the "collocation matrix". The algorithm is particularly suited for linear model, e.g. semi-loglinearized models with occasionally binding constraint. However, the algorithm can be easily adapted to completely non-linear models.

I show that, in a linear environment, the model can be represented as a system of linear difference equations over the grid. Let $\boldsymbol{C}$ be the vector that contains the unknown values of the policy functions. Then, the solution of the model can be represented as:

$$\boldsymbol{C} = \boldsymbol{P}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{\boldsymbol{E}})\boldsymbol{C} + \boldsymbol{K} \tag{4.1}$$

where $\boldsymbol{P}$ is a matrix containing the model's parameters and $\boldsymbol{K}$ is a matrix containing the constants of the model. The occasionally binding constraint will be entirely reflected by $\boldsymbol{P}$ and $\boldsymbol{K}$. $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{\boldsymbol{E}})$ is the collocation matrix for the expected state variables $\overline{\boldsymbol{S}}^{\boldsymbol{E}}$. For the application of the algorithm, I assume that the exogenous processes are continuous and stochastic processes. To discretize the exogenous processes, I use a quadrature scheme with $\widehat{\boldsymbol{\Omega}}$ containing the constant quadrature weights.

The goal of the algorithm is to find the solution for the policy function $\boldsymbol{C}$. I propose an iterative approach to find $\boldsymbol{C}$. First, I guess the solution for $\boldsymbol{C}$. Based on the guess, the algorithm checks for grid points on which the constraint is binding and computes $\boldsymbol{P}$ and $\boldsymbol{K}$, accordingly. Second, I solve the system of difference equations in (4.1) for $\boldsymbol{C}$. Using the solution for $\boldsymbol{C}$, the algorithm identifies the grid points with binding constraint and updates $\boldsymbol{P}$ and $\boldsymbol{K}$. Third, the algorithm repeats the previous steps until it converges to the final solution for $\boldsymbol{C}$.

All routines required to implement the algorithm are written in MATLAB using the Parallel Computing Toolbox. In this paper, I focus on a New Keynesian model with price and wage rigidities, habit formation and a ZLB constraint on the nominal interest rate. However, the suite of MATLAB programs can be easily adapted to solve models with, for example, (i) binding collateral constraints, (ii) endogenous regime switches, or (iii) heterogeneous agents.

One important characteristic of a good solution method is the accuracy of the solution. I analyze the trade-off between accuracy and speed of the solution method using alternative specifications for the model. Overall, the method provides accurate results in line with the findings in Aruoba et al. (2006) and Richter et al. (2014). Accuracy decreases if the number of quadrature weights for the numerical integration is lower (<10) or future shock uncertainty is higher. For the latter, a potential remedy would be to increase the number of grid points for the discretized exogenous process. However, this comes at the cost of higher computation times.

Another section of this Chapter is devoted to the question how shock uncertainty affects equilibrium dynamics. A increasing body of work has analyzed the effects of the ZLB constraint in a stochastic environment (e.g. Lindé and Trabandt (2018*b*), Adam and Billi (2006), Adam and Billi (2007), Fernández-Villaverde et al. (2015), Gust et al. (2017), Nakata (2016) and Richter and Throckmorton (2017)). They find that shock uncertainty may or may not alter macroeconomic dynamics. I find that the stochastic model can substantially differ from the deterministic model depending on the degree of shock uncertainty. Higher standard deviations for the exogenous processes would be associated with a higher probability of reaching the ZLB. In a recession with binding ZLB, economic activity deteriorates more in the stochastic economy than in the deterministic economy. I find that the differences are quantitatively non-negligible.

The remainder of this Chapter is organized as follows. Section 4.2 describes the New Keynesian model that I use to demonstrate the algorithm. Section 4.3 provides the benchmark calibration of the model. Section 4.4 describes the algorithm and the solution steps for different specifications of the model. Section 4.5 reports solution times and the results of accuracy tests. Section 4.6 discusses the effects of shock uncertainty for the model solution. In Section 4.7, I adapt the algorithm to solve the completely non-linear New Keynesian model. Finally, Section 4.8 concludes.

4.2 The Model

This section describes the model economy. The model is formulated in discrete time with an infinite horizon. The economy is populated by households, a final good producer, a continuum of intermediate good producers and a monetary policy authority.

Households. There is a continuum of households indexed by $i \in [0,1]$. Households are subject to wage-setting frictions as in Erceg et al. (2000). More specifically, assume that there exists a continuum of monopolistically competitive households, each of which supplies a differentiated labor service $N(i)$ to the production sector. A representative labor aggregator combines households' labor services in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index, N_t, is given by:[1]

$$N_t = \left[\int_0^1 N_t(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{4.2}$$

where ε_w represents the elasticity of substitution between labor types.

Let $W_t(i)$ denote the nominal wage for labor of type i. Wages are set by workers of each type and taken as given by all firms. Cost minimization yields the demand schedule for each labor type i:

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} N_t \tag{4.3}$$

where W_t is the aggregate wage index defined as

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\varepsilon_w} dj\right]^{\frac{1}{1-\varepsilon_w}}. \tag{4.4}$$

The utility of household i is given by

$$E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \left\{\log\left(C_t(i) - hC_{t-1}\right) - \frac{N_t(i)^{1+\chi}}{1+\chi}\right\} \tag{4.5}$$

where the variables $C_t(i)$ is consumption of an individual household and C_t is aggregate consumption. The parameter $h \in [0,1]$ disciplines the degree of external habit formation. δ_t is a discount factor shock following an AR(1) process:

$$\log \delta_t = \rho_d \log \delta_{t-1} + \varepsilon_t^d, \; \varepsilon_t^d \sim \mathcal{N}(0, \sigma_d^2). \tag{4.6}$$

[1] As in Erceg et al. (2000), I assume that the representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands.

The household has the following period t budget constraint:

$$P_t C_t(i) + B_t(i) \leq B_{t-1}(i) R_{t-1} + W_t(i) N_t(i) + P_t \Gamma_t \tag{4.7}$$

where P_t is the price of the consumption good, $W_t(i)$ is the individual nominal wage rate and $B_t(i)$ is a stock of risk-free bonds, paying gross interest R_t in the following period. Γ_t are firms' profits that are paid out to the households through dividends.

Each household maximizes expected lifetime utility with respect to consumption, labor supply and holdings of bonds subject to its budget constraint (4.7) and the labor demand function (4.3). Combining the first-order conditions for consumption and bond holdings yields the Euler equation:[2]

$$\frac{1}{C_t - hC_{t-1}} = \beta E_t \, \delta_t \frac{1}{C_{t+1} - hC_t} \frac{R_t}{\Pi_{t+1}} \tag{4.8}$$

with $\Pi_t = \frac{P_t}{P_{t-1}}$ representing (gross) inflation.

Households set nominal wages as staggered contracts. In particular, assume that a constant fraction $(1 - \theta_w)$ of households can renegotiate its wage in each period. The remainder θ_w stays with the past wage $W_{t-1}(i)$. The first-order condition associated with the choice of the optimal wage $W_t^*(i)$ is given by

$$E_t \sum_{k=0}^{\infty} (\beta \, \theta_w)^k \left[\prod_{s=0}^{k} \delta_{t+s} \right] \left\{ \frac{1}{C_{t+k} - hC_{t+k-1}} N_{t+k}(i) \left(\frac{W_t^*(i)}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{t+k} \right) \right\} = 0 \tag{4.9}$$

where $MRS_t \equiv N_t(i)^\chi (C_t - hC_{t-1})$.

Producers. There is a representative final good producer and a continuum of intermediate good produces indexed by $j \in [0,1]$. The final good producer purchases intermediate goods, processes them into the final good using a CES technology, and sells it to the households. The problem of the final good producer is given by

[2]Erceg et al. (2000) show that if there exist state contingent claims that insure households against idiosyncratic wage risk, and if preferences are separable in consumption and leisure, households will be identical in their choice of consumption and bond-holdings. They will only differ in their wage and in their labor supply.

$$\max_{\{Y_t(j)\}} P_t Y_t - \int_0^1 P_t(j) Y_t(j) \, dj \tag{4.10}$$

subject to (4.11)

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\gamma_t - 1}{\gamma_t}} dj \right]^{\frac{\gamma_t}{\gamma_t - 1}}$$

where $Y_t(j)$ is the quantity intermediate good of type j. γ_t is stochastic and determines the time-varying markup in the goods market. I assume that γ_t follows an AR(1) process:

$$\log \gamma_t = (1-\rho_\gamma) \log \gamma + \rho_\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma, \quad \varepsilon_t^\gamma \sim \mathcal{N}(0, \sigma_\gamma^2) \tag{4.12}$$

Intermediate good producers use labor to produce imperfectly substitutable intermediate goods according to the linear production function

$$Y_t(j) = N_t(j). \tag{4.13}$$

As in Calvo (1983), each firm faces a constant probability θ of not being able to adjust its price and stays with the price from the previous period $P_{t-1}(j)$. Hence, an intermediate firm sets the optimal price, $P_t^*(j)$, of its own good in order to maximize the expected discounted sum of future profits. Its problem is given by

$$\max_{P_t^*(i)} E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left[\prod_{s=0}^{k} \delta_{t+s} \right] \Lambda_{t,t+k} \left(P_t^*(j) Y_t(j) - W_t N_t(i) \right) \tag{4.14}$$

subject to Eq. (4.13) and demand $Y_t(j) = [P_t(j)^*/P_t]^{-\gamma_t} Y_t$. Here, $\Lambda_{t,t+k}$ is the stochastic discount factor.

Monetary Policy. Monetary policy follows a Taylor-type rule subject to the zero lower bound constraint,

$$R_t = \max \left\{ 1, \overline{R} \left(\Pi_t^{\varphi} \nu_t \right) \right\} \tag{4.15}$$

where $\overline{R}$ is the steady state (gross) nominal interest rate and ν_t is a monetary policy shock which follows an AR(1) process:

$$\log \nu_t = \rho_\nu \log \nu_{t-1} + \varepsilon_t^\nu, \quad \varepsilon_t^\nu \sim \mathcal{N}(0, \sigma_\nu^2). \tag{4.16}$$

Market Clearing. I assume market clearing in the final goods market, $Y_t = C_t$, and in the bond market, $B_t = 0$. Furthermore, the labor market clearing condition is given by

$$N_t = \int_0^1 N_t(j)\, dj. \tag{4.17}$$

For intermediate good firm j, it holds that $[P_t(j)/P_t]^{-\gamma_t} Y_t = N_t(j)$. Integrating over j and rearranging yields:

$$Y_t = \frac{N_t}{s_t} \tag{4.18}$$

with

$$s_t = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\gamma_t} dj \tag{4.19}$$

where the ratio $s_t \geq 1$ denotes Yun's (1999) price dispersion measure.

4.2.1 The Log-Linearized Equilibrium Conditions

The equilibrium can be characterized by the following set of equilibrium conditions which are log-linearized around the steady state with zero inflation. The $\widehat{\ }$ denotes log-deviations from steady state. $\widehat{\pi}_t$ and $\widehat{R}_t$ are log-deviations of (gross) inflation and the (gross) nominal interest rate, respectively. Furthermore, the variable w_t denotes the real wage and $\widehat{\pi}_t^w$ is wage inflation.

$$\text{Euler Equation:} \quad \widehat{C}_t = \frac{1}{1+h} E_t \widehat{C}_{t+1} + \frac{h}{1+h} \widehat{C}_{t-1} - \frac{1-h}{1+h} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) + \frac{1-h}{1+h} \widehat{\delta}_t \tag{4.20}$$

$$\text{Market Clearing:} \quad \widehat{Y}_t = \widehat{C}_t \tag{4.21}$$

$$\text{Production:} \quad \widehat{Y}_t = \widehat{N}_t \tag{4.22}$$

$$\text{Wage Phillips Curve:} \quad \widehat{\pi}_t^w = \beta E_t \widehat{\pi}_{t+1}^w + \kappa_w \left(\chi \widehat{N}_t + \frac{1}{1-h} \widehat{C}_t - \frac{h}{1-h} \widehat{C}_{t-1} - \widehat{w}_t \right) \tag{4.23}$$

$$\text{with } \kappa_w \equiv \frac{(1-\theta_w \beta)(1-\theta_w)}{\theta_w (1+\chi \varepsilon_w)} \tag{4.24}$$

$$\text{Wage Inflation:} \quad \widehat{\pi}_t^w = \widehat{w}_t + \widehat{\pi}_t - \widehat{w}_{t-1} \tag{4.25}$$

$$\text{Phillips curve:} \quad \widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \widehat{w}_t + \widehat{\gamma}_t \quad \text{with } \kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta} \tag{4.26}$$

$$\text{Monetary Policy:} \quad \widehat{R}_t = \max\{\log\beta, \varphi \widehat{\pi}_t + \widehat{v}_t\} \tag{4.27}$$

The model consists of 7 model equations in 7 variables $\left\{\widehat{\pi}_t, \widehat{C}_t, \widehat{Y}_t, \widehat{N}_t, \widehat{w}_t, \widehat{\pi}_t^w, \widehat{R}_t\right\}$ and 3 exogenous processes $\widehat{\delta}_t, \widehat{\gamma}_t, \widehat{v}_t$. Furthermore, there are 2 endogenous state variables $\left\{\widehat{Y}_{t-1}, \widehat{w}_{t-1}\right\}$.

4.3 Benchmark Calibration

Table 4.1 lists the baseline parameterization for the full model. The model is parameterized at quarterly frequency. The discount factor β is set such that the implied annualized steady state interest rate is 2%. Throughout the analysis, I assume the same AR(1) coefficients for the exogenous processes. In particular, I choose $\rho_d = 0.8$ (discount factor shock), $\rho_\gamma = 0.5$ (mark-up shock) and $\rho_v = 0.5$ (monetary policy shock). The unconditional standard deviations of the shocks σ_d, σ_γ and σ_v determine the probability of reaching the ZLB and will be determined separately for each exercise. All parameters are fairly in line with the literature.

Table 4.1 Benchmark Parameterization

Parameter	Value	Description
β	0.995	Discount factor (quarterly)
h	0.5	Consumption habit
χ	1	(Inverse) labor supply elasticity
θ	0.75	Prob. of not resetting price
θ_w	0.75	Prob. of not resetting wage
ε_w	0.33	Substitution elasticity for labor input
φ	1.5	Taylor rule: Inflation response coefficient
ρ_d	0.8	AR(1)-coefficient discount factor shock
ρ_γ	0.5	AR(1)-coefficient markup shock
ρ_v	0.5	AR(1)-coefficient monetary policy shock

4.4 The Numerical Algorithm

This section outlines the algorithm to solve the New Keynesian model. I use a collocation method with linear splines to obtain approximation of the unknown policy function. Generally, a collocation method approximates a functional equation so that the approximated function is exact at certain pre-specified points of the domain (Judd (1998), p. 384). I use a collocation method that approximates a function with a linear combination of n basis functions using n prescribed points of the domain, called the collocation nodes.

The algorithm is particularly suited for linear model, e.g. semi-loglinearized models with occasionally binding constraint. However, the algorithm can be easily adapted to non-linear models. I will show in this section that, in a linear environment, the model can be represented as a system of difference equations over the pre-defined grid. Suppose that the vector $\boldsymbol{C}$ contains the unknown values of the policy functions. Then, the solution of the model can be represented as:

$$\boldsymbol{C} = \boldsymbol{P}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{\boldsymbol{E}})\boldsymbol{C} + \boldsymbol{K} \tag{4.28}$$

where $\boldsymbol{P}$ is a matrix containing the model's parameters, $\boldsymbol{K}$ is a vector of constants, $\widehat{\boldsymbol{\Phi}}(\overline{S}^{\boldsymbol{E}})$ is the collocation matrix for expected states $\overline{\boldsymbol{S}}^{\boldsymbol{E}}$. I use a Gaussian quadrature scheme to discretize and integrate across the expected realizations of the state. $\widehat{\boldsymbol{\Omega}}$ contains the Gaussian quadrature weights. The non-linearity from the ZLB constraint on the nominal interest rate will be entirely reflected in the parameter matrix $\boldsymbol{P}$ and the vector of constants $\boldsymbol{K}$.

The goal of the algorithm is to find the unknown policy function $\boldsymbol{C}$. I propose an iterative approach to solve for $\boldsymbol{C}$. The algorithm starts with a guess for $\boldsymbol{C}$ and checks the grid points on which the ZLB is binding. It computes $\boldsymbol{P}$ and $\boldsymbol{K}$, accordingly. Then, the algorithm solves the difference equation in (4.28) for $\boldsymbol{C}$. Based on the updated policy function, $\boldsymbol{C}$, the algorithm checks for grid points with binding ZLB again and updates $\boldsymbol{P}$ and $\boldsymbol{K}$. The algorithm repeats the previous steps until it converges to the final solution of the policy function $\boldsymbol{C}$.

4.4.1 Basic Formulation of the Algorithm

In this section, I present a basic formulation of the algorithm: I consider a reduced form of the model to make it easier to understand the key steps of the algorithm. Section 4.4.3 describes the solution approach for the full model. Furthermore, I focus on the semi- loglinear model. As summarized in Section 4.2.1, all equilibrium equations are log-linearized around a zero inflation steady state, except for the max operator that truncates the nominal interest rate.

Assume that the discount factor shock is the only shock in the economy, i.e. $\gamma_t = \gamma$ and $\nu_t = \nu$. Moreover, $\theta_w = 0$ such that wages are fully flexible. Then, the equilibrium conditions reduce to:

$$\widehat{Y}_t = \frac{1}{1+h} E_t \widehat{Y}_{t+1} + \frac{h}{1+h} \widehat{Y}_{t-1} - \frac{1-h}{1+h} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) + \frac{1-h}{1+h} \widehat{\delta}_t \tag{4.29}$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \tilde{\kappa} \widehat{Y}_t \tag{4.30}$$

$$\widehat{R}_t = \max\{\log \beta, \varphi \widehat{\pi}_t\} \tag{4.31}$$

where $\tilde{\kappa} \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}(1+\chi)$. To further reduce the set of equations, I substitute Eq. (4.31) into Eq. (4.29). Let $\mathbf{Z} = \left[\widehat{\pi}', \widehat{\mathbf{Y}}'\right]'$ be the vector of decision rules for the two variables of the model, inflation and output. I approximate $\mathbf{Z}$ over the states $\boldsymbol{s}$ which are defined over the state variables $\widehat{Y}_{t-1}$ and $\widehat{\delta}_t$ by a linear combination of n known basis functions ϕ_i for $i = 1,...n$:

$$\mathbf{Z}(\boldsymbol{s}) \approx \widehat{\boldsymbol{\Phi}}(\boldsymbol{s})\,\mathbf{C} \tag{4.32}$$

where $\mathbf{C} = [\mathbf{c}^{\pi}\ \mathbf{c}^{\mathbf{y}}]'$ and $c^j = [c_1^j\ ...\ c_n^j]$, $j = \{\pi, y\}$ and

$$\widehat{\boldsymbol{\Phi}}(\boldsymbol{s}) = \begin{bmatrix} \boldsymbol{\Phi}(\boldsymbol{s}) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}(\boldsymbol{s}) \end{bmatrix} \tag{4.33}$$

where $\boldsymbol{\Phi}(\boldsymbol{s})$ is defined as $\boldsymbol{\Phi}(\boldsymbol{s}) = [\phi_1(\boldsymbol{s}) \dots \phi_n(\boldsymbol{s}))]$. $\phi_i(\boldsymbol{s})$, $i = 1,..,n$ is a vector of basis functions over the states $\boldsymbol{s}$. The coefficient vector $\boldsymbol{C}$ is set such that Eq. (4.32) holds exactly at n selected collocation nodes. Let the column vectors $\overline{\boldsymbol{d}}$ and $\overline{\mathbf{y}}$ contain the grid points of the discount factor shock and output, respectively. The vectors have length n_l, $l \in \{d, y\}$. The total number of grid points is $n = n_d \times n_y$. The grid is defined by matrix $\overline{\mathbf{S}}$:

$$\overline{\mathbf{S}} = \begin{bmatrix} [\mathbf{1}_{n_y} \otimes \overline{\boldsymbol{d}}]' \\ [\overline{\mathbf{y}} \otimes \mathbf{1}_{n_d}]' \end{bmatrix}' \tag{4.34}$$

where $\overline{\mathbf{S}}$ has dimension $n \times 2$ and $\mathbf{1}_h$ is a column vector of ones of length h. $\otimes$ denotes the Kronecker product. Then, it holds:

$$\mathbf{Z}(\overline{\boldsymbol{S}}) = \widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}})\mathbf{C} \tag{4.35}$$

Note that $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}})$ is a block matrix

$$\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}) = \begin{bmatrix} \boldsymbol{\Phi}(\overline{\boldsymbol{S}}) & 0 & \dots & 0 \\ 0 & \boldsymbol{\Phi}(\overline{\boldsymbol{S}}) & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \boldsymbol{\Phi}(\overline{\boldsymbol{S}}) \end{bmatrix} \tag{4.36}$$

where

$$\boldsymbol{\Phi}(\overline{\boldsymbol{S}}) = \begin{bmatrix} \phi_1(\overline{\boldsymbol{S}}_{(1,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}_{(1,:)}) \\ \vdots & \dots & \vdots \\ \phi_1(\overline{\boldsymbol{S}}_{(n,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}_{(n,:)}) \end{bmatrix} \tag{4.37}$$

$\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}})$ has dimension $2n \times 2n$ and $\boldsymbol{\Phi}(\overline{\boldsymbol{S}})$ is $n \times n$. $\overline{\boldsymbol{S}}_{(k,:)}$ refers to the elements in row k of matrix $\overline{\boldsymbol{S}}$. For example, the value of the policy function for inflation and output on grid point $\{\overline{d}_j, \overline{y}_k\}$ will be approximated by

$$\widehat{\pi}(\overline{d}_j, \overline{y}_k) = \sum_{i=1}^{n} \phi_i(\overline{d}_j, \overline{y}_k)\, c_i^{\pi}$$
$$\widehat{y}(\overline{d}_j, \overline{y}_k) = \sum_{i=1}^{n} \phi_i(\overline{d}_j, \overline{y}_k)\, c_i^{y}.$$

I use linear spline basis functions where the breakpoints coincide with the collocation nodes. This implies that $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}})$ is an identity matrix. Thus, the policy function approximations are identical to the vector coefficients $\boldsymbol{C}$:

$$\mathbf{Z}(\overline{\boldsymbol{S}}) = \mathbf{C}. \tag{4.38}$$

The discount factor shock follows an AR(1) process: $\widehat{\delta}_t = \rho_d \widehat{\delta}_{t-1} + \varepsilon_t^d$ with $\varepsilon_t^d \sim \mathcal{N}(0, \sigma_d^2)$. I use a Gaussian quadrature scheme to discretize normally distributed random variables. Assume that $\boldsymbol{\varepsilon} = [\varepsilon_1 \ldots \varepsilon_m]'$ is a column vector with m quadrature nodes and $\boldsymbol{\omega} = [\omega_1 \ldots \omega_m]'$ is a column vector of length m containing the quadrature weights. For the expected functions, the basis functions ϕ_i are evaluated for the matrix $\overline{\boldsymbol{S}}^E$:

$$\overline{\boldsymbol{S}}^E = \begin{bmatrix} [\mathbf{1}_{n_y} \otimes \overline{\boldsymbol{d}}^E]' \\ [\boldsymbol{c}^y \otimes \mathbf{1}_m]' \end{bmatrix}' \tag{4.39}$$

where

$$\overline{\boldsymbol{d}}^E = \begin{bmatrix} (\rho_d \overline{d}_1 \otimes \mathbf{1}_m) + \boldsymbol{\varepsilon} \\ \vdots \\ (\rho_d \overline{d}_{n_d} \otimes \mathbf{1}_m) + \boldsymbol{\varepsilon} \end{bmatrix}$$

Note that the basis function will be evaluated at the policy function approximations for output, $\boldsymbol{c}^y$. $\overline{\boldsymbol{S}}^E$ has dimension $nm \times 2$ and $\overline{\boldsymbol{d}}^E$ is a column vector of length $n_d m$. Hence, the approximation of expected inflation and output on grid point $\{\overline{d}_j, c_k^y\}$ is given by:

$$E\widehat{\pi}(\overline{d}_j, c^y_k) = \sum_{l=1}^{m} \sum_{i=1}^{n} \omega_l \, \phi_i(\rho_d \overline{d}_j + \varepsilon_l, c^y_k) \, c^\pi_i$$
$$E\widehat{y}(\overline{d}_j, c^y_k) = \sum_{l=1}^{m} \sum_{i=1}^{n} \omega_l \, \phi_i(\rho_d \overline{d}_j + \varepsilon_l, c^y_k) \, c^y_i$$

Now, the expected functions can be written compactly as:

$$EZ(\overline{S}) = \widehat{\mathbf{\Omega}} \widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E) \boldsymbol{C} \tag{4.40}$$

where

$$\widehat{\mathbf{\Omega}} = \begin{bmatrix} \mathbf{\Omega} & \mathbf{0}_{n \times nm} \\ \mathbf{0}_{n \times nm} & \mathbf{\Omega} \end{bmatrix} \qquad \mathbf{\Omega} = \begin{bmatrix} \boldsymbol{\omega}' & 0 & \dots & 0 \\ 0 & \boldsymbol{\omega}' & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \boldsymbol{\omega}' \end{bmatrix}$$

$$\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E) = \begin{bmatrix} \mathbf{\Phi}(\overline{\boldsymbol{S}}^E) & \mathbf{0}_{nm \times n} \\ \mathbf{0}_{nm \times n} & \mathbf{\Phi}(\overline{\boldsymbol{S}}^E) \end{bmatrix} \qquad \mathbf{\Phi}(\overline{\boldsymbol{S}}^E) = \begin{bmatrix} \phi_1(\overline{\boldsymbol{S}}^E_{(1,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}^E_{(1,:)}) \\ \vdots & \dots & \vdots \\ \phi_1(\overline{\boldsymbol{S}}^E_{(mn,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}^E_{(mn,:)}) \end{bmatrix}$$

$\widehat{\mathbf{\Omega}}$ is of dimension $2n \times 2mn$, $\mathbf{\Omega}$ is $n \times mn$, $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E)$ is $2mn \times 2n$ and $\mathbf{\Phi}(\overline{\boldsymbol{S}}^E)$ is $mn \times n$.

Now, it is possible to write a system of log-linearized equations over the grid $\overline{\boldsymbol{S}}$. In matrix notation, the system of difference equations over the grid $\overline{\mathbf{S}}$ is given by:

$$\boldsymbol{A}\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{B}E\boldsymbol{Z}(\overline{\boldsymbol{S}}) + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}}. \tag{4.41}$$

Or equivalently,

$$\boldsymbol{A}\boldsymbol{C} = \boldsymbol{B}\widehat{\mathbf{\Omega}}\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E)\boldsymbol{C} + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}} \tag{4.42}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{D}$ and $\boldsymbol{F}$ are matrices that contain the parameters or constants of the model equations. The goal of the algorithm is to find the policy function approximation $\boldsymbol{C}$ using an iterative approach. In each iteration step, I use the current solution for $\boldsymbol{C}$ to check for grid points where the ZLB is binding. Recall the truncated Taylor rule: $\widehat{R}_t = \max\{\log\beta, \varphi\,\widehat{\pi}_t\}$.

The column vector $\boldsymbol{D}$ has elements equal to $\log\beta$ at the grid points for which the ZLB is binding and 0 otherwise. At these grid points the Taylor rule will be basically "turned off" and the Taylor coefficient, φ is effectively zero. Similarly, I set $\varphi = 0$ in parameter matrix $\boldsymbol{A}$ at all points where the ZLB is binding. I use the following iterative approach to find the coefficient vector $\boldsymbol{C}$:

1. Start with a guess $\boldsymbol{C}^0$. Compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^0$.
2. Given $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$, solve for $\boldsymbol{C}^{\text{new}}$:

$$\boldsymbol{C}^{\text{new}} = [\boldsymbol{A} - \boldsymbol{B}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)]^{-1}[\boldsymbol{D} + \boldsymbol{F}\,\overline{\boldsymbol{S}}] \tag{4.43}$$

3. Update $\boldsymbol{C}^1 = \lambda\boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^1$.
4. Redo the previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

I set the convergence criteria to $z = 10^{-8}$ and the maximum number of iterations to $s = 300$. All routines required to implement the algorithm are written in MATLAB using the Parallel Computing Toolbox. The details of the implementation is provided in Appendix C.0.1.

4.4.2 A Special Case: No Endogenous State Variables

The New Keynesian model discussed in Section 4.2 nests a special case with no endogenous state variables if $h = 0$ and $\theta_w = 0$. This parameter choice would completely eliminate consumption habit formation and restores flexible wages. The solution algorithm simplifies significantly for models without continuous state variables. The key difference between this set up and the formulation of the algorithm in Section 4.4.1 goes back to the computation of the collocation matrix $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$. If the model has no endogenous state variables, it is sufficient to compute $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$ only once because $\overline{\boldsymbol{S}}^E$ is defined only over the dicretized exogenous variables. Here, $\overline{\boldsymbol{S}}^E$ is fixed and does not change in each iteration based on the current approximation of the policy function. Consider the model from Section 4.4.1 with $h = 0$. In this case, $\overline{\boldsymbol{S}}^E$ is given by:

$$\overline{S}^E = \left[[\mathbf{1}_{n_y} \otimes \overline{d}^E]' \right]' \tag{4.44}$$

where

$$\overline{d}^E = \begin{bmatrix} (\rho_d \overline{d}_1 \otimes \mathbf{1}_m) + \boldsymbol{\varepsilon} \\ \vdots \\ (\rho_d \overline{d}_{n_d} \otimes \mathbf{1}_m) + \boldsymbol{\varepsilon} \end{bmatrix}$$

Again, I use the following iterative approach to find the coefficient vector $\boldsymbol{C}$:

1. Start with a guess $\boldsymbol{C}^0$ and define $\overline{\boldsymbol{S}}^E$ and $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$. Compute $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^0$.

2. Given $\boldsymbol{A}$ and $\boldsymbol{D}$, solve for $\boldsymbol{C}^{\text{new}}$:

$$\boldsymbol{C}^{\text{new}} = [\boldsymbol{A} - \boldsymbol{B}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)]^{-1}[\boldsymbol{D} + \boldsymbol{F}\overline{\boldsymbol{S}}] \tag{4.45}$$

3. Update $\boldsymbol{C}^1 = \lambda \boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

4.4.3 General Formulation of the Algorithm

The full New Keynesian model discussed in Section 4.2 has two continuous state variables, $\widehat{Y}_{t-1}$ and $\widehat{w}_{t-1}$ and three continuous stochastic variables, $\widehat{\delta}_t, \widehat{\mu}_t$ and $\widehat{\nu}_t$. The total grid is defined over all state variables. Let $\overline{\mathbf{S}}$ be the array for each state variable where every position represents a unique permutation of the discretized state variables. The size of $\overline{\mathbf{S}}$ is $n \times 5$ where n is the product of the number of nodes for each state variable and 5 is the total number of state variables. Similarly, $\overline{\mathbf{S}}^E$ represents the array for the expected realizations of the stochastic variables in $t+1$. Every position in $\overline{\mathbf{S}}^E$ is a unique perturbation of the expected realization of the discretized state variable in time $t+1$ given the solutions of the endogenous state variables in time t. Assume that n_j, $j \in \{d, m, mp, y, w\}$, is the number of nodes for the discount factor shock, markup shock, monetary policy shock, lagged output and lagged real wage, respectively. m_j, $j \in \{d, m, mp\}$ is the number of quadrature nodes for the realizations of the discount factor shock, markup shock and monetary policy shock, respectively. Then, $n = n_d \times n_m \times n_{mp} \times n_y \times n_w$ and $m = m_y \times m_w$ and the total size of the array $\overline{\mathbf{S}}^E$ is $nm \times 5$.

There is a delicate balance between the size of the grid points and the tractability of the problem. More specifically, the dimension of $\overline{\mathbf{S}}^E$ is increasing not only in n but in nm. I apply parallel computing to avoid the storage of large matrices. In particular, I break down $\overline{\mathbf{S}}^E$ such that the basis functions ϕ_i for $i = 1, ...n$ can be evaluated for different parts of the grid simultaneously. Then, the basis functions are multiplied by the respective combination of quadrature weights in each parallel process. Finally, the different outcomes are combined such that it yields the collocation matrix:

$$\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)^{final} = \widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E) \tag{4.46}$$

which has dimension $n \times n$. Parallel computing leads to substantial speed gains for large models.

The rest of the algorithm follows an iterative approach to find the approximation of the policy functions, $\boldsymbol{C}$. Again, the iterative approach has the following steps:

1. Start with a guess $\boldsymbol{C}^0$. Compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^0$.

2. Given $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$, solve for $\boldsymbol{C}^{\text{new}}$:

$$\boldsymbol{C}^{\text{new}} = [\boldsymbol{A} - \boldsymbol{B}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)]^{-1}[\boldsymbol{D} + \boldsymbol{F}\,\overline{\boldsymbol{S}}] \tag{4.47}$$

3. Update $\boldsymbol{C}^1 = \lambda\boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

4.5 Solution Times and Accuracy

In this section, I report solution times and accuracy diagnostics for the New Keynesian model. The solution times depend on the size of the model (i.e. the number of grid points) and the number of endogenous state variables. The full model has two continuous state variables, $\widehat{Y}_{t-1}$ and $\widehat{w}_{t-1}$, and three continuous stochastic variables, $\widehat{\delta}_t$, $\widehat{\mu}_t$ and $\widehat{v}_t$. Depending on the choice of parameter values, this setup nests reduced forms of the model. In particular, consumption habit is ruled out if $h = 0$. Likewise, wages are perfectly flexible if $\theta_w = 0$.

Table 4.2 reports the solution times across the different specifications of the New Keynesian model and modifications, i.e. multi-core processing. All routines were computed with a 4-core processor (3.5 GHz) using MATLAB's Parallel Computing Toolbox. I ran all computations also on a 2-core processor (2.4 GHz). The results are reported in Appendix C.0.2. The different models vary in the number of exogenous processes and the number of endogenous state variables. The smallest model features the discount factor shock (denoted by DF in Table 4.2) and exhibits no consumption habit and wage rigidities (denoted as "0 lagged" in Table 4.2). In this case, I set the parameters $h = 0$ and $\theta_w = 0$. This model can be solved in 0.019 seconds. Computation times increase significantly if this model has two exogenous processes (discount factor shock (DF) and markup shock (M)) and three exogenous processes (discount factor shock (D), markup shock (M) and monetary policy shock (MP)). The only reason for this is the larger grid size. As stressed in Section 4.4.2, models with no endogenous state variables represent a special case in which the collocation matrix $\boldsymbol{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}})$ is constant and is evaluated only once before the algorithm starts. Hence, multi-core processing does not provide any speed gains for models with no endogenous variables because parallel computing is used only to update $\boldsymbol{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}})$.

Solution times also increase significantly with the number of endogenous state variables. In Table 4.2, the model with 1 lagged variable accounts for consumption habit ($h = 0.5$) but assumes no wage rigidities ($\theta_w = 0$). Finally, the specification with 2 lagged variables considers both consumption habit ($h = 0.5$) and wage rigidities ($\theta_w = 0.75$). Using additional processors provides relative speed gains (denoted as 1, 2 or 4 processors in Table 4.2) . Notice that solution times do not decrease one-to-one with the number of processors. There are two apparent reasons for this result. First, only parts of the code use parallel computing. Second, there is minimal communication overhead that cannot be entirely eliminated.

Following Judd (1998), Aruoba et al. (2006), Caldara et al. (2012) or Richter et al. (2014), I provide Euler equation errors and the integral of Euler equation errors as a measure of accuracy of the model solutions. All accuracy diagnostics are based on the solution of the large New Keynesian model with consumption habit formation ($h = 0.5$) and wage rigidities ($\theta_w = 0.75$). I calculated Euler equation errors for the consumption Euler equation and the New Keynesian Phllips curve. Generally, the model solution is exact if

$$E_t \left[f(\boldsymbol{x}_{t-1}, \boldsymbol{x}_t, \boldsymbol{y}_{t-1}, \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) \right] = 0 \tag{4.48}$$

Table 4.2 Solution Times for Selected Models (in Seconds)

		DF	DF and M	DF, M and MP
	Processors	$n_d = 20$	$n_d = n_m = 20$	$n_d = n_m = n_{mp} = 10$
0 lagged	1	0.019	0.4238	8.5775
		$n_d = 20$, $m_y = 10$	$n_d = n_m = 20$, $m_y = 10$	$n_d = n_m = n_{mp} = 7$, $m_y = 5$
1 lagged	1	0.4872	46.3535	524.3196
	2	1.3672	38.3587	403.6988
	4	1.5707	24.8325	325.894
		$n_d = 20$, $m_y = m_w = 10$	$n_d = n_m = 10$, $m_y = m_w = 5$	$n_d = n_m = n_{mp} = 5$, $m_y = m_w = 5$
2 lagged	1	45.8439	177.9769	589.0075
	2	36.3547	167.137	438.6066
	4	41.3928	158.6442	349.5804

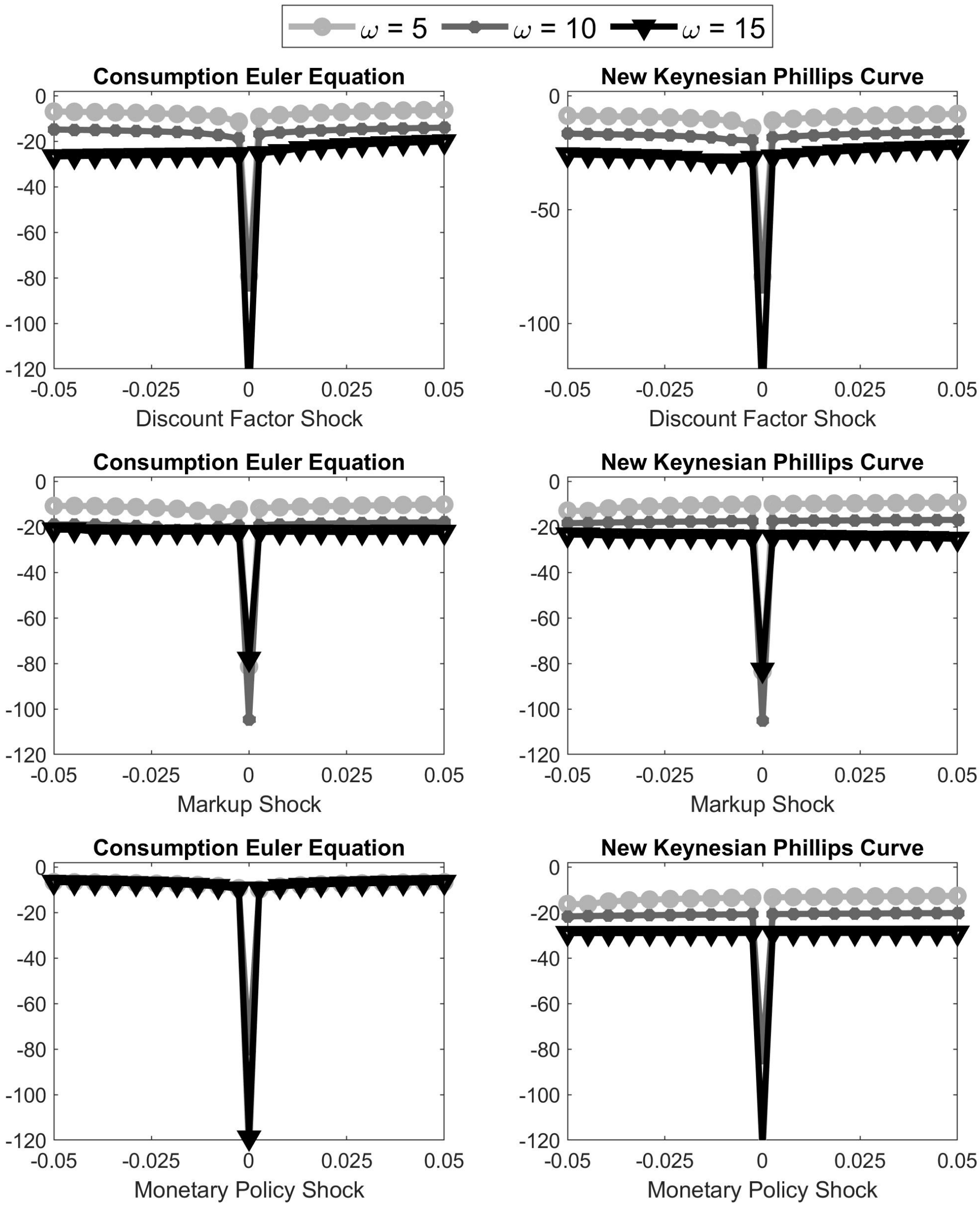
Note: DF: discount factor shock, M: markup shock, MP: monetary policy shock, 0 lagged: no endogenous state variables, 1 lagged: consumption habit formation, 2 lagged: consumption habit and wage rigidities. n_d, n_m, n_{mp}: no. of grid points for discount factor shock, markup shock and monetary policy shock. m_y, m_w: no. of gridpoints for output and real wage. I use 10 quadrature nodes for the numerical integration in all models.

where $\boldsymbol{x_t}$ is the vector of exogenous variables and $\boldsymbol{y}_t$ the vector of endogenous variables. $f(\cdot)$ is the forecast error based on the candidate solution of the model. More specifically, I calculate $f(\cdot)$ as the difference of the left hand side and the right hand side of the consumption Euler equation and the Phillips curve using the approximations of the policy functions. Moreover, I use a Gaussian Hermite quadrature method to approximate the value of $E_t[f(\cdot)]$.

Figure 4.1 compares Euler equation errors of the New Keynesian model with consumption habit ($h = 0$) and wage rigidities ($\theta_w = 0.75$) for the consumption Euler equation and the New Keynesian Phillips curve. For the ease of the presentation, absolute errors are reported in base 10 logarithms. The plots in Figure 4.1 show the Euler equation errors for each node of the discretized discount factor shock, markup shock and monetary policy shock, respectively.[3] Across all specifications in Figure 4.1, I assume that solutions are based on a state space of 2,000 nodes (10 points on $\widehat{Y}_{t-1}$ and $\widehat{w}_{t-1}$, 20 on one exogenous process). The unconditional standard deviation of all exogenous processes is set to $\sigma = 0.1/100$. The only difference between the models in Figure 4.1 is the number of quadrature nodes for the numerical integration. Using 5 quadrature nodes yields the least accurate solution. The approximation is worse towards the end of the grid because it heavily relies on extrapolation outside of the state space. Euler equation errors lower significantly with the number of quadrature nodes. However, this comes at the cost of higher computation times. In general, all numbers are in line with Aruoba et al. (2006) and Richter et al. (2014). As a complementary measure of accuracy, Table 4.3 reports the integral of the Euler equation errors (in base 10 logarithms) across different specifications of the model with consumption habit and wage rigidities, i.e. $h = 0.5$ and $\theta_w = 0.75$. To compute the integral, I simulate each model for 10,000 periods and compute the Euler equation error for the consumption Euler equation and the New Keynesian Phillips curve in every period. I use 10 quadrature nodes for all specifications. Again, all numbers are fairly in line with the findings in Aruoba et al. (2006) and Richter et al. (2014). The solutions become less accurate the higher the standard deviation of the shock. However, this can be partly compensated by increasing the number of grid points for the shock which comes at the cost of higher computation times. Note that the Euler equation error integrals are largest for the discount factor shock and smallest for the monetary policy shock. This inaccuracy goes back to the approximation errors when the ZLB constraint is binding. Table C.2 in Appendix C.0.3 shows Euler Error integrals for the New Keynesian model that ignores the ZLB constraint. In this case, the size of the integrals of the Euler equation errors is nearly identical for every shock across all model specifications.

[3] Lagged output and lagged real wage, both in log-deviations from steady state, are assumed to be zero.

Fig. 4.1 Euler Equation Errors: Consumption Euler Equation and New Keynesian Phillips Curve



Note: Equation Errors are reported in base 10 logarithms. $n_d = n_m = n_{mp} = 20$ and $n_y = n_w = 10$ for all models.

Table 4.3 Euler Equation Error Integrals

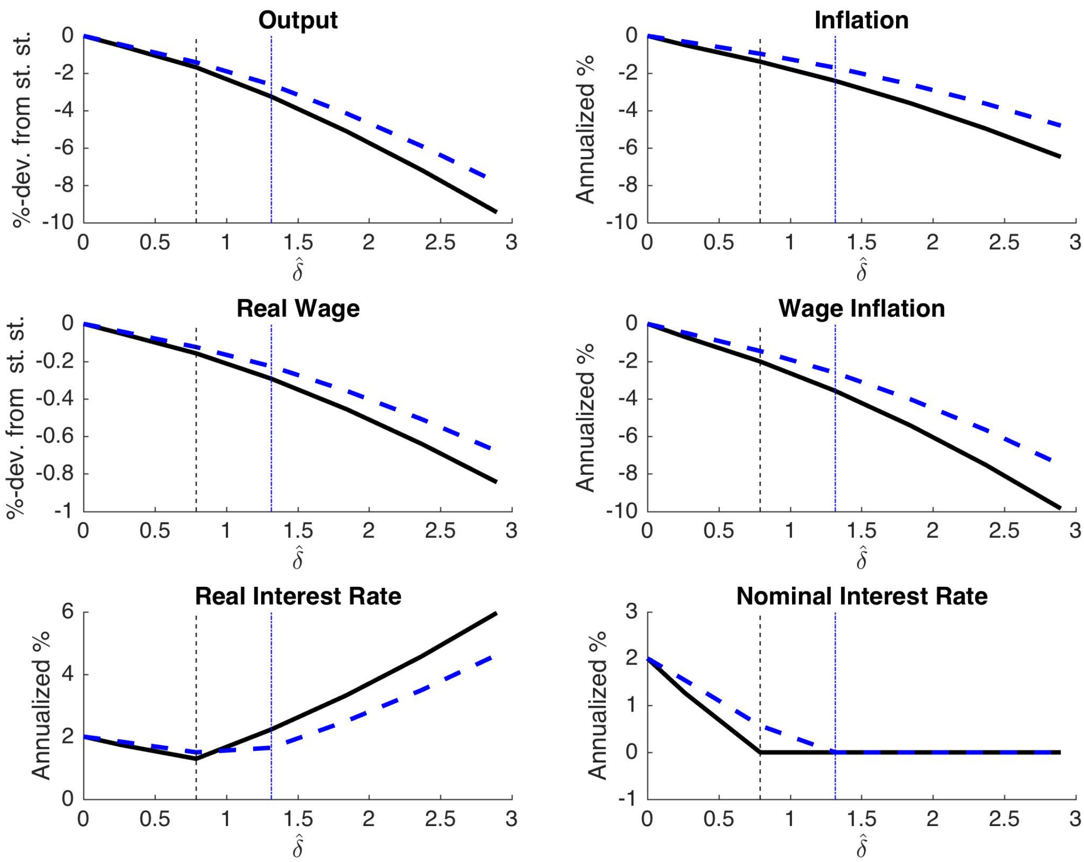
		$\sigma = 0.05/100$			$\sigma = 0.1/100$			$\sigma = 0.2/100$		
	$n =$	10	20	50	10	20	50	10	20	50
Euler Equation	**DF**	-11.60	-11.42	-19.28	-7.81	-8.82	-13.29	-4.99	-6.64	-8.49
	M	-20.91	-21.63	-22.12	-8.37	-8.91	-10.28	-5.29	-5.46	-6.08
	MP	-19.06	-31.18	-31.95	-31.21	-32.35	-32.04	-31.57	-31.76	-31.66
Phillips Curve	**DF**	-12.78	-12.17	-20.23	-8.99	-10.06	-14.09	-6.62	-7.52	-9.02
	M	-21.11	-21.18	-21.18	-9.84	-9.56	-9.67	-6.26	-5.68	-5.65
	MP	-19.89	-31.62	-37.62	-36.94	-37.04	-36.79	-36.29	-36.44	-36.40

Note: All Euler Equation Error Integrals are reported in base 10 logarithms. DF: discount factor shock, M: markup shock, MP: monetary policy shock, σ: unconditional standard deviation of respective shock, n: number of grid points for respective shock.

4.6 The Effects from Uncertainty

This section analyzes to which extent the presence of uncertainty alters the policy functions of the model variables. Numerous papers have documented the effects of shock uncertainty for macroeconomic analysis. Examples are Lindé and Trabandt (2018*b*), Adam and Billi (2006), Adam and Billi (2007), Fernández-Villaverde et al. (2015), Gust et al. (2017), Nakata (2016) and Richter and Throckmorton (2017). These authors show that future shock uncertainty can have non-negligible implications for equilibrium dynamics.

Fig. 4.2 Policy Functions: Stochastic vs. Deterministic Model



Note: Blue dashed lines: deterministic model. Black solid lines: mean responses from stochastic model. Dashed black and dash-dotted blue vertical lines indicate the level of $\widehat{\delta}_t$ above which the ZLB is binding in the stochastic model and the deterministic model, respectively.

Figure 4.2 shows the policy functions in the stochastic economy (solid black lines) and those in the deterministic economy (blue dashed lines) in response to the discount factor shock. The only difference between the two economies is that the unconditional standard deviation of the shock is set to $\sigma_d = 0$ in the deterministic model while it is $\sigma_d = 0.269/100$ in the stochastic model. An increase in the discount factor shock, $\widehat{\delta}$, means that the household becomes more impatient and, hence, wants to save more and spend less. Consequently, current demand falls and leads to lower inflation. Following the truncated Taylor rule, a reduction in inflation is accompanied by a reduction in the nominal interest rate. For a sufficiently large increase in $\widehat{\delta}_t$, the nominal interest rate reaches the ZLB and cannot further decline. As a result, the declines in output and inflation are larger when the ZLB is binding in both the deterministic and the stochastic economies.

The policy functions for inflation and wage inflation are substantially different even before the ZLB constraint is binding. In particular, inflation and wage inflation decline by a larger amount in the stochastic model than in the deterministic model. The possibility of reaching the ZLB in future periods lowers expected marginal cost leading forward-looking firms to reduce prices today. When the nominal interest rate declines more than one-for-one with inflation, the real interest rate decreases and dampens the fall in output.

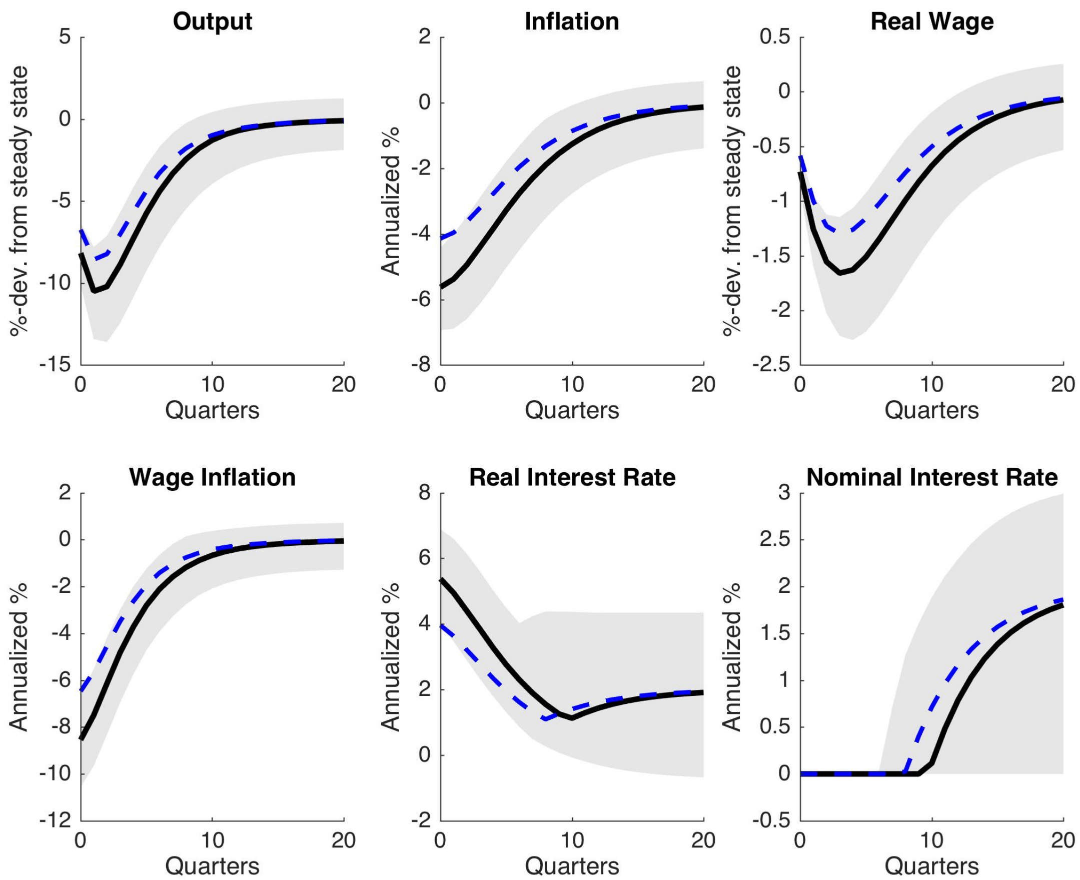
Table 4.4 Simulation Results: Stochastic vs. Deterministic Model

		ZLB		Standard Deviation		
		Probability	Duration	$\widehat{Y}$	$\widehat{\pi}$	$\widehat{R}$
$\sigma_d = \frac{0.269}{100}$	stoch. model	10.22%	3.23 q.	0.20%	1.39%	0.14%
	determ. model	3.76%	2.63 q.	0.17%	1.23%	0.13%
$\sigma_d = \frac{0.37}{100}$	determ. model	10.40%	3.32 q.	0.24%	1.78%	0.17%

Table 4.4 compares the results of stochastic simulations of the discount factor shock in the two economies. For the stochastic simulations, I simulate 6000 samples of 100 periods for each model as in Kiley and Roberts (2017) or Reifschneider and Williams (2000). Table 4.4

shows the results of the stochastic simulations with $\sigma_d = 0.269/100$ for both economies. In particular, it shows the average probability of being at the ZLB, the mean duration at the ZLB and standard deviations for output, inflation and the nominal interest rate. The probability of reaching the ZLB is substantially higher in the stochastic economy than in the deterministic economy. Likewise, periods at the ZLB tend to be longer in the stochastic model. The last row in Table 4.4 provides an alternative look at the implications of uncertainty at the ZLB. It shows the results of simulations of the deterministic model based on $\widehat{\delta}$ with a unconditional standard deviation σ_z chosen such that the ZLB frequency corresponds to the one in the stochastic model. It requires a substantially higher $\sigma_z = 0.37/100$ to replicate the same ZLB frequency as in the stochastic economy. As a result, the standard deviations of output, inflation and the nominal interest rate are larger and periods at the ZLB become longer on average.

Fig. 4.3 Impulse Response Functions: Stochastic vs. Deterministic Model

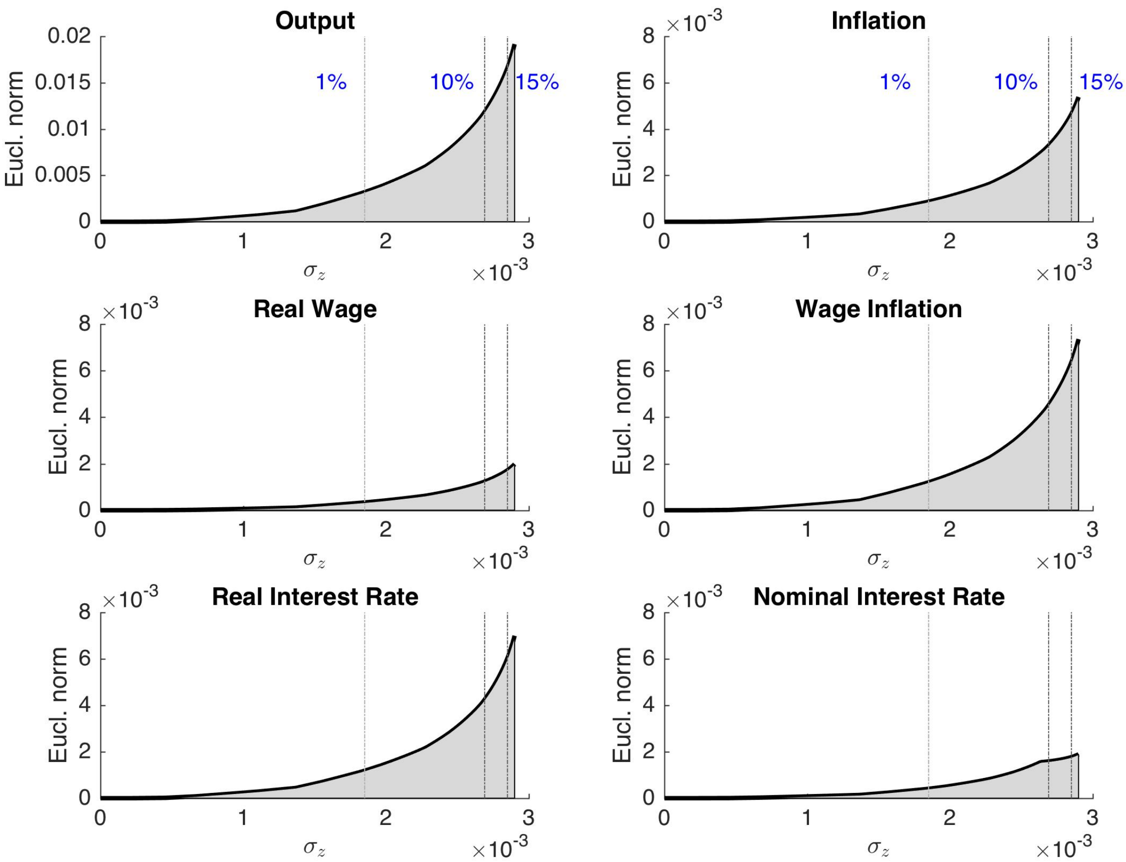


Note: Blue dashed lines: deterministic model. Black solid lines: mean responses from stochastic model. Shaded gray areas: 95% probability distribution at time t.

Figure 4.3 shows the impulse responses of the model's variables in response to a large initial increase in the discount factor shock so that the ZLB is expected to hold for 8 quarters in the stochastic model. The dashed blue lines depict the impulse responses for the deterministic model, while the solid black lines are the mean responses for the stochastic model. The shaded gray areas represent the variable's probability distribution at time t, covering 95% of the distribution. Figure 4.3 shows that the decline in inflation, output, wage inflation and real wages are larger in the stochastic model. The differences are quantitatively important. For example, output decreases by initially 8 percent in the stochastic model, while only by roughly 6 percent in the deterministic model. Inflation initially drops by 5.8 percent in the stochastic model but only by 4 in the deterministic model. As a result, the expected duration at the ZLB in the deterministic economy is 1 quarter shorter than in the stochastic economics. The differences between the impulse responses are large during the periods in which the ZLB is binding but becomes smaller over time. Due to the presence of wage rigidities and price rigidities, the differences in the expected paths for inflation and the real wage diminish only after 20 quarters when the nominal interest rate is well above zero.

The quantitative importance of uncertainty at the ZLB depends on the standard deviation of the shock σ_d. Figure 4.4 documents the differences in the policy functions for the model's variables of the stochastic model relative to the deterministic model. To measure the distance between the policy functions, I calculate the vector norms of the differences between the policy functions of the stochastic model and those of the deterministic model. The relative importance of uncertainty increases with σ_d. For relatively small σ_d, the policy functions are fairly the same in both models. However, the differences are exponentially increasing in σ_d for all variables.

Fig. 4.4 The Effect of σ_d: Stochastic vs. Deterministic Model



Note: Outer contour of curves: Euclidean norm of policy functions from stochastic vs. deterministic model. Gray dash-dotted vertical lines: level of σ_d associated with 1%, 10% and 15% probability to reach the ZLB (from left to right).

4.7 Solving the Non-Linear Model

The solution approach for the non-linear model is fairly similar to the one for the semi-loglinear model. One important difference is that the non-linear model will be represented as a system of non-linear equations. For the purpose of illustration, consider the non-linear model described in Section 4.2 with $h = 0$ (no habit formation) and $\theta_w = 0$ (no wage rigidities). Furthermore, assume that $\nu_t = \nu$ and $\gamma_t = \gamma$ for all t so that the discount factor shock δ_t is the only relevant shock. For this model, the grid will be defined over the pre-defined values for the discount factor shock, δ, and price dispersion, s. Let $\overline{\boldsymbol{d}}$ and $\overline{\boldsymbol{s}}$ be the column vectors that represent the values of the discount factor shock and the values of lagged price dispersion. Then, the grid is defined by matrix $\overline{\boldsymbol{S}}$:

$$\overline{\mathbf{S}} = \begin{bmatrix} [\mathbf{1}_{n_s} \otimes \overline{\boldsymbol{d}}]' \\ [\overline{\boldsymbol{s}} \otimes \mathbf{1}_{n_d}]' \end{bmatrix}' \tag{4.49}$$

where n_l with $l \in \{d,s\}$ is the number of collocation nodes for the discount factor shock and price dispersion, respectively. The system of equations for a single grid point $\{\overline{d}_j, \overline{s}_k\}$ can be written as:

$$\text{Euler Equation:} \quad \frac{1}{Y(\overline{d}_j,\overline{s}_k)} = \frac{1}{EY(\overline{d}_j,\overline{s}_k)}\overline{d}_j \frac{R(\overline{d}_j,\overline{s}_k)}{E\Pi(\overline{d}_j,\overline{s}_k)} \tag{4.50}$$

$$\text{Labor Supply:} \quad w(\overline{d}_j,\overline{s}_k) = Y(\overline{d}_j,\overline{s}_k)N(\overline{d}_j,\overline{s}_k) \tag{4.51}$$

$$\text{Market clearing:} \quad Y(\overline{d}_j,\overline{s}_k)) = \frac{N(\overline{d}_j,\overline{s}_k)}{s(\overline{d}_j,\overline{s}_k)} \tag{4.52}$$

$$\text{Optimal Price:} \quad P(\overline{d}_j,\overline{s}_k) = \frac{\gamma}{\gamma-1}\frac{K_1(\overline{d}_j,\overline{s}_k)}{K_2(\overline{d}_j,\overline{s}_k)} \tag{4.53}$$

$$\text{Price Setting 1:} \quad K_1(\overline{d}_j,\overline{s}_k) = w(\overline{d}_j,\overline{s}_k) + \theta\beta E\Pi(\overline{d}_j,\overline{s}_k)^{\gamma} EK_1(\overline{d}_j,\overline{s}_k) \tag{4.54}$$

$$\text{Price Setting 2:} \quad K_2(\overline{d}_j,\overline{s}_k) = 1 + \theta\beta E\Pi(\overline{d}_j,\overline{s}_k)^{\gamma-1} EK_2(\overline{d}_j,\overline{s}_k) \tag{4.55}$$

$$\text{Price Setting 2:} \quad P(\overline{d}_j,\overline{s}_k) = \left(\frac{1-\theta\Pi(\overline{d}_j,\overline{s}_k)^{\gamma-1}}{1-\theta}\right)^{\frac{1}{1-\gamma}} \tag{4.56}$$

$$\text{Price Dispersion:} \quad s(\overline{d}_j,\overline{s}_k) = (1-\theta)P(\overline{d}_j,\overline{s}_k)^{-\gamma} + \theta\Pi(\overline{d}_j,\overline{s}_k)^{\gamma}\overline{s}_k \tag{4.57}$$

$$\text{Monetary Policy:} \quad R(\overline{d}_j,\overline{s}_k) = \max\left\{1, \overline{R}\Pi(\overline{d}_j,\overline{s}_k)^{\varphi}\nu\right\} \tag{4.58}$$

where $EY(\overline{d}_j,\overline{s}_k)$ and $E\Pi(\overline{d}_j,\overline{s}_k)$, are 1-period ahead expected output and inflation, respectively. $EK_1(\overline{d}_j,\overline{s}_k)$ and $EK_2(\overline{d}_j,\overline{s}_k)$ are the 1-period ahead expectations for the variables K_1 and K_2 which are associated with the price setting condition. The solution for each variable can be written as a column vector over the grid. Denote the column vector for output $\boldsymbol{Y}$, for inflation $\boldsymbol{\Pi}$, for nominal interest rate $\boldsymbol{R}$, for real wage $\boldsymbol{w}$, for labor hours $\boldsymbol{N}$, for price dispersion $\boldsymbol{s}$, for optimal price $\boldsymbol{P}$, for the first price setting condition $\boldsymbol{K_1}$ and for the second price setting condition $\boldsymbol{K_2}$. Then, I stack the column vectors to define the policy function $\boldsymbol{C}$ so that $\boldsymbol{C} = \left[\boldsymbol{Y}',\boldsymbol{\Pi}',\boldsymbol{R}',\boldsymbol{w}',\boldsymbol{N}',\boldsymbol{s}',\boldsymbol{P}',\boldsymbol{K}_1',\boldsymbol{K}_2'\right]'$.

The continuous process for the discount factor shock is given by $\delta_t = \delta_{t-1}^{\rho_d}\exp\varepsilon_t^d$ with $\varepsilon_t^d \sim \mathcal{N}(0,\sigma_d^2)$. Again, I use a Gaussian quadrature scheme to discretize δ_t. Assume that $\boldsymbol{\varepsilon} = [\varepsilon_1 \ldots \varepsilon_m]'$ is a column vector with m quadrature nodes and $\boldsymbol{\omega} = [\omega_1 \ldots \omega_m]'$ is a column vector of length m containing the quadrature weights. The expected functions will be evaluated over matrix $\overline{\boldsymbol{S}}^E$:

$$\overline{\boldsymbol{S}}^E = \begin{bmatrix} [\mathbf{1}_{n_s} \otimes \overline{\boldsymbol{d}}^E]' \\ [\boldsymbol{s} \otimes \mathbf{1}_m]' \end{bmatrix}' \tag{4.59}$$

with

$$\overline{\boldsymbol{d}}^E = \begin{bmatrix} (\overline{d}_1^{\rho_d} \otimes \mathbf{1}_m)\exp[\boldsymbol{\varepsilon}] \\ \vdots \\ (\overline{d}_{n_d}^{\rho_d} \otimes \mathbf{1}_m)\exp[\boldsymbol{\varepsilon}] \end{bmatrix}$$

where $\exp[\boldsymbol{\varepsilon}]$ is an elementwise exponential. Then, I can approximate the expected variables as:

$$E\boldsymbol{Y} = \boldsymbol{\Omega}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)\,Y \tag{4.60}$$

$$E\boldsymbol{\Pi} = \boldsymbol{\Omega}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)\,\Pi \tag{4.61}$$

$$E\boldsymbol{K_1} = \boldsymbol{\Omega}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)\,K_1 \tag{4.62}$$

$$E\boldsymbol{K_2} = \boldsymbol{\Omega}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)\,K_2 \tag{4.63}$$

where $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$ is the matrix containing the basis functions evaluated for $\overline{\boldsymbol{S}}^E$ and $\boldsymbol{\Omega}$ contains the quadrature weights.

The model equations defined over the grid $\overline{\boldsymbol{S}}$ can be represented as:

$$0 = f\left(\boldsymbol{C}, \widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E}), \overline{\boldsymbol{S}}^{E}\right) \tag{4.64}$$

given the model parameters and the matrix $\boldsymbol{\Omega}$ containing the quadrature weights. I use a root finding method (Newton-Raphson-method) to solve for the unknown policy function $\boldsymbol{C}$. A particular advantage of the non-linear setting as compared to the linear setting is that the solution method can directly account for the occasionally binding constraint. In the linear model, the algorithm checks for the grid points on which the ZLB is binding and updates the parts of the system of equations that are affected by the occasionally binding constraint accordingly. Specifically, the algorithm in Section 4.4 updates the parameter matrix and the vector of constants. This step is irrelevant for the non-linear model. Hence, the algorithm will only update the expected state matrix $\overline{\boldsymbol{S}}^{E}$ and the corresponding matrix of basis functions $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E})$ as the interpolation and extrapolation points depend on the current solution $\boldsymbol{C}$. The algorithm reduces to the following steps:

1. Start with a guess $\boldsymbol{C}^0$. Compute $\overline{\boldsymbol{S}}^{E}$ and $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E})$ based on $\boldsymbol{C}^0$.

2. Given $\overline{\boldsymbol{S}}^{E}$ and $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E})$, solve the root-finding problem to determine $\boldsymbol{C}^{\text{new}}$:

$$0 = f\left(\boldsymbol{C}^{\text{new}}, \widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E}), \overline{\boldsymbol{S}}^{E}\right) \tag{4.65}$$

3. Update $\boldsymbol{C}^1 = \lambda \boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}^{E}$ and $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E})$ based on $\boldsymbol{C}^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

Generally, I find that the initial $\boldsymbol{C}^0$ is important for the algorithm to converge to the final solution of the model. A reasonable first guess $\boldsymbol{C}^0$ is the solution of the model in the steady state.

4.8 Conclusion

Models with occasionally binding constraints have become important for contemporary macroeconomic analysis. Policy function iteration methods represent a reliable way to solve models with non-linearities accurately. However, these methods often suffer from a severe drawback: they rely on grid-based techniques which impose non-negligible computational costs that are even increasing in the dimension of the model.

Another possible concern is the degree of curvature in the policy functions. It can be difficult to obtain solutions to the New Keynesian model if the ZLB is reached too often across the state space. One approach to tackle this problem could be a good initial conjecture of the policy function. In particular, one can start solving the model with a parameterization that induces less curvature in the policy functions and iterate on the parameters of interest.

This paper presents an algorithm which helps to reduce the computational burden for the solution of models with occasionally binding constraints. One key difference of my approach as opposed to other policy function iteration methods is that it solves for the whole policy function in one computation step. This makes it unnecessary to solve for the candidate value of the policy function on each grid node, which would lead to higher computational times. Parallel computing can provide additional speed gains. However, the key improvement of the algorithm does not come from parallelizing different processes and, hence, is still considerably fast without parallel computing.

References

Adam, K. and Billi, R. M. (2006), 'Optimal monetary policy under commitment with a zero bound on nominal interest rates', *Journal of Money, Credit, and Banking* **38**(7), 1877–1905.

Adam, K. and Billi, R. M. (2007), 'Discretionary monetary policy and the zero lower bound on nominal interest rates', *Journal of Monetary Economics* **54**(3), 728–752.

Altig, D., Christiano, L. J., Eichenbaum, M. and Lindé, J. (2011), 'Firm-specific capital, nominal rigidities and the business cycle', *Review of Economic Dynamics* **14**(2), 225–247.

Alvarez, F. E., Lippi, F. and Paciello, L. (2011), 'Optimal price setting with observation and menu costs', *The Quarterly Journal of Economics* **126**(4), 1909–1960.

Arias, J., Ascari, G., Branzoli, N. and Castelnuovo, E. (2017), 'Positive trend inflation and determinacy in a medium-sized new keynesian model', *FRB of Philadelphia Working Paper No. 17-16* .

Aruoba, S., Fernandez-Villaverde, J. and Rubio-Ramirez, J. (2006), 'Comparing solution methods for dynamic equilibrium economies', *Journal of Economic Dynamics and Control* **30**, 2477–2508.

Ascari, G. and Ropele, T. (2009), 'Trend inflation, taylor principle and indeterminacy', *Journal of Money, Credit and Banking* **41**(8), 1557–1584.

Ascari, G. and Sbordone, A. M. (2014), 'The macroeconomics of trend inflation', *Journal of Economic Literature* **53**(3), 679–739.

Bachmann, R., Berg, T. O. and Sims, E. R. (2015), 'Inflation expectations and readiness to spend: Cross-sectional evidence', *American Economic Journal: Economic Policy* **7**(1), 1–35.

Bakhshi, H., Burriel-Llombart, P., Khan, H. and Rudolf, B. (2007), 'The new keynesian phillips curve under trend inflation and strategic complementarity', *Journal of Macroeconomics* **29**(1), 37–59.

Ball, L. (2013), 'The case for four percent inflation', *Working Paper* .

Barsky, R. B., Boehm, C. E., House, C. L. and Kimball, M. S. (2015), 'Monetary policy and durable goods', *Working Paper* .

Barsky, R. B., House, C. L. and Kimball, M. S. (2007), 'Sticky-price models and durable goods', *American Economic Review* **97**(3), 984–998.

Basu, S. and Di Leo, P. (2017), 'Should central banks target investment prices?', *Boston College Working Paper Series No. 910* .

Bernanke, B. (2010), 'The economic outlook and monetary policy', *Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming* .

Bernedek, D., De Mooj, R. and Wingender, P. (2015), 'Estimating vat pass through', *IMF Working Paper* .

Bils, M. and Klenow, P. (2004), 'Some evidence on the importance of sticky prices', *Journal of Political Economy* pp. 947–985.

Blanchard, O., Dell'Ariccia, G. and Mauro, P. (2010), 'Rethinking macroeconomic policy', *IMF Staff Position Note* .

Blanchard, O. J., Erceg, C. and Lindé, J. (2016), 'Jump-starting the euro area recovery: Would a rise in core fiscal spending help the periphery?', *NBER Macroeconomics Annual* **31**.

Boehm, C. E. (2019), 'Government consumption and investment: Does the composition of purchases affect the multiplier', *forthcoming in the Journal of Monetary Economics* .

Boneva, L. M., Braun, R. A. and Waki, Y. (2016), 'Some unpleasant properties of loglinearized solutions when the nominal rate is zero', *Journal of Monetary Economics* **84**, 216–232.

Bouakez, H., Cardia, E. and Ruge-Murcia, F. J. (2005), 'Habit formation and the persistence of monetary shocks', *Journal of Monetary Economics* **52**, 1073–1088.

Brooks, B. P. (2004), 'Linear stability conditions for first-order three dimensional discrete dynamic', *Applied Mathematical Letters* **17**(4), 463–466.

Bundesbank (2018), 'Monthly report, june 2018'.

Caldara, D., Fernandez-Villaverde, J., Rubio-Ramirez, J. and Yao, W. (2012), 'Computing dsge models with recursive preferences and stochastic volatility', *Review of Economic Dynamics* **15**, 188–206.

Calvo, G. A. (1983), 'Staggered prices in a utility-maximizing framework', *Journal of Monetary Economics* **12**(3), 383–398.

Carare, A. and Danninger, S. (2008), 'Inflation smoothing and the modest effect of vat in germany', *IMF Working Paper WP/08/175* .

Carlstrom, C. and Fuerst, T. (2010), 'Nominal rigidities, residential investment, and adjustment costs', *Macroeconomic Dynamics* **14**(1), 136–148.

Cenesiz, M. A. and Guimaraes, L. (2018), 'Sticky price models, durable goods, and real wage rigidities', *Journal of Money, Credit and Banking* **(forthcoming)**.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of Political Econ- omy* **12**, 383–398.

Christiano, L. J., Eichenbaum, M. and Rebelo, S. (2011), 'When is the government spending multiplier large?', *Journal of Political Economy* **119**(1), 78–121.

Coenen, G., Orphanides, A. and Wieland, V. (2004), 'Price stability and monetary policy effectiveness when nominal interest rates are bounded at zero', *Advances in Macroeconomics* **4**(1).

Cogley, T. W., Primiceri, G. and Sargent, T. (2010), 'Inflation-gap persistence in the u.s.', *American Economic Journal: Macroeconomics* **2**(1), 43–69.

Cogley, T. W. and Sbordone, A. M. (2008), 'Trend inflation, indexation and inflation persistence in the new keynesian phillips curve.', *American Economic Review* **98**(5), 2101–2126.

Coibion, O., Gorodnichenko, Y. and Wieland, J. (2012), 'The optimal inflation rate in new keynesian model:: Should central banks raise their inflation targets in light of the zero lower bound?', *Review of Economics Studies* **79**, 1371–1406.

Correia, I., Farhi, E., Nicolini, J. P. and Teles, P. (2013), 'Unconventional fiscal policy at the zero bound', *American Economic Review* **103**(4), 1172–1211.

Crossley, T., Low, H. and Sleeman, C. (2014), 'Using a temporary indirect tax cut as a fiscal stimulus: evidence from the uk', *IFS Working Paper W14/16* .

D'Acunto, F., Hoang, D. and Weber, M. (2017), 'Inflation expectations and consumption expenditure', *Working Paper* .

De Michelis, A. and Iacoviello, M. (2016), 'Raising an inflation target: The japanese experience with abenomics', *European Economic Review* **88**, 67–87.

Del Negro, M., Giannone, D., Giannoni, M. and Tambalotti, A. (2017), 'Safety, liquidity, and the natural rate of interest', *Brookings Papers on Economic Activity* .

Dotsey, M., King, G. and Wolman, A. L. (1999), 'State-dependent pricing and the general equilibrium dynamics of money and output', *Quarterly Journal of Economics* **114**(2), 655–590.

Eggertsson, G. B. (2010), 'What fiscal policy is effective at zero interst rates?', *NBER Macroeconomics Annual* **25**.

Eggertsson, G. B., Mehrotra, N. and Robbins, J. (2017), 'A model of secular stagnation: Theory and quantitative evaluation', *NBER Working Paper No. 23093* .

Eggertsson, G. B. and Woodford, M. (2003), 'The zero bound on interest rates and optimal monetary policy', *Brookings Papers on Economic Activity* **1**, 139–211.

Erceg, C. J., Henderson, D. W. and Levin, A. T. (2000), 'Optimal monetary policy with staggered wage and price contracts', *Journal of Monetary Economics* **46**(2), 281–313.

Erceg, C. and Levin, A. (2006), 'Optimal monetary policy with durable consumption goods', *Journal of Monetary Economics* **53**(7), 1341–1359.

Erceg, C. and Lindé, J. (2014), 'Is there a fiscal free lunch in a liquidity trap?', *Journal of the European Economic Association* **12**(1), 73–107.

Fernández-Villaverde, J., Gordon, G., Guerrón-Quintana, P. and Rubio-Ramírez, J. F. (2015), 'Nonlinear adventures at the zero lower bound', *Journal of Economic Dynamics and Control* **57**, 182–204.

Fernández-Villaverde, J. and Rubio-Ramírez, J. F. (2008), 'How structural are struc- tural parameter values?', *NBER Macroeconomics Annual 2007* pp. 83–137.

Fernández-Villaverde, J. and Zarruk Valencia, D. (2018), 'A practical guide to parallelization in economics', *Manuscript, University of Pennsylvania* .

Gagnon, E., Kramer Johannsen, B. and Lopez-Salido, D. (2016), 'Understanding the new normal: The role of demographics', *FEDS Working Paper No. 2016-080* .

Galí, J. (2011), 'The return of the wage Phillips curve', *Journal of European Economic Association* **9**(3), 436–461.

Gust, C., Herbst, E., López-Salido, D. and Smith, M. E. (2017), 'The empirical implications of the interest-rate lower bound', *American Economic Review* **107**(7), 1971–2006.

Hall, R. E. (2009), 'By how much does gdp rise if the government buys more output?', *Brookings Papers on Economic Activity* **40**(2), 183–249.

Hamilton, J. D., Harris, E. S. and West, K. D. (2015), 'The equilibrium real funds rate: Past, present, and future', *presented at the US Monetary Policy Forum, New York, February 27, 2015* .

Ichiue, H. and Nishiguchi, S. (2015), 'Inflation expectations and consumer spending at the zero bound: Micro evidence', *Economic Inquiry* **53**(2), 1086–1107.

Judd, K. L. (1998), *Numerical methods in economics*, Cambridge, MA, MIT Press.

Kiley, M. (2007), 'Is moderate-to-high inflation inherently unstable?', *International Journal of Central Banking* **3**(2), 173–201.

Kiley, M. (2016), 'Policy paradoxes in the New Keynesian model', *Review of Economic Dynamics* **21**, 1–15.

Kiley, M. and Roberts, J. (2017), 'Monetary policy in a low interest rate world', *Brookings Papers on Economic Activity, Fall,* pp. 317–396.

Kimball, M. S. (1995), 'The quantitative analytics of the basic neomonetarist model', *Journal of Money, Credit and Banking* **27**(4), 1241–1277.

Kimball, M. S. and Shapiro, M. D. (2008), 'Labor supply: Are the income and substitution effects both large or both small?', *Nber Working Papers, National Bureau of Economic Research* .

Lindé, J. and Trabandt, M. (2018*a*), 'Resolving the missing deflation puzzle', *Working Paper* .

Lindé, J. and Trabandt, M. (2018*b*), 'Should we use linearized models to calculate fiscal multipliers?', *Journal of Applied Econometrics* **33**(7), 937– 965.

McKay, A., Nakamura, E. and Steinsson, J. (2016), 'The power of forward guidance revisited', *American Economic Review* **106**(10), 3133–31358.

Mertens, K. and Ravn, M. (2010), 'Fiscal policy in an expectations driven liquidity trap', *Discussion Paper No. 7931, Center for Economic Policy Research* .

Miranda, M. J. and Fackler, P. L. (2002), *Applied Computational Economics and Finance*, Cambridge, MA, MIT Press.

Monacelli, T. (2009), 'New keynesian models, durable goods, and collateral constraints', *Journal of Monetary Economics* **56**(2), 242–254.

Nakamura, E. and Steinsson, J. (2008), 'Five facts about prices: a reevaluation of menu cost models', *Quarterly Journal of Economics* **123**(4), 1415–1464.

Nakamura, E., Steinsson, J., Sun, P. and Villar, D. (2018), 'The elusive costs of inflation: Price dispersion during the u.s. great inflation', *Quarterly Journal of Economics (forthcoming)* .

Nakata, T. (2016), 'Uncertainty at the zero lower bound', *Working Paper* .

Ngo, P. (2018), 'The risk of hitting the zero lower bound and the optimal inflation target', *Macroeconomic Dynamics* **22**(2), 402–425.

Orphanides, A. and Solow, R. M. (1990), 'Money, inflation, and growth', *in Benjamin M. Friedman and Frank H. Hann, eds, Handbook of Monetary Economics, Amsterdam N. Holland* .

Pike, R., M. L. and Turner, D. (2009), 'Impact of vat reduction on the consumer price indices', *Economic & Labour Market Review* **3**(8), 17–21.

Ravina, E. (2007), 'Habit persistence and keeping up with the joneses: Evidence from micro data', *Manuscript, New York University.* .

Reifschneider, D. L. (2016), 'Gauging the ability of the fomc to respond to future recessions,', *Finance and Economics Discussion Series 2016-068. Washington: Board of Governors of the Federal Reserve System* .

Reifschneider, D. L. and Roberts, J. (2006), 'Expectations formation and the effectiveness of strategies for limiting the consequences of the zero bound', *Journal of the Japanese and International Economies* **20**(3), 314 – 37.

Reifschneider, D. L. and Williams, J. C. (2000), 'Three lessons for monetary policy in a low-inflation era', *Journal of Money, Credit, and Banking* **4**(2), 936 – 66.

Richter, A., Throckmorten, N. and Walker, T. (2014), 'Accuracy, speed and robustness of policy function iteration', *Computational Economics* **44**(4), 445–476.

Richter, A. W. and Throckmorton, N. A. (2017), 'Are nonlinear methods necessary at the zero lower bound?', *FRB Dallas Working Paper* **No. 1606**.

Rotemberg, J. J. and Woodford, M. (1997), 'An optimization-based econometric framework for the evaluation of monetary policy', *NBER Macroeconomics Annual 1997* pp. 297–346.

Rouwenhorst, G. K. (1995), *Asset Pricing Implications of Equilibrium Business Cycle Models*, Princeton University Press.

Schmidt, S. (2013), 'Optimal monetary and fiscal policy with a zero bound on nominal interest rates', *Journal of Money, Credit and Banking* **45**(7), 1335–1350.

Schmidt, S. (2017), 'Fiscal activism and the zero nominal interest rate bound', *Journal of Money, Credit and Banking* **49**(4), 695–732.

Schmitt-Grohé, S. and Uribe, M. (2007), 'Optimal simple and implementable monetary and fiscal rules', *Journal of Monetary Economics* **54**, 1702–1725.

Schmitt-Grohé, S. and Uribe, M. (2010), 'The optimal rate of inflation', *NBER Working Paper No. 16054* .

Smets, F. and Wouters, R. (2003), 'An estimated dynamic stochastic general equilibrium model of the euro area', *Journal of the European Economic Association* **1**(5), 1123–1175.

Smets, F. and Wouters, R. (2007), 'Shocks and frictions in us business cycles: A bayesian dsge approach', *American Economic Review* **97**(3), 586–606.

Sterk, V. (2010), 'Credit frictions and the comovement between durable and non- durable consumption', *Journal of Monetary Economics* **57**(2), 217–225.

Summers, L. (1991), ‘Panel discussion: Price stability: How should long-term monetary policy be determined?’, *Journal of Money, Credit and Banking* **23**, 625–31.

Taylor, J. B. (1993), ‘Discretion versus policy rules in practice’, *Carnegie-Rochester Conference Series on Public Policy* **39**, 195–214.

Treasury, H. (2008), ‘Pre-budget report statement’.
URL: *http://webarchive.nationalarchives.gov.uk/20100407010852/*

Williams, J. C. (2009), ‘Heeding daedalus: Optimal inflation and the zero lower bound’, *Brookings Papers on Economic Activity* **40**(2), 1–49.

Woodford, M. (2011), ‘Simple analytics of the government spending multiplier’, *American Economic Journal: Macroeconomics* **3**(1), 1–35.

Yun, T. (1996), ‘Nominal price rigidity’, *Money supply endogeneity, and business cycles* **37**(2), 345–370.

Appendix A

Appendix to Chapter 2

A.0.1 Derivation of the New Keynesian Phillips Curve Under a Positive Inflation Target

Suppose that in each period a firm can re-optimize its nominal price with fixed probability $1-\theta$, while with probability θ it charges the price of the previous period. The firm j which re-optimizes its price in period t chooses $P^*_{j,t}$ to maximize expected profits:

$$E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t+i} \left(P^*_{j,t} Y_{j,t+i} - \Gamma(Y_{j,t})\right) \tag{A.1}$$

subject to the demand function

$$Y_{j,t+i} = \frac{P^*_{j,t}}{P_{t+i}}^{-\varepsilon} Y_{t+i} \tag{A.2}$$

where Λ_{t+i} is the stochastic discount factor, $\Gamma(Y_{j,t}) = mc_{t+i} Y_{j,t+i}$ is total cost and $mc_{t+i} = w_{t+i}$ is marginal cost.

Let $p^*_{j,t} = \frac{P^*_{j,t}}{P_t}$ be the relative price of the optimizing firm in period t. The first-order condition of the firm's problem can be written as:

$$p^*_{j,t} = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t+i} Y_{t+i} \Pi^{\varepsilon}_{t,t+i} mc_{t+i}}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t+i} Y_{t+i} \Pi^{\varepsilon-1}_{t,t+i}} = \frac{\varepsilon}{\varepsilon-1} \frac{\psi_t}{\phi_t} \tag{A.3}$$

where $\Pi_{t,t+i}$ denotes the cumulated gross inflation rate over i periods:

$$\Pi_{t,t+i} = \begin{cases} 1 & \text{for } i = 0 \\ \pi_{t+1} \times \ldots \times \pi_{t+i} & \text{for } i = 1, 2, \ldots \end{cases} \tag{A.4}$$

with $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. We can rewrite ψ_t and ϕ_t recursively:

$$\psi_t = \Lambda_t mc_t Y_t + \theta\beta E_t[\pi_{t+1}^{\varepsilon}\psi_{t+1}] \tag{A.5}$$

$$\phi_t = \Lambda_t Y_t + \theta\beta E_t[\pi_{t+1}^{\varepsilon-1}\phi_{t+1}] \tag{A.6}$$

Let P_t be the price index associated with the final good y_t. It is a CES aggregate of the prices of the intermediate goods, $P_{j,t}$. It evolves as follows:

$$P_t = \left[\int_0^1 P_{j,t}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_{j,t}^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.7}$$

Rearraging the equation above gives:

$$p_{j,t}^{*} = \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{A.8}$$

Price dispersion between intermediate goods prices $P_{i,t}$ affects the relationship between employment, N_t and output Y_t. Assume each firm produces with a linear production technology: $Y_{j,t} = N_{j.t}$. Using the demand function in Eq. (3.3), aggregate labor demand is:

$$N_t = \int_0^1 N_{j,t} dj \tag{A.9}$$

$$= \int_0^1 Y_{j,t} dj = Y_t \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} dj \tag{A.10}$$

Let s_t be a measure of price dispersion:

$$s_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} dj, \tag{A.11}$$

aggregate output can be expressed as:

$$Y_t = \frac{N_t}{s_t}. \tag{A.12}$$

Using Eq. (A.7), we can rewrite s_t:

$$\begin{aligned} s_t =&(1-\theta)\left(\frac{P^*_{j,t}}{P_t}\right)^{-\varepsilon} + \theta\pi_t^{\varepsilon} \\ &\times\left\{(1-\theta)\left(\frac{P^*_{t-1}}{P_{t-1}}\right)^{-\varepsilon} + \theta^2(1-\theta)\left(\frac{P^*_{t-2}}{P_{t-1}}\right)^{-\varepsilon} + \ldots.\right\} \\ =&(1-\theta)(p^*_{j,t})^{-\varepsilon} + \theta\pi_t^{\varepsilon} s_{t-1}. \end{aligned} \tag{A.13}$$

We can now log-linearize Eqs. (A.3), (A.5), (A.6) and (A.8) around a deterministic steady state with trend inflation rate $\overline{\pi}$. Let $\widehat{}$ denote log-deviations from steady state. We do not model the household side explicitly. Note that $\widehat{\Lambda}_t = -\widehat{Y}_t$. We obtain the following expression for the dynamics of inflation:

$$\widehat{\pi}_t = \beta\omega(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})\widehat{mc}_t + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} \tag{A.14}$$

$$\text{with} \quad \widehat{\psi}_t = b(\overline{\pi})\Big((\varphi+1)\widehat{Y}_t + \varphi\widehat{s}_t\Big) + (1-b(\overline{\pi}))E_t\left(\widehat{\psi}_{t+1} + \widehat{\pi}_{t+1}\right) \tag{A.15}$$

$$\begin{aligned} \omega(\overline{\pi}) &= 1 + \varepsilon\overline{\pi}(1-\theta(1+\overline{\pi})^{\varepsilon-1}) \\ \kappa(\overline{\pi}) &= \frac{(1-\theta\beta(1+\overline{\pi})^{\varepsilon})(1-\theta(1+\overline{\pi}^{\varepsilon-1}))}{\theta(1+\overline{\pi}^{\varepsilon-1})} \\ \eta(\overline{\pi}) &= \beta\overline{\pi}(1-\theta(1+\overline{\pi}^{\varepsilon-1})) \\ b(\overline{\pi}) &= 1-\theta\beta(1+\overline{\pi}^{\varepsilon}) \end{aligned}$$

Furthermore, we have following labor supply equation: $\widehat{w}_t = \widehat{Y}_t + \varphi\widehat{N}_t$. Using Eq. (A.12), we can rewrite $\widehat{mc}_t$:

$$\widehat{mc}_t = (1+\varphi)\widehat{Y}_t + \varphi\widehat{s}_t. \tag{A.16}$$

We can use the expression above to obtain the generalized New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta\omega(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})[(1+\varphi)\widehat{Y}_t + \varphi\widehat{s}_t] + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} \tag{A.17}$$

Finally, we can use Eqs. (A.13) and (A.8) to show that price dispersion evolves as:

$$\widehat{s}_t = c(\overline{\pi})\widehat{s}_{t-1} + d(\overline{\pi})\widehat{\pi}_t \tag{A.18}$$

with $c(\overline{\pi}) = \theta\overline{\pi}^{\varepsilon}$ and $d(\overline{\pi}) = \frac{\varepsilon\theta\overline{\pi}^{\varepsilon-1}}{1-\theta\overline{\pi}^{\varepsilon-1}}(\overline{\pi}-1)$. Note that in a zero inflation steady state, i.e. $\overline{\pi} = 1$, we have:

$$\widehat{s}_t = \theta\widehat{s}_{t-1}. \tag{A.19}$$

Hence, small perturbations have a zero first-order impact on $\widehat{s}_t$. This implies that for $\overline{\pi} = 1$ that Eq. (A.17) becomes the familiar New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \kappa\widehat{Y}_t. \tag{A.20}$$

with $\kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta}$.

A.0.2 Determinacy Conditions

In the long-run (i.e. $\widehat{\delta}_t = 0$), the matrix representation of the model is given by

$$\begin{bmatrix} \widehat{Y}_t \\ \widehat{\pi}_t \\ \widehat{\psi}_t \end{bmatrix} = A \times \begin{bmatrix} \widehat{Y}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{\psi}_{t+1} \end{bmatrix} \tag{A.21}$$

with

$$A = \begin{bmatrix} 1+\phi_Y & \phi_\pi & 0 \\ -\kappa(\overline{\pi}) & 1 & 0 \\ 1-\theta\,\beta\,\overline{\pi}^{\varepsilon} & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & \beta\,\alpha(\overline{\pi}) & \eta(\overline{\pi}) \\ 0 & \theta\,\beta\,\overline{\pi}^{\varepsilon}\,\varepsilon & \theta\,\beta\,\overline{\pi}^{\varepsilon} \end{bmatrix} \tag{A.22}$$

In general, the characteristic polynomial associated with the cubic matrix, A, is:

$$-\lambda^3 + T\lambda^2 - M\lambda + D, \tag{A.23}$$

where T, M, D denote the trace, the sum of principal minors of order two, and the determinant of matrix A, respectively. Here, it holds that

$$T = \frac{1}{1+\phi_Y+\kappa(\overline{\pi})\,\phi_\pi}\left[1+\kappa(\overline{\pi})+\beta\,\alpha(\overline{\pi})\,(1+\phi_Y)-b(\overline{\pi})\,\phi_\pi\,\eta(\overline{\pi})-(b(\overline{\pi})-1)\,(1+\phi_Y+\kappa(\overline{\pi})\,\phi_\pi)\right] \tag{A.24}$$

$$M = \frac{(\alpha(\overline{\pi})\,\beta-\eta(\overline{\pi})\,b1(\overline{\pi}))-(b(\overline{\pi})-1)\,\kappa(\overline{\pi})-(b(\overline{\pi})-1)\,(1+\phi_Y)\,(\alpha(\overline{\pi})\,\beta-\eta(\overline{\pi})\,\varepsilon)}{1+\phi_Y+\kappa(\overline{\pi})\,\phi_\pi} \tag{A.25}$$

$$D = \frac{-\beta\,(b(\overline{\pi})-1)}{1+\phi_Y+\kappa(\overline{\pi})\,\phi_\pi} \tag{A.26}$$

Brooks (2004) states that a necessary and sufficient condition for determinacy is that all eigenvalues of A lie inside the unit circle. The necessary and sufficient conditions in terms of T, M and D are:

$$|D| < 1 \tag{A.27}$$

$$|T+D| < M+1 \tag{A.28}$$

$$D^2 - T\,D + M < 1 \tag{A.29}$$

Substituting the expressions for T, M and D in the conditions above returns the expressions stated in the text.

A.0.3 Proof of Proposition 1

First, note that $\partial\alpha(\overline{\pi})/\partial\overline{\pi} > 0$, $\partial\kappa(\overline{\pi})/\partial\overline{\pi} < 0$ and $\partial\eta(\overline{\pi})/\partial\overline{\pi} > 0$ hold if Assumption 2 is satisfied (the proof can be find in Ascari and Ropele (2009)). Define $b2(\overline{\pi}) = -(b(\overline{\pi})-1)$. It can be easily seen that $\partial b2(\overline{\pi})/\partial\overline{\pi} = \beta\,\theta\,\varepsilon\,\overline{\pi}^{\varepsilon-1} > 0$. If the determinacy conditions are fulfilled, we can solve for the aggregate supply curve:

$$\widehat{\pi}_S = \frac{\tilde{\kappa}(\overline{\pi})}{1-\beta\,\tilde{\alpha}(\overline{\pi})\,\mu}\,\widehat{Y}_S. \tag{A.30}$$

Substituting the expressions for $\tilde{\alpha}(\overline{\pi})$ and $\tilde{\kappa}(\overline{\pi})$ gives

$$\widehat{\pi}_S = \left[1 - \frac{\mu^2\,\eta(\overline{\pi})\,b2(\overline{\pi})\,\varepsilon}{1-\beta\,\alpha(\overline{\pi})\,\mu}\right]^{-1}\left[\frac{\kappa(\overline{\pi})}{1-\beta\,\alpha(\overline{\pi})\,\mu} + \frac{\eta(\overline{\pi})\,\mu\,b(\overline{\pi})}{(1-\beta\,\alpha(\overline{\pi})\,\mu)\,(1-\mu\,b2(\overline{\pi}))}\right]\widehat{Y}_S. \tag{A.31}$$

We can rewrite the expression above as:

$$\widehat{\pi}_S = \left[\frac{1-\beta\,\alpha(\overline{\pi})\,\kappa(\overline{\pi})}{\kappa(\overline{\pi})}\,\frac{1-\mu\,\theta\,\beta\,\overline{\pi}^{\varepsilon}}{1-\beta\,\mu\,\theta\,\overline{\pi}^{\varepsilon-1}} - \mu^2\,\theta^2\,\beta^2\,\varepsilon\,\overline{\pi}^{\varepsilon}\,\frac{\overline{\pi}^{\varepsilon}-\overline{\pi}^{\varepsilon-1}}{(1-\beta\,\theta\,\overline{\pi}^{\varepsilon})\,(1-\beta\,\mu\,\theta\,\overline{\pi}^{\varepsilon-1})}\right]^{-1}\widehat{Y}_S. \tag{A.32}$$

Recall that $\varepsilon > 1$. Denote $B = \frac{\kappa(\overline{\pi})}{1-\beta\,\alpha(\overline{\pi})\,\mu}$. Then, a sufficient condition to ensure that $\partial\widehat{\pi}_t/\partial\widehat{Y}_S > 0$ is

$$\frac{\partial B}{\partial\overline{\pi}} = \frac{\partial\kappa(\overline{\pi})/\partial\overline{\pi}}{1-\beta\,\alpha(\overline{\pi})\,\mu} + \frac{\kappa(\overline{\pi})\,\beta\,\mu\partial\alpha(\overline{\pi})/\partial\overline{\pi}}{(1-\beta\,\alpha(\overline{\pi})\,\mu)^2} \geq 0 \tag{A.33}$$

Rearranging yields

$$\beta\,\alpha(\overline{\pi})\,\mu \geq \frac{-\frac{\partial\kappa(\overline{\pi})/\partial\overline{\pi}}{\kappa(\overline{\pi})}}{-\frac{\partial\kappa(\overline{\pi})/\partial\overline{\pi}}{\kappa(\overline{\pi})} + \frac{\partial\alpha(\overline{\pi})/\partial\overline{\pi}}{\alpha(\overline{\pi})}} > 0. \tag{A.34}$$

A.0.4 Proof of Proposition 2

The solutions for inflation are:

Case 1: Normal times $$\widehat{\pi}_S^{NT} = -\omega_\pi(\overline{\pi})\,\widehat{z}_S \tag{A.35}$$

Case 2: ZLB $$\widehat{\pi}_S^{ZLB} = -\omega'_\pi(\overline{\pi})\,\widehat{z}_S \tag{A.36}$$

with $\omega_\pi(\overline{\pi}) = \frac{\tilde{K}(\overline{\pi})}{1-\mu+\phi_Y+\tilde{K}(\overline{\pi})\,(\phi_\pi-\mu)}$ and $\omega'_\pi(\overline{\pi}) = \frac{\tilde{K}(\overline{\pi})}{1-\mu-\tilde{K}(\overline{\pi})\,\mu}$.

Provided that $\partial\tilde{K}(\overline{\pi})/\partial\overline{\pi} > 0$ and $\phi_\phi > 1$, it holds that $\frac{\partial\omega_\pi(\overline{\pi})}{\partial\overline{\pi}} > 0$ and $\frac{\partial\omega'_\pi(\overline{\pi})}{\partial\overline{\pi}} > 0$ that is

Case 1: Normal times $$\frac{\partial\tilde{K}(\overline{\pi})}{\partial\overline{\pi}}\left[\frac{1-\mu+\phi_Y}{1-\mu+\phi_Y+\tilde{K}(\overline{\pi})\,(\phi_\pi-\mu)}\right] > 0 \tag{A.37}$$

Case 2: ZLB $$\frac{\partial\tilde{K}(\overline{\pi})}{\partial\overline{\pi}}\left[\frac{1-\mu}{1-\mu-\tilde{K}(\overline{\pi})\,\mu}\right] > 0 \tag{A.38}$$

Note that $\frac{\partial\omega'_\pi(\overline{\pi})}{\partial\overline{\pi}} > \frac{\partial\omega_\pi(\overline{\pi})}{\partial\overline{\pi}}$ as

$$\tilde{K}(\overline{\pi})\,\phi_\pi\,(1-\mu) > 0. \tag{A.39}$$

Hence, an increase in $\overline{\pi}$ leads to a higher inflation response in the model at the ZLB.

A.0.5 Proof of Proposition 3

The solutions for output are:

$$\text{Case 1: Normal times} \qquad \widehat{Y}_S^{NT} = -\omega_Y(\overline{\pi})\widehat{z}_S \tag{A.40}$$

$$\text{Case 2: ZLB} \qquad \widehat{Y}_S^{ZLB} = -\omega_Y'(\overline{\pi})\widehat{z}_S \tag{A.41}$$

with $\omega_Y(\overline{\pi}) = \frac{1}{1-\mu+\phi_Y+\tilde{K}(\overline{\pi})(\phi_\pi-\mu)}$ and $\omega_Y'(\overline{\pi}) = \frac{1}{1-\mu-\tilde{K}(\overline{\pi})\mu}$.

Provided that $\partial\tilde{K}(\overline{\pi})/\partial\overline{\pi} > 0$ and $\phi_\phi > 1$, it holds that $\frac{\partial\omega_Y(\overline{\pi})}{\partial\overline{\pi}} < 0$ and $\frac{\partial\omega_Y'(\overline{\pi})}{\partial\overline{\pi}} > 0$ that is

$$\text{Case 1: Normal times} \qquad -\frac{\partial\tilde{K}(\overline{\pi})}{\partial\overline{\pi}}(\phi_\pi-\mu) < 0 \tag{A.42}$$

$$\text{Case 2: ZLB} \qquad \frac{\partial\tilde{K}(\overline{\pi})}{\partial\overline{\pi}}\mu > 0 \tag{A.43}$$

A.0.6 Numerical Algorithm

The solution method to solve the stochastic models follows closely the algorithm presented in Chapter 3. However, I use the Rouwenhorst (1995) method to discretize the exogenous process. The Rouwenhorst (1995) method approximates the continuous process through a finite-state Markov chain that mimics closely the underlying process. The Rouwenhorst (1995) method is an efficient method to discretize, in particular, continuous processes with very high persistence, near unit root.

This section describes the solution method for the model presented in Section 2.5. Assume the following process for the discount factor shock: $\widehat{\delta}_t = \rho\,\widehat{\delta}_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0,\sigma^2)$. The Rouwenhorst (1995) method approximates the discount factor shock through a discrete-spaced Markov chain over an evenly spaced grid $\overline{\delta}$ of length n_d for the discount factor shock. The values for the shock over the grid will be determined by the unconditional standard deviation σ. I choose $n_d = 45$ and $\sigma = 0.125/100$. The method generates the equally spaced grid $\overline{\boldsymbol{d}}$ for the discount factor shock and an $n_d \times n_d$ transition matrix for the discount factor shock. Let $\boldsymbol{\Omega}^R$ be the transition matrix. It is defined as:

$$\boldsymbol{\Omega}^R = \begin{bmatrix} \boldsymbol{\omega}^R(\overline{d}_1)' \\ \vdots \\ \boldsymbol{\omega}^R(\overline{d}_{n_d})' \end{bmatrix} \tag{A.44}$$

where $\boldsymbol{\omega}^R(\overline{d}_j)'$ is a column vector containing the transition probabilities associated with the element $\overline{d}_j$ of vector $\overline{\boldsymbol{d}}$.

In addition, define an equally spaced grid for the price dispersion variable s_t. The variable s_t is an endogenous state variable. Denote the grid for price dispersion as column vector $\overline{\boldsymbol{s}}$. I set the length of $\overline{\boldsymbol{s}}$ to $n_s = 11$. The number of total grid points is $n = n_d \times n_s$. Then, the complete grid is defined by the matrix $\overline{\boldsymbol{S}}$:

$$\overline{\mathbf{S}} = \begin{bmatrix} [\mathbf{1}_{n_d} \otimes \overline{\boldsymbol{d}}]' \\ [\overline{\boldsymbol{s}} \otimes \mathbf{1}_{(n_d)}]' \end{bmatrix}' \tag{A.45}$$

where c^s are the elements in $\boldsymbol{C}$ associated with the solution of the price dispersion variable. $\mathbf{1}_{n_l}$ is a column vector of ones with length n_l where $l \in \{d, s\}$. Let $\boldsymbol{Z} = \left[\widehat{\boldsymbol{\pi}}', \widehat{\boldsymbol{Y}}', \widehat{\boldsymbol{\psi}}', \widehat{\boldsymbol{s}}'\right]'$ be the vector of policy functions of 4 variables which are inflation, output, marginal cost and price dispersion. I approximate $\boldsymbol{Z}$ over $\overline{\boldsymbol{S}}$. Similar to the approach in Chapter 3, it is possible to represent the model as a system of difference equations over the grid $\overline{\boldsymbol{S}}$:

$$\boldsymbol{A}\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{B}E\boldsymbol{Z}(\overline{\boldsymbol{S}}) + \boldsymbol{D}\,\boldsymbol{F}\overline{\boldsymbol{S}} \tag{A.46}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{F}$ are matrices containing the model parameters and $\boldsymbol{D}$ is a vector of constants which has nonzero elements on all grid points for which the ZLB is binding. Similar to the exercise in Chapter 3, I use linear spline basis functions where the breakpoints coincide with the collocation nodes. Then, $\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{C}$ is the policy function approximations for the pre-defined grid. For the expected function, $E\boldsymbol{Z}(\overline{\boldsymbol{S}})$, the basis functions are evaluated for the matrix $\overline{\boldsymbol{S}}^{E,R}$:

$$\overline{S}^{E,R} = \begin{bmatrix} [\left(\mathbf{1}_{n_d} \otimes \overline{\boldsymbol{d}}\right) \otimes \mathbf{1}_{n_s}]' \\ [\boldsymbol{c}^s \otimes (\mathbf{1}_{n_d} \otimes \mathbf{1}_{n_d})]' \end{bmatrix}' \tag{A.47}$$

Then, the expected function, $E\boldsymbol{Z}(\overline{\boldsymbol{S}})$, can be represented as:

$$EZ(\overline{S}) = \widehat{\boldsymbol{\Omega}}^R\,\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E,R})\,\boldsymbol{C} \tag{A.48}$$

where

$$\widehat{\mathbf{\Omega}}^R = \begin{bmatrix} \tilde{\mathbf{\Omega}}^R & \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n\times n_d^2 n_s} \\ \mathbf{0}_{n\times n_d^2 n_s} & \tilde{\mathbf{\Omega}}^R & \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n\times n_d^2 n_s} \\ \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n\times nm} & \tilde{\mathbf{\Omega}}^R & \mathbf{0}_{n\times n_d^2 n_s} \\ \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n\times n_d^2 n_s} & \tilde{\mathbf{\Omega}}^R \end{bmatrix} \qquad \tilde{\mathbf{\Omega}}^R = \begin{bmatrix} \tilde{\boldsymbol{\omega}}^R & 0 & \dots & 0 \\ 0 & \tilde{\boldsymbol{\omega}}^R & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \tilde{\boldsymbol{\omega}}^R \end{bmatrix}$$

$$\tilde{\boldsymbol{\omega}}^R = \begin{bmatrix} \boldsymbol{\omega}^R(\overline{d}_1)' & 0 & \dots & 0 \\ 0 & \boldsymbol{\omega}^R(\overline{d}_2)' & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \boldsymbol{\omega}^R(\overline{d}_{n_d})' \end{bmatrix}$$

$$\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^{E,R}) = \begin{bmatrix} \overline{\boldsymbol{S}}^{E,R} & \mathbf{0}_{n_d^2 n_s\times n} & \mathbf{0}_{n_d^2 n_s\times n} & \mathbf{0}_{n_d^2 n_s\times n} \\ \mathbf{0}_{n_d^2 n_s\times n} & \overline{\boldsymbol{S}}^{E,R} & \mathbf{0}_{n\times n_d^2 n_s} & \mathbf{0}_{n_d^2 n_s\times n} \\ \mathbf{0}_{n_d^2 n_s\times n} & \mathbf{0}_{n_d^2 n_s\times n} & \overline{\boldsymbol{S}}^{E,R} & \mathbf{0}_{n_d^2 n_s\times n} \\ \mathbf{0}_{n_d^2 n_s\times n} & \mathbf{0}_{n_d^2 n_s\times n} & \mathbf{0}_{n_d^2 n_s\times n} & \overline{\boldsymbol{S}}^{E,R} \end{bmatrix} \qquad \mathbf{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}}) = \begin{bmatrix} \phi_1(\overline{\boldsymbol{S}}^E_{(1,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}^E_{(1,:)}) \\ \vdots & \dots & \vdots \\ \phi_1(\overline{\boldsymbol{S}}^E_{(n_d^2 n_s,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}^E_{(n_d^2 n_s,:)}) \end{bmatrix}$$

$\widehat{\mathbf{\Omega}}^R$ is of dimension $4n \times 4n_d^2 n_s$, $\tilde{\mathbf{\Omega}}^R$ is $n \times n_d^2 n_s$, $\tilde{\boldsymbol{\omega}}^R$ is $n_d \times n_d^2$, $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^{E,R})$ is $4n_d^2 n_s \times 4n$ and $\mathbf{\Phi}(\overline{\boldsymbol{S}}^{E,R})$ is $n_d^2 n_s \times n$.

In matrix notation, the system of difference equations over the grid $\overline{\mathbf{S}}$ is given by:

$$\boldsymbol{A}\,\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{B}\,E\boldsymbol{Z}(\overline{\boldsymbol{S}}) + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}}. \tag{A.49}$$

Or equivalently,

$$\boldsymbol{A}\boldsymbol{C} = \boldsymbol{B}\widehat{\mathbf{\Omega}}^R\,\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^{E,R})\boldsymbol{C} + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}} \tag{A.50}$$

I use the following iterative approach to find the coefficient vector $\boldsymbol{C}$:

1. Start with a guess $\boldsymbol{C}^0$. Compute $\overline{\boldsymbol{S}}^{E,R}$, $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^{E,R})$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^0$.

2. Given $\overline{\boldsymbol{S}}^{E,R}$, $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^{E,R})$, $\boldsymbol{A}$ and $\boldsymbol{D}$, solve for $\boldsymbol{C}^{\text{new}}$:

$$\boldsymbol{C}^{\text{new}} = [\boldsymbol{A} - \boldsymbol{B}\widehat{\mathbf{\Omega}}^{\boldsymbol{R}}\,\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^{E,R})]^{-1}[\boldsymbol{D} + \boldsymbol{F}\,\overline{\boldsymbol{S}}] \tag{A.51}$$

3. Update $\boldsymbol{C}^1 = \lambda \boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}^{E,R}$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E,R})$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

Appendix B

Appendix to Chapter 3

B.0.1 Model Equations

Below I provide the log-linearized equilibrium conditions of the benchmark model and the two-sector model. The equilibrium equations are log-linearized around a zero inflation steady state. Define $\Pi_{S,t} = 1 + \pi_{S,t}$ as the gross inflation rate in sector S.

The Benchmark Model

Euler equation (non-durables): $$\widehat{C}_t - \frac{h_C}{1+h_C}\widehat{C}_{t-1} = \frac{1}{1+\sigma_C} E_t \widehat{C}_{t+1} - \frac{1}{1-h_C^2}\left(\widehat{i}_t - E_t \widehat{\Pi}_{C,t+1} + \widehat{\mu}_{C,t}\right) \tag{B.1}$$

Wage inflation: $$\widehat{\Pi}_{w,t} = \widehat{w}_t - \widehat{w}_{t-1} + \widehat{\Pi}_{C,t} \tag{B.2}$$

Wage Phillips Curve: $$\widehat{\Pi}_{w,t} = \beta E_t \widehat{\Pi}_{w,t+1} + \kappa_w \left(\phi \widehat{N}_t + \frac{1}{1-h_C}\left(\widehat{C}_t - h_C \widehat{C}_{t-1}\right) - \widehat{w}_t\right)$$
$$\text{with } \kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varepsilon_w \phi)\theta_w} \tag{B.3}$$

Non-durables production: $$\widehat{Y}_{C,t} = \widehat{N}_{C,t} \tag{B.4}$$

Non-durables market clearing: $$\widehat{Y}_{C,t} = \widehat{C}_t \tag{B.5}$$

Phillips curve (non-durables): $$\widehat{\Pi}_{C,t} = \beta E_t \widehat{\Pi}_{C,t+1} + \kappa_C \left(\widehat{W}_t + \frac{\overline{\tau}}{1+\overline{\tau}}\widehat{\tau}_t\right)$$
$$\text{with } \kappa_C = \frac{(1-\theta_C)(1-\beta\theta_C)}{\theta_C} \tag{B.6}$$

Taylor rule: $$\widehat{i}_t = \max\left\{\ln\beta, \gamma_\pi \widehat{\Pi}_{C,t}\right\} \tag{B.7}$$

Taste shock (non-durables): $$\widehat{\mu}_{C,t} = \rho_C \widehat{\mu}_{C,t-1} + \varepsilon_t \tag{B.8}$$

VAT shock: $$\widehat{\tau}_t = \varepsilon_t^{\tau} \tag{B.9}$$

The Two-Sector Model

Euler equation (non-durables): $$\widehat{C}_t - \frac{h_C}{1+h_C}\widehat{C}_{t-1} = \frac{1}{1+h_C} E_t \widehat{C}_{t+1} - \frac{1}{1-h_C^2}\left(\widehat{i}_t - E_t \widehat{\Pi}_{C,t+1} + \widehat{\mu}_{C,t}\right) \tag{B.10}$$

Demand for durables: $$\widehat{q}_t - \frac{1}{1-h_C}\left(\widehat{C}_t - h_C\widehat{C}_{t-1}\right) = -\frac{1-h_D-\beta(1-\delta)}{1-h_D}\left(\widehat{D}_t - h_D\widehat{D}_{t-1}\right) + \beta(1-\delta)E_t\left\{\widehat{q}_{t+1} - \widehat{C}_{t+1}\right\} + \widehat{\nu}_{D,t} \tag{B.11}$$

Wage inflation: $$\widehat{\Pi}_{w,t} = \widehat{w}_t - \widehat{w}_{t-1} + \widehat{\Pi}_{C,t} \tag{B.12}$$

Wage Phillips Curve: $$\widehat{\Pi}_{w,t} = \beta E_t \widehat{\Pi}_{w,t+1} + \kappa_w\left(\phi \widehat{N}_t + \frac{1}{1-h_C}\left(\widehat{C}_t - h_C\widehat{C}_{t-1}\right) - \widehat{w}_t\right)$$
$$\text{with } \kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{(1+\varepsilon_w\phi)\theta_w} \tag{B.13}$$

Non-durables production: $$\widehat{Y}_{C,t} = \widehat{N}_{C,t} \tag{B.14}$$

Durables production: $$\widehat{Y}_{D,t} = \widehat{N}_{D,t} \tag{B.15}$$

Relative price: $$\widehat{q}_t = \widehat{\Pi}_{D,t} - \widehat{\Pi}_{C,t} + \widehat{q}_{t-1} \tag{B.16}$$

Non-durables market clearing: $$\widehat{Y}_{C,t} = \widehat{C}_t \tag{B.17}$$

Durables market clearing $$\widehat{Y}_{D,t} = \frac{1}{\delta}\left(\widehat{D}_t - (1-\delta)\widehat{D}_{t-1}\right) \tag{B.18}$$

Labor market clearing: $$\widehat{N}_t = (1-\alpha)\widehat{N}_{C,t} + \alpha\widehat{N}_{D,t} \tag{B.19}$$

Phillips curve (non-durables): $$\widehat{\Pi}_{C,t} = \beta E_t \widehat{\Pi}_{C,t+1} + \kappa_C\left(\widehat{w}_t + \frac{\overline{\tau}}{1+\overline{\tau}}\widehat{\tau}_t\right)$$
$$\text{with } \kappa_C = \frac{(1-\theta_C)(1-\beta\theta_C)}{\theta_C} \tag{B.20}$$

Phillips curve (durables): $$\widehat{\Pi}_{D,t} = \beta E_t \widehat{\Pi}_{D,t+1} + \kappa_D\left(\widehat{w}_t + \frac{\overline{\tau}}{1+\overline{\tau}}\widehat{\tau}_t\right)$$
$$\text{with } \kappa_D = \frac{(1-\theta_D)(1-\beta\theta_D)}{\theta_D} \tag{B.21}$$

Taylor rule: $$\widehat{i}_t = \max\left\{\ln\beta, \gamma_\pi\left[(1-\alpha)\widehat{\Pi}_{C,t} + \alpha\widehat{\Pi}_{D,t}\right]\right\} \tag{B.22}$$

Preference shock (non-durables): $$\widehat{\mu}_{C,t} = \rho_C\widehat{\mu}_{C,t-1} + \varepsilon_t \tag{B.23}$$

Preference shock (durables): $$\widehat{\nu}_{D,t} = \widehat{\mu}_{C,t} \tag{B.24}$$

VAT shock: $$\widehat{\tau}_t = \varepsilon_t^\tau \tag{B.25}$$

B.0.2 Numerical Algorithm

The solution method to solve the stochastic models follows closely the algorithm presented in Chapter 3. The semi-loglinear models can be represented as a system of difference equations over the grid $\overline{\mathbf{S}}$:

$$\boldsymbol{A}\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{B}E\boldsymbol{Z}(\overline{\boldsymbol{S}}) + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}}. \tag{B.26}$$

Similar to the exercise in Chapter 3, I use linear spline basis functions where the breakpoints coincide with the collocation nodes. Then, $\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{C}$ is the policy function approximations for the pre-defined grid. For the expected functions, the basis functions ϕ_i are evaluated for the matrix $\overline{\boldsymbol{S}}^E$ which contains the expected values of all state variables. Then, the system of equations can be written as

$$\boldsymbol{A}\boldsymbol{C} = \boldsymbol{B}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)\boldsymbol{C} + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}} \tag{B.27}$$

where $\widehat{\boldsymbol{\Omega}}$ contains the Gaussian quadrature weights and matrix $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$ contains the basis functions evaluated for $\overline{\boldsymbol{S}}^E$.

The VAT cut can be interpreted as a deterministic shock. Assuming that the VAT cut is perfectly credible, households and firms are able to foresee the evolution of the VAT rate perfectly. In Section 3.5.2, I assume that the VAT cut lasts for 4 periods. I propose an iterative scheme to solve for the policy functions for each of the 4 periods of the VAT cut. In the last period of the VAT cut in $t = 4$, the grid for the shock associated with the VAT has values 0 or 1. 0 means that the VAT cut has reversed to steady state while 1 implies that the VAT cut is active. However, in period $t = 4$, we perfectly know that the VAT cut will be over in the next period. Hence, the expected functions of the model will be evaluated at those points of the grid for which the VAT cut is inactive. I use the following iterative approach to find the policy function $\boldsymbol{C}_4$ in period $t = 4$:

1. Start with a guess $\boldsymbol{C}_4^0$. Compute $\overline{\boldsymbol{S}}_4^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_4^E)$, $\boldsymbol{A}_4$ and $\boldsymbol{D}_4$ based on $\boldsymbol{C}_4^0$.

2. Given $\overline{\boldsymbol{S}}_4^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_4^E)$, $\boldsymbol{A}_4$ and $\boldsymbol{D}_4$, solve for $\boldsymbol{C}_4^{\text{new}}$:

$$\boldsymbol{C}_4^{\text{new}} = [\boldsymbol{A}_4 - \boldsymbol{B}_4\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_4^E)]^{-1}[\boldsymbol{D}_4 + \boldsymbol{F}\,\overline{\boldsymbol{S}}_4] \tag{B.28}$$

3. Update $\boldsymbol{C}_4^1 = \lambda \boldsymbol{C}_4^0 + (1-\lambda)\boldsymbol{C}_4^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}_4^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_4^E)$, $\boldsymbol{A}_4$ and $\boldsymbol{D}_4$ based on $\boldsymbol{C}_4^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}_4^s - \boldsymbol{C}_4^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

In period $t = 3$, agents correctly anticipate that the VAT cut will continue to be active in the next period. Hence, the grid for the shock associated with the VAT only has value 1. Given the solution $\boldsymbol{C}_4$, it is now possible to solve for the policy function in period $t = 3$. The system of equations in period $t = 3$ is given by:

$$\boldsymbol{A}_3 \boldsymbol{C}_3 = \boldsymbol{B} \widehat{\boldsymbol{\Omega}} \widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_3^E) \boldsymbol{C}_4 + \boldsymbol{D}_3 + \boldsymbol{F}\, \overline{\mathbf{S}}_3 \tag{B.29}$$

I solve for $\boldsymbol{C}_3$ using an iterative approach given $\boldsymbol{C}_4$:

1. Start with a guess $\boldsymbol{C}_3^0$. Compute $\overline{\boldsymbol{S}}_3^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_3^E)$, $\boldsymbol{A}_3$ and $\boldsymbol{D}_3$ based on $\boldsymbol{C}_3^0$.

2. Given $\overline{\boldsymbol{S}}_3^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_3^E)$, $\boldsymbol{A}_3$ and $\boldsymbol{D}_3$, solve for $\boldsymbol{C}_3^{\text{new}}$:

$$\boldsymbol{C}_4^{\text{new}} = \boldsymbol{A}_4^{-1} [\boldsymbol{B}_3 \widehat{\boldsymbol{\Omega}} \widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_3^E) \boldsymbol{C}_4 + \boldsymbol{D}_3 + \boldsymbol{F}\, \overline{\boldsymbol{S}}_3] \tag{B.30}$$

3. Update $\boldsymbol{C}_3^1 = \lambda \boldsymbol{C}_3^0 + (1-\lambda)\boldsymbol{C}_3^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}_3^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}_3^E)$, $\boldsymbol{A}_3$ and $\boldsymbol{D}_3$ based on $\boldsymbol{C}_3^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}_3^s - \boldsymbol{C}_3^{s-1})|| < z$ with $z = 10^{-8}$ or $s >= 300$.

Equivalently, we can find the solution for period $t = 2$ given $\boldsymbol{C}_3$ and the solution for period $t = 1$ given $\boldsymbol{C}_2$.

The two-sector models has in total 6 state variables: preference shock, VAT shock, non-durable goods consumption, stock of durable goods, real wage and the relative price. For this model, I assume the following numbers of breakpoints: $n_\mu = 15$ (preference shock), $n_\tau = 2$ (VAT shock), $n_c = 5$ (non-durable goods consumption), $n_d = 5$ (stock of durable goods) , $n_w = 8$ (real wage) and $n_q = 6$ (relative price). Therefore, the total grid size is $n = 36,000$. Furthermore, the grid points are unequally distributed for $\overline{\boldsymbol{c}}$ (non-durable goods consumption), $\overline{\boldsymbol{d}}$ (stock of durable goods), $\overline{\boldsymbol{w}}$ (real wage) and $\overline{\boldsymbol{q}}$ (relative price) with twice as many grid points for negative values. Finally, I set the number of quadrature nodes to $m = 10$.

B.0.3 Data

Price data is taken from Eurostat[1] and is disaggregated for individual good categories according to the Classification of Individual Consumption by Purpose (COICOP).[2] Prices used in the CPI calculation includes taxes such as the VAT. All data is provided on a monthly basis.

I exploit the detailed classification of household consumption to split the data into non-VAT goods (goods that do not attract VAT or the reduced rate of VAT, e.g. food, energy or education) and VAT goods (goods that attract the standard VAT rate) according to HMRC guidelines.[3] In the regression analysis, I restrict attention to VAT goods. Furthermore, I aggregate the goods into 2 main consumption categories: non-durable goods (non-durables and services) and durable consumption goods (semi-durables and durables).[4]

Data on household consumption is available only on quarterly or annual basis. To account for consumer purchases on a higher frequency, I use retail sales as a proxy. Information on retail trade and sales is available on a monthly basis and is taken from the statistical offices of every country in the sample. In particular, I use the (nominal) turnover index for retail sales with base year 2005. The index is provided for a series of retail categories that allows to construct an aggregate index for turnover on non-durables, semi-durables, durables and services. The sample covers data from January 2002 to December 2010.[6]

[1]Eurostat uses the data to compute the Harmonized Index of Consumer Prices (http://ec.europa.eu/eurostat/data/database).

[2]See for further information http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=5.

[3]For details, see http://www.hmrc.gov.uk/vat/forms-rates/rates/goods-services.htm.

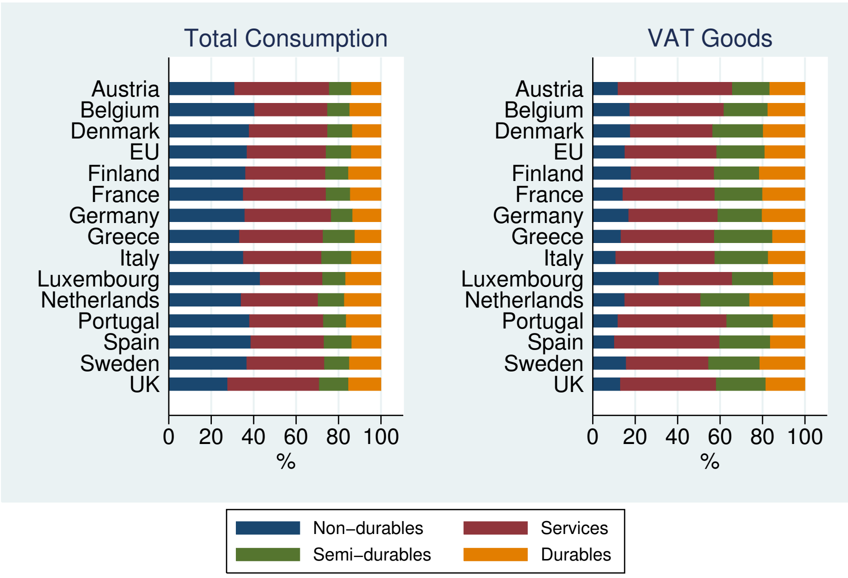
[4]I use Eurostat's data on item weights for the Harmonized Index of Consumer Prices to aggregate the individual data. The item weights are used to compute a price index for a representative consumption basket. Item weights are updated every year by Eurostat. The official definition for non-durable goods, semi-durable goods, durable goods and services are taken from the OECD. According to their definition, a non-durable good is "one that is used up entirely after less than a year while semi-durables can be used repeatedly or continuously over a period longer than a year"[5]. The latter differs from a durable good in its expected lifetime which is still significantly shorter and its price is substantially lower in most cases.

[6]In 2011 the VAT rate was permanently increased to 20% in the UK.

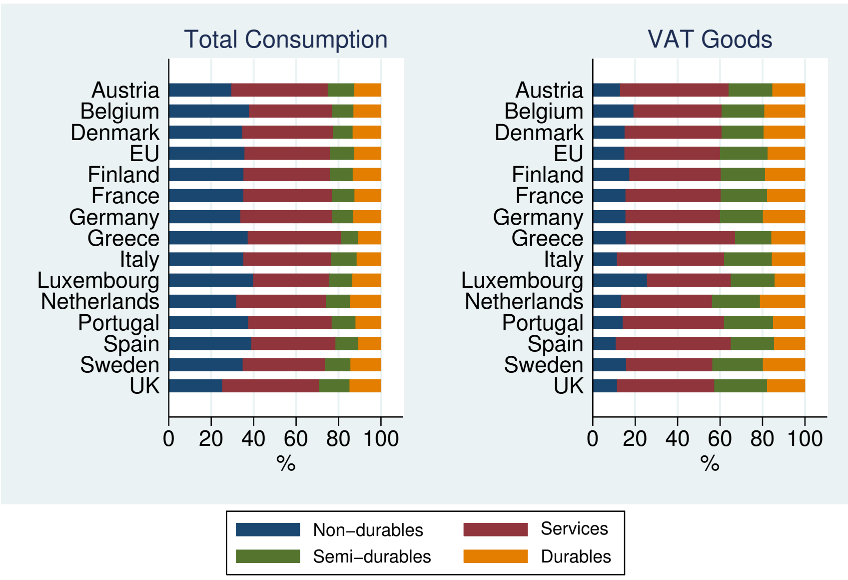
B.0.4 Figures

Fig. B.1 Representative Consumption Basket of Goods and Services for Selected Countries and EU in 2008 and in 2017 (left: all goods and services; right: goods and services that are subject to standard VAT rate)

(a) 2008



(b) 2017



Appendix C

Appendix to Chapter 4

C.0.1 Implementation of the Algorithm and MATLAB Routines

All of the routines required to implement the algorithm are written in MATLAB. All codes are available at https://tinyurl.com/Budianto-CollocationCodes. To compute the basis functions $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}})$, I use the functions `fspace.m`, `funnode.m` and `funbas.m` provided in the CompEcon Toolbox developed by Miranda and Fackler (2002). The toolbox can be downloaded at https://pfackler.wordpress.ncsu.edu/compecon/154-2/.

Note that the log-linearized model with n exogenous state variables and m endogenous state variables can be rewritten as a system of linear difference equations over the pre-defined grid:

$$\boldsymbol{A}\,\boldsymbol{C} = \boldsymbol{B}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E})\,\boldsymbol{C} + \boldsymbol{D} + \boldsymbol{F}\,\overline{\mathbf{S}} \tag{C.1}$$

In order to find the policy function approximations $\boldsymbol{C}$, I use the following iterative approach:

1. Start with a guess $\boldsymbol{C}^0$. Compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^0$.

2. Given $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$, solve for $\boldsymbol{C}^{\text{new}}$:

$$\boldsymbol{C}^{\text{new}} = [\boldsymbol{A} - \boldsymbol{B}\widehat{\boldsymbol{\Omega}}\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^{E})]^{-1}[\boldsymbol{D} + \boldsymbol{F}\,\overline{S}] \tag{C.2}$$

3. Update $\boldsymbol{C}^1 = \lambda\boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ where λ is between 0 and 1. Then, compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{A}$ and $\boldsymbol{D}$ based on $\boldsymbol{C}^1$.

4. Redo the previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < \delta$. with $\delta = 10^{-8}$ or $s >= 300$.

The algorithm is broken down into a sequence of easily implementable steps that are executed with the following set of functions:

- `mainscript.m` - The main script that executes all functions and saves the results in the structure `M`.
- `vars.m` - Outputs a structure `SV`, containing specifications of the names of the model variables.
- `sstate.m` - Outputs a structure `SZ`, containing spefications of the state variables, e.g. names, number of break points or boundaries of the grid.
- `xcoll.m` - Outputs a structure `GZ`, containing the matrices $\overline{S}$, $\overline{d}^{\boldsymbol{E}}$ and the Gaussian-Hermite quadrature weights $\boldsymbol{\omega}$.
- `params.m` - Outputs a structure `Params`, containing the parameter matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{D}$ and $\boldsymbol{F}$ for the initial step.
- `xiter.m` -Outputs the structure `D`, containing specifications for the iteration approach, e.g. number of maximal iteration steps or convergence criteria.

  The algorithm starts by specifying the initial $\boldsymbol{C}^0$ and $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$. By default, $\boldsymbol{C}^0$ is a zero vector and $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$ is evaluated at the grid points. Depending on the size of the grid and the number of state variables, computing $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$ can be fairly involved. To speed up the computation and avoid storing large matrices, I break down $\overline{S}^E$ into n/n_d matrices of size $n_d m \times 2$ and parallelize the computation of the basis functions. This is performed with `collmat2.m`.

C.0.2 Solution Times Using Dual-Core (2.4 GHz) Processors

Table C.1 reports solution times for the different models computed with a 2-core processor (2.4 GHz) using MATLAB's Parallel Computing Toolbox.

Table C.1 Solution Times for Selected Models (in Seconds)

		DF	DF and M	DF, M and MP
	Processors	$n_d = 20$	$n_d = n_m = 20$	$n_d = n_m = n_{mp} = 10$
0 lagged	1	0.0461	0.4915	9.8548
		$n_d = 20$, $m_y = 10$	$n_d = n_m = 20$, $m_y = 10$	$n_d = n_m = n_{mp} = 7$, $m_y = 5$
1 lagged	1	0.7457	55.7491	628.3464
	2	2.2899	47.2253	489.9326
		$n_d = 20$, $m_y = m_w = 10$	$n_d = n_m = 10$, $m_y = m_w = 5$	$n_d = n_m = n_{mp} = 5$, $m_y = m_w = 5$
2 lagged	1	55.1337	225.0477	745.0218
	2	57.8905	227.3864	618.8270

Note: DF: discount factor shock, M: markup shock, MP: monetary policy shock, 0 lagged: no endogenous state variables, 1 lagged: consumption habit formation, 2 lagged: consumption habit and wage rigidities. n_d, n_m, n_{mp}: no. of grid points for discount factor shock, markup shock and monetary policy shock. m_y, m_w: no. of gridpoints for output and real wage. I use 10 quadrature nodes for the numerical integration in all models.

C.0.3 Euler Equation Error Intergrals for the New Keynesian Model without ZLB

Table C.2 reports Euler equation error integrals for the New Keynesian model with consumption habit formation ($h = 0.5$) and sticky wages ($\theta_w = 0.75$). I assume no ZLB constraint on the nominal interest rate. The integrals were computed over the Euler equation errors in every period based on a simulation of the models for 10,000 periods.

Table C.2 Euler Equation Error Integrals (in base 10 logarithms)

		$\sigma = 0.05/100$			$\sigma = 0.1/100$			$\sigma = 0.2/100$		
	$n =$	10	20	50	10	20	50	10	20	50
Euler Equation	**DF**	-22.51	-22.89	-23.24	-22.50	-22.65	-22.94	-22.26	-22.02	-22.05
	M	-19.87	-20.10	-20.65	-18.85	-19.20	-19.55	-18.79	-19.12	-19.48
	MP	-22.33	-22.84	-23.11	-19.03	-19.02	-19.64	-18.92	-19.24	-19.62
Phillips Curve	**DF**	-24.95	-25.29	-25.60	-24.66	-24.76	-24.99	-24.31	-24.06	-24.07
	M	-22.43	-22.66	-23.18	-21.59	-21.96	-22.32	-21.56	-21.91	-22.31
	MP	-24.86	-25.37	-25.65	-21.74	-21.73	-22.35	-21.65	-21.95	-22.33

Note: DF: discount factor shock, M: markup shock, MP: monetary policy shock, σ: unconditional standard deviation of respective shock, n: number of grid points for respective shock.