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A study on flow decomposition methods for scheduling of electric buses in public transport based on aggregated time–space network models

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Abstract

Over the past few years, many public transport companies have launched pilot projects testing the operation of electric buses. The basic objective of these projects is to substitute diesel buses with electric buses within the companies' daily operations. Despite an extensive media coverage, the share of electric buses deployed still remains very small in practice. In this context, new challenges arise for a company's planning process due to the considerably shorter ranges of electric buses compared to traditional combustion engine buses and to the necessity to recharge their batteries at charging stations. Vehicle scheduling, an essential planning task within the planning process, is especially affected by these additional challenges. In this paper, we define the *mixed fleet vehicle scheduling problem with electric vehicles*. We extend the traditional vehicle scheduling problem by considering a mixed fleet consisting of electric buses with limited driving ranges and rechargeable batteries as well as traditional diesel buses without such range limitations. To solve the problem, we introduce a three-phase solution approach based on an aggregated time–space network consisting of an exact solution method for the vehicle scheduling problem without range limitations, innovative flow decomposition methods, and a novel algorithm for the consideration of charging procedures. Through a computational study using real-world bus timetables, we show that our solution approach meets the requirements of a first application of electric buses in practice. Since the employment of electric buses is mainly influenced by the availability of charging infrastructure, which is determined by the distribution of charging stations within the route network, we particularly focus on the influence of the charging infrastructure.

Keywords Electric vehicles · Vehicle scheduling · Public transport · Time–space network · Flow decomposition

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1 Introduction

Scheduling a fleet of vehicles is an essential task within the planning process of public transport companies. The mathematical optimization problem that arises from this task is widely known as the *Vehicle Scheduling Problem* (VSP). The objective is to determine the assignment of a company's vehicles to a set of timetabled service trips at minimum cost. In general, the costs consist of fixed costs for the acquisition of the buses used and costs for the buses' operation. Service trips denote trips for transporting passengers from a departure stop to an arrival stop at specific times. A vehicle can also perform deadhead trips without passengers in order to change its location. The set of all trips executed by a vehicle is denoted as its rotation. Vehicle rotations need to satisfy some basic constraints. (1) The trips of a vehicle rotation must be mutually compatible, that is, the trips have to be executable without time overlaps. (2) Every trip is covered exactly once, and (3) a vehicle begins and ends its rotation at one specific depot. Depending on the number of depots, the resulting problem is denoted as the *Single* or *Multi Depot Vehicle Scheduling Problem*. Moreover, multiple vehicle types may be considered (cf. Ferland and Michelon 1988). The VSP and its extensions are well studied problems in the research community and have been widely analyzed (cf. Bertossi et al. 1987; Daduna et al. 1995 or Bunte and Kliewer 2009).

Driven by the social and political trend towards sustainable management of resources and the subsequent rejection of fossil energy sources in favor of renewable energies, the importance of alternative engines in urban traffic and public transportation has increased strongly. Electric vehicles (EVs) occupy a special position within the range of vehicles with alternative engines, since they have numerous important advantages. First, electric engines have a much higher degree of efficiency compared to combustion engines. Second, EVs are locally emission-free, which means that almost no greenhouse gases, fine particles, and nitrogen oxides are being emitted during their operation. Nowadays, where thresholds for these emissions are largely exceeded, especially in urban areas, the use of EVs represents a key factor in order to reduce the negative effects on public health. Furthermore, electric buses enable a significant reduction of noise, which is especially important for urban areas (cf. Schallaböck 2012).

Currently, three main different types of EVs exist: (I) fuel cell electric vehicles containing an electric engine as well as a fuel cell, which generates electric energy directly from hydrogen or methanol, (II) hybrid electric vehicles containing an electric engine and a traditional combustion engine, which can be switched on when required, and (III) battery electric vehicles (BEV), which merely contain an electric engine. The latter type of vehicle has the shortest range of the aforementioned vehicle types, because no additional engines can be switched on. The last two vehicle types contain a battery to store the electric energy needed for powering their engines. In this paper we consider BEVs, since this type of vehicle implies the strongest restrictions for vehicle scheduling.

To compensate for their range limitations, BEVs perform detours to charging stations during their operations in order to recharge their batteries. There are three main different options for this. First, a vehicle battery can be recharged overnight during longer idle times at the depot. Second, a battery can be recharged during smaller breaks within a vehicle's operation, which is called opportunity charging. Lastly, a vehicle battery can be swapped for a fully charged battery. Depending on the charging option and the waiting time at a charging station, a battery can be fully or partially charged. In this context, the current of a charging system is particularly important because it determines the charging time. Different charging technologies are available for transferring energy into the batteries. Nowadays, this transfer is mainly performed either by a wire (conductively) or inductively.

Many companies have launched pilot projects testing the operation of electric buses during the provision of their services. For example, the German cities of Munich, Leipzig, and Dresden started in 2009 with deploying hybrid electric buses.[1] In 2011, the first BEVs started operations in Germany within the public transport system of Osnabrück. Since 2015, the Berliner Verkehrsbetriebe (BVG) is carrying out the pilot project *E-Bus Berlin*[2] whereby BEVs operate on a single line in the city center of Berlin. An extension to include other bus lines is being considered. The buses used are partially charged by inductive charging systems at intermediate stops on service trips. To oppose battery aging effects, the vehicle batteries are charged conductively up to 70% of their capacities at terminal stations (cf. Millner 2020; Pelletier et al. 2017).

As things stand, companies in public transportation face considerable challenges when deploying electric buses for their daily services. Electric buses have much shorter ranges compared to traditional diesel buses due to their restricted battery capacities, and they need to make detours to charging stations to recharge their batteries in order to overcome this disadvantage (cf. Wang et al. 2016). Within the pilot project *E-Bus Berlin*, electric buses (Solaris Urbino 12 electric), equipped with a lithium-ion-battery capable of storing 90 kWh, are deployed. Assuming consumptions of about 1.5–1.8 kWh (depending on several influencing factors), this results in a range of approximately 54 km.[3] The same bus type with a traditional diesel engine (Solaris Urbino 12) is able to cover a distance of about 450 km. Another challenge of electric buses is the significant increase in costs for their deployment. The reasons for this are the additional need for vehicles due to their lower ranges, high acquisition costs due to high battery costs, and necessary charging stations within the route network (cf. Pihlatie et al. 2014). According to a study by *Transport & Environment*[4] from 2018, the acquisition costs for a BEV are approximately 60% higher than the traditional combustion engine alternative. For that reason, the electrification of public transport systems still remains a very slow, gradual process. It is presumed that the proportion of BEVs will increase in the future. Accordingly, companies in public transport must nowadays deploy a fleet of vehicles consisting of both combustion engine vehicles and BEVs for their daily operations.

In this paper, we introduce a three-phase solution approach based on an aggregated time–space network (TSN) for scheduling a mixed fleet of vehicles consisting of BEVs

[1] https://www.xn--starterset-elektromobilitt-4hc.de/content/1-Bausteine/5-OEPNV/2016-projektuebersicht-20152016-hybrid-und-elektrobusprojekte-in-deutschland.pdf [Online accessed on 19-March-2020, in German].

[2] https://www.mpm.tu-berlin.de/menue/forschung/projekte/e_bus_berlin [Online accessed on 16-March-2020, in German].

[3] https://www.bsvg.net/fileadmin/user_upload/downloads/Emil/Datenblatt_E12.pdf [Online accessed on 22-March-2020, in German].

[4] https://www.euractiv.com/wp-content/uploads/sites/2/2018/11/2018_11_electric_bus_paper_final.pdf [Online accessed on 24-March-2020].

with limited driving ranges and traditional combustion engine vehicles without range restrictions. To do so, we define the *mixed fleet vehicle scheduling problem with electric vehicles* (MF-(E)VSP) as an extension of the traditional VSP. The solution approach consists of an exact solution method for the VSP without range limitations, based on a TSN in the form of a mixed-integer linear program, followed by a second phase, in which limited driving ranges will be taken into account by applying innovative flow decomposition methods, and a third phase in which charging procedures are inserted into the vehicle rotations. The approach aims at maximizing the proportion of feasible vehicle rotations for BEVs within the full set of vehicle rotations while retaining optimal numbers of vehicles used and deadhead trips required. The numbers of vehicles used and deadhead trips are obtained by solving the standard VSP without range limitations. Vehicle rotations that are infeasible for BEVs continue to be served by traditional combustion engine vehicles. The TSN based solution method has been proven as highly efficient and has already been used for real-world applications. Since the charging infrastructure has a significant influence on the deployment of BEVs, we also analyse the impact of different settings on generated solutions. With this in mind, the experiments conducted and their results may help to speed up the switch from combustion engine to BEVs in public transport.

The paper is organized as follows: in Sect. 2 we present related literature before defining the MF-(E)VSP (Sect. 3). Then, we introduce the three-phase solution approach based on an aggregated time–space network in Sect. 4. In Sect. 5 we perform a computational study and evaluate the solution approach with regard to proportions of applicable BEVs and changes in the charging infrastructure. Concluding this paper, Sect. 6 provides a summary and a prospect for further research.

2 E-VSP and related problems in the literature

In the following, we provide an overview of related literature. There is a wide range of literature dealing with vehicle scheduling for public transport. For an overview, we refer to Bunte and Kliewer (2009). With regard to the contribution of this paper, solution approaches addressing the deployment of EVs are especially relevant. In recent years, a variety of optimization problems have been introduced that incorporate limited driving ranges of the vehicles used and the possibility to restore their ranges. The literature presented below mainly differ in their way of incorporating the additional restrictions caused by the deployment of EVs as well as the level of reality they reflect regarding electric issues.

First, Desrosiers et al. (1995) and Haghani and Banihashemi (2002) introduced the *Time Window Constraint Scheduling Problem* as an extension of the traditional VSP by restricting the lengths and durations of vehicle rotations. The authors use a definition of the VSP from Bodin et al. (1978). For this purpose, they added constraints to the problem formulation that restrict fuel consumption of the vehicles deployed. However, the authors neglected the possibility to recharge a vehicle's battery at some charging stations within its rotation. The authors present exact and heuristic solution methods. In order to solve even larger-scale instances, they propose techniques for decreasing the problem size. Wang and Shen (2007) defined the *Vehicle Scheduling*

Problem with Route and Fueling Time Constraints as a first approach to incorporate both vehicles' limited ranges and the option to recharge a battery. They develop a heuristic solution method that incorporates route time constraints and finds vehicle rotations starting and ending at the depot. Subsequently, they use a bipartite graph model to connect these rotations in relation to fuel time restrictions. In general, the term *Electric Vehicle Scheduling Problem* (E-VSP) has been established when considering both limited driving ranges of vehicles and the opportunity to recharge their batteries at specific charging stations. Li (2013) proposed the VSP with limited energy using time-expanded station nodes, thus considering the possibility to recharge and the capacities of charging stations. The author presents a construction heuristic producing vehicle rotations which serve as initial solutions for different column generation based solution approaches. Chao and Xiaohong (2013) proposed a heuristic method based on a Non-dominated Sorting Genetic Algorithm (NSGA-II) which they tested on a real-world instance with 119 service trips. They aim at minimizing vehicle costs as well as total charging demand. Besides a limited range, the authors consider the possibility of swapping a vehicle's battery. After the removal, a fully charged battery is inserted. Adler and Mirchandani (2016) presented a column-generation approach for the E-VSP. In order to obtain initial solutions for the solution method the concurrent scheduler algorithm by Bodin et al. (1978) is extended to take into account the additional restrictions caused by BEVs. The solution method is tested on real-world instances with up to 4000 service trips.

All of the solution approaches discussed have in common that charging processes are performed within constant time windows. The assumption of constant time windows for charging implies that vehicles remain idle at a charging station for a fixed time period, whether or not the vehicle batteries have already been fully charged. This assumption leads to a substantial simplification because the actual charging process of modern batteries is very complex (Montoya et al. 2017). As a first solution towards a more realistic reflection of battery charging processes, van Kooten Niekerk et al. (2017) developed a column-generation approach, which considers partial chargings in linear time in order to adapt this aspect. Linear time windows for charging refer to a linear increase in energy depending on the waiting time of a vehicle at a charging station. In technical terms, this means that vehicle batteries are charged with a constant current during the entire charging process (Olsen and Kliewer 2020). Janoveca and Kohnia (2019) presented an exact solution model for the E-VSP based on a mixed-integer linear program. For solving, they use standard optimization software libraries. Regarding technical aspects, they also consider linear charging times of the vehicle batteries. Yao et al. (2020) proposed a heuristic solution method based on a genetic algorithm for the E-VSP with multiple vehicle types. They analyse the impact of different driving ranges, recharging durations, and energy consumptions of vehicles on the solution quality. Even though the authors consider a significant higher level of technical characteristics in comparison to previous work, they also assume that chargings are performed in linear time. Regarding further literature, there is no work at all dealing with the impact of different scenarios of the charging infrastructure on resulting vehicle rotations. Furthermore, homogeneous vehicle fleets basically consisting of only one major type of propulsion are assumed. Within the solution approach presented in this paper, we consider a heterogeneous fleet of vehicles, apply linear time windows for battery charging, and evaluate different settings of the charging infrastructure to point out interrelations.

3 Problem description

In this section, we introduce the MF-(E)VSP as the essential problem of this paper and present the TSN based solution approach together with methods for flow decomposition.

The objective of the traditional VSP is to assign a given set of timetabled service trips to a set of vehicles at minimum costs while satisfying the following constraints:

- each service trip is assigned exactly once,
- each vehicle starts and ends its rotation at the same depot,
- each vehicle rotation contains a feasible sequence of trips.

A vehicle rotation represents a sequence of trips that a vehicle executes consecutively. The trips may be pull-out or pull-in trips from or to the depot, deadhead trips, and service trips. The public transportation network is assumed to be given by a set of stop points including the vehicle depots. Each service trip is defined precisely by its departure time, arrival time, departure stop, and arrival stop. Distances and travel times between any two stop points in the network are each given by a matrix. The distances and travel times may differ between service and deadhead trips. Although travel times may vary, depending on the time of the day, we will assume fixed durations between any two stop points.

Any solution of the VSP generated is assessed by the total costs caused, consisting of operational and fixed costs. Each vehicle in use causes fixed costs, independently of the rotation to be performed. The fixed costs represent the vehicle's acquisition costs. Operational costs comprise costs per hour in order to reflect the drivers' wages and costs per kilometer to take into account buses' maintenance and wear.

The use of BEVs leads to additional restrictions that have to be satisfied in order to enable regular operations:

- a BEV's residual energy cannot fall below zero and cannot exceed its battery capacity,
- a BEV can only be recharged at specified charging stations.

The residual energy of a battery is often denoted as its *State of Charge* (SoC) respectively *Depth of Discharge* (DoD). In order to incorporate BEVs, the network is extended by introducing a set of charging stations representing stop points equipped with charging technology. The charging technology determines the time which is needed for the intake of energy, the *charging time*. This is due to the current, which may differ between different charging technologies. We assume that charging procedures start immediately on arrival at a stop point without buffer times. Possible turning times at final stops and changeover times at charging stations are assumed to be part of previous trips. In order to take charging procedures into account, we assume specific costs for charging arising from energy prices and maintenance. Each vehicle contains a battery, which is mainly characterized by its capacity, denoting the maximum amount of energy that can be stored. Furthermore, a vehicle consumes a specific amount of energy per kilometer driven, which differs on service and deadhead trips due to the greater weight when passengers are being carried. A vehicle rotation is termed feasible for BEVs if the restrictions introduced are satisfied. If every vehicle used satisfies the restrictions, the problem is denoted as the E-VSP.

As indicated by the real-world project in Berlin, many companies in public transport deploy a mixed fleet consisting of both BEVs with range limitations and traditional combustion engine vehicles without range restrictions. Consequently, neither a pure form of the VSP nor the E-VSP can be used for operational planning. This challenge leads to the MF-(E)VSP as the essential problem of this paper, which considers a mixed form of these two problems. Formally, the set of vehicles now consists of two major subsets: The first subset contains combustion engine vehicles and the second subset BEVs. Range restrictions must be satisfied for each vehicle of the second subset.

4 Three-phase solution approach based on an aggregated time–space network

We now discuss our three-phase solution approach for solving the MF-(E)VSP based on an aggregated TSN. Kliewer et al. (2006) introduced a modeling approach for the multi depot VSP with multiple vehicle types using a TSN. This solution method generally comprises three consecutive steps: First, the TSN is constructed, based on the underlying public transportation network and the timetable. Second, optimal flow values through the TSN are computed by solving a multi-commodity flow problem. Last, decomposition strategies are applied in order to obtain executable vehicle rotations from the flow values.

As previously described, the aim of the MF-(E)VSP is to maximize the proportion of feasible vehicle rotations for BEVs within the entire set while retaining optimal numbers of vehicles used and deadhead trips required obtained by solving the standard VSP. Consequently, the first two steps of the solution procedure remain unchanged. However, the step of flow decomposition needs to be modified to consider challenges arising from the use of BEVs. In addition, charging procedures have to be inserted into the vehicle rotations. This results in the following three phases of our solution approach:

Phase I: Construction of the TSN and determination of optimal flow values without consideration of range limitations,
Phase II: Decomposition of the flow into executable vehicle rotations,
Phase III: Insertion of charging procedures.

The following sections describe the specific phases of the solution approach.

4.1 Phase I: Aggregated time–space network and exact solution method for the VSP without range limitations

A TSN generally shows activities in time and space. A TSN for multi-depot vehicle scheduling consists of multiple layers, whereby each layer corresponds to a combination of depot and vehicle type. A layer basically consists of time lines, arcs, and nodes.

For each stop point of the route network, a time line is created representing all possible arrival and departure events at the specific stop. Arcs represent service trips, deadhead trips, and idle times of the vehicles. Deadhead trips starting at the depot are denoted as pull-out trips and deadhead trips ending at the depot as pull-in trips. Thereby, the vertical axis of the network describes the spatial and the horizontal axis the temporal component. A node of the TSN connects a group of possible arrivals to a subsequent group of possible departures. The arcs of both groups are sorted in ascending order by the arrival/departure times. Hence, all stop points are represented as ordered sets of nodes that are connected by waiting arcs. To set up a TSN model, an arc is added between the corresponding time lines for each service trip that can be served by a layer's vehicle type. Then, a node is inserted for every group of consecutive arrival and departure events on a time line. For each arc, the horizontal distance between the arrival and departure node of a trip represents its duration. The nodes of each time line are linked by waiting-arcs to represent the vehicles' idle times. Series of compatible trips from different time lines are linked by aggregated deadhead-arcs between the corresponding nodes, representing possible deadhead trips. Possible pull-out and pull-in-arcs from/to the depot are inserted for every service trip. As it must be ensured that each vehicle returns to its original depot at the end of a day, a circulation-arc from the last node to the first node of the time line, belonging to the depot, is added to each layer.

The concept of time lines enables a significant reduction of the problem's complexity by aggregating the deadhead-arcs into groups of compatible connections, which represents the main advantage of the TSN formulation. The concept of transitivity in the compatibility of trips is used to do this. A deadhead-arc can be omitted if the same connection can be reached using a combination of other deadhead- and waiting-arcs. For further details of the procedure for reducing deadhead-arcs, we refer to Kliewer et al. (2006). Figure 1 illustrates an example of a TSN after applying the reduction procedure. The figure shows one time line that represents the depot and two time lines that represent stop points. There are three service trips that operate between the two stops. However, the directions of travel are different. As this figure illustrates by way of example, a deadhead-arc to connect service trip 1 with service trip 3 is not necessary because this connection is provided by a sequence of waiting-arcs within the time line of stop point 1. Likewise, it does not need a deadhead-arc from the depot to the departure of service trip 3 as this node can be reached by the deadhead-arc from the depot to service trip 2 and a waiting-arc on the time line of stop point 1. The application of this procedure to the entire set of service trips enables a major reduction in the number of deadhead-arcs. Following Kliewer et al. (2006), a reduction of up to 97% can be achieved for real-world timetables.

The resulting TSN model corresponds to a multi-commodity flow problem (according to Kliewer et al. 2008). Let $N = \{1, 2, \ldots, n\}$ be the set of trips and D the set of depots. For each depot $d \in D$, a network $G^d = (V^d, A^d)$ is defined which consists of nodes V^d and arcs A^d. Let $N^d(n) \in A^d$ be the arc that corresponds to trip n of the network G^d. Let $u^d \in \mathbb{N}$ be the maximum number of available vehicles within a depot d and $M \in \mathbb{N}$ the total number of available vehicles over all depots. Let the parameters $c^d_{ij} \geq 0$ be vehicle costs of arcs $(i, j) \in A^d$ reflecting travel and idle times. The costs of waiting arcs in the depot is set to 0. On the circulation arc of the network

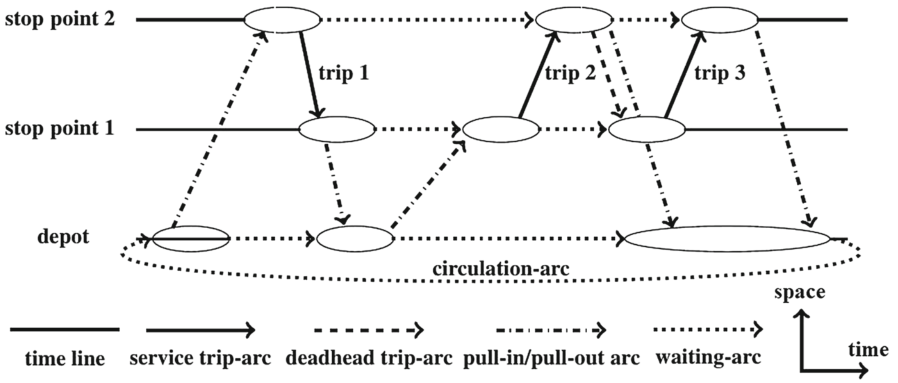

Fig. 1 Example of a time–space network with three time lines and three service trips (according to Kliewer et al. 2006)

a fixed cost for using a vehicle is set. Decision variables $x_{ij}^d \in \mathbb{N}$ indicate whether an arc (i, j) is used and assigned to the depot d or not. Therefore, the following upper bound is defined for each decision variable:

$$u_{ij}^d = \begin{cases} 1, & \text{if } x_{ij}^d \text{ corresponds to a service trip} \\ u^d, & \text{if } x_{ij}^d \text{ corresponds to a circulation arc} \\ M, & \text{otherwise} \end{cases}$$

With this we can formulate the multi-commodity flow problem as the following mixed-integer linear program (MIP) (1)–(5). Due to the significant reduction of connections within the TSN, even real-world instances with very large networks and timetables can be solved to optimality using standard optimization software libraries.

$$min \sum_{d \in D} \sum_{(i,j) \in A^d} c_{ij}^d x_{ij}^d \tag{1}$$

$$\sum_{\{j:(i,j) \in A^d\}} x_{ij}^d - \sum_{\{j:(j,i) \in A^d\}} x_{ji}^d = 0 \quad \forall\, i \in V^d, \forall\, d \in D \tag{2}$$

$$\sum_{d \in D, (i,j) \in N^d(n)} x_{ij}^d = 1 \quad \forall\, n \in N \tag{3}$$

$$0 \leq x_{ij}^d \leq u_{ij}^d \quad \forall\, (i, j) \in A^d, \forall\, d \in D \tag{4}$$

$$x_{ij}^d \in \mathbb{N} \quad \forall\, (i, j) \in A^d, \forall\, d \in D \tag{5}$$

The objective (1) is to minimize the sum of total vehicle costs. Constraint (2) ensures the flow conservation, indicating that the flow into each node equals the flow out of each node. Constraint (3) secures that each trip is covered by exactly one vehicle. Constraint (4) ensures that the upper bound of each decision variable is not exceeded. According to constraint (5), all decision variables are non-negative integers.

Due to the formulation of the VSP as a multi-commodity flow problem, solutions provide optimal flow values for each arc of the network. Consequently, no path-related constraints can be considered because the problem formulation does not contain an optimization of the paths. The flow values allow many different paths through the network. All of them represent optimal solutions with regard to the number of vehicles needed and deadhead trips required, but differ in the distribution of waiting times. In order to obtain executable vehicle rotations, flow decomposition methods are used to break down the optimal flow.

4.2 Phase II: Flow decomposition methods for the deployment of BEVs

To divide the optimal flow values into executable paths through the TSN, we propose eight decomposition methods. All of the methods are local procedures since they solve a decision-making problem at each node of the TSN without considering the entire network. For all methods, incoming arcs are connected to outgoing arcs within each node of the TSN. The first two decomposition methods presented in Sects. 4.2.1 and 4.2.2 are taken from Kliewer et al. (2006) whereas the other strategies are novel procedures explicitly designed for the consideration of electric vehicles' characteristics.

4.2.1 FIFO

The widely known, simple procedure *FirstIn-FirstOut* (`FIFO`), which is often used within database applications, can also be used for flow decomposition. `FIFO` combines the first incoming arc within each node of a time line with the first outgoing arc, the second incoming with the second outgoing, etc. (all of them with positive flow values). Figure 2 shows an example of a node within a time line of a TSN. On the left side of the figure, the procedure `FIFO` is illustrated by an example with three incoming resp. outgoing arcs.

4.2.2 LIFO

The procedure *LastIn-FirstOut* (`LIFO`) proceeds contrarily: The last incoming arc is linked to the first outgoing one. On the right side of Fig. 2, the procedure `LIFO` is illustrated. Although the decomposition strategies `FIFO` and `LIFO` are not directly related to the use of BEVs, we use them in our computational study in order to compare standard to more complex decomposition methods.

4.2.3 MaxMinChargingTime

The procedures `FIFO` and `LIFO` have in common that they are fairly simple and do not consider any characteristics of BEVs, such as charging times or energy consumption. In contrast, we propose the novel strategy `MaxMinChargingTime`, which aims at maximizing the minimum waiting times at charging stations within vehicle rotations in order to enable vehicles to recharge. Since overlong time windows for charging for some vehicles would result in too short charging times for other vehicles, we solve

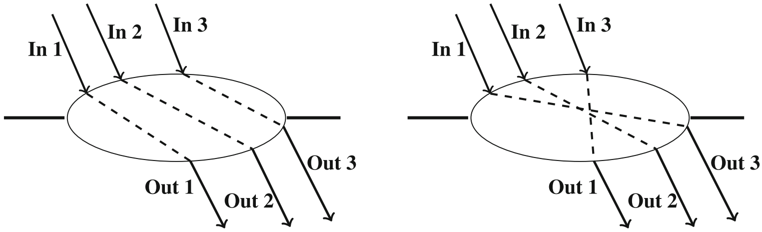

Fig. 2 `FIFO` (left) and `LIFO` (right) illustrated by one node of a time line within a time–space network

an assignment problem that maximizes the minimum waiting time for each possible connection.

Let $n \in \mathbb{N}$ be the number of incoming arcs for a node and $m \in \mathbb{N}$ the number of outgoing arcs. Let $b_{ij} \geq 0 \; \forall i = 1, \ldots, n, \;\; j = 1, \ldots, m$ be parameters that reflect waiting times at the stop point of the node between each incoming arc i and each outgoing arc j. Decision variables $x_{i,j} \in \{0, 1\}$ indicate whether an incoming arc i is connected with an outgoing arc j $(x_{i,j} = 1)$ or not $(x_{i,j} = 0)$. The assignment problem can be represented by the following mathematical optimization problem:

$$\max \min_{i,j} b_{ij} \cdot x_{ij} \tag{6}$$

$$\text{s.t.} \sum_{j=1}^{m} x_{ij} = 1, \quad i = 1, \ldots, n \tag{7}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, \ldots, m \tag{8}$$

$$x_{ij} \in \{0, 1\}, \quad i = 1, \ldots, n, \;\; j = 1, \ldots, m. \tag{9}$$

The objective (6) is to maximizing the minimum waiting times. Constraint (7) of the problem formulation ensures that each incoming arc is connected with precisely one outgoing arc. Constraint (8) ensures the same for each outgoing and incoming arc. According to constraint (9), all decision variables are binary. The optimization problem is solved using standard optimization software libraries, since the number of choices is small even for large real-world instances.

4.2.4 BalanceConsumption

An alternative view enables the decomposition method `BalanceConsumption`. The main idea of this procedure is to consider the energy consumption of the vehicle rotations with regard to potential connections of incoming and outgoing arcs. The objective is to balance the consumption of the different vehicle rotations. To this purpose, a bottleneck problem is solved. In this way, the maximum sum of consumption over every possible connection between incoming and outgoing arcs is minimized. The mathematical optimization problem is identical to the proposed with regard to

`MaxMinChargingTime` but now the parameters b_{ij} reflect the sum of consumption of any connection between an incoming arc i and an outgoing arc j.

4.2.5 MaxMinChargingTime-BalanceConsumption

As an extension of the previous strategies, `MaxMinChargingTime-BalanceConsumption` combines the methods `MaxMinChargingTime` and `BalanceConsumption`, so that the first strategy is applied at every charging station and the second at every non-charging station. This should maximize waiting times of vehicles at charging stations and balance the vehicles' consumption at every non-charging station. This way, the benefits of the two decomposition methods can be combined.

4.2.6 Extended-MaxMinChargingTime-BalanceConsumption

For some instances it might be advantageous to consider the consumption even at charging stations. Therefore, the `Extended-MaxMinChargingTime-BalanceConsumption` strategy solves a bottleneck problem at every node but includes both possible waiting times for charging and consumption of the different vehicle rotations. Thus, a weighted sum of both components is considered. Besides the adjusted objective function, the mathematical optimization problem is identical to the model used within `MaxMinChargingTime` and again solved by standard software libraries.

4.2.7 Extended-BalanceConsumption

Within the strategy of `BalanceConsumption` it might be useful to consider only the consumption between two charging stations within a vehicle rotation instead of the entire vehicle rotation. Furthermore, it is likely beneficial to link two already infeasible parts of vehicle rotations to avoid additional infeasibilities. These two components are considered within `Extended-BalanceConsumption`. For this purpose, the sum of consumption for all pairs of incoming and outgoing arcs is computed but now considering the consumption between any two directly consecutive charging stations. If both the incoming and outgoing part of the corresponding vehicle rotations are infeasible for BEVs, the consumption of the specific connection is set to a sufficiently high value in order to rule out this connection for BEVs. In that way, the procedure aims at connecting infeasible parts of vehicle rotations.

4.2.8 MaxMinChargingTime-Extended-BalanceConsumption

As a last strategy, we use *MaxMinChargingTime-Extended-BalanceConsumption* which is a combination of the previously introduced strategies. At this point, the assignment of arcs at charging stations is done by `MaxMinChargingTime` and at non-charging stations by `Extended-BalanceConsumption`. Analogous to `MaxMinChargingTime-BalanceConsumption`, the benefits of the two strategies should be exploited.

Table 1 illustrates the main characteristics of the methods presented for flow decomposition.

Table 1 Overview of the main characteristics of the flow decomposition methods

Decomposition method	Chrg. time	Energy con.	Multiple strategies	Feas. of rot.
`FIFO`				
`LIFO`				
`MaxMinChargingTime`	•	•		
`BalanceConsumption`		•		
`MaxMinChargingTime-BalanceConsumption`	•	•	•	
`Extended-MaxMinChargingTime-BalanceConsumption`	•	•	•	
`Extended-BalanceConsumption`		•	•	•
`MaxMinChargingTime-Extended-BalanceConsumption`	•	•	•	•

4.3 Phase III: Charging insertion procedure

After dividing the optimal flow values into executable paths considering charging times and energy consumption, charging procedures have to be inserted into the vehicle rotations in order to enable operation by BEVs. Therefore, we now introduce an algorithm that adds charging procedures to vehicle rotations. Since waiting times at intermediate stops of service trips are determined by the timetable, we focus on waiting times between consecutive service trips. The basic procedure is illustrated by Algorithm 1.

Algorithm 1 Charging Insertion Procedure

Input: vehicle rotations V, charging stations S, lower bound $\underline{E}$ and upper bound $\overline{E}$ for the SoC
Output: vehicle rotations V with feasibility for all $v \in V$

1: **for each** $v \in V$ **do**
2: Set v as $feasible$;
3: **for each** $t \in v$ **do**
4: Update SoC after executing t w.r.t. energy consumption and opportunity charging;
5: **if** Departure stop of t is in S **and** updated SoC $< \overline{E}$ **then**
6: **if** $previous(t)$ is a deadhead trip **then**
7: **if** Waiting time before $previous(t)$ plus after $previous(t)$ is positive **then**
8: Shift $previous(t)$ backwards;
9: Add charging procedure before t;
10: Update SoC;
11: **else if** Waiting time before t is positive **then**
12: Add charging procedure before t;
13: Update SoC;
14: **if** SoC $< \underline{E}$ **then**
15: Set v as $infeasible$;
16: Delete all charging procedures within v;
17: **break**;
18: **return** V;

The set of vehicle rotations V obtained by flow decomposition, the set of charging stations S, and a specific lower and upper bound for the SoC serve as the input data. The bounds for the SoC will be used within our computational study to incorporate battery aging effects. Therefore, we assume that the SoC of a vehicle battery cannot fall below the lower bound and cannot exceed the upper bound after leaving the depot.

The set V of vehicle rotations is processed consecutively and the current rotation v is considered (l. 1). Initially, each vehicle rotation is assumed to be feasible for BEVs (l. 2). Then, after each trip t of v the SoC is computed by subtracting the energy consumption of t. If trip t is a service trip, the amount of energy being charged by opportunity charging at intermediate stops is added (l. 4). If the current vehicle rotation remains feasible after executing t waiting times before and after the trip can be used for charging if corresponding time windows are positive and the current departure stop point is a charging station (l. 5). Since service trips are fixed by their departure and arrival times and deadhead trips can be shifted, we use a case differentiation to insert charging procedures. To do this, we consider the previous trip $previous(t)$ of t as the first case. If $previous(t)$ is a deadhead trip, we check whether the waiting time before trip $previous(t)$ plus the waiting time before trip t is positive (l. 6 and l. 7). If this is the case, trip $previous(t)$ is shifted backwards in order to increase possible charging times, a charging procedure is added before executing trip t, and the SoC is updated (l. 8–l. 10). Within this step, we take into account charging procedures already inserted by the algorithm in order to prevent that earlier charging procedures are shortened or even removed. If $previous(t)$ is a service trip, we perform this procedure by considering the waiting time before trip t, add a possible charging procedure, and update the SoC (l. 12 and l.13). In all cases where charging is possible the specific upper bounds for the SoC of the batteries are considered. If the updated SoC falls below the lower bound, the current vehicle rotation is infeasible and, thus, cannot be executed by BEVs (l. 15). In this case, charging procedures already inserted into the rotation are removed and the next vehicle rotation is processed (l. 16). After each vehicle rotation has been processed the algorithm returns the modified vehicle rotations and their feasibility resp. infeasibility (l. 18).

5 Computational study

In this section, we present the results of our computational study. We start by introducing the instances to be solved and the experimental parameters. Then, we look at the results of solving the MF-(E)VSP according to the procedure introduced in Section 4. In this context, we analyze the percentages of feasible vehicle rotations for BEVs as the crucial aspect of this paper. In particular, we investigate the impact of the proposed decomposition methods on resulting vehicle rotations considering different settings of the charging infrastructure.

5.1 Problem instances and parameters

Our computational experiments are performed on six real-world instances, with up to 10,000 service trips, which differ in their number of service trips, their distributions over the day, and numbers of stop points. The instances are based on real-world data from German public transport companies. The names of the instances contain the total number of service trips. The instances are characterized by different kinds of distribution of the numbers of simultaneously performed timetabled trips over the day, see Fig. 3, and differ in the number of stop points. The different profiles of service trips cover the most popular patterns in public transport, since the instances $t867$, $t1135$, and $t3067$ can be associated with urban areas comprising peak times in the morning and afternoon, whereas $t1296$, $t2633$, and $t10710$ represent rather rural areas characterized by constant services throughout the day. For our study, the instances' original data have been adapted in order to address the requirements of BEVs.

Within this study, we assume two major engine types of the vehicles deployed: BEVs with range limitations and traditional combustion engine vehicles without range limitations. For reasons of simplification, we consider a single vehicle depot. Consequently, every vehicle starts and ends its rotation at the depot regardless of the vehicle

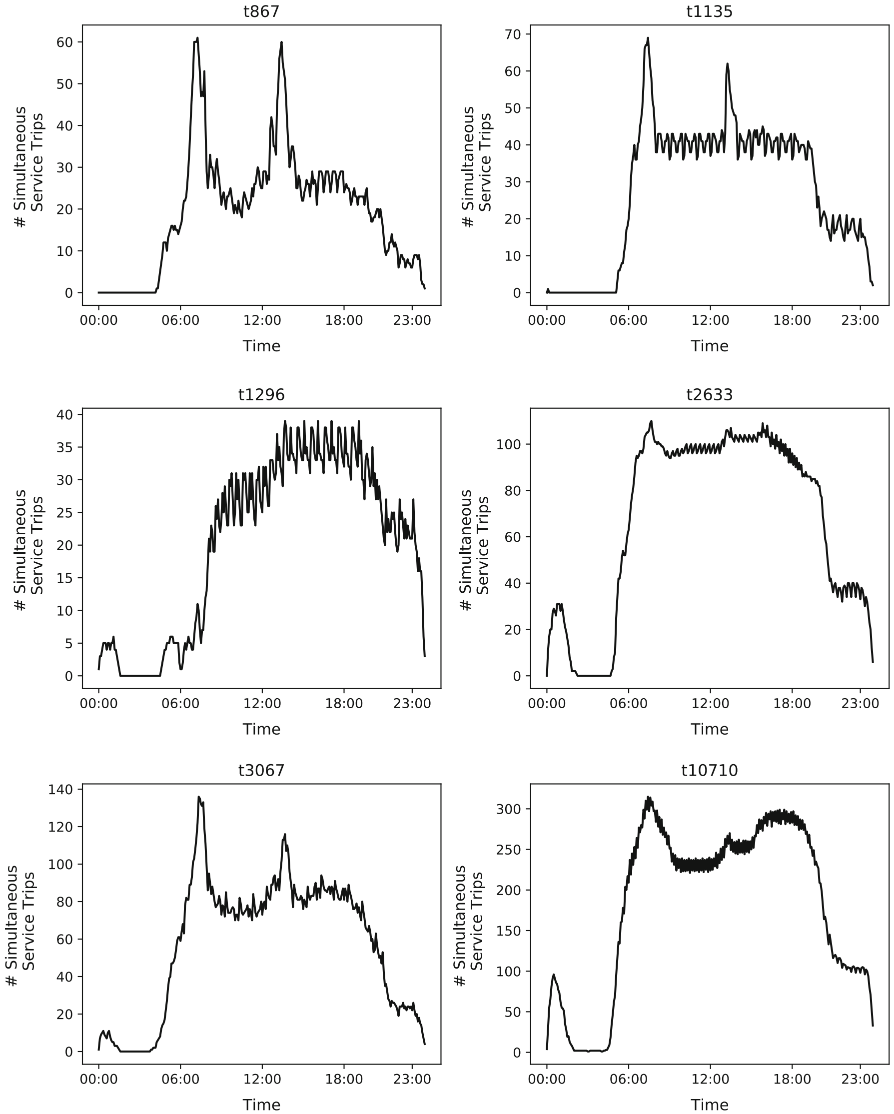

Fig. 3 Temporal distribution of timetabled service trips over the day for each instance

type. Furthermore, we assume that both types of vehicles are able to cover every timetabled service trip.

Inspired by the real-world project in Berlin, we assume a battery capacity of 90 kWh for all BEVs. A BEV always leaves the depot with a fully charged battery. Therefore, we assume a sufficiently large number of charging systems in the depot and a sufficient period of time between arrivals and departures of the vehicles. To take battery aging effects into account, we assume a lower and upper bound within a battery's SoC ranges during its operations after leaving the depot (cf. Jossen 2005). We will use

20% of the battery capacity for the lower and 80% for the upper bound. Although the consumption of BEVs is influenced by several factors such as line topologies, road gradients, and traffic conditions, we assume constant consumption by a BEV per kilometer driven. However, we assume that consumption per kilometer differ on service (1.8 kWh/km) and deadhead trips (1.5 kWh/km). This leads to a maximum range of 60 km on deadhead trips and 50 km on service trips for each BEV.

An important part of this study will be the analysis of different settings of the charging infrastructure. Therefore, we consider different scenarios that differ with regard to the proportion of charging stations at highly frequented stops in the full set of stop points. We use proportions of charging stations of 10%, 20%, and 50% within the following study. To achieve this, stop points are ordered by the number of service trips departing or arriving at the respective stop points and the corresponding subset of stop points is equipped with charging systems. We assume unbounded capacities of charging stations, which means the number of simultaneous charging procedures and amounts of energy that can be charged at a charging station are unbounded. As this assumption represents a broad generalization, especially with regard to highly frequented traffic hubs, we investigate this issue in greater detail within our study.

A battery can be charged either between two successive service trips or at intermediate stop points during the execution of a service trip if corresponding stop points are equipped with charging technology. As described earlier, different charging systems exist at the present time, mainly differing in terms of the energy transfer and the current provided during a charging process. To incorporate this crucial aspect, we consider different currents provided at the charging stations. We use currents of 1.8, 3, and 9 kWh/min. Since we consider only one type of BEV at the same time, we conduct our study for each current. Despite having explained the need for complex models to incorporate the nonlinear charging process of modern lithium-ion batteries, we assume a constant current during the entire charging process of BEVs for each charging system, and thus linear charging times. Following the real-world project in Berlin, we first assume 50 min for charging a battery to full capacity if it is completely empty, which leads to a current of 90 kWh/50 min = 1.8 kWh/min. Building on this, we assume 30 and 10 min for a complete charging of the battery leading to 3 kWh/min and 9 kWh/min to represent more efficient fast-charging systems. Since we focus on the feasibility of vehicle rotations for BEVs computed at minimum costs for traditional diesel busses, we do not consider any additional cost parameters arising from the use of BEVs, like energy costs or fixed costs for charging stations. Finally, as we consider a single type of BEVs, we assume that each BEV can be charged at every available charging station.

5.2 Results of solving the MF-(E)VSP using the three-phase solution approach

We now discuss the results of solving the MF-(E)VSP. With regard to the implementation of BEVs in public transport, not only the percentages of feasible vehicle rotations for BEVs are important but also related aspects such as percentages of service trips covered by BEVs, kilometers driven by BEVs, and characteristics of the charging procedures. In the following, we discuss each of the specified aspects. The solution approach provided is implemented in C# under .Net using the optimization library of IBM ILOG CPLEX 12.5. All of the results have been obtained by using a CPU with a 2.7 GHz processor. We receive acceptable run times for all instances with the standard optimizer of CPLEX. The maximum runtime over all instances was approximately 30 s.

5.2.1 Percentages of feasible vehicle rotations for BEVs

We first analyse the percentages of feasible vehicle rotations for BEVs within solutions of the MF-(E)VSP. Table 2 provides an overview of the results, according to the assumed distribution of charging stations, the current provided by the charging systems, and the method used for flow decomposition. Additionally, the number of vehicles needed in the optimal solution of the standard VSP is given for each instance.

Looking at the detailed results, we can observe that the entire set of vehicle rotations cannot be served by BEVs in any of the cases examined. Furthermore, the percentage of feasible vehicle rotations for BEVs differs significantly according to the assumed distribution of charging stations, the current provided, and the instances. In all cases, an increasing distribution of charging stations as well as increasing currents both lead to an increase in feasible vehicle rotations for BEVs. However, the impacts of these two factors differ significantly. It can be stated that, in general, the charging stations' influence on the feasibility of rotations for BEVs depends strongly on the instances themselves. If an instance's distribution of timetabled trips over the day contains peak times (see Fig. 3), the influence of an increasing distribution of charging stations is significantly higher than in the case of an almost unvarying offer of service trips. With regard to the specific data of this study, an increase in charging stations leads to more feasible vehicle rotations when solving instances $t867$, $t1135$, and $t3067$ that can be associated with urban areas but has very little impact on the solutions for instances $t1296$, $t2633$, and $t10710$ that correspond to rural areas. Similar observations can be made regarding the currents provided at charging stations. Again, an increase in the current leads to higher percentages of feasible rotations when the corresponding instances contain peak times in service trips over the day than in the case without peak times. Independently of the respective instance, we can observe that the higher the current provided at charging stations, the higher is the impact of an increasing distribution of charging stations on the solutions. This is reasonable because longer charging times caused by lower currents almost entirely cancel out the higher degrees of freedom caused by a greater number of charging stations. However, in the case of 9 kWh/min as the current, these advantages can be used to obtain better solutions.

With regard to the different flow decomposition methods, we identify that the use of more complex methods especially designed for the deployment of BEVs generally leads to a higher percentage of feasible vehicle rotations by comparison to methods not considering the special features of BEVs. The traditional methods `FIFO` and `LIFO` achieve worse results than any other method in all of the cases examined. It is worth noting that the lower the number of charging stations is, the better results are obtained when using more specific methods for flow decomposition. This is reasonable because more charging stations distributed within the network enable more degrees of freedom and thus compensate for the unspecific procedures of traditional methods such

Table 2 Summary of percentages of feasible vehicle rotations for all instances divided by the decomposition method used, the assumed charging infrastructure, and the current provided at charging stations

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t867 (69 vehicles in optimal solution)									
`FIFO`	38.2	40.3	42.6	39.7	44.9	50.9	41.2	47.6	59.8
`LIFO`	37.3	39.7	39.9	38.7	41.4	42.0	39.8	43.7	44.9
`MaxMinChargingTime`	36.7	38.9	42.3	37.2	40.2	44.2	39.6	42.5	46.3
`BalanceConsumption`	36.9	38.2	42.1	37.4	40.7	43.9.2	40.1	42.7	45.3
`MaxMinChargingTime-BalanceConsumption`	40.3	43.2	46.8	42.7	47.3	62.7	44.9	50.3	66.8
`Extended-MaxMinChargingTime-BalanceConsumption`	40.0	43.7	45.2	42.7	48.2	63.7	48.6	62.9	69.4
`Extended-BalanceConsumption`	43.7	49.4	50.9	45.2	50.8	66.7	49.4	64.9	70.3
`MaxMinChargingTime-Extended-BalanceConsumption`	44.3	51.5	55.3	48.3	53.9	68.3	52.2	64.9	72.1
Instance t1135 (75 vehicles in optimal solution)									
`FIFO`	35.5	36.8	39.4	36.8	40.7	57.9	39.4	56.6	61.8
`LIFO`	35.5	39.5	39.5	36.8	40.8	57.9	42.1	56.6	61.8
`MaxMinChargingTime`	36.8	39.5	42.1	38.2	42.1	63.2	43.4	61.8	63.2
`BalanceConsumption`	36.8	39.5	40.8	38.2	40.8	63.2	40.8	59.2	64.5
`MaxMinChargingTime-BalanceConsumption`	38.2	43.4	46.1	39.5	46.1	61.8	43.4	60.5	65.8
`Extended-MaxMinChargingTime-BalanceConsumption`	39.5	43.4	44.7	42.1	47.4	61.8	47.4	60.5	65.8
`Extended-BalanceConsumption`	42.1	48.7	48.7	43.4	47.4	67.1	44.7	61.8	65.8
`MaxMinChargingTime-Extended-BalanceConsumption`	42.1	48.7	48.7	43.4	46.1	67.1	44.7	63.2	67.1

Table 2 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t1296 (47 vehicles in optimal solution)									
`FIFO`	05.8	06.6	07.1	08.2	08.9	09.4	09.1	9.5	19.2
`LIFO`	05.4	05.7	06.1	08.1	08.9	09.3	08.7	11.2	18.3
`MaxMinChargingTime`	06.0	06.4	06.9	07.5	08.5	10.9	09.3	12.4	23.1
`BalanceConsumption`	06.2	07.5	08.2	07.9	09.1	10.3	09.2	11.4	23.1
`MaxMinChargingTime-BalanceConsumption`	07.7	08.3	09.6	08.7	09.8	010.3	010.8	12.7	20.9
`Extended-MaxMinChargingTime-BalanceConsumption`	07.3	07.9	08.4	08.4	09.7	010.2	09.7	11.8	21.7s
`Extended-BalanceConsumption`	09.0	09.2	09.2	010.6	010.8	11.9	010.5	12.7	24.1
`MaxMinChargingTime-Extended-BalanceConsumption`	09.0	09.2	09.2	011.0	011.8	12.2	09.6	12.9	24.6
Instance t2633 (125 vehicles in optimal solution)									
`FIFO`	05.6	06.4	06.4	08.0	08.8	09.6	08.8	10.4	20.8
`LIFO`	05.6	05.6	05.6	08.0	08.0	09.6	08.0	10.4	21.6
`MaxMinChargingTime`	05.6	06.4	06.4	08.0	08.8	10.4	08.8	12.0	22.4
`BalanceConsumption`	06.4	07.2	08.0	08.0	08.8	10.4	08.8	10.4	22.4
`MaxMinChargingTime-BalanceConsumption`	05.6	08.0	08.8	08.0	08.8	09.6	08.8	12.0	22.4
`Extended-MaxMinChargingTime-BalanceConsumption`	05.6	08.0	08.8	08.0	08.8	09.6	09.6	11.2	22.4
`Extended-BalanceConsumption`	08.0	08.0	08.0	08.8	09.6	10.4	09.6	10.4	22.4
`MaxMinChargingTime-Extended-BalanceConsumption`	08.0	08.0	08.0	09.6	09.6	10.4	09.6	12.0	22.4

Table 2 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t3067 (165 vehicles in optimal solution)									
`FIFO`	30.3	32.1	42.4	32.7	38.8	42.4	47.9	48.5	61.8
`LIFO`	31.5	32.7	42.4	32.7	38.8	43.0	48.5	49.1	62.4
`MaxMinChargingTime`	31.5	35.2	43.0	32.7	39.4	44.2	49.1	49.7	63.6
`BalanceConsumption`	31.5	32.1	42.4	35.2	33.9	44.8	49.7	49.7	65.5
`MaxMinChargingTime-BalanceConsumption`	33.3	37.0	42.4	35.8	39.4	47.3	49.1	49.7	63.6
`Extended-MaxMinChargingTime-BalanceConsumption`	34.5	37.0	42.4	35.8	38.8	47.9	49.7	50.9	65.5
`Extended-BalanceConsumption`	37.0	40.0	43.6	38.2	40.0	49.1	50.3	50.9	66.1
`MaxMinChargingTime-Extended-BalanceConsumption`	37.0	40.0	43.0	38.2	40.6	48.5	50.9	51.5	66.1
Instance t10710 (349 vehicles in optimal solution)									
`FIFO`	9.7	10.9	12.3	11.2	12.0	16.9	14.3	17.5	27.2
`LIFO`	10.6	11.2	12.3	11.2	13.2	14.9	14.9	18.1	27.2
`MaxMinChargingTime`	12.3	13.2	14.3	14.9	15.5	16.6	14.9	18.3	28.1
`BalanceConsumption`	11.2	12.3	14.0	11.7	15.2	14.6	15.5	18.3	27.8
`MaxMinChargingTime-BalanceConsumption`	12.3	12.6	14.3	12.9	14.6	16.6	16.0	18.1	28.4
`Extended-MaxMinChargingTime-BalanceConsumption`	11.7	12.0	14.3	13.5	14.9	15.2	16.6	19.8	28.4
`Extended-BalanceConsumption`	14.0	16.3	19.2	14.9	16.3	18.6	17.2	18.6	28.4
`MaxMinChargingTime-Extended-BalanceConsumption`	14.6	16.3	18.9	15.2	15.5	18.9	16.6	18.3	28.9

as `FIFO` or `LIFO`. The application of more complex methods leads to particularly good results when solving the instances $t867$, $t1135$, and $t3067$. This is because these instances contain peak times of timetabled trips over the day, which allow the vehicles to recharge their batteries during times with reduced offers. At this point, methods that consider charging times and energy consumptions take better advantage of these conditions. Regarding the instances $t1296$, $t2633$, and $t10710$, more specific methods still provide better solutions but have less impact. In all the cases examined, the method `MaxMinChargingTime-Extended-BalanceConsumption` provides the best results. This is because this method covers most aspects of BEVs.

5.2.2 Percentage of service trips covered by BEVs

Another interesting aspect is the percentage of service trips covered by BEVs because service trips represent a core service of public transport companies. Table 3 contains the percentage of service trips covered by BEVs, again divided according to the assumed distribution of charging stations, the current provided by the charging systems, and the method used for flow decomposition.

Basically, we can observe that the proportion of service trips covered by BEVs is very similar to the proportion of feasible vehicle rotations. The statements previously made can also be justified with regard to Table 3. However, we can observe that in all cases, the percentage of service trips covered is smaller than the corresponding percentage of feasible vehicle rotations. This may be explained by the lengths of the vehicle rotations. Feasible rotations tend to be shorter than infeasible ones and, thus, contain less service trips. In contrast to the data of Table 2, we now observe significant improvements in the distribution of charging stations. Now, increasing numbers of charging stations cause a steady increase in service trips covered by BEVs, even with the same current provided. In concrete terms, this means that longer vehicle rotations become feasible when the number of charging stations is increased. Regarding the methods used for flow decomposition, we observe that more specific methods lead to higher percentages of service trips covered by BEVs. As in the previous case, the method `MaxMinChargingTime-Extended-BalanceConsumption` achieves the best results. Again, the use of traditional methods without considering the vehicles' limited ranges and the possibility to recharge batteries leads to the worst results.

5.2.3 Percentage of kilometers covered by BEVs

Especially interesting for companies in public transport, particularly in urban areas, is the share of kilometers that can be covered by BEVs. This is because every kilometer that is served by an BEV leads to a reduction of noise, gases, and fine particles and may contribute to public health. Table 4 shows percentages of driven kilometers by BEVs.

The observations with regard to the share of kilometers driven by BEVs essentially correspond to the previously obtained statements. The use of decomposition methods taking into account the special features of BEVs is still preferable by comparison to traditional methods. The more complex methods achieve higher percentages of kilometers driven by BEVs in all cases. However, throughout the results, we can

Table 3 Summary of percentages of covered service trips by BEVs for all instances divided by the decomposition method used, the assumed charging infrastructure, and the current provided at charging stations

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t867 (867 service trips)									
`FIFO`	18.9	20.7	21.3	22.4	25.9	43.8	24.9	47.3	48.9
`LIFO`	21.6	23.2	24.6	24.2	24.2	43.9	27.1	44.2	53.1
`MaxMinChargingTime`	19.4	23.3	24.9	20.9	24.4	48.9	24.2	48.7	51.3
`BalanceConsumption`	24.2	24.7	25.7	25.9	32.8	52.1	29.3	47.0	54.2
`MaxMinChargingTime-BalanceConsumption`	20.1	26.4	27.3	23.7	28.1	48.9	30.1	47.4	52.9
`Extended-MaxMinChargingTime-BalanceConsumption`	21.7	26.8	28.4	26.2	30.0	48.1	29.9	39.4	52.1
`Extended-BalanceConsumption`	25.8	29.9	30.8	28.1	38.6	51.9	34.1	45.9	52.7
`MaxMinChargingTime-Extended-BalanceConsumption`	26.1	30.9	32.1	29.6	31.6	52.5	35.0	47.8	53.0
Instance t1135 (1135 service trips)									

Table 3 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
`FIFO`	18.7	20.4	20.4	21.6	25.2	43.0	24.4	46.6	48.4
`LIFO`	21.2	22.8	24.0	24.0	23.0	43.1	26.3	43.5	51.8
`MaxMinChargingTime`	18.7	22.6	24.4	20.7	23.9	48.2	23.8	48.0	50.7
`BalanceConsumption`	23.8	24.1	25.0	25.0	31.6	50.9	29.0	46.3	53.6
`MaxMinChargingTime-BalanceConsumption`	20.0	26.2	27.1	23.7	27.9	48.3	29.7	46.9	54.4
`Extended-MaxMinChargingTime-BalanceConsumption`	20.9	26.2	27.8	25.9	29.9	47.8	29.5	38.1	53.0
`Extended-BalanceConsumption`	25.4	29.7	28.8	27.8	37.5	51.3	33.7	45.2	51.1
`MaxMinChargingTime-Extended-BalanceConsumption`	25.3	27.9	29.9	27.6	29.4	51.5	29.0	47.5	51.4

Table 3 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t1296 (1296 service trips)									
`FIFO`	01.8	02.1	02.8	04.2	04.8	06.2	04.5	06.8	18.1
`LIFO`	01.7	02.2	02.3	04.2	04.4	06.8	04.4	06.6	17.8
`MaxMinChargingTime`	02.2	02.4	02.9	04.5	05.0	07.1	05.2	08.2	18.0
`BalanceConsumption`	03.5	04.3	04.5	05.2	05.8	07.0	05.5	06.9	19.1
`MaxMinChargingTime-BalanceConsumption`	02.1	04.8	05.4	04.3	05.6	06.9	05.2	06.9	19.1
`Extended-MaxMinChargingTime-BalanceConsumption`	02.0	04.5	05.1	04.5	05.3	06.5	05.3	07.5	18.6

Table 3 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
`Extended-BalanceConsumption`	03.5	04.1	04.2	05.2	05.8	06.4	05.8	07.4	19.0
`MaxMinChargingTime-Extended-BalanceConsumption`	03.5	03.8	04.2	05.5	06.1	06.6	05.8	08.9	17.9
Instance t2633 (2633 service trips)									
`FIFO`	01.7	02.1	02.6	04.2	04.9	06.0	04.6	06.2	17.2
`LIFO`	01.7	02.1	02.1	04.1	04.4	06.6	04.4	06.5	17.3
`MaxMinChargingTime`	02.1	02.1	02.7	04.4	04.9	06.8	05.0	07.9	18.3
`BalanceConsumption`	03.6	04.5	04.1	04.9	05.6	06.8	05.1	06.7	18.5
`MaxMinChargingTime-BalanceConsumption`	02.1	04.7	05.2	04.5	05.4	06.4	05.0	06.5	18.4
`Extended-MaxMinChargingTime-BalanceConsumption`	02.1	04.7	05.2	04.6	05.4	06.7	05.2	07.3	18.2
`Extended-BalanceConsumption`	03.6	04.0	04.0	05.0	05.6	06.0	05.6	07.2	18.8
`MaxMinChargingTime-Extended-BalanceConsumption`	03.6	03.9	04.0	05.6	06.0	06.4	05.6	08.1	18.4

Table 3 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t3067 (3067 service trips)									
`FIFO`	15.4	18.8	28.6	16.1	22.3	29.3	33.8	32.2	53.4
`LIFO`	18.7	18.9	24.8	20.3	21.2	29.8	32.8	32.6	53.8
`MaxMinChargingTime`	15.4	18.8	27.3	16.1	22.3	30.0	30.1	36.6	57.3
`BalanceConsumption`	15.8	17.2	24.0	17.8	20.2	30.7	35.7	36.8	60.9
`MaxMinChargingTime-BalanceConsumption`	16.8	20.9	24.4	19.1	22.9	33.2	35.9	35.6	57.2
`Extended-MaxMinChargingTime-BalanceConsumption`	18.8	20.9	26.8	19.1	23.5	34.0	37.2	37.2	61.0
`Extended-BalanceConsumption`	17.2	21.1	26.4	20.4	21.3	33.5	34.9	36.9	58.0

Table 3 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
`MaxMinChargingTime-Extended-BalanceConsumption`	17.3	21.3	22.9	20.9	22.3	32.6	33.5	37.2	59.9
Instance t10710 (10,710 service trips)									
`FIFO`	4.3	4.7	5.5	4.1	5.9	6.6	5.8	8.1	17.6
`LIFO`	4.0	3.9	5.5	4.3	4.7	6.6	5.8	7.8	17.2
`MaxMinChargingTime`	4.3	4.7	6.0	4.3	5.9	7.3	6.3	8.1	17.8
`BalanceConsumption`	4.4	5.4	6.0	4.8	6.2	7.7	6.4	8.8	17.9
`MaxMinChargingTime-BalanceConsumption`	04.5	05.8	06.1	05.3	06.5	08.9	07.6	08.3	18.1
`Extended-Max BalanceConsumption MinChargingTime-`	04.4	05.5	06.2	04.6	07.0	07.0	07.6	09.3	18.8
`Extended-BalanceConsumption`	05.8	06.8	08.7	06.5	07.9	09.3	07.8	08.6	18.6
`MaxMinChargingTime-Extended-BalanceConsumption`	06.2	06.8	08.2	06.5	07.1	09.3	06.8	09.9	19.6

Table 4 Summary of percentages of kilometers driven by BEVs for all instances divided by the decomposition method used, the assumed charging infrastructure, and the current provided at charging stations

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t867 (10,144.6 km in optimal solution)									
FIFO	15.2	16.8	17.1	15.8	21.3	40.1	17.5	45.6	46.9
LIFO	15.2	16.3	20.9	22.0	19.1	39.3	19.2	43.7	48.9
MaxMinChargingTime	17.8	22.9	24.1	18.3	23.3	46.9	28.3	45.4	49.8
BalanceConsumption	20.6	21.8	25.0	21.4	22.5	49.4	24.1	43.6	52.4
MaxMinChargingTime-BalanceConsumption	16.7	20.1	27.4	22.1	27.1	39.2	28.1	42.6	54.0
Extended-MaxMinChargingTime-BalanceConsumption	17.1	22.2	26.0	21.8	25.7	40.2	28.4	34.5	49.8
Extended-BalanceConsumption	23.3	24.5	26.0	23.8	26.2	49.1	27.5	42.5	51.0
MaxMinChargingTime-Extended-BalanceConsumption	25.0	27.8	28.4	28.7	32.0	47.5	29.9	44.2	52.8
Instance t1135 (13,192.7 km in optimal solution)									
FIFO	15.5	16.3	17.1	15.9	21.7	39.4	17.3	45.1	46.2
LIFO	15.7	16.1	20.7	22.5	18.8	38.7	18.9	43.4	48.1
MaxMinChargingTime	17.3	22.3	23.6	18.2	23.3	46.6	28.6	46.9	48.6
BalanceConsumption	20.4	21.6	24.9	21.3	22.5	49.1	23.8	43.2	52.0
MaxMinChargingTime-BalanceConsumption	16.9	20.4	27.0	22.1	26.7	40.0	27.8	41.1	53.1
Extended-MaxMinChargingTime-BalanceConsumption	17.0	22.2	25.9	21.6	25.3	40.3	28.2	34.1	49.7
Extended-BalanceConsumption	23.3	24.1	25.9	23.9	26.6	49.6	27.3	42.0	50.4
MaxMinChargingTime-Extended-BalanceConsumption	24.9	27.3	28.0	27.7	31.8	47.0	28.9	43.4	50.8

Table 4 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t1296 (15,623.8 km in optimal solution)									
`FIFO`	01.5	01.8	02.5	03.8	04.4	05.8	04.2	06.2	16.4
`LIFO`	01.2	01.6	01.9	03.7	04.1	05.2	04.3	06.5	16.2
`MaxMinChargingTime`	01.5	01.9	02.3	04.0	04.6	06.2	04.3	07.5	17.7
`BalanceConsumption`	03.5	03.8	04.2	04.3	05.4	05.8	04.8	06.8	18.1
`MaxMinChargingTime-BalanceConsumption`	02.0	04.5	05.2	03.9	05.2	06.0	04.8	08.0	17.3
`Extended-MaxMinChargingTime-BalanceConsumption`	01.8	04.6	05.2	04.2	05.0	06.2	05.2	07.5	18.4
`Extended-BalanceConsumption`	03.3	03.7	03.8	04.9	05.5	06.4	05.8	07.2	18.4
`MaxMinChargingTime-Extended-BalanceConsumption`	03.4	04.7	05.8	05.6	06.5	06.9	06.1	06.9	18.9
Instance t2633 (30,905.7 km in optimal solution)									
`FIFO`	01.4	01.8	02.4	03.7	04.3	05.8	04.1	06.2	16.4
`LIFO`	01.4	01.6	01.8	03.9	04.0	05.2	04.2	06.4	16.7
`MaxMinChargingTime`	01.6	02.0	02.5	04.2	04.5	06.5	04.5	07.8	18.0
`BalanceConsumption`	03.6	03.8	04.3	04.4	05.3	05.9	04.7	06.6	17.7
`MaxMinChargingTime-BalanceConsumption`	02.0	04.4	05.0	03.8	05.3	06.0	04.5	07.9	17.8
`Extended-MaxMinChargingTime-BalanceConsumption`	01.8	04.5	05.0	04.1	04.9	05.8	05.0	06.9	17.8
`Extended-BalanceConsumption`	03.3	03.6	03.9	04.7	05.3	06.0	05.4	06.8	17.1
`MaxMinChargingTime-Extended-BalanceConsumption`	03.2	04.5	05.4	05.2	05.9	06.0	05.4	06.2	17.9

Table 4 continued

Decomposition method	10% charging stations			20% charging stations			50% charging stations		
	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)	1.8 kWh/m (%)	3 kWh/m (%)	9 kWh/m (%)
Instance t3067 (31,111.5 km in optimal solution)									
`FIFO`	13.4	16.3	21.1	14.3	18.0	25.7	26.2	28.8	45.9
`LIFO`	13.8	15.4	21.3	14.0	18.7	26.5	28.8	28.9	46.4
`MaxMinChargingTime`	16.3	16.5	23.9	17.7	19.3	26.1	29.2	29.9	49.2
`BalanceConsumption`	14.0	16.5	25.1	15.7	19.7	26.0	31.2	32.0	52.9
`MaxMinChargingTime-BalanceConsumption`	14.6	18.2	21.2	16.6	20.2	29.0	31.2	31.8	49.4
`Extended-MaxMinChargingTime-BalanceConsumption`	16.5	18.2	23.2	16.7	20.5	29.3	32.0	34.7	52.4
`Extended-BalanceConsumption`	15.4	18.2	22.7	18.0	19.4	29.3	30.2	31.8	50.2
`MaxMinChargingTime-Extended-BalanceConsumption`	15.3	18.3	22.1	18.2	19.7	28.1	33.0	34.9	51.9
Instance t10710 (85,807.2 km in optimal solution)									
`FIFO`	03.9	04.3	05.5	04.0	05.3	06.5	05.4	07.6	17.4
`LIFO`	03.5	03.7	05.1	04.3	04.4	06.6	05.5	07.5	17.5
`MaxMinChargingTime`	04.1	04.4	05.6	04.1	05.5	07.3	06.2	07.9	17.9
`BalanceConsumption`	04.3	04.9	05.8	04.7	05.9	06.8	06.2	08.7	17.8
`MaxMinChargingTime-BalanceConsumption`	03.9	05.4	05.5	05.0	06.2	08.5	07.1	07.8	18.1
`Extended-MaxMinChargingTime-BalanceConsumption`	04.0	05.3	05.7	04.6	06.8	06.8	07.3	09.1	19.4
`Extended-BalanceConsumption`	05.3	06.4	08.5	06.5	07.7	09.0	07.6	08.3	18.2
`MaxMinChargingTime-Extended-BalanceConsumption`	06.1	06.7	08.1	06.5	07.0	08.9	06.8	07.4	19.5

identify a further reduction in the percentages compared to our previous results. The vehicle rotations that are feasible for BEVs tend to contain shorter service trips than the rotations that cannot be executed by BEVs. Consequently, the total numbers of kilometers driven by BEVs are reduced.

5.2.4 Charging characteristics

So far, the implementation of resulting vehicle rotations in practice was not of major importance. Thereby, it is especially important that the charging procedures performed by BEVs are reasonably distributed over the day for all charging stations. Uneven distribution may lead to significant problems concerning the practical operation of BEVs because building sites for charging systems are usually restricted, especially in urban areas. Therefore, we analyse the charging data of the vehicle rotations at this point. Table 5 provides the maximum, minimum, and average maximum numbers of simultaneous charging procedures at the same charging station over all decomposition methods, for all instances, distributions of charging stations, and currents. Furthermore, the average numbers of charging stations actually used over all BEVs deployed are specified. Here, we do not address each flow decomposition method separately, since we are analyzing the general usability of our solution approach. Moreover, the following data does not consider opportunity charging during the execution of service trips. This is because corresponding charging times are usually very short and therefore do not have a significant impact on practical operations. In order to be able to better assess the findings, we additionally indicated the number of stop points for each instance.

With regard to the average numbers of charging stations that are actually used over all BEVs deployed, we can see from the data in Table 5 that the more stop points are equipped with charging systems, the more are actually used; however, in a disproportionately limited way considering corresponding percentages. Furthermore, the higher the assumed current at a charging station, the more charging stations are used. Both observations can be explained by the increased number of feasible vehicle rotations for BEVs. If this proportion rises, the more longer rotations can be executed by BEVs and, thus, more charging stations are used. However, this behaviour depends on the instances' distributions of timetabled service trips. If instances contain peak times, both the increasing proportion of charging stations available as well as increasing currents lead to higher percentages of charging stations used. If this is not the case, the number of charging stations used is mainly increased by a rise of charging stations available but not significantly by increasing currents.

Regarding the maximum number of simultaneous charging procedures at the same charging station, we can make similar observations when considering instances with peak times of service trips. Both increasing proportions of available charging stations and increasing currents lead to higher numbers of simultaneous chargings. When solving instance $t867$, the average maximum number of chargings varies between 2.1 and 5.0, regarding instance $t1135$ between 2.3 and 5.3, and with regard to instance $t3067$ between 4.1 and 8.9. When we look at the results of instance $t10710$ without peak times of timetabled service trips, we obtain rather contrary observations. The lower the current at a charging station, the higher is the average maximum number

Table 5 Maximum, minimum, and average maximum numbers of simultaneous chargings at the same charging station over all decomposition methods divided by the assumed charging infrastructure and the current provided at charging stations

Instance 867 (207 stop points)						**Instance t1135** (101 stop points)					
Chrg.stat.	Current	Avg.			Chrg.stat.	Chrg.stat.	Current	Avg.			Chrg.stat.
(%)	(kWh/min)	max.	min	max	used	(%)	(kWh/min)	max.	min	max	used
10	1.8	2.1	1	3	6.1/20	10	1.8	2.3	1	4	3.3/14
10	3	2.5	2	3	7.9/20	10	3	2.9	2	4	4.9/14
10	9	3.1	2	4	12.3/20	10	9	4.9	3	6	7/14
20	1.8	2.2	2	4	7.8/40	20	1.8	2.5	1	4	4/28
20	3	2.8	2	4	10.2/40	20	3	3.8	3	4	5.6/28
20	9	3.8	3	5	14.7/40	20	9	5	3	6	7.1/28
50	1.8	2.9	2	4	9.3/100	50	1.8	3	2	4	4.9/70
50	3	3.7	3	4	15.8/100	50	3	4.3	4	6	8.6/70
50	9	5.0	4	5	19.7/100	50	9	5.3	3	6	13.3/70

Instance 1296 (88 stop points)						**Instance t2633** (67 stop points)					
Chrg.stat.	Current	Avg.			Chrg.stat.	Chrg.stat.	Current	Avg.			Chrg.stat.
(%)	(kWh/min)	Max.	Min	Max	used	(%)	(kWh/min)	Max.	Min	Max	used
10	1.8	1.9	1	3	3.3/9	10	1.8	2.3	1	4	2.6/8
10	3	1.6	1	2	4.9/9	10	3	1.9	1	2	2.9/8
10	9	1.2	1	2	7/9	10	9	1.4	1	2	3.3/8
20	1.8	1.2	1	2	4/18	20	1.8	1.6	1	2	4.6/16
20	3	1.5	1	2	5.6/18	20	3	1.9	1	2	4.9/16
20	9	1.9	1	3	9.3/18	20	9	2	1	3	6.3/16
50	1.8	1.6	1	2	4.9/27	50	1.8	1.8	1	2	5.3/40
50	3	1.8	1	2	8.6/27	50	3	2	2	2	5.5/40
50	9	1.9	2	3	13.3/27	50	9	2.1	2	3	10.5/40

Table 5 continued

Instance t3067 (209 stop points)						**Instance t10710** (140 stop points)					
Chrg.stat.	Current	Avg.			Chrg.stat.	Chrg.stat.	Current	Avg.			Chrg.stat.
(%)	(kWh/min)	Max.	Min	Max	used	(%)	(kWh/min)	Max.	Min	Max	used
10	1.8	5.9	4	7	5.6/21	10	1.8	14.8	13	17	5.4/15
10	3	4.1	3	5	6.6/21	10	3	12	10	15	5.5/15
10	9	4.8	3	6	9.5/21	10	9	7.4	7	9	11.5/15
20	1.8	5.4	4	7	7.3/42	20	1.8	12	10	17	7.4/30
20	3	5.9	4	7	10.1/42	20	3	9.5	8	12	7.6/30
20	9	5.8	5	7	19.3/42	20	9	6.4	5	8	14.6/30
50	1.8	7	6	9	15.4/105	50	1.8	11.4	10	13	13.3/75
50	3	7.6	5	11	19/105	50	3	10.6	9	13	13.9/75
50	9	8.9	8	9	38.6/105	50	9	8.9	8	11	31.3/75

of chargings. In addition, a rise in the distribution of charging stations reduces the resulting maximum numbers independently of the assumed current. In the worst case, with a current of 1.8 kWh/min and 10% charging stations, we obtain 14.8 and in the best case 6.4 simultaneous chargings on average. Looking at the instances $t1296$ and $t2633$, we observe only slight changes without any apparent relation. In contrast to previous instances, the corresponding numbers are much lower and vary between 1.2 and 1.9 and between 1.4 and 2.3 simultaneous chargings respectively.

The results may give the impression that some solutions generated cannot be realized in practice due to the particularly high numbers of simultaneous charging procedures. The exact maximum number of simultaneous chargings that can be carried out at a charging station depends on several factors. For example, space limitations, numbers of charging points, or restrictions imposed by the electricity grid are of importance. However, the absolute numbers of simultaneous chargings can be reduced by subsequently optimizing buffer times entailed in the vehicle rotations. Each vehicle rotation contains service trips, deadhead trips, and charging procedures, of which the last two can be shifted, whereas service trips are fully fixed. By shifting deadheads and chargings, the maximum number of simultaneous chargings may be balanced and, thus, may be reduced, which generally enables a better realization in practice.

6 Summary and further research

In this paper we introduced the MF-(E)VSP as an extension of the traditional VSP to consider a heterogeneous fleet of vehicles consisting of BEVs with limited driving ranges and traditional combustion engine vehicles without range limitations. To solve the problem, we proposed a three-phase solution approach based on an aggregated time–space network. The approach consists of an exact solution method for the VSP without range limitations in the form of a mixed-integer linear program, followed by a second phase, in which limited driving ranges are considered by applying flow decomposition methods and a third phase, in which charging procedures are inserted into the vehicle rotations. The aim is to maximize the proportion of feasible vehicle rotations for BEVs within the entire set of vehicle rotations while retaining the optimal number of vehicles used and deadhead trips required obtained by solving a standard VSP. Our approach was evaluated by solving real-world instances with up to 10,000 service trips.

Essentially, it can be stated that the percentage of feasible vehicle rotations for BEVs within solutions generated, together with corresponding characteristics, meet the requirements of a first application of BEVs in practice, especially when considering the slow shift towards their use in public transport. However, there are remarkable differences between the solutions generated. The results show that the performance of the solution approach presented depends strongly on the instances' distributions of service trips and the methods used for flow decomposition. If an instance's distribution contains peak times we generate significantly better results than in the case of an almost unvarying distribution. Similarly, the impact on resulting percentages of increasing numbers of charging stations and increasing currents provided by the charging systems is much greater when timetabled trips have peak times. Furthermore, flow decomposition methods especially developed for the use of BEVs achieve significantly better results than traditional methods not considering the special features of BEVs. With regard to the percentages, we are able to cover up to 72.1% of vehicle rotations with BEVs when peak times exist, and merely up to 28.9% when this is not the case. This is mainly because instances that contain peak times of timetabled trips over the day allow the vehicles to recharge their batteries during times with reduced offers. Nevertheless, we cover a minimum of 37% in the first and 8.0% in the second case. Compared to this, traditional methods for flow decomposition without considering the limited driving ranges of BEVs have significantly poorer outcomes. Furthermore, the shares of service trips covered and kilometers driven by BEVs have a strong positive correlation to the previous aspect. Particularly, with regard to environmental issues such as noise, dust, and air pollution, this coherence plays a significant role, since public transport companies are aiming to reduce negative effects on public health by deploying BEVs.

In summary, this study remains only a first step towards more realistic concepts, models, and solution approaches for the application of BEVs in public transport companies' practice. Subsequent research should conduct further analysis with regard to the input parameters as well as the underlying assumptions about the vehicle and charging technology in order to evaluate our findings and gain further insights into the problem. The determination of the underlying charging infrastructure in particular represents an essential research topic, which may likely lead to further optimization potentials. This aspect of electro-mobility may be considered as a stand-alone problem or may be integrated into vehicle scheduling. In addition, it would be interesting to see how the consideration of multiple vehicle types with different driving ranges affect the solutions to be generated. Finally, one could extend the solution methods used for solving the E-VSP. In addition to the development of exact solution methods, heuristic solution methods capable of solving extremely large real-world problem instances with many depots and vehicle types are particularly interesting. This has already been developed for the traditional VSP without range-limited vehicles by Gintner et al. (2005).

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