**Fachbereich Erziehungswissenschaften und Psychologie der Freien Universität Berlin**

# Structural Equation Modeling of Multitrait-Multimethod-Multioccasion Data

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# 1 Introduction

## 1.1 Multimethod Measurement in Psychology

What is *multimethod measurement*? Multimethod measurement is the use of more than one method to assess some or all of the constructs or traits of interest in a study. For example, a psychologist interested in aggressive behavior of children might use different sources (raters) to measure aggression. The different sources could be self-reports (e.g., through items like “I am often involved in fights at school”), parent, peer, and teacher reports (e.g., by using modified items like “My / This child is often involved in fights at school”). In addition, the psychologist might use observational data to study aggressive behavior. Note that the notion of a “method” is a rather heterogeneous and not very well-defined concept in psychology. In other words, almost anything can serve as a method: A particular test or rater, an observation, a physiological measure, or a specific questionnaire. Even single items are sometimes conceived of as different methods. For example, positively and negatively worded items supposed to measure the same latent trait are often found to produce method effects that can be attributed to the differences in wording (see, e.g., Maydeu-Olivares & Coffman, 2006; Rauch, Schweizer, & Moosbrugger, 2007). In addition, as discussed in detail in Chapter 3.2.1, method effects often occur in terms of indicator-specific effects in longitudinal studies where the same item or test is repeatedly measured. Others have considered different time points or situations to be different methods (e.g., Biesanz & West, 2004).

Eid and Diener (2006) recently noticed that multimethod research strategies become more and more popular in almost all areas of psychology and are nowadays often preferred to designs that employ only a single method. The reasons for the growing interest in multimethod assessments are obvious. Studies that rely on a single method are often less informative than are studies that combine multiple sources of information. Results based on a single method may be specific to that particular measurement instrument (e.g., item set, test, rater, or observation). As a consequence, the generalizability of findings obtained from single method investigations might be limited. For example, the construct *aggressive behavior* might not be reliably captured by children’s self-report alone since children might respond in a socially desirable manner. The degree of method-specificity (“method bias”) of a particular method can only be estimated when multiple methods are employed. For a comprehensive overview of multimethod research strategies in psychology see Eid and Diener (2006).

## 1.2 Need for MTMM Longitudinal Models

Many phenomena in psychology cannot be satisfactorily investigated by using cross-sectional research designs. Researchers are often interested in the development and change of psychological attributes over time. Therefore, longitudinal research designs are often employed. In these designs, the constructs of interest are assessed on at least two occasions of measurement. With regard to longitudinal studies, the same issues mentioned above for cross-sectional research designs apply: A multimethod longitudinal design is likely to be more informative than a monomethod longitudinal design and should be preferred if possible. As Burns and Haynes (2006) note “A single method (rating scale) with a single source (parent) at a single time point provide little information about the time course of the particular problem” (p. 417). Otherwise stated, the most comprehensive insights can be obtained if *multiple methods*, *multiple constructs*, and *multiple occasions of measurement* are used. As I will explain below, it is also beneficial to use *multiple indicators* (observed variables such as items or scales) per construct-method-occasion unit (CMOU). In the next section, I will briefly review existing models for analyzing MTMM data.

## 1.3 Available Models for Analyzing MTMM Data

In my discussion of existing approaches for analyzing MTMM data, I will focus on SEM-based models (so called *confirmatory factor analysis* [CFA] models for MTMM data [CFA-MTMM models]) for two reasons. First, CFA-MTMM models are nowadays the most popular models for analyzing MTMM data (Eid, Lischetzke, & Nussbeck, 2006). Second, the parameters of the MTMM-MO models that will be presented in this thesis can also be estimated using SEM. I start with existing models for cross-sectional MTMM data and then provide an overview of models for longitudinal MTMM data that have been developed so far.

### *1.3.1 MTMM Models for Cross-Sectional Data*

#### 1.3.1.1 Single Indicator Models

Figure 1 shows the four CFA-MTMM models that are probably the best known models for analyzing cross-sectional MTMM data. Following the common conventions for path diagrams, the observed variables are shown in boxes and the latent variables (trait factors, method factors, and error variables [“unique factors”] $E_{jk}$) are shown in ellipses. Single headed arrows indicate linear regression paths (in this case factor loadings), whereas doubleheaded arrows symbolize covariances (or correlations). The models shown in Figure 1 are in line with the "classical" MTMM design, in which three traits and three methods are used.

In cross-sectional MTMM models for single indicators, there is only one observed variable per trait-method-unit (TMU), for example only one depression self-report score. Hence, only two indices are needed for the observed variables $Y_{jk}$. The index $j$ denotes the trait and $k$ indicates the method used to measure the trait (e.g., type of rater). Figure 1A shows the *Correlated Trait-Correlated Uniqueness (CT-CU) model* (Kenny, 1976; Marsh, 1989; Marsh & Bailey, 1991). In the CT-CU model, each observed variable is influenced by a trait factor and an error variable (unique factor) $E_{jk}$. All trait factors can be correlated. The correlations of the trait factors indicate discriminant validity. No method factors are included in this model. Instead, method effects due to the same method $k$ are captured by correlations among error variables ("correlated uniquenesses"). Note that in the CT-CU model, the error variables $E_{jk}$ comprise influences due to indicator-specificity, method-specificity, and random measurement error. As shown in Figure 1A, all error variables with the same method index $k$ are allowed to correlate. These residual correlations mirror common (and reliable) variance specific to indicators pertaining to the same method. (The idea is that all variables $Y_{jk}$ with the same method index $k$ share common variance due to the same method.) The CT-CU model is a straightforward MTMM model that is often used in substantive applications. It is easy to specify and avoids complications (e.g., estimation and convergence problems) that frequently occur in models in which method factors are included. Thus it is often preferred to other MTMM models. However, important limitations of the CT-CU model are that (1) model parsimony decreases as the number of traits and methods increases given that more error covariances need to be estimated (Lance, Noble, & Scullen, 2002), (2) it confounds random measurement error, indicator-specific variance, and method effects and thus underestimates the reliabilities of the indicators, (3) it does not permit correlations between *different* methods, (4) it does not "explain" method effects in terms of latent factors that could be related to other variables in the model (e.g., in order to explain method effects), and (5) it does not allow for a decomposition into trait, method, and unique components of variance. See Lance et al. (2002) for a detailed critique of the CT-CU model.

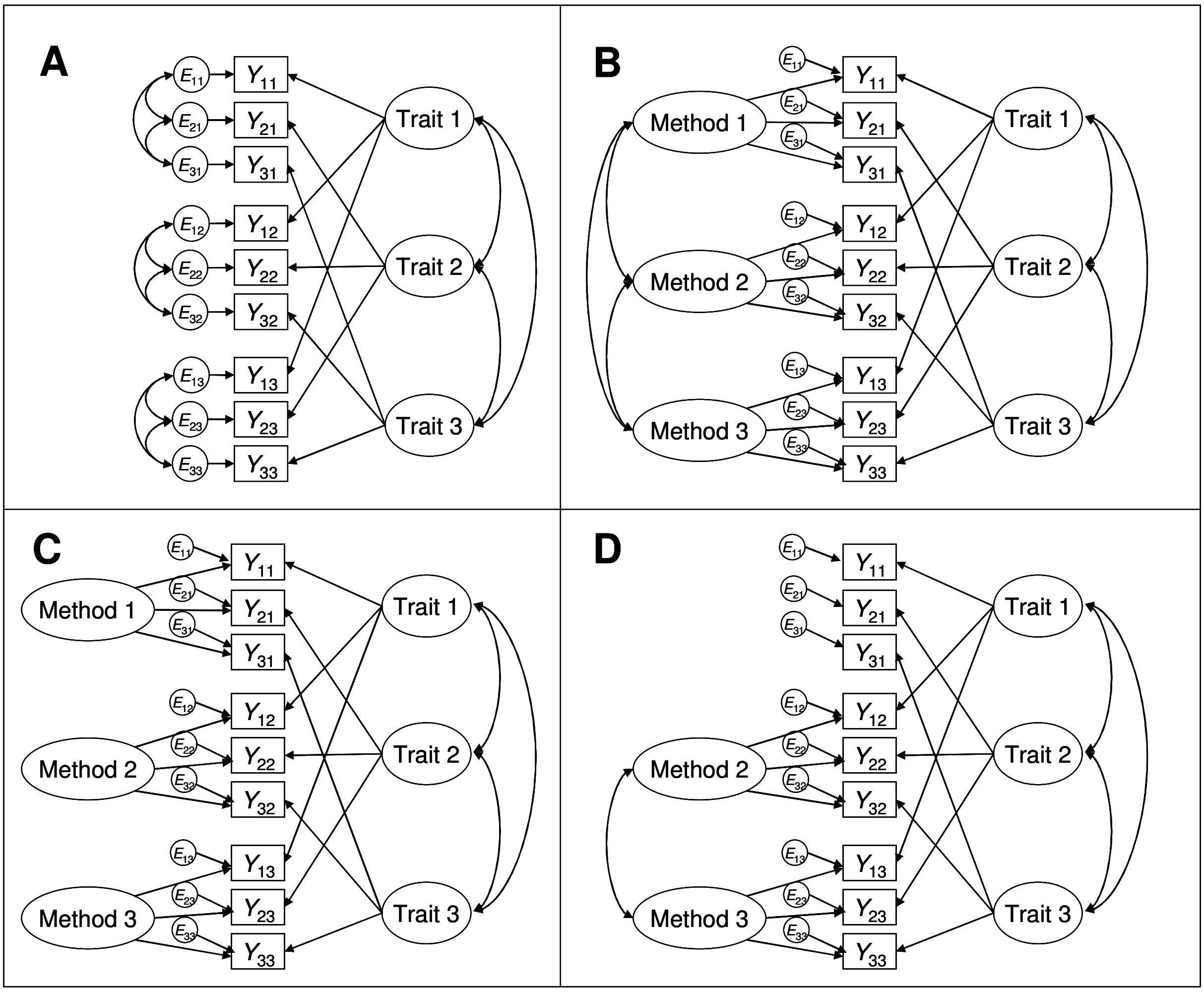


*Figure 1.* Single indicator CFA-MTMM models for analyzing cross-sectional MTMM data obtained from a design with three traits and three methods. $Y_{jk}$ = observed variable ($j$ = trait, $k$ = method); $E_{jk}$ = unique factor (error variable). A: CT-CU model. B: CT-CM model. C: CT-UM model. D: CT-C($M$–1) model. Detailed explanations are provided in the text.

In the *Correlated Trait-Correlated Method (CT-CM) model* (Marsh & Grayson, 1995; Widaman, 1985; see Figure 1B), the correlations among error variables are replaced by common method factors that account for the residual covariation among indicators sharing the same method. There is a trait factor for each trait and a method factor for each method. Trait and method factors are assumed to be uncorrelated, whereas correlations among traits and correlations among methods can be estimated. The CT-CM model overcomes several shortcomings of the CT-CU model. For a larger number of traits and methods, the CT-CM model tends to be more parsimonious than the CT-CU model. The reason is that the CT-CM model imposes a unidimensional method structure on the residual variables, whereas in the CT-CU model, all error correlations are freely estimated. (For a detailed comparison of the number of estimated parameters in both models for different MTMM designs see Lance et al., 2002, Table 4.) The CT-CM model does not confound unique (indicator-specific + error) sources of variance with method variance and, in contrast to the CT-CU model, it allows for a separation of variance components due to trait, method, and error influences to quantify the degree of convergent validity, method-specificity, and reliability of the observed variables. Moreover, in the CT-CM model, method effects can be correlated, and they can be related to external variables. The advantage is that one can study associations between different methods (e.g., do different raters share a common view of a target?), and that attempts can be made to relate method effects to covariates. Given these advantages, the CT-CM model also is a very popular model for analyzing MTMM data. Unfortunately, despite its greater theoretical soundness (compared to the CT-CU model), researchers applying the CT-CM model in practice often encounter serious problems. The CT-CM model is not globally identified and thus does not always lead to a convergent solution (Eid, 2000; Grayson & Marsh, 1994; Kenny & Kashy, 1992). In cases where the model does not converge, no parameter estimates are available and the user needs to specify a different type of model to obtain a solution for the data. Even if the model estimation procedure converges such that parameter estimates are available, *improper solutions* occur frequently in applications of the CT-CM model (Lance et al., 2002; Marsh & Bailey, 1991; Marsh, Byrne, & Craven, 1992). Improper solutions (so-called *Heywood cases*, Chen, Bollen, Paxton, Curran, & Kirby, 2001) are solutions with out-of-range parameter estimates such as negative variances or correlations estimated to be larger than $|1|$. Improper solutions pose serious problems, as they might indicate model misspecification(s) or serious estimation problems. Even if one attributes out-of-range parameter estimates to random sampling fluctuations (see e.g., Chen et al., 2001) and accepts an improper solution as valid, it is unclear how one should handle these improper parameter estimates and how they should be interpreted. Another weakness of the CT-CM model is that the interpretation of the method factors becomes dubious if all method factors are substantially correlated (Eid, Lischetzke, Nussbeck, & Trierweiler, 2003; Widaman, 1985). The method factors would then mirror general trait effects rather than method-specific influences.

A model nested within the CT-CM model that overcomes several of the shortcomings of the CT-CM model is the *Correlated Trait-Uncorrelated Method (CT-UM) model* depicted in Figure 1C. The CT-UM model is a special case of the CT-CM model in which the correlations among the method factors are set to zero. This constraint enhances the identification status of the model (unless there are fewer than three indicators for each method factor) and thus reduces the likelihood of estimation problems that are common in applications of the CT-CM model. Another consequence of the CT-UM specification is that interpretation problems due to correlated method factors are avoided. At the same time, one can still separate method effects from uniqueness and the estimation of variance components is still possible. However, the CT-UM model does not permit correlations between *any* of the method factors. From a theoretical point of view, it is an appropriate model if method effects are orthogonal. This is likely the case if the methods considered represent a random sample from a set of interchangeable methods (e.g., students randomly chosen from the “set” of all students attending a lecture to rate their professor; Eid, Nussbeck, Geiser, Cole, Gollwitzer, & Lischetzke, in press). Yet this assumption is questionable if methods are not interchangeable, but differ structurally from one another. Structurally different methods are also referred to as fixed methods (Eid et al., in press). A method is considered fixed if it cannot be replaced by another method from the same set of equivalent methods. For example, a given self-report cannot be replaced by another self-report. Likewise, for a given individual (target), there is no set of fathers or mothers from which one could randomly draw different mothers or fathers to rate traits of their son or daughter. Mother and father are fixed for each individual—at least in our western society. Given that fixed methods probably represent the most frequently used type of method in psychology, the CT-UM model appears to be less appropriate for most psychological MTMM applications.

The *Correlated Trait-Correlated (Method Minus One)* [CT-C($M$–1)] *model* (Eid, 2000; see Figure 1D) overcomes limitations of both the CT-CM and CT-CU model, while retaining most of the advantages of these two models. The term in parentheses ($M$–1) indicates that the CT-C($M$–1) model sets the method factor of one method to zero (in Figure 1D the first method, $k = 1$). That is, one uses one method factor less than methods considered. The remaining $M - 1$ method factors can be correlated as in the CT-CM model. Setting one method to zero implies that one method is selected as *reference* (or *standard*) *method*. In general, the most outstanding or established method should be taken as reference. For example, in many MTMM studies that use multiple raters as methods, the targets’ self-report will be the most prominent choice for the reference method. The reports of other raters (*non-reference methods*; e.g., parent, peer, or teacher ratings) or observational data would then serve as non-reference methods to be contrasted against the self-report. This provides an answer to the question in which way the other-reports deviate from the values that would be predicted on the basis of the target’s own view of him or herself. In other cases, one might select the method judged to be the most objective or “gold standard” measure of a construct as reference method (e.g., a physiological measure or a well-established scale). For instance, a researcher who has developed a computerized version of a well-established intelligence test might contrast the scores obtained from the paper-and-pencil version of this test against the computer version by using the paper-and-pencil version as reference method in the CT-C($M$–1) model. Such an application of the CT-C($M$–1) approach has been presented by Feigenspan (2005).

The method factors in the CT-C($M$–1) model are defined as residual factors in a true score regression analysis. In this latent regression analysis, the true score variables of the non-reference methods are regressed on the common true score of the reference method (Eid, 2000). The residuals of this regression are the method factors. As a consequence, correlations between trait and method factors are not admitted by definition of the model and must be set to zero in empirical applications. As the CT-CM and the CT-UM model, the CT-C($M$–1) model can be used to separate variance components due to trait, method, and error influences to study the convergent validity, method-specificity, and reliability of the measures. As in the CT-CM model, all method factors can be correlated to investigate similarities between methods. An important advantage of the CT-C($M$–1) model is that although it allows method factors to be correlated, it is globally identified and appears to be less prone to estimation problems and improper solutions than the CT-CM model. A limitation of the CT-C($M$–1) model is that a reference method needs to be selected and that it is not a symmetric model (as are the CT-CM and CT-UM models). Geiser, Eid, & Nussbeck (2008) provide detailed guidelines concerning the proper choice of a reference method when using the CT-C($M$–1) model. However, it might not always be easy or even possible to choose an appropriate reference method based on theoretical considerations. If methods are interchangeable (i.e., none of the methods has any specific properties that makes it different from the remaining methods), the CT-UM model or a multilevel CFA approach would be more appropriate (Eid et al., in press).

Given the asymmetry of the CT-C($M$–1) model, its fit to a given data set as well as the parameter estimates are not invariant across different reference methods. For instance, a CT-C($M$–1) model in which self-report is selected as reference method might fit worse than an alternative version in which mother or father ratings are defined as reference method. However, as Geiser et al. (2008) have shown, this limitation can be overcome if an alternative restricted model that is conceptually similar and nested within the multiple indicator CT-C($M$–1) model is used. In this restricted model variant, specific constraints on the trait factor loadings of indicators pertaining to non-reference methods are imposed. As a consequence of these restrictions, the fit of the model remains invariant if an alternative reference method is selected. Given its theoretical and practical advantages and its apparent robustness to convergence and estimation problems, the CT-C($M$–1) model can be considered one of the most useful models currently available to analyze cross-sectional MTMM data.

#### 1.3.1.2 Multiple Indicator Models

One important limitation of all MTMM models discussed so far is that each TMU is represented by only one observed variable $Y_{jk}$ in these models. The models in Figure 1 are therefore referred to as *single indicator models* in contrast to *multiple indicator models* that make use of multiple indicators per TMU. As an example, consider a classical MTMM design with three traits (depression, anxiety, and competence) and three methods (self, parent, and teacher report). In single indicator models, there would be *only one* $Y_{jk}$ variable for self-reported depression, *one* $Y_{jk}$ variable for self-reported anxiety, *one* $Y_{jk}$ variable for self-reported competence, *one* $Y_{jk}$ variable for parent-reported depression, and so on. In multiple indicator models, there would be *multiple* measures (e.g., several different items or scales) for self-reported depression, self-reported anxiety, self-reported competence, and so on.

A consequence of model specifications with single indicators per TMU is the implicit assumption that method effects generalize perfectly across traits. The main problem of single indicator models is that the (rather restrictive) assumption of general method effects is not testable in these models. There is empirical evidence that this assumption is almost never tenable (e.g., Eid et al., 2003; in press). Therefore, several researchers have proposed extensions of single indicator models to models allowing for multiple indicators per TMU (Eid et al., 2003; Marsh, 1993; Marsh & Hocevar, 1988). Multiple indicator models enable researchers to test whether method effects are trait-specific or not.

To illustrate this, consider the multiple indicator CT-C($M$–1) model proposed by Eid et al. (2003), which is shown in Figure 2. We can see that the observed variables $Y_{ijk}$ (and the error variables $E_{ijk}$) now have three indices: $i$ for the indicator, $j$ for the trait, and $k$ for the method. In the model shown in Figure 2, there are two traits, three methods, and three indicators per TMU. (An example with two traits has been selected simply to save space, but the model is not limited to only two traits.) The first method ($k$ = 1) serves as reference method. Except for this reference method, there is a (trait-specific) method factor $M_{jk}$ for each TMU. Correlations among the trait factors $T_j$ again indicate discriminant validity with respect to the reference method. Correlations between trait and method factors belonging to the same TMU are not permitted by definition of the model. (For example, the method factor $M_{12}$ is not allowed to correlate with the trait factor $T_1$ in Figure 2.)

Correlations among method factors belonging to the same method but different traits can be studied to determine the degree to which method effects are trait-specific. For example, one could assess the correlation of $M_{12}$ and $M_{22}$ in Figure 2 to estimate the generalizability of the specificity of Method 2 across Trait 1 and Trait 2. Correlations close to unity indicate that the assumption of general method effects made in single indicator models is reasonable. In contrast, if the correlations are substantially smaller than unity, this would mean that method effects are to some degree trait-specific. For example, the method effects of computerized testing on the scores of an intelligence test might be different for a reasoning subscale (Trait 1) than for a mental speed subscale (Trait 2). Zero correlations would indicate that method effects for one trait could not be used to predict method effects of the same method for another trait.

The correlations of method factors belonging to the same trait, but different methods can be used to study the degree to which non-reference methods show a common deviation from the reference method. For example, parents and teachers (non-reference methods) might share a common view of a child that is not shared with the child's own view and thus is not predictable by the self-report (reference method). The shared view of parents and teachers would express itself in the correlation among method factors belonging to the same trait and different methods, for example $M_{12}$ and $M_{13}$ in Figure 2. A more detailed discussion of all available correlations in the multiple indicator CT-C($M$–1) model can be found in Eid et al. (2003).

The main advantage of multiple indicator models is that they allow for a test of whether method effects are trait-specific. A statistical omnibus test for this question is a chi-square test of model fit for a multiple indicator CT-C($M$–1) model in which one specifies general method factors $M_k$ instead of trait-specific method factors $M_{jk}$ (see Figure 3). The specification of general method factors $M_k$ is tantamount to assuming that all method factors $M_{jk}$ with the same index $k$ are perfectly correlated. If the chi-square test of model fit is significant for the model with general method factors $M_k$ (but not significant for a model version with trait-specific method factors $M_{jk}$ as shown in Figure 2), the assumption of perfectly general method effects must be rejected in favor of the assumption of trait-specific method effects. Furthermore, the two model variants could be compared directly by using information criteria such as *Akaike's Information Criterion* (*AIC*; Akaike, 1974). According to this criterion, the model with the smaller AIC value would be preferred.

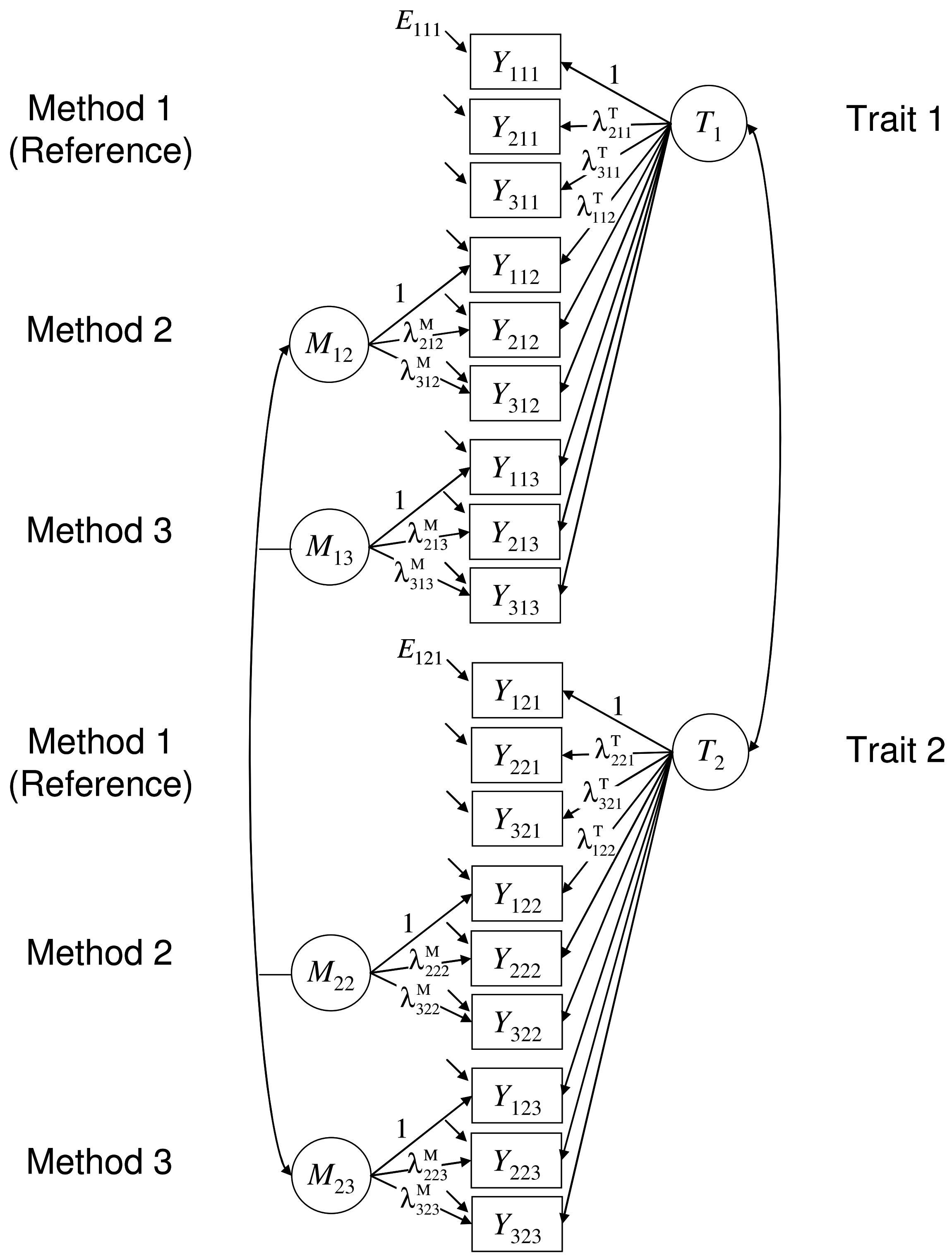


*Figure 2.* Multiple indicator CT-C($M$–1) model (Eid et al., 2003) with trait-specific method factors for two traits, three methods, and three indicators per TMU. $Y_{ijk}$ = observed variable ($i$ = indicator, $j$ = trait, $k$ = method). $T_j$ = trait factor. $M_{jk}$ = trait-specific method factor. $E_{ijk}$ = error variable. For reasons of clarity, factor loadings ($\lambda^{T}_{ijk}$, $\lambda^{M}_{ijk}$) and error variables are not shown for all indicators. Admissible correlations between trait and method factors belonging to different TMUs have also been omitted to avoid cluttering.

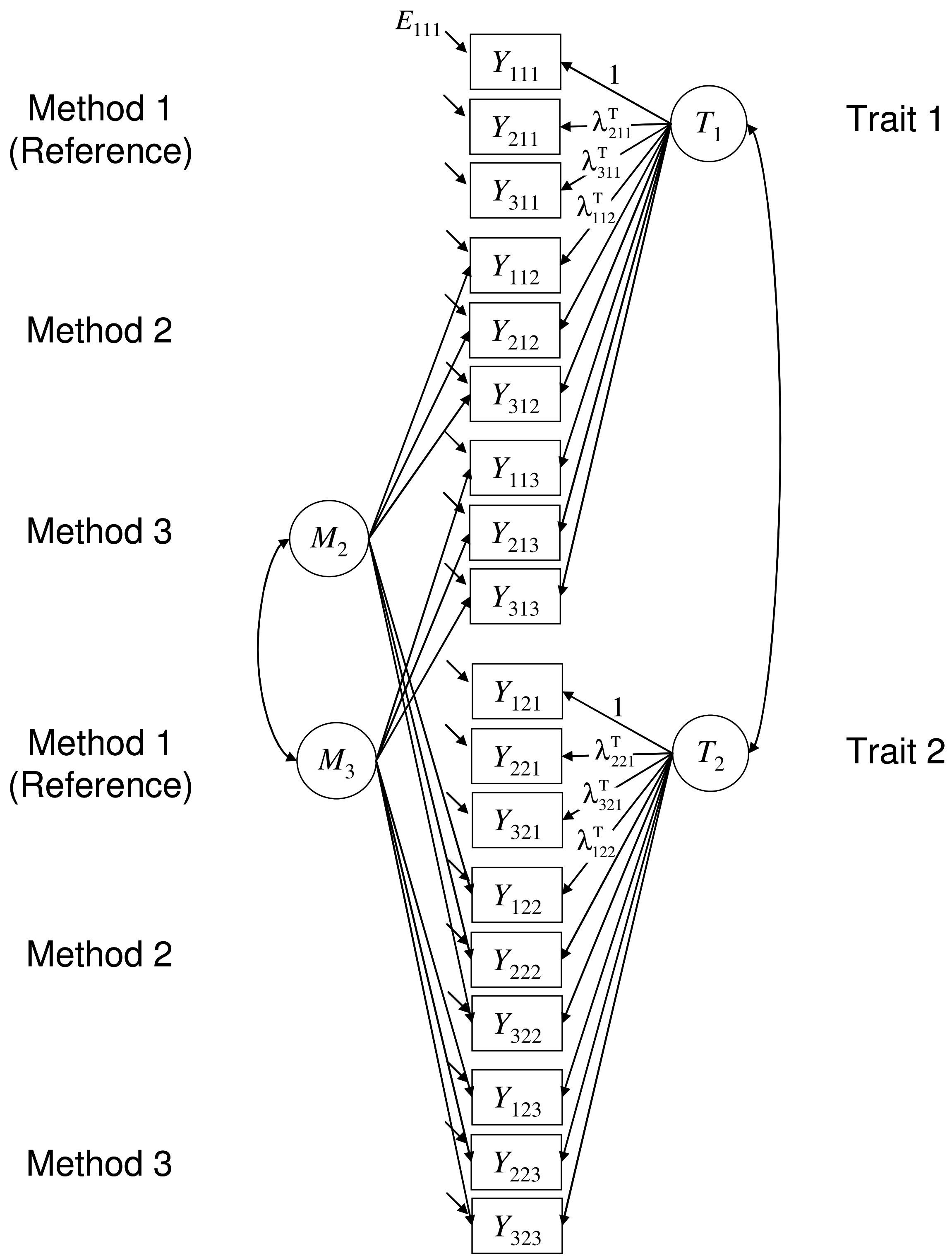


*Figure 3.* Multiple indicator CT-C(*M*–1) model (Eid et al., 2003) with general method factors for two traits, three methods, and three indicators per TMU. $Y_{ijk}$ = observed variable ($i$ = indicator, $j$ = trait, $k$ = method). $T_j$ = trait factor. $M_k$ = general method factor. $E_{ijk}$ = error variable. For reasons of clarity, trait factor loadings ($\lambda^{T}_{ijk}$) and error variables are not shown for all indicators. Method factor loadings ($\lambda^{M}_{ijk}$) as well as admissible correlations between trait and method factors belonging to different TMUs have also been omitted to avoid cluttering.

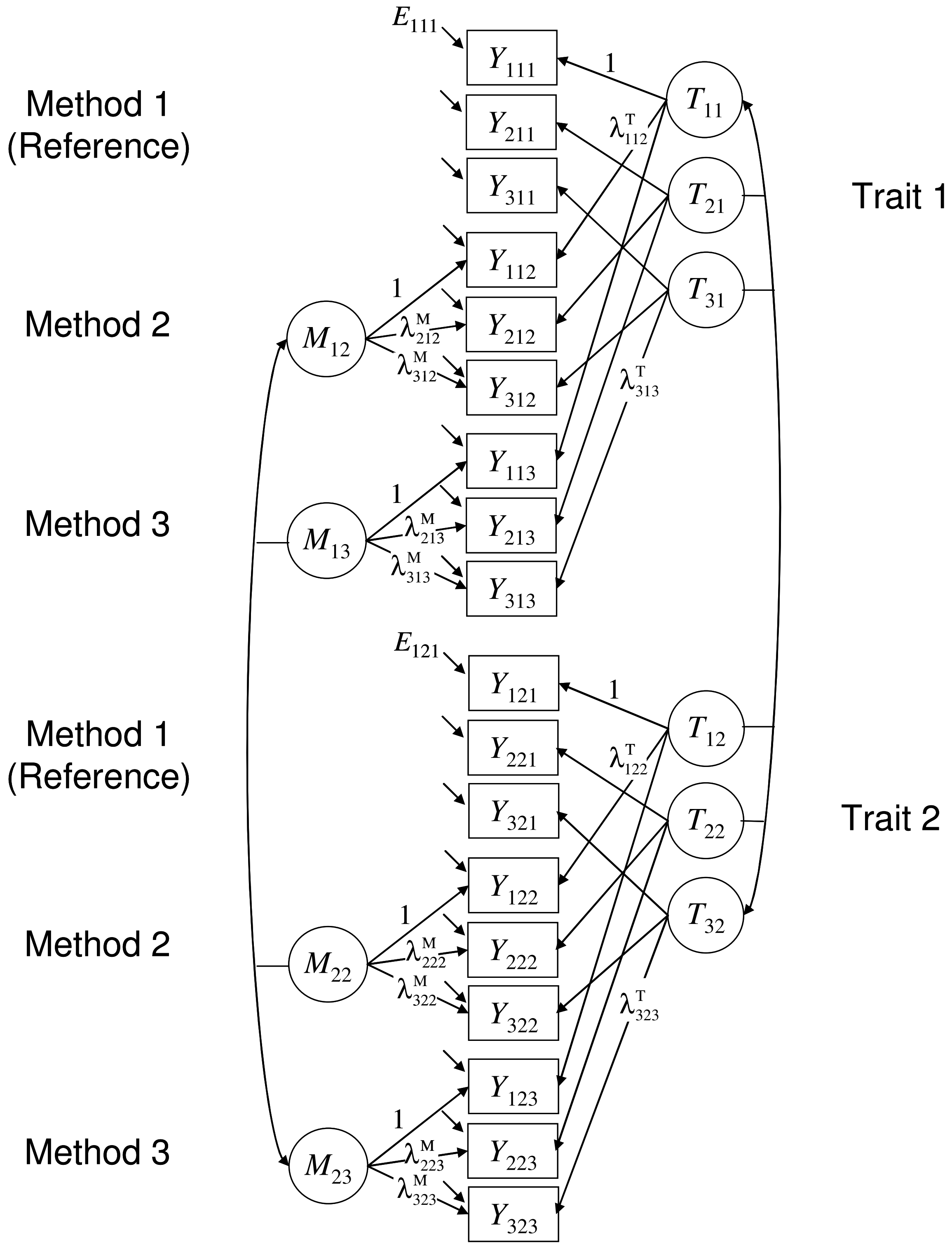


*Figure 4.* Multiple indicator CT-C($M$–1) model with trait-specific method factors and indicator-specific trait factors (Eid et al., in press) for two traits, three methods, and three indicators per TMU. $Y_{ijk}$ = observed variable ($i$ = indicator, $j$ = trait, $k$ = method). $T_{ij}$ = indicator-specific trait factor. $M_{jk}$ = trait-specific method factor. $E_{ijk}$ = error variable. For reasons of clarity, factor loadings ($\lambda^{T}_{ijk}$, $\lambda^{M}_{ijk}$) and error variables are not shown for all indicators. Admissible correlations between trait and method factors belonging to different TMUs have also been omitted to avoid cluttering.

If multiple indicators per TMU are used, these indicators may not be perfectly homogeneous within a trait. For example, each item or subscale supposed to measure the same trait might reflect a slightly different aspect or facet of the trait. These indicator-specific effects might generalize across different methods, in particular if equivalent items or scales are used across different groups of raters. If such indicator-specific effects occur, neither the model depicted in Figure 2 nor the model in Figure 3 might show an acceptable fit to the data (given sufficient statistical power to detect such effects).

An alternative, less restrictive specification of the multiple indicator CT-C($M$–1) model that takes indicator-specific effects into account has been presented by Eid et al. (in press; see Figure 4). This model variant uses indicator-specific trait variables. Figure 4 shows that there are now as many trait factors $T_{ij}$ as there are different indicators per TMU. This is also indicated by the additional index $i$ for the trait factors. The fit of the model in Figure 4 can be compared to the fit of the model in Figure 2 to find out whether the indicators are homogeneous or not. In case of very high (close to perfect) correlations among indicator-specific trait factors belonging to the same trait (e.g., between $T_{11}$, $T_{21}$, and $T_{31}$ in Figure 4), the more parsimonious solution with general trait factors $T_j$ (see Figure 2) should be preferred.

It should be noted that the multiple indicator CT-C($M$–1) model is not the only CFA-MTMM model for multiple indicators that has been proposed in the literature. Other approaches, such as the application of second order CFA, have for example been presented by Marsh and Hocevar (1988) as well as Marsh (1993). Empirical applications show that multiple indicator MTMM models are generally preferable to single indicator models given that method effects tend to be trait-specific—at least to some degree (see, e.g., Eid et al., in press).

### *1.3.2 MTMM Models for Longitudinal Data*

Despite the popularity and widespread use of MTMM analysis and CFA-MTMM models in psychological research, relatively few attempts have been made to develop and use appropriate models for longitudinal MTMM data. Almost all currently available CFA-MTMM models I know of are designed for MTMM analyses restricted to a single occasion of measurement. It has already been noted that many research questions in psychology cannot be satisfactorily answered by a cross-sectional research design. In evaluation research, for instance, the concepts of interest usually need to be measured on at least two occasions of measurement (e.g., before and after an intervention) and the focus is on *change* rather than on a single *state*. In clinical psychology, one is often interested in the stability and change of particular psychiatric symptoms in the course of a therapy (e.g., Burns, Walsh, & Gomez, 2003). In industrial psychology, employees may be evaluated on several occasions of measurement in order to study whether their performance changes over time. Likewise, many important research questions in developmental psychology are investigated by repeated assessments of children or adolescents (e.g., Cole, Martin, Peeke, Henderson, & Harwell, 1998; Cole, Martin, Powers, & Truglio, 1996; Cole, Cho, Martin, Seroczynski, Tram, & Hoffman, 2001; Nolen-Hoeksema, Girgus, & Seligman, 1992).

The tremendous number of publications devoted to statistical models for longitudinal data further underscores the importance of longitudinal research in the social sciences (e.g., Bollen & Curran, 2006; Collins & Sayer, 2001; Duncan, Duncan, Strycker, Li, & Alpert, 1999; Jöreskog, 1979a, 1979b; Little, Schnabel, & Baumert, 2000; Steyer, Ferring, & Schmitt, 1992; Steyer, Eid, & Schwenkmezger, 1997; Steyer, Schmitt, & Eid, 1999; Tisak & Tisak, 1996, 2000). However, as I noted above, only very few approaches to longitudinal data modeling have explicitly integrated the idea of multimethod measurement into their modeling framework (exceptions are discussed below). This is rather surprising given the large number of applied MTMM-MO studies that have been conducted over the past years (e.g., Biesanz & West, 2004; Burns et al., 2003; Cole et al., 1996, 1998, 2001; Conley, 1985; Corwyn, 2000; Lambert, Salzer, & Bickman, 1998; Zhou, Eisenberg, Losoya, Fabes, Reiser, Guthrie et al., 2002).

MTMM-MO designs are particularly popular in developmental psychology. Recent examples of MTMM-MO studies in developmental psychology include the multimethod investigation of adolescent popularity, social adaptation, and deviant behavior conducted by Allen, Porter, McFarland, Marsh, and McElhaney (2005), the assessment of childhood depression and anxiety by multi-informant designs (e.g., Cole et al., 1998; Tram & Cole, 2006), mediator analyses of the effects of positive parenting (measured by self- and parent reports) on mental health problems of bereaved children (Kwok, Haine, Sandler, Wolchik, Ayers, & Tein, 2005), the analysis of observed and parent-reported temperament in early childhood (Majdandzic & van den Boom, 2007), the study of the development of aggressive behavior in children as measured by observations and teacher reports (Ostrov & Crick, 2007), and Zhou et al.'s (2002) investigation of the relationship between parental warmth/positive expressiveness and children's empathy-related responding and social functioning using multiple raters.

One plausible explanation for the lack of comprehensive MTMM-MO measurement models may be that the covariance and mean structure associated with MTMM-MO data is complex, even if only a moderate number of constructs[1], methods, and time points are considered. This implies that measurement models for such type of data will be rather complicated, too. Many applied researchers may be overwhelmed by the difficulty of specifying and fitting an appropriate model in a simultaneous analysis of a complete MTMM-MO data set. Some researchers might consider not using CFA models at all and analyze manifest (observed variable) MTMM correlation matrices instead.

However, this approach has many serious limitations (Bollen, 1989). Observed variable correlation coefficients are likely attenuated by measurement error (e.g., Schmidt & Hunter, 1999). Therefore, inferences with respect to convergent and discriminant validity based on correlation tables will likely be biased. Furthermore, by analyzing manifest correlation tables, no overall theoretical model can be tested and the investigator cannot separate variance components due to trait, method, and error influences. Likewise, neither the stability of trait and method effects nor possible changes in the convergent and discriminant validity over time can be analyzed in an optimal way.

Another strategy that is sometimes applied to analyze MTMM-MO data is to use separate analyses for each time point (i.e., one specifies separate CFA-MTMM models for each wave). However, by specifying separate models, it is not possible to assess the fit of a comprehensive model including all observed variables. Therefore, no appropriate tests of measurement invariance over time can be conducted. Yet checking measurement invariance is very important in longitudinal studies in order to scrutinize whether the psychometric properties of the measures have changed over time.

Related to this question is the problem of whether the meaning of the latent variables (which represent the psychological constructs of interest and the method effects) can be considered the same on each occasion of measurement (Meredith, 1993; Tisak & Tisak, 2000). A second serious limitation of sequential CFA modeling approaches is that associations between constructs and method factors over time cannot be examined. Consequently, no statements about stability and change of constructs and method effects can be made. Moreover, questions with regard to possible changes in the convergent and discriminant validity cannot be studied in a satisfying way. Thus, in a sequential modeling

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[1] In my discussion of longitudinal MTMM approaches I use the term *construct* rather than *trait* given that in longitudinal modeling, a distinction can be made between states (i.e., the value on a construct on a specific measurement occasion comprising both stable and occasion-specific influences) and traits (i.e., the stable component of the state score; see e.g., Steyer et al., 1992, 1999). In cross-sectional data, strictly speaking, only one *state* score is available per construct. No distinction between stable (trait) and occasion-specific (state residual) influences is possible.

approach, valuable information inherent in MTMM-MO data is neglected and many important research questions cannot be examined. Therefore, there is a need for more adequate modeling techniques which consider the complex structure of an MTMM-MO matrix in a single model in order to analyze the data appropriately. In the following, I discuss three more comprehensive and sophisticated approaches to the analysis of MTMM-MO data.

#### 1.3.2.1 The Multi-Occasion CU Approach

In MTMM-MO designs, either three or four indices are needed for the observed variables, depending on whether a single indicator or a multiple indicator model is used. In single indicator models, three indices are needed ($Y_{jkl}$: $j$ = construct, $k$ = method, $l$ = occasion of measurement), whereas one needs four indices in multiple indicator models ($Y_{ijkl}$: $i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement).

In their discussion of methods for testing meditational hypotheses, Cole and Maxwell (2003) proposed a longitudinal correlated uniqueness (CU) approach in which three types of shared method variance can be represented by correlations among error variables (see Figure 5): (1) within-wave, cross-construct correlated uniquenesses for indicators pertaining to the same method mirror method effects on a given time point as in the CT-CU model (see Path A in Figure 5), (2) cross-wave, within-construct correlated uniquenesses for the same indicator capture method variance caused by stable indicator-specific effects (Path B in Figure 5), (3) cross-wave, cross-construct correlated uniquenesses may be admitted to account for additional effects due to the same method (Path C in Figure 5).

Cole and Maxwell's approach of modeling different types of shared method effects through error correlations parallels Kenny's (1976) CT-CU model for cross-sectional MTMM data (see Section 1.3.1.1 and Figure 1). The advantage of Cole and Maxwell's multi-occasion CU model is that method and error effects are taken into account so that structural (e.g., meditational) hypotheses can be more accurately tested.

On the other hand, the same disadvantages discussed above for the single-occasion CT-CU model apply to the multi-occasion CU model as well: (1) many uniqueness correlations need to be estimated in designs with many indicators, constructs, methods, and time points, (2) random measurement error cannot be separated from indicator-specific variance and shared method variance leading to an underestimation of the reliabilities of the measures, (3) variance components due to construct, method, indicator-specificity, and error are not available, (4) correlations between different methods cannot be estimated, and (5) method effects cannot be related to external variables. Another limitation is that Cole and Maxwell's model is a single indicator model that does not allow for construct-specific method effects.

A model that overcomes some of the limitations of the multi-occasion CU model is Burns et al.'s (2003) multi-occasion extension of the CT-CM model (see also Burns & Haynes, 2006). This model is discussed in the next section.

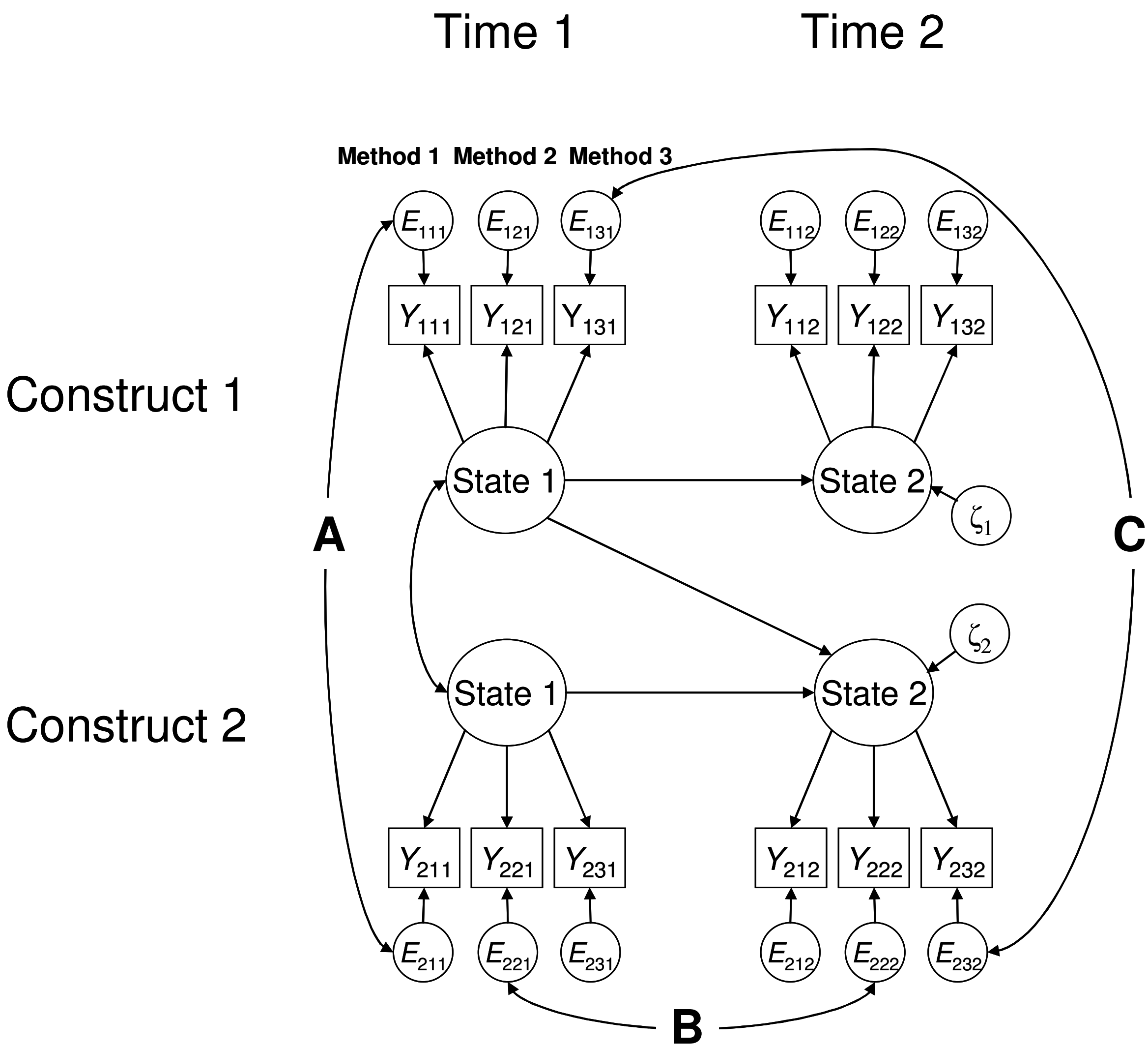


*Figure 5.* Multi-occasion CU model (Cole & Maxwell, 2003) for two constructs, three methods, and two occasions of measurement. $Y_{jkl}$ = observed variable ($j$ = construct, $k$ = method, $l$ = occasion of measurement). $E_{jkl}$ = error variable. Three types of correlated uniquenesses are shown. A: within-wave, cross-construct correlated uniquenesses, B: cross-wave, within-construct correlated uniquenesses for the same indicator, C: cross-wave, cross-construct correlated uniquenesses for the same method. Not all possible correlated uniquenesses are shown for reasons of clarity.

#### 1.3.2.2 The CS-CM Model

Figure 6 shows the multi-occasion extension of the single indicator CT-CM model proposed by Burns et al. (2003; see also Burns & Haynes, 2006) for three constructs, three methods, and two occasions of measurement. Instead of including various kinds of correlated uniquenesses, Burns et al.'s model uses occasion-specific method factors to capture cross-sectional method effects. The model can be seen as a special kind of multistate model (see Chapter 2) with one state factor for each construct and one method factor for each method on each occasion of measurement. Hence, I will refer to it as *Correlated State-Correlated Method* (CS-CM) model.

As in the CT-CM model for a single time point, all method factors can be correlated and all state factors can be correlated. (Note that a similar approach with uncorrelated method factors is discussed in Scherpenzeel & Saris, 2007.) In CS-CM model, correlations over time can also be examined. The admissible across-time correlations indicate the stability of inter-individual differences with respect to construct and method effects. Correlations between state and method factors are not admitted.

Burns et al.'s approach allows separating occasion-specific variance due to a construct from occasion-specific method variance and error influences. Furthermore, different methods can be correlated and external variables can be included to explain method effects.

A potential shortcoming is that the CS-CM model is based on the cross-sectional CT-CM model and thus might be prone to similar identification, estimation, and interpretation problems. Another limitation is that Burns et al. did not discuss whether a mean structure can be included in their model. Mean structures are generally of interest in longitudinal studies, as one often seeks to investigate mean changes over time. Furthermore, and related to the question of mean structures, Burns et al. did not address the issue of testing measurement invariance over time. Measurement invariance concerns the question of whether the same constructs are measured on each occasion of measurement (Meredith, 1993; Tisak & Tisak, 2000; Raykov, 2006) and is a very important issue in longitudinal modeling. Finally, as Cole and Maxwell's model, the CS-CM model is a single indicator model. Hence, it cannot be used to study construct-specific method effects. The next model overcomes this limitation.

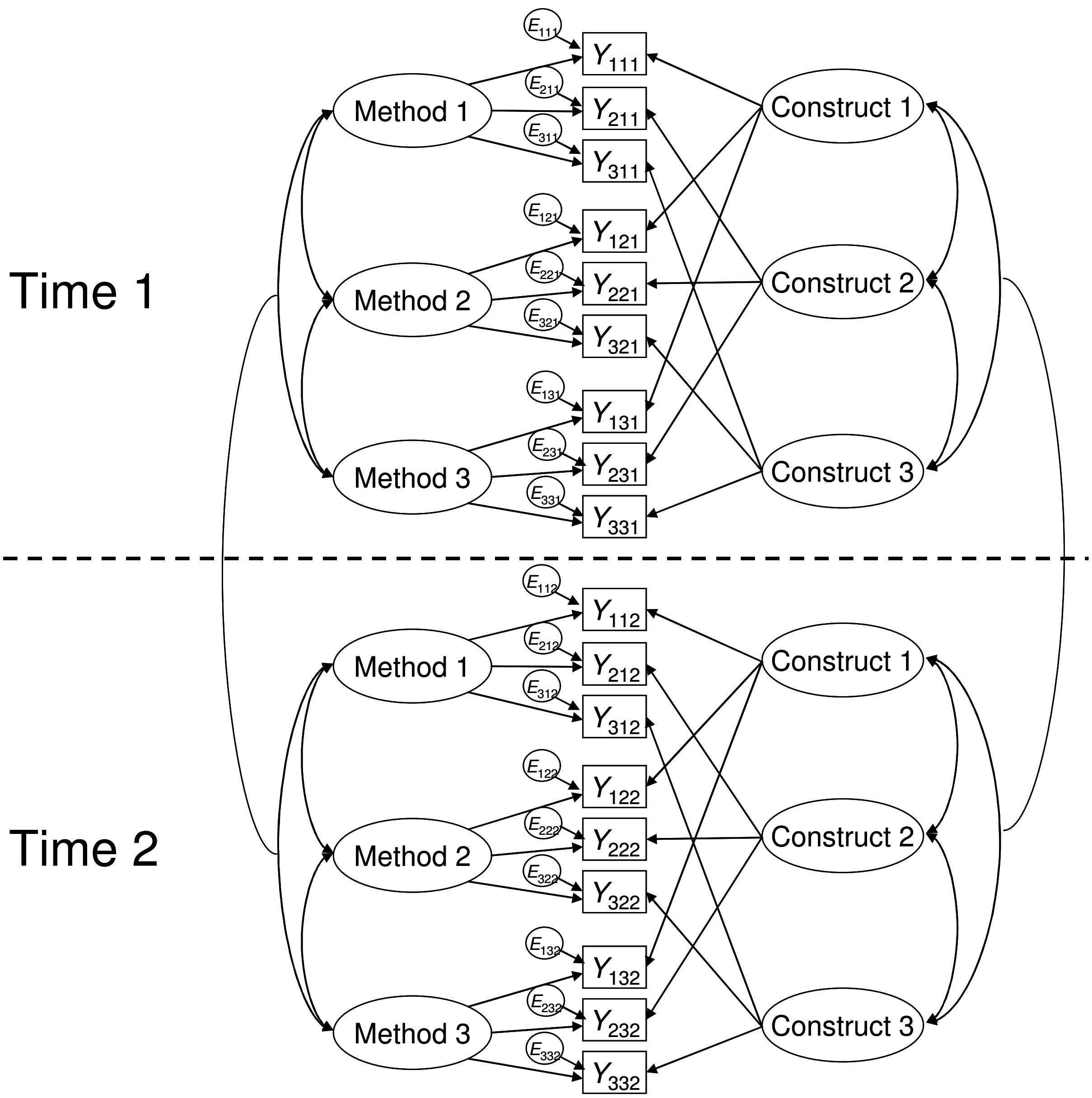


*Figure 6.* CS-CM model (Burns et al., 2003) for three constructs, three methods, and two occasions of measurement. $Y_{jkl}$ = observed variable ($j$ = construct, $k$ = method, $l$ = occasion of measurement). $E_{jkl}$ = error variable.

### 1.3.2.3 The Multi-Method Latent State-Trait Model

Courvoisier (2006; Courvoisier, Nussbeck, Eid, Geiser, & Cole, in press) recently presented a multiple indicator model for analyzing MTMM-MO data. Courvoisier's model represents an extension of the latent-state trait (LST) model (Steyer, 1988; Steyer et al., 1992, 1999) for mono-method data to a multi-method LST model (see Figure 7). In general, LST models allow for a separation of situation-specific (state-like) influences from stable (trait-like) and error components of variance (e.g., Steyer et al., 1999). Courvoisier's multi-method

LST model makes it possible to determine occasion-specific (state residual) and trait influences separately for different methods. In this way, a researcher can scrutinize whether the strength of occasion-specific influences on psychological measures differs for different methods.

Figure 7 shows an example in which one construct is measured by two methods on two occasions of measurement. Each construct-method-occasion unit (CMOU) is represented by two indicators $Y_{ijkl}$. As in Eid's (2000) CT-C($M$–1) approach, one method is selected as reference, here the first method ($k$ = 1). Each indicator pertaining to the reference method loads on an indicator-specific trait factor and an (occasion-specific) state residual factor (shown on the right hand side of Figure 7). Note that trait factors are shown in grey circles, whereas state residual factors are shown in white circles in Figure 7. The trait factors represent stable inter-individual differences with respect to the reference method. The state-residual factors capture reliable occasion-specific influences of the reference method. Indicators pertaining to the non-reference method also load on the trait and state residual factors of the reference method. In addition, these indicators are influenced by trait and state residual factors *specific* to the non-reference method (shown on the left hand side of Figure 7).

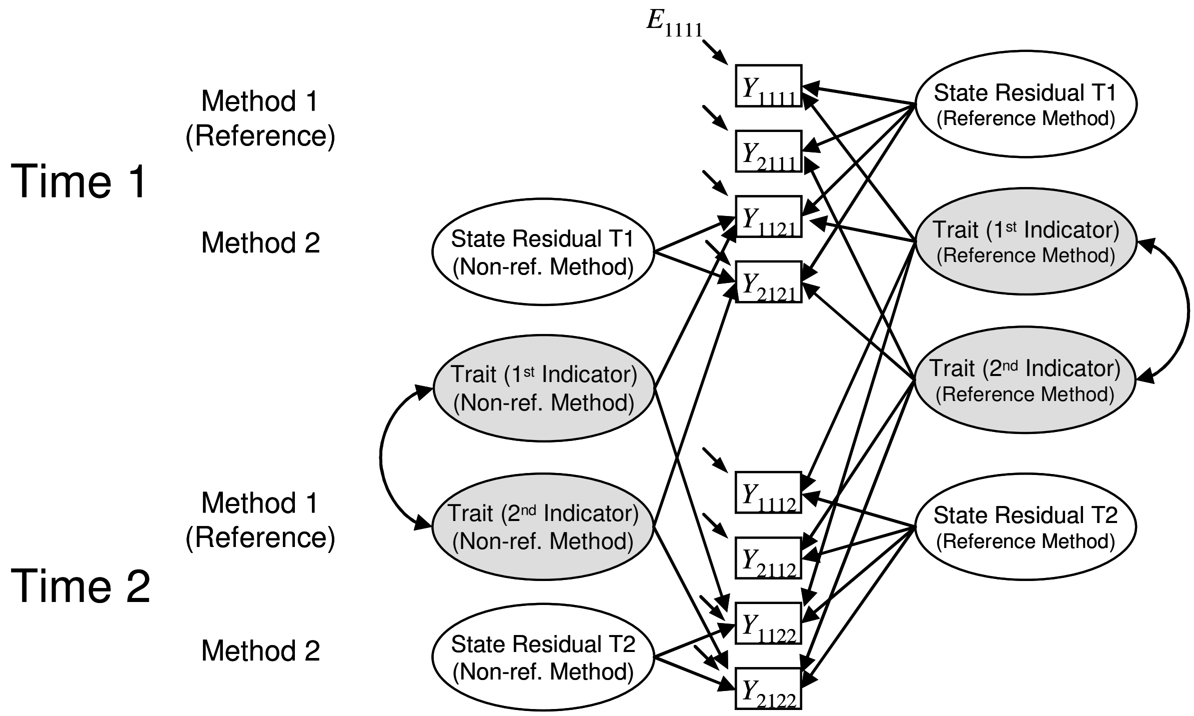


*Figure 7.* Multi-method LST model (Courvoisier, 2006) for one construct, two methods, and two occasions of measurement. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $E_{ijkl}$ = error variable.

The multi-method LST model allows researchers to contrast trait and occasion-specific effects in a reference method against trait and occasion-specific effects in non-reference methods. Five different components of variance can be separated: (1) the proportion of variance explained by the reference method trait factor, (2) the proportion of variance explained by the particular non-reference method trait factor, (3) the proportion of variance due to occasion-specific effects shared with the reference method, (4) the proportion of variance due to occasion-specific effects specific to the particular non-reference method, and (5) the proportion of variance due to measurement error.

The multi-method LST model can be estimated for a single or for multiple constructs. For perfectly homogeneous indicators, the indicator-specific trait factors can be replaced by general trait factors to make the model more parsimonious. Admissible and non-admissible correlations as well as applications of this model to multiple constructs are discussed in Courvoisier (2006; see also Courvoisier et al., in press). A related approach has been presented by Vautier (2004) who showed how method effects caused by bipolar items can be studied in a multi-method LST model. Scherpenzeel & Saris (2007) discussed how a multi-occasion version of the CT-UM model can be extended to a multi-method state-trait model, but they considered only one indicator per CMOU.

LST models assume that on the one hand, there is a stable construct (trait) that influences behavior, and on the other hand, there are occasion-specific fluctuations around the stable trait component (influences of the situation in which measurement takes place). The concept of a stable trait underlying behavior on the one hand and an occasion-specific part on the other hand has proven to be very useful in many areas of psychology (see, e.g., Eid, Schneider, and Schwenkmezger, 1999; Steyer et al., 1992, 1999). LST approaches are especially useful when the process to be analyzed is *variability*.

However, this assumption does not hold for all phenomena in psychology. There are many examples of psychological constructs for which it does not make sense to assume an underlying stable trait value that remains the same across the life span. For example, many attributes studied in developmental psychology are subject to *enduring changes* (e.g., language acquisition, social behavior, school achievement, etc.). They are not just fluctuating around a stable trait. Likewise, in most intervention studies, it is not variability around a stable value that is of interest. Most interventions actually aim at causing (hopefully lasting) trait changes (e.g., changes in mental health, substance abuse, health behavior, etc.).

Hence, researchers are often interested in trait changes over time rather than in separating trait effects from time-specific variability. An appropriate general MTMM model for analyzing trait changes over time has not yet been presented. There is a need for a model that does not make the (often too restrictive) assumption of an underlying stable trait and that can be used to assess trait changes over time in a multimethod context.

## 1.4 Aims and Structure of the Present Work

The aim of the present work is to develop appropriate measurement models for MTMM-MO data that are as general, flexible, and comprehensive as possible and that allow for an assessment of trait change over time. I will present two approaches: A general MTMM state model and a change version of this model. These models do not make the (sometimes too restrictive) assumption of an underlying stable trait. Instead, they focus on correlated latent *states* that may change over time. Hence, the models presented here are less restrictive than, for example, Courvoisier's (2006) multi-method LST model.

The new models will be formulated based on stochastic measurement theory (Steyer, 1989; Zimmermann, 1975). The advantage of this approach is that (1) all latent variables are clearly defined and have a clear psychometric meaning, (2) the assumptions of the model are clearly understandable, and (3) one can derive the implications of these assumptions for the identifiability of the model parameters and the testability of the model.

The difference between Courvoisier's (2006) and Vautier's (2004) multi-method LST approaches and the models presented here is that the models developed in this thesis are not concerned with a separation of trait, time-specific, and error components of variance for different methods. Rather, my goal is to develop a general MTMM measurement model that can be used to study the important question of measurement invariance over time and to propose an alternative parameterization of the model that can be used to study inter-individual differences in intra-individual change on the level of latent state and method variables. As mentioned above, the models presented here are more general and less restrictive than models in which state factors are further decomposed into trait and state residual factors (such as the multi-method LST model). The present models can for example be applied in evaluation studies in which the goal is to investigate change between different time points.

In the theoretical part, a detailed psychometric analysis of the models will be provided. In the empirical part, I will assess the applicability of the models to a real MTMM-MO data set and present results of a small Monte Carlo simulation study in which I studied the performance of the models for different sample sizes. In the final section, I will discuss advantages and limitations of the models. Furthermore, in the final discussion, I will provide a comparison of the models presented here to already established methods for analyzing MTMM-MO data as well as detailed guidelines for potential users of the new models.

The next chapter is an introduction to the basic concepts of classical test theory (CTT) and latent state (LS) theory. These concepts are reviewed here as they play a key role in the definition of the new MTMM-MO models described in Chapters 3 and 4. Readers may wonder why CTT is reviewed here, as it has little to do with longitudinal modeling or with measuring change. I nonetheless provide a brief introduction into CTT given that the concept of the *true score variable* plays an important role in the definition of the MTMM-MO models presented in this work.

# 2 Classical Test Theory and Latent State Theory

## 2.1 Classical Test Theory

In order to formulate psychometric models as stochastic measurement models, it is necessary to define a probability space for all variables considered in a model. A probability space consists of three components $(\Omega, \mathfrak{A}, P)$, where $\Omega$ denotes the set of possible outcomes (the elements of $\Omega$ are explained in detail below), $\mathfrak{A}$ denotes a $\sigma$-Algebra consisting of subsets of $\Omega$, and $P$ is a non-negative, countable additive set function on $\mathfrak{A}$ with $P(\Omega) = 1$ (i.e., $P$ is a probability function of $\Omega$; for a detailed explanation of the components of the probability space, see Eid, 1995; Steyer, 1988, 1989; or Steyer & Eid, 2001). The three components of the probability space describe the random experiment which transforms an empirical phenomenon into a statistical measurement model. In the random experiment a unit *u* is drawn from a set *U* of observational units. This unit *u* could for example be a child sampled from a group of children. Then, one may record the values of *u* with respect to certain attributes of interest. For example, the child may be asked to complete a self-report questionnaire that contains 6 binary items to measure depression. This experiment is considered *random* because neither do we know a priori *which* unit *u* (which child) will be drawn nor do we know *how* this child will respond to the items of the questionnaire. The set $\Omega$ of possible outcomes of this random experiment can be expressed as a set product

$$\Omega = U \times M, \tag{1}$$

where *M* represents the set of possible values (e.g., the possible scores on the depression self-report questionnaire, for example 0–6). An element $\omega \in \Omega$ is a possible outcome of the random experiment. For example, an outcome could be:

$$\omega = \langle \text{Jerry, } 3 \rangle. \tag{2}$$

In this random experiment *Jerry* was drawn from the set of observational units *U* and Jerry achieved a total score of 3 on the depression self-report questionnaire. On the other hand, the set of possible values *M* may contain more than a single outcome such that

$$M = M_1 \times \ldots \times M_m, \tag{3}$$

where $M_i,\ i = 1,...,m$, represents the set of possible values of the *i*th observed attribute (for example, *i* could indicate the items of the 6-item depression questionnaire). Hence, a possible outcome of a random experiment at the item level could be

$$\omega = \langle \text{Jerry, agree, agree, disagree, agree, disagree, disagree} \rangle, \tag{4}$$

where again Jerry was drawn and he agreed with the first, second and fourth item statements, whereas he disagreed with the third, fifth and sixth item statements.

The outcomes of such random experiments can also be expressed in terms of numerical random variables $Y_i : \Omega \to \mathbb{R},\ i = 1,...,m$, where $\mathbb{R}$ denotes the set of real numbers. For example, the binary responses to the six items of the depression questionnaire may be transformed to numerical values by assigning the value of 0 for the response *disagree* and and 1 for the response *agree*. Another possibility is that the variables $Y_i$ represent continuous test score variables (e.g., the sum score of the depression questionnaire).

In CTT, the mapping $p_U : \Omega \to U$ indicates which unit *u* has been drawn in the random experiment. In other words, the values of the mapping $p_U : \Omega \to U$ are the observational units $u \in U$ (e.g., persons). This mapping is used to define the *true score variables* $\tau_i,\ i = 1,...,m,$:

$$\tau_i := E(Y_i \mid p_U), \tag{5}$$

where $E(Y_i \mid p_U)$ is the conditional expectation (regression) of the variables $Y_i$ given $p_U$. The values of $E(Y_i \mid p_U)$ are the so-called *true scores* of the observational units *u* [i.e., $E(Y_i \mid p_U = u)$]. The *error variables* $E_i,\ i = 1,...,m,$ are defined as residuals of the regressions of the variables $Y_i$ on the true score variables $\tau_i$:

$$E_i := Y_i - E(Y_i \mid p_U) = Y_i - \tau_i. \tag{6}$$

A simple rearrangement of Equation 6 yields the well-know decomposition of an observed variable $Y_i$ in CTT:

$$Y_i = \tau_i + E_i. \tag{7}$$

The true score variables $\tau_i$ represent that part of the observed variables $Y_i$, which is due to true inter-individual differences. The residuals $E_i$ comprise (unsystematic) influences due to measurement error. The definition of $\tau_i$ and $E_i$ implies that the variables $E_i$ have an expectation of zero and that $\tau_i$ and $E_i$ are uncorrelated with each other (for a more detailed description of the properties of $\tau_i$ and $E_i$, see, e.g., Steyer, 1988, 1989; as well as Steyer & Eid, 2001). An important consequence of the uncorrelatedness of $\tau_i$ and $E_i$ is that the variances of the observed variables $Y_i$ can be additively decomposed into true score and error variance:

$$Var(Y_i) = Var(\tau_i) + Var(E_i). \tag{8}$$

Hence we may define the *reliability* $Rel(Y_i)$ as the ratio of true score variance to observed variance:

$$Rel(Y_i) = \frac{Var(\tau_i)}{Var(Y_i)}. \tag{9}$$

The reliability coefficient $Rel(Y_i)$ varies between 0 and 1, where 0 indicates a completely unreliable measure and 1 indicates perfect reliability (no measurement error).

To identify the parameters of the true score model, to separate measurement error from true inter-individual differences, and to test specific assumptions, special *models* of CTT have to be used. Furthermore, more than one observed variable of a construct needs to be assessed. An example of a just-identified CTT model is depicted in Figure 8. Figure 8 shows the *model of τ-congeneric variables* for three observed variables[2]. Figure 8A shows that each observed variable $Y_i$ is regressed on its own latent true score variable $\tau_i$, corresponding to the true score model in Equation 7. In the model of $\tau$-congeneric variables, the true score variables pertaining to indicators of the same construct are assumed to be linear functions of each other:

$$\tau_i = \alpha_{ii'} + \lambda_{ii'} \cdot \tau_{i'}, \tag{10}$$

[2] Not all models of CTT require three indicators to identify the parameters of the model. For example, for the model of essentially τ-equivalent variables with equal error variances (also know as model of τ-parallel variables) to be identified, two indicators are sufficient (see Steyer & Eid, 2001). I present the model of τ-congeneric variables here as it is a general CTT model that is often implicitly used as measurement model in latent variable SEMs for continuous outcomes.

where $i,\ i' = 1,...,m$, $\alpha_{ii'}$ is an intercept term, and $\lambda_{ii'}$ denotes the slope. This assumption implies that the true score variables $\tau_i$ are perfectly correlated and thus can be replaced by a common true score variable (or "common factor") η, where $\tau_i = \alpha_i + \lambda_i \cdot \eta$ (see Steyer & Eid, 2001). Furthermore, the assumption of uncorrelated error variables is made in Figure 8 [i.e., $Cov(E_i, E_{i'}) = 0$, for $i \neq i'$]. Note that $\lambda_1$ has been set equal to unity in Figure 8 to identify the model. Notice also that the true score variables $\tau_i$ have no associated residual terms because they are completely determined by the common factor η. One can therefore drop the $\tau_i$'s from the figure without loss of information, as is done in Figure 8B. In Figure 8B, the $Y_i$-variables load directly on η. As you can see, Figure 8B is in line with the reflective measurement model or "common factor model" commonly used in SEM (e.g., Bollen, 1989; Jöreskog, 1968, 1971a). The model in Figure 8 is just-identified. Nine pieces of information are available (three observed means, three variances, and three covariances), and nine parameters are estimated (three intercepts, two loadings, three residual variances, and the variance of η)[3]. Hence, the model has zero degrees of freedom, implying that the assumption of τ-congenerity is not testable with only three observed variables (it would be testable for four or more measures of a construct).

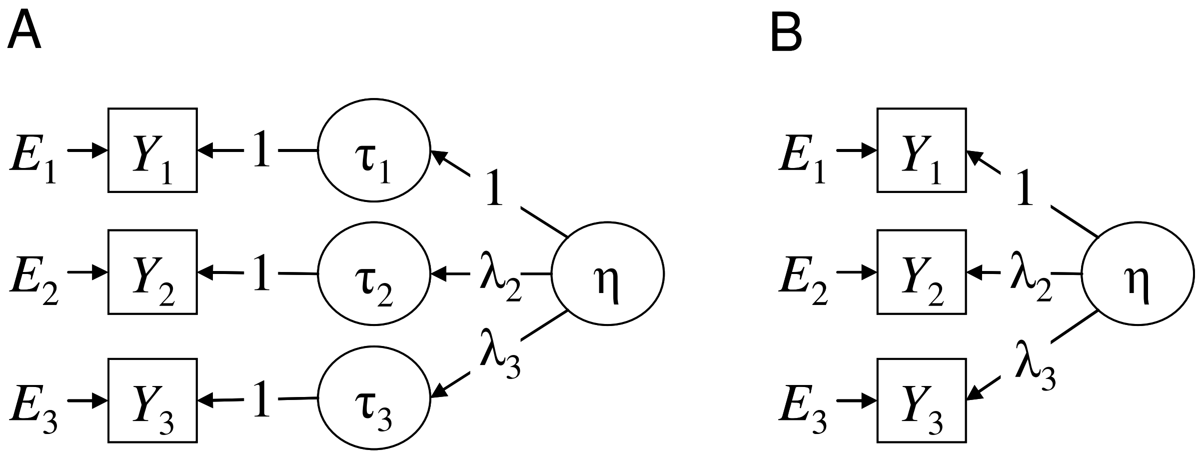


*Figure 8.* Path diagram illustrating the 1-factor model implied by the CTT model of τ-congeneric variables for three observed variables. $Y_i$ = observed variable. $\tau_i$ = indicator-specific true score variable. η = common true score variable. $E_i$ = error variable. Here, the loading $\lambda_1$ is set equal to 1 for identification. Intercepts $\alpha_i$ are not shown. A: All indicator-specific true score variables $\tau_i$ are included. B: The indicator-specific true score variables $\tau_i$ have been dropped.

[3] The mean of η is set to zero to identify the model. An alternative to this specification would be to set one $\alpha_i$ to zero to achieve identification and estimate the mean of η.

The model of $\tau$-congeneric variables is less restrictive than other models of CTT since the observed variables are allowed to differ in difficulty (they can have different intercepts $\alpha_i$) and in discrimination (they can have different loadings $\lambda_i$). Other, more restricted models of CTT are nested within the model of $\tau$-congeneric variables. For instance, the model of essentially $\tau$-equivalent variables can be derived from the model of $\tau$-congeneric variables by constraining the factor loadings $\lambda_i$ to be equal for all indicators. The model of fully parallel variables is obtained by setting the intercepts $\alpha_i$, the factor loadings $\lambda_i$, and the error variances $Var(E_i)$ equal for all indicators. For a more detailed discussion of different models of CTT see Steyer (1989) or Steyer and Eid (2001).

An important limitation of CTT is that it does not explicitly consider situation-specific influences on psychological scores. It is well-known that most psychological variables (even attributes commonly understood as stable traits) cannot be measured completely independently of the situation in which measurement takes place. If we take the example of the depression self-report questionnaire, this questionnaire likely does not measure a depression *trait* but rather a depression *state* (i.e., depression at the particular occasion on which measurement takes place). Occasion-specific effects are likely to influence most observed variables. Ignoring such effects would cause no serious problems as long as situations could be assumed to vary randomly for all observational units. In many cases, however, this is not a reasonable assumption. Instead, psychological assessment often takes place in the same situation (i.e., the situation during assessment is the same for all units *u*, e.g., all children are tested at the same time in the same stuffy class-room).

In order to account for the effects of situations and person-situation interactions on psychological measurement, CTT has been generalized to latent state (LS) and LST theory (Steyer, 1988; Steyer et al., 1992). The basic concepts of LS theory will be introduced in the next section. LST theory will not be treated in detail, as the concepts of LST theory are less important for the formulation of the MTMM-MO models defined in this thesis.

## 2.2 Latent State Theory

In LS theory, the random experiment considered in CTT is extended in order to take into account that persons are measured in situations. The set *U* of observational units has a different meaning. In LS theory, it is the set product

$$U = U_0 \times U_1 \times \ldots \times U_n, \tag{11}$$

where $U_0$ is the set of persons and the sets $U_l$, $l = 1, \ldots, n$, contain possible situations on a given occasion of measurement *l*. In other words, the observational units *u* in LS theory are not *persons*, but *persons-in-a-situation*. For example, Jerry is drawn and his score in the depression self-report questionnaire is recorded on a first occasion of measurement ($l = 1$). On this occasion, a certain situation from the set $U_1$ of situations is present (e.g., Jerry is in a particularly good mood because his grand-father was visiting him the day before his score on the depression questionnaire was recorded). On a second occasion of measurement ($l = 2$), Jerry is tested again. Now, he may be in a different situation (this time stemming from the set $U_2$ of situations), where he is in a bad mood because he obtained a bad grade at school. It is important to notice that the situations are inner states. These states may depend on outer influences, but also on inner influences. Therefore, they are difficult to measure. Moreover, they do not have to be the same for all participants, but can vary from individual to individual. The set of possible outcomes has to be extended in order to include the possible influences of *n* occasions of measurement:

$$\Omega = U_0 \times U_1 \times \ldots \times U_n \times M_1 \times \ldots \times M_n . \tag{12}$$

There are now two mappings. First, there is the mapping $p_0 : \Omega \to U_0$. The values of the mapping $p_0 : \Omega \to U_0$ are the observational units (persons) as in CTT. Second, there is the mapping $p_l : \Omega \to U_l$. The values of the mapping $p_0 : \Omega \to U_0$ are the situations in which persons are measured on a particular measurement occasion *l*. Finally, the values of $(p_0, p_l)$ are the *persons in situations* on measurement occasion *l*.

The counterpart to the true score variables $\tau_i$ in CTT are the *latent state variables* $S_{il}$ in LS theory, where the index *i* again denotes the observed variable and *l* denotes the occasion of measurement:

$$S_{il} := E(Y_{il} \mid p_0, p_l). \tag{13}$$

In Equation 13, $E(Y_{il} \mid p_0, p_l)$ is the conditional expectation (regression) of an observed variable $Y_{il}$ given the person and the situation. As in CTT, the measurement error variables $E_{il}$ are defined as residuals with respect to this regression:

$$E_{il} := Y_{il} - E(Y_{il} \mid p_0, p_l) = Y_{il} - S_{il} . \tag{14}$$

The decomposition of an observed variable in LS theory is analogous to the decomposition in CTT. Each observed variable is decomposed into a latent state variable (the occasion-specific true score) and an error variable:

$$Y_{il} = S_{il} + E_{il}, \tag{15}$$

where $E_{il}$ has an expected value of zero and is uncorrelated with $S_{il}$. The variance decomposition in LS theory is given by:

$$Var(Y_{il}) = Var(S_{il}) + Var(E_{il}). \tag{16}$$

The reliability coefficient may again be defined as the ratio of true score variance to observed variance:

$$Rel(Y_{il}) = \frac{Var(S_{il})}{Var(Y_{il})}. \tag{17}$$

In the extension of LS theory to LST theory, the latent state variables $S_{il}$ are further decomposed into stable and occasion-specific parts (Steyer, 1988). The details of this decomposition will not be addressed here as they are not relevant for an understanding of the MTMM-MO models defined in Chapters 3 and 4. Readers interested in a detailed treatment of LST theory can refer to Steyer (1988) as well as Steyer et al. (1992, 1999).

### *2.2.1 The Correlated State Model*

A well-known model derived from LS theory is the *correlated state* (CS) or *multistate model* (e.g., Steyer et al., 1992, see Figure 9). The CS model makes the assumption of *occasion-specific congenerity* (Steyer et al., 1992). Occasion-specific congenerity means that all latent state variables belonging to the same construct that are measured on the same occasion of measurement are linear functions of each other:

$$S_{il} = \alpha_{il} + \lambda_{il} \cdot S_{i'l}, \tag{18}$$

where $i$, $i' = 1, \ldots, m$, $\alpha_{il}$ is an intercept, and $\lambda_{il}$ denotes a slope parameter (factor loading). As a consequence of Equation 18, one may assume that there is a *common* occasion-specific state factor $S_l$ on each occasion of measurement. Under this assumption, one obtains the following measurement model for the observed variables (see also Figure 9):

$$Y_{il} = \alpha_{il} + \lambda_{il} \cdot S_l + E_{il} . \tag{19}$$

The common latent state factors $S_l$ can be correlated. The size of these correlations indicates the degree of (covariance) stability over time. The CS model can be seen as a basic measurement model for longitudinal data (Tisak & Tisak, 2000). A CS model for one construct measured by two observed variables on two occasions of measurement is depicted in Figure 9. Note that the loadings $\lambda_{11}$ and $\lambda_{12}$ are fixed to unity for identification. Furthermore, the assumption of uncorrelated error variables is made [i.e., $Cov(E_{il}, E_{i'l'}) = 0$, for $i, i' = 1,...,m$; $l, l' = 1,...,n$; and $(i,l) \neq (i',l')$]. Figure 9A and B show two equivalent model versions. In Figure 9A, the latent state true score variables $S_{il}$ are still included. In Figure 9B, the variables $S_{il}$ have been dropped, in line with the more commonly used "common factor" approach.

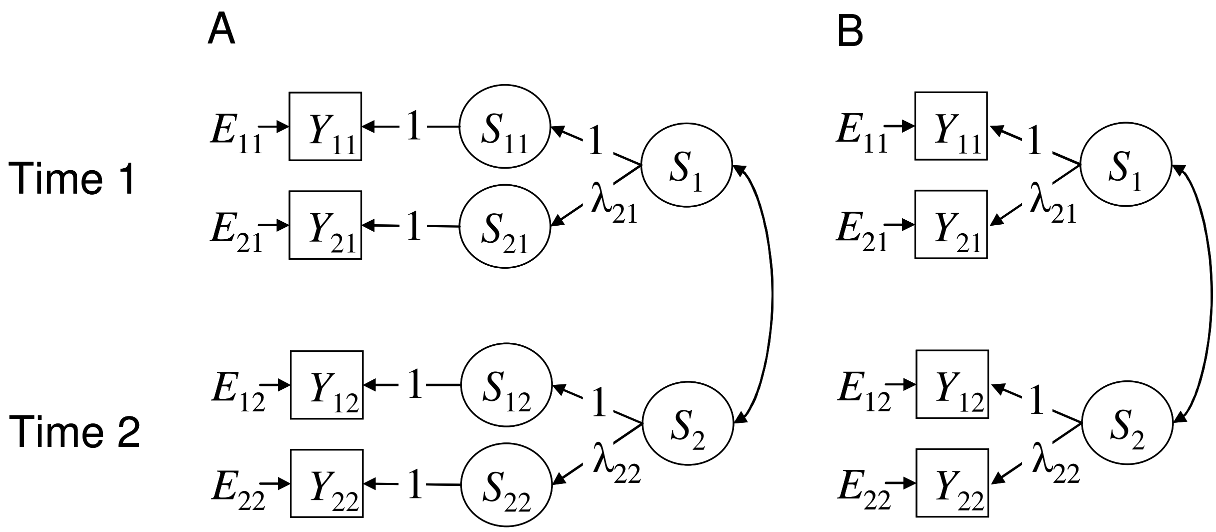


*Figure 9.* Path diagram of a CS model for one construct measured on two occasions of measurement. $Y_{il}$ = observed variable ($i$ = indicator, $l$ = occasion of measurement). $S_{il}$ = indicator-specific latent state variable. $S_l$ = latent state factor. $E_{il}$ = error variable. Here, the loadings $\lambda_{11}$ and $\lambda_{12}$ are set equal to 1 for identification. Intercepts $\alpha_{il}$ are not shown. A: All indicator-specific latent state variables $S_{il}$ are included. B: Indicator-specific latent state variables $S_{il}$ have been dropped.

The minimum requirement for applying the CS model is that there are two observed variables of a construct that have been measured on two time points (as in Figure 9). The CS model can easily be extended to multiple constructs (the only difference is that an index, e.g., *j*, is added for the construct). In Chapter 3, I describe an extension of this model to multiple constructs and multiple methods.

### 2.2.2 The Latent Difference Model

Steyer et al. (1997; Steyer, Partchev, & Shanahan, 2000) have shown how CS models with time invariant measurement parameters (intercepts and loadings) can be reformulated as latent difference (or latent change) models (see Figure 10). Latent difference models (see also McArdle & Hamagami, 1988) are straightforward models for studying inter-individual differences in intra-individual change. The basic idea of latent difference modeling is that any latent state factor $S_l$ can be decomposed into a preceding state factor (e.g., the initial state factor $S_1$) and a latent difference factor [e.g., $(S_l - S_1)$] representing latent change from time 1 (T1) to time *l*:

$$S_l = 1 \cdot S_1 + 1 \cdot (S_l - S_1)\,. \tag{20}$$

Equation 20 is a simple restatement and does not contain any restrictive assumptions. By implementing Equation 20 in the structural model (as shown in Figure 10B), the CS model shown in Figure 10A can easily be reformulated as a latent difference model (the measurement model remains unchanged). The latent difference factors $\left(S_l - S_1\right)$ represent true inter-individual differences in intra-individual change from T1 to time *l*. Here, the term “true” means that the difference scores are corrected for measurement error. Therefore, these models have also been referred to as “true change” models (Steyer et al., 1997).

Latent difference modeling offers a direct and flexible approach to investigating change. The latent difference model is statistically equivalent to the CS model. That is, one does not specify a new model but just makes the information about change more accessible (Steyer et al., 1997, 2000).

Latent difference models do not make any restrictive assumptions with regard to a specific functional form of change as do, for example, growth curve models. The most restrictive assumption made in latent difference models is the assumption of measurement invariance over time. For the latent difference scores to be meaningful, factor loadings and measurement intercepts need to be time-invariant. (The decomposition into initial status and change in Equation 19 only makes sense if the same attribute is measured on both occasions of measurement.) Time-invariant intercepts and loadings imply that the measurement structure of the construct does not change over time. The assumption of measurement invariance is a general requirement in longitudinal studies (e.g., Meredith & Horn, 2001; Tisak & Tisak, 2000). Fortunately, this assumption can be empirically tested. When invariance constraints are imposed on intercepts and loadings, the measurement model of the CS/latent difference model simplifies to (see also Figure 10A):

$$Y_{il} = \alpha_i + \lambda_i \cdot S_l + E_{il}, \tag{21}$$

where the occasion index $l$ has been dropped from the intercepts and loadings to express that these parameters do not vary over time. This reduced model can be tested against the more general measurement model in Equation 19 to investigate whether measurement invariance is tenable.

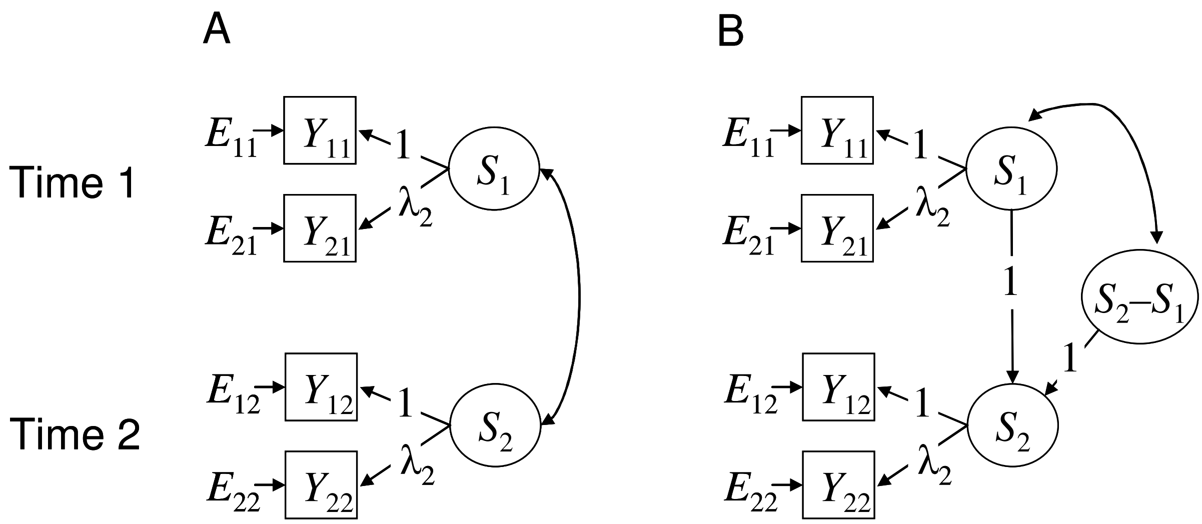


*Figure 10.* Path diagram of a CS model with invariant parameters for one construct measured on two occasions of measurement. $Y_{il}$ = observed variable ($i$ = indicator, $l$ = occasion of measurement). $S_l$ = latent state factor. $S_2 - S_1$ = latent difference variable. $E_{il}$ = error variable. A: State version. B: Latent difference version. The loading $\lambda_1$ of $Y_{11}$ and $Y_{12}$ is set equal to 1 for identification. Intercepts $\alpha_i$ are not shown. Note that there is no residual term for the latent state factor $S_2$ in Figure 10B.

Latent difference variables can be included for investigating change between a given occasion of measurement and the first occasion ("baseline change model"; Steyer et al., 2000) or for change between any specific pair of state factors that is of interest (e.g., to study change between adjacent time points in a "neighbor change model"). I will discuss the different possibilities of including latent difference variables in greater detail in Chapter 4 where I present multi-method change models.

### 2.2.3 *The CS Model Applied to MTMM-MO Data*

The CS model can directly be applied to MTMM-MO data. If multiple indicators are available for each CMOU, separate correlated state factors can be specified for each CMOU as shown in Figure 11. Such a *multi-method CS model* allows for the estimation of a latent MTMM-MO correlation matrix, containing the correlations among state factors pertaining to different constructs, methods, and time points. The latent MTMM-MO matrix is very useful to study the convergent and discriminant validity as well as the temporal stability of different methods over time, as the coefficients in this matrix are corrected for measurement error. An application of the multi-method CS model and its change version to MTMM-MO data has been described by Geiser, Eid, Nussbeck, Courvoisier, and Cole (2008).

Although the multi-method CS model deals with method effects in an appropriate way and provides very useful information, it has some limitations for the analysis of MTMM-MO data. Given that there is a separate state factor for each CMOU (and no method factors), the multi-method CS model does not allow determining the convergent validity and method-specificity in terms of variance components for the indicators. Each state factor represents a construct measured by a specific method. Hence, method effects are confounded with the state factors for all methods. Therefore, method-specific deviations cannot be isolated, and they cannot be related to external variables to explain these deviations. Geiser, Eid, Nussbeck, et al. (2008) offer a detailed comparison of the multi-method CS model and the CS-C($M$–1) approach to be discussed in the following chapter.

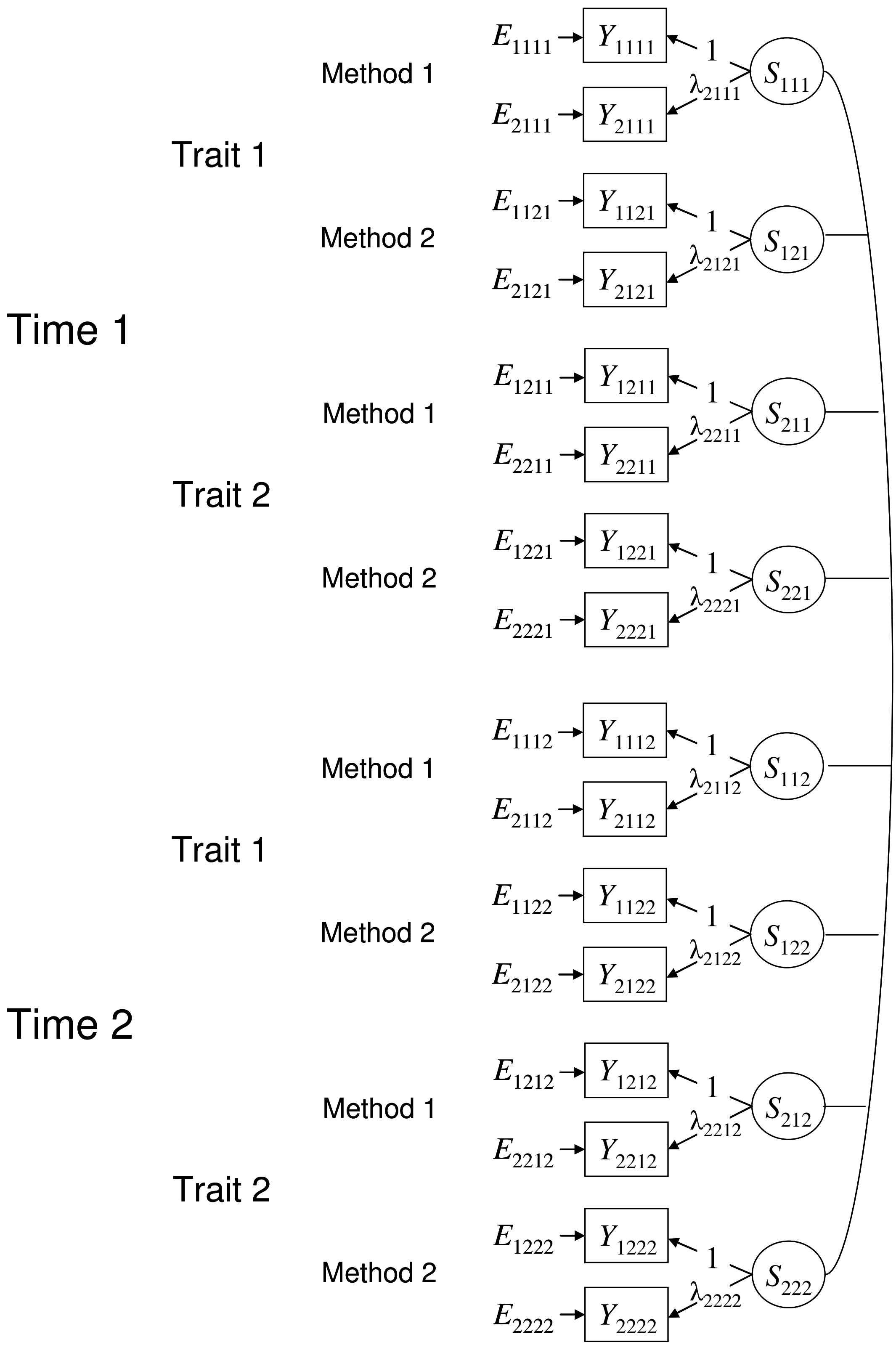


*Figure 11.* Multi-method CS model for two constructs measured by two methods on two occasions of measurement. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{jkl}$ = latent state factor. $E_{ijkl}$ = error variable. $\lambda_{ijkl}$ = state factor loading. All state factors can be correlated.

# 3 The Correlated State-Correlated (*Method*–1) Model

In this chapter, I show how the CS model can be extended to a multimethod model that allows for a detailed analysis of method effects in longitudinal studies. This extended model represents a combination of the CS model introduced in Chapter 2.2.1 and the multiple indicator CT-C(*M*–1) model described in Chapter 1.3.1.2. It is therefore called *Correlated State-Correlated (Method Minus One)* [CS-C(*M*–1)] *model*. In the following section, I present the definition of the CS-C(*M*–1) model in four steps. Subsequently, I will discuss important properties of the model as well as admissible and non-admissible factor correlations. In Section 3.2, a variant of the CS-C(*M*–1) model with indicator-specific factors across time will be introduced. This variant is a useful extension if the same indicators share systematic (but indicator-specific) variance across time. In Section 3.5, I offer a more technical treatment of both model variants. In the more technical part, I show how both models can be defined based on stochastic measurement theory and analyze questions of uniqueness, meaningfulness, covariance structure, and identification for the variables and parameters of the models.

## 3.1 The CS-C(*M*–1) Model

### *3.1.1 Definition of the CS-C(M–1) Model*

As I mentioned in Section 1.3.1.2, indicators (i.e., items, test scores, test halves, or item parcels) within a TMU might not be perfectly unidimensional. Each indicator might represent a slightly different facet of the construct. In addition, indicators with the same index *i*, but different method index *k*, might share common aspects not shared with the other indicators of the same construct. For example, the *Children's Depression Inventory* (CDI; Kovacs, 1985) is a self-report questionnaire for measuring childhood depression. Its parent-report version (CDI parent form) contains the same items as does the self-report form. Therefore, if item parcels are created in the same way for both the self-report and the parent form of the CDI, this may lead to shared indicator-specific variance across the two rater types. In order to account for such shared indicator-specific sources of variance, I will present a version of the CS-C(*M*–1) model in which the latent state factors are indicator-specific. The CS-C(*M*–1) model is thus a direct extension of the CT-C(*M*–1) model with indicator-specific trait factors discussed in Section 1.3.1.2 (see Figure 4). The model variant to be presented in Section 3.2 accounts for indicator-specific effects across time, while assuming homogeneity of indicators within the same time point.

*Step 1.* Basic Decomposition of Latent State Theory

As in the conventional CS model, the starting point is the basic decomposition of an observed variable $Y_{ijkl}$ into a latent state variable $S_{ijkl}$ and an error variable $E_{ijkl}$ (see also Chapter 2.2):

$$Y_{ijkl} = S_{ijkl} + E_{ijkl} \,. \tag{22}$$

Note that, in order to formulate a general MTMM-MO model, we need two additional indices: *j* for the construct and *k* for the method. The index *i* again denotes the indicator and *l* denotes the occasion of measurement. It is important to understand that at this point, the state variables $S_{ijkl}$ are *indicator-specific* (see also Figure 12A) That is, there is a separate state variable for each indicator. Therefore, the model in Equation 22 is not identified. Identified models with common factors are obtained by introducing specific homogeneity assumptions with regard to the latent variables in later steps.

*Step 2.* Choice of the Reference Method

As in the multiple indicator CT-C(*M*–1) model, one method is selected as the comparison standard (so-called *reference method*). As explained below, the latent state variables belonging to the reference method are then used as predictors in a latent regression analysis (Eid, 2000; Eid et al., 2003; see Step 3). Without loss of generality, the first method ($k$ = 1) is selected as the reference method[4]. The general measurement equation for all reference method indicators is given by:

$$Y_{ij1l} = S_{ij1l} + E_{ij1l} \,. \tag{23}$$

*Step 3.* Definition of Construct- and Occasion-Specific Method Factors

In the following, we consider the latent state variables $S_{ijkl}$ belonging to non reference method indicators ($k \neq 1$). How do these latent state variables relate to the latent state variables $S_{ij1l}$, pertaining to the reference method? I assume that the latent state variables $S_{ijkl}$ are linearly regressed on the variables $S_{ij1l}$. This latent regression is expressed by the following equation:

[4] For the sake of simplicity and consistency (and without loss of generality), I will assume throughout this work that the first method ($k$ = 1) serves as the reference method. In empirical applications, researchers may in principle select any method as reference method. For guidelines regarding the proper choice of the reference method in practical applications, see Geiser, Eid, and Nussbeck (2008).

$$E(S_{ijkl} \mid S_{ij1l}) = \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{ij1l}\text{, for } k \neq 1, \tag{24}$$

where $E(S_{ijkl} \mid S_{ij1l})$ denotes the conditional expectation (regression) of a latent state variable $S_{ijkl}$ belonging to a non-reference method on the comparison standard latent state variable $S_{ij1l}$. The residuals of this regression are the latent method variables $M_{ijkl}$. They are defined as

$$M_{ijkl} := S_{ijkl} - E(S_{ijkl} \mid S_{ij1l})\,. \tag{25}$$

The latent residual variables $M_{ijkl}$ represent that part of a non-reference state variable $S_{ijkl}$ that is not explained by the reference state variable $S_{ij1l}$. Hence, $M_{ijkl}$ represents the method-specific deviation of $S_{ijkl}$ from the expected value of $S_{ijkl}$ given $S_{ij1l}$. For example, $S_{ij1l}$ could represent the latent state true score variable associated with a self-rating (reference method) of depression. $S_{ijkl}$ could be the latent state true score variable of the corresponding friend rating. In this example, a score on $M_{ijkl}$ would represent the unique view of a friend that is not shared with the depression self-rating. The friend might over- or underestimate an individual's level of depression (with respect to the value predicted by the self-report), and this over- or underestimation would be expressed in the value of $M_{ijkl}$.

*Step 4.* Definition of Common Construct- and Occasion-Specific Method Factors

Note that, like the variables $S_{ijkl}$, the method-specific residual variables $M_{ijkl}$ are indicator-specific (see also Figure 12A). Each indicator pertaining to a non-reference method has its own method effect $M_{ijkl}$:

$$Y_{ijkl} = \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{ij1l} + M_{ijkl} + E_{ijkl}\text{, for } k \neq 1\,. \tag{26}$$

Of course, such a model is not identified. In order to obtain an identified model, I introduce a homogeneity assumption with regard to the indicator-specific method variables $M_{ijkl}$. I assume that all residuals $M_{ijkl}$, pertaining to the same construct *j*, method *k*, and occasion of measurement *l*, but different indicators *i* and *i*' are linear functions of each other:

$$M_{ijkl} = \lambda_{\text{M}ii'jkl} \cdot M_{i'jkl}\,. \tag{27}$$

This implies that all residuals $M_{ijkl}$ with the same indexes $j$, $k$, and $l$ are perfectly correlated (they differ only by a multiplicative constant, i.e., $\lambda_{\text{M}ii'jkl}$). Substantively, this means that I assume method effects to be homogeneous for all indicators supposed to measure the same construct by the same method on the same occasion of measurement. Note that there is no additive constant (intercept) in Equation 27. This is due to the method factors being defined as residuals (see Equation 25). Residuals have means of zero by definition and as a consequence, no intercept term appears in Equation 27.

A consequence of the homogeneity assumption made introduced in Equation 27 is that we can define *common* method factors $M_{jkl}$. (The proofs are provided in Section 3.5.1.2.) All indicators that belong to a non-reference method ($k \neq 1$) and the same CMOU ($j, k, l$) then measure (1) an indicator-specific reference state factor $S_{ij1l}$ and (2) a common occasion-specific, construct-specific method factor $M_{jkl}$:

$$Y_{ijkl} = \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{ij1l} + \lambda_{\text{M}ijkl} M_{jkl} + E_{ijkl} \,. \tag{28}$$

(For the indicators of the reference method, Equation 23 holds.) Figure 12 shows a path diagram of a CS-C(*M*–1) model for one construct measured by one reference and one non-reference method on two occasions of measurement. Note that there are two indicators per CMOU so that method effects can be construct-specific as well as occasion-specific. The model in Figure 12 can be identified by fixing one method factor loading per method factor to one (i.e., by setting $\lambda_{\text{M}1121} = \lambda_{\text{M}1122} = 1$). Possible factor covariances are not shown in Figure 12 in order to avoid cluttering. Admissible and Non-admissible factor covariances are discussed below. Some important properties of the CS-C(*M*–1) model can be summarized as follows:

1. As in the multiple indicator CT-C(*M*–1) model for cross-sectional data, a reference method is selected and there is no method factor for the reference method on any occasion of measurement.
2. Method effects can be *construct-specific*. Only if all method factors belonging to the same method *k*, but different constructs *j* (on the same occasion *l*) are perfectly correlated [i.e., $Cor(M_{jkl}, M_{j'kl}) = 1,\ j \neq j'$], one may replace the construct-specific

method factors $M_{jkl}$ by general method factors $M_{kl}$. The property of construct-specific method effects is also shared with the multiple indicator CT-C(*M*–1) model.

3. In the CS-C(*M*–1) model, method effects can not only be construct-specific, but also *occasion-specific*. Consequently, one can use the model to study the generalizability of method effects *across constructs* and *across situations / occasions of measurement*. An implication of this property is that homogeneous (i.e., non-construct-specific) *and* stable (i.e., time-invariant) method effects can only be assumed if method factors belonging to the same method but different constructs and different occasions of measurement are perfectly correlated [i.e., if $Cor(M_{jkl}, M_{j'kl'}) = 1,\ j \neq j',\ l \neq l'$]. In this case, it would be sufficient to specify general method factors $M_k$. Another, less restrictive assumption can also be tested. One may test whether method effects are perfectly stable over time. This hypothesis would be supported if the method factors belonging to the same method *and* the same construct but different occasions of measurement are perfectly correlated [i.e., if $Cor(M_{jkl}, M_{jkl'}) = 1,\ l \neq l'$]. In this case, one could specify a more parsimonious model with method factors $M_{jk}$ (instead of $M_{jkl}$).
4. The CS-C(*M*–1) model relaxes the rather restrictive assumption that indicators are homogeneous by allowing for indicator-specific latent state variables. This property is shared with the CT-C(*M*–1) model with indicator-specific trait variables (see Figure 4).

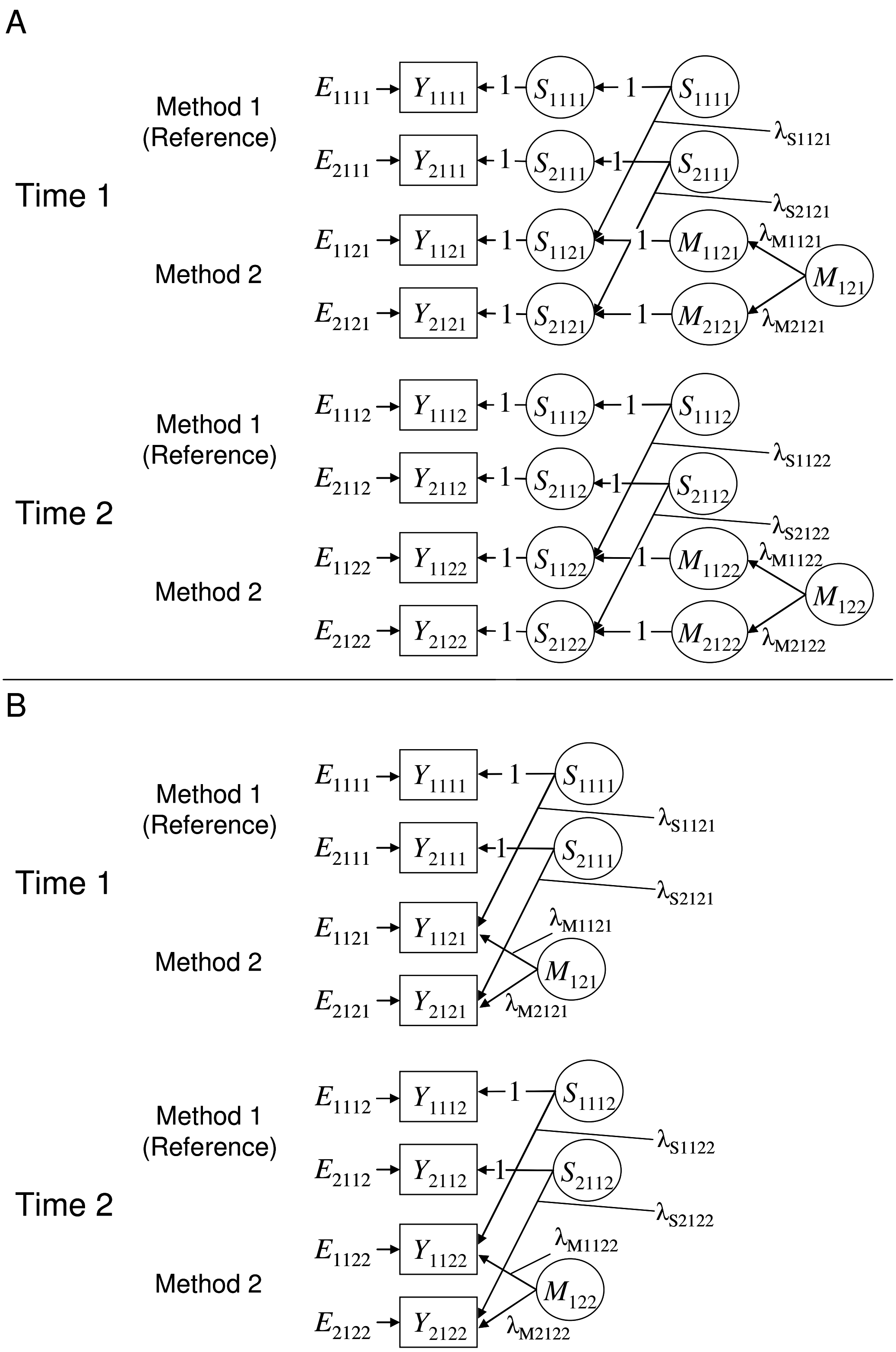


*Figure 12.* Path diagram of a CS-C($M$–1) model for one construct measured by two methods on two occasions of measurement. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{ijkl}$ = latent state variable. $M_{ijkl}$ = latent method variable. $M_{jkl}$ = common method factor. $E_{ijkl}$ = error variable. $\lambda_{Sijkl}$ = state factor loading. $\lambda_{Mijkl}$ = method factor loading. A: All indicator-specific latent state variables $S_{ijkl}$ are included. B: Latent state variables $S_{ijkl}$ have been dropped (except $S_{ij1l}$). For the sake of clarity, no factor covariances are shown (but see text and Figure 13).

In the following, I discuss the latent variable covariance structure in the CS-C($M$–1) model. In the first part, I explain which types of latent correlations are assumed to be zero or are equal to zero by definition of the model. In the second part, I discuss permissible latent correlations and their meaning for MTMM-MO analyses. In the third part, I present the variance decomposition of the observed and latent state variables and I show how coefficients for quantifying the convergent validity, method-specificity, and reliability of the indicators can be defined.

### *3.1.2 Covariance Structure of the Latent Variables*

#### 3.1.2.1 Non-Permissible Latent Correlations

The following correlations are *not* permitted in the CS-C($M$–1) model. In practical applications, researchers must think of fixing these correlations to zero.

(A) State factors are not allowed to correlate with any method factor that belongs to the same construct on the same occasion of measurement:

$$Cov(M_{jkl}, S_{ij1l}) = 0. \tag{29}$$

This follows from the definition of the method factors $M_{jkl}$ as residuals with respect to the state variables $S_{ij1l}$. Residuals are always uncorrelated with their regressors (see, e.g., Steyer, 1988; as well as Steyer & Eid, 2001).

(B) Error variables are assumed to be uncorrelated with all other error variables:

$$Cov(E_{ijkl}, E_{i'j'k'l'}) = 0, \text{ for } (i, j, k, l) \neq (i', j', k', l'). \tag{30}$$

(C) Correlations between error variables and other latent variables are not admissible either:

$$Cov(E_{ijkl}, S_{i'j'1l'}) = Cov(E_{ijkl}, M_{j'k'l'}) = 0. \tag{31}$$

#### 3.1.2.2 Permissible Latent Correlations

The following latent correlations can be estimated in the CS-C($M$–1) model. Note that examples of all these correlations are given in Figure 13. The numbers in Figure 13 correspond to the numeration in the text.

(1) The correlations $Cor(S_{ij1l}, S_{i'j1l})$, $i \neq i'$, between indicator-specific latent state factors belonging to the same construct on the same occasion of measurement can be interpreted as indexes of the degree of homogeneity of the indicators. Low correlations indicate that the observed variables (e.g., tests, test halves or item parcels) are rather heterogeneous (i.e., capture different aspects or facets of a construct). In contrast, correlations close to unity point to a high homogeneity of the indicators. If $Cor(S_{ij1l}, S_{i'j1l}) = 1$, it would be sufficient to specify a *single* common state factor $S_{j1l}$ for all indicators $Y_{ijkl}$ and $Y_{i'jkl}$ instead of multiple indicator-specific state factors.

(2) The correlations $Cor(S_{ij1l}, S_{i'j'1l})$, $j \neq j'$, between latent state factors belonging to different constructs on the same occasion of measurement can be interpreted as coefficients of discriminant validity with respect to the reference method. If the correlation is rather small, there is evidence for discriminant validity of the constructs on a given time point *l*.

(3) The correlations $Cor(M_{jkl}, M_{j'kl})$, $j \neq j'$, between method factors belonging to the same method but different constructs $j$ and $j'$ on the same occasion of measurement characterize the generalizability of method effects across constructs on a given measurement occasion. A correlation of zero indicates that a method effect is perfectly construct-specific (does not generalize at all across constructs). For example, the bias of a teacher rating with respect to a child's depression level might not generalize to an anxiety rating. In contrast, a correlation of unity means that a method effect is perfectly homogeneous across constructs. That would be the case, for example, if a teacher rating with respect to the construct *depression* perfectly generalized to the construct *anxiety*. In practice, correlations between .6 and .8 are often found, showing that method effects generalize across constructs to some degree, but not perfectly so.

(4) The correlations $Cor(M_{jkl}, M_{jk'l})$, $k \neq k'$, between method factors belonging to the same construct but different methods indicate the common deviation of non-reference methods from the reference method. To illustrate, imagine a study in which self, peer, and teacher ratings are used to measure children's anxiety. The self-report is selected as reference method. Then, the correlation between the method factors for the peer-report and the teacher-report of anxiety represents a partial correlation corrected for the influence of the self-report. A value of zero for this partial correlation means that peers and teachers do *not* share a common view of the target that is not shared with the

targets' own view. On the other hand, if the correlation between the method factors is substantial, this means that there is a consistent method bias that generalizes across non-reference methods. For instance, peers and teachers might share a common view of the targets' anxiety that is not shared with the targets' own rating.

(5) The correlations $Cor(M_{jkl}, S_{ij'1l})$, $j \neq j'$, between a method factor of a construct *j* and any state factor belonging to another construct $j'$ on the same occasion of measurement indicate "pure" discriminant validity corrected for method influences of the reference method. In practice, these correlations often do not significantly differ from zero.

(6) Correlations between method factors belonging to different TMU's on the same occasion of measurement [i.e., $Cor(M_{jkl}, M_{j'k'l})$, $j \neq j'$, $k \neq k'$] are also measures of discriminant validity between methods, corrected for the discriminant validity with respect to the reference method. Significant correlations indicate that the reference method cannot completely explain the associations between different methods. For example, an over- or underestimation of anxiety by teachers (compared to the reference method) might be associated with over- or underestimation of depression by peers.

So far, only admissible correlations between latent factors measured on the *same* occasion of measurement have been considered. All these correlations can also be investigated in the cross-sectional multiple indicator CT-C(*M*–1) model (see Chapter 1.3.1.2). In the following, I discuss additional correlation coefficients that cannot be examined in the CT-C(*M*–1) model, but can be calculated in the CS-C(*M*–1) model.

(7) The correlations between the latent state factors belonging to the same construct assessed on different measurement occasions [i.e., $Cor(S_{ij1l}, S_{i'j1l'})$, $l \neq l'$] represent coefficients of *construct stability*. Two different types can be distinguished: (a) the correlations $Cor(S_{ij1l}, S_{ij1l'})$, $l \neq l'$, are the correlations between the *same* indicator-specific state factors over time. These correlations are stability coefficients *not* corrected for indicator-specific effects (number 7a in Figure 13); (b) the correlations $Cor(S_{ij1l}, S_{i'j1l'})$, $i \neq i'$, $l \neq l'$, are the correlations between state factors of the same construct over time, but measured by *different* indicators. Hence, these state factors do

not share indicator-specific sources of variance. Their correlation thus represents construct stability corrected for indicator-specific influences (number 7b in Figure 13). One can expect the stability coefficients of type 7a to be higher than the coefficients of type 7b. The type-7a-coefficients might be inflated due to shared (construct-irrelevant) indicator-specific variance. In either case, if $Cor(S_{ij1l}, S_{i'j1l'}) = 1$ (for $l \neq l'$), inter-individual differences with respect to construct *j* remain perfectly stable between time *l* and time $l'$. Correlations smaller than unity indicate that some individuals have changed more than others between two occasions of measurement. In sum, these correlations allow investigating whether there has been true differential change between two time points.

(8) The correlations $Cor(S_{ij1l}, S_{i'j'1l'}),\ j \neq j',\ l \neq l'$, between latent state variables pertaining to different constructs *j* measured on different occasions *l* can be interpreted as discriminant validity coefficients with respect to the reference method that are corrected for common occasion-specific influences. In specific situations, these correlations may also be seen as coefficients of *predictive validity* with respect to the reference method.

(9) The correlations $Cor(M_{jkl}, S_{ij1l'}),\ l \neq l'$, between state factors and method factors belonging to the same construct, but different occasions of measurement are somewhat difficult to interpret. Significant correlations would indicate that the method-specific deviation of method *k* from the reference method at time *l* can predict the scores on the reference method state factor pertaining to the same construct at time $l'$. Although it is conceivable that, for example, a self-report on one measurement occasion might influence an other-report on another occasion, one would not generally expect these correlations to be substantial and might therefore consider fixing them to zero for reasons of parsimony.

(10) The correlations $Cor(M_{jkl}, S_{ij'1l'}),\ j \neq j',\ l \neq l'$, between construct-specific method factors and state factors belonging to another construct on another occasion of measurement are coefficients of discriminant validity corrected for common method effects *and* common occasion-specific influences. My experience is that these correlations generally do not differ significantly from zero in empirical applications.

(11) In order to assess the degree of stability of construct-specific method effects, one can investigate the correlations between method factors belonging to the same construct

and the same method, but different measurement occasions [i.e., $Cor(M_{jkl}, M_{jkl'})$, $l \neq l'$]. A high correlation indicates that the deviation of a method from the reference method is stable over time for a given construct. For example, teachers might consistently over- or underestimate the degree of anxiety of a child across different occasions of measurement.

(12) The generalizability of method effects across constructs corrected for common occasion-specific influences can be estimated by means of the correlations $Cor(M_{jkl}, M_{j'kl'})$, $j \neq j'$, $l \neq l'$. A high correlation indicates that the method-specific deviation of method $k$ from the reference method is both stable across constructs and stable over time. For instance, teachers might consistently over- or underestimate the level of anxiety *and* depression in children across different situations/occasions of measurement.

(13) The correlations between method factors belonging to different methods, but the same construct measured on different occasions [$Cor(M_{jkl}, M_{jk'l'})$, $k \neq k'$, $l \neq l'$] indicate the consistency of method effects across constructs corrected for common occasion-specific influences.

(14) The discriminant validity of methods, corrected for construct-specific and common occasion-specific influences, can also be estimated. One can therefore correlate the method factors of a method $k$ belonging to a construct $j$ at time $l$ with the method factors of other methods $k'$, belonging to different constructs $j'$ on different occasions of measurement $l'$, formally expressed as $Cor(M_{jkl}, M_{j'k'l'})$, $j \neq j'$, $k \neq k'$, $l \neq l'$.

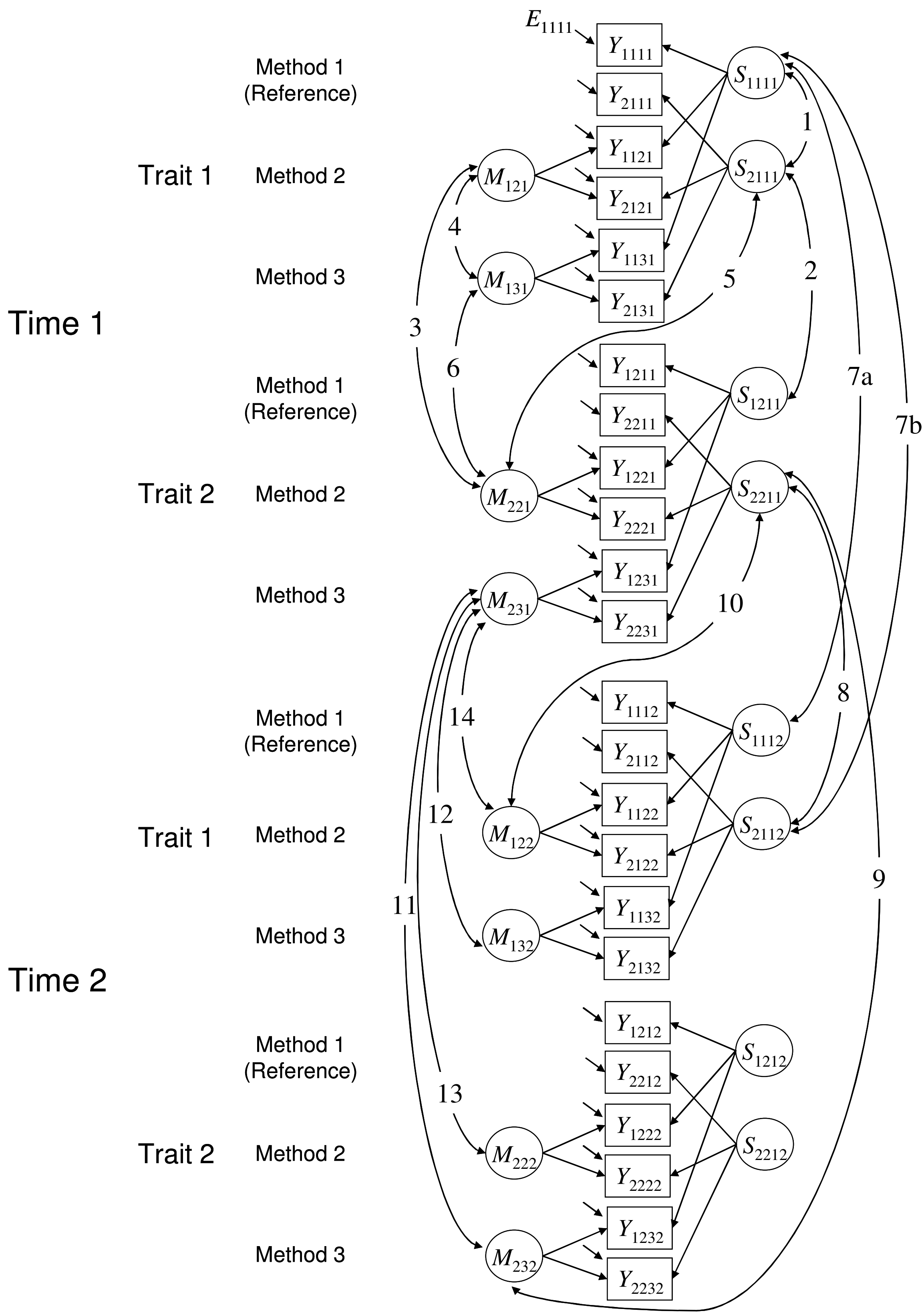


*Figure 13.* Path diagram of a CS-C($M$–1) model for two constructs, three methods, and two time points. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{ij1l}$ = latent state factor. $M_{jkl}$ = common method factor. $E_{ijkl}$ = error variable. The numbers 1–14 refer to various types of latent correlations that are discussed in the text.

### 3.1.3 Latent Variable Mean Structure

In the CS-C(*M*–1) model, not only the latent variable covariance structure, but also the latent variable mean structure can be analyzed. For example, it might be of interest whether the means of the latent state factors $S_{ij1l}$ change over time. The mean structure of the latent state factors is given by:

$$E(S_{ij1l}) = E(Y_{ij1l})\,. \tag{32}$$

Equation 32 follows given that (1) there is no intercept in the equation $Y_{ij1l} = S_{ij1l} + E_{ij1l}$ and (2) the error variables $E_{ij1l}$ have an expectation (mean) of zero by definition. The method factors $M_{jkl}$, being residual variables, also have means of zero:

$$E(M_{jkl}) = 0\,. \tag{33}$$

Consequently, only the means of the latent state factors can be tested for invariance across occasions of measurement in the CS-C(*M*–1) model. Mean differences with respect to the indicators of non-reference methods can be assessed by fixing the latent state factor means to zero and estimating the intercepts for all indicators. The intercepts are then identical to the observed variable means. Tests for mean differences across time for all methods can be conducted by testing a constrained model in which some or all intercepts are set equal over time for the same indicator.

### 3.1.4 Variance Decomposition and Variance Components

As a consequence of Equation 29, the variances of the indicator-specific state variables can be decomposed in the following way:

$$Var(S_{ijkl}) = \lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l}) + \lambda^2_{\mathrm{M}ijkl} Var(M_{jkl}),\ \text{for } k \neq 1. \tag{34}$$

Furthermore, as a consequence of Equations 29 to 31, the variances of the observed variables can be decomposed as follows:

$$Var(Y_{ijkl}) = \begin{cases} Var(S_{ij1l}) + Var(E_{ij1l}),\ \text{for } k = 1, \\ \lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l}) + \lambda^2_{\mathrm{M}ijkl} Var(M_{jkl}) + Var(E_{ijkl}),\ \text{for } k \neq 1. \end{cases} \tag{35}$$

The additive variance decomposition allows defining coefficients of *consistency*, *method specificity*, *reliability*, and *unreliability*. The consistency coefficient represents the proportion of variance of an observed variable (indicator) that is explained by the reference method state factor on a given occasion of measurement. It can be interpreted as an index of the convergent validity with respect to the reference method:

$$CO(Y_{ijkl}) = \frac{\lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l})}{Var(Y_{ijkl})}. \tag{36}$$

The method-specificity coefficient represents the proportion of the variance of an indicator that is due to method-specific influences on a given occasion of measurement:

$$MS(Y_{ijkl}) = \frac{\lambda^2_{\mathrm{M}ijkl} Var(M_{jkl})}{Var(Y_{ijkl})},\ k \neq 1. \tag{37}$$

Consistency and method-specificity coefficients add up to the reliability coefficient. The reliability coefficient represents the proportion of the variance of an indicator that is explained by the true score variable. It can also be calculated as the sum of consistency and method-specificity coefficients:

$$Rel(Y_{ijkl}) = \frac{Var(S_{ijkl})}{Var(Y_{ijkl})} = \frac{\lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l})}{Var(Y_{ijkl})} + \frac{\lambda^2_{\mathrm{M}ijkl} Var(M_{jkl})}{Var(Y_{ijkl})} = CO(Y_{ijkl}) + MS(Y_{ijkl}). \tag{38}$$

The consistency and method-specificity coefficients can also be defined with respect to the latent state variables:

$$CO(S_{ijkl}) = \frac{\lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l})}{Var(S_{ijkl})}, \tag{39}$$

$$MS(S_{ijkl}) = \frac{\lambda^2_{\mathrm{M}ijkl} Var(M_{jkl})}{Var(S_{ijkl})},\ k \neq 1. \tag{40}$$

Given that the variables $S_{ijkl}$ represent the "true states" (that do not contain measurement error), the coefficients $CO(S_{ijkl})$ and $MS(S_{ijkl})$ add up to unity. Table 1 summarizes the definition of the CS-C(*M*–1) model.

Table 1

*Summary of the CS-C(M–1) State Model*

| Definition | Equation |
|---|---|
| Basic decomposition of latent state theory | $Y_{ijkl} = S_{ijkl} + E_{ijkl}$ |
| True score regression | $E(S_{ijkl} \mid S_{ij1l}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij1l}$ (for $k \neq 1$) |
| Definition of method variables | $M_{ijkl} := S_{ijkl} - E(S_{ijkl} \mid S_{ij1l})$ |
| Definition of common method factors | $M_{ijkl} = \lambda_{\mathrm{M}ijkl} M_{jkl}$ |
| Covariances of method and state factors (same construct, same occasion) | $Cov(M_{jkl}, S_{ij1l}) = 0$ |
| Covariances of error variables | $Cov(E_{ijkl}, E_{i'j'k'l'}) = 0,\ (i,j,k,l) \neq (i',j',k',l')$ |
| Covariances between error variables and other latent variables | $Cov(E_{ijkl}, S_{i'j'1l'}) = Cov(E_{ijkl}, M_{j'k'l'}) = 0$ |
| Mean structure (state factors) | $E(S_{ij1l}) = E(Y_{ij1l})$ |
| Mean structure (method factors and error variables) | $E(M_{jkl}) = E(E_{ijkl}) = 0$ |
| Variance decomposition (observed variables) | $Var(Y_{ijkl}) = \begin{cases} Var(S_{ij1l}) + Var(E_{ij1l}), \text{ for } k = 1 \\ \lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l}) + \lambda^2_{\mathrm{M}ijkl} Var(M_{jkl}) + Var(E_{ijkl}), \text{ for } k \neq 1 \end{cases}$ |
| Consistency (observed variables) | $CO(Y_{ijkl}) = \dfrac{\lambda^2_{\mathrm{S}ijkl} Var(S_{ij1l})}{Var(Y_{ijkl})}$ |
| Method-specificity (observed variables) | $MS(Y_{ijkl}) = \dfrac{\lambda^2_{\mathrm{M}ijkl} Var(M_{jkl})}{Var(Y_{ijkl})}$ (for $k \neq 1$) |

*(Table continues)*

| Definition | Equation |
|---|---|

| | |
|---|---|
| Reliability | $Rel(Y_{ijkl}) = \frac{Var(S_{ijkl})}{Var(Y_{ijkl})} = \frac{\lambda^2_{Sijkl} Var(S_{ij1l})}{Var(Y_{ijkl})} + \frac{\lambda^2_{Mijkl} Var(M_{jkl})}{Var(Y_{ijkl})}$ $= CO(Y_{ijkl}) + MS(Y_{ijkl})$ |
| Variance decomposition (state variables) | $Var(S_{ijkl}) = \lambda^2_{Sijkl} Var(S_{ij1l}) + \lambda^2_{Mijkl} Var(M_{jkl}),\ \text{for } k \neq 1$ |
| Consistency (state variables) | $CO(S_{ijkl}) = \frac{\lambda^2_{Sijkl} Var(S_{ij1l})}{Var(S_{ijkl})}$ |
| Method-specificity (state variables) | $MS(S_{ijkl}) = \frac{\lambda^2_{Mijkl} Var(M_{jkl})}{Var(S_{ijkl})}$ (for $k \neq 1$) |

*Note.* $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). Without loss of generality, the first method ($k$ = 1) is selected as reference method. $S_{ijkl}$ = latent state variable. $E_{ijkl}$ = error variable. $E(S_{ijkl} \mid S_{ij1l})$ denotes the conditional expectation (regression) of $S_{ijkl}$ on $S_{ij1l}$. $\alpha_{ijkl}$ = intercept. $\lambda_{Sijkl}$ = state factor loading. $M_{ijkl}$ = latent method-specific residual variable. $M_{jkl}$ = common method factor. $\lambda_{Mijkl}$ = method factor loading.

## 3.2 The CS-C(*M*–1) Model With Indicator-Specific Factors Across Time

### *3.2.1 Indicator-Specific Effects Across Time*

I already mentioned in Chapters 1.3.1.2 and 3.1 that perfectly homogeneous indicators are rarely available in practice and that indicator-specific effects might generalize across methods and across time. Therefore, the CS-C(*M*–1) model introduced in Chapter 3.1 was defined as a model with indicator-specific state factors. The CS-C(*M*–1) model with indicator-specific state factors as defined in Chapter 3.1 accounts for indicator-specific effects that generalize across different methods within the same measurement occasion. It may not, however, appropriately capture indicator-specific effects of the same indicator that generalize over time. As I pointed out before, indicator-specific effects over time are often encountered in longitudinal studies when the same indicators are repeatedly measured (Sörbom, 1975). The CS-C(*M*–1) model defined in Chapter 3.1 assumes that all error variables $E_{ijkl}$ are uncorrelated. This assumption may be too restrictive if indicator-specific effects generalize across time. Failure to model shared indicator-specific effects over time can lead to model misspecification and biased parameter estimates. One possibility to deal with indicator-specific effects over time is to admit correlations between specific error variables, that is, estimate some or all correlations between error variables associated with the same variable over time (e.g., Cole & Maxwell, 2003; Sörbom, 1975).

A limitation of models with auto-correlated errors (correlated uniqueness models) is that in these models, indicator-specific effects are confounded with measurement error, leading to an underestimation of the reliabilities of the indicators (cp. Chapter 1.3). Hence, a better way to handle indicator-specific effects over time is to include *indicator-specific* factors in the model (e.g., Jöreskog, 1979; Marsh & Grayson, 1994; Raffalovich & Bohrnstedt, 1987; Tisak & Tisak, 2000). Models with indicator-specific factors make it possible to separate variance components due to indicator-specific effects from variance due to random measurement error. As a consequence, an underestimation of the observed variable reliabilities is avoided in these models.

According to Eid et al. (1999), it is not necessary to include an indicator-specific factor for *each* repeatedly measured indicator *i*. Eid et al. (1999) have shown that it is sufficient to use $i - 1$ indicator specific factors per construct. (Specifying as many indicator-specific factors as there are different indicators often leads to an overfactorization as well as identification and estimation problems.)

In the next section, I present an alternative variant of the CS-C($M$–1) model that includes indicator-specific factors over time. In order to avoid an over-factorization that might lead to identification and estimation problems, the CS-C($M$–1) model with indicator-specific factors across time is formulated as a model with general (instead of indicator-specific) latent state factors.

### *3.2.2 Definition of the CS-C(M–1) Model With Indicator-Specific Factors*

The definition of the CS-C($M$–1) model with indicator-specific factors is presented in five steps.

*Step 1.* Basic Decomposition of Latent State Theory

As in the CS-C($M$–1) model defined in Chapter 3.1, the starting point is the basic decomposition of an observed variable $Y_{ijkl}$ ($i$ = indicator, $j$ = trait, $k$ = method, $l$ = occasion of measurement) into a latent state variable $S_{ijkl}$ and an error variable $E_{ijkl}$ (cp. Chapter 2.2):

$$Y_{ijkl} = S_{ijkl} + E_{ijkl} . \tag{41}$$

The error variable $E_{ijkl}$ is again defined as a residual with respect to $S_{ijkl}$. As a consequence, $E_{ijkl}$ has zero expectation, and $E_{ijkl}$ and $S_{ijkl}$ are uncorrelated with each other.

*Step 2.* Choice of Reference Method, Reference Indicators, and Marker Indicators

In the CS-C(*M*–1) model with indicator-specific factors, one method is again selected as reference method. In addition, for each construct-occasion unit, one indicator belonging to the reference method is selected as *reference indicator*. Without loss of generality, I again select the first method ($k$ = 1) as the reference method, and I choose the first indicator ($i$ = 1) measured by the reference method as the reference indicator. All reference indicators belonging to the reference method can be decomposed as follows:

$$Y_{1j1l} = S_{1j1l} + E_{1j1l}\,. \tag{42}$$

The variables $S_{1j1l}$ can be interpreted as *reference state variables*. To introduce $(i-1)\cdot j\cdot k$ indicator-specific factors, *marker indicators* have to be defined for each construct-method unit for which no indicator-specific factors are specified. Without loss of generality, I select all indicators with index $i$ = 1 ($Y_{1jkl}$) as marker indicators. Hence, the variables $Y_{1j1l}$ are at the same time reference and marker indicators.

*Step 3.* Definition of Indicator-Specific Factors for the Reference Method

In the next step, I consider only the latent state variables $S_{ij1l}$, $i \neq 1$, belonging to indicators of the reference method ($k$ = 1). I assume that the variables $S_{ij1l}$ are linearly regressed on the reference state variables $S_{1j1l}$, pertaining to the same construct and the same measurement occasion:

$$E(S_{ij1l} \mid S_{1j1l}) = \alpha_{ij1l} + \lambda_{Sij1l} S_{1j1l}\,, \text{ for } i \neq 1, \tag{43}$$

where $E(S_{ij1l} \mid S_{1j1l})$ denotes the conditional expectation (regression) of $S_{ij1l}$ on $S_{1j1l}$, and $\alpha_{ij1l}$ as well as $\lambda_{Sij1l}$ are real constants. The residuals of this regression are the indicator-specific variables $IS_{ij1l}$ for the (non-marker) indicators of the reference method:

$$IS_{ij1l} := S_{ij1l} - E(S_{ij1l} \mid S_{1j1l})\,. \tag{44}$$

*Common* indicator-specific factors for the same indicators over time are obtained by assuming that all indicator-specific variables $IS_{ij1l}$ belonging to the same indicator and the same construct differ only by a multiplicative constant $\lambda_{\text{IS}ij1ll'}$:

$$IS_{ij1l} = \lambda_{\text{IS}ij1ll'} IS_{ij1l'} . \tag{45}$$

A consequence of this unidimensionality assumption is that the (indicator-specific) variables $IS_{ij1l}$ can be replaced by a common indicator-specific factor $IS_{ij1}$.

Hence, all latent state variables $S_{ij1l}$, $i \neq 1$, can be decomposed into (1) an intercept ($\alpha_{ij1l}$), (2) one part that is due to an occasion-specific state factor common to all indicators of the same construct measured on the same measurement occasion ($\lambda_{\text{S}ij1l} S_{1j1l}$), and (3) an occasion-unspecific (stable) indicator-specific part ($\lambda_{\text{IS}ij1l} IS_{ij1}$):

$$S_{ij1l} = \alpha_{ij1l} + \lambda_{\text{S}ij1l} S_{1j1l} + \lambda_{\text{IS}ij1l} IS_{ij1} . \tag{46}$$

*Step 4.* Definition of Trait- and Occasion-Specific Method Variables

In this step, I consider the latent state variables $S_{ijkl}$ belonging to non-reference method indicators ($k \neq 1$). I assume that the latent state variables $S_{ijkl}$, $k \neq 1$, are also linearly regressed on the reference state variables $S_{1j1l}$:

$$E(S_{ijkl} \mid S_{1j1l}) = \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{1j1l} \text{, for } k \neq 1 , \tag{47}$$

where $\alpha_{ijkl}$ and $\lambda_{\text{S}ijkl}$ are real constants. The residuals of this regression are defined as:

$$M_{ijkl} := S_{ijkl} - E(S_{ijkl} \mid S_{1j1l}) . \tag{48}$$

The variables $M_{ijkl}$ represent occasion-specific method-specific deviations of an indicator from the expected value given the reference state variable on measurement occasion $l$ (i.e., the method variables $M_{ijkl}$ represent that part of the reliable variance of an indicator that is not shared with the reference method).

*Step 5.* Definition of Indicator-Specific Factors for the Non-Reference Methods

To define indicator-specific factors also for the indicators pertaining to non-reference methods, I assume that the variables $M_{ijkl}$, $i \neq 1$, are linearly regressed on the variables $M_{1jkl}$ (which belong to the marker indicators $Y_{1jkl}$, $k \neq 1$):

$$E(M_{ijkl} \mid M_{1jkl}) = \lambda_{\mathrm{M}ijkl} M_{1jkl}\text{, for } i \neq 1, \tag{49}$$

where $\lambda_{\mathrm{M}ijkl}$ denotes a real constant. The residuals of this regression represent the indicator-specific effects pertaining to the indicators of the non-reference methods:

$$IS_{ijkl} := M_{ijkl} - E(M_{ijkl} \mid M_{1jkl})\,. \tag{50}$$

It is then assumed that all variables $IS_{ijkl}$ with the same indices *i*, *j*, and *k* differ only by a multiplicative constant ($\lambda_{\mathrm{IS}ijkll'}$) such that:

$$IS_{ijkl} = \lambda_{\mathrm{IS}ijkll'} IS_{ijkl'}\,. \tag{51}$$

Similarly to Equation 45, this assumption implies that indicator-specific effects are unidimensional for the same indicator over time. This is equivalent to assuming common occasion-unspecific factors $IS_{ijk}$ for all non-marker indicators pertaining to non-reference methods (the proofs are provided in Section 3.5.2.2). There is no additive constant (intercept) in Equation 51, given that the indicator-specific variables are residuals of a latent regression analysis.

In sum, the state variables $S_{1jkl}$, belonging to the marker indicators (*i* = 1) of the non-reference methods ($k \neq 1$), can be decomposed into (1) an intercept ($\alpha_{1jkl}$), (2) one part that is due to a common occasion-specific reference method state factor ($\lambda_{\mathrm{S}1jkl} S_{1j1l}$), and (3) a common occasion-specific method factor ($M_{1jkl}$):

$$S_{1jkl} = \alpha_{1jkl} + \lambda_{\mathrm{S}1jkl} S_{1j1l} + M_{1jkl}\,. \tag{52}$$

The state variables $S_{ijkl}$, belonging to the non-marker indicators ($i \neq 1$) of the non-reference methods ($k \neq 1$), are decomposed into (1) an intercept ($\alpha_{ijkl}$), (2) one part that is due to a reference method state factor ($\lambda_{\text{S}ijkl}S_{1j1l}$), (3) one part that is due to a method factor ($\lambda_{\text{M}ijkl}M_{1jkl}$), and (4) one part that is due to an indicator-specific factor ($\lambda_{\text{IS}ijkl}IS_{ijk}$):

$$S_{ijkl} = \alpha_{ijkl} + \lambda_{\text{S}ijkl}S_{1j1l} + \lambda_{\text{M}ijkl}M_{1jkl} + \lambda_{\text{IS}ijkl}IS_{ijk}. \tag{53}$$

The measurement equations for the extended CS-C(*M*–1) model with general state factors and *i* – 1 indicator-specific factors are given by:

$$Y_{ijkl} = \begin{cases} S_{1j1l} + E_{1j1l}, \text{ for } i, k = 1, \\ \alpha_{ij1l} + \lambda_{\text{S}ij1l}S_{1j1l} + \lambda_{\text{IS}ij1l}IS_{ij1} + E_{ij1l}, \text{ for } i \neq 1, k = 1, \\ \alpha_{1jkl} + \lambda_{\text{S}1jkl}S_{1j1l} + M_{1jkl} + E_{1jkl}, \text{ for } i = 1, k \neq 1, \\ \alpha_{ijkl} + \lambda_{\text{S}ijkl}S_{1j1l} + \lambda_{\text{M}ijkl}M_{1jkl} + \lambda_{\text{IS}ijkl}IS_{ijk} + E_{ijkl}, \text{ for } i, k \neq 1. \end{cases} \tag{54}$$

The CS-C(*M*–1) model with indicator-specific factors is illustrated in the path diagram in Figure 14. Note two important differences between this version of the CS-C(*M*–1) model and the version introduced in Section 3.1. First, the latent state factors in the CS-C(*M*–1) model with indicator-specific factors across time are general and not indicator-specific. Second, the CS-C(*M*–1) model with indicator-specific factors across time contains additional factors (the indicator-specific factors $IS_{ijk}$ shown in grey circles) for all but the marker indicators (*i* = 1) that capture indicator-specific effects over time. Admissible covariances between latent factors are not shown in Figure 14 for reasons of clarity. Admissible and non-admissible covariances are discussed in the next section.

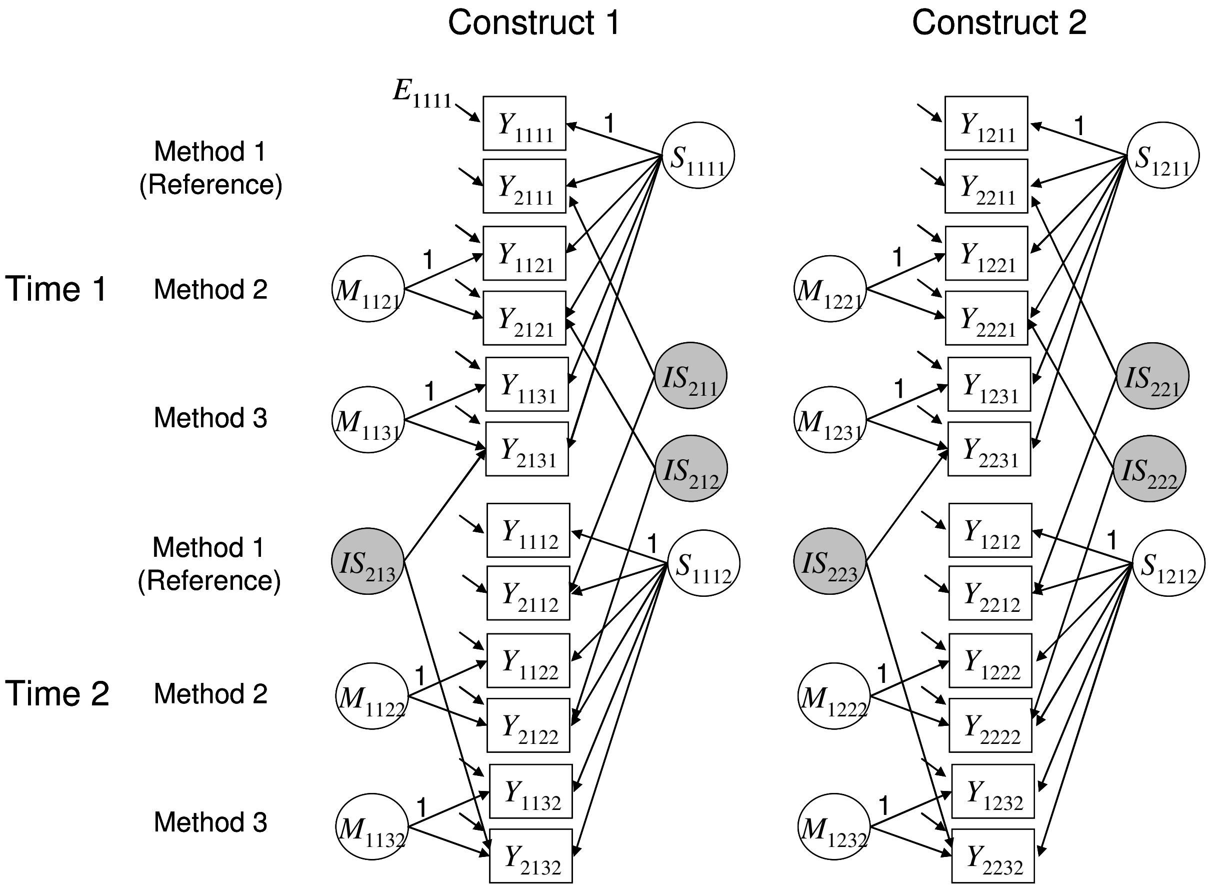


*Figure 14.* Extended CS-C(*M*–1) model with general state factors and indicator-specific factors over time for two constructs, three methods, and two time points. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{1j1l}$ = general latent state factor. $M_{1jkl}$ = latent method factor. $IS_{ijk}$ = indicator-specific factor. $E_{ijkl}$ = error variable. For reasons of clarity, permissible factor correlations are not shown.

### *3.2.3 Covariance Structure of the Latent Variables*

3.2.3.1 Non-Permissible Latent Correlations

The following latent factor correlations are not permitted in the CS-C(*M*–1) model and must be fixed to zero in empirical applications:

(A) State factors are not allowed to correlate with method factors belonging to the same construct on the same occasion of measurement:

$$Cov(M_{1jkl}, S_{1j1l}) = 0, \tag{55}$$

This follows from the definition of the method factors as residuals with respect to the state factors that pertain to the same construct on the same occasion of measurement.

(B) All indicator-specific factors are uncorrelated with their regressors:

$$Cor(IS_{ijk}, S_{1j1l}) = Cor(IS_{ijk}, M_{1jkl}) = 0. \tag{56}$$

(C) Error variables are uncorrelated with all other latent variables:

$$Cor(E_{ijkl}, S_{1j'1l'}) = Cor(E_{ijkl}, M_{1j'k'l'}) = Cor(E_{ijkl}, IS_{i'j'k'}) = 0. \tag{57}$$

(D) Error variables are not correlated with other error variables:

$$Cor(E_{ijkl}, E_{i'j'k'l'}) = 0, \quad (i, j, k, l) \neq (i', j', k', l'). \tag{58}$$

Equations 55 to 57 are direct consequences of the definition of the latent variables in the model. Therefore, they cannot be empirically tested. In contrast, Equation 58 is *not* a direct consequence of the model definition. Equation 58 represents a testable assumption that could be falsified in an empirical application of the model.

3.2.3.2 Permissible Latent Correlations

In this section, I discuss the most important and most meaningful types of permissible latent correlations in the CS-C(*M*–1) model with indicator-specific factors.

(1) The correlations $Cor(S_{1j1l}, S_{1j'1l}), j \neq j'$, between state factors belonging to different constructs on the same occasion of measurement can be interpreted as coefficients of *discriminant validity* with respect to the reference method. Discriminant validity requires that these correlations not be too high.

(2) *Construct stability* can be assessed by means of the correlations among state factors representing the same construct (measured by the reference method) on different measurement occasions [i.e., $Cor(S_{1j1l}, S_{1j1l'}), l \neq l'$, "construct stability coefficients"]. The finding of a strong positive correlation would suggest that individual differences with respect to the construct under study are stable over time (according to the reference method), and that situation-specific influences are negligible (i.e., that the attribute is trait-like rather than state-like). Weak to moderate correlations may be interpreted in terms of significant occasion-specific effects (i.e., the attribute is state-like rather than trait-like).

(3) The correlations $Cor(S_{1j1l}, S_{1j'1l'}), j \neq j', l \neq l'$, between latent state variables of different constructs measured on different measurement occasions can be interpreted as coefficients of discriminant validity with respect to the reference method that are corrected for common occasion-specific influences. Alternatively,

these correlations may be interpreted as coefficients of *predictive validity* with respect to the reference method.

(4) The correlations $Cor(S_{1j1l}, M_{1j'kl}),\ j \neq j'$, between a state factor of a construct $j$ and any method factor belonging to another construct $j'$ on the same occasion of measurement represent pure discriminant validity coefficients corrected for common method influences of the reference method.

(5) The correlations $Cor(M_{1jkl}, S_{1j'1l'}),\ l \neq l'$, between state factors and method factors belonging to different occasions of measurement are generally difficult to interpret. In most applications, these correlations are not theoretically meaningful and therefore, I recommend fixing them to zero a priori unless strong hypotheses exist as to why such a correlation should differ from zero.

(6) The correlations $Cor(M_{1jkl}, M_{1j'kl}),\ j \neq j'$, between method factors belonging to the same method $k$ but different constructs $j$ and $j'$ on the same occasion of measurement again indicate the degree of generalizability of method effects across constructs on a given occasion of measurement. A correlation of one would indicate a perfectly general (not at all trait-specific) method effect.

(7) The correlations $Cor(M_{1jkl}, M_{1jk'l}),\ k \neq k'$, between method factors belonging to the same construct but different methods are partial correlations between non-reference methods that are corrected for the influence of the reference method (the reference method has been partialled out). They indicate that method-specific deviations from the reference method generalize across different (non-reference) methods. As an example, imagine a study in which self, parent, and teacher ratings are used to measure children's anxiety, and the self-report is selected as the reference method. Then, the correlation between the method factors for the parent- and teacher report would be a partial correlation corrected for the influence of the self-report. A zero correlation would mean that parents and teachers do not share a common view of the children over and above the common view that is shared with the children's own view. On the other hand, if the correlation between the method factors was substantial, this would mean that there is a consistent method bias across methods. For example, two or more raters (e.g., parents, teachers) might share a common view of a target that is not shared with the target's (e.g., student's) own view.

(8) Correlations between method factors belonging to different construct-method units on the same occasion of measurement [i.e., $Cor(M_{1jkl}, M_{1j'k'l})$, $j \neq j'$, $k \neq k'$] also indicate the generalizability of method effects. For example, overestimation of anxiety by teachers (compared to the reference method) might be associated with overestimation of depression by peers.

(9) In order to assess the degree of stability of construct-specific method effects, one can investigate the correlations between method factors belonging to the same construct and the same method on *different* occasions of measurement [i.e., $Cor(M_{1jkl}, M_{1jkl'})$, $l \neq l'$]. A high correlation indicates high stability of method-specific deviations from the reference method. For example, parents might consistently underestimate the degree of anxiety of a child across different situations or occasions of measurement.

(10) The generalizability of method effects across constructs corrected for common occasion-specific influences is mirrored by the correlations $Cor(M_{1jkl}, M_{1j'kl'})$, $j \neq j'$, $l \neq l'$. A high correlation means that the bias of a given non-reference method $k$ is both stable across constructs and stable over time. For instance, teachers might consistently underestimate both the level of anxiety *and* the level of depression in children across different situations / occasions of measurement.

(11) The correlations between method factors belonging to different methods but the same construct measured on different occasions of measurement [i.e., $Cor(M_{1jkl}, M_{1jk'l'})$, $k \neq k'$, $l \neq l'$] indicate the common method bias of non-reference methods corrected for occasion-specific influences.

(12) The generalizability of method effects, corrected for construct-specific and occasion-specific influences, can be investigated by means of the correlations $Cor(M_{1jkl}, M_{1j'k'l'})$, $j \neq j'$, $k \neq k'$, $l \neq l'$.

(13) Indicator-specific factors $IS_{ijk}$ may in principle be correlated with all other indicator-specific factors $IS_{i'j'k'}$. In practical applications, it makes sense to admit the correlations $Cor(IS_{ijk}, IS_{ijk'})$ if highly similar measures are used across methods (e.g., similar questionnaire items). The higher the correlation, the greater is the generalization of indicator-specific effects across methods (e.g., due to similar item wording). In many other cases, correlations between indicator-specific factors and

other factors may not be theoretically meaningful or they may not be of substantial magnitude. For example, correlations between indicator-specific factors $IS_{ijk}$ and state factors $S_{1j'1l}$, as well as between indicator-specific factors $IS_{ijk}$ and method factors $M_{1j'kl}$ belonging to different constructs are permissible for $j \neq j'$. However, in most cases, these correlations will be difficult to interpret. In addition, they are often estimated to be close to zero in empirical applications. Therefore, one should consider fixing them to zero for reasons of parsimony.

### 3.2.4 *Latent Variable Mean Structure*

In the CS-C(*M*–1) model with indicator-specific factors, the means of the latent state factors, $E(S_{1j1l})$, are given by:

$$E(S_{1j1l}) = E(Y_{1j1l}) . \tag{59}$$

Equation 59 follows from Equation 54 given that $Y_{1j1l} = S_{1j1l} + E_{1j1l}$ and $E(E_{ijkl}) = 0$. (All error variables have an expectation of zero by definition.) The method factors and indicator-specific factors are defined as residual factors. Hence, they have zero means:

$$E(IS_{ijk}) \ = \ E(M_{1jkl}) \ = 0. \tag{60}$$

A consequence of Equation 60 is that neither $M_{1jkl}$ nor $IS_{ijk}$ contribute to the observed variable means. For any indicator (except the marker indicators of the reference method), the following mean structure holds:

$$E(Y_{ijkl}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} E(S_{1j1l}), \quad (i,k) \neq (1,1) . \tag{61}$$

### 3.2.5 *Variance Decomposition and Variance Components*

The variances of the latent state variables can be additively decomposed, given that their components are uncorrelated according to Equations 55 to 57:

$$Var(S_{ijkl}) = \begin{cases} \lambda^2_{\mathrm{S}ij1l} Var(S_{1j1l}) + \lambda^2_{\mathrm{IS}ij1l} Var(IS_{ij1}), \text{ for } i \neq 1,\ k = 1, \\ \lambda^2_{\mathrm{S}1jkl} Var(S_{1j1l}) + Var(M_{1jkl}), \text{ for } i = 1,\ k \neq 1, \\ \lambda^2_{\mathrm{S}ijkl} Var(S_{1j1l}) + \lambda^2_{\mathrm{M}ijkl} Var(M_{1jkl}) + \lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk}), \text{ for } i,k \neq 1. \end{cases} \tag{62}$$

For the observed variables, the following variance decomposition is obtained:

$$Var(Y_{ijkl}) = \begin{cases} Var(S_{1j1l}) + Var(E_{1j1l}), \text{ for } i,k=1, \\ \lambda^2_{\text{S}ij1l} Var(S_{1j1l}) + \lambda^2_{\text{IS}ij1l} Var(IS_{ij1}) + Var(E_{ij1l}), \text{ for } i \neq 1, \ k=1, \\ \lambda^2_{\text{S}1jkl} Var(S_{1j1l}) + Var(M_{1jkl}) + Var(E_{1jkl}), \text{ for } i=1, \ k \neq 1, \\ \lambda^2_{\text{S}ijkl} Var(S_{1j1l}) + \lambda^2_{\text{M}ijkl} Var(M_{1jkl}) + \lambda^2_{\text{IS}ijkl} Var(IS_{ijk}) + Var(E_{ijkl}), \text{ for } i,k \neq 1. \end{cases} \quad (63)$$

Due to the additive variance decomposition, coefficients of *consistency*, *method specificity*, *indicator-specificity*, and *reliability* can be defined. The *consistency coefficient* $CO(Y_{ijkl})$ represents the proportion of variance of an observed variable $Y_{ijkl}$ that can be explained by the corresponding state factor $S_{1j1l}$. Hence, $CO(Y_{ijkl})$ indicates the degree of convergent validity with respect to the reference method:

$$CO(Y_{ijkl}) = \frac{\lambda^2_{\text{S}ijkl} Var(S_{1j1l})}{Var(Y_{ijkl})}. \quad (64)$$

The consistency coefficient is occasion-specific. One can compare the consistency coefficients calculated for different time points in order to find out whether the convergent validity of an indicator has changed over time. Note that $\sqrt{CO(Y_{ijkl})} = Cor(Y_{ijkl}, S_{1j1l})$. This correlation can be interpreted as a *standardized validity coefficient* in the sense of Bollen (1989, p. 199).

The *method specificity coefficient* $MS(Y_{ijkl})$ is also occasion-specific and can be calculated for all indicators belonging to non-reference methods. $MS(Y_{ijkl})$ represents the proportion of variance of an indicator that is due to method-specific influences of (non-reference) method $k$ ($k \neq 1$) on a given occasion of measurement:

$$MS(Y_{ijkl}) = \frac{\lambda^2_{\text{M}ijkl} Var(M_{1jkl})}{Var(Y_{ijkl})}, \ k \neq 1. \quad (65)$$

The *indicator-specificity coefficient* $IS(Y_{ijkl})$ gives the proportion of variance that can be attributed to indicator-specific effects. It can be calculated for all non-marker indicators $Y_{ijkl}$, $i \neq 1$:

$$IS(Y_{ijkl}) = \frac{\lambda^2_{\text{IS}ijkl} Var(IS_{ijk})}{Var(Y_{ijkl})}, \ i \neq 1. \quad (66)$$

In contrast to the consistency and method-specificity coefficients, the indicator-specificity coefficient is *occasion-unspecific*. It represents the stable variable-specific part of a non-marker-indicator.

The sum of consistency, method-specificity, and indicator-specificity coefficients yields the *reliability coefficient* $Rel(Y_{ijkl})$. The reliability coefficient represents the proportion of the variance of an indicator that is *not* due to random measurement error:

$$\begin{aligned} Rel(Y_{ijkl}) &= \frac{Var(S_{ijkl})}{Var(Y_{ijkl})} \\ &= \frac{\lambda^2_{Sijkl} Var(S_{1j1l})}{Var(Y_{ijkl})} + \frac{\lambda^2_{Mijkl} Var(M_{1jkl})}{Var(Y_{ijkl})} + \frac{\lambda^2_{ISijkl} Var(IS_{ijk})}{Var(Y_{ijkl})} \\ &= CO(Y_{ijkl}) + MS(Y_{ijkl}) + IS(Y_{ijkl}). \end{aligned} \tag{67}$$

The consistency, method-specificity, and indicator-specificity coefficients can also be defined for the latent state variables $S_{ijkl}$:

$$CO(S_{ijkl}) = \frac{\lambda^2_{Sijkl} Var(S_{1j1l})}{Var(S_{ijkl})}, \tag{68}$$

$$MS(S_{ijkl}) = \frac{\lambda^2_{Mijkl} Var(M_{1jkl})}{Var(S_{ijkl})}, \; k \neq 1, \tag{69}$$

$$IS(S_{ijkl}) = \frac{\lambda^2_{ISijkl} Var(IS_{ijk})}{Var(S_{ijkl})}, \; i \neq 1. \tag{70}$$

Given that the latent state variables $S_{ijkl}$ do not contain measurement error, $CO(S_{ijkl}) + MS(S_{ijkl}) + IS(S_{ijkl}) = 1$. Table 2 summarizes the definition of the CS-C(*M*–1) model with indicator-specific factors.

Table 2

*Summary of the CS-C(M–1) State Model With Indicator-Specific Factors*

| Definition | Equation |
|---|---|
| Basic decomposition of latent state theory | $Y_{ijkl} = S_{ijkl} + E_{ijkl}$ |
| True score regression for state variables pertaining to the reference method | $E(S_{ij1l} \mid S_{1j1l}) = \alpha_{ij1l} + \lambda_{\mathrm{S}ij1l} S_{1j1l}$ (for $i \neq 1$) |
| Definition of indicator-specific variables for the reference method | $IS_{ij1l} := S_{ij1l} - E(S_{ij1l} \mid S_{1j1l})$ |
| Definition of common indicator-specific factors for the reference method | $IS_{ij1l} = \lambda_{\mathrm{IS}ij1l} IS_{ij1}$ |
| True score regression for state variables pertaining to non-reference methods | $E(S_{ijkl} \mid S_{1j1l}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{1j1l}$ (for $k \neq 1$) |
| Definition of method variables | $M_{ijkl} := S_{ijkl} - E(S_{ijkl} \mid S_{1j1l})$ |
| True score regression for method variables pertaining to non-marker indicators | $E(M_{ijkl} \mid M_{1jkl}) = \lambda_{\mathrm{M}ijkl} M_{1jkl}$ (for $i \neq 1$) |
| Definition of indicator-specific variables for the non-reference methods | $IS_{ijkl} := M_{ijkl} - E(M_{ijkl} \mid M_{1jkl})$ |
| Definition of common indicator-specific factors for the non-reference methods | $IS_{ijkl} = \lambda_{\mathrm{IS}ijkl} IS_{ijk}$ |

*(Table continues)*

| Definition | Equation |
| --- | --- |
| Covariances of indicator-specific factors and state factors (same construct) | $Cov(IS_{ijk}, S_{1j1l}) = 0$ |
| Covariances of method factors and state factors (same construct, same measurement occasion) | $Cov(M_{1jkl}, S_{1j1l}) = 0$ |
| Covariances of method factors and indicator-specific factors (same construct, same method) | $Cov(M_{1jkl}, IS_{ijk}) = 0$ |
| Covariances of error variables | $Cov(E_{ijkl}, E_{i'j'k'l'}) = 0,\ (i, j, k, l) \neq (i', j', k', l')$ |
| Covariances between error variables and other latent variables | $Cov(E_{ijkl}, S_{i'j'1l'}) = Cov(E_{ijkl}, M_{1j'k'l'}) = Cov(E_{ijkl}, IS_{i'j'k'}) = 0$ |
| Mean structure (state factors) | $E(S_{1j1l}) = E(Y_{1j1l})$ |
| Mean structure (method factors, indicator-specific factors, and error variables) | $E(M_{1jkl}) = E(IS_{ijk}) = E(E_{ijkl}) = 0$ |
| Variance decomposition (observed variables) | $Var(Y_{ijkl}) = \begin{cases} Var(S_{1j1l}) + Var(E_{1j1l}), \text{ for } i, k = 1, \\ \lambda^2_{\text{S}ij1l} Var(S_{1j1l}) + \lambda^2_{\text{IS}ij1l} Var(IS_{ij1}) + Var(E_{ij1l}), \text{ for } i \neq 1,\ k = 1, \\ \lambda^2_{\text{S}1jkl} Var(S_{1j1l}) + Var(M_{1jkl}) + Var(E_{1jkl}), \text{ for } i = 1,\ k \neq 1, \\ \lambda^2_{\text{S}ijkl} Var(S_{1j1l}) + \lambda^2_{\text{M}ijkl} Var(M_{1jkl}) + \lambda^2_{\text{IS}ijkl} Var(IS_{ijk}) + Var(E_{ijkl}), \text{ for } i, k \neq 1. \end{cases}$ |

*(Table continues)*

| Definition | Equation |
|---|---|
| Consistency (observed variables) | $CO(Y_{ijkl}) = \frac{\lambda^2_{\mathrm{S}ijkl} Var(S_{1j1l})}{Var(Y_{ijkl})}$ |
| Method-specificity (observed variables) | $MS(Y_{ijkl}) = \frac{\lambda^2_{\mathrm{M}ijkl} Var(M_{1jkl})}{Var(Y_{ijkl})}$ (for $k \neq 1$) |
| Indicator-specificity (observed variables) | $IS(Y_{ijkl}) = \frac{\lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk})}{Var(Y_{ijkl})}$, (for $i \neq 1$) |
| Reliability | $Rel(Y_{ijkl}) = \frac{Var(S_{ijkl})}{Var(Y_{ijkl})} = \frac{\lambda^2_{\mathrm{S}ijkl} Var(S_{1j1l})}{Var(Y_{ijkl})} + \frac{\lambda^2_{\mathrm{M}ijkl} Var(M_{1jkl})}{Var(Y_{ijkl})} + \frac{\lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk})}{Var(Y_{ijkl})}$ $= CO(Y_{ijkl}) + MS(Y_{ijkl}) + IS(Y_{ijkl}).$ |
| Variance decomposition (state variables) | $Var(S_{ijkl}) = \begin{cases} \lambda^2_{\mathrm{S}ij1l} Var(S_{1j1l}) + \lambda^2_{\mathrm{IS}ij1l} Var(IS_{ij1}), \text{ for } i \neq 1,\ k = 1, \\ \lambda^2_{\mathrm{S}1jkl} Var(S_{1j1l}) + Var(M_{1jkl}), \text{ for } i = 1,\ k \neq 1, \\ \lambda^2_{\mathrm{S}ijkl} Var(S_{1j1l}) + \lambda^2_{\mathrm{M}ijkl} Var(M_{1jkl}) + \lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk}), \text{ for } i,k \neq 1. \end{cases}$ |
| Consistency (state variables) | $CO(S_{ijkl}) = \frac{\lambda^2_{\mathrm{S}ijkl} Var(S_{1j1l})}{Var(S_{ijkl})}$ |

*(Table continues)*

| Definition | Equation |
|---|---|
| Method-specificity (state variables) | $MS(S_{ijkl}) = \frac{\lambda^2_{\mathrm{M}ijkl} Var(M_{1jkl})}{Var(S_{ijkl})}$ (for $k \neq 1$) |
| Indicator-specificity (state variables) | $IS(S_{ijkl}) = \frac{\lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk})}{Var(S_{ijkl})}$ (for $i \neq 1$) |

*Note.* $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). Without loss of generality, the first method ($k$ = 1) is selected as reference method and the first indicators ($i$ = 1) are selected as marker indicators. $S_{ijkl}$ = latent state variable. $E_{ijkl}$ = error variable. $E(S_{ijkl} \mid S_{1j1l})$ denotes the conditional expectation (regression) of $S_{ijkl}$ on $S_{1j1l}$. $\alpha_{ijkl}$ = intercept. $\lambda_{\mathrm{S}ijkl}$ = state factor loading. $IS_{ijkl}$ = latent indicator-specific residual variable. $IS_{ijk}$ = common indicator-specific factor. $\lambda_{\mathrm{IS}ijkl}$ = indicator-specific factor loading. $M_{ijkl}$ = latent method-specific residual variable. $\lambda_{\mathrm{M}ijkl}$ = method factor loading.

## 3.3 Modeling Strategies for Different Forms of Indicator-Specificity

In this chapter, I have introduced two variants of the CS-C(*M*–1) model: A CS-C(*M*–1) model with indicator-specific state factors and a CS-C(*M*–1) model with general state factors and indicator-specific factors across time. Each model allows analyzing specific forms of indicator-specificity. The CS-C(*M*–1) model with indicator-specific state factors is most appropriate if indicator-specific effects are not expected to generalize across time (e.g., due to long intervals between the measurement occasions), but are expected to generalize across methods within an occasion of measurement (e.g., because similar items have been used across methods). The CS-C(*M*–1) model with general state factors and indicator-specific factors across time is useful if one does not expect indicator-specific effects to generalize across methods, but across time for the same indicator. This latter case is likely the one that is more frequently encountered in longitudinal MTMM studies.

A useful modeling strategy is to estimate both versions of the CS-C(*M*–1) model in the first step of an MTMM-MO analysis. One may then compare the fit of both models (e.g., by means of information criteria) to decide which model more appropriately represents the data. In addition, one may estimate a parsimonious version of the CS-C(*M*–1) model with general state factors, in which the indicator-specific factors across time are dropped (see Figure 15). If this reduced model does not fit worse than the two other model variants, it should be selected given that it is more parsimonious than the two other CS-C(*M*–1) model variants. In Chapter 5, I illustrate the model selection issue in detail based on an empirical application to real data.

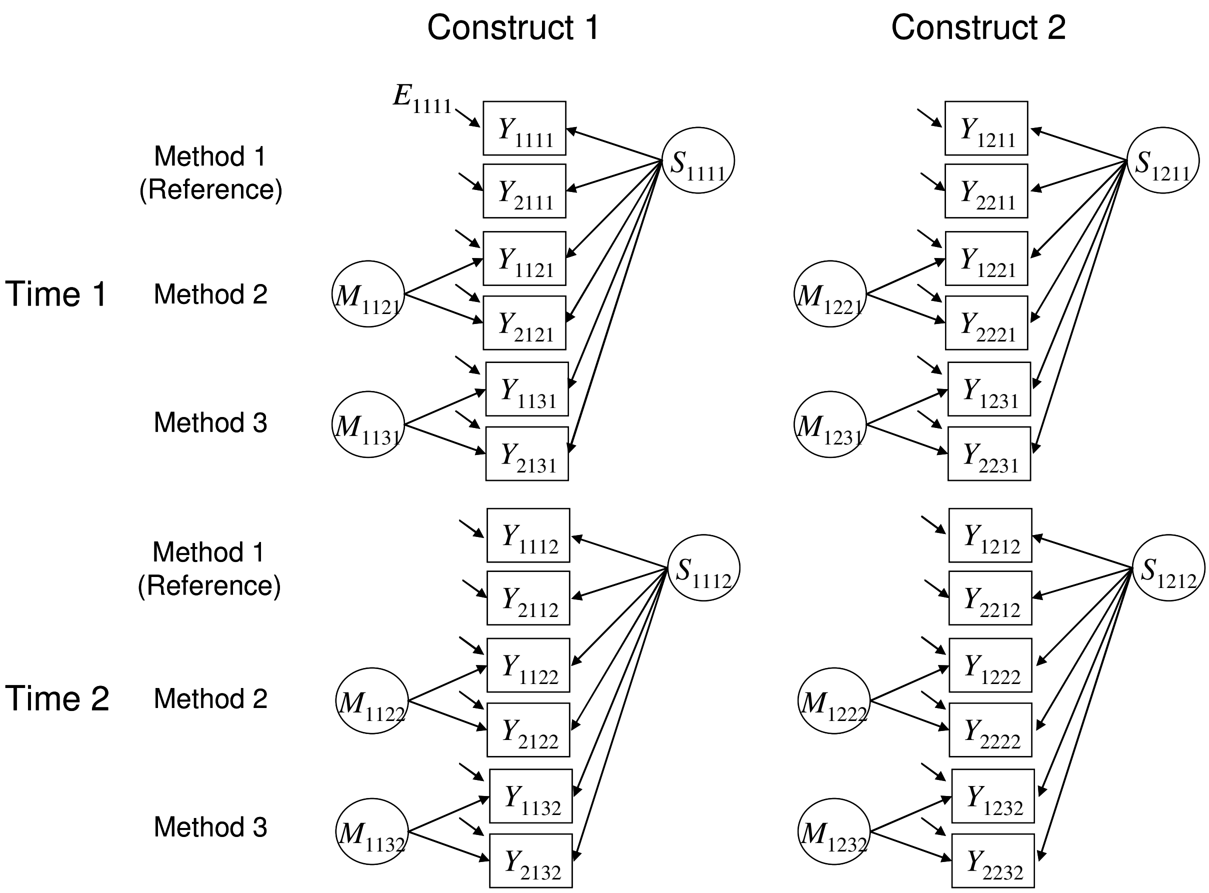


*Figure 15.* CS-C($M$–1) model with general state factors *without* indicator-specific factors $IS_{ijk}$ over time for two constructs, three methods, and two time points. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{1j1l}$ = latent state factor. $M_{1jkl}$ = latent method factor. $E_{ijkl}$ = error variable. For the sake of clarity, permissible factor correlations are not shown.

## 3.4 Measurement Invariance

An important issue in longitudinal modeling is the question of *measurement invariance* (Meredith & Horn, 2001). I have already briefly outlined this issue in Chapter 2.2.2 when I introduced the latent difference version of the CS model. Measurement invariance concerns the question of whether the latent variables to be measured are connected in the same way to their indicators on each measurement occasion (Tisak & Tisak, 2000; Raykov, 2006). In this respect, Meredith (1993) distinguishes between *configural*, *weak*, *strong*, and *strict factorial invariance*. *Configural invariance* only requires that the number of factors and the pattern of factor loadings is the same on each occasion of measurement—a very weak form of invariance. With respect to the CS-C($M$–1) model, this would mean that the indicators load on the same state and method factors on each occasion of measurement, but that the parameters of the measurement model (i.e., the loadings, intercepts, and error variances) need not be constant over time. *Weak factorial invariance* (or *metric invariance*) holds if, in addition to configural invariance, the links between the observed and latent variables (i.e., the loadings $\lambda_{\mathrm{S}ijkl}$, $\lambda_{\mathrm{M}ijkl}$, and $\lambda_{\mathrm{IS}ijkl}$) are equal over time. *Strong factorial invariance* additionally requires the observed variable measurement intercepts $\alpha_{ijkl}$ to be time-invariant. The condition of *strict factorial invariance* is satisfied if the measures show configural invariance as well as constant loadings, constant intercepts, and constant residual variances $Var(E_{ijkl})$ across time. In addition, invariance tests can be conducted with respect to the factor means, variances, and covariances. In practical applications, researchers usually aim at establishing at least weak factorial invariance (constant loadings) in order to make sure that the measurement structure of the latent variables remains the same over time. If weak factorial invariance does not hold, interpretations of certain model parameters might become ambiguous. For example, what do the correlations between state factors measured on different occasions mean if the state factors are not measured in the same way? Measurement invariance is especially important if the latent difference versions of the CS-C(*M*–1) model are considered (see Chapter 4) given that change scores can only be meaningfully interpreted if certain invariance conditions hold. I will thus return to the issue of measurement invariance in Chapter 4.

## 3.5 Formal Definition of the CS-C(*M*–1) Models

In this chapter, the CS-C(*M*–1) models (with and without indicator-specific factors across time) are formally defined on the basis of stochastic measurement theory (Steyer & Eid, 2001). That is, the assumptions of the CS-C(*M*–1) models which have already been introduced in the preceding section will be formalized. This formalization is necessary to define which assumptions must hold in order to logically deduce the existence of the latent variables and the associated parameters (e.g., loadings).

In addition, important questions of measurement theory are discussed for both models. In particular, I examine the questions of *uniqueness*, *meaningfulness*, *testability*, and *identifiability*. The uniqueness problem concerns the question of which kinds of transformations of these variables and parameters are admissible. Related to the uniqueness problem is the question of *meaningfulness*: Which statements about the variables/parameters are meaningful? (Meaningful here means *invariant with respect to the admissible transformations*.) The fourth important issue that will be treated for the models is *testability*. Testability means that one wants to derive consequences of the model definition for the covariance and mean structure of the observed variables. In other words, which are the restrictions imposed by the models? Which covariance and mean structure is implied by the models? Finally, I show under which conditions the parameters of the models can be uniquely determined and estimated from the empirical information (i.e., from the variances, covariances, and means of the observed variables). This concerns the question of *identifiability* of the model parameters.

### *3.5.1 CS-C(M-1) Model Without Indicator-Specific Factors Across Time*

#### 3.5.1.1 Model Definition

The assumptions of the CS-C(*M*–1) model without indicator-specific factors across time are formally expressed in the following definition.

**Definition 1: CS-C(*M*–1) Model**

The random variables $Y_{1111},...,Y_{ijkl},...,Y_{mnop}$, $i \in I := \{1,...,m\}$, $j \in J := \{1,...,n\}$, $k \in K := \{1,...,o\}$, $l \in L := \{1,..., p\}$, on a probability space $(\Omega, \mathfrak{A}, P)$ are variables of a CS-C(*M*–1) model if and only if the following conditions hold:

(a) $(\Omega, \mathfrak{A}, P)$ is a probability space such that $\Omega = U_0 \times U_1 \times ... \times U_n \times M_1 \times ... \times M_n$.

(b) The projections $p_0 : \Omega \rightarrow U_0$, $p_l : \Omega \rightarrow U_l$ are random variables on $(\Omega, \mathfrak{A}, P)$.

(c) The variables $Y_{ijkl} : \Omega \rightarrow \mathbb{R}$ are random variables on $(\Omega, \mathfrak{A}, P)$.

(d) Without loss of generality, the first method (*k* = 1) is selected as reference method. Then, the variables

$$S_{ijkl} := E(Y_{ijkl} \mid p_0, p_l), \tag{71}$$

$$M_{ijkl} := S_{ijkl} - E\left(S_{ijkl} \mid S_{ij1l}\right), \text{ and} \tag{72}$$

$$E_{ijkl} := Y_{ijkl} - S_{ijkl} \tag{73}$$

are random variables on $(\Omega, \mathfrak{A}, P)$, where $E(Y_{ijkl} \mid p_0, p_l)$ denotes the conditional expectation of $Y_{ijkl}$ given the person $(p_0)$ and the situation $(p_l)$, and $E\left(S_{ijkl} \mid S_{ij1l}\right)$ denotes the conditional expectation of $S_{ijkl}$ given the reference state variable $S_{ij1l}$. The variables $E_{ijkl}$ are the measurement error variables.

(e) For each quadruple $(i, j, k, l)$ $i \in I$, $j \in J$, $k \in K$, $l \in L$, $k \neq 1$ there is a constant $\alpha_{ijkl} \in \mathbb{R}$

as well as a constant $\lambda_{\mathrm{S}ijkl} \in \mathbb{R}$, $\lambda_{\mathrm{S}ijkl} > 0$, such that

$$E\left(S_{ijkl} \mid S_{ij1l}\right) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij1l}. \tag{74}$$

(f) For each quintuple $(i, i', j, k, l)$, $i, i' \in I$, $j \in J$, $k \in K$, $l \in L$, $k \neq 1$ there is a constant $\lambda_{\mathrm{M}ii'jkl} \in \mathbb{R}$, $\lambda_{\mathrm{M}ii'jkl} > 0$, such that

$$M_{ijkl} = \lambda_{\mathrm{M}ii'jkl} M_{i'jkl}. \tag{75}$$

*Explanations.* Each observed variable $Y_{ijkl}$ has its own associated latent state (true score) variable $S_{ijkl}$. According to Condition (e), all latent state variables $S_{ijkl}$ belonging to the same construct and the same occasion of measurement are positive linear functions of the reference state variables $S_{ij1l}$. The variables $S_{ij1l}$ are labeled *reference state variables* because they pertain to indicators measured by the reference method ($k$ = 1). Specifically, Condition (e) states that the latent state variables $S_{ijkl}$ are regressed on the respective reference state variables $S_{ij1l}$ and that these regressions $E\left(S_{ijkl} \mid S_{ij1l}\right)$ are linear. In other words, the reference state variables $S_{ij1l}$ are used to predict the latent state variables belonging to the same indicator, construct and occasion, and a *non-reference method* ($k \neq 1$) in a linear latent regression analysis. The residuals of this latent regression analysis are the method variables $M_{ijkl}$. These residuals represent inter-individual differences with respect to the variables $S_{ijkl}$ that cannot be explained by the corresponding reference state variable $S_{ij1l}$, but are due to construct- and occasion-specific method influences. Condition (f) states that the residuals $M_{ijkl}$ belonging to the same construct, method, and occasion are linear functions of each other (they may only differ by a multiplicative constant $\lambda_{\mathrm{M}ii'jkl}$). Hence, the residuals $M_{ijkl}$ are assumed to be perfectly correlated. There is no additive constant in Equation 75 given that the expectation (mean) of residual variables is always zero (see, e.g., Steyer & Eid, 2001, p. 357).

### 3.5.1.2 Existence of Common Method Factors $M_{jkl}$

The following corollary shows that the assumption of perfectly correlated $M_{ijkl}$ variables (Equation 75) is equivalent to assuming a *common method factor* $M_{jkl}$ for all indicators belonging to the same construct, (non-reference) method, and measurement occasion.

**Corollary 1: Existence of Common Latent Method Factors $M_{jkl}$**

The random variables $Y_{1111},...,Y_{ijkl},...,Y_{mnop}$, $i \in I := \{1,...,m\}$, $j \in J := \{1,...,n\}$, $k \in K := \{1,...,o\}$, $l \in L := \{1,..., p\}$, on a probability space $(\Omega, \mathfrak{A}, P)$ are variables of a CS-C($M$–1) model if

conditions (a) to (e) in Definition 1 hold and

(f') for each quadruple $(i, j, k, l)$, $i \in I$, $j \in J$, $k \in K$, $l \in L$, $k \neq 1$, there is a constant $\lambda_{\text{M}ijkl} \in \mathbb{R}$, $\lambda_{\text{M}ijkl} > 0$, and a real random variable $M_{jkl}$ on $(\Omega, \mathfrak{A}, P)$ such that:

$$M_{ijkl} = \lambda_{\text{M}ijkl} M_{jkl}. \tag{76}$$

*Proof*. If one defines, for example, $M_{jkl} := M_{1jkl}$ as well as $\lambda_{\text{M}ijkl} := \lambda_{\text{M}i1jkl}$ and inserts these parameters in Equation 75 (see Definition 1), this results in $M_{ijkl} = \lambda_{\text{M}ijkl} \cdot M_{jkl}$ (Equation 76). Furthermore, according to Equation 76, $M_{jkl}$ can be expressed as $M_{jkl} = \frac{M_{ijkl}}{\lambda_{\text{M}ijkl}}$ as well as $M_{jkl} = \frac{M_{i'jkl}}{\lambda_{\text{M}i'jkl}}$. By setting both equations equal, it follows that $M_{ijkl} = \frac{\lambda_{\text{M}ijkl}}{\lambda_{\text{M}i'jkl}} M_{i'jkl}$. By defining $\lambda_{\text{M}ii'jkl} := \frac{\lambda_{\text{M}ijkl}}{\lambda_{\text{M}i'jkl}}$ one obtains Equation 75.

*Explanations.* Corollary 1 shows an important implication of Condition (f) in Definition 1, namely that all residuals $M_{ijkl}$ measure a common method factor $M_{jkl}$. This implication is obvious given that Condition (f) in Definition 1 postulates that all residuals $M_{ijkl}$ belonging to the same construct, method, and measurement occasion differ only by a multiplicative constant ($\lambda_{\text{M}ii'jkl}$). It follows that all indicators $Y_{ijkl}$ pertaining to the same construct, method, and measurement occasion measure a latent state factor $S_{ij1l}$ and a common construct- and occasion-specific method factor $M_{jkl}$. In sum, the measurement equations for the observed variables are given by:

$$Y_{ijkl} = \begin{cases} S_{ij1l} + E_{ij1l}, & \text{for } k = 1, \text{ and} \\ \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{ij1l} + \lambda_{\text{M}ijkl} M_{jkl} + E_{ijkl}, & \text{for } k \neq 1. \end{cases} \tag{77}$$

The proof of Corollary 1 makes clear that the variables $M_{jkl}$ are *not* uniquely defined. Any of the residual variables $M_{ijkl}$ (or even any similarity transformation of a method variable $M_{ijkl}$) could be chosen to play the role of $M_{jkl}$. The question of the uniqueness of the variables $M_{jkl}$ as well as the associated $\lambda_{\text{M}ijkl}$-parameters is treated in more detail in the next section.

3.5.1.3 Admissible Transformations and Uniqueness

After having defined the CS-C(*M*–1) model, it is useful to examine how *unique* the parameters of this model are defined given the assumptions in Definition 1 and Corollary 1. For example, the uniqueness issue concerns the question of whether there is only one possible "version" of the method factors given the above assumptions or whether there are many different versions all of which fulfill these assumptions. In general, the variables and parameters in a measurement model are not uniquely defined, as there are usually many different admissible versions of the variables and parameters. Hence, it is important to assess which *transformations* of the variables and model parameters are admissible (i.e., which kinds of transformations lead to other versions of a given parameter that also fulfill the model assumptions). The analysis of the degree of uniqueness is also important to determine which statements with respect to the variables are *meaningful* (the concept of meaningfulness will be explained in the next section). A synonym for "degree of uniqueness" is *scale level*. Thus, one could also say that with the question of uniqueness one seeks to determine the scale level of the variables of a given measurement model (for a more detailed discussion of the uniqueness problem see Steyer & Eid, 2001, Chapter 7.3).

With respect to the CS-C(*M*–1) model, questions of uniqueness and meaningfulness need to be investigated only for the latent method factors $M_{jkl}$ and the method factor loadings $\lambda_{\text{M}ijkl}$. The reason is that the latent state variables $S_{ij1l}$ *are* uniquely defined in the CS-C(*M*–1) model. This can be seen from Definition 1, in which the variables $S_{ij1l}$ were defined as the conditional expectations of the variables $Y_{ij1l}$ given the person and the situation [see Condition (d) in Definition 1]. Hence, the variables $S_{ij1l}$ are uniquely defined as the latent state true score variables of the variables $Y_{ij1l}$. In contrast, the variables $M_{jkl}$ and the coefficients $\lambda_{\text{M}ijkl}$ are not uniquely defined. Recall that in Corollary 1 I have shown that method effects are common for all variables measuring the same construct *j* by the same non-reference method $(k \neq 1)$ on the same measurement occasion *l*: $M_{ijkl} = \lambda_{\text{M}ijkl} M_{jkl}$. Thus, the variables $M_{jkl}$ (as well as the coefficients $\lambda_{\mathrm{M}ijkl}$) are uniquely defined only up to *similarity transformations*, that is, up to a multiplication with a positive real number. This is shown in Corollary 2:

**Corollary 2: Admissible Transformations**

If (a) $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl} \right\rangle$ is a CS-C($M$–1) model and (b) the variables $M^*_{jkl}$ as well as the coefficients $\lambda^*_{\mathrm{M}ijkl}$, are defined as follows for all $i \in I,\ j \in J,\ k \in K,\ l \in L,$ and $k \neq 1$:

$$M^*_{jkl} := \delta_{jkl} M_{jkl}, \tag{78}$$

$$\lambda^*_{\mathrm{M}ijkl} := \left( \frac{1}{\delta_{jkl}} \right) \lambda_{\mathrm{M}ijkl}, \tag{79}$$

where $\delta_{jkl} \in \mathbb{R}$, $\delta_{jkl} > 0$, then (c) $M^* := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M^*_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda^*_{\mathrm{M}ijkl} \right\rangle$ is a CS-C($M$–1) model, too, given that the following equation holds for all $i \in I,\ j \in J,\ k \in K,\ l \in L,$ and $k \neq 1$:

$$M_{ijkl} = \lambda^*_{\mathrm{M}ijkl} M^*_{jkl}. \tag{80}$$

*Proof*. In Equation 46, $M_{jkl}$ can be replaced by $M^*_{jkl}$ if the constant $\lambda_{\mathrm{M}ijkl}$ is also replaced by $\lambda^*_{\mathrm{M}ijkl}$:

$$M_{ijkl} = \lambda_{\mathrm{M}ijkl} M_{jkl} = \lambda^*_{\mathrm{M}ijkl} M^*_{jkl} = \left( \frac{1}{\delta_{jkl}} \right) \lambda_{\mathrm{M}ijkl} \delta_{jkl} M_{jkl} = \lambda_{\mathrm{M}ijkl} M_{jkl}.$$

Consequently, there is a whole “family” of latent method factors $M_{jkl}$ with associated coefficients $\lambda_{\mathrm{M}ijkl}$. All members of this family are similarity transformations of each other. Therefore, $M_{jkl}$ as well as $\lambda_{\mathrm{M}ijkl}$ are measured on a ratio scale.

### 3.5.1.4 Meaningfulness

As shown in Corollary 2, the variables $M_{jkl}$ as well as the parameters $\lambda_{\mathrm{M}ijkl}$ are uniquely defined only up to similarity transformations. Therefore, it has to be shown which statements concerning these model parameters are meaningful. A *meaningful statement* can be understood as a statement that remains true even if the parameter under consideration has been subject to one of the admissible transformation. The most important meaningful statements with respect to $M_{jkl}$ and $\lambda_{\mathrm{M}ijkl}$ are provided in Corollary 3:

**Corollary 3: Meaningfulness**

If both $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl} \right\rangle$ and $M^* := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M^*_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda^*_{\mathrm{M}ijkl} \right\rangle$ are CS-C(*M*–1) models, then for

(1) $i, i' \in I,\ j \in J,\ k \in K,\ l \in L$:

$$\frac{\lambda_{\mathrm{M}ijkl}}{\lambda_{\mathrm{M}i'jkl}} = \frac{\lambda^*_{\mathrm{M}ijkl}}{\lambda^*_{\mathrm{M}i'jkl}}, \tag{81}$$

(2) $\omega_1, \omega_2 \in \Omega,\ j \in J,\ k \in K,\ l \in L$:

$$\frac{M_{jkl}(\omega_1)}{M_{jkl}(\omega_2)} = \frac{M^*_{jkl}(\omega_1)}{M^*_{jkl}(\omega_2)}, \tag{82}$$

(3) $\omega_1, \omega_2 \in \Omega,\ j, j' \in J,\ k, k' \in K,\ l, l' \in L$:

$$\frac{M_{jkl}(\omega_1)}{M_{jkl}(\omega_2)} - \frac{M_{j'k'l'}(\omega_1)}{M_{j'k'l'}(\omega_2)} = \frac{M^*_{jkl}(\omega_1)}{M^*_{jkl}(\omega_2)} - \frac{M^*_{j'k'l'}(\omega_1)}{M^*_{j'k'l'}(\omega_2)}, \tag{83}$$

(4) $i \in I,\ j \in J,\ k \in K,\ l \in L$:

$$\lambda^2_{\mathrm{M}ijkl} Var\left(M_{jkl}\right) = \lambda^{*2}_{\mathrm{M}ijkl} Var\left(M^*_{jkl}\right), \tag{84}$$

(5) $j, j' \in J,\ k, k' \in K,\ l, l' \in L$:

$$Corr\left(M_{jkl}, M_{j'k'l'}\right) = Corr\left(M^*_{jkl}, M^*_{j'k'l'}\right). \tag{85}$$

(6) $i \in I,\ j, j' \in J,\ k \in K,\ l, l' \in L,\ j \neq j'$:

$$Corr\left(M_{jkl}, S_{ij'1l'}\right) = Corr\left(M^{*}_{jkl}, S_{ij'1l'}\right), \tag{86}$$

where $Var(.)$ denotes the variance and $Corr(.,.)$ denotes the correlation.

*Proofs*. By inserting $\lambda_{\mathrm{M}ijkl}\left(\frac{1}{\delta_{jkl}}\right)$ for $\lambda^{*}_{\mathrm{M}ijkl}$ as well as $\delta_{jkl} M_{jkl}$ for $M^{*}_{jkl}$ one can easily show that Equations 81 to 86 hold.

*Explanations.* For the factor loadings $\lambda_{\mathrm{M}ijkl}$, statements with regard to absolute values of these parameters are *not* meaningful, given that the multiplication with a positive real number is permissible and would result in different values of the $\lambda_{\mathrm{M}ijkl}$-parameters. The same argument holds for specific values of the variables $M_{jkl}$. However, statements with respect to the *ratio* of two coefficients $\lambda_{\mathrm{M}ijkl}$ and $\lambda_{\mathrm{M}i'jkl}$ are meaningful (see Equations 81). Furthermore, statements with respect to the *ratio* of specific values of the method factors $M_{jkl}$ are meaningful, too (Equation 82). Consequently, it is meaningful to say, for instance, that the value of a person A on a common method factor is *x*-times larger that the value of a person B on the same method factor. Meaningful statements can also be made with respect to the differences between ratios of values of two different method factors $M_{jkl}$ and $M_{j'k'l'}$ (Equation 83). Since the product $\lambda^{2}_{\mathrm{M}ijkl} Var\left(M_{jkl}\right)$ is also invariant under similarity transformations (Equation 84), it follows that statements with respect to the *method specificity coefficients* (see Table 1) are meaningful, too. Moreover, correlations between method factors as well as correlations between method factors and state factors are meaningful—as far as they are admissible (see Equations 85 and 86).

#### 3.5.1.5 Covariance Structure

In order to derive testable consequences of the CS-C(*M*–1) model for the covariance structure of the observed variables, it is necessary to introduce further assumptions that are not included in Definition 1 (cf. Steyer, 1988). These assumptions define a more restrictive variant of the CS-C(*M*–1) model, which I call *CS-C(M–1) model with conditional regressive independence* (see Definition 2).

**Definition 2: CS-C($M$–1) Model With Conditional Regressive Independence**

If $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl} \right\rangle$ is a CS-C($M$–1) model, then $M$ is called a *CS-C(M–1) model with conditional regressive independence* if and only if the following assumption holds for all $i, i' \in I := \{1,...,m\}$, $j, j' \in J := \{1,...,n\}$, $k, k' \in K := \{1,...,o\}$, $l, l' \in L := \{1,..., p\}$:

$$E\left[Y_{ijkl} \mid p_0, p_1, ..., p_p, \left(Y_{i'j'k'l'}, (i', j', k', l') \neq (i, j, k, l)\right)\right] = E\left(Y_{ijkl} \mid p_0, p_l\right). \tag{87}$$

*Explanations.* Equation 87 states that given a person and a situation on a measurement occasion $l$, an observed variable $Y_{ijkl}$ does neither depend on other situations (on different measurement occasions $l'$, $l \neq l'$), nor on the values of other observed variables. As I will show, this assumption has important consequences for the covariance structure of the observed variables. Theorem 1 summarizes the implications of the CS-C($M$–1) model with conditional regressive independence for the covariance structure of the observed variables.

**Theorem 1: Covariance Structure**

If $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl} \right\rangle$ is a CS-C($M$–1) model with conditional regressive independence and, without loss of generality, $k = 1$ is chosen as the reference method, then the following covariance structure holds for all $i, i' \in I := \{1,...,m\}$, $j, j' \in J := \{1,...,n\}$, $k, k' \in K := \{1,...,o\}$, $l, l' \in L := \{1,..., p\}$ and

(a) for all observed variables:

$$\begin{aligned} Cov(Y_{ijkl}, Y_{i'j'k'l'}) = {} & \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'j'k'l'}Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'j'k'l'}Cov(S_{ij1l}, M_{j'k'l'}) \\ & + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'j'k'l'}Cov(M_{jkl}, S_{i'j'1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'j'k'l'}Cov(M_{jkl}, M_{j'k'l'}) \\ & + Cov(E_{ijkl}, E_{i'j'k'l'}) \end{aligned} \tag{88}$$

(b) for all latent variables:

$$Cov(S_{ij1l}, M_{jkl}) = 0, \tag{89}$$

$$Cov(S_{ij1l}, E_{i'j'k'l'}) = 0, \tag{90}$$

$$Cov(M_{jkl}, E_{i'j'k'l'}) = 0, \tag{91}$$

$$Cov(E_{ijkl}, E_{i'j'k'l'}) = 0, \quad (i,j,k,l) \neq (i',j',k',l'), \tag{92}$$

where $Cov(.,.)$ denotes the covariance.

*Proofs.*

*Equation 88*

The covariance structure of the observed variables follows from Equations 77, 90, and 91 by applying rules of covariance algebra (see, e.g., Bollen, 1989; Steyer & Eid, 2001, Box F.1):

$$
\begin{aligned}
& Cov(Y_{ijkl}, Y_{i'j'k'l'}) \\
&= Cov\left[(\alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij1l} + \lambda_{\mathrm{M}ijkl} M_{jkl} + E_{ijkl}), (\alpha_{i'j'k'l'} + \lambda_{\mathrm{S}i'j'k'l'} S_{i'j'1l'} + \lambda_{\mathrm{M}i'j'k'l'} M_{j'k'l'} + E_{i'j'k'l'})\right] \\
&= Cov(\alpha_{ijkl}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{S}i'j'k'l'} Cov(\alpha_{ijkl}, S_{i'j'1l'}) + \lambda_{\mathrm{M}i'j'k'l'} Cov(\alpha_{ijkl}, M_{j'k'l'}) + Cov(\alpha_{ijkl}, E_{i'j'k'l'}) \\
&\quad + \lambda_{\mathrm{S}ijkl} Cov(S_{ij1l}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}i'j'k'l'} Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{M}i'j'k'l'} Cov(S_{ij1l}, M_{j'k'l'}) \\
&\quad + \lambda_{\mathrm{S}ijkl} Cov(S_{ij1l}, E_{i'j'k'l'}) \\
&\quad + \lambda_{\mathrm{M}ijkl} Cov(M_{jkl}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{S}i'j'k'l'} Cov(M_{jkl}, S_{i'j'1l'}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}i'j'k'l'} Cov(M_{jkl}, M_{j'k'l'}) \\
&\quad + \lambda_{\mathrm{M}ijkl} Cov(M_{jkl}, E_{i'j'k'l'}) \\
&\quad + Cov(E_{ijkl}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{S}i'j'k'l'} Cov(E_{ijkl}, S_{i'j'1l'}) + \lambda_{\mathrm{M}i'j'k'l'} Cov(E_{ijkl}, M_{j'k'l'}) + Cov(E_{ijkl}, E_{i'j'k'l'}).
\end{aligned}
$$

Given that constants cannot covary with other constants or variables, $Cov(\alpha_{ijkl}, \alpha_{i'j'k'l'})$ = $\lambda_{\mathrm{S}i'j'k'l'} Cov(\alpha_{ijkl}, S_{i'j'1l'})$ = $\lambda_{\mathrm{M}i'j'k'l'} Cov(\alpha_{ijkl}, M_{j'k'l'})$ = $Cov(\alpha_{ijkl}, E_{i'j'k'l'})$ = $\lambda_{\mathrm{S}ijkl} Cov(S_{ij1l}, \alpha_{i'j'k'l'})$ = $\lambda_{\mathrm{M}ijkl} Cov(M_{jkl}, \alpha_{i'j'k'l'})$ = $Cov(E_{ijkl}, \alpha_{i'j'k'l'}) = 0$. The terms $\lambda_{\mathrm{S}ijkl} Cov(S_{ij1l}, E_{i'j'k'l'})$, $\lambda_{\mathrm{M}ijkl} Cov(M_{jkl}, E_{i'j'k'l'})$, $\lambda_{\mathrm{S}i'j'k'l'} Cov(E_{ijkl}, S_{i'j'1l'})$, and $\lambda_{\mathrm{M}i'j'k'l'} Cov(E_{ijkl}, M_{j'k'l'})$ are equal to zero according to Equations 90 and 91.

*Equation 89*

The uncorrelatedness of the latent state variables with all method factors that belong to the same construct on the same measurement occasion follows since $M_{jkl} = \frac{1}{\lambda_{\mathrm{M}ijkl}} M_{ijkl}$ (see Equation 76). Therefore, $Cov(S_{ij1l}, M_{jkl}) = \frac{1}{\lambda_{\mathrm{M}ijkl}} Cov(S_{ij1l}, M_{ijkl})$.

The covariance between $S_{ij1l}$ and $M_{ijkl}$ is zero because $M_{ijkl}$ is a residual with respect to $S_{ij1l}$

(see Equation 72; residuals are always uncorrelated with their regressors, see Steyer & Eid, 2001, Box G.1). Hence, $Cov(S_{ij1l}, M_{jkl}) = 0,$ too.

*Equations 90–92*

These equations follow from the independence assumption introduced in Definition 2. Equation 92 can be rewritten as

$Cov(E_{ijkl}, E_{i'j'k'l'}) = Cov\left\{\left[Y_{ijkl} - E(Y_{ijkl} \mid p_0, p_l)\right], \left[Y_{i'j'k'l'} - E(Y_{i'j'k'l'} \mid p_0, p_{l'})\right]\right\}$. According to Bauer (1978, p. 54, Satz 9.4) $E_{i'j'k'l'} = Y_{i'j'k'l'} - E(Y_{i'j'k'l'} \mid p_0, p_l)$ is a $(p_0, p_l, Y_{i'j'k'l'})$-measurable function (cf. Steyer, 1988, p. 368-369). The supposition made in Definition 2 allows replacing $E(Y_{ijkl} \mid p_0, p_l)$ by

$E\left[Y_{ijkl} \mid p_0, p_1, ..., p_p, \left(Y_{i'j'k'l'}, \left(i', j', k', l'\right) \neq \left(i, j, k, l\right)\right)\right]$. Hence, for $\left(i', j', k', l'\right) \neq \left(i, j, k, l\right)$, $E_{ijkl}$ is a residual also with respect to the regressors $p_0, p_l$ and $Y_{i'j'k'l'}$. Given that a residual (here: $E_{ijkl}$) is always uncorrelated with each numerically measurable function (here: $E_{i'j'k'l'}$) of his regressors, $Cov(E_{ijkl}, E_{i'j'k'l'}) = 0$ for $(i, j, k, l) \neq (i', j', k', l')$. The derivation of equation (90) follows a similar logic:

$Cov(S_{ijkl}, E_{i'j'k'l'}) = Cov\left\{E(Y_{ijkl} \mid p_0, p_l), \left[Y_{i'j'k'l'} - E(Y_{i'j'k'l'} \mid p_0, p_{l'})\right]\right\}$. According to Definition 2, $E(Y_{i'j'k'l'} \mid p_0, p_{l'})$ can be replaced by

$E\left[Y_{i'j'k'l'} \mid p_0, p_1, ..., p_p, \left(Y_{ijkl}, \left(i, j, k, l\right) \neq \left(i', j', k', l'\right)\right)\right]$. The variable $S_{ijkl} := E(Y_{ijkl} \mid p_0, p_l)$ is a $(p_0, p_l)$-measurable function and $E_{i'j'k'l'}$ is a residual with respect to the regressors $p_0$ and $p_l$. As stated before, a residual (here: $E_{i'j'k'l'}$) is always uncorrelated with each numerically measurable function [in this case $E(Y_{ij1l} \mid p_0, p_l)$ ] of his regressors. Therefore, Equation 90 holds, too.

Equation 91: By using Equations 76, 72 and 74, we may rewrite Equation 91 as follows:

$$Cov(M_{jkl}, E_{i'j'k'l'}) = Cov\left[\frac{1}{\lambda_{\text{M}ijkl}} M_{ijkl}, E_{i'j'k'l'}\right] = Cov\left[\frac{1}{\lambda_{\text{M}ijkl}}\left(S_{ijkl} - E\left(S_{ijkl} \mid S_{ij1l}\right)\right), E_{i'j'k'l'}\right]$$

$$= \frac{1}{\lambda_{\text{M}ijkl}} Cov\left(S_{ijkl}, E_{i'j'k'l'}\right) - Cov\left(E\left(S_{ijkl} \mid S_{ij1l}\right), E_{i'j'k'l'}\right)$$

$$= \frac{1}{\lambda_{\text{M}ijkl}} Cov\left(S_{ijkl}, E_{i'j'k'l'}\right) - Cov\left(\left(\alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{ij1l}\right), E_{i'j'k'l'}\right)$$

$$= \frac{1}{\lambda_{\text{M}ijkl}} Cov\left(S_{ijkl}, E_{i'j'k'l'}\right) - \lambda_{\text{S}ijkl} Cov\left(S_{ij1l}, E_{i'j'k'l'}\right).$$

Given that both $S_{ijkl}$ and $S_{ij1l}$ are uncorrelated with $E_{i'j'k'l'}$, it follows that $M_{jkl}$ and $E_{i'j'k'l'}$ are also uncorrelated.

*Explanations.* Theorem 1 shows the implications of the model definition for the observed and latent variable covariance structure. Only the most general covariance structure equation for the observed variables is shown in Theorem 1 (Equation 88). To illustrate in more detail in which way the observed variances and covariances are functions of the parameters of the model, I provide the most important special cases of Equation 88 in Corollary 4.

The independence of method factors and state factors belonging to the same construct on the same measurement occasion (Equation 89) is not a newly introduced assumption, but a direct consequence of the definition of the method factors $M_{jkl}$ as residuals with respect to $S_{ij1l}$ (Definition 1). The independence of state factors and error variables (Equation 90), method factors and error variables (Equation 91), as well as error variables and other error variables (Equation 62) is a consequence of the conditional regressive independence assumption in Definition 2. Note that in empirical applications of the model, the respective covariances must be set to zero.

**Corollary 4: Covariance Structure of the Observed Variables**

If $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\text{S}ijkl}, \lambda_{\text{M}ijkl} \right\rangle$ is a CS-C(*M*–1) model with conditional regressive independence and, without loss of generality, $k = 1$ is chosen as the reference method, then the following covariance structure holds for all $i, i' \in I := \{1, ..., m\}$, $j, j' \in J := \{1, ..., n\}$, $k, k' \in K := \{1, ..., o\}$, $l, l' \in L := \{1, ..., p\}$ and

(a) for all observed variables measured on the same occasion of measurement ($l = l'$):

$$Cov(Y_{ijkl}, Y_{i'j'k'l}) = \begin{cases} Var(S_{ij1l}) + Var(E_{ij1l}),\ i = i', j = j',\ k, k' = 1, \\ \lambda^2_{\text{S}ijkl} Var(S_{ij1l}) + \lambda^2_{\text{M}ijkl} Var(M_{jkl}) + Var(E_{ijkl}),\ i = i', j = j',\ k = k',\ k' \neq 1, \\ \lambda_{\text{S}ijk'l} Var(S_{ij1l}),\ i = i', j = j',\ k = 1,\ k' \neq 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}ijk'l} Var(S_{ij1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}ijk'l} Cov(M_{jkl}, M_{jk'l}),\ i = i', j = j',\ k \neq k',\ k, k' \neq 1, \\ Cov(S_{ij1l}, S_{i'j1l}),\ i \neq i', j = j',\ k, k' = 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}i'jkl} Cov(S_{ij1l}, S_{i'j1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}i'jkl} Var(M_{jkl}),\ i \neq i', j = j',\ k = k',\ k' \neq 1, \\ \lambda_{\text{S}i'jk'l} Cov(S_{ij1l}, S_{i'j1l}),\ i \neq i', j = j',\ k = 1,\ k' \neq 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}i'jk'l} Cov(S_{ij1l}, S_{i'j1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}i'jk'l} Cov(M_{jkl}, M_{jk'l}),\ i \neq i', j = j',\ k \neq k',\ k, k' \neq 1, \\ Cov(S_{ij1l}, S_{ij'1l}),\ i = i', j \neq j',\ k, k' = 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}ij'kl} Cov(S_{ij1l}, S_{ij'1l}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}ij'kl} Cov(S_{ij1l}, M_{j'kl}) \\ \quad + \lambda_{\text{M}ijkl}\lambda_{\text{S}ij'kl} Cov(M_{jkl}, S_{ij'1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}ij'kl} Cov(M_{jkl}, M_{j'kl}),\ i = i', j \neq j',\ k = k',\ k' \neq 1, \\ \lambda_{\text{S}ij'k'l} Cov(S_{ij1l}, S_{ij'1l}) + \lambda_{\text{M}ij'k'l} Cov(S_{ij1l}, M_{j'k'l}),\ i = i', j \neq j',\ k = 1,\ k' \neq 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}ij'k'l} Cov(S_{ij1l}, S_{ij'1l}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}ij'k'l} Cov(S_{ij1l}, M_{j'k'l}) \\ \quad + \lambda_{\text{M}ijkl}\lambda_{\text{S}ij'k'l} Cov(M_{jkl}, S_{ij'1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}ij'k'l} Cov(M_{jkl}, M_{j'k'l}),\ i = i', j \neq j',\ k \neq k',\ k, k' \neq 1, \\ Cov(S_{ij1l}, S_{i'j'1l}),\ i \neq i', j \neq j',\ k, k' = 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}i'j'kl} Cov(S_{ij1l}, S_{i'j'1l}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}i'j'kl} Cov(S_{ij1l}, M_{j'kl}) \\ \quad + \lambda_{\text{M}ijkl}\lambda_{\text{S}i'j'kl} Cov(M_{jkl}, S_{i'j'1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}i'j'kl} Cov(M_{jkl}, M_{j'kl}),\ i \neq i', j \neq j',\ k = k',\ k' \neq 1, \\ \lambda_{\text{S}i'j'k'l} Cov(S_{ij1l}, S_{i'j'1l}) + \lambda_{\text{M}i'j'k'l} Cov(S_{ij1l}, M_{j'k'l}),\ i \neq i', j \neq j',\ k = 1,\ k' \neq 1, \\ \lambda_{\text{S}ijkl}\lambda_{\text{S}i'j'k'l} Cov(S_{ij1l}, S_{i'j'1l}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}i'j'k'l} Cov(S_{ij1l}, M_{j'k'l}) \\ \quad + \lambda_{\text{M}ijkl}\lambda_{\text{S}i'j'k'l} Cov(M_{jkl}, S_{i'j'1l}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}i'j'k'l} Cov(M_{jkl}, M_{j'k'l}),\ i \neq i', j \neq j',\ k \neq k',\ k, k' \neq 1, \end{cases} \tag{93}$$

(b) for all observed variables measured on different occasions of measurement ($l \neq l'$):

$$
Cov(Y_{ijkl}, Y_{i'j'k'l'}) = \begin{cases}
Cov(S_{ij1l}, S_{ij1l'}), \;\; i = i', \, j = j', \, k, k' = 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}ijkl'} Cov(S_{ij1l}, S_{ij1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}ijkl'} Cov(S_{ij1l}, M_{jkl'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}ijkl'} Cov(M_{jkl}, S_{ij1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}ijkl'} Cov(M_{jkl}, M_{jkl'}), \;\; i = i', \, j = j', \, k = k', \, k' \neq 1, \\
\lambda_{\mathrm{S}ijk'l'} Cov(S_{ij1l}, S_{ij1l'}) + \lambda_{\mathrm{M}ijk'l'} Cov(S_{ij1l}, M_{jk'l'}), \; i = i', \, j = j', \, k = 1, \, k' \neq 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}ijk'l'} Cov(S_{ij1l}, S_{ij1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}ijk'l'} Cov(S_{ij1l}, M_{jk'l'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}ijk'l'} Cov(M_{jkl}, S_{ij1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}ijk'l'} Cov(M_{jkl}, M_{jk'l'}), \;\; i = i', \, j = j', \, k \neq k', \, k, k' \neq 1, \\
Cov(S_{ij1l}, S_{i'j1l'}), \;\; i \neq i', \, j = j', \, k, k' = 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'jkl'} Cov(S_{ij1l}, S_{i'j1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'jkl'} Cov(S_{ij1l}, M_{jkl'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'jkl'} Cov(M_{jkl}, S_{i'j1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'jkl'} Cov(M_{jkl}, M_{jkl'}), \;\; i \neq i', \, j = j', \, k = k', \, k' \neq 1, \\
\lambda_{\mathrm{S}ij1l}\lambda_{\mathrm{S}i'jk'l'} Cov(S_{ij1l}, S_{i'j1l'}) + \lambda_{\mathrm{S}ij1l}\lambda_{\mathrm{M}i'jk'l'} Cov(S_{ij1l}, M_{jk'l'}), \; i \neq i', \, j = j', \, k = 1, \, k' \neq 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'jk'l'} Cov(S_{ij1l}, S_{i'j1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'jk'l'} Cov(S_{ij1l}, M_{jk'l'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'jk'l'} Cov(M_{jkl}, S_{i'j1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'jk'l'} Cov(M_{jkl}, M_{jk'l'}), \;\; i \neq i', \, j = j', \, k \neq k', \, k, k' \neq 1, \\
Cov(S_{ij1l}, S_{ij'1l'}), \;\; i = i', \, j \neq j', \, k, k' = 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}ij'kl'} Cov(S_{ij1l}, S_{ij'1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}ij'kl'} Cov(S_{ij1l}, M_{j'kl'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}ij'kl'} Cov(M_{jkl}, S_{ij'1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}ij'kl'} Cov(M_{jkl}, M_{j'kl'}), \;\; i = i', \, j \neq j', \, k = k', \, k' \neq 1, \\
\lambda_{\mathrm{S}ij'k'l'} Cov(S_{ij1l}, S_{ij'1l'}) + \lambda_{\mathrm{M}ij'k'l'} Cov(S_{ij1l}, M_{j'k'l'}), \; i = i', \, j \neq j', \, k = 1, \, k' \neq 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}ij'k'l'} Cov(S_{ij1l}, S_{ij'1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}ij'k'l'} Cov(S_{ij1l}, M_{j'k'l'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}ij'k'l'} Cov(M_{jkl}, S_{ij'1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}ij'k'l'} Cov(M_{jkl}, M_{j'k'l'}), \;\; i = i', \, j \neq j', \, k \neq k', \, k, k' \neq 1, \\
Cov(S_{ij1l}, S_{i'j'1l'}), \;\; i \neq i', \, j \neq j', \, k, k' = 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'j'kl'} Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'j'kl'} Cov(S_{ij1l}, M_{j'kl'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'j'kl'} Cov(M_{jkl}, S_{i'j'1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'j'kl'} Cov(M_{jkl}, M_{j'kl'}), \;\; i \neq i', \, j \neq j', \, k = k', \, k' \neq 1, \\
\lambda_{\mathrm{S}i'j'k'l'} Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{\mathrm{M}i'j'k'l'} Cov(S_{ij1l}, M_{j'k'l'}), \; i \neq i', \, j \neq j', \, k = 1, \, k' \neq 1, \\
\lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'j'k'l'} Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'j'k'l'} Cov(S_{ij1l}, M_{j'k'l'}) \\
\quad + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'j'k'l'} Cov(M_{jkl}, S_{i'j'1l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'j'k'l'} Cov(M_{jkl}, M_{j'k'l'}), \;\; i \neq i', \, j \neq j', \, k \neq k', \, k, k' \neq 1.
\end{cases} \tag{94}
$$

*Proof*. Equations 63 and 64 directly follow from Equation 58 by applying Equations 59 to 62.

*Explanations.* As I will illustrate below, Equations 63 and 64 are useful to prove the identifiability of the unknown model parameters.

### 3.5.1.6 Mean Structure

Theorem 2 shows the consequences of the model definition for the observed and latent variable mean structure.

**Theorem 2: Mean Structure**

If $M := \langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl} \rangle$ is a CS-C($M$–1) model and, without loss of generality, $k = 1$ is chosen as the reference method, then the following mean structure holds for all $i \in I := \{1,...,m\}$, $j \in J := \{1,...,n\}$, $k \in K := \{1,...,o\}$, and $l \in L := \{1,...,p\}$:

$$E(Y_{ijkl}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} E(S_{ij1l}), \quad (95)$$

$$E(S_{ij1l}) = E(Y_{ij1l}), \quad (96)$$

$$E(M_{jkl}) = 0, \quad (97)$$

$$E(E_{ijkl}) = 0, \quad (98)$$

where $E(.)$ denotes the expected value (mean).

*Proof*. According to Equation 77, $Y_{ijkl} = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij1l} + \lambda_{\mathrm{M}ijkl} M_{jkl} + E_{ijkl}$. Hence, $E(Y_{ijkl}) = E(\alpha_{ijkl}) + E(\lambda_{\mathrm{S}ijkl} S_{ij1l}) + E(\lambda_{\mathrm{M}ijkl} M_{jkl}) + E(E_{ijkl})$. The terms $E(\lambda_{\mathrm{M}ijkl} M_{jkl})$ and $E(E_{ij1l})$ are zero according to Equations 97 and 98 so that this equation simplifies to Equation 95. Equation 96 follows from Equation 95, given that $\alpha_{ij1l} = 0$ and $\lambda_{\mathrm{S}ij1l} = 1$ (see Equation 77). Equations 97 and 98 follow from the definition of $M_{jkl}$ and $E_{ijkl}$ as residuals (see Equations 72 and 73). Residuals always have an expected value of zero (Steyer & Eid, 2001, Box G.1).

*Explanations.* Equation 95 shows that the mean of an observed variable is identical to the mean of the corresponding state factor if and only if $\alpha_{ijkl} = 0$ and $\lambda_{\mathrm{S}ijkl} = 1$. According to Equation 96, the means of the latent state factors are identical to the means of the indicators pertaining to the reference method. Equations 97 and 98 show an important implication of the model definition, namely that the method factors and error variables, being defined as residuals, have means of zero. Therefore, in empirical applications of the model, the means of the method factors and error variables have to be set to zero. Note that this is *not* a testable assumption, but a direct consequence of the model definition.

#### 3.5.1.7 Identification

In this section, I show how the parameters of the CS-C(*M*–1) model can be identified. In general, the parameters of a SEM (i.e., the intercepts, loading parameters, latent means, latent variances, and latent covariances) are identified if it can be shown that they can be uniquely determined from the means, variances, and covariances of the observed variables (Bollen, 1989). *Uniquely determined* means that there is one and only one mathematical solution for each model parameter. The relevant parameters for which identification needs to be proven in the CS-C(*M*–1) model are the intercepts ($\alpha_{ijkl}$), the state factor loadings ($\lambda_{Sijkl}$), the method factor loadings ($\lambda_{Mijkl}$), the variances of the common state factors [$Var(S_{ij1l})$], the variances of the method factors [$Var(M_{jkl})$], the admissible covariances between the latent factors and the variances of the error variables [$Var(E_{ijkl})$].

A prerequisite for the identification of latent variable SEMs is that each latent factor is assigned a scale (Bollen, 1989). From Definition 1, it follows that $\alpha_{ij1l} = 0$ and $\lambda_{Sij1l} = 1$ (see also Equation 47). These constraints identify the scale of the indicator-specific state factors $S_{ij1l}$. In order to assign a scale to the method factors, one factor loading $\lambda_{Mijkl}$ per method factor must be fixed to a non-zero value. Alternatively, one may fix the method factor variances to a positive value. Fixing one loading is generally preferable in longitudinal SEMs, given that one is often interested in estimating and comparing the factor variances over time. To simplify the present identification corollary, I assume without loss of generality that the method factor loading of the first indicator is set to one for each method factor (i.e., $\lambda_{M1jkl} = 1$). Corollary 5 shows how each parameter of the CS-C(*M*–1) model is identified under this condition. Note that parameters are either expressed in terms of observed means, variances, and covariances or in terms of other identified model parameters. The latter is done in cases where the terms would become very complicated if all parameters were replaced by observed covariances.

**Corollary 5: Identification**

If $M := \langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{jkl}, E_{ijkl}, \alpha_{ijkl}, \lambda_{Sijkl}, \lambda_{Mijkl} \rangle$ is a CS-C(*M*–1) model with conditional regressive independence, *k* = 1 is chosen as the reference method without loss of generality, and all method factor loadings $\lambda_{M1jkl}$ are set to 1, then for all

$i, i' \in I := \{1,...,m\},\ j, j' \in J := \{1,...,n\},\ k, k' \in K := \{1,...,o\},\ l, l' \in L := \{1,..., p\}$:

$$\alpha_{ijkl} = E(Y_{ijkl}) - \frac{Cov(Y_{ijkl}, Y_{i'j1l})}{Cov(Y_{1j1l}, Y_{i'j1l})} E(Y_{ij1l}), \qquad (99)$$

$$\lambda_{\mathrm{S}ijkl} = \frac{Cov(Y_{ijkl}, Y_{i'j1l})}{Cov(Y_{ij1l}, Y_{i'j1l})},\ i \neq i',\ k \neq 1, \tag{100}$$

$$E(S_{ij1l}) = E(Y_{ij1l}), \tag{101}$$

$$Var(S_{ij1l}) = \frac{Cov(Y_{ij1l}, Y_{ijkl})Cov(Y_{ij1l}, Y_{i'j1l})}{Cov(Y_{ijkl}, Y_{i'j1l})},\ i \neq i',\ k \neq 1, \tag{102}$$

$$Cov(S_{ij1l}, S_{i'j'1l'}) = Cov(Y_{ij1l}, Y_{i'j'1l'}),\ i \neq i',\ (i,j,l) \neq (i',j',l'), \tag{103}$$

$$\lambda_{\mathrm{M}ijkl} = \frac{\left[Cov(Y_{ijkl}, Y_{1jk'l}) - \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}1jk'l}Cov(S_{ij1l}, S_{1j1l})\right]}{Cov(M_{jkl}, M_{jk'l})},\ i \neq 1,\ k \neq k',\ k,k' \neq 1, \tag{104}$$

$$Var(M_{jkl}) = \frac{Cov(Y_{ijkl}, Y_{i'jkl}) - \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'jkl}Cov(S_{ij1l}, S_{i'j1l})}{\lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'jkl}},\ i \neq i',\ k \neq 1, \tag{105}$$

$$Cov(M_{jkl}, M_{jk'l}) = Cov(Y_{1jkl}, Y_{1jk'l}) - \frac{Cov(Y_{1jk'l}, Y_{i'j1l})Cov(Y_{1j1l}, Y_{1jkl})}{Cov(Y_{1j1l}, Y_{i'j1l})},\ i' \neq 1,\ k \neq k',\ k,k' \neq 1, \tag{106}$$

$$\begin{aligned} &Cov(M_{jkl}, M_{j'k'l'}) \\ &= [Cov(Y_{ijkl}, Y_{i'j'k'l'}) - \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'j'k'l'}Cov(S_{ij1l}, S_{i'j'1l'}) \\ &\quad - \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'j'k'l'}Cov(S_{ij1l}, M_{j'k'l'}) - \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'j'k'l'}Cov(M_{jkl}, S_{i'j'1l'})]\frac{1}{\lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'j'k'l'}}, \\ &\qquad k,k' \neq 1, \end{aligned} \tag{107}$$

$$Cov(S_{ij1l}, M_{j'kl'}) = \frac{\left[Cov(Y_{ij1l}, Y_{i'j'kl'}) - \lambda_{\mathrm{S}i'j'kl'}Cov(S_{ij1l}, S_{i'j'1l'})\right]}{\lambda_{\mathrm{M}i'j'kl'}}, \quad \text{for } j \neq j' \text{ if } l = l',\ \text{and } k \neq 1, \tag{108}$$

$$Var(E_{ij1l}) = Var(Y_{ij1l}) - \frac{Cov(Y_{ij1l}, Y_{ijkl})Cov(Y_{ij1l}, Y_{i'j1l})}{Cov(Y_{ijkl}, Y_{i'j1l})},\ i \neq i',\ k \neq 1, \tag{109}$$

$$Var(E_{ijkl}) = Var(Y_{ijkl}) - \lambda^2_{\mathrm{S}ijkl}Var(S_{ij1l}) - \lambda^2_{\mathrm{M}ijkl}Var(M_{jkl}),\ \text{for } k \neq 1. \tag{110}$$

*Proofs*. In order to make the proofs more easily understandable, I present them in the order in which the parameters are most easily identified and not in the same order as in Corollary 4.

*Equation 101: Identifiability of* $E(S_{ij1l})$

$E(S_{ij1l}) = E(Y_{ij1l})$ directly follows from Theorem 3 (Equation 96).

*Equation 99: Identifiability of* $\alpha_{ijkl}$

According to Equation 95 (Theorem 3), $E(Y_{ijkl}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} E(S_{ij1l})$. Hence,

$$\begin{aligned}\alpha_{ijkl} &= E(Y_{ijkl}) - \lambda_{\mathrm{S}ijkl} E(S_{ij1l}) \\ &= E(Y_{ijkl}) - \frac{Cov(Y_{ijkl}, Y_{i'j1l})}{Cov(Y_{1j1l}, Y_{i'j1l})} \cdot E(Y_{ij1l}).\end{aligned}$$

*Equation 103: Identifiability of* $Cov(S_{ij1l}, S_{i'j'1l'})$

According to Equation 94 (Theorem 2), $Cov(S_{ij1l}, S_{i'j'1l'}) = Cov(Y_{ij1l}, Y_{i'j'1l'})$ for $i \neq i'$ and $(i, j, l) \neq (i', j', l')$.

*Equation 100: Identifiability of* $\lambda_{\mathrm{S}ijkl}$ *(for* $k \neq 1$*)*

According to Equation 93 (Theorem 2), $Cov(Y_{ijkl}, Y_{i'j1l}) = \lambda_{\mathrm{S}ijkl} Cov(S_{ij1l}, S_{i'j1l})$, for $i \neq i'$, $k \neq 1$.

Therefore, $\lambda_{\mathrm{S}ijkl} = \dfrac{Cov(Y_{ijkl}, Y_{i'j1l})}{Cov(S_{ij1l}, S_{i'j1l})}$, for $i \neq i'$, $k \neq 1$, and $Cov(S_{ij1l}, S_{i'j1l}) \neq 0$. Equation 103 allows replacing the interstate covariances $Cov(S_{ij1l}, S_{i'j1l})$ by $Cov(Y_{ij1l}, Y_{i'j1l})$ which leads to Equation 100.

*Equation 102: Identifiability of* $Var(S_{ij1l})$

According to Equation 93 (Theorem 2), $Cov(Y_{ij1l}, Y_{ijkl}) = \lambda_{\mathrm{S}ijkl} Var(S_{ij1l})$, where $k \neq 1$. Therefore

$$Var(S_{ij1l}) = \frac{Cov(Y_{ij1l}, Y_{ijkl})}{\lambda_{\mathrm{S}ijkl}} = \frac{Cov(Y_{ij1l}, Y_{ijkl})}{\dfrac{Cov(Y_{ijkl}, Y_{i'j1l})}{Cov(Y_{ij1l}, Y_{i'j1l})}} = \frac{Cov(Y_{ij1l}, Y_{ijkl}) Cov(Y_{ij1l}, Y_{i'j1l})}{Cov(Y_{ijkl}, Y_{i'j1l})},$$

for $i \neq i'$, $k \neq 1$, and $Cov(Y_{ijkl}, Y_{i'j1l}) \neq 0$.

*Equation 106:* Identifiability of $Cov(M_{jkl}, M_{jk'l})$, $k \neq k'$

According to Equation 93 (Theorem 2),

$Cov(Y_{ijkl}, Y_{ijk'l}) = \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}ijk'l} Var(S_{ij1l}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}ijk'l} Cov(M_{jkl}, M_{jk'l})$. For $i = 1$ and $k \neq k'$, we obtain:

$Cov(Y_{1jkl}, Y_{1jk'l}) = \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1jk'l} Var(S_{1j1l}) + Cov(M_{jkl}, M_{jk'l})$, given that $\lambda_{\mathrm{M}1jkl} = \lambda_{\mathrm{M}1jk'l} = 1$. Solving for $Cov(M_{jkl}, M_{jk'l})$ and inserting the observed information for the already identified parameters yields:

$$Cov(M_{jkl}, M_{jk'l}) = Cov(Y_{1jkl}, Y_{1jk'l}) - \lambda_{S1jkl}\lambda_{S1jk'l}Var(S_{1j1l})$$

$$= Cov(Y_{1jkl}, Y_{1jk'l}) - \frac{Cov(Y_{1jkl}, Y_{i'j1l})}{Cov(Y_{1j1l}, Y_{i'j1l})}\frac{Cov(Y_{1jk'l}, Y_{i'j1l})}{Cov(Y_{1j1l}, Y_{i'j1l})}\frac{Cov(Y_{1j1l}, Y_{1jkl})Cov(Y_{1j1l}, Y_{i'j1l})}{Cov(Y_{1jkl}, Y_{i'j1l})}$$

$$= Cov(Y_{1jkl}, Y_{1jk'l}) - \frac{Cov(Y_{1jk'l}, Y_{i'j1l})Cov(Y_{1j1l}, Y_{1jkl})}{Cov(Y_{1j1l}, Y_{i'j1l})},$$

where $i \neq 1$, $k \neq k'$, and $Cov(Y_{1j1l}, Y_{i'j1l}) \neq 0$.

*Equation 104: Identifiability of $\lambda_{Mijkl}$ (for $i \neq 1$)*

According to Equation 93 (Theorem 2),

$Cov(Y_{ijkl}, Y_{i'jk'l}) = \lambda_{Sijkl}\lambda_{Si'jk'l}Cov(S_{ij1l}, S_{i'j1l}) + \lambda_{Mijkl}\lambda_{Mi'jk'l}Cov(M_{jkl}, M_{jk'l})$, where $i \neq i'$, $k \neq k'$ and $k, k' \neq 1$. For $i' = 1$, we obtain: $Cov(Y_{ijkl}, Y_{1jk'l}) = \lambda_{Sijkl}\lambda_{S1jk'l}Cov(S_{ij1l}, S_{1j1l}) + \lambda_{Mijkl}Cov(M_{jkl}, M_{jk'l})$, given that $\lambda_{M1jk'l} = 1$. Solving for $\lambda_{Mijkl}$ yields:

$$\lambda_{Mijkl} = \frac{\left[Cov(Y_{ijkl}, Y_{1jk'l}) - \lambda_{Sijkl}\lambda_{S1jk'l}Cov(S_{ij1l}, S_{1j1l})\right]}{Cov(M_{jkl}, M_{jk'l})}, \text{where } Cov(M_{jkl}, M_{jk'l}) \neq 0.$$

(All terms on the right hand side have already been shown to be identified.)

*Equation 108: Identifiability of $Cov(S_{ij1l}, M_{j'kl'})$ (for $j \neq j'$ if $l = l'$)*

According to Equation 94 (Theorem 2),

$Cov(Y_{ij1l}, Y_{i'j'kl'}) = \lambda_{Si'j'kl'}Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{Mi'j'kl'}Cov(S_{ij1l}, M_{j'kl'})$, for $j \neq j'$ if $l = l'$ $k \neq 1$. Solving for $Cov(S_{ij1l}, M_{j'kl'})$ yields:

$$Cov(S_{ij1l}, M_{j'kl'}) = \frac{\left[Cov(Y_{ij1l}, Y_{i'j'kl'}) - \lambda_{Si'j'kl'}Cov(S_{ij1l}, S_{i'j'1l'})\right]}{\lambda_{Mi'j'kl'}}, \text{ where } \lambda_{Mi'j'kl'} \neq 0.$$

(All terms on the right hand side have already been shown to be identified.)

*Equation 107: Identifiability of $Cov(M_{jkl}, M_{j'k'l'})$*

According to Equation 94 (Theorem 2),

$$Cov(Y_{ijkl}, Y_{i'j'k'l'}) = \lambda_{Sijkl}\lambda_{Si'j'k'l'}Cov(S_{ij1l}, S_{i'j'1l'}) + \lambda_{Sijkl}\lambda_{Mi'j'k'l'}Cov(S_{ij1l}, M_{j'k'l'})$$
$$+\lambda_{Mijkl}\lambda_{Si'j'k'l'}Cov(M_{jkl}, S_{i'j'1l'}) + \lambda_{Mijkl}\lambda_{Mi'j'k'l'}Cov(M_{jkl}, M_{j'k'l'}),$$

where $k, k' \neq 1$. Solving for $Cov(M_{jkl}, M_{j'k'l'})$ yields:

$$Cov(M_{jkl}, M_{j'k'l'}) = [Cov(Y_{ijkl}, Y_{i'j'k'l'}) - \lambda_{Sijkl}\lambda_{Si'j'k'l'}Cov(S_{ij1l}, S_{i'j'1l'})$$
$$-\lambda_{Sijkl}\lambda_{Mi'j'k'l'}Cov(S_{ij1l}, M_{j'k'l'}) - \lambda_{Mijkl}\lambda_{Si'j'k'l'}Cov(M_{jkl}, S_{i'j'1l'})]\frac{1}{\lambda_{Mijkl}\lambda_{Mi'j'k'l'}},$$

where $\lambda_{Mijkl}\lambda_{Mi'j'k'l'} \neq 0$. (All terms on the right hand side have already been shown to be identified.)

*Equation 105: Identifiability of $Var(M_{jkl})$*

According to Equation 93 (Theorem 2),
$Cov(Y_{ijkl}, Y_{i'jkl}) = \lambda_{Sijkl}\lambda_{Si'jkl}Cov(S_{ij1l}, S_{i'j1l}) + \lambda_{Mijkl}\lambda_{Mi'jkl}Var(M_{jkl})$, where $i \neq i'$, $k \neq 1$.
Solving for $Var(M_{jkl})$ yields:

$$Var(M_{jkl}) = \frac{\left[Cov(Y_{ijkl}, Y_{i'jkl}) - \lambda_{Sijkl}\lambda_{Si'jkl}Cov(S_{ij1l}, S_{i'j1l})\right]}{\lambda_{Mijkl}\lambda_{Mi'jkl}}, \text{ where } i \neq i',\ k \neq 1, \text{ and } \lambda_{Mijkl}\lambda_{Mi'jkl} \neq 0.$$

(All terms on the right hand side have already been shown to be identified.)

*Equation 109 and 110: Identifiability of $Var(E_{ijkl})$*

According to Equation 93 (Theorem 2), $Var(Y_{ij1l}) = Var(S_{ij1l}) + Var(E_{ij1l})$ and

$Var(Y_{ijkl}) = \lambda^2_{Sijkl}Var(S_{ij1l}) + \lambda^2_{Mijkl}Var(M_{jkl}) + Var(E_{ijkl})$, for $k \neq 1$. Hence,

$$Var(E_{ij1l}) = Var(Y_{ij1l}) - Var(S_{ij1l})$$
$$= Var(Y_{ij1l}) - \frac{Cov(Y_{ij1l}, Y_{ijkl})Cov(Y_{ij1l}, Y_{i'j1l})}{Cov(Y_{ijkl}, Y_{i'j1l})}, \text{ where } i \neq i' \text{ and } k \neq 1, \text{ and}$$

$$Var(E_{ijkl}) = Var(Y_{ijkl}) - \lambda^2_{Sijkl}Var(S_{ij1l}) - \lambda^2_{Mijkl}Var(M_{jkl}), \text{ for } k \neq 1.$$

(All terms on the right hand side have already been shown to be identified.)

The minimal condition for identification of the CS-C($M$–1) model is that there is a 2x1x2x2 MTMM-MO design, that is, one construct ($n$ = 1) assessed by two methods ($o$ = 2) on two measurement occasions ($p = 2$), with two indicators per method ($m = 2$). This design is sufficient for obtaining an identified model, given substantial parameter values. In particular, under this design, there would be six latent variables (four latent state factors and two latent method factors), each of which would be measured by only two indicators. Therefore, a requirement for the identification of the "minimal model" is that each latent variable is substantially correlated (has a non-zero covariance) with at least one other latent variable (or an external covariate) in the model. If a latent variable in this minimal model does *not* covary with another variable in the model, the factor loadings of both indicators of this latent variable must be fixed to a non-zero value to identify the model.

### *3.5.2 CS-C(M–1) Model With Indicator-Specific Factors Across Time*

#### 3.5.2.1 Model Definition

The assumptions of the CS-C(*M*–1) model with indicator-specific factors are summarized in Definition 3.

**Definition 3: CS-C(*M*–1) Model With Indicator-Specific Factors**

The random variables $Y_{1111},...,Y_{ijkl},...,Y_{mnop}$, $i \in I := \{1,...,m\}$, $j \in J := \{1,...,n\}$, $k \in K := \{1,...,o\}$, $l \in L := \{1,...,p\}$, on a probability space $(\Omega, \mathfrak{A}, P)$ are variables of a CS-C(*M*–1) model with indicator-specific factors if and only if the following conditions hold:

(a) $(\Omega, \mathfrak{A}, P)$ is a probability space such that $\Omega = U_0 \times U_1 \times ... \times U_n \times M_1 \times ... \times M_n$.

(b) The projections $p_0 : \Omega \rightarrow U_0$, $p_l : \Omega \rightarrow U_l$ are random variables on $(\Omega, \mathfrak{A}, P)$.

(c) The variables $Y_{ijkl} : \Omega \rightarrow \mathbb{R}$ are random variables on $(\Omega, \mathfrak{A}, P)$.

(d) Without loss of generality, the first method (*k* = 1) is selected as reference method.

(e) Without loss of generality, the first indicators (*i* = 1) are selected as marker indicators.

Then, the variables

$$S_{ijkl} := E(Y_{ijkl} \mid p_0, p_l), \tag{111}$$

$$IS_{ij1l} := S_{ij1l} - E\left(S_{ij1l} \mid S_{1j1l}\right), \tag{112}$$

$$M_{ijkl} := S_{ijkl} - E\left(S_{ijkl} \mid S_{1j1l}\right),\ k \neq 1, \tag{113}$$

$$IS_{ijkl} := M_{ijkl} - E\left(M_{ijkl} \mid M_{1jkl}\right),\ k \neq 1, \text{ and} \tag{114}$$

$$E_{ijkl} := Y_{ijkl} - S_{ijkl} \tag{115}$$

are random variables on $(\Omega, \mathfrak{A}, P)$, where $E(Y_{ijkl} \mid p_0, p_l)$ denotes the conditional expectation of $Y_{ijkl}$ given the person $(p_0)$ and the situation $(p_l)$, the variables $IS_{ijkl}$ represent the indicator-specific effects, $E\left(S_{ijkl} \mid S_{1j1l}\right)$ denotes the conditional expectation of $S_{ijkl}$ given the marker state variable $S_{1j1l}$, and $E\left(M_{ijkl} \mid M_{1jkl}\right)$ denotes the conditional expectation of a method variable $M_{ijkl}$ given the method variable for the first indicator $M_{1jkl}$. The variables $E_{ijkl}$ are measurement error variables.

(f) For each quadruple $(i, j, k, l)$ $i \in I,\ j \in J,\ k \in K,\ l \in L$ there is a constant $\alpha_{ijkl} \in \mathbb{R}$ as well as a constant $\lambda_{\mathrm{S}ijkl} \in \mathbb{R}$, $\lambda_{\mathrm{S}ijkl} > 0$, such that

$$E\left(S_{ijkl} \mid S_{1j1l}\right) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{1j1l}. \tag{116}$$

(g) For each quadruple $(i, j, k, l)$ $i \in I,\ j \in J,\ k \in K,\ l \in L,\ k \neq 1$ there is a constant $\lambda_{\mathrm{M}ijkl} \in \mathbb{R},\ \lambda_{\mathrm{M}ijkl} > 0$, such that

$$E\left(M_{ijkl} \mid M_{1jkl}\right) = \lambda_{\mathrm{M}ijkl} M_{1jkl}. \tag{117}$$

(h) For each quintuple $(i, j, k, l, l')$, $i \in I,\ j \in J,\ k \in K,\ l, l' \in L$ there is a constant $\lambda_{\mathrm{IS}ijkll'} \in \mathbb{R},\ \lambda_{\mathrm{IS}ijkll'} > 0$, such that

$$IS_{ijkl} = \lambda_{\mathrm{IS}ijkll'} IS_{ijkl'}. \tag{118}$$

*Explanations.* As in the CS-C(*M*–1) model without indicator-specific factors (see Definition 1 in Section 3.5.1), the basis for the model definition are the observed variables $Y_{ijkl}$, each of which has its own associated latent state (true score) variable $S_{ijkl}$. According to Condition (f), all latent state variables $S_{ijkl}$ belonging to the same construct and the same occasion of measurement are linearly regressed on the state variables $S_{1j1l}$, pertaining to the first indicator (i.e., the marker indicator) of the reference method.

Note that *all* state variables within the same construct-occasion unit are now regressed on the *same* state variable $S_{1j1l}$, belonging to the *marker indicator* of the reference method. In the model without indicator-specific factors across time defined in Section 3.5.1, the regressions were assumed to be *indicator-specific*. That is, the state variables $S_{ijkl}$ were regressed on the corresponding reference method state variable $S_{ij1l}$, with the same index *i*, instead of a single "common" state variable. As a result of this difference, we obtain *general* state factors in the CS-C(*M*–1) model with indicator-specific factors, whereas indicator-specific state factors were obtained in the model defined in Section 3.5.1.

The second difference is that in contrast to the model defined in Section 3.5.1, there are now *two* types of latent residual variables of the latent regression analysis $E(S_{ijkl} \mid S_{1j1l}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{1j1l}$. For the state variables $S_{ij1l}$, pertaining to the reference method ($k$ = 1), the residuals are the indicator-specific variables $IS_{ij1l}$. These variables mirror that part of the state variables $S_{ij1l}$, $i \neq 1$, that cannot be predicted from the state variable pertaining to the marker indicator $S_{1j1l}$. For the state variables $S_{ijkl}$, $k \neq 1$, the residuals of this regression are the method variables $M_{ijkl}$ (as in the model defined in Section 3.5.1).

The third difference is that a second latent regression analysis is assumed in addition to the regression $E(S_{ijkl} \mid S_{1j1l}) = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{1j1l}$. In this second latent regression analysis, the method variables $M_{ijkl}$, $i \neq 1$, are regressed on the method variables belonging to the first indicator of a non-reference method, $M_{1jkl}$: $E(M_{ijkl} \mid M_{1jkl}) = \lambda_{\mathrm{M}ijkl} M_{1jkl}$. The residuals of this second latent regression analysis are the indicator-specific variables $IS_{ijkl}$, $k \neq 1$, for state variables belonging to the non-reference methods.

Finally, in Condition (h) I introduce the assumption that all indicator-specific variables $IS_{ijkl}$ that belong to the same indicator $i$, the same construct $j$, and the same method $k$, but different measurement occasions $l$ and $l'$ are unidimensional (i.e., they differ only by a multiplicative constant $\lambda_{\mathrm{IS}ijkll'}$). Hence, in Condition (h) a homogeneity assumption with regard to the indicator-specific effects across time is made. There are no additive constants in Equations 117 and 118 given that the expectation (mean) of residual variables is always zero (see, e.g., Steyer & Eid, 2001, p. 357).

#### 3.5.2.2 Existence of Common Indicator-Specific Factors $IS_{ijk}$

Corollary 6 shows that Condition (h) in Definition 3 implies the existence of *common* indicator-specific factors $IS_{ijk}$.

**Corollary 6: Existence of Common Indicator-Specific Factors $IS_{ijk}$**

The random variables $Y_{1111}, \ldots, Y_{ijkl}, \ldots, Y_{mnop}$, $i \in I := \{1, \ldots, m\}$, $j \in J := \{1, \ldots, n\}$, $k \in K := \{1, \ldots, o\}$, $l \in L := \{1, \ldots, p\}$, on a probability space $(\Omega, \mathfrak{A}, P)$ are variables of a CS-C($M$–1) model with indicator-specific factors if conditions (a) to (g) in Definition 1 hold and

(f') for each quadruple $(i, j, k, l)$, $i \in I$, $j \in J$, $k \in K$, $l \in L$, $i \neq 1$, there is a constant $\lambda_{\text{IS}ijkl} \in \mathbb{R}$, $\lambda_{\text{IS}ijkl} > 0$, and a real random variable $IS_{ijk}$ on $(\Omega, \mathfrak{A}, P)$ such that:

$$IS_{ijkl} = \lambda_{\text{IS}ijkl} IS_{ijk}. \tag{120}$$

*Proof*. If one defines, for example, $IS_{ijk} := IS_{ijk1}$ as well as $\lambda_{\text{IS}ijkl} := \lambda_{\text{IS}ijkl1}$ and inserts these parameters in Equation 118 (see Definition 3), this results in $IS_{ijkl} = \lambda_{\text{IS}ijkl} IS_{ijk}$ (Equation 120). Furthermore, according to Equation 120, $IS_{ijk}$ can be expressed as $IS_{ijk} = \frac{IS_{ijkl}}{\lambda_{\text{IS}ijkl}}$ as well as $IS_{ijk} = \frac{IS_{ijkl'}}{\lambda_{\text{IS}ijkl'}}$. By setting both equations equal, it follows that $IS_{ijkl} = \frac{\lambda_{\text{IS}ijkl}}{\lambda_{\text{IS}ijkl'}} IS_{ijkl'}$. By defining $\lambda_{\text{IS}ijkll'} := \frac{\lambda_{\text{IS}ijkl}}{\lambda_{\text{IS}ijkl'}}$ one obtains Equation 118.

*Explanations.* The similarity of Corollary 1 and Corollary 6 is obvious. In Corollary 1, the existence of common method factors was shown based on the assumption of homogeneous method effects (see Definition 1). In Corollary 6, it is shown that the assumption of homogeneous indicator-specific effects across time (Equation 118) implies the existence of common indicator-specific factors. In sum, the following measurement equations for the observed variables result from the assumptions made in Definition 3:

$$Y_{ijkl} = \begin{cases} S_{1j1l} + E_{1j1l}, \text{ for } i = k = 1, \\ \alpha_{ij1l} + \lambda_{\text{S}ij1l} S_{1j1l} + \lambda_{\text{IS}ij1l} IS_{ij1} + E_{ij1l}, \text{ for } i \neq 1, k = 1, \\ \alpha_{1jkl} + \lambda_{\text{S}1jkl} S_{1j1l} + M_{1jkl} + E_{1jkl}, \text{ for } i = 1, k \neq 1, \\ \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{1j1l} + \lambda_{\text{M}ijkl} \cdot M_{1jkl} + \lambda_{\text{IS}ijkl} IS_{ijk} + E_{ijkl}, \text{ for } i, k \neq 1 \end{cases} \tag{121}$$

It is obvious from Corollary 6 that the variables $IS_{ijk}$ are not uniquely defined. One could choose any variable $IS_{ijkl}$ to serve as "common indicator-specific factor" $IS_{ijk}$. Note again the similarity to Corollary 1, which revealed that the method factors $M_{jkl}$ in the model defined in Section 3.5.1 were not uniquely defined.

3.5.2.3 Admissible Transformations and Uniqueness

In the CS-C($M$–1) model with indicator-specific factors, both the common state variables $S_{1j1l}$ and the common method variables $M_{1jkl}$ are uniquely defined. In Definition 3, the variables $S_{1j1l}$ were defined as the conditional expectations of the variables $Y_{1j1l}$ given the person and the situation [see Condition (e) in Definition 3]. Therefore, the variables $S_{1j1l}$ are uniquely defined as the latent state true score variables of the variables $Y_{1j1l}$. According to Condition (e) in Definition 3, the common method variables $M_{1jkl}$ are defined as $M_{1jkl} := S_{1jkl} - E(S_{1jkl} \mid S_{1j1l})$, that is, as the residuals of the latent regression $E(S_{1jkl} \mid S_{1j1l}) = \alpha_{1jkl} + \lambda_{\text{S}1jkl} S_{1j1l}$. This shows that the common method variables $M_{1jkl}$ are also uniquely defined. They represent that part of the state variables $S_{1jkl}$, $k \neq 1$, which cannot be predicted from $S_{1j1l}$. Given that both $S_{1jkl}$ and $S_{1j1l}$ are uniquely defined, the residual of the regression of $S_{1jkl}$ on $S_{1j1l}$ (i e., $M_{1jkl}$) is also uniquely defined.

I already noted that the common indicator-specific factors $IS_{ijk}$ are *not* uniquely defined. As shown in Corollary 6, the variables $IS_{ijk}$ and the coefficients $\lambda_{\text{IS}ijkl}$ are uniquely defined only up to similarity transformations (i.e., up to a multiplication with a positive real number). This is shown in Corollary 7:

**Corollary 7: Admissible Transformations**

If (a) $M := \langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\text{S}ijkl}, \lambda_{\text{M}ijkl}, \lambda_{\text{IS}ijkl} \rangle$ is a CS-C($M$–1) model with indicator-specific factors and (b) the variables $IS^*_{ijk}$ as well as the coefficients $\lambda^*_{\text{IS}ijkl}$, are defined as follows for all $i \in I$, $j \in J$, $k \in K$, $l \in L$, and $i \neq 1$:

$$IS^*_{ijk} := \delta_{ijk} IS_{ijk}, \tag{122}$$

$$\lambda^*_{\text{IS}ijkl} := \left(\frac{1}{\delta_{ijk}}\right) \lambda_{\text{IS}ijkl}, \tag{123}$$

where $\delta_{ijk} \in \mathbb{R}$, $\delta_{ijk} > 0$, then (c) $M^* := \langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS^*_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\text{S}ijkl}, \lambda_{\text{M}ijkl}, \lambda^*_{\text{IS}ijkl} \rangle$

is a CS-C($M$–1) model with indicator-specific factors, too, given that the following equation holds for all $i \in I$, $j \in J$, $k \in K$, $l \in L$, and $i \neq 1$:

$$IS_{ijkl} = \lambda^{*}_{\text{IS}ijkl} IS^{*}_{ijk}. \tag{124}$$

*Proof*. $IS_{ijk}$ can be replaced by $IS^{*}_{ijk}$ in Equation 120 if the constant $\lambda_{\text{IS}ijkl}$ is also replaced by $\lambda^{*}_{\text{IS}ijkl}$:

$$IS_{ijkl} = \lambda_{\text{IS}ijkl} IS_{ijk} = \lambda^{*}_{\text{M}ijkl} IS^{*}_{ijk} = \left(\frac{1}{\delta_{ijk}}\right) \lambda_{\text{IS}ijkl} \delta_{ijk} IS_{ijk} = \lambda_{\text{IS}ijkl} IS_{ijk}.$$

*Explanations.* Corollary 7 shows that similarity transformations of $IS_{ijk}$ and $\lambda_{\text{IS}ijkl}$ are admissible. Hence, the scale level for $IS_{ijk}$ and $\lambda_{\text{IS}ijkl}$ is a ratio scale.

#### 3.5.2.4 Meaningfulness

Important meaningful statements that can be made with respect to $IS_{ijk}$ and $\lambda_{\text{IS}ijkl}$ are presented in Corollary 8.

**Corollary 8: Meaningfulness**

If both $M := \langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\text{S}ijkl}, \lambda_{\text{M}ijkl}, \lambda_{\text{IS}ijkl} \rangle$ and $M^{*} := \langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS^{*}_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\text{S}ijkl}, \lambda_{\text{M}ijkl}, \lambda^{*}_{\text{IS}ijkl} \rangle$ are CS-C($M$–1) models with indicator-specific factors, then for

(1) $i \in I$, $j \in J$, $k \in K$, $l, l' \in L$:

$$\frac{\lambda_{\text{IS}ijkl}}{\lambda_{\text{IS}ijkl'}} = \frac{\lambda^{*}_{\text{IS}ijkl}}{\lambda^{*}_{\text{IS}ijkl'}}, \tag{125}$$

(2) $\omega_1, \omega_2 \in \Omega$, $i \in I$, $j \in J$, $k \in K$:

$$\frac{IS_{ijk}(\omega_1)}{IS_{ijk}(\omega_2)} = \frac{IS^{*}_{ijk}(\omega_1)}{IS^{*}_{ijk}(\omega_2)}, \tag{126}$$

(3) $\omega_1, \omega_2 \in \Omega,\ i, i' \in I,\ j, j' \in J,\ k, k' \in K$:

$$\frac{IS_{ijk}(\omega_1)}{IS_{ijk}(\omega_2)} - \frac{IS_{i'j'k'}(\omega_1)}{IS_{i'j'k'}(\omega_2)} = \frac{IS^*_{ijk}(\omega_1)}{IS^*_{ijk}(\omega_2)} - \frac{IS^*_{i'j'k'}(\omega_1)}{IS^*_{i'j'k'}(\omega_2)}, \tag{127}$$

(4) $i \in I,\ j \in J,\ k \in K,\ l \in L$:

$$\lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk}) = \lambda^{*2}_{\mathrm{IS}ijkl} Var(IS^*_{ijk}), \tag{128}$$

(5) $i, i' \in I,\ j, j' \in J,\ k, k' \in K$:

$$Corr(IS_{ijk}, IS_{i'j'k'}) = Corr(IS^*_{ijk}, IS^*_{i'j'k'}). \tag{129}$$

where $Var(.)$ denotes the variance and $Corr(.,.)$ denotes the correlation.

*Proofs*. To show that Equations 125 to 129 hold, one simply needs to replace $\lambda^*_{\mathrm{IS}ijkl}$ by $\lambda_{\mathrm{IS}ijkl}\left(\frac{1}{\delta_{ijk}}\right)$ and $IS^*_{ijk}$ by $\delta_{ijk} IS_{ijk}$.

*Explanations.* The explanations are essentially the same as for Corollary 3. Statements that concern absolute values of $\lambda_{\mathrm{IS}ijkl}$ and $IS_{ijk}$ are not meaningful, given that the multiplication with a positive real number is permissible and that $\lambda_{\mathrm{IS}ijkl}$ as well as $IS_{ijk}$ would take on different values after such a transformation. In contrast, statements with respect to the *ratio* of two coefficients $\lambda_{\mathrm{IS}ijkl}$ and $\lambda_{\mathrm{IS}ijkl'}$ as well as statements with respect to the *ratio* of specific values of the indicator-specific factors $IS_{ijk}$ are meaningful (see Equations 125 and 126). Meaningful statements can also be made with respect to the differences between ratios of values of two different indicator-specific factors $IS_{ijk}$ and $IS_{i'j'k'}$ (Equation 127). Given that the term $\lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk})$ can be meaningfully interpreted according to Equation 128, it follows that statements concerning the indicator-specificity coefficient (see Table 2) are also meaningful. Furthermore, meaningful statements can be made with respect to the correlations between indicator-specific factors (see Equation 129).

#### 3.5.2.5 Covariance Structure

To derive testable consequences for the covariance structure implied by the CS-C($M$–1) model with indicator-specific factors, it is necessary to make the assumption of conditional regressive independence for this model variant, too. The assumption of conditional regressive independence for this model is introduced in Definition 4.

**Definition 4: CS-C(*M*–1) Model With Indicator-Specific Factors And Conditional Regressive Independence**

If $M := \langle(\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl}, \lambda_{\mathrm{IS}ijkl}\rangle$ is a CS-C(*M*–1) model with indicator-specific factors, then *M* is called a *CS-C(M–1) model with indicator-specific factors and conditional regressive independence* if and only if the following assumptions hold

(a) for all $i, i' \in I := \{1,...,m\}$, $j, j' \in J := \{1,...,n\}$, $k, k' \in K := \{1,...,o\}$, $l, l' \in L := \{1,..., p\}$:

$$E\left[Y_{ijkl} \mid p_0, p_1, ..., p_p, \left(Y_{i'j'k'l'}, (i', j', k', l') \neq (i, j, k, l)\right)\right] = E\left(Y_{ijkl} \mid p_0, p_l\right). \qquad (130)$$

*Explanations.* Definition 4 is equivalent to Definition 2. As will be shown below, a consequence of the assumption of conditional regressive independence is that correlations among error variables as well as correlations between error variables and other latent variables are excluded. Theorem 3 summarizes the implications of the CS-C(*M*–1) model with indicator-specific factors and conditional regressive independence for the covariance structure of the observed variables.

**Theorem 3: Covariance Structure**

If $M := \langle(\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl}, \lambda_{\mathrm{IS}ijkl}\rangle$ is a CS-C(*M*–1) model with indicator-specific factors and conditional regressive independence, without loss of generality $k = 1$ is chosen as the reference method, and without loss of generality all indicators $Y_{1jkl}$ have been selected as marker indicators, then the following covariance structure holds for all $i, i' \in I := \{1,...,m\}$, $j, j' \in J := \{1,...,n\}$, $k, k' \in K := \{1,...,o\}$, $l, l' \in L := \{1,..., p\}$ and

(a) for all observed variables:

$$
\begin{aligned}
Cov(Y_{ijkl}, Y_{i'j'k'l'}) = {} & \lambda_{\text{S}ijkl}\lambda_{\text{S}i'j'k'l'}Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}i'j'k'l'}Cov(S_{1j1l}, M_{1j'k'l'}) \\
& + \lambda_{\text{S}ijkl}\lambda_{\text{IS}i'j'k'l'}Cov(S_{1j1l}, IS_{i'j'k'}) \\
& + \lambda_{\text{M}ijkl}\lambda_{\text{S}i'j'k'l'}Cov(M_{1jkl}, S_{1j'1l'}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}i'j'k'l'}Cov(M_{1jkl}, M_{1j'k'l'}) \\
& + \lambda_{\text{M}ijkl}\lambda_{\text{IS}i'j'k'l'}Cov(M_{1jkl}, IS_{i'j'k'}) \\
& + \lambda_{\text{IS}ijkl}\lambda_{\text{S}i'j'k'l'}Cov(IS_{ijk}, S_{1j'1l'}) + \lambda_{\text{IS}ijkl}\lambda_{\text{M}i'j'k'l'}Cov(IS_{ijk}, M_{1j'k'l'}) \\
& + \lambda_{\text{IS}ijkl}\lambda_{\text{IS}i'j'k'l'}Cov(IS_{ijk}, IS_{i'j'k'}) + Cov(E_{ijkl}, E_{i'j'k'l'}),
\end{aligned} \tag{131}
$$

(b) for all latent variables:

$$Cov(S_{1j1l}, M_{1jkl}) = 0, \tag{132}$$

$$Cov(S_{1j1l}, IS_{ijk}) = 0, \tag{133}$$

$$Cov(S_{1j1l}, E_{i'j'k'l'}) = 0, \tag{134}$$

$$Cov(M_{1jkl}, IS_{ijk}) = 0, \tag{135}$$

$$Cov(M_{1jkl}, E_{i'j'k'l'}) = 0, \tag{136}$$

$$Cov(IS_{ijk}, E_{i'j'k'l'}) = 0, \tag{137}$$

$$Cov(E_{ijkl}, E_{i'j'k'l'}) = 0, \quad (i, j, k, l) \neq (i', j', k', l'), \tag{138}$$

where $Cov(.,.)$ denotes the covariance.

*Proofs.*

*Equation 131*

The covariance structure of the observed variables follows from Equations 121, 134, 136, and 137 by applying rules of covariance algebra (see, e.g., Bollen, 1989; Steyer & Eid, 2001, Box F.1):

$$
\begin{aligned}
Cov(Y_{ijkl}, Y_{i'j'k'l'}) = Cov[&(\alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{1j1l} + \lambda_{\mathrm{M}ijkl} M_{1jkl} + \lambda_{\mathrm{IS}ijkl} IS_{ijk} + E_{ijkl}),\\
&(\alpha_{i'j'k'l'} + \lambda_{\mathrm{S}i'j'k'l'} S_{1j'1l'} + \lambda_{\mathrm{M}i'j'k'l'} M_{1j'k'l'} + \lambda_{\mathrm{IS}i'j'k'l'} IS_{i'j'k'} + E_{i'j'k'l'})]\\
= &Cov(\alpha_{ijkl}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{S}i'j'k'l'} Cov(\alpha_{ijkl}, S_{1j'1l'}) + \lambda_{\mathrm{M}i'j'k'l'} Cov(\alpha_{ijkl}, M_{1j'k'l'})\\
&+ \lambda_{\mathrm{IS}i'j'k'l'} Cov(\alpha_{ijkl}, IS_{i'j'k'}) + Cov(\alpha_{ijkl}, E_{i'j'k'l'})\\
&+ \lambda_{\mathrm{S}ijkl} Cov(S_{1j1l}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'j'k'l'} Cov(S_{1j1l}, S_{1j'1l'})\\
&+ \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{M}i'j'k'l'} Cov(S_{1j1l}, M_{1j'k'l'}) + \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{IS}i'j'k'l'} Cov(S_{1j1l}, IS_{i'j'k'})\\
&+ \lambda_{\mathrm{S}ijkl} Cov(S_{1j1l}, E_{i'j'k'l'})\\
&+ \lambda_{\mathrm{M}ijkl} Cov(M_{1jkl}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{S}i'j'k'l'} Cov(M_{1jkl}, S_{1j'1l'})\\
&+ \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'j'k'l'} Cov(M_{1jkl}, M_{1j'k'l'}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{IS}i'j'k'l'} Cov(M_{1jkl}, IS_{i'j'k'})\\
&+ \lambda_{\mathrm{M}ijkl} Cov(M_{1jkl}, E_{i'j'k'l'})\\
&+ \lambda_{\mathrm{IS}ijkl} Cov(IS_{ijk}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{IS}ijkl}\lambda_{\mathrm{S}i'j'k'l'} Cov(IS_{ijk}, S_{1j'1l'})\\
&+ \lambda_{\mathrm{IS}ijkl}\lambda_{\mathrm{M}i'j'k'l'} Cov(IS_{ijk}, M_{1j'k'l'}) + \lambda_{\mathrm{IS}ijkl}\lambda_{\mathrm{IS}i'j'k'l'} Cov(IS_{ijk}, IS_{i'j'k'})\\
&+ \lambda_{\mathrm{IS}ijkl} Cov(IS_{ijk}, E_{i'j'k'l'})\\
&+ Cov(E_{ijkl}, \alpha_{i'j'k'l'}) + \lambda_{\mathrm{S}i'j'k'l'} Cov(E_{ijkl}, S_{1j'1l'}) + \lambda_{\mathrm{M}i'j'k'l'} Cov(E_{ijkl}, M_{1j'k'l'})\\
&+ \lambda_{\mathrm{IS}i'j'k'l'} Cov(E_{ijkl}, IS_{i'j'k'}) + Cov(E_{ijkl}, E_{i'j'k'l'}).
\end{aligned}
$$

Given that constants cannot covary with other constants or variables, $Cov(\alpha_{ijkl}, \alpha_{i'j'k'l'})$ = $\lambda_{\mathrm{S}i'j'k'l'} Cov(\alpha_{ijkl}, S_{1j'1l'})$ = $\lambda_{\mathrm{M}i'j'k'l'} Cov(\alpha_{ijkl}, M_{1j'k'l'})$ = $\lambda_{\mathrm{IS}i'j'k'l'} Cov(\alpha_{ijkl}, IS_{i'j'k'})$ = $Cov(\alpha_{ijkl}, E_{i'j'k'l'})$ = $\lambda_{\mathrm{S}ijkl} Cov(S_{1j1l}, \alpha_{i'j'k'l'})$ = $\lambda_{\mathrm{M}ijkl} Cov(M_{1jkl}, \alpha_{i'j'k'l'})$ = $\lambda_{\mathrm{IS}ijkl} Cov(IS_{ijk}, \alpha_{i'j'k'l'})$ = $Cov(E_{ijkl}, \alpha_{i'j'k'l'})$ = 0. Furthermore, the terms $\lambda_{\mathrm{S}ijkl} Cov(S_{1j1l}, E_{i'j'k'l'})$, $\lambda_{\mathrm{M}ijkl} Cov(M_{1jkl}, E_{i'j'k'l'})$, $\lambda_{\mathrm{IS}ijkl} Cov(IS_{ijk}, E_{i'j'k'l'})$, $\lambda_{\mathrm{S}i'j'k'l'} Cov(E_{ijkl}, S_{1j'1l'})$, $\lambda_{\mathrm{M}i'j'k'l'} Cov(E_{ijkl}, M_{1j'k'l'})$, and $\lambda_{\mathrm{IS}i'j'k'l'} Cov(E_{ijkl}, IS_{i'j'k'})$ are equal to zero according to Equations 134, 136, and 137.

*Equation 132*

The uncorrelatedness of the latent state variables with all method factors that pertain to the same construct on the same measurement occasion follows given that the variables $M_{1jkl}$ are residuals with respect to $S_{1j1l}$ (as defined in Equation 132). Residuals are always uncorrelated with their regressors (see Steyer & Eid, 2001, Box G.1).

*Equation 133*

According to Equation 120, $IS_{ijk} = \frac{1}{\lambda_{\mathrm{IS}ijkl}} IS_{ijkl}$. Therefore, $Cov(S_{1j1l}, IS_{ijk})$ can be rewritten as

$Cov(S_{1j1l}, IS_{ijk}) = \frac{1}{\lambda_{\text{IS}ijkl}} Cov(S_{1j1l}, IS_{ijkl})$. According to Equation 112, $IS_{ij1l}$ is a residual with respect to $S_{1j1l}$. Hence, $Cov(S_{1j1l}, IS_{ij1}) = 0$. According to Equation 114, $IS_{ijkl}$, $k \neq 1$, can be replaced by $M_{ijkl} - E(M_{ijkl} \mid M_{1jkl})$, leading to

$Cov(S_{1j1l}, IS_{ijk}) = \frac{1}{\lambda_{\text{IS}ijkl}} Cov\{S_{1j1l}, [M_{ijkl} - E(M_{ijkl} \mid M_{1jkl})]\}$. Using Equation 117, we can replace $M_{ijkl} - E(M_{ijkl} \mid M_{1jkl})$ by $\lambda_{\text{M}ijkl} M_{1jkl}$:

$$
\begin{aligned}
Cov(S_{1j1l}, IS_{ijk}) &= \frac{1}{\lambda_{\text{IS}ijkl}} Cov\{S_{1j1l}, [M_{ijkl} - \lambda_{\text{M}ijkl} M_{1jkl}]\} \\
&= \frac{1}{\lambda_{\text{IS}ijkl}} [Cov(S_{1j1l}, M_{ijkl}) - \lambda_{\text{M}ijkl} Cov(S_{1j1l}, M_{1jkl})].
\end{aligned}
$$

Given that both $M_{ijkl}$ and $M_{1jkl}$ are residuals with respect to $S_{1j1l}$ according to Equation 113, $Cov(S_{1j1l}, M_{ijkl}) = Cov(S_{1j1l}, M_{1jkl}) = 0$. Hence, $Cov(S_{1j1l}, IS_{ijk}) = 0$, too.

*Equation 135*

According to Equation 120, $IS_{ijk} = \frac{1}{\lambda_{\text{IS}ijkl}} IS_{ijkl}$. Therefore, $Cov(M_{1jkl}, IS_{ijk})$ can be rewritten as $Cov(M_{1jkl}, IS_{ijk}) = \frac{1}{\lambda_{\text{IS}ijkl}} Cov(M_{1jkl}, IS_{ijkl})$. Given that $IS_{ijkl}$ is a residual with respect to $M_{1jkl}$ according to Equation 114, it follows that $Cov(M_{1jkl}, IS_{ijkl}) = 0$. Hence, $Cov(M_{1jkl}, IS_{ijk}) = 0$, too.

*Equation 138*

The uncorrelatedness of the error variables follows from the independence assumption introduced in Definition 4. Equation 138 can be rewritten as $Cov(E_{ijkl}, E_{i'j'k'l'}) = Cov\{[Y_{ijkl} - E(Y_{ijkl} \mid p_0, p_l)], [Y_{i'j'k'l'} - E(Y_{i'j'k'l'} \mid p_0, p_{l'})]\}$. According to Bauer (1978, p. 54, Satz 9.4) $E_{i'j'k'l'} = Y_{i'j'k'l'} - E(Y_{i'j'k'l'} \mid p_0, p_l)$ is a $(p_0, p_l, Y_{i'j'k'l'})$-measurable function (cf. Steyer, 1988, p. 368-369). The assumption made in Definition 4 allows replacing $E(Y_{ijkl} \mid p_0, p_l)$ by $E[Y_{ijkl} \mid p_0, p_1, ..., p_p, (Y_{i'j'k'l'}, (i', j', k', l') \neq (i, j, k, l))]$. Hence, for $(i', j', k', l') \neq (i, j, k, l)$, $E_{ijkl}$ is a residual also with respect to the regressors $p_0, p_l$ and $Y_{i'j'k'l'}$. Given that a residual (here: $E_{ijkl}$ ) is always uncorrelated with each numerically measurable function (here: $E_{i'j'k'l'}$ ) of his regressors, $Cov(E_{ijkl}, E_{i'j'k'l'}) = 0$ for $(i, j, k, l) \neq (i', j', k', l')$.

*Equation 134*

The derivation of Equation 134 follows a similar logic:

$Cov(S_{ijkl}, E_{i'j'k'l'}) = Cov\left\{E(Y_{ijkl} \mid p_0, p_l), \left[Y_{i'j'k'l'} - E(Y_{i'j'k'l'} \mid p_0, p_{l'})\right]\right\}$. According to Definition 4, $E(Y_{i'j'k'l'} \mid p_0, p_{l'})$ can be replaced by $E\left[Y_{i'j'k'l'} \mid p_0, p_1, ..., p_p, \left(Y_{ijkl}, \left(i, j, k, l\right) \neq \left(i', j', k', l'\right)\right)\right]$. The variable $S_{ijkl} := E(Y_{ijkl} \mid p_0, p_l)$ is a $(p_0, p_l)$-measurable function and $E_{i'j'k'l'}$ is a residual with respect to the regressors $p_0$ and $p_l$. As stated before, a residual (here: $E_{i'j'k'l'}$ ) is always uncorrelated with each numerically measurable function [in this case $E(Y_{ij1l} \mid p_0, p_l)$ ] of his regressors. Therefore, $Cov(S_{1j1l}, E_{i'j'k'l'}) = 0$.

*Equation 136*

By using Equations 113 and 116, we may rewrite Equation 136 as follows:

$$
\begin{aligned}
Cov(M_{1jkl}, E_{i'j'k'l'}) &= Cov\left[\left(S_{1jkl} - E\left(S_{1jkl} \mid S_{1j1l}\right)\right), E_{i'j'k'l'}\right] \\
&= Cov\left(S_{1jkl}, E_{i'j'k'l'}\right) - Cov\left(E\left(S_{1jkl} \mid S_{1j1l}\right), E_{i'j'k'l'}\right) \\
&= Cov\left(S_{1jkl}, E_{i'j'k'l'}\right) - Cov\left(\left(\alpha_{1jkl} + \lambda_{S1jkl} S_{1j1l}\right), E_{i'j'k'l'}\right) \\
&= Cov\left(S_{1jkl}, E_{i'j'k'l'}\right) - \lambda_{S1jkl} Cov\left(S_{1j1l}, E_{i'j'k'l'}\right).
\end{aligned}
$$

Given that both $S_{1jkl}$ and $S_{1j1l}$ are uncorrelated with $E_{i'j'k'l'}$ as has been shown above, it follows that $M_{1jkl}$ and $E_{i'j'k'l'}$ are also uncorrelated.

*Equation 137*

For the non-marker indicators of the reference method ($k$ = 1), we may rewrite Equation 137 as follows (by using Equations 120, 112, and 116):

$$Cov(IS_{ij1}, E_{i'j'k'l}) = Cov\left[\frac{1}{\lambda_{\text{IS}ij1l}} IS_{ij1l}, E_{i'j'k'l'}\right] = Cov\left[\frac{1}{\lambda_{\text{IS}ij1l}}\left(S_{ij1l} - E\left(S_{ij1l} \mid S_{1j1l}\right)\right), E_{i'j'k'l'}\right]$$

$$= \frac{1}{\lambda_{\text{IS}ij1l}} Cov\left(S_{ij1l}, E_{i'j'k'l'}\right) - Cov\left(E\left(S_{ij1l} \mid S_{1j1l}\right), E_{i'j'k'l'}\right)$$

$$= \frac{1}{\lambda_{\text{IS}ij1l}} Cov\left(S_{ij1l}, E_{i'j'k'l'}\right) - Cov\left(\left(\alpha_{ij1l} + \lambda_{\text{S}ij1l} S_{1j1l}\right), E_{i'j'k'l'}\right)$$

$$= \frac{1}{\lambda_{\text{IS}ij1l}} Cov\left(S_{ij1l}, E_{i'j'k'l'}\right) - \lambda_{\text{S}ij1l} Cov\left(S_{1j1l}, E_{i'j'k'l'}\right).$$

Given that both $S_{1jkl}$ and $S_{1j1l}$ are uncorrelated with $E_{i'j'k'l'}$ as has been shown above, it follows that $M_{1jkl}$ and $E_{i'j'k'l'}$ are also uncorrelated.

For the non-marker indicators of the non-reference method ($k \neq 1$), we may rewrite Equation 137 as follows (by using Equations 120, 114, and 117):

$$Cov(IS_{ijk}, E_{i'j'k'l}) = Cov\left[\frac{1}{\lambda_{\text{IS}ijkl}} IS_{ijkl}, E_{i'j'k'l'}\right] = Cov\left[\frac{1}{\lambda_{\text{IS}ijkl}}\left(M_{ijkl} - E\left(M_{ijkl} \mid M_{1jkl}\right)\right), E_{i'j'k'l'}\right]$$

$$= \frac{1}{\lambda_{\text{IS}ijkl}} Cov\left(M_{ijkl}, E_{i'j'k'l'}\right) - Cov\left(E\left(M_{ijkl} \mid M_{1jkl}\right), E_{i'j'k'l'}\right)$$

$$= \frac{1}{\lambda_{\text{IS}ijkl}} Cov\left(M_{ijkl}, E_{i'j'k'l'}\right) - \lambda_{\text{M}ijkl} Cov\left(M_{1jkl}, E_{i'j'k'l'}\right).$$

Given that both $M_{ijkl}$ and $M_{1jkl}$ are uncorrelated with $E_{i'j'k'l'}$ as has been shown above, it follows that $IS_{ijk}$, $k \neq 1$, and $E_{i'j'k'l'}$ are also uncorrelated.

*Explanations.* Theorem 3 shows the implications of the model definition for the observed and latent variable covariance structure. Only the most general covariance structure equation for the observed variables is shown in Theorem 3 (Equation 131). To illustrate in more detail how the observed variances and covariances can be expressed in terms of the parameters of the model, I provide the most important special cases in Corollary 8.

The independence of method factors and state factors belonging to the same construct on the same measurement occasion (Equation 132) is again a direct consequence of the definition of the method factors $M_{1jkl}$ as residuals with respect to $S_{1j1l}$ (see Definition 3). The independence of state factors $S_{1j1l}$ and indicator-specific factors $IS_{ij1}$ belonging to the same construct also follows directly from the definition of the variables $IS_{ij1}$ as residuals with respect to $S_{1j1l}$. In contrast, the uncorrelatedness of $IS_{ijk}$, $k \neq 1$, and $S_{1j1l}$ follows indirectly, given that $IS_{ijk}$ is a residual in a regression analysis including two method variables $M_{ijkl}$ and

$M_{1jkl}$, both of which are residuals with respect to $S_{1j1l}$. Furthermore, the variables $IS_{ijk}$ are uncorrelated with the method factors $M_{1jkl}$, belonging to the same construct and the same method, given that $IS_{ijk}$ is a residual with respect to $M_{1jkl}$.

The independence of state factors and error variables (Equation 134), method factors and error variables (Equation 136), indicator-specific factors and error variables (Equation 137), as well as error variables and other error variables (Equation 138) is a consequence of the assumption of conditional regressive independence made in Definition 4. Note that in empirical applications of the model, the respective covariances must be fixed to zero.

**Corollary 8: Covariance Structure of the Observed Variables**

If $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl}, \lambda_{\mathrm{IS}ijkl} \right\rangle$ is a CS-C($M$–1) model with indicator-specific factors and conditional regressive independence, without loss of generality $k = 1$ is chosen as the reference method, and without loss of generality all indicators $Y_{1jkl}$ are selected as marker indicators, then the following covariance structure holds for all $i, i' \in I := \{1, ..., m\}$, $j, j' \in J := \{1, ..., n\}$, $k, k' \in K := \{1, ..., o\}$, $l, l' \in L := \{1, ..., p\}$ and

(a) for all observed variables measuring the same construct ($j = j'$) on the same measurement occasion ($l = l'$):

$$Cov(Y_{ijkl}, Y_{i'jk'l}) = \begin{cases} Var(S_{1j1l}) + Var(E_{1j1l}),\ i, i' = 1,\ k, k' = 1, \\ \lambda_{\mathrm{S}i'j1l} Var(S_{1j1l}),\ i = 1,\ i' \neq 1,\ k, k' = 1, \\ \lambda_{\mathrm{S}1jk'l} Var(S_{1j1l}),\ i, i' = 1,\ k = 1,\ k' \neq 1 \\ \lambda_{\mathrm{S}i'jk'l} Var(S_{1j1l}),\ i = 1,\ i' \neq 1,\ k = 1,\ k' \neq 1, \\ \lambda^2_{\mathrm{S}ij1l} Var(S_{1j1l}) + \lambda^2_{\mathrm{IS}ij1l} Var(IS_{ij1}) + Var(E_{ij1l}),\ i = i',\ i, i' \neq 1,\ k, k' = 1, \\ \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}i'j1l} Var(S_{1j1l}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}i'j1l} Cov(IS_{ij1}, IS_{i'j1}),\ i \neq i',\ i, i' \neq 1,\ k, k' = 1, \\ \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}1jk'l} Var(S_{1j1l}) + \lambda_{\mathrm{IS}ij1l} Cov(IS_{ij1}, M_{1jk'l}),\ i \neq 1,\ i' = 1,\ k = 1,\ k' \neq 1, \\ \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}i'jk'l} Var(S_{1j1l}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{M}i'jk'l} Cov(IS_{ij1}, M_{1jk'l}) \\ \quad + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}i'jk'l} Cov(IS_{ij1}, IS_{i'jk'}),\ i, i' \neq 1,\ k = 1,\ k' \neq 1, \\ \lambda^2_{\mathrm{S}1jkl} Var(S_{1j1l}) + Var(M_{1jkl}) + Var(E_{1jkl}),\ i, i' = 1,\ k = k',\ k, k' \neq 1, \\ \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}i'jkl} Var(S_{1j1l}) + \lambda_{\mathrm{M}i'jkl} Var(M_{1jkl}),\ i = 1,\ i' \neq 1,\ k = k',\ k, k' \neq 1, \\ \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1jk'l} Var(S_{1j1l}) + Cov(M_{1jkl}, M_{1jk'l}),\ i, i' = 1,\ k \neq k',\ k, k' \neq 1, \\ \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}i'jk'l} Var(S_{1j1l}) + \lambda_{\mathrm{M}i'jk'l} Cov(M_{1jkl}, M_{1jk'l}) \\ \quad + \lambda_{\mathrm{IS}i'j1l} Cov(M_{1jkl}, IS_{i'jk'}),\ i = 1,\ i' \neq 1,\ k \neq k',\ k, k' \neq 1, \\ \lambda^2_{\mathrm{S}ijkl} Var(S_{1j1l}) + \lambda^2_{\mathrm{M}ijkl} Var(M_{1jkl}) + \lambda^2_{\mathrm{IS}ijkl} Var(IS_{ijk}) + Var(E_{ijkl}),\ i = i',\ i, i' \neq 1,\ k = k',\ k, k' \neq 1, \\ \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}i'jkl} Var(S_{1j1l}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}i'jkl} Var(M_{1jkl}) \\ \quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{IS}i'jkl} Cov(IS_{ijk}, IS_{i'jk}),\ i \neq i',\ i, i' \neq 1,\ k = k',\ k, k' \neq 1, \\ \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}i'jk'l} Var(S_{1j1l}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}i'jk'l} Cov(M_{1jkl}, M_{1jk'l}) \\ \quad + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{IS}i'jk'l} Cov(M_{1jkl}, IS_{i'jk'}) + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{M}i'jk'l} Cov(IS_{ijk}, M_{1jk'l}) \\ \quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{IS}i'jk'l} Cov(IS_{ijk}, IS_{i'jk'}),\ i, i' \neq 1,\ k \neq k',\ k, k' \neq 1, \end{cases} \tag{139}$$

(b) for all observed variables measuring different constructs ( $j \neq j'$ ) on the same measurement occasion ( $l = l'$ ):

$$
Cov(Y_{ijkl}, Y_{i'j'k'l}) = \begin{cases}
Cov(S_{1j1l}, S_{1j'1l}), \; i,i'=1, \; k,k'=1, \\
\lambda_{\mathrm{S}i'j'1l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{IS}i'j'1l} Cov(S_{1j1l}, IS_{i'j'1}), \; i=1, \; i' \neq 1, \; k,k'=1, \\
\lambda_{\mathrm{S}1j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + Cov(S_{1j1l}, M_{1j'k'l}), \; i,i'=1, \; k=1, \; k' \neq 1, \\
\lambda_{\mathrm{S}i'j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{M}i'j'k'l} Cov(S_{1j1l}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{IS}i'j'k'l} Cov(S_{1j1l}, IS_{i'j'k'}), \; i=1, \; i' \neq 1, \; k=1, \; k' \neq 1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}i'j'1l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{IS}i'j'1l} Cov(S_{1j1l}, IS_{ij'1}) \\
\quad + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{S}i'j'1l} Cov(IS_{ij1}, S_{i'j'1l}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}i'j'1l} Cov(IS_{ij1}, IS_{i'j'1}), \; i,i' \neq 1, \; k,k'=1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}1j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{S}ij1l} Cov(S_{1j1l}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{S}1j'k'l} Cov(IS_{ij1}, S_{1j'1l}) + \lambda_{\mathrm{IS}ij1l} Cov(IS_{ij1}, M_{1j'k'l}), \; i \neq 1, \; i'=1, \; k=1, \; k' \neq 1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}i'j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{M}i'j'k'l} Cov(S_{1j1l}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{IS}i'j'k'l} Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{S}i'j'k'l} Cov(IS_{ij1}, S_{1j'1l}) \\
\quad + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{M}i'j'k'l} Cov(IS_{ij1}, M_{1j'k'l}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}i'j'k'l} Cov(IS_{ij1}, IS_{i'j'k'}), \; i,i' \neq 1, \; k=1, \; k' \neq 1, \\
\lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{S}1jkl} Cov(S_{1j1l}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{S}1j'k'l} Cov(M_{1jkl}, S_{1j'k'l}) + Cov(M_{1jkl}, M_{1j'k'l}), \; i,i'=1, \; k,k' \neq 1, \\
\lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}i'j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{M}i'j'k'l} Cov(S_{1j1l}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{IS}i'j'k'l} Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{\mathrm{S}i'j'k'l} Cov(M_{1jkl}, S_{1j'1l}) \\
\quad + \lambda_{\mathrm{M}i'j'k'l} Cov(M_{1jkl}, M_{1j'k'l}) + \lambda_{\mathrm{IS}i'j'k'l} Cov(M_{1jkl}, IS_{i'j'k'}), \; i=1, \; i' \neq 1, \; k,k' \neq 1, \\
\lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}i'j'k'l} Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{M}i'j'k'l} Cov(S_{1j1l}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{IS}i'j'k'l} Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{S}i'j'k'l} Cov(M_{1jkl}, S_{1j'1l}) \\
\quad + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}i'j'k'l} Cov(M_{1jkl}, M_{1j'k'l}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{IS}i'j'k'l} Cov(M_{1jkl}, IS_{i'j'k'}) \\
\quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{S}i'j'k'l} Cov(IS_{ijk}, S_{1j'1l}) + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{M}i'j'k'l} Cov(IS_{ijk}, M_{1j'k'l}) \\
\quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{IS}i'j'k'l} Cov(IS_{ijk}, IS_{i'j'k'}), \; i,i' \neq 1, \; k,k' \neq 1,
\end{cases} \tag{140}
$$

(c) for all observed variables measuring the same construct ( $j = j'$ ) on different measurement occasions ( $l \neq l'$ ):

$$
Cov(Y_{ijkl}, Y_{i'jk'l'}) = \begin{cases}
Cov(S_{1j1l}, S_{1j1l'}), \; i,i'=1, \; k,k'=1, \\
\lambda_{\mathrm{S}i'j1l'} Cov(S_{1j1l}, S_{1j1l'}), \; i=1, \; i'\neq 1, \; k,k'=1, \\
\lambda_{\mathrm{S}1jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + Cov(S_{1j1l}, M_{1jk'l'}), \; i,i'=1, \; k=1, \; k'\neq 1, \\
\lambda_{\mathrm{S}i'jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{M}i'jk'l'} Cov(S_{1j1l}, M_{1jk'l'}), \; i=1, \; i'\neq 1, \; k=1, \; k'\neq 1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}ij1l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}ij1l'} Var(IS_{ij1}), \; i=i', \; i,i'\neq 1, \; k,k'=1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}i'j1l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}i'j1l'} Cov(IS_{ij1}, IS_{i'j1}), \; i\neq i', \; i,i'\neq 1, \; k,k'=1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}1jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}ij1l} Cov(S_{1j1l}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{IS}ij1l} Cov(IS_{ij1}, M_{1jk'l'}), \; i\neq 1, \; i'=1, \; k=1, \; k'\neq 1, \\
\lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{S}i'jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}ij1l} \lambda_{\mathrm{M}i'jk'l'} Cov(S_{1j1l}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{M}i'jk'l'} Cov(IS_{ij1}, M_{1jk'l'}) + \lambda_{\mathrm{IS}ij1l} \lambda_{\mathrm{IS}i'jk'l'} Cov(IS_{ij1}, IS_{i'jk'}), \; i,i'\neq 1, \; k=1, \; k'\neq 1, \\
\lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1jkl'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}1jkl} Cov(S_{1j1l}, M_{1jkl'}) \\
\quad + \lambda_{\mathrm{S}1jkl'} Cov(M_{1jkl}, S_{1j1l'}) + Cov(M_{1jkl}, M_{1jkl'}), \; i,i'=1, \; k=k', \; k,k'\neq 1, \\
\lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}1jkl} Cov(S_{1j1l}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{S}1jk'l'} Cov(M_{1jkl}, S_{1j1l'}) + Cov(M_{1jkl}, M_{1jk'l'}), \; i,i'=1, \; k\neq k', \; k,k'\neq 1, \\
\lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}i'jkl'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{M}i'jkl'} Cov(S_{1j1l}, M_{1jkl'}) \\
\quad + \lambda_{\mathrm{S}i'jkl'} Cov(M_{1jkl}, S_{1j1l'}) + \lambda_{\mathrm{M}i'jkl'} Cov(M_{1jkl}, M_{1jkl'}), \; i=1, \; i'\neq 1, \; k=k', \; k,k'\neq 1, \\
\lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}i'jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{M}i'jk'l'} Cov(S_{1j1l}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{S}i'jk'l'} Cov(M_{1jkl}, S_{1j1l'}) + \lambda_{\mathrm{M}i'jk'l'} Cov(M_{1jkl}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{IS}i'jk'l'} Cov(M_{1jkl}, IS_{i'jk'}), \; i=1, \; i'\neq 1, \; k\neq k', \; k,k'\neq 1, \\
\lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}ijkl'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{M}ijkl'} Cov(S_{1j1l}, M_{1jkl'}) \\
\quad + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{S}ijkl'} Cov(M_{1jkl}, S_{1j1l'}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}ijkl'} Cov(M_{1jkl}, M_{1jkl'}) \\
\quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{IS}ijkl'} Var(IS_{ijk}), \; i=i', \; i,i'\neq 1, \; k=k', \; k,k'\neq 1, \\
\lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}i'jkl'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{M}i'jkl'} Cov(S_{1j1l}, M_{1jkl'}) \\
\quad + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{S}i'jkl'} Cov(M_{1jkl}, S_{1j1l'}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}i'jkl'} Cov(M_{1jkl}, M_{1jkl'}) \\
\quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{IS}i'jkl'} Cov(IS_{ijk}, IS_{i'jk}), \; i\neq i', \; i,i'\neq 1, \; k=k', \; k,k'\neq 1, \\
\lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{S}i'jk'l'} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{S}ijkl} \lambda_{\mathrm{M}i'jk'l'} Cov(S_{1j1l}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{S}i'jk'l'} Cov(M_{1jkl}, S_{1j1l'}) + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{M}i'jk'l'} Cov(M_{1jkl}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{M}ijkl} \lambda_{\mathrm{IS}i'jk'l'} Cov(M_{1jkl}, IS_{i'jk'}) + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{M}i'jk'l'} Cov(IS_{ijk}, M_{1jk'l'}) \\
\quad + \lambda_{\mathrm{IS}ijkl} \lambda_{\mathrm{IS}i'jk'l'} Cov(IS_{ijk}, IS_{i'jk'}), \; i,i'\neq 1, \; k\neq k', \; k,k'\neq 1,
\end{cases} \tag{141}
$$

(d) for all observed variables measuring different constructs ( $j \neq j'$ ) on different measurement occasions ( $l \neq l'$ ):

$$
Cov(Y_{ijkl}, Y_{i'j'k'l'}) = \begin{cases}
Cov(S_{1j1l}, S_{1j'1l'}), \; i, i' = 1, \; k, k' = 1, \\
\lambda_{\text{S}i'j'1l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{IS}i'j'1l'} Cov(S_{1j1l}, IS_{i'j'1}), \; i = 1, \; i' \neq 1, \; k, k' = 1, \\
\lambda_{\text{S}1j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + Cov(S_{1j1l}, M_{1j'k'l'}), \; i, i' = 1, \; k = 1, \; k' \neq 1, \\
\lambda_{\text{S}i'j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{M}i'j'k'l'} Cov(S_{1j1l}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{IS}i'j'k'l'} Cov(S_{1j1l}, IS_{i'j'k'}), \; i = 1, \; i' \neq 1, \; k = 1, \; k' \neq 1, \\
\lambda_{\text{S}ij1l} \lambda_{\text{S}i'j'1l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}ij1l} \lambda_{\text{IS}i'j'1l'} Cov(S_{1j1l}, IS_{i'j'1}) \\
\quad + \lambda_{\text{IS}ij1l} \lambda_{\text{S}i'j'1l'} Cov(IS_{ij1}, S_{1j'1l'}) + \lambda_{\text{IS}ij1l} \lambda_{\text{IS}i'j'1l'} Cov(IS_{ij1}, IS_{i'j'1}), \; i, i' \neq 1, \; k, k' = 1, \\
\lambda_{\text{S}ij1l} \lambda_{\text{S}1j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}ij1l} Cov(S_{1j1l}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{IS}ij1l} \lambda_{\text{S}1j'k'l'} Cov(IS_{ij1}, S_{1j'k'l'}) + \lambda_{\text{IS}ij1l} Cov(IS_{ij1}, M_{1j'k'l'}), \; i \neq 1, \; i' = 1, \; k = 1, \; k' \neq 1, \\
\lambda_{\text{S}ij1l} \lambda_{\text{S}i'j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}ij1l} \lambda_{\text{M}i'j'k'l'} Cov(S_{1j1l}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{S}ij1l} \lambda_{\text{IS}i'j'k'l'} Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{\text{IS}ij1l} \lambda_{\text{S}i'j'k'l'} Cov(IS_{ij1}, S_{1j'1l'}) \\
\quad + \lambda_{\text{IS}ij1l} \lambda_{\text{M}i'j'k'l'} Cov(IS_{ij1}, M_{1j'k'l'}) + \lambda_{\text{IS}ij1l} \lambda_{\text{IS}i'j'k'l'} Cov(IS_{ij1}, IS_{i'j'k'}), \; i, i' \neq 1, \; k = 1, \; k' \neq 1, \\
\lambda_{\text{S}1jkl} \lambda_{\text{S}1j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}1jkl} Cov(S_{1j1l}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{S}1j'k'l'} Cov(M_{1jkl}, S_{1j'1l'}) + Cov(M_{1jkl}, M_{1j'k'l'}), \; i, i' = 1, \; k, k' \neq 1, \\
\lambda_{\text{S}1jkl} \lambda_{\text{S}i'j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}1jkl} \lambda_{\text{M}i'j'k'l'} Cov(S_{1j1l}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{S}1jkl} \lambda_{\text{IS}i'j'k'l'} Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{\text{S}i'j'k'l'} Cov(M_{1jkl}, S_{1j'1l'}) \\
\quad + \lambda_{\text{M}i'j'k'l'} Cov(M_{1jkl}, M_{1j'k'l'}) + \lambda_{\text{IS}i'j'k'l'} Cov(M_{1jkl}, IS_{i'j'k'}), \; i = 1, \; i' \neq 1, \; k, k' \neq 1, \\
\lambda_{\text{S}ijkl} \lambda_{\text{S}i'j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{S}ijkl} \lambda_{\text{M}i'j'k'l'} Cov(S_{1j1l}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{S}ijkl} \lambda_{\text{IS}i'j'k'l'} Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{\text{M}ijkl} \lambda_{\text{S}i'j'k'l'} Cov(M_{1jkl}, S_{1j'1l'}) \\
\quad + \lambda_{\text{M}ijkl} \lambda_{\text{M}i'j'k'l'} Cov(M_{1jkl}, M_{1j'k'l'}) + \lambda_{\text{M}ijkl} \lambda_{\text{IS}i'j'k'l'} Cov(M_{1jkl}, IS_{i'j'k'}) \\
\quad + \lambda_{\text{IS}ijkl} \lambda_{\text{S}i'j'k'l'} Cov(IS_{ijk}, S_{1j'1l'}) + \lambda_{\text{IS}ijkl} \lambda_{\text{M}i'j'k'l'} Cov(IS_{ijk}, M_{1j'k'l'}) \\
\quad + \lambda_{\text{IS}ijkl} \lambda_{\text{IS}i'j'k'l'} Cov(IS_{ijk}, IS_{i'j'k'}), \; i, i' \neq 1, \; k, k' \neq 1,
\end{cases} \tag{142}
$$

*Proof*. Equations 139 to 142 directly follow from Equation 131 by applying Equations 132 to 138.

#### 3.5.2.6 Mean Structure

Theorem 4 shows the consequences of the model definition for the observed and latent variable mean structure.

**Theorem 4: Mean Structure**

If $M := \langle(\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{Sijkl}, \lambda_{Mijkl}, \lambda_{ISijkl}\rangle$ is a CS-C($M$–1) model with indicator-specific factors, without loss of generality $k = 1$ is chosen as the reference method, and without loss of generality all indicators $Y_{1jkl}$ are selected as marker indicators, then the following mean structure holds for all $i \in I := \{1,...,m\}$, $j \in J := \{1,...,n\}$, $k \in K := \{1,...,o\}$, and $l \in L := \{1,..., p\}$:

$$E(Y_{ijkl}) = \alpha_{ijkl} + \lambda_{Sijkl} E(S_{1j1l}), \tag{143}$$

$$E(S_{1j1l}) = E(Y_{1j1l}), \tag{144}$$

$$E(M_{1jkl}) = 0, \tag{145}$$

$$E(IS_{ijk}) = 0, \tag{146}$$

$$E(E_{ijkl}) = 0, \tag{147}$$

where $E(.)$ denotes the expected value (mean).

*Proof*. According to Equation 121, $Y_{ijkl} = \alpha_{ijkl} + \lambda_{Sijkl} S_{1j1l} + \lambda_{Mijkl} M_{1jkl} + \lambda_{ISijkl} IS_{ijk} + E_{ijkl}$ for $i, k \neq 1$. Hence, $E(Y_{ijkl}) = E(\alpha_{ijkl}) + E(\lambda_{Sijkl} S_{1j1l}) + E(\lambda_{Mijkl} M_{1jkl}) + E(\lambda_{ISijkl} IS_{ijk}) + E(E_{ijkl})$. The terms $E(\lambda_{Mijkl} M_{jkl})$, $E(\lambda_{ISijkl} IS_{ijk})$, and $E(E_{ij1l})$ are zero according to Equations 145, 146, and 147 so that this equation simplifies to Equation 143. Equation 144 follows from Equation 143, given that $\alpha_{1j1l} = 0$ and $\lambda_{S1j1l} = 1$ (see Equation 121). Equations 145, 146, and 147 follow from the definition of $M_{1jkl}$, $IS_{ijk}$, and $E_{ijkl}$ as residuals in a latent regression analysis (see Equations 112, 113, and 114). Residuals have an expected value (mean) of zero by definition (see Steyer & Eid, 2001, Box G.1).

*Explanations.* Equation 143 shows that the mean of an observed variable is identical to the mean of the corresponding state factor if and only if $\alpha_{ijkl} = 0$ and $\lambda_{\mathrm{S}ijkl} = 1$. According to Equation 144, the means of the latent state factors are identical to the means of the marker indicators ($i$ = 1) pertaining to the reference method. Equations 145, 146, and 147 show an important implication of the model definition, namely that the method factors, indicator-specific factors, and error variables, being defined as residuals, have means of zero. Therefore, in empirical applications of the model, the means of the method factors, indicator-specific factors, and error variables have to be fixed to zero. Note that this is *not* a testable assumption, but a direct consequence of the model definition.

3.5.2.7 Identification

The relevant parameters for which identification needs to be proven in the CS-C($M$–1) model with indicator-specific factors are the intercepts ($\alpha_{ijkl}$), the state factor loadings ($\lambda_{\mathrm{S}ijkl}$), the method factor loadings ($\lambda_{\mathrm{M}ijkl}$), the indicator-specific factor loadings ($\lambda_{\mathrm{IS}ijkl}$), the variances of the state factors [$Var(S_{1j1l})$], the variances of the method factors [$Var(M_{1jkl})$], the variances of the indicator-specific factors [$Var(IS_{ijk})$], the admissible covariances between the latent factors and the variances of the error variables [$Var(E_{ijkl})$].

A prerequisite for the identification is that each latent factor is assigned a scale (Bollen, 1989). From Definition 3, it follows that $\alpha_{1j1l} = 0$ and $\lambda_{\mathrm{S}1j1l} = \lambda_{\mathrm{M}1jkl} = 1$ (see also Equation 121). These implicit constraints identify the scales of the state factors $S_{1j1l}$ and of the method factors $M_{1jkl}$. In order to assign a scale to the indicator-specific factors $IS_{ijk}$, one factor loading $\lambda_{\mathrm{IS}ijkl}$ per indicator-specific factor must be fixed to a non-zero value. Alternatively, one may fix $Var(IS_{ijk})$ to a positive value. To simplify the present identification corollary, I assume without loss of generality that the indicator-specific factor loadings are set to one at the first measurement occasion for all indicators (i.e., $\lambda_{\mathrm{IS}ijk1} = 1$). Corollary 9 shows how each parameter of the CS-C($M$–1) model with indicator-specific factors can be identified under this condition. Note that the unknown parameters to be identified are either expressed in terms of observed means, variances, and covariances or in terms of other known-to-be-identified model parameters. The latter is done in cases in which the terms would become very complicated if all parameters were replaced by observed covariances.

**Corollary 9: Identification**

If $M := \left\langle (\Omega, \mathfrak{A}, P), S_{ijkl}, M_{ijkl}, IS_{ijk}, E_{ijkl}, \alpha_{ijkl}, \lambda_{\mathrm{S}ijkl}, \lambda_{\mathrm{M}ijkl}, \lambda_{\mathrm{IS}ijkl} \right\rangle$ is a CS-C(*M*−1) model with indicator-specific factors and conditional regressive independence, the first indicators ($Y_{1jkl}$) are chosen as marker indicators, $k = 1$ is chosen as the reference method, and all indicator-specific factor loadings $\lambda_{\mathrm{IS}ijk1}$ are set to 1, then for all

$i, i' \in I := \{1, ..., m\},\ \ j, j' \in J := \{1, ..., n\},\ k, k' \in K := \{1, ..., o\},\ \ l, l' \in L := \{1, ..., p\}$:

$$\alpha_{ijkl} = E(Y_{ijkl}) - \lambda_{\mathrm{S}ijkl} E(Y_{1j1l}), \tag{148}$$

$$\lambda_{\mathrm{S}ij1l} = \frac{Cov(Y_{ij1l}, Y_{1j1l'})}{Cov(Y_{1j1l}, Y_{1j1l'})},\ i \neq 1,\ l \neq l', \tag{149}$$

$$\lambda_{\mathrm{S}ijkl} = \frac{Cov(Y_{1j1l}, Y_{ijkl})}{Var(S_{1j1l})},\ k \neq 1, \tag{150}$$

$$E(S_{1j1l}) = E(Y_{1j1l}), \tag{151}$$

$$Var(S_{1j1l}) = \frac{Cov(Y_{1j1l}, Y_{ij1l}) Cov(Y_{1j1l}, Y_{1j1l'})}{Cov(Y_{ij1l}, Y_{1j1l'})},\ i \neq 1,\ l \neq l', \tag{152}$$

$$Cov(S_{1j1l}, S_{1j'1l'}) = Cov(Y_{1j1l}, Y_{1j'1l'}),\ (j,l) \neq (j',l'), \tag{153}$$

$$Cov(S_{1j1l}, M_{1j'kl'}) = Cov(Y_{1j1l}, Y_{1j'kl'}) - \lambda_{\mathrm{S}1j'kl'} Cov(S_{1j1l}, S_{1j'1l'}),\ (j,l) \neq (j',l'),\ k \neq 1, \tag{154}$$

$$Cov(S_{1j1l}, IS_{ij'1}) = Cov(Y_{1j1l}, Y_{ij'11}) - \lambda_{\mathrm{S}ij'11} Cov(S_{1j1l}, S_{1j'11}),\ i \neq 1,\ j \neq j', \tag{155}$$

$$\begin{aligned} Cov(S_{1j1l}, IS_{ij'k}) &= Cov(Y_{1j1l}, Y_{ij'k1}) - \lambda_{\mathrm{S}ij'k1} Cov(S_{1j1l}, S_{1j'11}) \\ &\quad - \lambda_{\mathrm{M}ij'k1} Cov(S_{1j1l}, M_{1j'k1}),\ i,k \neq 1,\ j \neq j', \end{aligned} \tag{156}$$

$$\lambda_{\mathrm{M}ijkl} = \frac{\left[ Cov(Y_{ijkl}, Y_{1j1l'}) - \lambda_{\mathrm{S}ijkl} Cov(S_{1j1l}, S_{1j1l'}) \right]}{Cov(M_{1jkl}, S_{1j1l'})},\ i,k \neq 1,\ l \neq l', \tag{157}$$

$$Var(M_{1jkl}) = \frac{\left[Cov(Y_{1jkl}, Y_{ijkl}) - \lambda_{\text{S}1jkl}\lambda_{\text{S}ijkl}Var(S_{1j1l})\right]}{\lambda_{\text{M}ijkl}},\ i,k \neq 1, \tag{158}$$

$$Cov(M_{1jkl}, M_{1jk'l}) = Cov(Y_{1jkl}, Y_{1jk'l}) - \lambda_{\text{S}1jkl}\lambda_{\text{S}1jk'l}Var(S_{1j1l}),\ k \neq k', \tag{159}$$

$$\begin{aligned} Cov(M_{1jkl}, M_{1j'k'l'}) = {} & Cov(Y_{1jkl}, Y_{1jk'l}) - \lambda_{\text{S}1jkl}\lambda_{\text{S}1j'k'l'}Cov(S_{1j1l}, S_{1j'1l'}) \\ & - \lambda_{\text{S}1jkl}Cov(S_{1j1l}, M_{1j'k'l'}) - \lambda_{\text{S}1j'k'l'}Cov(M_{1jkl}, S_{1j'1l'}), \\ & k,k' \neq 1,\ (j,l) \neq (j',l'), \end{aligned} \tag{160}$$

$$Cov(M_{1jkl}, IS_{ij1}) = \frac{[Cov(Y_{ij1l}, Y_{1jkl}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}1jkl}Var(S_{1j1l})]}{\lambda_{\text{IS}ij1l}},\ i,k \neq 1, \tag{161}$$

$$\begin{aligned} Cov(M_{1jkl}, IS_{ij'1}) = {} & [Cov(Y_{1jkl}, Y_{ij'1l}) - \lambda_{\text{S}1jkl}\lambda_{\text{S}ij'1l}Cov(S_{1j1l}, S_{1j'1l}) \\ & - \lambda_{\text{S}ij'1l}Cov(M_{1jkl}, S_{1j'1l}) \\ & - \lambda_{\text{S}1jkl}\lambda_{\text{IS}ij'1l}Cov(S_{1j1l}, IS_{ij'1})]\frac{1}{\lambda_{\text{IS}ij'1l}},\ i,k \neq 1,\ j \neq j', \end{aligned} \tag{162}$$

$$\begin{aligned} Cov(M_{1jk1}, IS_{ijk'}) = {} & Cov(Y_{1jk1}, Y_{ijk'1}) - \lambda_{\text{S}1jk1}\lambda_{\text{S}ijk'1}Var(S_{1j11}) \\ & - \lambda_{\text{M}ijk'1}Cov(M_{1jk1}, M_{1jk'1}),\ i \neq 1,\ k \neq k',\ k,k' \neq 1, \end{aligned} \tag{163}$$

$$\begin{aligned} Cov(M_{1jkl}, IS_{ijk'}) = {} & Cov(Y_{1jkl}, Y_{ijk'1}) - \lambda_{\text{S}1jkl}\lambda_{\text{S}ijk'1}Cov(S_{1j1l}, S_{1j11}) \\ & - \lambda_{\text{S}1jkl}\lambda_{\text{M}ijk'1}Cov(S_{1j1l}, M_{1jk'1}) - \lambda_{\text{S}ijk'1}Cov(M_{1jkl}, S_{1j11}) \\ & - \lambda_{\text{M}ijk'1}Cov(M_{1jkl}, M_{1jk'1}),\ i \neq 1,\ k \neq k',\ k,k' \neq 1,\ l \neq l', \end{aligned} \tag{164}$$

$$\begin{aligned} Cov(M_{1jkl}, IS_{ij'k'}) = {} & [Cov(Y_{1jkl}, Y_{ij'k'l}) - \lambda_{\text{S}1jkl}\lambda_{\text{S}ij'k'l}Cov(S_{1j1l}, S_{1j'1l}) \\ & - \lambda_{\text{S}1jkl}\lambda_{\text{M}ij'k'l}Cov(S_{1j1l}, M_{1j'k'l}) - \lambda_{\text{S}1jkl}\lambda_{\text{IS}ij'k'l}Cov(S_{1j1l}, IS_{ij'k'}) \\ & - \lambda_{\text{S}ij'k'l}Cov(M_{1jkl}, S_{1j'1l}) - \lambda_{\text{M}ij'k'l}Cov(M_{1jkl}, M_{1j'k'l})]\frac{1}{\lambda_{\text{IS}ij'k'l}}, \\ & i \neq 1,\ j \neq j',\ k \neq k',\ k,k' \neq 1, \end{aligned} \tag{165}$$

$$\lambda_{\text{IS}ij1l} = \frac{[Cov(Y_{ij1l}, Y_{i'j11}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}i'j11}Cov(S_{1j1l}, S_{1j11})]}{Cov(IS_{ij1}, IS_{i'j1})},\ i \neq i',\ i,i' \neq 1,\ l \neq 1, \tag{166}$$

$$\begin{aligned}\lambda_{\text{IS}ijkl} = [&Cov(Y_{ijkl}, Y_{i'jk1}) - \lambda_{\text{S}ijkl}\lambda_{\text{S}i'jk1}Cov(S_{1j1l}, S_{1j11}) \\ &- \lambda_{\text{S}ijkl}\lambda_{\text{M}i'jk1}Cov(S_{1j1l}, M_{1jk1}) - \lambda_{\text{M}ijkl}\lambda_{\text{S}i'jk1}Cov(M_{1jkl}, S_{1j11}) \\ &- \lambda_{\text{M}ijkl}\lambda_{\text{M}i'jk1}Cov(M_{1jkl}, M_{1jk1})]\frac{1}{Cov(IS_{ijk}, IS_{i'jk})},\ i \neq i',\ i,i' \neq 1,\ k,l \neq 1,\end{aligned} \tag{167}$$

$$Var(IS_{ij1}) = [Cov(Y_{ij1l}, Y_{ij1l'}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}ij1l'}Cov(S_{ij1l}, S_{ij1l'})]\frac{1}{\lambda_{\text{IS}ij1l}\lambda_{\text{IS}ij1l'}},\ i \neq 1,\ l \neq l', \tag{168}$$

$$\begin{aligned}Var(IS_{ijk}) = [&Cov(Y_{ijkl}, Y_{ijkl'}) - \lambda_{\text{S}ijkl}\lambda_{\text{S}ijkl'}Cov(S_{ij1l}, S_{ij1l'}) \\ &- \lambda_{\text{S}ijkl}\lambda_{\text{M}ijkl'}Cov(S_{ij1l}, M_{1jkl'}) - \lambda_{\text{M}ijkl}\lambda_{\text{S}ijkl'}Cov(M_{1jkl}, S_{ij1l'}) \\ &- \lambda_{\text{M}ijkl}\lambda_{\text{M}ijkl'}Cov(M_{1jkl}, M_{1jkl'})]\frac{1}{\lambda_{\text{IS}ijkl}\lambda_{\text{IS}ijkl'}},\ i,k \neq 1,\ l \neq l',\end{aligned} \tag{169}$$

$$Cov(IS_{ij1}, IS_{i'j1}) = Cov(Y_{ij11}, Y_{i'j11}) - \lambda_{\text{S}ij11}\lambda_{\text{S}i'j11}Var(S_{1j11}),\ i \neq i',\ i,i' \neq 1, \tag{170}$$

$$\begin{aligned}Cov(IS_{ij1}, IS_{i'j'1}) = [&Cov(Y_{ij1l}, Y_{i'j'1l'}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}i'j'1l'}Cov(S_{1j1l}, S_{1j'1l'}) \\ &- \lambda_{\text{S}ij1l}\lambda_{\text{IS}i'j'1l'}Cov(S_{1j1l}, IS_{i'j'1}) \\ &- \lambda_{\text{IS}ij1l}\lambda_{\text{S}i'j'1l'}Cov(IS_{ij1}, S_{1j'1l'})]\frac{1}{\lambda_{\text{IS}ij1l}\lambda_{\text{IS}i'j'1l'}},\ i,i' \neq 1,\ j \neq j',\ l \neq l',\end{aligned} \tag{171}$$

$$\begin{aligned}Cov(IS_{ij1}, IS_{i'jk}) = [&Cov(Y_{ij1l}, Y_{i'jkl'}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}i'jkl'}Cov(S_{1j1l}, S_{1j1l'}) \\ &- \lambda_{\text{S}ij1l}\lambda_{\text{M}i'jkl'}Cov(S_{1j1l}, M_{1jkl'}) \\ &- \lambda_{\text{IS}ij1l}\lambda_{\text{M}i'jkl'}Cov(IS_{ij1}, M_{1jkl'})]\frac{1}{\lambda_{\text{IS}ij1l}\lambda_{\text{IS}i'jkl'}},\ i,i' \neq 1,\ k \neq 1,\ l \neq l',\end{aligned} \tag{172}$$

$$\begin{aligned}Cov(IS_{ij1}, IS_{i'j'k}) = [&Cov(Y_{ij1l}, Y_{i'j'kl'}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}i'j'kl'}Cov(S_{1j1l}, S_{1j'1l'}) \\ &- \lambda_{\text{S}ij1l}\lambda_{\text{M}i'j'kl'}Cov(S_{1j1l}, M_{1j'kl'}) - \lambda_{\text{S}ij1l}\lambda_{\text{IS}i'j'kl'}Cov(S_{1j1l}, IS_{i'j'k}) \\ &- \lambda_{\text{IS}ij1l}\lambda_{\text{S}i'j'kl'}Cov(IS_{ij1}, S_{1j'1l'}) \\ &- \lambda_{\text{IS}ij1l}\lambda_{\text{M}i'j'kl'}Cov(IS_{ij1}, M_{1j'kl'})]\frac{1}{\lambda_{\text{IS}ij1l}\lambda_{\text{IS}i'j'kl'}}, \\ &i,i' \neq 1,\ j \neq j',\ k \neq 1,\ l \neq l',\end{aligned} \tag{173}$$

$$\begin{aligned}Cov(IS_{ijk}, IS_{i'jk}) = &Cov(Y_{ijk1}, Y_{i'jk1}) - \lambda_{\text{S}ijk1}\lambda_{\text{S}i'jk1}Var(S_{1j11}) \\ &- \lambda_{\text{M}ijk1}\lambda_{\text{M}i'jk1}Var(M_{1jk1}),\ i \neq i',\ i,i' \neq 1,\ k \neq 1,\end{aligned} \tag{174}$$

$$\begin{aligned} Cov(IS_{ijk}, IS_{i'jk'}) = [&Cov(Y_{ijkl}, Y_{i'jk'l}) - \lambda_{\text{S}ijkl}\lambda_{\text{S}i'jk'l}Var(S_{1j1l}) \\ &- \lambda_{\text{M}ijkl}\lambda_{\text{M}i'jk'l}Cov(M_{1jkl}, M_{1jk'l}) - \lambda_{\text{M}ijkl}\lambda_{\text{IS}i'jk'l}Cov(M_{1jkl}, IS_{i'jk'}) \\ &- \lambda_{\text{IS}ijkl}\lambda_{\text{M}i'jk'l}Cov(IS_{ijk}, M_{1jk'l})]\frac{1}{\lambda_{\text{IS}ijkl}\lambda_{\text{IS}i'jk'l}}, \\ &i, i' \neq 1,\ k \neq k',\ k, k' \neq 1, \end{aligned} \tag{175}$$

$$\begin{aligned} Cov(IS_{ijk}, IS_{i'j'k'}) = [&Cov(Y_{ijkl}, Y_{i'j'k'l'}) - \lambda_{\text{S}ijkl}\lambda_{\text{S}i'j'k'l'}Cov(S_{1j1l}, S_{1j'1l'}) \\ &- \lambda_{\text{S}ijkl}\lambda_{\text{M}i'j'k'l'}Cov(S_{1j1l}, M_{1j'k'l'}) - \lambda_{\text{S}ijkl}\lambda_{\text{IS}i'j'k'l'}Cov(S_{1j1l}, IS_{i'j'k'}) \\ &- \lambda_{\text{M}ijkl}\lambda_{\text{S}i'j'k'l'}Cov(M_{1jkl}, S_{1j'1l'}) - \lambda_{\text{M}ijkl}\lambda_{\text{M}i'j'k'l'}Cov(M_{1jkl}, M_{1j'k'l'}) \\ &- \lambda_{\text{M}ijkl}\lambda_{\text{IS}i'j'k'l'}Cov(M_{1jkl}, IS_{i'j'k'}) - \lambda_{\text{IS}ijkl}\lambda_{\text{S}i'j'k'l'}Cov(IS_{ijk}, S_{1j'1l'}) \\ &- \lambda_{\text{IS}ijkl}\lambda_{\text{M}i'j'k'l'}Cov(IS_{ijk}, M_{1j'k'l'})]\frac{1}{\lambda_{\text{IS}ijkl}\lambda_{\text{IS}i'j'k'l'}}, \\ &i, i' \neq 1,\ j \neq j',\ k, k' \neq 1, \end{aligned} \tag{176}$$

$$Var(E_{1j1l}) = Var(Y_{1j1l}) - Var(S_{1j1l}) \tag{177}$$

$$Var(E_{ij1l}) = Var(Y_{ij1l}) - \lambda^2_{\text{S}ij1l}Var(S_{1j1l}) - \lambda^2_{\text{IS}ij1l}Var(IS_{ij1}),\ i \neq 1, \tag{178}$$

$$Var(E_{1jkl}) = Var(Y_{1jkl}) - \lambda^2_{\text{S}1jkl}Var(S_{1j1l}) - Var(M_{1jkl}),\ \text{for } k \neq 1, \tag{179}$$

$$Var(E_{ijkl}) = Var(Y_{ijkl}) - \lambda^2_{\text{S}ijkl}Var(S_{1j1l}) - \lambda^2_{\text{M}ijkl}Var(M_{1jkl}) - \lambda^2_{\text{IS}ijkl}Var(IS_{ijk}),\ \text{for } i, k \neq 1. \tag{180}$$

*Proofs*. In order to make the proofs more accessible, I present them in the order in which the unknown parameters are most easily identified and not in the same order as in Corollary 9.

*Equation 151: Identifiability of $E(S_{1j1l})$*

Equation 151 is the same as Equation 144. The proof of Equation 144 is available in Theorem 4.

*Equation 148: Identifiability of $\alpha_{ijkl}$*

According to Equation 143 (Theorem 4), $E(Y_{ijkl}) = \alpha_{ijkl} + \lambda_{\text{S}ijkl}E(S_{1j1l})$. According to Equation 151, $E(S_{1j1l}) = E(Y_{1j1l})$. Hence, $\alpha_{ijkl} = E(Y_{ijkl}) - \lambda_{\text{S}ijkl}E(Y_{1j1l})$.

*Equation 153: Identifiability of $Cov(S_{1j1l}, S_{1j'1l'})$*

According to Equations 140–142, $Cov(Y_{1j1l}, Y_{1j'1l'}) = Cov(S_{1j1l}, S_{1j'1l'})$ for $(j, l) \neq (j', l')$.

*Equation 149: Identifiability of $\lambda_{\text{S}ij1l}$ (for $i \neq 1$)*

According to Equation 141, $Cov(Y_{ij1l}, Y_{1j1l'}) = \lambda_{\text{S}ij1l} Cov(S_{1j1l}, S_{1j1l'})$, for $i \neq 1$. Therefore,

$\lambda_{\text{S}ij1l} = \dfrac{Cov(Y_{ij1l}, Y_{1j1l'})}{Cov(S_{1j1l}, S_{1j1l'})}$, for $i \neq 1$, $l \neq l'$, and and $Cov(S_{1j1l}, S_{1j1l'}) \neq 0$. Equation 153 allows replacing $Cov(S_{1j1l}, S_{1j1l'})$ by $Cov(Y_{1j1l}, Y_{1j1l'})$ which leads to Equation 149.

*Equation 152: Identifiability of $Var(S_{1j1l})$*

According to Equation 139, $Cov(Y_{1j1l}, Y_{ij1l}) = \lambda_{\text{S}ij1l} Var(S_{1j1l})$, for $i \neq 1$. Therefore,

$Var(S_{1j1l}) = \dfrac{Cov(Y_{1j1l}, Y_{ij1l})}{\lambda_{\text{S}ij1l}} = \dfrac{Cov(Y_{1j1l}, Y_{ij1l})}{\dfrac{Cov(Y_{ij1l}, Y_{1j1l'})}{Cov(Y_{1j1l}, Y_{1j1l'})}} = \dfrac{Cov(Y_{1j1l}, Y_{ij1l}) Cov(Y_{1j1l}, Y_{1j1l'})}{Cov(Y_{ij1l}, Y_{1j1l'})}$, for $i \neq 1$, $l \neq l'$, and $Cov(Y_{ij1l}, Y_{1j1l'}) \neq 0$.

*Equation 150: Identifiability of $\lambda_{\text{S}ijkl}$ (for $k \neq 1$)*

According to Equation 139, $Cov(Y_{1j1l}, Y_{ijkl}) = \lambda_{\text{S}ijkl} Var(S_{1j1l})$, where $k \neq 1$. Hence,

$\lambda_{\text{S}ijkl} = \dfrac{Cov(Y_{1j1l}, Y_{ijkl})}{Var(S_{1j1l})}$, for $Var(S_{1j1l}) \neq 0$. $Var(S_{1j1l})$ is identified according to Equation 152.

*Equation 154: Identifiability of $Cov(S_{1j1l}, M_{1j'kl'})$ [for $(j,l) \neq (j',l')$ ]*

According to Equations 140–142, $Cov(Y_{1j1l}, Y_{1j'kl'}) = \lambda_{\text{S}1j'kl'} Cov(S_{1j1l}, S_{1j'1l'}) + Cov(S_{1j1l}, M_{1j'kl'})$ for $(j,l) \neq (j',l')$ and $k \neq 1$. Hence, $Cov(S_{1j1l}, M_{1j'kl'}) = Cov(Y_{1j1l}, Y_{1j'kl'}) - \lambda_{\text{S}1j'kl'} Cov(S_{1j1l}, S_{1j'1l'})$. The parameters $\lambda_{\text{S}1j'kl'}$ and $Cov(S_{1j1l}, S_{1j'1l'})$ are identified according to Equations 150 and 153.

*Equation 155: Identifiability of $Cov(S_{1j1l}, IS_{ij'1})$ (for $j \neq j'$ )*

According to Equations 140 and 142,

$Cov(Y_{1j1l}, Y_{ij'1l'}) = \lambda_{\text{S}ij'1l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\text{IS}ij'1l'} Cov(S_{1j1l}, IS_{ij'1})$ for $i \neq 1$ and $j \neq j'$. For $l'$ = 1, $\lambda_{\text{IS}ij'1l'}$ is set to unity (i.e., $\lambda_{\text{IS}ij'11} = 1$), so that we obtain:

$Cov(S_{1j1l}, IS_{ij'1}) = Cov(Y_{1j1l}, Y_{ij'11}) - \lambda_{\text{S}ij'11} Cov(S_{1j1l}, S_{1j'11})$. The parameters $\lambda_{\text{S}ij'11}$ and $Cov(S_{1j1l}, S_{1j'11})$ are identified according to Equations 149 and 153.

*Equation 157: Identifiability of $\lambda_{\text{M}ijkl}$*

According to Equation 141, $Cov(Y_{ijkl}, Y_{1j1l'}) = \lambda_{\text{S}ijkl} Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\text{M}ijkl} Cov(M_{1jkl}, S_{1j1l'})$ for

$i,k \neq 1$ and $l \neq l'$. Hence, $\lambda_{\mathrm{M}ijkl} = \left[Cov(Y_{ijkl}, Y_{1j1l'}) - \lambda_{\mathrm{S}ijkl} Cov(S_{1j1l}, S_{1j1l'})\right] \frac{1}{Cov(M_{1jkl}, S_{1j1l'})}$, for $Cov(M_{1jkl}, S_{1j1l'}) \neq 0$. The parameters $\lambda_{\mathrm{S}ijkl}$, $Cov(S_{1j1l}, S_{1j1l'})$, and $Cov(M_{1jkl}, S_{1j1l'})$ are identified according to Equations 150, 153, and 154.

*Equation 156: Identifiability of $Cov(S_{1j1l}, IS_{ij'k})$ (for $k \neq 1$ and $j \neq j'$)*

According to Equations 140 and 142,

$$Cov(Y_{1j1l}, Y_{ij'kl'}) = \lambda_{\mathrm{S}ij'kl'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\mathrm{M}ij'kl'} Cov(S_{1j1l}, M_{1j'kl'}) + \lambda_{\mathrm{IS}ij'kl'} Cov(S_{1j1l}, IS_{ij'k}) \quad \text{for}$$

$i,k \neq 1$ and $j \neq j'$. For $l' = 1$, $\lambda_{\mathrm{IS}ij'kl'}$ is set to unity (i.e., $\lambda_{\mathrm{IS}ij'k1} = 1$), so that we obtain: $Cov(S_{1j1l}, IS_{ij'k}) = Cov(Y_{1j1l}, Y_{ij'k1}) - \lambda_{\mathrm{S}ij'k1} Cov(S_{1j1l}, S_{1j'11}) - \lambda_{\mathrm{M}ij'k1} Cov(S_{1j1l}, M_{1j'k1})$. The parameters $\lambda_{\mathrm{S}ij'k1}$, $\lambda_{\mathrm{M}ij'k1}$, $Cov(S_{1j1l}, S_{1j'11})$, and $Cov(S_{1j1l}, M_{1j'k1})$ are identified according to Equations 150, 153, 154, and 157.

*Equation 158: Identifiability of $Var(M_{1jkl})$*

According to Equation 139, $Cov(Y_{1jkl}, Y_{ijkl}) = \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}ijkl} Var(S_{1j1l}) + \lambda_{\mathrm{M}ijkl} Var(M_{1jkl})$ for $i \neq 1$ and $k \neq 1$. Hence, $Var(M_{1jkl}) = \left[Cov(Y_{1jkl}, Y_{ijkl}) - \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}ijkl} Var(S_{1j1l})\right] \frac{1}{\lambda_{\mathrm{M}ijkl}}$, for $\lambda_{\mathrm{M}ijkl} \neq 0$. The parameters $\lambda_{\mathrm{S}1jkl}$, $\lambda_{\mathrm{S}ijkl}$, $Var(S_{1j1l})$, and $\lambda_{\mathrm{M}ijkl}$ are identified according to Equations 150, 152, and 157.

*Equation 159: Identifiability of $Cov(M_{1jkl}, M_{1jk'l})$ (for $k \neq k'$)*

According to Equation 139, $Cov(Y_{1jkl}, Y_{1jk'l}) = \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1jk'l} Var(S_{1j1l}) + Cov(M_{1jkl}, M_{1jk'l})$ for $k \neq k'$. Hence, $Cov(M_{1jkl}, M_{1jk'l}) = Cov(Y_{1jkl}, Y_{1jk'l}) - \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1jk'l} Var(S_{1j1l})$. The parameters $\lambda_{\mathrm{S}1jkl}$, $\lambda_{\mathrm{S}1jk'l}$, and $Var(S_{1j1l})$ are identified according to Equations 150 and 152.

*Equation 160: Identifiability of $Cov(M_{1jkl}, M_{1j'k'l'})$, [for $(j,l) \neq (j',l')$]*

According to Equations 140–142,

$$\begin{aligned} Cov(Y_{1jkl}, Y_{1j'k'l'}) = {} & \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{\mathrm{S}1jkl} Cov(S_{1j1l}, M_{1j'k'l'}) + \lambda_{\mathrm{S}1j'k'l'} Cov(M_{1jkl}, S_{1j'1l'}) \\ & + Cov(M_{1jkl}, M_{1j'k'l'}), \text{ for } (j,l) \neq (j',l'). \end{aligned}$$

Hence,

$$\begin{aligned} Cov(M_{1jkl}, M_{1j'k'l'}) = {} & Cov(Y_{1jkl}, Y_{1j'k'l'}) - \lambda_{\mathrm{S}1jkl} \lambda_{\mathrm{S}1j'k'l'} Cov(S_{1j1l}, S_{1j'1l'}) - \lambda_{\mathrm{S}1jkl} Cov(S_{1j1l}, M_{1j'k'l'}) \\ & - \lambda_{\mathrm{S}1j'k'l'} Cov(M_{1jkl}, S_{1j'1l'}), \text{ for } (j,l) \neq (j',l'). \end{aligned}$$

All parameters on the right hand side of Equation 160 are identified according to Equations 150, 153, and 154.

*Equation 170: Identifiability of $Cov(IS_{ij1}, IS_{i'j1})$ (for $i \neq i'$)*

According to Equation 139, $Cov(Y_{ij1l}, Y_{i'j1l}) = \lambda_{\mathrm{S}ij1l}\lambda_{\mathrm{S}i'j1l}Var(S_{1j1l}) + \lambda_{\mathrm{IS}ij1l}\lambda_{\mathrm{IS}i'j1l}Cov(IS_{ij1}, IS_{i'j1})$ for $i \neq i'$ and $i, i' \neq 1$. For $l$ = 1, $\lambda_{\mathrm{IS}ij1l}$ and $\lambda_{\mathrm{IS}i'j1l}$ are set to unity (i.e., $\lambda_{\mathrm{IS}ij11} = \lambda_{\mathrm{IS}i'j11} = 1$), so that we obtain: $Cov(IS_{ij1}, IS_{i'j1}) = Cov(Y_{ij11}, Y_{i'j11}) - \lambda_{\mathrm{S}ij11}\lambda_{\mathrm{S}i'j11}Var(S_{1j11})$. The parameters $\lambda_{\mathrm{S}ij11}$, $\lambda_{\mathrm{S}i'j11}$, and $Var(S_{1j11})$ are identified according to Equations 149 and 152.

*Equation 166: Identifiability of $\lambda_{\mathrm{IS}ij1l}$*

According to Equation 141,

$Cov(Y_{ij1l}, Y_{i'j1l'}) = \lambda_{\mathrm{S}ij1l}\lambda_{\mathrm{S}i'j1l'}Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\mathrm{IS}ij1l}\lambda_{\mathrm{IS}i'j1l'}Cov(IS_{ij1}, IS_{i'j1})$ for $i \neq i'$, $i, i' \neq 1$, and $l \neq l'$. For $l'$ = 1, $\lambda_{\mathrm{IS}i'j1l'}$ is set to unity (i.e., $\lambda_{\mathrm{IS}i'j11} = 1$), so that we obtain: $\lambda_{\mathrm{IS}ij1l} = [Cov(Y_{ij1l}, Y_{i'j11}) - \lambda_{\mathrm{S}ij1l}\lambda_{\mathrm{S}i'j11}Cov(S_{1j1l}, S_{1j11})]\dfrac{1}{Cov(IS_{ij1}, IS_{i'j1})}$, for $l \neq 1$ and $Cov(IS_{ij1}, IS_{i'j1}) \neq 0$. The parameters $\lambda_{\mathrm{S}ij1l}$, $\lambda_{\mathrm{S}i'j11}$, $Cov(S_{1j1l}, S_{1j11})$, and $Cov(IS_{ij1}, IS_{i'j1})$ are identified according to Equations 149, 153, and 170.

*Equation 174: Identifiability of $Cov(IS_{ijk}, IS_{i'jk})$ (for $i \neq i'$ and $k \neq 1$)*

According to Equation 139,

$$Cov(Y_{ijkl}, Y_{i'jkl}) = \lambda_{\mathrm{S}ijkl}\lambda_{\mathrm{S}i'jkl}Var(S_{1j1l}) + \lambda_{\mathrm{M}ijkl}\lambda_{\mathrm{M}i'jkl}Var(M_{1jkl}) + \lambda_{\mathrm{IS}ijkl}\lambda_{\mathrm{IS}i'jkl}Cov(IS_{ijk}, IS_{i'jk}),$$

for $i \neq i'$, $i, i' \neq 1$, and $k \neq 1$. For $l$ = 1, $\lambda_{\mathrm{IS}ijkl}$ and $\lambda_{\mathrm{IS}i'jkl}$ are set to unity (i.e., $\lambda_{\mathrm{IS}ijk1} = \lambda_{\mathrm{IS}i'jk1} = 1$), so that we obtain:

$$Cov(IS_{ijk}, IS_{i'jk}) = Cov(Y_{ijk1}, Y_{i'jk1}) - \lambda_{\mathrm{S}ijk1}\lambda_{\mathrm{S}i'jk1}Var(S_{1j11}) - \lambda_{\mathrm{M}ijk1}\lambda_{\mathrm{M}i'jk1}Var(M_{1jk1})$$

All parameters on the right hand side of Equation 174 are are identified according to Equations 150, 152, 157, and 158.

*Equation 167: Identifiability of $\lambda_{\mathrm{IS}ijkl}$ (for $k \neq 1$)*

According to Equation 141,

$$Cov(Y_{ijkl}, Y_{i'jkl'}) = \lambda_{\text{S}ijkl}\lambda_{\text{S}i'jkl'}Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}i'jkl'}Cov(S_{1j1l}, M_{1jkl'})$$
$$+ \lambda_{\text{M}ijkl}\lambda_{\text{S}i'jkl'}Cov(M_{1jkl}, S_{1j1l'}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}i'jkl'}Cov(M_{1jkl}, M_{1jkl'})$$
$$+ \lambda_{\text{IS}ijkl}\lambda_{\text{IS}i'jkl'}Cov(IS_{ijk}, IS_{i'jk}),$$

for $i \neq i'$, $i, i' \neq 1$, $k \neq 1$, and $l \neq l'$. For $l' = 1$, $\lambda_{\text{IS}i'jkl'}$ is set to unity (i.e., $\lambda_{\text{IS}i'jk1} = 1$), so that we obtain:

$$\lambda_{\text{IS}ijkl} = [Cov(Y_{ijkl}, Y_{i'jk1}) - \lambda_{\text{S}ijkl}\lambda_{\text{S}i'jk1}Cov(S_{1j1l}, S_{1j11}) - \lambda_{\text{S}ijkl}\lambda_{\text{M}i'jk1}Cov(S_{1j1l}, M_{1jk1})$$
$$- \lambda_{\text{M}ijkl}\lambda_{\text{S}i'jk1}Cov(M_{1jkl}, S_{1j11}) - \lambda_{\text{M}ijkl}\lambda_{\text{M}i'jk1}Cov(M_{1jkl}, M_{1jk1})]\frac{1}{Cov(IS_{ijk}, IS_{i'jk})},$$

for $l \neq 1$ and $Cov(IS_{ijk}, IS_{i'jk}) \neq 0$. All parameters on the right hand side of Equation 167 are are identified according to Equations 150, 153, 154, 157, 160, and 174.

*Equation 168: Identifiability of $Var(IS_{ij1})$*

According to Equation 141,

$Cov(Y_{ij1l}, Y_{ij1l'}) = \lambda_{\text{S}ij1l}\lambda_{\text{S}ij1l'}Cov(S_{ij1l}, S_{ij1l'}) + \lambda_{\text{IS}ij1l}\lambda_{\text{IS}ij1l'}Var(IS_{ij1})$, for $i \neq 1$ and $l \neq l'$. Hence

$Var(IS_{ij1}) = [Cov(Y_{ij1l}, Y_{ij1l'}) - \lambda_{\text{S}ij1l}\lambda_{\text{S}ij1l'}Cov(S_{ij1l}, S_{ij1l'})]\frac{1}{\lambda_{\text{IS}ij1l}\lambda_{\text{IS}ij1l'}}$, for $\lambda_{\text{IS}ij1l}\lambda_{\text{IS}ij1l'} \neq 0$. All parameters on the right hand side of Equation 168 are identified according to Equations 149, 153, and 166.

*Equation 169: Identifiability of $Var(IS_{ijk})$ (for $k \neq 1$)*

According to Equation 141,

$$Cov(Y_{ijkl}, Y_{ijkl'}) = \lambda_{\text{S}ijkl}\lambda_{\text{S}ijkl'}Cov(S_{ij1l}, S_{ij1l'}) + \lambda_{\text{S}ijkl}\lambda_{\text{M}ijkl'}Cov(S_{ij1l}, M_{1jkl'})$$
$$+ \lambda_{\text{M}ijkl}\lambda_{\text{S}ijkl'}Cov(M_{1jkl}, S_{ij1l'}) + \lambda_{\text{M}ijkl}\lambda_{\text{M}ijkl'}Cov(M_{1jkl}, M_{1jkl'})$$
$$+ \lambda_{\text{IS}ijkl}\lambda_{\text{IS}ijkl'}Var(IS_{ijk}),$$

for $i, k \neq 1$ and $l \neq l'$. Hence

$$Var(IS_{ijk}) = [Cov(Y_{ijkl}, Y_{ijkl'}) - \lambda_{\text{S}ijkl}\lambda_{\text{S}ijkl'}Cov(S_{ij1l}, S_{ij1l'})$$
$$- \lambda_{\text{S}ijkl}\lambda_{\text{M}ijkl'}Cov(S_{ij1l}, M_{1jkl'}) - \lambda_{\text{M}ijkl}\lambda_{\text{S}ijkl'}Cov(M_{1jkl}, S_{ij1l'})$$
$$- \lambda_{\text{M}ijkl}\lambda_{\text{M}ijkl'}Cov(M_{1jkl}, M_{1jkl'})]\frac{1}{\lambda_{\text{IS}ijkl}\lambda_{\text{IS}ijkl'}},$$

for $\lambda_{\text{IS}ijkl}\lambda_{\text{IS}ijkl'} \neq 0$. All parameters on the right hand side of Equation 169 are identified according to Equations 150, 153, 154, 157, 160, and 167.

*Equation 161: Identifiability of $Cov(M_{1jkl}, IS_{ij1})$*

According to Equation 139, $Cov(Y_{ij1l}, Y_{1jkl}) = \lambda_{Sij1l}\lambda_{S1jkl}Var(S_{1j1l}) + \lambda_{ISij1l}Cov(IS_{ij1}, M_{1jkl})$ for $i \neq 1$ and $k \neq 1$. Hence, $Cov(M_{1jkl}, IS_{ij1}) = [Cov(Y_{ij1l}, Y_{1jkl}) - \lambda_{Sij1l}\lambda_{S1jkl}Var(S_{1j1l})]\dfrac{1}{\lambda_{ISij1l}}$, for $\lambda_{ISij1l} \neq 0$. The parameters $\lambda_{Sij1l}$, $\lambda_{S1jkl}$, $Var(S_{1j1l})$, and $\lambda_{ISij1l}$ are identified according to Equations 149, 150, 152, and 166.

*Equation 162: Identifiability of $Cov(M_{1jkl}, IS_{ij'1})$ (for $j \neq j'$)*

According to Equation 140,

$$Cov(Y_{ij1l}, Y_{1j'kl}) = \lambda_{Sij1l}\lambda_{S1j'kl}Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{Sij1l}Cov(S_{1j1l}, M_{1j'kl}) + \lambda_{ISij1l}\lambda_{S1j'kl}Cov(IS_{ij1}, S_{1j'1l}) + \lambda_{ISij1l}Cov(IS_{ij1}, M_{1j'kl}),$$

for $i \neq 1$, $k \neq 1$, and $j \neq j'$. Hence,

$$Cov(IS_{ij1}, M_{1j'kl}) = [Cov(Y_{ij1l}, Y_{1j'kl}) - \lambda_{Sij1l}\lambda_{S1j'kl}Cov(S_{1j1l}, S_{1j'1l}) - \lambda_{Sij1l}Cov(S_{1j1l}, M_{1j'kl}) - \lambda_{ISij1l}\lambda_{S1j'kl}Cov(IS_{ij1}, S_{1j'1l})]\frac{1}{\lambda_{ISij1l}},$$

for $\lambda_{ISij1l} \neq 0$. This is equivalent to

$$Cov(M_{1jkl}, IS_{ij'1}) = [Cov(Y_{1jkl}, Y_{ij'1l}) - \lambda_{S1jkl}\lambda_{Sij'1l}Cov(S_{1j1l}, S_{1j'1l}) - \lambda_{Sij'1l}Cov(M_{1jkl}, S_{1j'1l}) - \lambda_{S1jkl}\lambda_{ISij'1l}Cov(S_{1j1l}, IS_{ij'1})]\frac{1}{\lambda_{ISij'1l}}.$$

All parameters on the right hand side of Equation 162 are identified according to Equations 149, 150, 153, 154, 155, and 166.

*Equation 163: Identifiability of $Cov(M_{1jk1}, IS_{ijk'})$ (for $k \neq k'$ and $k' \neq 1$)*

According to Equation 139,

$Cov(Y_{1jkl}, Y_{ijk'l}) = \lambda_{S1jkl}\lambda_{Sijk'l}Var(S_{1j1l}) + \lambda_{Mijk'l}Cov(M_{1jkl}, M_{1jk'l}) + \lambda_{ISij1l}Cov(M_{1jkl}, IS_{ijk'})$ for $i \neq 1$, $k \neq k'$, and $k, k' \neq 1$. For $l = 1$, $\lambda_{ISij1l}$ is set to unity (i.e., $\lambda_{ISij11} = 1$) so that we obtain:

$$Cov(M_{1jk1}, IS_{ijk'}) = Cov(Y_{1jk1}, Y_{ijk'1}) - \lambda_{S1jk1}\lambda_{Sijk'1}Var(S_{1j11}) - \lambda_{Mijk'1}Cov(M_{1jk1}, M_{1jk'1}).$$

All parameters on the right hand side of Equation 163 are identified according to Equations 150, 152, 157, and 159.

*Equation 164: Identifiability of $Cov(M_{1jkl}, IS_{ijk'})$ (for $k \neq k'$ and $k' \neq 1$)*

According to Equation 141,

$$Cov(Y_{1jkl}, Y_{ijk'l'}) = \lambda_{S1jkl}\lambda_{Sijk'l'}Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{S1jkl}\lambda_{Mijk'l'}Cov(S_{1j1l}, M_{1jk'l'}) + \lambda_{Sijk'l'}Cov(M_{1jkl}, S_{1j1l'})$$
$$+ \lambda_{Mijk'l'}Cov(M_{1jkl}, M_{1jk'l'}) + \lambda_{ISijk'l'}Cov(M_{1jkl}, IS_{ijk'}),$$

for $i \neq 1$, $k \neq k'$, $k, k' \neq 1$, and $l \neq l'$. For $l'$ = 1, $\lambda_{ISijk'l'}$ is set to unity (i.e., $\lambda_{ISijk'1} = 1$), so that we obtain:

$$Cov(M_{1jkl}, IS_{ijk'}) = Cov(Y_{1jkl}, Y_{ijk'1}) - \lambda_{S1jkl}\lambda_{Sijk'1}Cov(S_{1j1l}, S_{1j11}) - \lambda_{S1jkl}\lambda_{Mijk'1}Cov(S_{1j1l}, M_{1jk'1})$$
$$- \lambda_{Sijk'1}Cov(M_{1jkl}, S_{1j11}) - \lambda_{Mijk'1}Cov(M_{1jkl}, M_{1jk'1}).$$

All parameters on the right hand side of Equation 164 are identified according to Equations 150, 153, 154, 157, and 160.

*Equation 165: Identifiability of $Cov(M_{1jkl}, IS_{ij'k'})$ (for $j \neq j'$, $k \neq k'$, and $k' \neq 1$)*

According to Equation 140,

$$Cov(Y_{1jkl}, Y_{ij'k'l}) = \lambda_{S1jkl}\lambda_{Sij'k'l}Cov(S_{1j1l}, S_{1j'1l}) + \lambda_{S1jkl}\lambda_{Mij'k'l}Cov(S_{1j1l}, M_{1j'k'l})$$
$$+ \lambda_{S1jkl}\lambda_{ISij'k'l}Cov(S_{1j1l}, IS_{ij'k'}) + \lambda_{Sij'k'l}Cov(M_{1jkl}, S_{1j'1l})$$
$$+ \lambda_{Mij'k'l}Cov(M_{1jkl}, M_{1j'k'l}) + \lambda_{ISij'k'l}Cov(M_{1jkl}, IS_{ij'k'}),$$

for $i \neq 1$, $j \neq j'$, $k \neq k'$, and $k, k' \neq 1$. Hence,

$$Cov(M_{1jkl}, IS_{ij'k'}) = [Cov(Y_{1jkl}, Y_{ij'k'l}) - \lambda_{S1jkl}\lambda_{Sij'k'l}Cov(S_{1j1l}, S_{1j'1l})$$
$$- \lambda_{S1jkl}\lambda_{Mij'k'l}Cov(S_{1j1l}, M_{1j'k'l}) - \lambda_{S1jkl}\lambda_{ISij'k'l}Cov(S_{1j1l}, IS_{ij'k'})$$
$$- \lambda_{Sij'k'l}Cov(M_{1jkl}, S_{1j'1l}) - \lambda_{Mij'k'l}Cov(M_{1jkl}, M_{1j'k'l})]\frac{1}{\lambda_{ISij'k'l}},$$

for $\lambda_{ISij'k'l} \neq 0$. All parameters on the right hand side of Equation 165 are identified according to Equations 150, 153, 154, 156, 157, 160, and 167.

*Equation 171: Identifiability of $Cov(IS_{ij1}, IS_{i'j'1})$ (for $j \neq j'$)*

According to Equation 142,

$$Cov(Y_{ij1l}, Y_{i'j'1l'}) = \lambda_{Sij1l}\lambda_{Si'j'1l'}Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{Sij1l}\lambda_{ISi'j'1l'}Cov(S_{1j1l}, IS_{i'j'1})$$
$$+ \lambda_{ISij1l}\lambda_{Si'j'1l'}Cov(IS_{ij1}, S_{1j'1l'}) + \lambda_{ISij1l}\lambda_{ISi'j'1l'}Cov(IS_{ij1}, IS_{i'j'1}),$$

for $i, i' \neq 1$, $j \neq j'$, and $l \neq l'$. Hence,

$$Cov(IS_{ij1}, IS_{i'j'1}) = [Cov(Y_{ij1l}, Y_{i'j'1l'}) - \lambda_{Sij1l}\lambda_{Si'j'1l'}Cov(S_{1j1l}, S_{1j'1l'})$$
$$- \lambda_{Sij1l}\lambda_{ISi'j'1l'}Cov(S_{1j1l}, IS_{i'j'1}) - \lambda_{ISij1l}\lambda_{Si'j'1l'}Cov(IS_{ij1}, S_{1j'1l'})]\frac{1}{\lambda_{ISij1l}\lambda_{ISi'j'1l'}},$$

for $\lambda_{ISij1l}\lambda_{ISi'j'1l'} \neq 0$. All parameters on the right hand side of Equation 171 are identified according to Equations 149, 153, 155, and 166.

*Equation 172: Identifiability of $Cov(IS_{ij1}, IS_{i'jk})$ (for $k \neq 1$)*

According to Equation 141,

$$Cov(Y_{ij1l}, Y_{i'jkl'}) = \lambda_{Sij1l}\lambda_{Si'jkl'}Cov(S_{1j1l}, S_{1j1l'}) + \lambda_{Sij1l}\lambda_{Mi'jkl'}Cov(S_{1j1l}, M_{1jkl'}) + \lambda_{ISij1l}\lambda_{Mi'jkl'}Cov(IS_{ij1}, M_{1jkl'}) + \lambda_{ISij1l}\lambda_{ISi'jkl'}Cov(IS_{ij1}, IS_{i'jk}),$$

for $i, i' \neq 1$, $k \neq 1$, and $l \neq l'$. Hence,

$$Cov(IS_{ij1}, IS_{i'jk}) = [Cov(Y_{ij1l}, Y_{i'jkl'}) - \lambda_{Sij1l}\lambda_{Si'jkl'}Cov(S_{1j1l}, S_{1j1l'}) - \lambda_{Sij1l}\lambda_{Mi'jkl'}Cov(S_{1j1l}, M_{1jkl'}) - \lambda_{ISij1l}\lambda_{Mi'jkl'}Cov(IS_{ij1}, M_{1jkl'})]\frac{1}{\lambda_{ISij1l}\lambda_{ISi'jkl'}},$$

for $\lambda_{ISij1l}\lambda_{ISi'jkl'} \neq 0$. All parameters on the right hand side of Equation 172 are identified according to Equations 149, 150, 153, 154, 157, 161, 166, and 167.

*Equation 173: Identifiability of $Cov(IS_{ij1}, IS_{i'j'k})$ (for $j \neq j'$ and $k \neq 1$)*

According to Equation 142,

$$Cov(Y_{ij1l}, Y_{i'j'kl'}) = \lambda_{Sij1l}\lambda_{Si'j'kl'}Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{Sij1l}\lambda_{Mi'j'kl'}Cov(S_{1j1l}, M_{1j'kl'}) + \lambda_{Sij1l}\lambda_{ISi'j'kl'}Cov(S_{1j1l}, IS_{i'j'k}) + \lambda_{ISij1l}\lambda_{Si'j'kl'}Cov(IS_{ij1}, S_{1j'1l'}) + \lambda_{ISij1l}\lambda_{Mi'j'kl'}Cov(IS_{ij1}, M_{1j'kl'}) + \lambda_{ISij1l}\lambda_{ISi'j'kl'}Cov(IS_{ij1}, IS_{i'j'k}),$$

for $i, i' \neq 1$, $k \neq 1$, $j \neq j'$, and $l \neq l'$. Hence,

$$Cov(IS_{ij1}, IS_{i'j'k}) = [Cov(Y_{ij1l}, Y_{i'j'kl'}) - \lambda_{Sij1l}\lambda_{Si'j'kl'}Cov(S_{1j1l}, S_{1j'1l'}) - \lambda_{Sij1l}\lambda_{Mi'j'kl'}Cov(S_{1j1l}, M_{1j'kl'}) - \lambda_{Sij1l}\lambda_{ISi'j'kl'}Cov(S_{1j1l}, IS_{i'j'k}) - \lambda_{ISij1l}\lambda_{Si'j'kl'}Cov(IS_{ij1}, S_{1j'1l'}) - \lambda_{ISij1l}\lambda_{Mi'j'kl'}Cov(IS_{ij1}, M_{1j'kl'})]\frac{1}{\lambda_{ISij1l}\lambda_{ISi'j'kl'}},$$

for $\lambda_{ISij1l}\lambda_{ISi'j'kl'} \neq 0$. All parameters on the right hand side of Equation 173 are identified according to Equations 149, 150, 153, 154, 155, 156, 157, 162, 166, and 167.

*Equation 175: Identifiability of $Cov(IS_{ijk}, IS_{i'jk'})$ (for $k \neq k'$ and $k, k' \neq 1$)*

According to Equation 139,

$$Cov(Y_{ijkl}, Y_{i'jk'l}) = \lambda_{Sijkl}\lambda_{Si'jk'l}Var(S_{1j1l}) + \lambda_{Mijkl}\lambda_{Mi'jk'l}Cov(M_{1jkl}, M_{1jk'l}) + \lambda_{Mijkl}\lambda_{ISi'jk'l}Cov(M_{1jkl}, IS_{i'jk'}) + \lambda_{ISijkl}\lambda_{Mi'jk'l}Cov(IS_{ijk}, M_{1jk'l}) + \lambda_{ISijkl}\lambda_{ISi'jk'l}Cov(IS_{ijk}, IS_{i'jk'}),$$

for $i, i' \neq 1$, $k \neq k'$, and $k, k' \neq 1$. Hence,

$$Cov(IS_{ijk}, IS_{i'jk'}) = [Cov(Y_{ijkl}, Y_{i'jk'l}) - \lambda_{Sijkl}\lambda_{Si'jk'l}Var(S_{1j1l}) - \lambda_{Mijkl}\lambda_{Mi'jk'l}Cov(M_{1jkl}, M_{1jk'l}) - \lambda_{Mijkl}\lambda_{ISi'jk'l}Cov(M_{1jkl}, IS_{i'jk'}) - \lambda_{ISijkl}\lambda_{Mi'jk'l}Cov(IS_{ijk}, M_{1jk'l})]\frac{1}{\lambda_{ISijkl}\lambda_{ISi'jk'l}},$$

for $\lambda_{ISijkl}\lambda_{ISi'jk'l} \neq 0$. All parameters on the right hand side of Equation 175 are identified according to Equations 150, 152, 157, 159, 164, and 167.

*Equation 176: Identifiability of $Cov(IS_{ijk}, IS_{i'j'k'})$ (for $j \neq j'$)*

According to Equations 140 and 142,

$$Cov(Y_{ijkl}, Y_{i'j'k'l'}) = \lambda_{Sijkl}\lambda_{Si'j'k'l'}Cov(S_{1j1l}, S_{1j'1l'}) + \lambda_{Sijkl}\lambda_{Mi'j'k'l'}Cov(S_{1j1l}, M_{1j'k'l'}) + \lambda_{Sijkl}\lambda_{ISi'j'k'l'}Cov(S_{1j1l}, IS_{i'j'k'}) + \lambda_{Mijkl}\lambda_{Si'j'k'l'}Cov(M_{1jkl}, S_{1j'1l'}) + \lambda_{Mijkl}\lambda_{Mi'j'k'l'}Cov(M_{1jkl}, M_{1j'k'l'}) + \lambda_{Mijkl}\lambda_{ISi'j'k'l'}Cov(M_{1jkl}, IS_{i'j'k'}) + \lambda_{ISijkl}\lambda_{Si'j'k'l'}Cov(IS_{ijk}, S_{1j'1l'}) + \lambda_{ISijkl}\lambda_{Mi'j'k'l'}Cov(IS_{ijk}, M_{1j'k'l'}) + \lambda_{ISijkl}\lambda_{ISi'j'k'l'}Cov(IS_{ijk}, IS_{i'j'k'}),$$

for $i, i' \neq 1$, $j \neq j'$, and $k, k' \neq 1$. Hence,

$$Cov(IS_{ijk}, IS_{i'j'k'}) = [Cov(Y_{ijkl}, Y_{i'j'k'l'}) - \lambda_{Sijkl}\lambda_{Si'j'k'l'}Cov(S_{1j1l}, S_{1j'1l'}) - \lambda_{Sijkl}\lambda_{Mi'j'k'l'}Cov(S_{1j1l}, M_{1j'k'l'}) - \lambda_{Sijkl}\lambda_{ISi'j'k'l'}Cov(S_{1j1l}, IS_{i'j'k'}) - \lambda_{Mijkl}\lambda_{Si'j'k'l'}Cov(M_{1jkl}, S_{1j'1l'}) - \lambda_{Mijkl}\lambda_{Mi'j'k'l'}Cov(M_{1jkl}, M_{1j'k'l'}) - \lambda_{Mijkl}\lambda_{ISi'j'k'l'}Cov(M_{1jkl}, IS_{i'j'k'}) - \lambda_{ISijkl}\lambda_{Si'j'k'l'}Cov(IS_{ijk}, S_{1j'1l'}) - \lambda_{ISijkl}\lambda_{Mi'j'k'l'}Cov(IS_{ijk}, M_{1j'k'l'})]\frac{1}{\lambda_{ISijkl}\lambda_{ISi'j'k'l'}},$$

for $\lambda_{ISijkl}\lambda_{ISi'j'k'l'} \neq 0$. All parameters on the right hand side of Equation 176 are identified according to Equations 150, 153, 154, 156, 157, 160, 165, and 167.

*Equation 177: Identifiability of $Var(E_{1j1l})$*

According to Equation 139, $Cov(Y_{1j1l}, Y_{1j1l}) = Var(Y_{1j1l}) = Var(S_{1j1l}) + Var(E_{1j1l})$. Hence, $Var(E_{1j1l}) = Var(Y_{1j1l}) - Var(S_{1j1l})$. The parameter $Var(S_{1j1l})$ is identified according to Equation 152.

*Equation 178: Identifiability of $Var(E_{ij1l})$ (for $i \neq 1$)*

According to Equation 139,

$Cov(Y_{ij1l}, Y_{ij1l}) = Var(Y_{ij1l}) = \lambda^2_{Sij1l}Var(S_{1j1l}) + \lambda^2_{ISij1l}Var(IS_{ij1}) + Var(E_{ij1l})$, for $i \neq 1$. Hence, $Var(E_{ij1l}) = Var(Y_{ij1l}) - \lambda^2_{Sij1l}Var(S_{1j1l}) - \lambda^2_{ISij1l}Var(IS_{ij1})$. All parameters on the right hand side of

Equation 178 are identified according to Equations 149, 152, 166, and 168.

*Equation 179: Identifiability of $Var(E_{1jkl})$ (for $k \neq 1$)*

According to Equation 139,

$Cov(Y_{1jkl}, Y_{1jkl}) = Var(Y_{1jkl}) = \lambda^2_{S1jkl} Var(S_{1j1l}) + Var(M_{1jkl}) + Var(E_{1jkl})$, for $k \neq 1$. Hence, $Var(E_{1jkl}) = Var(Y_{1jkl}) - \lambda^2_{S1jkl} Var(S_{1j1l}) - Var(M_{1jkl})$. All parameters on the right hand side of Equation 179 are identified according to Equations 150, 152, and 158.

*Equation 180: Identifiability of $Var(E_{ijkl})$ (for $i, k \neq 1$)*

According to Equation 139,

$Cov(Y_{ijkl}, Y_{ijkl}) = Var(Y_{ijkl}) = \lambda^2_{Sijkl} Var(S_{1j1l}) + \lambda^2_{Mijkl} Var(M_{1jkl}) + \lambda^2_{ISijkl} Var(IS_{ijk}) + Var(E_{ijkl})$, for $i, k \neq 1$. Hence, $Var(E_{ijkl}) = Var(Y_{ijkl}) - \lambda^2_{Sijkl} Var(S_{1j1l}) - \lambda^2_{Mijkl} Var(M_{1jkl}) - \lambda^2_{ISijkl} Var(IS_{ijk})$. All parameters on the right hand side of Equation 180 are identified according to Equations 150, 152, 157, 158, 167 and 169.

The minimal condition for identification of the CS-C($M$–1) model with general state factors and indicator-specific factors across time is again a 2x1x2x2 MTMM-MO design (one construct [$n$ = 1] measured by two methods [$o$ = 2] on two measurement occasions [$p$ = 2], with two indicators per method [$m$ = 2]). This design is sufficient for obtaining an identified model, given substantial parameter values. In particular, under this minimal condition, both the method factors and the indicator-specific factors are measured by only two indicators, respectively. Therefore, each method factor and each indicator-specific factor must have a substantial (non-zero) covariance with at least one other latent variable (or an external covariate) in the model. If a method factor or an indicator-specific factor in this 2x1x2x2 MTMM-MO design does *not* covary with another variable in the model, the factor loadings of both indicators must be fixed to a non-zero value to identify the model. Furthermore, identification problems can occur in this "minimal model" if the indicator-specific effects are not substantial. In this case, one possible solution is to drop the indicator-specific factors.

## 3.6 Summary and Discussion

The CS-C($M$–1) model represents a general longitudinal MTMM measurement model for multiple, potentially heterogeneous indicators per CMOU. Depending on the type of indicator-specificity present in the data (generalization of indicator-specific effects across methods vs. across time points), users can choose between a model version with indicator-specific state variables and a version with general state factors in conjunction with indicator-specific factors across time. If there are no indicator-specific effects at all, a parsimonious version with general state factors without additional indicator-specific factors can be used (see Figure 15).

Either variant of the CS-C($M$–1) model enables researchers to investigate the convergent and discriminant validity of their measures in a longitudinal context. By comparing the consistency and method-specificity coefficients over time, researchers can check whether the convergent validity of different methods changes in the course of a longitudinal investigation. Furthermore, (changes in) the discriminant validity, associations among different constructs and methods, as well as the stability of inter-individual differences with regard to constructs and methods can be studied through latent correlations. By including a mean structure in the analysis, also hypotheses with regard to mean differences between different methods and across time can be tested.

An important advantage of the CS-C($M$–1) approach is that the entire information of the MTMM-MO covariance matrix and mean vector of the observed variables are analyzed in a single model. Hence, researchers do not need to (a) analyze separate models for each wave and (b) do not loose important information through a fragmentation of the data. Furthermore, as in MTMM models for cross-sectional data, measurement error influences are taken into account. Due to the specification of indicator-specific state factors (or indicator-specific factors across time), indicators do not necessarily have to be perfectly homogeneous, which is also an advantage given that perfectly unidimensional measures of a construct are rarely available in psychology. Note, however, that I nonetheless strongly recommend that indicators be selected that are as homogeneous as possible. I return the problem of indicator-specific effects in the final discussion.

An additional advantage of the CS-C($M$–1) model is that important assumptions can be scrutinized—particularly the assumption of construct-specific method effects and questions of measurement invariance over time. A number of more specific longitudinal MTMM models can be derived from the CS-C($M$–1) model. For example, the multi-method latent difference models that I will present in the next chapter are directly obtained from the basic CS-C($M$–1) model by simple reformulations. Furthermore, the CS-C($M$–1) model can easily be extended to a latent autoregressive model (e.g., Jöreskog, 1979a, 1979b; Hertzog & Nesselroade, 1987) by specifying an autoregressive structure among the latent state and/or latent method factors. The model can also be extended to analyze latent growth curves by imposing a second-order growth structure on the latent state factors (the principle of extending CS models to second-order growth models is described in Ferrer, Balluerka, & Widaman, 2008; Hancock, Kuo, & Lawrence, 2001; McArdle, 1988; as well as Sayer & Cumsille, 2001). In the next chapter, I show how researchers can study latent change in MTMM-MO studies by including latent difference variables in the CS-C($M$–1) model.

# 4 The CS-C(*M*–1) Change Model

Steyer et al. (1997, 2000; see also Steyer, 1988, 2005) have demonstrated how the conventional (mono-method) CS model can be reformulated as a latent difference model (see Section 2.2.2). Here, I show how the same principle can be applied to the CS-C(*M*–1) model with indicator-specific state variables defined in Chapter 3.1. [The CS-C(*M*–1) model with indicator-specific factors across time introduced in Section 3.2 can also be formulated as a latent difference model. As the principle is exactly the same and can easily be transferred, I describe this possibility only for the CS-C(*M*–1) model defined in Chapter 3.1.]

The change version of the CS-C(*M*–1) model [henceforth referred to as CS-C(*M*–1) change model] can be used to study inter-individual differences in intra-individual change simultaneously for different methods. It is a particularly useful model for intervention and evaluation studies that employ a multi-method design (e.g., multiple informants rating behavior problems in children before and after an intervention as in Barquero, Scheithauer, Bondü, & Mayer, 2007).

## 4.1 Introducing Latent Difference Variables in the CS-C(*M*–1) Model

First, I consider the measurement equations for the indicators pertaining to the reference method, using two indicators $Y_{ij1l}$ and $Y_{ij1l'}$, measured on two measurement occasions $l$ and $l'$, where $l' > l$:

$$Y_{ij1l} = S_{ij1l} + E_{ij1l}, \tag{181}$$

$$Y_{ij1l'} = S_{ij1l'} + E_{ij1l'}. \tag{182}$$

To include the latent difference between state $l'$ and state $l$ as a latent variable, Equation 182 can be rewritten as follows without making any restrictive assumptions:

$$Y_{ij1l'} = S_{ij1l} + (S_{ij1l'} - S_{ij1l}) + E_{ij1l'}. \tag{183}$$

Hence, $S_{ij1l'}$ is decomposed into the preceding latent state $S_{ij1l}$ plus the latent difference $(S_{ij1l'} - S_{ij1l})$, representing change from the preceding state $S_{ij1l}$ to state $S_{ij1l'}$:

$$S_{ij1l'} = S_{ij1l} + (S_{ij1l'} - S_{ij1l}). \tag{184}$$

The principle is exactly the same as for the mono-method CS model discussed in Chapter 2.2.2. To illustrate, let us assume that $l$ = 1 (the first measurement occasion) and $l' = 2$ (the second measurement occasion). Then, $S_{ij11}$ represents the initial status (the latent state at T1) as measured by the reference method, and the latent difference variable $(S_{ij12} - S_{ij11})$ represents change (growth or decline) from T1 to T2 as measured by the reference method.

A value of zero on $(S_{ij12} - S_{ij11})$ would indicate that an individual's latent score has *not* changed over time (according to the reference method). A positive value of $(S_{ij12} - S_{ij11})$ would indicate growth (a larger latent state score at T2 than at T1), whereas a negative value of $(S_{ij12} - S_{ij11})$ indicates a decline (a smaller latent state score at T2 than at T1). If there is no change for *any* individual [i.e., *all* individuals have values of zero on $(S_{ij12} - S_{ij11})$], or all individuals change by the same amount [i.e., *all* individuals have identical values on $(S_{ij12} - S_{ij11})$], $S_{ij11}$ and $S_{ij12}$ would be perfectly correlated and hence it would be sufficient to specify a model with a single (trait) factor (e.g., $S_{ij11}$). This shows that the latent difference model is a model for measuring *differential change*. The latent difference variables will have non-zero variances only if some individuals change more (or less) than do others.

Next, I consider the measurement equations for two indicators $Y_{ijkl}$ and $Y_{ijkl'}$ pertaining to a non-reference method *k*, $k \neq 1$, measured on two measurement occasions $l$ and $l'$, $l' > l$:

$$Y_{ijkl} = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij1l} + \lambda_{\mathrm{M}ijkl} M_{jkl} + E_{ijkl}, \tag{185}$$

$$Y_{ijkl'} = \alpha_{ijkl'} + \lambda_{\mathrm{S}ijkl'} S_{ij1l'} + \lambda_{\mathrm{M}ijkl'} M_{jkl'} + E_{ijkl'}, \text{ where } k \neq 1. \tag{186}$$

Given that we can replace $S_{ij1l'}$ by $\left[S_{ij1l} + (S_{ij1l'} - S_{ij1l})\right]$ and $M_{jkl'}$ by $\left[M_{jkl} + (M_{jkl'} - M_{jkl})\right]$ without making any restrictive assumptions, Equation 186 can be restated as:

$$\begin{aligned} Y_{ijkl'} &= \alpha_{ijkl'} + \lambda_{\mathrm{S}ijkl'} \left[S_{ij1l} + (S_{ij1l'} - S_{ij1l})\right] + \lambda_{\mathrm{M}ijkl'} \left[M_{jkl} + (M_{jkl'} - M_{jkl})\right] + E_{ijkl'} \\ &= \alpha_{ijkl'} + \lambda_{\mathrm{S}ijkl'} S_{ij1l} + \lambda_{\mathrm{S}ijkl'} (S_{ij1l'} - S_{ij1l}) + \lambda_{\mathrm{M}ijkl'} M_{jkl} + \lambda_{\mathrm{M}ijkl'} (M_{jkl'} - M_{jkl}) + E_{ijkl'}. \end{aligned} \tag{187}$$

In the following, I will refer to the variables $(S_{ij1l'} - S_{ij1l})$ as *latent state difference variables* and to the variables $(M_{jkl'} - M_{jkl})$ as *latent method difference variables*. The latent state difference variables represent inter-individual differences in intra-individual change with respect to the reference method. The latent method difference variables mirror inter-individual differences in intra-individual change with respect to method-specific deviations from the reference method. That is, the latent method difference variables represent residual change in the non-reference methods not accounted for by change in the reference method (the over- or underestimation by non-reference methods with respect to the reference method). The latent method difference variables can for example be used to study the question of why different methods diverge in the assessment of change. One can introduce potential explanatory variables in the model that might explain the deviation of the change scores from the reference method. Figure 16 shows the state and change versions of a CS-C($M$–1) model for one construct measured by two methods on two measurement occasions.

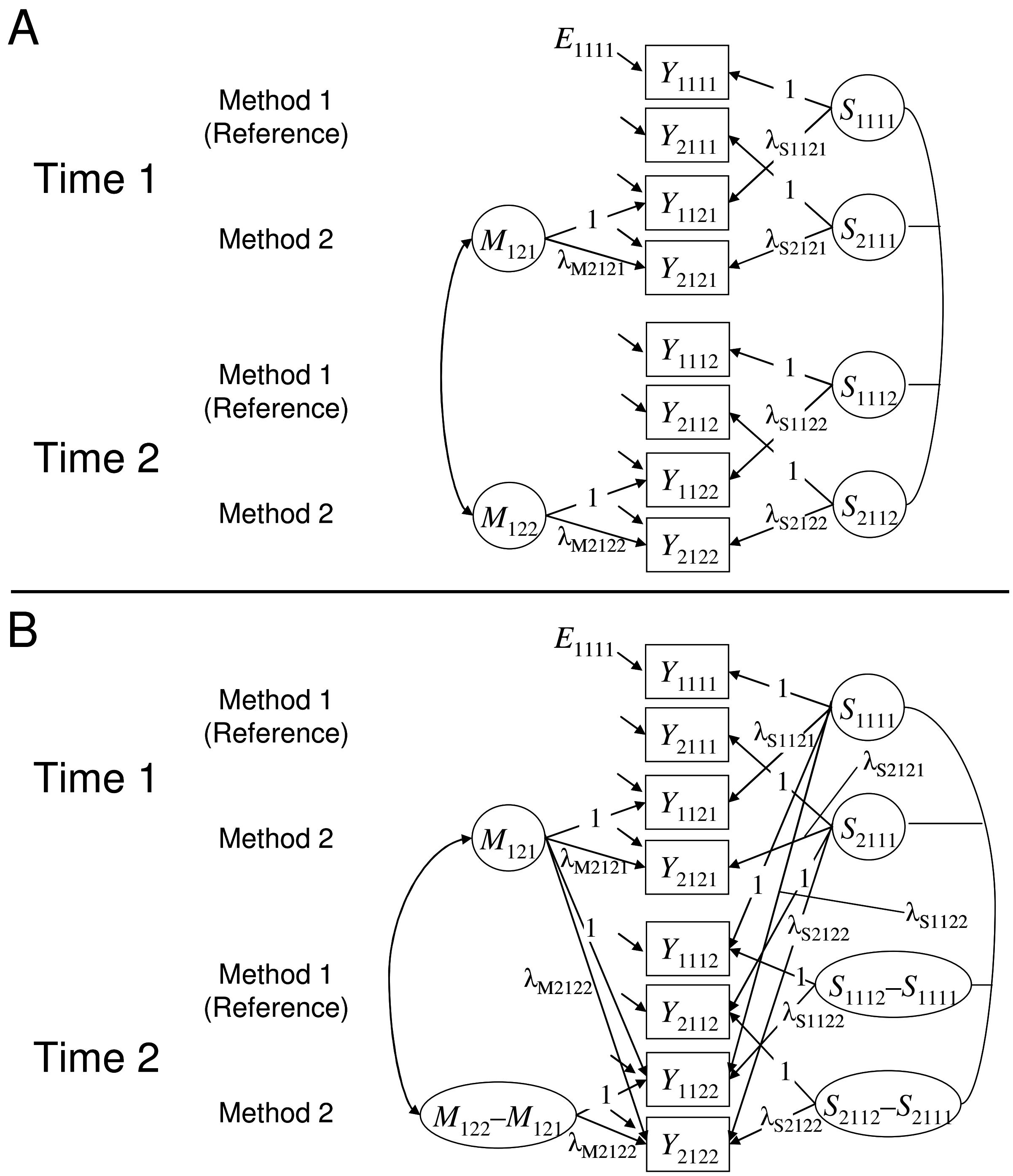


*Figure 16.* Path diagram of a CS-C(*M*–1) model for one construct measured by two methods on two measurement occasions. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{ij1l}$ = latent state factor. $S_{ij1l'} - S_{ij1l}$ = latent state difference factor. $M_{jkl}$ = common method factor. $M_{jkl'} - M_{jkl}$ = latent method difference factor. $E_{ijkl}$ = error variable. A: state version. B: latent difference version. For the sake of clarity, not all possible factor correlations are shown.

## 4.2 Baseline Versus Neighbor Change Version

The principle of introducing latent difference variables can be applied in different ways. Steyer et al. (1997, 2000) presented a *baseline change version* and a *neighbor change version*. In the baseline change version, the latent difference variables represent change between a given measurement occasion $l$ and the first measurement occasion ($l = 1$). Hence, T1 serves as the "baseline" against which change is assessed. In the neighbor change version, the latent difference variables represent true change between adjacent ("neighbored") occasions of measurement. The same distinction can be made for the CS-C($M$–1) change model. I will first present the baseline change version of the CS-C($M$–1) change model.

### *4.2.1 Baseline Change Version*

The general measurement equations for the CS-C($M$–1) baseline change model can be written as follows:

(a) for the reference method ($k = 1$):

$$Y_{ij1l} = S_{ij11} + (S_{ij1l} - S_{ij11}) + E_{ij1l}, \text{ and} \tag{188}$$

(b) for the non-reference methods ($k \neq 1$):

$$Y_{ijkl} = \alpha_{ijkl} + \lambda_{\text{S}ijkl} S_{ij11} + \lambda_{\text{S}ijkl} (S_{ij1l} - S_{ij11}) + \lambda_{\text{M}ijkl} M_{jk1} + \lambda_{\text{M}ijkl} (M_{jkl} - M_{jk1}) + E_{ijkl}. \tag{189}$$

### *4.2.2 Neighbor Change Version*

As mentioned above, in the neighbor change version, the change variables represent change between adjacent measurement occasions. For $l = 3, 4, ..., p$, the general measurement equations for the neighbor change version can be expressed as[5]:

(a) for the reference method ($k = 1$):

$$Y_{ij1l} = S_{ij11} + (S_{ij1l} - S_{ij1(l-1)}) + (S_{ij1(l-1)} - S_{ij1(l-2)}) + ... + (S_{ij12} - S_{ij11}) + E_{ij1l}, \text{ and} \tag{190}$$

(b) for the non-reference methods ($k \neq 1$):

[5] The neighbor change version can also be used if there are fewer than three time points. However, for only two time points, the neighbor change model is identical to the baseline change model.

$$\begin{aligned} Y_{ijkl} &= \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij11} + \lambda_{\mathrm{S}ijkl}(S_{ij1l} - S_{ij1(l-1)}) + \lambda_{\mathrm{S}ijkl}(S_{ij1(l-1)} - S_{ij1(l-2)}) \\ &+ \ldots + \lambda_{\mathrm{S}ijkl}(S_{ij12} - S_{ij11}) \\ &+ \lambda_{\mathrm{M}ijkl} M_{jk1} + \lambda_{\mathrm{M}ijkl}(M_{jkl} - M_{jk(l-1)}) + \lambda_{\mathrm{M}ijkl}(M_{jk(l-1)} - M_{jk(l-2)}) \\ &+ \ldots + \lambda_{\mathrm{M}ijkl}(M_{jk2} - M_{jk1}) + E_{ij1l}. \end{aligned} \tag{191}$$

To further illustrate the difference between the baseline and neighbor change versions imagine that there are three occasions of measurement, $l$ = 1, 2, 3. In this case, the measurement equations for the baseline version of the model are given by:

(a) for the reference method ($k = 1$):

$$Y_{ij11} = S_{ij11} + E_{ij11}, \tag{192}$$

$$Y_{ij12} = S_{ij11} + (S_{ij12} - S_{ij11}) + E_{ij12}, \text{ and} \tag{193}$$

$$Y_{ij13} = S_{ij11} + (S_{ij13} - S_{ij11}) + E_{ij13}. \tag{194}$$

(b) for the non-reference methods ($k \neq 1$):

$$Y_{ijk1} = \alpha_{ijk1} + \lambda_{\mathrm{S}ijk1} S_{ij11} + \lambda_{\mathrm{M}ijk1} M_{jk1} + E_{ijk1}, \tag{195}$$

$$Y_{ijk2} = \alpha_{ijk2} + \lambda_{\mathrm{S}ijk2} S_{ij12} + \lambda_{\mathrm{S}ijk2}(S_{ij12} - S_{ij11}) + \lambda_{\mathrm{M}ijk2} M_{jk2} + \lambda_{\mathrm{M}ijk2}(M_{jk2} - M_{jk1}) + E_{ijk2}, \tag{196}$$

$$Y_{ijk3} = \alpha_{ijk3} + \lambda_{\mathrm{S}ijk3} S_{ij13} + \lambda_{\mathrm{S}ijk3}(S_{ij13} - S_{ij11}) + \lambda_{\mathrm{M}ijk3} M_{jk3} + \lambda_{\mathrm{M}ijk3}(M_{jk3} - M_{jk1}) + E_{ijk3}. \tag{197}$$

For the neighbor change version, we obtain in this case:

(a) for the reference method ($k = 1$):

$$Y_{ij11} = S_{ij11} + E_{ij11}, \tag{198}$$

$$Y_{ij12} = S_{ij11} + (S_{ij12} - S_{ij11}) + E_{ij12}, \text{ and} \tag{199}$$

$$Y_{ij13} = S_{ij11} + (S_{ij13} - S_{ij12}) + (S_{ij12} - S_{ij11}) + E_{ij13}. \tag{200}$$

(b) for the non-reference methods ($k \neq 1$):

$$Y_{ijk1} = \alpha_{ijk1} + \lambda_{\mathrm{S}ijk1} S_{ij11} + \lambda_{\mathrm{M}ijk1} M_{jk1} + E_{ijk1}, \tag{201}$$

$$Y_{ijk2} = \alpha_{ijk2} + \lambda_{\text{S}ijk2} S_{ij12} + \lambda_{\text{S}ijk2} (S_{ij12} - S_{ij11}) + \lambda_{\text{M}ijk2} M_{jk2} + \lambda_{\text{M}ijk2} (M_{jk2} - M_{jk1}) + E_{ijk2}, \tag{202}$$

$$\begin{aligned} Y_{ijk3} = \alpha_{ijk3} &+ \lambda_{\text{S}ijk3} S_{ij11} + \lambda_{\text{S}ijk3} (S_{ij13} - S_{ij12}) + \lambda_{\text{S}ijk3} (S_{ij12} - S_{ij11}) \\ &+ \lambda_{\text{M}ijk3} M_{jk1} + \lambda_{\text{M}ijk3} (M_{jk3} - M_{jk2}) + \lambda_{\text{M}ijk3} (M_{jk2} - M_{jk1}) + E_{ij1l}. \end{aligned} \tag{203}$$

Figure 17 shows the baseline and neighbor change versions for one construct, two methods, and three time points as path diagrams.

## 4.3 Measurement Invariance

In Chapters 2.2.2 and 3.4, I already pointed out that measurement invariance over time is a crucial issue in longitudinal modeling. For the models of latent change presented here, the question of measurement invariance is of particular importance, given that we are studying the differences in latent variable scores. What do the latent difference scores mean if we are not measuring the same construct on each measurement occasion? We need to assure that we are not "subtracting apples from oranges". To be more concrete, we can only meaningfully interpret the latent difference variables if the factor loadings and measurement intercepts are time-invariant. Time-invariant intercepts and loadings imply that the measurement structure of the construct has not changed over time (so-called stationarity condition; Tisak & Tisak, 2000). As for the CS model, the stationarity condition can be tested. When invariance constraints are imposed on all intercepts and loadings, the general measurement model of the CS-C(*M*–1) model (cp. Equation 77) simplifies to

$$Y_{ijkl} = \begin{cases} S_{ij1l} + E_{ij1l}, & \text{for } k = 1, \text{ and} \\ \alpha_{ijk} + \lambda_{\text{S}ijk} S_{ij1l} + \lambda_{\text{M}ijk} M_{jkl} + E_{ijkl}, & \text{for } k \neq 1, \end{cases} \tag{204}$$

where the occasion index $l$ has been dropped from the intercepts and loadings to express that these parameters are time-invariant. The fit of the invariance model in Equation 204 can be tested against the fit of the more general model in Equation 77 to investigate whether the assumption of measurement invariance is tenable.

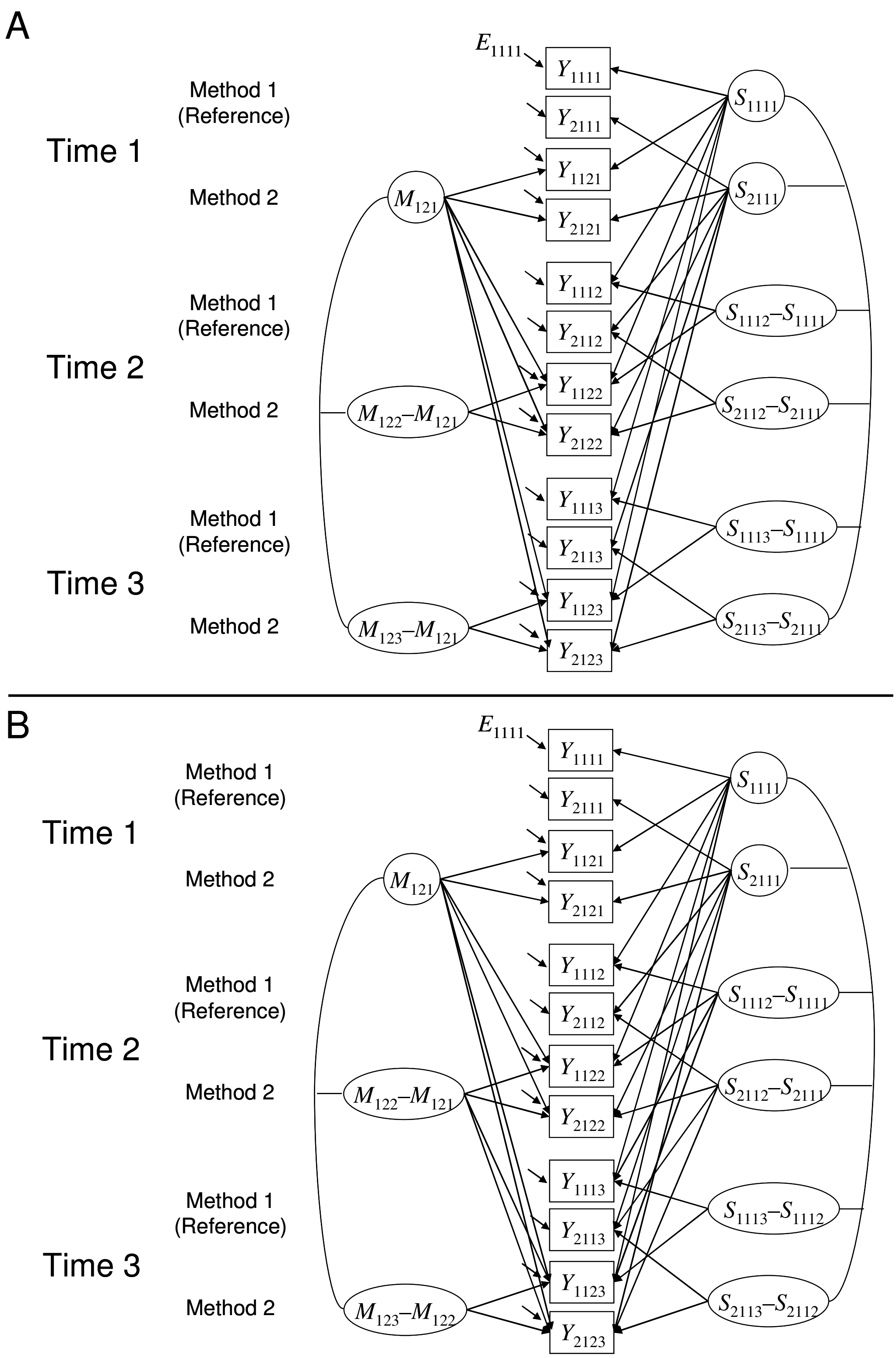


*Figure 17.* Path diagram of a CS-C(*M*–1) change model for one construct measured by two methods on three measurement occasions. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{ij1l}$ = latent state factor. $S_{ij1l'} - S_{ij1l}$ = latent state difference factor. $M_{jkl}$ = common method factor. $M_{jkl'} - M_{jkl}$ = latent method difference factor. $E_{ijkl}$ = error variable. A: baseline change version. B: neighbor change version. For the sake of clarity, not all possible factor correlations are shown.

The reduced model with time-invariant parameters should be used as the base for specifying the change versions of the CS-C(*M*–1) model, as shown in Equation 205 for the non-reference method (the equation for *k* = 1 remains unchanged given that $\alpha_{ij1l} = 0$ and $\lambda_{\text{S}ij1l} = 1$):

$$Y_{ijkl'} = \alpha_{ijk} + \lambda_{\text{S}ijk} S_{ij1l} + \lambda_{\text{S}ijk}(S_{ij1l'} - S_{ij1l}) + \lambda_{\text{M}ijk} M_{jkl} + \lambda_{\text{M}ijk}(M_{jkl'} - M_{jkl}) + E_{ijkl'}. \qquad (205)$$

If measurement invariance is not tenable, the latent difference scores should not be interpreted.

## 4.4 Non-Permissible Latent Correlations

The same types of latent correlations that are assumed to be zero in the state version of the CS-C(*M*–1) model (or that are zero by definition of the model) also have to be constrained to zero in the CS-C(*M*–1) change model:

$$Cov(M_{jkl}, S_{ij1l}) = 0, \qquad (29,\ \textit{repeated})$$

$$Cov(E_{ijkl}, E_{i'j'k'l'}) = 0, \text{ for } (i, j, k, l) \neq (i', j', k', l'), \qquad (30,\ \textit{repeated})$$

$$Cov(E_{ijkl}, S_{i'j'1l'}) = Cov(E_{ijkl}, M_{j'k'l'}) = 0. \qquad (31,\ \textit{repeated})$$

Given that the state and method factors are uncorrelated with the error variables, the latent state and latent method *difference* variables are also uncorrelated with the error variables:

$$Cov[E_{ijkl}, (M_{j'k'l''} - M_{j'k'l'}),] = Cov(E_{ijkl}, M_{j'k'l''}) - Cov(E_{ijkl}, M_{j'k'l'}) = 0, \qquad (206)$$

$$Cov[E_{ijkl}, (S_{i'j'1l''} - S_{i'j'1l'}),] = Cov(E_{ijkl}, S_{i'j'1l''}) - Cov(E_{ijkl}, S_{i'j'1l'}) = 0. \qquad (207)$$

In addition, I recommend that the following covariances be fixed to zero in empirical applications:

$$Cov(M_{jkl}, S_{ij1l'}) = 0. \qquad (208)$$

$$Cov[(M_{jkl'} - M_{jkl}), (S_{ij1l'''} - S_{ij1l''})] = 0. \qquad (209)$$

According to Equation 208, state factors are not correlated with *any* method factors belonging to the same construct, irrespective of the measurement occasion. Equation 209 is a direct consequence of Equation 208. According to Equation 209, state difference variables are not correlated with any method difference variables belonging to the same construct, irrespective of the measurement occasion. These additional independence assumptions are important for several reasons. First, they make the model more parsimonious, since fewer parameters have to be estimated. Second, in most practical applications, the correlations between state and method factors pertaining to different measurement occasions are estimated to be close to (and not significantly different from) zero anyway. Third, if one constrains these correlations to zero, the state version and the change version of the CS-C($M$–1) model are equivalent models that produce exactly the same fit for a given data set. This is practical, given that one may be interested in parameter estimates from both types of models. If the state version with the additional independence assumptions fits the data, one can be sure that any change version will show the same fit. Fourth, and most important, variance components for quantifying the convergent validity and method-specificity of observed difference variables (see Section 4.7) can only be separated if the state and method difference variables are uncorrelated.

## 4.5 Permissible Latent Correlations

Using the latent difference parameterization of the CS-C($M$–1) model, one can estimate a number of interesting correlations that are not directly available in the state version of the model. In the following, some of these correlations are discussed:

(1) The correlations $Cor[S_{ij11}, (S_{ij1l'} - S_{ij1l})]$, $l' > l$, between initial (T1) state factors and state difference variables belonging to the same construct, indicate the association between initial status and change. Positive correlations imply that individuals with higher latent state scores at T1 tend to have higher change scores between time $l$ and $l'$ than individuals with lower latent T1 scores.

(2) The correlations $Cor[S_{ij11}, (S_{i'j'1l'} - S_{i'j'1l})]$, $j \neq j'$ and $l' > l$, between T1 state factors and state difference variables belonging to a different construct, can be used to find out whether the initial state with respect to one construct can be used to predict change in another construct. For example, in therapy studies, it might be interesting to correlate background variables such as the degree of social support at T1 (before psychotherapy) with symptom change (e.g., the degree of decline in depressive symptoms). A negative correlation would indicate that the higher the social support at T1, the greater the decline in depression.

(3) The correlations $Cor[(S_{ij1l'} - S_{ij1l}), (S_{ij1l'''} - S_{ij1l''})]$, $l' > l$ and $l''' > l''$, between state difference factors belonging to the same construct, characterize the association between change scores pertaining to different measurement occasions. Positive values of these correlations indicate that individuals with higher change scores between time $l$ and $l'$ tend to have higher change scores also between time $l''$ and $l'''$.

(4) The correlations $Cor[(S_{ij1l'} - S_{ij1l}), (S_{i'j'1l'} - S_{i'j'1l})]$, $j \neq j'$ and $l' > l$, between state difference factors belonging to different constructs, indicate to which degree inter-individual differences in change with respect to one construct are associated with inter-individual differences in change with respect to another construct. Positive correlations indicate that individuals with higher change scores (e.g., with respect to depression) tend to have higher change scores also with regard to a second construct (e.g., anxiety). Hence, these correlations can be used to investigate the discriminant validity of change with respect to the reference method. High correlations indicate low discriminant validity of change. (One can distinguish between correlations among difference variables capturing change between the *same* measurement occasions and correlations among difference variables capturing change between *different* measurement occasions.)

(5) The correlations $Cor[M_{jk1}, (M_{jkl'} - M_{jkl})]$, $l' > l$, between T1 method factors and method difference factors belonging to the same construct and the same method, indicate the association between the method-specific deviation from the reference method at T1 with the method-specific deviation in change.

(6) The correlations $Cor[M_{jk1}, (M_{j'kl'} - M_{j'kl})]$, $j \neq j'$ and $l' > l$, between T1 method factors and method difference factors belonging to a different construct, indicate the association between the method-specific deviation from the reference method at T1 with the method-specific deviation in change for the same method but a different construct. These correlations are relatively difficult to interpret and probably not substantial in most applications.

(7) The correlations $Cor[M_{jk1}, (M_{jk'l'} - M_{jk'l})]$, $k \neq k'$ and $l' > l$, between T1 method factors and method difference factors belonging to the same construct, but a different method, indicate the association between the method-specific deviation of a method $k$ from the reference method at T1 with the method-specific deviation in change for a different method $k'$. These correlations are also relatively difficult to interpret.

(8) The correlations $Cor[M_{jk1}, (M_{j'k'l'} - M_{j'k'l})]$, $j \neq j'$, $k \neq k'$, and $l' > l$, between initial T1 method factors and method difference factors belonging to a different construct and a

different method, indicate the association between the method-specific deviation of a method $k$ from the reference method at T1 with the method-specific deviation in change for a different construct $j'$ and a different method $k'$. For most applications, one would not expect these correlations to be substantial.

(9) The correlations $Cor[(M_{jkl'} - M_{jkl}),(M_{jkl'''} - M_{jkl''})]$, $l' > l$ and $l''' > l''$, between method difference factors belonging to the same construct and the same method, characterize the association between the method-specific deviation in change scores pertaining to different measurement occasions. Positive values of these correlations indicate that individuals with higher method-specific residual change scores between time $l$ and $l'$ also tend to have higher method-specific residual change scores between time $l''$ and $l'''$.

(10) The correlations $Cor[(M_{jkl'} - M_{jkl}),(M_{j'kl'} - M_{j'kl})]$, $j \neq j'$ and $l' > l$, between method difference factors belonging to the same method, but different constructs, can be used to investigate the discriminant validity of change corrected for influences of the reference method. High correlations indicate low discriminant validity of change with respect to the non-reference methods.

(11) The correlations $Cor[(M_{jkl'} - M_{jkl}),(M_{jk'l'} - M_{jk'l})]$, $k \neq k'$ and $l' > l$, between method difference factors belonging to the same construct, but different methods, indicate the degree to which different methods agree in the assessment of change over and above what they have in common with the reference method. High positive correlations indicate a common "view of change" of different non-reference methods that is not shared with the reference method. For example, change scores based on parent and teacher ratings might deviate from change scores based on the self-report (reference method). If, in addition, the parent and teacher ratings lead to a common view of change that is not shared with the self-report, the latent method difference factors for parents and teachers will be correlated.

(12) The correlations $Cor[(M_{jkl'} - M_{jkl}),(M_{j'k'l'''} - M_{j'k'l''})]$, $j \neq j'$, $k \neq k'$, $l' > l$, and $l''' > l''$, between method difference factors belonging to different constructs, different methods, and different time points indicate to which degree inter-individual differences in method-specific residual change with respect to one construct are associated with inter-individual differences in method-specific residual change with respect to another construct. It is likely that these correlations are close to zero in most applications.

(13) For different constructs ( $j \neq j'$ ), the correlations between (a) initial state factors and method difference factors { $Cov[S_{ij11},(M_{j'kl'} - M_{j'kl})]$ }, (b) initial method factors and state difference factors { $Cov[M_{jk1},(S_{ij'1l'} - S_{ij'1l})]$ }, and (c) state difference and method difference factors { $Cov[(S_{ij1l'} - S_{ij1l}),(M_{j'kl'''} - M_{j'kl''})]$ } are admissible. However, in most research contexts, these correlations can be expected to be negligible and not of substantive interest. One might therefore consider constraining them to zero in empirical applications for reasons of model parsimony.

## 4.6 Mean Structure

The latent change versions of the CS-C(*M*–1) model enable researchers to study latent mean change over time in a straightforward way. For this purpose, the means of the latent state difference variables can be estimated. These means represent the difference in latent state factor means between time $l'$ and time *l*:

$$E(S_{ij1l'} - S_{ij1l}) = E(S_{ij1l'}) - E(S_{ij1l}) = E(Y_{ij1l'}) - E(Y_{ij1l}). \tag{210}$$

Hence, if $E(S_{ij1l'} - S_{ij1l})$ is significantly different from zero, this indicates that there has been mean change (mean growth or decline) over time with respect to the reference method. A positive value of $E(S_{ij1l'} - S_{ij1l})$ implies average *growth* (an increase in the latent state means over time), whereas a negative value implies average *decline* (a decrease in the latent state means over time). The means of the latent method difference variables are always zero, given that the method factor means are zero on all occasions of measurement:

$$E(M_{jkl'} - M_{jkl}) = E(M_{jkl'}) - E(M_{jkl}) = 0. \tag{211}$$

As in the state version of the model, mean changes regarding non-reference methods can be studied by comparing the intercepts over time (see Chapter 3.1.3).

## 4.7 Variance Decomposition and Variance Components

Given (a) independence between state and method difference variables (Equation 209) and (b) strong factorial invariance[6] (i.e., equal intercepts $\alpha_{ijkl} = \alpha_{ijkl'} = \alpha_{ijk}$ , equal state factor

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[6] For the variance decomposition, the assumption of time-invariant intercepts is not mandatory. Being constants, the intercepts drop out in the variance decomposition, no matter whether they are time-invariant or not. However, given that it is strongly advisable to establish strong factorial invariance in latent difference modeling

loadings $\lambda_{\mathrm{S}ijkl} = \lambda_{\mathrm{S}ijkl'} = \lambda_{\mathrm{S}ijk}$, and equal method factor loadings $\lambda_{\mathrm{M}ijkl} = \lambda_{\mathrm{M}ijkl'} = \lambda_{\mathrm{M}ijk}$; Meredith, 1993), the variance of a *latent state* difference score can be decomposed as:

$$Var(S_{ijkl'} - S_{ijkl}) = \lambda^2_{\mathrm{S}ijk} Var(S_{ij1l'} - S_{ij1l}) + \lambda^2_{\mathrm{M}ijk} Var(M_{jkl'} - M_{jkl}), \text{ for } k \neq 1. \tag{212}$$

*Proof*. In the case of strong factorial invariance, for two latent state variables, we obtain :

$$S_{ijkl} = \alpha_{ijk} + \lambda_{\mathrm{S}ijk} S_{ij1l} + \lambda_{\mathrm{M}ijk} M_{jkl}, \text{ as well as}$$

$$S_{ijkl'} = \alpha_{ijk} + \lambda_{\mathrm{S}ijk} S_{ij1l'} + \lambda_{\mathrm{M}ijk} M_{jkl'}.$$

The state difference score is then given by:

$$\begin{aligned}(S_{ijkl'} - S_{ijkl}) &= (\alpha_{ijk} + \lambda_{\mathrm{S}ijk} S_{ij1l'} + \lambda_{\mathrm{M}ijk} M_{jkl'}) - (\alpha_{ijk} + \lambda_{\mathrm{S}ijk} S_{ij1l} + \lambda_{\mathrm{M}ijk} M_{jkl}) \\ &= \alpha_{ijk} - \alpha_{ijk} + \lambda_{\mathrm{S}ijk}(S_{ij1l'} - S_{ij1l}) + \lambda_{\mathrm{M}ijk}(M_{jkl'} - M_{jkl}).\end{aligned}$$

Equation 212 follows by applying rules of covariance algebra, since $\alpha_{ijk}$ drops out and $(S_{ij1l'} - S_{ij1l})$ and $(M_{jkl'} - M_{jkl})$ are assumed to be uncorrelated according to Equation 176.

Given the independence conditions in Equations 30, 173, 174, and 176, as well as strong factorial invariance, the variance of an *observed* difference score can be decomposed as:

$$Var(Y_{ijkl'} - Y_{ijkl}) = \begin{cases} Var(S_{ij1l'} - S_{ij1l}) + Var(E_{ij1l'}) + Var(E_{ij1l}), \text{ for } k = 1, \\ \lambda^2_{\mathrm{S}ijk} Var(S_{ij1l'} - S_{ij1l}) + \lambda^2_{\mathrm{M}ijk} Var(M_{jkl'} - M_{jkl}) \\ + Var(E_{ijkl'}) + Var(E_{ijkl}), \text{ for } k \neq 1. \end{cases} \tag{213}$$

*Proof*. In the case of strong factorial invariance, for two observed variables, we obtain :

$$Y_{ijkl} = \alpha_{ijk} + \lambda_{\mathrm{S}ijk} S_{ij1l} + \lambda_{\mathrm{M}ijk} M_{jkl} + E_{ijkl}, \text{as well as}$$

$$Y_{ijkl'} = \alpha_{ijk} + \lambda_{\mathrm{S}ijk} S_{ij1l'} + \lambda_{\mathrm{M}ijk} M_{jkl'} + E_{ijkl'}.$$

(i.e., time-invariant loadings *and* time-invariant intercepts, see discussion below), I assume intercept invariance here as well.

The observed difference score is then given by:

$$\begin{aligned}(Y_{ijkl'} - Y_{ijkl}) &= (\alpha_{ijk} + \lambda_{Sijk} S_{ij1l'} + \lambda_{Mijk} M_{jkl'} + E_{ijkl'}) - (\alpha_{ijk} + \lambda_{Sijk} S_{ij1l} + \lambda_{Mijk} M_{jkl} + E_{ijkl}) \\ &= \alpha_{ijk} - \alpha_{ijk} + \lambda_{Sijk}(S_{ij1l'} - S_{ij1l}) + \lambda_{Mijk}(M_{jkl'} - M_{jkl}) + E_{ijkl'} - E_{ijkl}.\end{aligned}$$

Equation 213 follows by applying rules of covariance algebra, since $\alpha_{ijk}$ drops out, and all variables on the right hand side of the equation are assumed to be uncorrelated according to Equations 30, 206, 207, and 209.

An important difference between the observed and the latent state difference score variance decomposition should be noted: For the observed difference score variances, the error variances of *both* time points are part of the equation.

On the basis of the additive variance decomposition, we can define coefficients of consistency, method-specificity, and reliability for observed change scores. The consistency coefficient indicates the proportion of variance of an observed change score that is determined by change in the reference method state factor and can thus be interpreted as an index of the *convergent validity of change*. It represents that part of the variance of a change score that is shared with the reference method:

$$CO(Y_{ijkl'} - Y_{ijkl}) = \frac{\lambda^2_{Sijk} Var(S_{ij1l'} - S_{ij1l})}{Var(Y_{ijkl'} - Y_{ijkl})}. \tag{214}$$

The method-specificity coefficient represents the proportion of variance of an observed change score that is due to method-specific deviations from change as measured by the reference method (i.e., change score variance that is specific to a particular non-reference method):

$$MS(Y_{ijkl'} - Y_{ijkl}) = \frac{\lambda^2_{Mijk} Var(M_{jkl'} - M_{jkl})}{Var(Y_{ijkl'} - Y_{ijkl})}. \tag{215}$$

The reliability of an observed change score can be calculated as:

$$\begin{aligned}Rel(Y_{ijkl'} - Y_{ijkl}) &= 1 - \frac{Var(E_{ijkl'}) + Var(E_{ijkl})}{Var(Y_{ijkl'} - Y_{ijkl})} \\ &= CO(Y_{ijkl'} - Y_{ijkl}) + MS(Y_{ijkl'} - Y_{ijkl}).\end{aligned} \tag{216}$$

The reliability coefficient indicates that part of the variance of an observed change score that is not due to measurement error. It is well-known that observed difference scores are often less reliable than are conventional scores. The reason becomes clear from Equation 183: The sum of the two error variances of both time points $l$ and $l'$ are in the numerator of the reliability formula. Hence, error influences of both time points have an impact on observed difference scores. This underscores the need for latent variable models of change: We should study change through *latent* difference scores [as is done in the CS-C(*M*–1) change model] rather than through *observed* difference scores that may be very unreliable indicators of change.

The consistency and method-specificity coefficients can also be defined for the latent state difference variables $Var(S_{ijkl'} - S_{ijkl})$:

$$CO(S_{ijkl'} - S_{ijkl}) = \frac{\lambda^2_{\mathrm{S}ijk} Var(S_{ij1l'} - S_{ij1l})}{Var(S_{ijkl'} - S_{ijkl})}, \tag{217}$$

$$MS(S_{ijkl'} - S_{ijkl}) = \frac{\lambda^2_{\mathrm{M}ijk} Var(M_{jkl'} - M_{jkl})}{Var(S_{ijkl'} - S_{ijkl})}. \tag{218}$$

$CO(S_{ijkl'} - S_{ijkl})$ and $MS(S_{ijkl'} - S_{ijkl})$ add up to unity. Table 3 summarizes the most important equations for the CS-C(*M*–1) change models.

## 4.8 Extensions of the CS-C(*M*–1) Change Model

The CS-C(*M*–1) change model can be extended by including external variables in the model. For example, researchers can add potential explanatory variables of change or study the effect of change in one construct on (change in) other constructs. In other words, the latent difference variables can serve as endogenous variables (outcomes) or predictor variables in extended SEMs including observed and/or latent covariates. For example, one might be interested in finding predictor variables that explain why intervention programs (e.g., watching Sesame street) cause greater changes in cognitive abilities in some children than in others. On the other hand, change scores may themselves be used as predictors of other variables (e.g., change in one variable, say *cognitive abilities*, may cause [or at least correlate with] change in another variable, for example *school grades*). The latent difference variables might also serve as mediator variables. For example, in intervention studies, a certain treatment (exogenous variable) might cause change in a construct $j$ which in turn might trigger change in another construct $j'$. The latent change score for construct $j$ might then mediate the influence of the treatment on change in construct $j'$. For example, an intervention designed to modify behavior problems in Kindergarten may cause change aggressive behavior in children. Change in aggressive behavior might have a causal impact on distal outcomes such as achievement and the number of friends in primary school, or alcohol and drug abuse in adolescence.

## 4.9 Summary and Discussion

In this Chapter, I demonstrated how Steyer et al.'s (1997, 2000) approach of analyzing change in terms of latent (difference) variables can be transferred to the multi-method situation. Latent difference modeling offers a direct and flexible approach to investigating change. When the additional assumption of independence between all state and method factors belonging to the same construct is made, the latent difference model is equivalent to the state version. That is, one does not specify a "new" model but just makes the information about change inherent in state models more accessible through reparameterization (Steyer et al., 1997, 2000). In contrast to growth curve models, latent difference models do not make any restrictive assumptions regarding the specific functional form of change.

The CS-C(*M*–1) change model enables researchers to investigate change simultaneously for different methods. In particular, this model makes it possible to contrast change as assessed by a reference method against change as measured by other (non-reference) methods. Researchers can use the model to study the convergent and discriminant validity of change. The convergent validity and method-specificity of change can be assessed in terms of variance components for the observed variables. The discriminant validity of change can be quantified by means of latent correlations. Mean differences over time are captured by the means of the latent difference variables and the intercepts of the observed variables.

Depending on the research question, researchers can choose between a baseline and a neighbor change version of the CS-C(*M*–1) model. The baseline change version is most useful if a researcher is interested in change with respect to a baseline occasion of measurement (in many studies the initial status, i.e., T1). The neighbor change version should be applied if a researcher has specific hypotheses regarding change between adjacent time points. For example, in an intervention study, there might be specific phases, in which an effect is to be expected. The neighbor change version can be used to investigate change between two or more specific time points. It should be noted that the CS-C(*M*–1) change model is more general and not limited to a baseline or neighbor assessment of change. The specification can be modified to study change between any specific measurement occasions of interest.

A potentially limiting factor in latent difference modeling is the required assumption of measurement invariance over time. For the latent difference scores to be meaningful, factor loadings and measurement intercepts should be time-invariant. The condition of strong factorial invariance may not always be tenable, especially when the intervals between the measurement occasions are large or when individuals are assessed at different stages in their development (e.g., from childhood to adolescence). As I already mentioned, an advantage of the models presented here is that the assumption of measurement invariance is testable for all methods. If full invariance for all indicators is not tenable, it might at least be possible to establish partial invariance (Byrne, Shavelson, & Muthén, 1989). Partial invariance means that invariance holds only for some, but not all indicators. Under specific conditions, partial invariance may be sufficient to warrant proper interpretation of the latent difference variables (see Byrne et al., 1989).

Given that latent change models focus on inter-individual differences in intra-individual change, these models are especially useful for analyzing treatment effects in intervention and evaluation studies. For this purpose, the CS-C(*M*–1) change model can easily be extended to a multiple group CFA model (e.g., Jöreskog, 1971b, Thompson & Green, 2006), in which the parameters of the model are simultaneously estimated in (and can be compared across) several groups. For example, one might be interested in comparing change in behavior problems of children in a control group and an intervention group in which a special treatment is applied (e.g., Barquero et al., 2007). If a multi-method design is used, a multi-group CS-C(*M*–1) change analysis can be used to test various hypotheses with regard to (a) measurement invariance across time and groups as well as (b) differences in structural parameters (difference factor means, variances, and covariances) across time and groups. Steyer (2005) discusses such a multi-group latent difference approach for the mono-method case.

Note that the principle of reformulating the CS-C(*M*–1) state model as a change model was shown here only for the CS-C(*M*–1) model variant presented in Chapters 3.1 and 3.5.1. However, this principle can equally well be applied to the CS-C(*M*–1) model version with general state factors and indicator-specific factors across time introduced in Chapters 3.2 and 3.5.2. In fact, in Chapter 5.5, I present an application of the CS-C(*M*–1) model with general state factors and indicator-specific factors across time, in which this model variant is parameterized as a change model.

Table 3

*Summary of the CS-C(M–1) Change Models*

| Definition | Equation |
|---|---|
| General decomposition of state factors on measurement occasions $l$ and $l'$, $l' \geq l$ | $S_{ij1l'} = S_{ij1l} + (S_{ij1l'} - S_{ij1l})$ |
| General decomposition of method factors on measurement occasions $l$ and $l'$, $l' \geq l$ | $M_{jkl'} = M_{jkl} + (M_{jkl'} - M_{jkl})$ |
| Measurement equation for indicators pertaining to the reference method on measurement occasions $l$ and $l'$, $l' \geq l$ | $Y_{ij1l'} = S_{ij1l} + (S_{ij1l'} - S_{ij1l}) + E_{ij1l'}$ |
| Measurement equation for indicators pertaining to non-reference methods on measurement occasions $l$ and $l'$, $l' \geq l$ | $Y_{ijkl'} = \alpha_{ijkl'} + \lambda_{\text{S}ijkl'} S_{ij1l} + \lambda_{\text{S}ijkl'}(S_{ij1l'} - S_{ij1l}) + \lambda_{\text{M}ijkl'} M_{jkl} + \lambda_{\text{M}ijkl'}(M_{jkl'} - M_{jkl}) + E_{ijkl'}$ |
| Covariances of method factors and state factors | $Cov(M_{jkl}, S_{ij1l'}) = 0$ |
| Covariances of state difference variables and method difference variables | $Cov[(M_{jkl'} - M_{jkl}), (S_{ij1l'''} - S_{ij1l''})] = 0$ |
| Covariances of error variables | $Cov(E_{ijkl}, E_{i'j'k'l'}) = 0, \quad (i, j, k, l) \neq (i', j', k', l')$ |
| Covariances of error variables and other latent variables | $Cov(E_{ijkl}, S_{i'j'1l'}) = Cov(E_{ijkl}, M_{j'k'l'})$ $= Cov[E_{ijkl}, (S_{i'j'1l''} - S_{i'j'1l'})] = Cov[E_{ijkl}, (M_{j'k'l''} - M_{j'k'l'})] = 0$ |

*(Table continues)*

| Definition | Equation |
|---|---|
| Mean structure (state difference variables) | $E(S_{ij1l'} - S_{ij1l}) = E(Y_{ij1l'}) - E(Y_{ij1l})$ |
| Mean structure (method difference variables and error variables) | $E(M_{jkl'} - M_{jkl}) = E(E_{ijkl}) = 0$ |
| Variance decomposition (observed difference variables) | $Var(Y_{ijkl'} - Y_{ijkl}) = \begin{cases} Var(S_{ij1l'} - S_{ij1l}) + Var(E_{ij1l'}) + Var(E_{ij1l}), \text{ for } k = 1, \\ \lambda^2_{\text{S}ijk} Var(S_{ij1l'} - S_{ij1l}) + \lambda^2_{\text{M}ijk} Var(M_{jkl'} - M_{jkl}) + Var(E_{ijkl'}) + Var(E_{ijkl}), \text{ for } k \neq 1 \end{cases}$ |
| Consistency (observed difference variables) | $CO(Y_{ijkl'} - Y_{ijkl}) = \frac{\lambda^2_{\text{S}ijk} Var(S_{ij1l'} - S_{ij1l})}{Var(Y_{ijkl'} - Y_{ijkl})}$ |
| Method-specificity (observed difference variables) | $MS(Y_{ijkl'} - Y_{ijkl}) = \frac{\lambda^2_{\text{M}ijk} Var(M_{jkl'} - M_{jkl})}{Var(Y_{ijkl'} - Y_{ijkl})}$ |
| Reliability | $Rel(Y_{ijkl'} - Y_{ijkl}) = 1 - \frac{Var(E_{ijkl'}) + Var(E_{ijkl})}{Var(Y_{ijkl'} - Y_{ijkl})} = CO(Y_{ijkl'} - Y_{ijkl}) + MS(Y_{ijkl'} - Y_{ijkl})$ |
| Variance decomposition (state difference variables) | $Var(S_{ijkl'} - S_{ijkl}) = \lambda^2_{\text{S}ijk} Var(S_{ij1l'} - S_{ij1l}) + \lambda^2_{\text{M}ijk} Var(M_{jkl'} - M_{jkl}), \text{ for } k \neq 1$ |
| Consistency (state difference variables) | $CO(S_{ijkl'} - S_{ijkl}) = \frac{\lambda^2_{\text{S}ijk} Var(S_{ij1l'} - S_{ij1l})}{Var(S_{ijkl'} - S_{ijkl})}$ |

*(Table continues)*

| Definition | Equation |
|---|---|
| Method-specificity (state difference variables) | $MS(S_{ijkl'} - S_{ijkl}) = \frac{\lambda^2_{\mathrm{M}ijk} Var(M_{jkl'} - M_{jkl})}{Var(S_{ijkl'} - S_{ijkl})}$ |
| | Baseline change model |
| Decomposition of state factors on measurement occasions $l$, $l \geq 1$ | $S_{ij1l} = S_{ij11} + (S_{ij1l} - S_{ij11})$ |
| Decomposition of method factors on measurement occasions $l$, $l \geq 1$ | $M_{jkl} = M_{jk1} + (M_{jkl} - M_{jk1})$ |
| Measurement equation for indicators pertaining to the reference method on measurement occasions $l$, $l \geq 1$ | $Y_{ij1l} = S_{ij11} + (S_{ij1l} - S_{ij11}) + E_{ij1l}$ |
| Measurement equation for indicators pertaining to non-reference methods on measurement occasions $l$, $l \geq 1$ | $Y_{ijkl} = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij11} + \lambda_{\mathrm{S}ijkl}(S_{ij1l} - S_{ij11}) + \lambda_{\mathrm{M}ijkl} M_{jk1} + \lambda_{\mathrm{M}ijkl}(M_{jkl} - M_{jk1}) + E_{ijkl}$ |
| | Neighbor change model |
| Decomposition of state factors on measurement occasions $l = 3, 4, ..., p$ | $S_{ij1l} = S_{ij11} + (S_{ij1l} - S_{ij1(l-1)}) + (S_{ij1(l-1)} - S_{ij1(l-2)}) + ... + (S_{ij12} - S_{ij11})$ |
| Decomposition of method factors on measurement occasions $l = 3, 4, ..., p$ | $M_{jkl} = \lambda_{\mathrm{M}ijkl} \cdot M_{jk1} + \lambda_{\mathrm{M}ijkl} \cdot (M_{jkl} - M_{jk(l-1)}) + \lambda_{\mathrm{M}ijkl} \cdot (M_{jk(l-1)} - M_{jk(l-2)}) + ... + \lambda_{\mathrm{M}ijkl} \cdot (M_{jk2} - M_{jk1})$ |

*(Table continues)*

| Definition | Equation |
|---|---|
| Measurement equation for indicators pertaining to the reference method on measurement occasions $l = 3, 4, \ldots, p$ | $Y_{ij1l} = S_{ij11} + (S_{ij1l} - S_{ij1(l-1)}) + (S_{ij1(l-1)} - S_{ij1(l-2)}) + \ldots + (S_{ij12} - S_{ij11}) + E_{ij1l}$ |
| Measurement equation for indicators pertaining to non-reference methods on measurement occasions $l = 3, 4, \ldots, p$ | $Y_{ijkl} = \alpha_{ijkl} + \lambda_{\mathrm{S}ijkl} S_{ij11} + \lambda_{\mathrm{S}ijkl}(S_{ij1l} - S_{ij1(l-1)}) + \lambda_{\mathrm{S}ijkl}(S_{ij1(l-1)} - S_{ij1(l-2)}) + \ldots + \lambda_{\mathrm{S}ijkl}(S_{ij12} - S_{ij11})$ $+ \lambda_{\mathrm{M}ijkl} M_{jk1} + \lambda_{\mathrm{M}ijkl}(M_{jkl} - M_{jk(l-1)}) + \lambda_{\mathrm{M}ijkl}(M_{jk(l-1)} - M_{jk(l-2)}) + \ldots + \lambda_{\mathrm{M}ijkl}(M_{jk2} - M_{jk1}) + E_{ij1l}$ |

*Note.* Without loss of generality, the first method ($k$ = 1) is selected as reference method. $Y_{ijkl}$ = observed variable ($i$ = indicator, $j$ = construct, $k$ = method, $l$ = occasion of measurement). $S_{ij1l}$ = latent state factor. $M_{jkl}$ = common method factor. $E_{ijkl}$ = error variable. $\alpha_{ijkl}$ = intercept. $\lambda_{\mathrm{S}ijkl}$ = state factor loading. $\lambda_{\mathrm{M}ijkl}$ = method factor loading.

# 5 Applications

In this chapter, I illustrate the practical use of the CS-C($M$–1) state and change models by discussing applications to a real MTMM-MO data set provided by Prof. David A. Cole from the Vanderbilt University (USA). His longitudinal MTMM study focused on three different constructs (i.e., *Depression*, *Anxiety*, and *Competence*) in several hundred American school children over several years. He used four different methods (i.e., *self*, *parent*, *teacher*, and *peer reports*) to assess these constructs on a total of eight occasions of measurement (for details concerning these studies, see Cole, Martin, & Powers, 1997; Cole, Martin, Powers, & Truglio, 1996; as well as Cole, Truglio, & Peeke, 1997). All constructs were assessed by repeatedly administered questionnaires. The subset of data used for the present analyses is described in some detail in the following section.

## 5.1 Description of the Data Set

### *5.1.1 Sample*

A subset of Cole et al.'s data including two constructs (depression and anxiety), three methods (self-report, parent rating, and teacher rating), and four occasions of measurement was analyzed. (I used only two constructs to keep the analyses as simple as possible. Two constructs allow illustrating all relevant features of the models.) The total sample size was $N = 906$. The children were nested within 49 school classrooms. Assessments took place every six months during a period of two years. The children were not rated by the same teacher on all measurement occasions. The same teachers rated the children on the first and the second measurement occasion; different teachers provided ratings for the third and fourth occasion. This has consequences for the model specification as discussed below.

### *5.1.2 Measures*

Depression was measured by the self-report and parent form of the *Child Depression Inventory* (CDI and CDI-PF; Kovacs, 1985), as well as the *Teacher Report Index of Depression* (TRID; Cole, 1995). Anxiety was assessed by the child and parent form of the *Revised Children's Manifest Anxiety Scale* (RCMAS-CF and RCMAS-PF; Reynolds & Richmond, 1978), as well as the *Teacher Report Index of Anxiety* (TRIA; Cole, 1995). Table 4 provides an overview of these measures (for more details see Cole, Truglio, & Peeke, 1997). As discussed in Chapter 3.1, the CS-C($M$–1) model is a multiple indicator MTMM model (i.e., at least two indicators are required for each CMOU). Multiple indicators per

CMOU allow researchers to analyze construct- and occasion-specific method effects. In Cole's MTMM-MO study, each construct was measured by only one scale. However, the scales consisted of multiple items. To illustrate the CS-C($M$–1) model as a multiple indicator model at least two continuous indicators per CMOU were necessary. Therefore, I created two continuous indicators for each CMOU by splitting each scale into two parcels (test halves). For each scale, the parcels were compiled by calculating the mean of half of the items, respectively. Care was taken to create parcels that were as homogeneous as possible[7]. Furthermore, the parcels consisted of identical items across raters[8] (as far as possible) and across time.

Table 4

*Questionnaires Used in the Cole et al. Studies*

| | Self report | Parent report | Teacher report |
|---|---|---|---|
| Depression | Child Depression Inventory (CDI; Kovacs, 1985; 26 items measured on a 3-point rating scale) * | Child Depression Inventory – Parent Form (CDI-PF; Kovacs, 1985; 26 items measured on a 3-point rating scale) * | Teacher Report Index of Depression (TRID; Cole, 1995; 13 items measured on a 4-point rating scale) |
| Anxiety | Revised Children's Manifest Anxiety Scale (RCMAS; Reynolds & Richmond, 1978; 28 items measured on a 3-point rating scale) # | Revised Children's Manifest Anxiety Scale – Parent Form (RCMAS-PF; Reynolds & Richmond, 1978; 28 items measured on a 3-point rating scale) # | Teacher Report Index of Anxiety (TRIA; Cole, 1995; 12 items measured on a 4-point rating scale) |

*Note.* Only the scales relevant to the present application are shown. Scales that are equivalent (consist of the same items) across methods are indicated with the same symbol (* or #).

Another possibility to obtain multiple indicators per CMOU would have been to use the items themselves as indicators. However, as the item responses were based on relatively few response categories (see Table 4), this would have added the additional complication of dealing with ordinal variables. Ordinal variables require special CFA models, the treatment of

[7] For this purpose, I conducted principal component analyses separately for each scale. I then distributed the items evenly according to their loadings on the first unrotated principal component.

[8] As mentioned above, the teachers used different scales than did the children and the parents. Hence, the parcels for the teacher report consisted of different items than the parcels for the self- and parent report.

which is beyond the scope of the present work. Furthermore, the large number of items per construct would have led to extremely large models with an excessive number of estimated parameters.

The use of item parcels is a somewhat controversial issue (Little, Cunningham, Shahar, & Widaman, 2002), as item parcels often do not consist of perfectly unidimensional, tau-equivalent items (as it should be). If multidimensional parcels are used, it is unclear what a parcel score really means and whether different parcels should be used as indicators of a single common factor.

In the present application, I conducted preliminary item level factor analyses. These analyses revealed that the items of all scales almost exclusively measured a single common factor, although unidimensionality was not perfectly achieved—which is not surprising given the large number of items per scale. Hence, I believe that the use of item parcels was justified in this case—bearing in mind that the present analyses primarily serve illustrative purposes. In actual applications, researchers should either conduct the CS-C($M$–1) analyses using items as indicators or use item parcels only if the items show no or only marginal departures from unidimensionality.

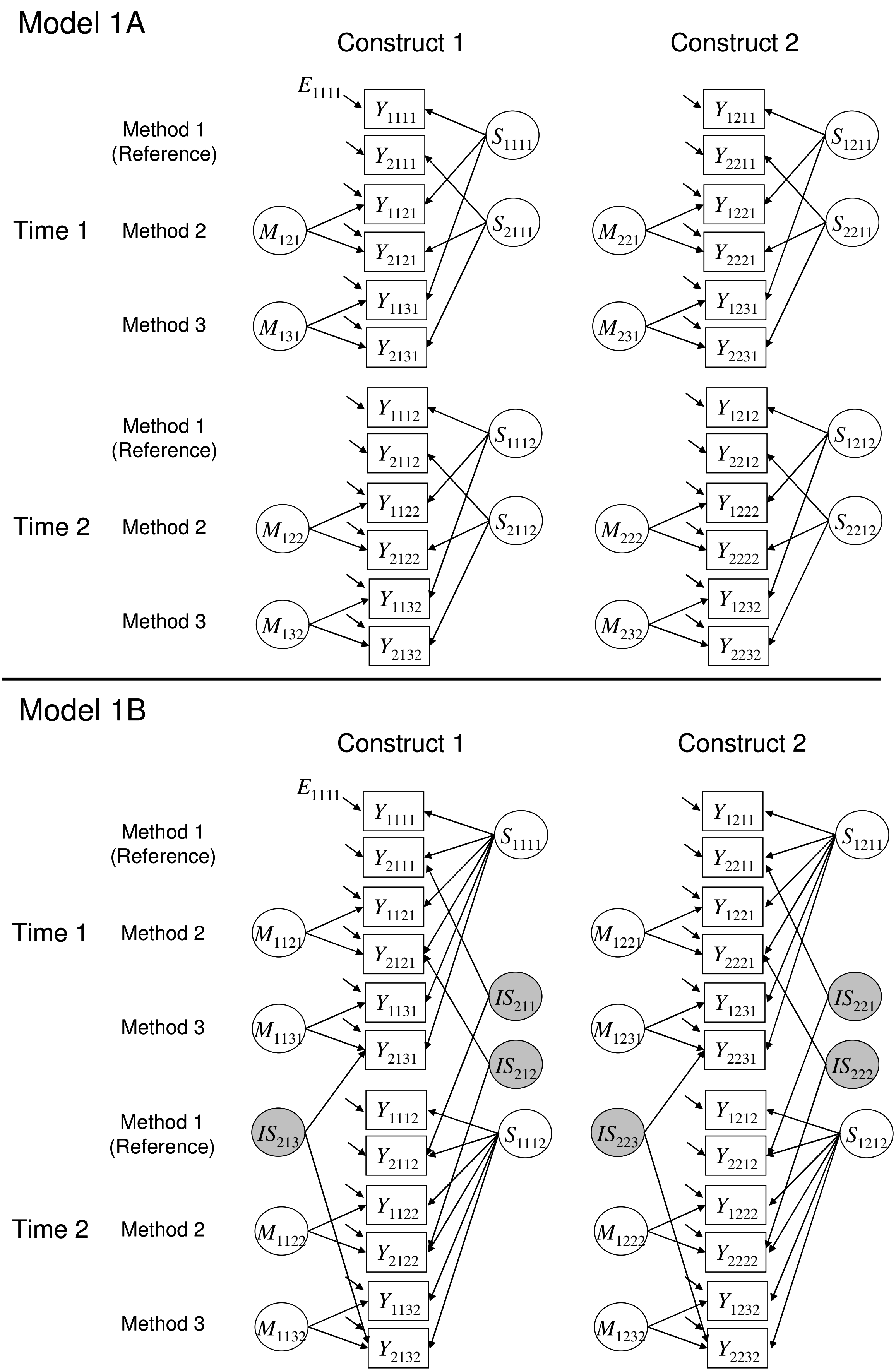
In all models, the children's self-report was used as the reference method. The parent report and teacher ratings were used as non-reference methods to be contrasted against the self-report. As such, the contrast in methods represents the degree to which parent and teacher reports deviate from children's self-perceptions.

## 5.2 3-Step Approach to Model Testing

In this section, I propose a 3-Step approach to model testing using the CS-C($M$–1) approach. I suggest that a "top down" strategy be used, in which one begins with two rather unrestrictive model variants in the first step. Starting with rather general and unrestrictive baseline models is a useful strategy to approach complex MTMM-MO data.

The analysis steps discussed in the following serve three basic goals: (1) determining whether a CS-C($M$–1) structure fits the data at all, (2) finding out whether indicator-specific effects are present and if so how these are most appropriately modeled, and (3) testing for measurement invariance over time and determining the degree of invariance that is tenable for the data.

Note that the 3-Step approach presented here does not cover *all* possible constellations one might find in real data situations, as there are simply too many. However, I believe that such an idealized framework is nonetheless useful as a guide for practical applications.




*(Figure continues)*

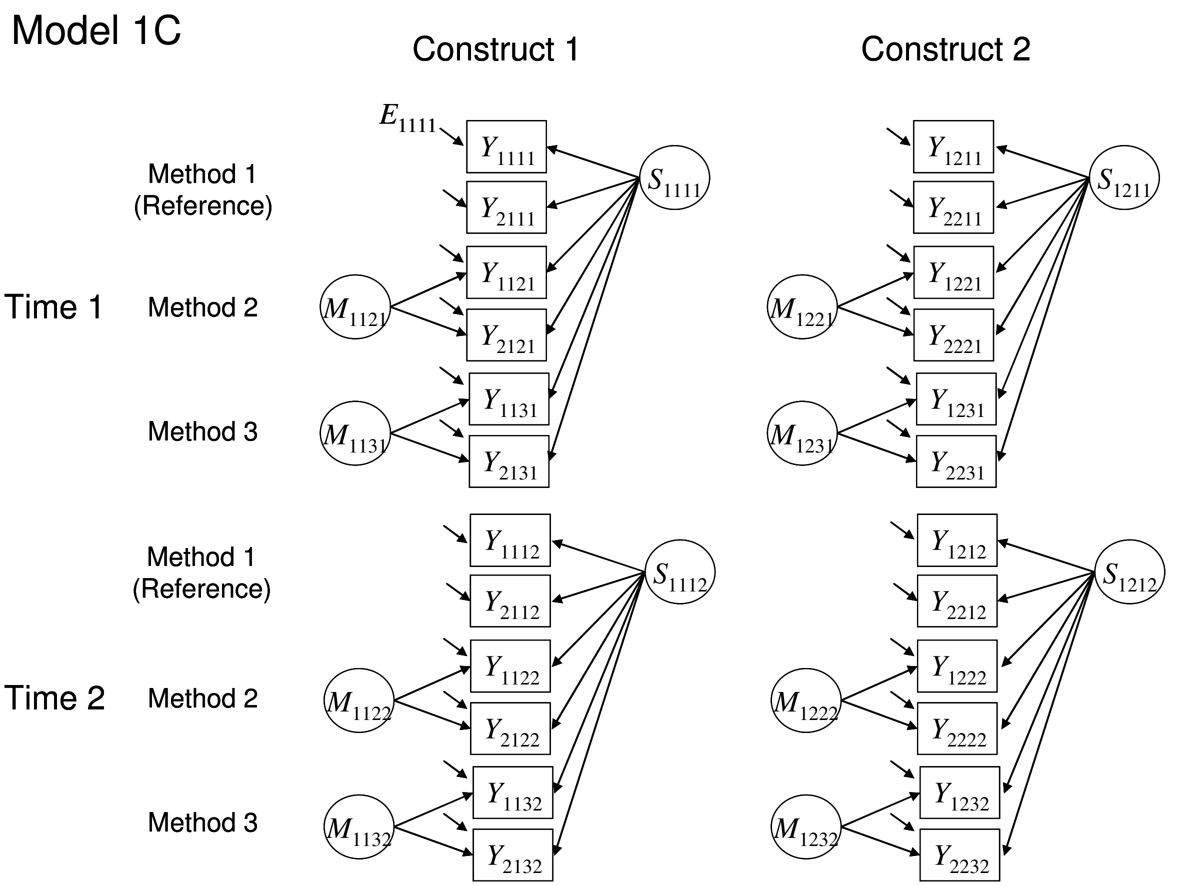



*Figure 18.* CS-C(*M*–1) model variants tested in Step 1 of the MTMM-MO analysis. Each model is shown for the case of two constructs, three methods, and two time points. $Y_{ijkl}$ = observed variable (*i* = indicator, *j* = construct, *k* = method, *l* = occasion of measurement). $S_{ij1l}$ = latent state factor. $M_{jkl}$ and $M_{1jkl}$ = common method factors. $IS_{ijk}$ = indicator-specific factor. $E_{ijkl}$ = error variable. Model 1A: CS-C(*M*–1) model with indicator-specific state variables. Model 1B: CS-C(*M*–1) model with general state factors and indicator-specific factors across time. Model 1C: CS-C(*M*–1) model with general state factors *without* indicator-specific factors across time. For the sake of clarity, factor correlations are not shown in the path diagrams.

### *5.2.1 Step 1: Determination of a Baseline Model*

Step 1 serves (1) to test whether a CS-C(*M*–1) structure fits the data and (2) to establish the type of indicator-specificity present in the data (if any). Are there indicator-specific effects at all? If yes, is it more important to model indicator-specific effects across methods or across time? For answering these questions, I recommend that three variants of the CS-C(*M*–1) model be estimated in Step 1 (see Figure 18): Model 1A is the CS-C(*M*–1) model with indicator-specific state variables presented in Section 3.1. Model 1B is the CS-C(*M*–1) model with general state factors and indicator-specific factors across time presented in Section 3.2. Model 1C is the CS-C(*M*–1) model with general state factors *without* indicator-specific factors across time (first presented in Figure 15). For convenience, these three models are shown once again in Figure 18.

The three models considered in Step 1 should be estimated without imposing any parameter invariance constraints over time other than those implied by the model definition[9]. Substantively, this means that in the models estimated in Step 1, the observed variables are allowed to change their psychometric properties over time (i.e., only *configural invariance* is assumed to hold, i.e., the same pattern of loadings for all variables; Meredith, 1993). In addition, there are no constraints on latent variable variances or means.

The absolute fit of the three models (1A-1C) is then evaluated to find out whether a CS-C($M$–1) structure adequately fits the data. Moreover, the fit of the three model variants is compared to determine the type of indicator-specificity present in the data. Note that the fit of the three models (1A–1C) should *not* be compared by a likelihood ratio $\chi^2$ test ($\chi^2$ difference test), because the more restricted variants without indicator-specific factors are obtained from the less restricted models by fixing variances or correlations of factors to their boundary values of zero or one, respectively. This violates regularity assumptions of the likelihood ratio $\chi^2$ test (e.g., Takane, van der Heijden, & Browne, 2003). Instead, information criteria such as the *AIC* measure (Akaike, 1974) can be used to compare the models estimated in Step 1.

If Model 1C does not fit worse than the two other models, then either type of indicator-specificity is negligible and Model 1C should be selected as the base for the following analysis steps as it is the most parsimonious model. If Model 1A is the best-fitting model, this means that indicator-specific effects are present that generalize across methods. In this case, Model 1A should be retained for further analyses. If Model 1B fits best, then indicator-specific effects generalize across time rather than across methods. This is probably the case that researchers will most frequently encounter in practice. In this case, one should retain Model 1B as the baseline model for the invariance tests conducted in Steps 2 and 3.

If none of the three model variants fits the data, it would be useful to first study the structure for each occasion of measurement and / or for each construct separately to detect the source(s) of the lack of fit. In this case, it is possible that other models than those presented here would have to be looked for.

The next logical step (Step 2) is to test for measurement invariance of the indicators belonging to the reference method. In general, different hypotheses about measurement

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[9] In its most general form, the CS-C($M$–1) model (with or without indicator-specific factors across time) does not impose any parameter invariance constraints over time. The marker indicators are an exception. Their loadings and/or intercepts are fixed to identify the metric of the latent factors. If the same indicators are selected as markers on each occasion of measurement, this implies measurement invariance for these indicators.

invariance over time can be tested by specifying different versions of the CS-C($M$–1) model, in which different sets of parameter equality constraints are imposed. These model versions are nested within the baseline model selected in Step 1 and can be tested against the baseline model and against each other using $\chi^2$ difference testing.

### *5.2.2 Step 2: Assessing Invariance for the Reference Method*

Step 2 allows determining (1) whether the psychometric properties of the reference method indicators have changed over time and (2) whether the measurement structure (intercepts and loadings) of the state factors is the same on each measurement occasion (as the state factors are defined by the reference method). A useful sequence of invariance tests for the reference method is:

- 2A Test for loading invariance (possible change in regression slopes / scale discrimination).
- 2B If loading invariance holds, additionally test for intercept invariance (possible change in origin / scale difficulty).
- 2C If at least loading invariance holds, additionally test for invariance of error variances (possible change in indicator-specific / measurement error influences).
- 2D If at least loading invariance holds, additionally test for invariance of the state factor variances.
- 2E If a model with indicator-specific factors $IS_{ijk}$ across time was selected as the baseline model in Step 1, one can also test whether the loadings on the indicator-specific factors are invariant over time.
- 2F If at least state factor loading and intercept invariance is tenable (Case #2B), an additional test of equality of latent state means over time for the same construct can be performed. This would imply a test of mean change over time with respect to the reference method.

If strict invariance holds (i.e., if loadings, intercepts, residual variances, and state factor variances are equal over time), the reliabilities of the reference method indicators would also be constant over time.

### *5.2.3 Step 3: Assessing Invariance for the Non-Reference Methods*

If at least loading and intercept invariance has been established for the indicators of the reference method, one may want to study invariance with respect to the indicators pertaining to the non-reference methods. A useful strategy is to study invariance for one non-reference method at a time, following the sequence of invariance tests proposed for the reference method in Step 2.

It is important to repeat that the three steps proposed above represent an idealized framework for testing measurement invariance in a MTMM-MO context. In practical applications, one may find less clear-cut situations in which for instance indicator-specific effects are present for some but not all indicators or in which invariance can be established only for some indicators within a method or construct, or only across some but not all measurement occasions. Clearly, not all possible constellations of partial invariance can be discussed here. However, researchers may refer to Byrne, Shavelson, and Muthén (1989) who proposed a framework of partial measurement invariance for standard multigroup CFA models. This framework can be transferred to the MTMM-MO situation and to analyses using the CS-C(*M*–1) approach.

## 5.3 Details on the Statistical Analysis

### *5.3.1 Estimation and Software*

All analyses were conducted based on the raw data (i.e., individual data were used, not summary data such as, e.g., a covariance matrix). Both the observed variable covariances and means were included in the analysis. Robust Maximum Likelihood (ML) estimation was used for all models due to non-normality and clustering of the data (see discussion below). All models were analyzed using the Mplus program (Muthén & Muthén, 1998–2007). All covariances between state and method factors belonging to the same construct were constrained to zero [i.e., $Cor(S_{1j1l}, M_{jkl'}) = 0$ and $Cor(S_{1j1l}, M_{1jkl'}) = 0$].

### *5.3.2 Multilevel Structure of the Data*

A complication that often arises in psychological studies is that observations are not independent, but clustered within a hierarchical structure. In the present data set, the children were nested within school classrooms. Such complex or cluster sample structure is also referred to as multilevel structure and requires a special treatment. When the non-independence of observations is ignored, standard errors and test statistics in conventional covariance structure analyses can be biased (Julian, 2000). In the present analysis, I handled this problem by using an appropriate robust ML estimator. This estimator is referred to as MLR estimator in the software Mplus (Muthén & Satorra, 1995; Muthén & Muthén, 1998-2007). The MLR estimator provides the conventional ML parameter estimates, but computes corrected standard errors and fit statistics that allow for a more accurate statistical inference with clustered and non-normal data (option TYPE = COMPLEX in Mplus; Muthén & Muthén, 1998-2007).

### *5.3.3 Handling of Missing Data*

I used full information maximum likelihood (FIML) estimation in order to handle missing data (Arbuckle, 1996; Little & Rubin, 2003; Wothke, 2000). The FIML method is generally preferred to listwise deletion and other ad hoc approaches to handling missing data (Schafer & Graham, 2002; Wothke, 2000). (In Mplus, FIML estimation is also available in conjunction with the MLR estimator.)

### *5.3.4 Goodness-of-Fit Assessment*

Goodness-of-fit was assessed using the $\chi^2$ test of model fit, the *Root Mean Square Error of Approximation* (*RMSEA*; Steiger, 1990), and the *Comparative Fit Index* (*CFI*; Bentler, 1990). A nonsignificant $\chi^2$ value indicates that the assumption of exact fit in the population is not rejected, suggesting a good fit of the model to the data. The *RMSEA* coefficient is a measure of approximate fit. *RMSEA* values smaller than .05 point to an acceptable fit. The *CFI* compares the fit of the target model with the fit of a baseline model. The baseline model is a null model that assumes zero covariation among the observed variables. For a good model, the *CFI* should be greater than .95 (Schermelleh-Engel, Moosbrugger, & Müller, 2003). I performed $\chi^2$ difference tests in order to compare nested models. Given the large sample size and the large number of model comparisons, I considered a $\chi^2$ difference as significant only if the $p$-value was $< .01$. As an additional index for model comparisons, I report the *AIC* measure (Akaike, 1974). According to this criterion, the model with the smallest *AIC* value fits the data best.

## 5.4 Application of the CS-C(*M*–1) State Model

### *5.4.1 Assessment of Indicator-Specific Effects*

I used the 3-Step approach to model testing described in Section 5.2 to determine the most appropriate model variant for the Cole et al. data set. In the first step, I tested the three models shown in Figure 18 to find out whether indicator-specific effects were present. I specified the indicator-specific factors to capture the indicator-specificity of the second parcels ($i$ = 2), respectively.

Given that the children were not rated by the same teacher on all occasions of measurement, the assumption of a single indicator-specific factor for each construct seemed too strong for the teacher rating. I therefore included school-year specific factors instead. That is, I specified four (instead of two) indicator-specific factors for the teacher rating (i.e., for both constructs there were separate correlated indicator-specific factors for time one [T1] and T2 as well as for T3 and T4, respectively.) Table 5 contains the goodness-of-fit measures for all analysis steps. The $\chi^2$ difference test for a given step always represents a test against the less restricted model mentioned in parentheses (italicized).

All three baseline model variants (1A-1C) showed a good fit according to the *CFI* and *RMSEA* coefficients. Model 1A contained indicator-specific state factors, but no indicator-specific factors over time. Although this model fit relatively well, it returned parameter estimates that clearly indicated model misspecifications due to an overfactorization. In this model, *all* correlations between the indicator-specific state variables pertaining to the same construct on the same measurement occasion were estimated to be between $r = .95$ and $r = 1.04$. These perfect or close to perfect latent correlations showed that the indicator-specific state factors were homogeneous, implying that indicator-specific effects did not generalize across different methods. This result indicated that it was more appropriate to specify *general* instead of *indicator-specific* state factors for each construct on each measurement occasion (as done in Model 1B).

In Model 1B, the indicator-specific state factors belonging to the same construct on the same measurement occasion were assumed to be homogeneous, whereas indicator-specific effects over time were modeled through indicator-specific factors $IS_{ijk}$. The estimation of Model 1B returned no offending parameter estimates and it showed the best (smallest) *AIC* value of all three models. Dropping the indicator-specific factors over time (Model 1C) to make the model still more parsimonious led to a strong increase in the $\chi^2$ value (indicating worse fit) and a decrease in fit also according to the *AIC* index.

In sum, Step 1 clearly revealed that the generalization of indicator-specificity *across methods within the same measurement occasion* was negligible in the present data, whereas the generalization of indicator-specificity for the same indicator *over time* was not. Given these unequivocal results, I proceeded to Step 2 (measurement invariance testing for the reference method) using Model 1B as the baseline model.

### *5.4.2 Assessment of Measurement Invariance and Model Selection*

Step 2 of the analysis revealed that strict measurement invariance was tenable for the self-report measures, as indicated by the non-significant $\chi^2$ difference values for Models 2A–2E. Step 2F yielded a large and significant $\chi^2$ difference value, implying that the assumption of constant state factor means over time had to be rejected. Detailed analyses showed that the latent means were not invariant, neither for anxiety nor for depression. I thus proceeded to Step 3 (invariance testing for the non-reference methods) using Model 2E (with unconstrained state factor means for both constructs) as the base.

In Step 3, I first considered the parent report variables. It turned out that invariance was tenable for the state and method factor loadings, as well as for the error variances of the parent report measures (*p*s > .01 for the $\chi^2$ difference test in Steps 3A_1, 3B_1, and 3D_1). The variances of the method factors pertaining to the parent report were also invariant over time (Step 3E_1). In contrast, the intercepts of the parent report indicators, as well as the loadings on the indicator-specific factors did not show invariance over time (see Steps 3C_1 and 3F_1). Hence, I continued my analyses with Model 3E_1 and tested for invariance of the teacher report variables. The analysis of the teacher variables revealed that invariance was tenable only for the method factor loadings and error variances (Model 3D_2). All other invariance constraints led to a significant increase in the $\chi^2$ values ($p < .01$). This was also the case when I tested for invariance separately across T1 – T2 and T3 – T4 (where the teachers were the same, respectively). The implication is that for the teachers, there was a change in *structural bias* as will be discussed below.

To summarize, strict measurement invariance was not rejected for the self-report measures, whereas weaker forms of invariance were established for the parent and teacher report indicators. One explanation for the non-invariance of the parameters pertaining to the teacher variables may be that different teachers rated the children on the last two measurement occasions.

In the following, I report detailed outcomes for Model 3D_2 because (1) it was the most parsimonious model that was not rejected by a $\chi^2$ difference test and (2) it was still acceptable when tested against the baseline Model 1B [$\chi^2_{diff}(90) = 117.47$, $p = .03$]. An annotated Mplus input script for estimating this model is provided in the appendix (see Section 13.1).

Table 5

*Goodness-of-Fit Measures for Different CS-C(M–1) Model Variants*

| | | $\chi^2$ test | | | $\chi^2$ difference test | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Step | | value | *df* | *p* | value | *df* | *p* | *CFI* | *RMSEA* | *AIC* |
| Step 1: Determination of the baseline model[a] | | | | | | | | | | |
| 1A | (indicator-specific state factors + no indicator-specific factors across time) | 1,257.05[b] | 680[b] | <.01[b] | —[c] | —[c] | —[c] | .98[b] | .03[b] | –97.63[b] |
| 1B | (general state factors + indicator-specific factors across time) | 707.55 | 636 | .03 | —[c] | —[c] | —[c] | 1.00 | .01 | –580.59 |
| 1C | (general state factors + no indicator-specific factors across time) | 1,611.03 | 836 | <.01 | —[c] | —[c] | —[c] | .97 | .03 | –7.40 |
| Step 2: Assessment of measurement invariance and mean change for the reference method (self-report = SR) | | | | | | | | | | |
| 2A | (*Model 1B* + loadings invariant for SR) | 722.70 | 642 | .02 | 12.99 | 6 | .04 | 1.00 | .01 | –574.14 |
| 2B | (*Model 2A* + intercepts invariant for SR) | 725.66 | 648 | .02 | 2.94 | 6 | .82 | 1.00 | .01 | –582.82 |
| 2C | (*Model 2B* + error variances invariant for SR) | 743.67 | 660 | .01 | 16.42 | 12 | .17 | 1.00 | .01 | –578.90 |
| 2D | (*Model 2C* + state factor variances invariant for SR) | 756.03 | 666 | .01 | 10.86 | 6 | .09 | 1.00 | .01 | –574.60 |
| 2E | (*Model 2D* + loadings on indicator-specific factors invariant) | 765.54 | 672 | .01 | 8.95 | 6 | .18 | 1.00 | .01 | –574.01 |
| 2F | (*Model 2E* + state factor means invariant) | 932.90 | 678 | <.01 | 176.06 | 6 | <.01 | .99 | .02 | –402.91 |
| Step 3: Assessment of measurement invariance for the non-reference methods (parent report = PR, teacher report = TR) | | | | | | | | | | |
| 3A_1 | (*Model 2E* + state factor loadings invariant for PR) | 774.45 | 684 | .01 | 9.05 | 12 | .70 | 1.00 | .01 | –587.35 |

*(Table continues)*

| Step | | $\chi^2$ test | | | $\chi^2$ difference test | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | value | *df* | *p* | value | *df* | *p* | *CFI* | *RMSEA* | *AIC* |
| 3B_1 | (*Model 3A_1* + method factor loadings invariant for PR) | 776.84 | 690 | .01 | 2.77 | 6 | .84 | 1.00 | .01 | –595.94 |
| 3C_1 | (*Model 3B_1* + intercepts invariant for PR) | 801.69 | 702 | .01 | 26.84 | 12 | <.01 | 1.00 | .01 | –593.94 |
| 3D_1 | (*Model 3B_1* + error variances invariant for PR) | 789.81 | 702 | .01 | 13.10 | 12 | .36 | 1.00 | .01 | –602.12 |
| 3E_1 | (*Model 3D_1* + method factor variances invariant PR) | 800.53 | 708 | .01 | 9.27 | 6 | .16 | 1.00 | .01 | –596.35 |
| 3F_1 | (*Model 3E_1* + indicator-specific factor loadings invariant for PR) | 823.39 | 714 | <.01 | 21.04 | 6 | <.01 | 1.00 | .01 | –582.60 |
| 3A_2 | (*Model 3E_1* + state factor loadings invariant for TR) | 830.07 | 720 | <.01 | 28.94 | 12 | <.01 | 1.00 | .01 | –587.33 |
| 3B_2 | (*Model 3E_1* + method factor loadings invariant for TR) | 802.01 | 714 | .01 | 3.01 | 6 | .81 | 1.00 | .01 | –603.64 |
| 3C_2 | (*Model 3B_2* + intercepts invariant for TR) | 836.71 | 726 | <.01 | 27.76 | 12 | <.01 | 1.00 | .01 | –582.17 |
| 3D_2 | (*Model 3B_2* + error variances invariant for TR) | 831.07 | 726 | <.01 | 22.74 | 12 | .03 | 1.00 | .02 | –585.68 |
| 3E_2 | (*Model 3D_2* + method factor variances invariant for TR) | 854.96 | 732 | <.01 | 18.24 | 6 | <.01 | 1.00 | .01 | –566.61 |

*Note.* *N* = 906. SR = self-report. PR = parent report. TR = teacher report. [a]All models tested in Step 1 assume non-invariant parameters, except for the marker indicators. [b]Improper solution with .95 ≤ *r* ≤ 1.04 for eight state factors. [c]$\chi^2$ difference test not applicable due to violation of regularity conditions (see discussion in the text). $\chi^2$ = robust (MLR) chi-square value computed under the complex sample option in Mplus. The procedure described in Satorra & Bentler (1999) was used in order to calculate the correctly scaled $\chi^2$ difference value for the MLR estimator. The $\chi^2$ difference test for a given step is always a test against the less restricted model in mentioned in parentheses (italicized). In case of a significant $\chi^2$ difference, the more restricted model was rejected and invariance testing in the next step was continued using the less restricted model. *CFI* = Comparative Fit Index. *RMSEA* = Root Mean Square Error of Approximation. *AIC* = Akaike's Information Criterion.

### 5.4.3 Convergent Validity and Variance Components

The estimated intercepts, factor loadings, and error variances are presented in Table 6 (for depression) and Table 7 (for anxiety). An important finding is that the state factor loadings of the parent and teacher report measures are much smaller than the loadings of the self-report indicators. This indicates a rather low degree of convergent validity between the self-rating and the other ratings. The lack of convergent validity is still more clearly seen from the variance components (see Table 8 and Table 9). All other-report indicators show very low consistency, but high method-specificity coefficients. For depression, all consistency coefficients associated with the parent and teacher report measures are below .10, indicating that the reference method (self-report) accounts for less than 10% of the variance of these measures. The consistencies of the other-report indicators are even smaller for anxiety (< .05). In fact, for the teacher report, most of them are not significantly different from zero. One might conclude that self-reported anxiety and teacher-reported anxiety represent two almost independent constructs in this age group. An alternative or additional explanation may be that the questionnaires completed by the teachers were different from the questionnaires administered to the children and their parents. It is also likely that the teacher ratings mirror a different facet of anxiety (i.e., anxiety at school/in class) than do the self- and parent reports. However, convergent validity is not much higher for the parent ratings either (although equivalent items were used for the self- and parent report of depression and anxiety). This might indicate that the use of a different questionnaire cannot fully explain the lack of convergence.

With regard to changes in the convergent validity over time, an interesting finding is the slight tendency for the teacher ratings to show somewhat higher convergent validity at the end of a school year (i.e., at the $2^{nd}$ and $4^{th}$ occasion of measurement)—whereas the consistency coefficients for the parent ratings remain stable over time. A possible explanation is that teachers get to know their students better in the course of a school year.

In general, the indicator-specific factors over time account for less than 10% of the variance of the indicators (the values are higher for some of the parent indicators). This shows that the observed variables are rather (but not perfectly) homogeneous indicators of the latent variables. Measurement error plays a minor role in the present application as shown by the high reliability coefficients for all observed variables.

Table 6

*Unstandardized Parameter Estimates for Depression (Model 3D_2)*

| | Intercept ($\alpha_{ijkl}$) | | State factor loading ($\lambda_{Sijkl}$) | | Method factor loading ($\lambda_{Mijkl}$) | | Indicator-specific factor loading ($\lambda_{ISijkl}$) | | Error variance [$Var(E_{ijkl})$] | | Model-implied mean |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | |
| DS11 | 0.00* | — | 1.00* | — | — | — | — | — | 0.01 | 0.00 | 0.32 |
| DS21 | –0.01 | 0.01 | 0.93 | 0.02 | — | — | 1.00* | — | 0.01 | 0.00 | 0.29 |
| DP11 | 0.15 | 0.01 | 0.16 | 0.04 | 1.00* | — | — | — | 0.01 | 0.00 | 0.20 |
| DP21 | 0.19 | 0.01 | 0.16 | 0.03 | 1.03 | 0.04 | 1.00* | — | 0.01 | 0.00 | 0.24 |
| DT11 | 0.62 | 0.06 | 0.29 | 0.07 | 1.00* | — | — | — | 0.02 | 0.00 | 0.71 |
| DT21 | 0.70 | 0.06 | 0.34 | 0.07 | 1.06 | 0.02 | 1.00* | — | 0.04 | 0.00 | 0.81 |
| DS12 | 0.00* | — | 1.00* | — | — | — | — | — | 0.01 | 0.00 | 0.29 |
| DS22 | –0.01 | 0.01 | 0.93 | 0.02 | — | — | 1.00* | — | 0.01 | 0.00 | 0.26 |
| DP12 | 0.14 | 0.01 | 0.16 | 0.04 | 1.00* | — | — | — | 0.01 | 0.00 | 0.18 |
| DP22 | 0.18 | 0.01 | 0.16 | 0.03 | 1.03 | 0.04 | 0.92 | 0.12 | 0.01 | 0.00 | 0.22 |
| DT12 | 0.71 | 0.06 | 0.49 | 0.07 | 1.00* | — | — | — | 0.02 | 0.00 | 0.85 |
| DT22 | 0.82 | 0.06 | 0.54 | 0.07 | 1.06 | 0.02 | 1.00* | — | 0.04 | 0.00 | 0.98 |
| DS13 | 0.00* | — | 1.00* | — | — | — | — | — | 0.01 | 0.00 | 0.29 |
| DS23 | –0.01 | 0.01 | 0.93 | 0.02 | — | — | 1.00* | — | 0.01 | 0.00 | 0.27 |
| DP13 | 0.14 | 0.01 | 0.16 | 0.04 | 1.00* | — | — | — | 0.01 | 0.00 | 0.19 |
| DP23 | 0.18 | 0.01 | 0.16 | 0.03 | 1.03 | 0.04 | 0.74 | 0.12 | 0.01 | 0.00 | 0.22 |

*(Table continues)*

| | Intercept ($\alpha_{ijkl}$) | | State factor loading ($\lambda_{Sijkl}$) | | Method factor loading ($\lambda_{Mijkl}$) | | Indicator-specific factor loading ($\lambda_{ISijkl}$) | | Error variance [$Var(E_{ijkl})$] | | Model-implied mean |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | |
| DT13 | 0.60 | 0.03 | 0.30 | 0.07 | 1.00* | — | — | — | 0.02 | 0.00 | 0.69 |
| DT23 | 0.70 | 0.04 | 0.33 | 0.08 | 1.06 | 0.02 | 1.00* | — | 0.04 | 0.00 | 0.80 |
| DS14 | 0.00* | — | 1.00* | — | — | — | — | — | 0.01 | 0.00 | 0.27 |
| DS24 | –0.01 | 0.01 | 0.93 | 0.02 | — | — | 1.00* | — | 0.01 | 0.00 | 0.25 |
| DP14 | 0.16 | 0.01 | 0.16 | 0.04 | 1.00* | — | — | — | 0.01 | 0.00 | 0.20 |
| DP24 | 0.19 | 0.01 | 0.16 | 0.03 | 1.03 | 0.04 | 0.59 | 0.14 | 0.01 | 0.00 | 0.24 |
| DT14 | 0.62 | 0.04 | 0.57 | 0.06 | 1.00* | — | — | — | 0.02 | 0.00 | 0.78 |
| DT24 | 0.78 | 0.05 | 0.58 | 0.08 | 1.06 | 0.02 | 1.00* | — | 0.04 | 0.00 | 0.94 |

*Note.* DS = depression self-report; DP = depression parent report; DT = depression teacher report; the first number refers to the indicator, whereas the second number indicates the occasion of measurement. Intercepts are time-invariant for the self-report indicators. State factor loadings are time-invariant for the self- and parent report indicators. Method factor loadings are time-invariant for the parent- and teacher report measures. Indicator-specific factor loadings are time-invariant for the self- and teacher indicators. Error variances are time-invariant for all indicators. Fixed parameters are marked with asterisks (*). Dashes (—) indicate factor loadings fixed to zero and standard errors not estimated due to a fixed parameter. The model-implied means can be directly compared across self- and parent indicators, but they are not comparable to the teacher report means given that the teachers used different questionnaires.

Table 7

*Unstandardized Parameter Estimates for Anxiety (Model 3D_2)*

| | Intercept ( $\alpha_{ijkl}$ ) | | State factor loading ( $\lambda_{Sijkl}$ ) | | Method factor loading ( $\lambda_{Mijkl}$ ) | | Indicator-specific factor loading ( $\lambda_{ISijkl}$ ) | | Error variance [ $Var(E_{ijkl})$ ] | | Model-implied mean |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | |
| AS11 | 0.00* | — | 1.00* | — | — | — | — | — | 0.02 | 0.00 | 0.72 |
| AS21 | 0.05 | 0.01 | 0.96 | 0.02 | — | — | 1.00* | — | 0.02 | 0.00 | 0.74 |
| AP11 | 0.41 | 0.02 | 0.12 | 0.03 | 1.00* | — | — | — | 0.01 | 0.00 | 0.50 |
| AP21 | 0.38 | 0.02 | 0.12 | 0.03 | 1.07 | 0.03 | 1.00* | — | 0.01 | 0.00 | 0.47 |
| AT11 | 1.49 | 0.06 | 0.13 | 0.05 | 1.00* | — | — | — | 0.03 | 0.00 | 1.58 |
| AT21 | 1.57 | 0.06 | 0.04 | 0.05 | 0.94 | 0.02 | 1.00* | — | 0.03 | 0.00 | 1.60 |
| AS12 | 0.00* | — | 1.00* | — | — | — | — | — | 0.02 | 0.00 | 0.62 |
| AS22 | 0.05 | 0.01 | 0.96 | 0.02 | — | — | 1.00* | — | 0.02 | 0.00 | 0.64 |
| AP12 | 0.39 | 0.02 | 0.12 | 0.03 | 1.00* | — | — | — | 0.01 | 0.00 | 0.46 |
| AP22 | 0.35 | 0.02 | 0.12 | 0.03 | 1.07 | 0.03 | 0.79 | 0.06 | 0.01 | 0.00 | 0.43 |
| AT12 | 1.59 | 0.07 | 0.19 | 0.04 | 1.00* | — | — | — | 0.03 | 0.00 | 1.70 |
| AT22 | 1.64 | 0.07 | 0.14 | 0.05 | 0.94 | 0.02 | 1.00* | — | 0.03 | 0.00 | 1.72 |
| AS13 | 0.00* | — | 1.00* | — | — | — | — | — | 0.02 | 0.00 | 0.56 |
| AS23 | 0.05 | 0.01 | 0.96 | 0.02 | — | — | 1.00* | — | 0.02 | 0.00 | 0.59 |
| AP13 | 0.37 | 0.02 | 0.12 | 0.03 | 1.00* | — | — | — | 0.01 | 0.00 | 0.44 |
| AP23 | 0.36 | 0.02 | 0.12 | 0.03 | 1.07 | 0.03 | 0.87 | 0.10 | 0.01 | 0.00 | 0.43 |

*(Table continues)*

| | Intercept ($\alpha_{ijkl}$) | | State factor loading ($\lambda_{Sijkl}$) | | Method factor loading ($\lambda_{Mijkl}$) | | Indicator-specific factor loading ($\lambda_{ISijkl}$) | | Error variance [$Var(E_{ijkl})$] | | Model-implied mean |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | Estimate | *SE* | |
| AT13 | 1.56 | 0.03 | 0.06 | 0.04 | 1.00* | — | — | — | 0.03 | 0.00 | 1.59 |
| AT23 | 1.56 | 0.04 | 0.05 | 0.05 | 0.94 | 0.02 | 1.00* | — | 0.03 | 0.00 | 1.59 |
| AS14 | 0.00* | — | 1.00* | — | — | — | — | — | 0.02 | 0.00 | 0.52 |
| AS24 | 0.05 | 0.01 | 0.96 | 0.02 | — | — | 1.00* | — | 0.02 | 0.00 | 0.55 |
| AP14 | 0.39 | 0.02 | 0.12 | 0.03 | 1.00* | — | — | — | 0.01 | 0.00 | 0.45 |
| AP24 | 0.36 | 0.02 | 0.12 | 0.03 | 1.07 | 0.03 | 0.68 | 0.07 | 0.01 | 0.00 | 0.42 |
| AT14 | 1.56 | 0.05 | 0.21 | 0.07 | 1.00* | — | — | — | 0.03 | 0.00 | 1.67 |
| AT24 | 1.58 | 0.05 | 0.19 | 0.06 | 0.94 | 0.02 | 1.00* | — | 0.03 | 0.00 | 1.68 |

*Note.* AS = anxiety self-report; AP = anxiety parent report; AT = anxiety teacher report; the first number refers to the indicator whereas the second number indicates the occasion of measurement. Intercepts are time-invariant for the self-report indicators. State factor loadings are time-invariant for the self- and parent report indicators. Method factor loadings are time-invariant for the parent- and teacher report measures. Indicator-specific factor loadings are time-invariant for the self- and teacher indicators. Error variances are time-invariant for all indicators. Fixed parameters are marked with asterisks (*). Dashes (—) indicate factor loadings fixed to zero and standard errors not estimated due to a fixed parameter. The model-implied means can be directly compared across self- and parent indicators, but they are not comparable to the teacher report means given that the teachers used different questionnaires.

Table 8

*Variance Components for Depression (Model 3D_2)*

| | Observed variables $Y_{ijkl}$ | | | | Latent state variables $S_{ijkl}$ | | |
|---|---|---|---|---|---|---|---|
| | Consistency $CO(Y_{ijkl})$ | Method-specificity $MS(Y_{ijkl})$ | Indicator-specificity $IS(Y_{ijkl})$ | Reliability $Rel(Y_{ijkl})$ | Consistency $CO(S_{ijkl})$ | Method-specificity $MS(S_{ijkl})$ | Indicator-specificity $IS(S_{ijkl})$ |
| DS11 | .86 | | | .86 | 1.00 | | |
| DS21 | .82 | | .04 | .87 | .95 | | .05 |
| DP11 | .05 | .80 | | .85 | .06 | .94 | |
| DP21 | .04 | .71 | .13 | .87 | .04 | .81 | .15 |
| DT11 | .02 | .91 | | .93 | .02 | .98 | |
| DT21 | .02 | .83 | .06 | .91 | .03 | .91 | .06 |
| DS12 | .86 | | | .86 | 1.00 | | |
| DS22 | .82 | | .04 | .87 | .95 | | .05 |
| DP12 | .05 | .80 | | .85 | .06 | .94 | |
| DP22 | .04 | .72 | .11 | .87 | .04 | .83 | .13 |
| DT12 | .05 | .88 | | .94 | .06 | .94 | |
| DT22 | .05 | .81 | .05 | .92 | .06 | .88 | .06 |
| DS13 | .86 | | | .86 | 1.00 | | |
| DS23 | .82 | | .04 | .87 | .95 | | .05 |
| DP13 | .05 | .80 | | .85 | .06 | .94 | |
| DP23 | .04 | .75 | .07 | .86 | .05 | .87 | .09 |
| DT13 | .03 | .89 | | .92 | .03 | .97 | |
| DT23 | .03 | .81 | .06 | .89 | .03 | .90 | .07 |
| DS14 | .86 | | | .86 | 1.00 | | |
| DS24 | .82 | | .04 | .87 | .95 | | .05 |
| DP14 | .05 | .80 | | .85 | .06 | .94 | |
| DP24 | .04 | .77 | .05 | .86 | .05 | .90 | .06 |
| DT14 | .10 | .82 | | .92 | .11 | .89 | |
| DT24 | .08 | .75 | .06 | .89 | .09 | .84 | .07 |

*Note.* DS = depression self-report; DP = depression parent report; DT = depression teacher report; the first number refers to the indicator, whereas the second number indicates the occasion of measurement. Rounding errors may prevent the consistency, method-specificity, and indicator-specificity coefficients to exactly add up to the reliability coefficient for the observed variables. For the same reason, the consistency, method-specificity, and indicator-specificity coefficients may not exactly add up to one for the latent state variables.

Table 9

*Variance Components for Anxiety (Model 3D_2)*

| | Observed variables $Y_{ijkl}$ | | | | Latent state variables $S_{ijkl}$ | | |
|---|---|---|---|---|---|---|---|
| | Consistency $CO(Y_{ijkl})$ | Method-specificity $MS(Y_{ijkl})$ | Indicator-specificity $IS(Y_{ijkl})$ | Reliability $Rel(Y_{ijkl})$ | Consistency $CO(S_{ijkl})$ | Method-specificity $MS(S_{ijkl})$ | Indicator-specificity $IS(S_{ijkl})$ |
| AS11 | .89 | | | .89 | 1.00 | | |
| AS21 | .84 | | .05 | .90 | .94 | | .06 |
| AP11 | .03 | .85 | | .88 | .03 | .97 | |
| AP21 | .02 | .70 | .17 | .90 | .02 | .79 | .19 |
| AT11 | .01 | .89 | | .90 | .01 | .99 | |
| AT21 | .00 | .80 | .10 | .90 | 1.00 | .89 | .11 |
| AS12 | .89 | | | .89 | .00 | | |
| AS22 | .84 | | .05 | .90 | .94 | | .06 |
| AP12 | .03 | .85 | | .88 | .03 | .97 | |
| AP22 | .02 | .75 | .11 | .89 | .03 | .85 | .13 |
| AT12 | .02 | .90 | | .91 | .02 | .98 | |
| AT22 | .01 | .81 | .08 | .91 | .01 | .90 | .09 |
| AS13 | .89 | | | .89 | 1.00 | | |
| AS23 | .84 | | .05 | .90 | .94 | | .06 |
| AP13 | .03 | .85 | | .88 | .03 | .97 | |
| AP23 | .02 | .73 | .13 | .89 | .03 | .82 | .15 |
| AT13 | .00 | .88 | | .88 | .00 | 1.00 | |
| AT23 | .00 | .83 | .04 | .88 | .00 | .95 | .05 |
| AS14 | .89 | | | .89 | 1.00 | | |
| AS24 | .84 | | .05 | .90 | .94 | | .06 |
| AP14 | .03 | .85 | | .88 | .03 | .97 | |
| AP24 | .02 | .77 | .09 | .88 | .03 | .88 | .10 |
| AT14 | .03 | .87 | | .90 | .03 | .97 | |
| AT24 | .02 | .83 | .04 | .89 | .03 | .93 | .04 |

*Note.* AS = anxiety self-report; AP = anxiety parent report; AT = anxiety teacher report; the first number refers to the indicator, whereas the second number indicates the occasion of measurement. Rounding errors may prevent the consistency, method-specificity, and indicator-specificity coefficients to exactly add up to the reliability coefficient for the observed variables. For the same reason, the consistency, method-specificity, and indicator-specificity coefficients may not exactly add up to one for the latent state variables.

### 5.4.4 Discriminant Validity

The latent state factors representing depression and anxiety are highly correlated (see Table 10). The correlations are particularly high on the same occasion of measurement, indicating the presence of occasion-specific effects. One plausible explanation for the high correlations is that depression and anxiety constitute two closely related concepts. An alternative interpretation is that the measures do not sufficiently discriminate between the two constructs (low discriminant validity of the scales).

### 5.4.5 Construct Stability and Mean Change

Correlations between latent state factors belonging to the same construct measured on different occasions are also rather high, indicating that inter-individual differences with regard to both constructs are stable over time (see Table 10). However, none of the stability coefficients is equal to one, which means that neither depression nor anxiety can be conceived of as perfectly stable traits.

As indicated by the series of invariance tests, the state factor means changed for both, depression and anxiety. The latent means reported in Table 10 reveal a decrease in the mean level of latent self-reported depression and anxiety over time. Interestingly, the mean decrease appears to be stronger for anxiety than for depression, indicating discriminant validity with regard to mean change over time. A detailed analysis of the means of the parent report indicators revealed that the depression means did not significantly change over time, whereas the means of the parent indicators of anxiety differed significantly over time. For the teachers, the means of both the depression and anxiety indicators changed significantly over time.

The estimated model-implied means for the parent report (see Table 6 and Table 7) indicate that parents on average underestimated the depression and anxiety level of their children (as compared to the children's self-report). Furthermore, the model-implied means of the parent ratings were almost constant over time for both constructs. Hence, the mean trajectories derived from self-report do not match the mean trajectories according to the parent ratings. This can be interpreted as a lack of "convergent validity of mean change". On average, parents appeared to be rather insensitive to changes in their children's depression and anxiety levels. The mean trajectories derived from the teacher ratings are not directly comparable to the self- and parent report means given that the teachers used different scales and given that different teachers rated the children on the last two measurement occasions.

Table 10

*State Factor Covariances, Correlations, Means, and Variances (Model 3D_2)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1. $S_{11}$ (Depression T1) | — | 0.06 (0.01) | 0.05 (0.01) | 0.05 (0.01) | 0.09 (0.01) | 0.07 (0.01) | 0.07 (0.01) | 0.07 (0.01) |
| 2. $S_{12}$ (Depression T2) | .72 | — | 0.07 (0.01) | 0.06 (0.01) | 0.07 (0.01) | 0.09 (0.01) | 0.08 (0.01) | 0.08 (0.01) |
| 3. $S_{13}$ (Depression T3) | .62 | .75 | — | 0.07 (0.01) | 0.06 (0.01) | 0.08 (0.01) | 0.10 (0.01) | 0.09 (0.01) |
| 4. $S_{14}$ (Depression T4) | .58 | .68 | .81 | — | 0.06 (0.01) | 0.07 (0.01) | 0.08 (0.01) | 0.11 (0.01) |
| 5. $S_{21}$ (Anxiety T1) | .74 | .59 | .52 | .48 | — | 0.13 (0.01) | 0.12 (0.01) | 0.11 (0.01) |
| 6. $S_{22}$ (Anxiety T2) | .60 | .74 | .62 | .58 | .75 | — | 0.14 (0.01) | 0.13 (0.01) |
| 7. $S_{23}$ (Anxiety T3) | .56 | .65 | .78 | .66 | .67 | .80 | — | 0.15 (0.01) |
| 8. $S_{24}$ (Anxiety T4) | .57 | .65 | .71 | .85 | .61 | .76 | .84 | — |
| Means | 0.32 (0.01) | 0.29 (0.01) | 0.29 (0.01) | 0.27 (0.01) | 0.72 (0.02) | 0.62 (0.02) | 0.56 (0.02) | 0.52 (0.02) |
| Variances | 0.09 (0.01) | 0.09 (0.01) | 0.09 (0.01) | 0.09 (0.01) | 0.18 (0.01) | 0.18 (0.01) | 0.18 (0.01) | 0.18 (0.01) |

*Note.* Correlations are printed below, and covariances above the main diagonal. Values in parentheses are standard errors. The state factor variances were constrained to be time-invariant.

### *5.4.6 Generalizability and Stability of Method Effects*

The correlations and covariances among the method factors are shown in Table 11. It can be seen that method effects generalized to a large degree across constructs in the present application. This is shown by the high correlations among method factors belonging to the same method (i.e., parent or teacher report), but different constructs, on the same measurement occasion. These correlations are smaller for method factors belonging to the same method, different constructs, and *different* occasions (for the teacher report, I consider only the correlations between the T1 and T2, as well as T3 and T4 method factors since teachers changed from T2 to T3). Another important result is that method effects were highly stable over time. This is shown by the high correlations between method factors belonging to the same construct-method unit, but different occasions of measurement. In sum, the high correlations found for the same type of rater across constructs and across time indicate the presence of general rater-specific influences.

The correlations between method factors belonging to the same construct, the same measurement occasion, and different methods are all positive and of small to medium size. This shows that parents and teachers exhibit a *common* rater bias: There is a tendency for parents and teachers to have a common view of a child that is not shared with the child's own view. However, the correlations are not very large, indicating that the common bias is rather small compared to the specific bias of each method.

Finally, indicator-specific effects generalize to some degree across self- and parent-ratings as indicated by medium-size correlations between the indicator-specific factors pertaining to the self- and parent-ratings and the same construct (for depression: $r = .262$; for anxiety: $r = .270$; not shown in a table). Correlations including indicator-specific factors pertaining to the teacher-rating were not considered given that the teacher-rating scales of depression and anxiety differed from the self- and parent report scales.

Table 11

*Method Factor Covariances, Correlations, and Variances (Model 3D_2)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. $M_{121}$ (PR depression 1) | — | 0.03 (0.00) | 0.02 (0.00) | 0.02 (0.00) | 0.04 (0.00) | 0.03 (0.00) | 0.03 (0.00) | 0.03 (0.00) | 0.03 (0.01) | 0.03 (0.01) | 0.02 (0.01) | 0.01 (0.01) | 0.03 (0.01) | 0.03 (0.01) | 0.01 (0.01) | 0.01 (0.01) |
| 2. $M_{122}$ (PR depression 2) | .82 | — | 0.03 (0.00) | 0.03 (0.00) | 0.04 (0.00) | 0.04 (0.00) | 0.03 (0.01) | 0.03 (0.01) | 0.02 (0.01) | 0.03 (0.01) | 0.01 (0.01) | 0.01 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.01 (0.01) | 0.01 (0.01) |
| 3. $M_{123}$ (PR depression 3) | .67 | .70 | — | 0.03 (0.00) | 0.03 (0.00) | 0.03 (0.00) | 0.04 (0.00) | 0.03 (0.00) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.01 (0.01) |
| 4. $M_{124}$ (PR depression 4) | .64 | .73 | .79 | — | 0.03 (0.00) | 0.03 (0.00) | 0.04 (0.00) | 0.04 (0.00) | 0.01 (0.01) | 0.01 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.01 (0.01) | 0.02 (0.01) | 0.01 (0.01) | 0.01 (0.01) |
| 5. $M_{221}$ (PR anxiety 1) | .76 | .70 | .53 | .60 | — | 0.06 (0.01) | 0.06 (0.01) | 0.05 (0.01) | 0.04 (0.01) | 0.04 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.04 (0.01) | 0.03 (0.01) | 0.01 (0.01) | 0.01 (0.01) |
| 6. $M_{222}$ (PR anxiety 2) | .65 | .83 | .59 | .61 | .79 | — | 0.06 (0.01) | 0.06 (0.01) | 0.03 (0.01) | 0.03 (0.01) | 0.01 (0.01) | 0.02 (0.01) | 0.03 (0.01) | 0.03 (0.01) | 0.01 (0.01) | 0.01 (0.01) |
| 7. $M_{223}$ (PR anxiety 3) | .57 | .62 | .81 | .69 | .73 | .76 | — | 0.06 (0.01) | 0.04 (0.01) | 0.03 (0.01) | 0.03 (0.01) | 0.02 (0.01) | 0.03 (0.01) | 0.03 (0.01) | 0.02 (0.01) | 0.01 (0.01) |
| 8. $M_{224}$ (PR anxiety 4) | .54 | .61 | .63 | .83 | .70 | .75 | .83 | — | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.02 (0.01) | 0.03 (0.01) | 0.02 (0.01) | 0.02 (0.01) |
| 9. $M_{131}$ (TR depression 1) | .30 | .23 | .21 | .10 | .27 | .22 | .23 | .14 | — | 0.23 (0.02) | 0.07 (0.01) | 0.06 (0.02) | 0.24 (0.03) | 0.20 (0.02) | 0.03 (0.01) | 0.03 (0.02) |
| 10. $M_{132}$ (TR depression 2) | .31 | .23 | .19 | .13 | .24 | .17 | .19 | .13 | .72 | — | 0.07 (0.01) | 0.06 (0.02) | 0.19 (0.02) | 0.26 (0.02) | 0.05 (0.01) | 0.03 (0.02) |
| 11. $M_{133}$ (TR depression 3) | .16 | .15 | .25 | .20 | .12 | .08 | .20 | .15 | .24 | .23 | — | 0.16 (0.02) | 0.05 (0.01) | 0.05 (0.02) | 0.21 (0.01) | 0.13 (0.02) |
| 12. $M_{134}$ (TR depression 4) | .13 | .14 | .20 | .22 | .13 | .11 | .17 | .13 | .21 | .20 | .65 | — | 0.03 (0.02) | 0.04 (0.02) | 0.14 (0.02) | 0.19 (0.02) |
| 13. $M_{231}$ (TR anxiety 1) | .29 | .22 | .22 | .12 | .25 | .21 | .22 | .15 | .82 | .63 | .17 | .11 | — | 0.23 (0.02) | 0.02 (0.02) | 0.02 (0.02) |
| 14. $M_{232}$ (TR anxiety 2) | .24 | .23 | .19 | .15 | .22 | .21 | .20 | .16 | .62 | .80 | .18 | .14 | .77 | — | 0.03 (0.02) | 0.03 (0.02) |
| 15. $M_{233}$ (TR anxiety 3) | .12 | .12 | .18 | .13 | .04 | .05 | .12 | .14 | .12 | .16 | .85 | .58 | .09 | .11 | — | 0.16 (0.02) |
| 16. $M_{234}$ (TR anxiety 4) | .08 | .10 | .11 | .12 | .07 | .09 | .09 | .12 | .10 | .10 | .52 | .78 | .07 | .09 | .62 | — |
| Variances | 0.04 (0.00) | 0.04 (0.00) | 0.04 (0.00) | 0.04 (0.00) | 0.08 (0.01) | 0.08 (0.01) | 0.08 (0.01) | 0.08 (0.01) | 0.31 (0.03) | 0.33 (0.02) | 0.25 (0.02) | 0.23 (0.02) | 0.28 (0.03) | 0.32 (0.03) | 0.24 (0.01) | 0.26 (0.01) |

*Note.* PR = parent report. TR = teacher report. Correlations are printed below, and covariances above the main diagonal. Values in parentheses are standard errors.

## 5.5 Application of the CS-C(*M*–1) Change Models

For the illustration of the CS-C(*M*–1) change models, I analyzed a reduced set of variables. Depression and anxiety were again selected as constructs. I did not consider the teacher report indicators for the change models given that the teacher rating scales had failed to show measurement invariance over time in the application of the CS-C(*M*–1) state model reported in Section 5.4. Furthermore, I did not consider the fourth occasion of measurement given that three waves are sufficient to illustrate both the baseline and the neighbor change version. I again used the children's self-report as the reference method and the parent report as non-reference method to be contrasted against the self-report.

### *5.5.1 Assessment of Measurement Invariance and Model Selection*

In the first step, I again tested for measurement invariance over time by comparing three models. A model with general state factors and indicator-specific factors over time for the second indicator served as the base, as this model had shown the best fit to the data in the application of the state version of the CS-C(*M*–1) model. The first version of this model included no equality constraints for any parameter over time (except for the marker indicators). In the second model, I constrained the state, method, and indicator-specific factor loadings as well as the measurement intercepts of all indicators to be time-invariant. In the third model, also the error variances were set equal over time for all indicators.

Table 12 shows goodness-of-fit statistics for the three model variants. (Note that the fit measures are shown only once given that the state, baseline change, and neighbor change versions are statistically equivalent and produce the same fit to the data.) As shown in Table 12, all three models fit the data very well according to the $\chi^2$ test, the *CFI* coefficient, and the *RMSEA* coefficient. According to the $\chi^2$ difference test, the most restricted version, in which all parameters of the measurement model were constrained to be time-invariant (Model 3), did not fit significantly worse than the less constrained version, in which the error variances were allowed to take on any value (Model 2). The *AIC* values for Model 2 and Model 3 differed only marginally. Therefore, I will report detailed outcomes for Model 3. Annotated Mplus input scripts for estimating the change versions of Model 3 are available from the Appendix (see Section 13.1.2).

### *5.5.2 Convergent Validity and Variance Components*

Table 13 provides the parameter estimates for the CS-C(*M*–1) measurement model. These parameters can be obtained either from the state or from the change versions of the model (and they are identical across these model variants). Table 14 contains the variance components for the observed variables calculated from the state version of the CS-C(*M*–1) model. Again, for both depression and anxiety, the parent rating indicators show very low consistency and very high method-specificity coefficients. Parents' views of the children strongly diverged from the children's own perspective. Indicator-specific effects play a larger role in the parent than in the self-report variables. An explanation may be that parents differentiate more strongly between different facets of depression and anxiety than do the children themselves. The highest indicator-specificity coefficient is .16. This shows that although indicator-specific effects are present, they play a relatively minor role (the indicators of a construct are rather homogeneous).

Table 12

*Goodness-of-Fit Measures for Different CS-C(M–1) Change Model Variants*

| | $\chi^2$ test | | | $\chi^2$ difference test | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | value | *df* | *p* | value | *df* | *p* | *CFI* | *RMSEA* | *AIC* |
| Model 1 (Configural invariance) | 171.24 | 150 | .11 | | | | 1.00 | .01 | -5,489.09 |
| Model 2 (Strong factorial invariance) | 203.43 | 186 | .18 | 32.51 | 36 | .64 | 1.00 | .01 | -5,521.26 |
| Model 3 (Strict factorial invariance) | 227.83 | 202 | .10 | 23.43 | 16 | .10 | 1.00 | .01 | -5,520.78 |

*Note.* $N = 906$. Configural invariance = no invariance constraints on any parameters, except for the marker indicators. Strong factorial invariance = time invariant loadings and intercepts for all indicators. Strict factorial invariance = time invariant loadings, intercepts, and error variances for all indicators. The $\chi^2$ difference test for a given model is always a test against the previous, less restricted model. *CFI* = Comparative Fit Index. *RMSEA* = Root Mean Square Error of Approximation. *AIC* = Akaike's Information Criterion.

Table 15 shows the estimated variance components for the observed change scores from both the baseline and the neighbor change version (shaded cells). The estimates in Table 15 provide information on the convergent validity of change of the indicators. The observed change scores based on the parent ratings show very low consistencies and very high method-specificities. The reliability estimates for the observed change scores are clearly lower than the reliabilities of the variables $Y_{ijkl}$ on a single time point (see Table 14). The reason is that measurement error influences of *both* time points ($l$ and $l'$) have an impact on the change scores $(Y_{ijkl} - Y_{ijkl'})$, as discussed in Chapter 4.7. This is not the case for the observed variables

$Y_{ijkl}$ on a single time point. The unreliability of the observed change scores clearly demonstrates the advantages of using SEM for studying change. As I noted before, *observed* change scores are greatly distorted by measurement error. In contrast, *latent* change scores are corrected for measurement error.

Table 13

*Estimated Intercepts, Factor Loadings, and Error Variances (Model 3)*

| | Intercept ($\alpha_{ijkl}$) | State factor loading ($\lambda_{Sijkl}$) | | | Method factor loading ($\lambda_{Mijkl}$) | | | Indicator-specific factor loading ($\lambda_{ISijkl}$) | | | Error variance [$Var(E_{ijkl})$] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Estimate | *SE* | Standardized estimate | Estimate | *SE* | Standardized estimate | Estimate | *SE* | Standardized estimate | |
| DS11 | 0.00* | 1.00* | — | .94 | — | — | — | — | — | — | 0.01 |
| DS21 | –0.00 | 0.92 | 0.02 | .91 | — | — | — | 1.00* | — | .21 | 0.01 |
| DP11 | 0.13 | 0.18 | 0.03 | .26 | 1.00* | — | .89 | — | — | — | 0.01 |
| DP21 | 0.17 | 0.19 | 0.03 | .24 | 1.01 | 0.03 | .83 | 1.00* | — | .33 | 0.01 |
| DS12 | 0.00* | 1.00* | — | .93 | — | — | — | — | — | — | 0.01 |
| DS22 | –0.00 | 0.92 | 0.02 | .90 | — | — | — | 1.00* | — | .22 | 0.01 |
| DP12 | 0.13 | 0.18 | 0.03 | .27 | 1.00* | — | .87 | — | — | — | 0.01 |
| DP22 | 0.17 | 0.19 | 0.03 | .25 | 1.01 | 0.03 | .80 | 1.00* | — | .36 | 0.01 |
| DS13 | 0.00* | 1.00* | — | .93 | — | — | — | — | — | — | 0.01 |
| DS23 | –0.00 | 0.92 | 0.02 | .90 | — | — | — | 1.00* | — | .22 | 0.01 |
| DP13 | 0.13 | 0.18 | 0.03 | .25 | 1.00* | — | .89 | — | — | — | 0.01 |
| DP23 | 0.17 | 0.19 | 0.03 | .23 | 1.01 | 0.03 | .83 | 1.00* | — | .33 | 0.01 |
| AS11 | 0.00* | 1.00* | — | .95 | — | — | — | — | — | — | 0.01 |
| AS21 | 0.06 | 0.94 | 0.02 | .91 | — | — | — | 1.00* | — | .24 | 0.02 |
| AP11 | 0.36 | 0.16 | 0.02 | .22 | 1.00* | — | .92 | — | — | — | 0.01 |

*(Table continues)*

| | Intercept ($\alpha_{ijkl}$) | State factor loading ($\lambda_{Sijkl}$) | | | Method factor loading ($\lambda_{Mijkl}$) | | | Indicator-specific factor loading ($\lambda_{ISijkl}$) | | | Error variance [$Var(E_{ijkl})$] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Estimate | *SE* | Standardized estimate | Estimate | *SE* | Standardized estimate | Estimate | *SE* | Standardized estimate | |
| AP21 | 0.33 | 0.17 | 0.03 | .20 | 1.07 | 0.04 | .85 | 1.00* | — | .36 | 0.01 |
| AS12 | 0.00* | 1.00* | — | .95 | — | — | — | — | — | — | 0.01 |
| AS22 | 0.06 | 0.94 | 0.02 | .92 | — | — | — | 1.00* | — | .23 | 0.02 |
| AP12 | 0.36 | 0.16 | 0.02 | .24 | 1.00* | — | .91 | — | — | — | 0.01 |
| AP22 | 0.33 | 0.17 | 0.03 | .22 | 1.07 | 0.04 | .83 | 1.00* | — | .37 | 0.01 |
| AS13 | 0.00* | 1.00* | — | .95 | — | — | — | — | — | — | 0.01 |
| AS23 | 0.06 | 0.94 | 0.02 | .91 | — | — | — | 1.00* | — | .24 | 0.02 |
| AP13 | 0.36 | 0.16 | 0.02 | .22 | 1.00* | — | .91 | — | — | — | 0.01 |
| AP23 | 0.33 | 0.17 | 0.03 | .20 | 1.07 | 0.04 | .84 | 1.00* | — | .37 | 0.01 |

*Note.* DS = depression self report indicator; DP = depression parent report indicator; AS = anxiety self report indicator; AP = anxiety parent report indicator; the first number refers to the indicator, whereas the second number refers to the occasion of measurement. Fixed parameters are marked with an asterisk (*). Standard errors are not available for fixed parameters. All unstandardized parameters are time-invariant.

Table 14

*Variance Components (Model 3)*

| | Observed variables $Y_{ijkl}$ | | | | Latent state variables $S_{ijkl}$ | | |
|---|---|---|---|---|---|---|---|
| | Consistency $CO(Y_{ijkl})$ | Method-specificity $MS(Y_{ijkl})$ | Indicator-specificity $IS(Y_{ijkl})$ | Reliability $Rel(Y_{ijkl})$ | Consistency $CO(S_{ijkl})$ | Method-specificity $MS(S_{ijkl})$ | Indicator-specificity $IS(S_{ijkl})$ |
| DS11 | .87 | | | .87 | 1.00 | | |
| DS21 | .83 | | .04 | .88 | .95 | | .05 |
| DP11 | .07 | .78 | | .85 | .08 | .92 | |
| DP21 | .06 | .69 | .11 | .85 | .07 | .80 | .13 |
| DS12 | .86 | | | .86 | 1.00 | | |
| DS22 | .82 | | .04 | .87 | .95 | | .05 |
| DP12 | .07 | .76 | | .83 | .09 | .91 | |
| DP22 | .06 | .65 | .12 | .84 | .07 | .78 | .15 |
| DS13 | .86 | | | .86 | 1.00 | | |
| DS23 | .82 | | .05 | .86 | .95 | | .05 |
| DP13 | .06 | .79 | | .85 | .07 | .93 | |
| DP23 | .05 | .70 | .11 | .86 | .06 | .81 | .13 |
| AS11 | .91 | | | .91 | 1.00 | | |
| AS21 | .83 | | .06 | .89 | .94 | | .06 |
| AP11 | .05 | .84 | | .89 | .05 | .95 | |
| AP21 | .04 | .72 | .13 | .89 | .05 | .81 | .14 |
| AS12 | .91 | | | .91 | 1.00 | | |
| AS22 | .84 | | .05 | .89 | .94 | | .06 |
| AP12 | .06 | .82 | | .87 | .06 | .94 | |
| AP22 | .05 | .69 | .14 | .88 | .05 | .79 | .16 |
| AS13 | .90 | | | .90 | 1.00 | | |
| AS23 | .82 | | .06 | .88 | .93 | | .07 |
| AP13 | .05 | .83 | | .88 | .05 | .95 | |
| AP23 | .04 | .71 | .13 | .88 | .04 | .80 | .15 |

*Note.* DS = depression self-report indicator; DP = depression parent report indicator; AS = anxiety self-report indicator; AP = anxiety parent report indicator; the first number refers to the indicator, whereas the second number refers to the occasion of measurement. Rounding errors may prevent the consistency, method-specificity, and indicator-specificity coefficients to exactly add up to the reliability coefficient for the observed variables. For the same reason, the consistency, method-specificity, and indicator-specificity coefficients may not exactly add up to one for the latent state variables.

Table 15

*Variance Components for Change Scores (Model 3)*

| | Observed difference variables $(Y_{ijkl'} - Y_{ijkl})$ | | | Latent difference variables $(S_{ijkl'} - S_{ijkl})$ | |
|---|---|---|---|---|---|
| | Consistency $CO(Y_{ijkl'} - Y_{ijkl})$ | Method-specificity $MS(Y_{ijkl'} - Y_{ijkl})$ | Reliability $Rel(Y_{ijkl'} - Y_{ijkl})$ | Consistency $CO(S_{ijkl'} - S_{ijkl})$ | Method-specificity $MS(S_{ijkl'} - S_{ijkl})$ |
| (DS12-DS11) | .64 | | .64 | 1.00 | |
| (DS22-DS21) | .64 | | .64 | 1.00 | |
| (DP12-DP11) | .06 | .43 | .49 | .12 | .88 |
| (DP22-DP21) | .06 | .41 | .47 | .12 | .88 |
| (DS13-DS11) | .71 | | .71 | 1.00 | |
| (DS23-DS21) | .71 | | .71 | 1.00 | |
| (DS13-DS12) | .62 | | .62 | 1.00 | |
| (DS23-DS22) | .61 | | .61 | 1.00 | |
| (DP13-DP11) | .06 | .60 | .66 | .09 | .91 |
| (DP23-DP21) | .05 | .58 | .64 | .09 | .91 |
| (DP13-DP12) | .04 | .59 | .63 | .06 | .94 |
| (DP23-DP22) | .04 | .56 | .60 | .06 | .94 |
| (AS12-AS11) | .70 | | .70 | 1.00 | |
| (AS22-AS21) | .65 | | .65 | 1.00 | |
| (AP12-AP11) | .04 | .57 | .61 | .07 | .93 |
| (AP22-AP21) | .04 | .54 | .58 | .07 | .93 |
| (AS13-AS11) | .75 | | .75 | 1.00 | |
| (AS23-AS21) | .69 | | .69 | 1.00 | |
| (AS13-AS11) | .65 | | .65 | 1.00 | |
| (AS23-AS21) | .59 | | .59 | 1.00 | |
| (AP13-AP11) | .04 | .64 | .68 | .06 | .94 |
| (AP23-AP21) | .04 | .62 | .66 | .06 | .94 |
| (AP13-AP12) | .03 | .61 | .64 | .05 | .95 |
| (AP23-AP22) | .03 | .59 | .62 | .04 | .96 |

*Note.* DS = depression self-report indicator; DP = depression parent report indicator; AS = anxiety self-report indicator; AP = anxiety parent report indicator; the first number refers to the indicator, whereas the second number refers to the occasion of measurement. Shaded cells indicate estimates derived from the neighbor change version. Rounding errors may prevent the consistency, method-specificity, and indicator-specificity coefficients to exactly add up to the reliability coefficient for the observed variables. For the same reason, the consistency, method-specificity, and indicator-specificity coefficients may not exactly add up to one for the latent state variables.

### 5.5.3 Structural Model: Discriminant Validity of Change

Table 16 and Table 17 contain the estimated covariances, correlations, variances, and means for the latent state and latent difference version, respectively. The variances of all latent difference factors are statistically significant. This shows that there are inter-individual differences in intra-individual change for all constructs and method effects in the present application. This can also be seen from the state version, in which the stability coefficients for the state and method factors are all smaller than one, see Table 16.

The means of the state difference factors are all negative indicating that on average there is a decline in both depression and anxiety over time. All means are significantly different from zero except the mean of the latent state difference factor for depression T3–T2. The standardized mean differences are rather small for depression (effect size measure Cohen's *d*: –0.20 [T2–T1], –0.16 [T3–T1], and .01 [T3–T2][10]) and moderate for anxiety [*d* values: –0.48 [T2–T1], –0.66 [T3–T1], and –0.30 [T3–T2]).

The estimated covariances and correlations between the latent state and latent state difference factors (see Table 17) show that for both constructs, the latent difference variables are negatively correlated with the initial status (T1) state factors. Hence, on average, children with smaller T1 scores have larger change scores than children with larger T1 scores. The correlations among the latent difference variables are positive, indicating that individuals with greater change scores for one time interval tend to have greater change scores also for the remaining intervals. The inter-correlations of latent difference scores *across* constructs indicate discriminant validity of change processes. In the present case, these correlations are positive. Hence, there is a tendency for children showing an increase (or decrease) in depression over time to show an increase (or decrease) in anxiety as well. The correlations are relative high indicating low discriminant validity of change.

An interesting finding is that some of the correlations between latent difference variables belonging to the same construct are negative: The correlation between *Depression 2 – Depression 1* and *Depression 3 – Depression 2* is –.27, and between *Anxiety 2 – Anxiety 1* and *Anxiety 3 – Anxiety 2* the correlation is estimated to be –.10. These negative correlations show that although there is a slight mean decrease of depression and anxiety over time, change scores for adjacent time points do not generally agree with respect to the direction of change. For example, there is a tendency for a child with an *increasing* depression score from T1 to

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[10] Cohen's *d* was calculated using the following formula: $d = \frac{\sqrt{2} \cdot [E(S_{j1l'} - S_{j1l})]}{\sqrt{Var(S_{j1l'} - S_{j1l})}}$, where $l' > l$.

T2 to show a *decrease* in depression from T2 to T3 rather than a continuing increase. This indicates that the underlying process is *variability* rather than general growth or decline, especially for depression.

The correlations among the method and method difference factors show a similar pattern. Like the state difference factors, the method change scores are also negatively correlated with the corresponding T1 method factors. In contrast, the method difference factors are positively correlated with other method difference factors belonging to the same TMU. Furthermore, method difference factors are highly positively correlated with other method difference factors belonging to the same method and different constructs for the *same* change period.

### 5.5.4 Structural Model: Correlations With Sex

In order to demonstrate the inclusion of covariates, I added the variable *sex* as a correlate of the latent state and latent difference factors. The extended model also fit the data well ($\chi^2 = 229.61$, $df = 210$, $p = .17$; $RMSEA = .01$; $CFI = 1.00$). The Mplus input files for estimating the baseline and neighbor change versions of this model are provided in the appendix (see Sections 13.1.2 and 13.1.3).

Given that (1) I had no specific a priori hypotheses and (2) there were 12 correlations of interest (I did not attempt to interpret the correlations between sex and the indicator-specific factors), I used two-tailed *z*-tests in conjunction with a Bonferroni adjusted alpha level of .05/12 = .004 to identify significant correlations with sex (i.e., I did not interpret correlations with sex as significantly different from zero unless the actually calculated two-tailed *p*-value was smaller than .004.) According to this criterion, five correlations in the state version and two correlations in the change version were significantly different from zero (all $p$s < .001). For the state version, these were the correlations between sex and (1) the depression state factor at T3 ($r = .17$, $SE = .04$; $z = 3.95$), (2) the method factor for depression at T1 ($r = -.14$, $SE = .04$; $z = -3.57$), and (3) the anxiety state factors at all three time points T1–T3 ($r$s = .23, .27, .28; all $SE$s = .03; $z$-scores = 6.97, 8.52, 7.99, respectively). For the change version, the only significant correlations with sex were found for the anxiety state factor at T1 and the T1 method factor for depression. There were no significant correlations between sex and any of the latent change factors. The negative correlation of the T1 parent report method factor for depression with sex is interesting as it shows that parents' specific view in their assessment of children's depression depends in part on the children's sex. To understand this, recall that the method factors are residual factors from which the effect of self-report has been partialled out. Hence, the correlations between the method factors and external variables are *semipartial correlations*. The method factors represent the "pure" (specific) parent method, not shared with the self-report method. The weak negative correlation of the method factor with sex indicates that parents tend to overestimate the depression more strongly if the child is a boy than they do if the child is a girl. (This finding is small, however, and should not be over-interpreted.)

Table 16

*State and Method Factor Covariances, Correlations, Means, and Variances (Model 3, State Model)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. $S_{11}$ (Depression T1) | — | 0.07 (0.01) | 0.06 (0.01) | 0.10 (0.01) | 0.08 (0.01) | 0.07 (0.01) | — | — | — | 0.01 (0.00) | 0.00 (0.00) | 0.00 (0.00) |
| 2. $S_{12}$ (Depression T2) | .73 | — | 0.06 (0.01) | 0.08 (0.01) | 0.10 (0.01) | 0.08 (0.01) | — | — | — | 0.00 (0.00) | 0.01 (0.00) | 0.00 (0.00) |
| 3. $S_{13}$ (Depression T3) | .62 | .74 | — | 0.06 (0.01) | 0.08 (0.01) | 0.09 (0.01) | — | — | — | 0.00 (0.00) | 0.00 (0.00) | 0.00 (0.00) |
| 4. $S_{21}$ (Anxiety T1) | .75 | .59 | .50 | — | 0.14 (0.01) | 0.12 (0.01) | –0.00 (0.00) | –0.00 (0.00) | –0.00 (0.00) | — | — | — |
| 5. $S_{22}$ (Anxiety T2) | .61 | .75 | .60 | .76 | — | 0.14 (0.01) | –0.00 (0.00) | 0.00 (0.00) | –0.00 (0.00) | — | — | — |
| 6. $S_{23}$ (Anxiety T3) | .55 | .65 | .77 | .68 | .81 | — | –0.01 (0.00) | –0.00 (0.00) | 0.00 (0.00) | — | — | — |
| 7. $M_{121}$ (PR depression T1) | — | — | — | –.03 | –.02 | –.07 | — | 0.03 (0.00) | 0.03 (0.00) | 0.04 (0.00) | 0.03 (0.00) | 0.03 (0.00) |
| 8. $M_{122}$ (PR depression T2) | — | — | — | –.02 | .02 | –.01 | .83 | — | 0.02 (0.00) | 0.03 (0.01) | 0.04 (0.00) | 0.03 (0.00) |
| 9. $M_{123}$ (PR depression T3) | — | — | — | –.02 | –.05 | .01 | .66 | .68 | — | 0.03 (0.00) | 0.03 (0.00) | 0.04 (0.01) |
| 10. $M_{221}$ (PR anxiety T1) | .08 | .05 | .04 | — | — | — | .76 | .69 | .50 | — | 0.06 (0.01) | 0.06 (0.01) |

*(Table continues)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11. $M_{222}$ (PR anxiety T2) | .04 | .06 | .02 | — | — | — | .64 | .81 | .56 | .79 | — | 0.05 (0.01) |
| 12. $M_{223}$ (PR anxiety T3) | .03 | .04 | .02 | — | — | — | .58 | .61 | .80 | .72 | .74 | — |
| Means | 0.32 (0.01) | 0.29 (0.01) | 0.29 (0.01) | 0.72 (0.02) | 0.62 (0.02) | 0.56 (0.02) | — | — | — | — | — | — |
| Variances | 0.10 (0.01) | 0.09 (0.01) | 0.08 (0.01) | 0.18 (0.01) | 0.19 (0.01) | 0.17 (0.01) | 0.04 (0.00) | 0.03 (0.00) | 0.04 (0.01) | 0.08 (0.01) | 0.07 (0.01) | 0.08 (0.01) |

*Note.* Correlations are printed below, and covariances above the main diagonal. Values in parentheses are standard errors. All correlations between state and method factors were fixed to zero a priori.

Table 17

*State and Method Factor Covariances, Correlations, Means, and Variances (Model 3, Change Models)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. $S_{11}$ (Depression T1) | — | –0.03 (0.00) | –0.04 (0.00) | –0.01 (0.00) | 0.10 (0.01) | –0.02 (0.00) | –0.03 (0.01) | –0.01 (0.01) | — | — | — | — | 0.01 (0.00) | –0.00 (0.00) | –0.00 (0.00) | –0.00 (0.00) |
| 2. $S_{12} - S_{11}$ (Depression T2-T1) | –.41 | — | 0.04 (0.00) | –0.01 (0.00) | –0.02 (0.01) | 0.04 (0.01) | 0.03 (0.01) | –0.01 (0.00) | — | — | — | — | –0.00 (0.00) | 0.00 (0.00) | 0.00 (0.00) | –0.00 (0.00) |
| 3. $S_{13} - S_{11}$ (Depression T3-T1) | –.49 | .63 | — | — | –0.04 (0.01) | 0.03 (0.01) | 0.06 (0.01) | — | — | — | — | — | –0.00 (0.00) | 0.00 (0.00) | 0.00 (0.00) | — |
| 4. $S_{13} - S_{12}$ (Depression T3-T2) | –.17 | –.27 | — | — | –0.01 (0.00) | –0.01 (0.00) | | 0.03 (0.00) | — | — | — | — | –0.00 (0.00) | –0.00 (0.00) | — | 0.00 (0.00) |
| 5. $S_{21}$ (Anxiety T1) | .75 | –.24 | –.32 | –.15 | — | –0.04 (0.01) | –0.06 (0.01) | –0.02 (0.01) | –0.00 (0.00) | 0.00 (0.00) | 0.00 (0.00) | 0.00 (0.00) | — | — | — | — |
| 6. $S_{22} - S_{21}$ (Anxiety T2-T1) | –.18 | .56 | .38 | –.12 | –.33 | — | 0.07 (0.01) | –0.02 (0.01) | 0.00 (0.00) | 0.00 (0.00) | –0.00 (0.00) | –0.01 (0.00) | — | — | — | — |
| 7. $S_{23} - S_{21}$ (Anxiety T3-T1) | –.28 | .43 | .65 | — | –.45 | .66 | — | — | –0.00 (0.00) | 0.00 (0.00) | 0.01 (0.00) | — | — | — | — | — |
| 8. $S_{23} - S_{22}$ (Anxiety T3-T2) | –.14 | –.10 | — | .60 | –.19 | –.30 | — | — | –0.00 (0.00) | 0.00 (0.00) | 0.01 (0.00) | 0.01 (0.00) | — | — | — | — |
| 9. $M_{121}$ (PR depression T1) | — | — | — | — | –.03 | .01 | –.05 | –.07 | — | –0.01 (0.00) | –0.01 (0.00) | –0.00 (0.00) | 0.04 (0.00) | –0.01 (0.00) | –0.01 (0.00) | –0.00 (0.00) |
| 10. $M_{122} - M_{121}$ (PR depression T2-T1) | — | — | — | — | .01 | .08 | .11 | .05 | –.44 | — | 0.01 (0.00) | –0.00 (0.00) | –0.01 (0.00) | 0.01 (0.00) | 0.01 (0.00) | –0.01 (0.00) |
| 11. $M_{123} - M_{121}$ (PR depression T3-T1) | — | — | — | — | .01 | –.07 | .10 | — | –.41 | .44 | — | — | –0.01 (0.00) | 0.01 (0.00) | –0.03 (0.00) | — |
| 12. $M_{123} - M_{122}$ (PR depression T3-T2) | — | — | — | — | .00 | –.13 | .18 | .18 | –.11 | –.27 | — | — | –0.01 (0.00) | –0.00 (0.00) | — | 0.02 (0.00) |

*(Table continues)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13. $M_{221}$ (PR anxiety T1) | .08 | –.04 | –.04 | –.01 | — | — | — | — | .76 | –.24 | –.32 | –.17 | — | –0.02 (0.00) | –0.03 (0.01) | –0.00 (0.00) |
| 14. $M_{222} - M_{221}$ (PR anxiety T2-T1) | –.07 | .11 | .04 | –.06 | — | — | — | — | –.26 | .63 | .36 | –.08 | –.42 | — | 0.02 (0.00) | –0.01 (0.00) |
| 15. $M_{223} - M_{221}$ (PR anxiety T3-T1) | –.07 | .08 | .05 | — | — | — | — | — | –.28 | .26 | .79 | — | –.42 | .51 | — | — |
| 16. $M_{223} - M_{222}$ (PR anxiety T3-T2) | –.01 | –.01 | — | 0.03 | — | — | — | — | –.06 | –.30 | — | .78 | –.06 | –.37 | — | — |
| Means | 0.32 (0.01) | –0.03 (0.01) | –0.03 (0.01) | 0.00 (0.01) | 0.72 (0.02) | –0.10 (0.01) | –0.16 (0.02) | –0.06 (0.01) | — | — | — | — | — | — | — | — |
| Variances | 0.10 (0.01) | 0.05 (0.01) | 0.07 (0.01) | 0.05 (0.01) | 0.18 (0.01) | 0.09 (0.01) | 0.11 (0.01) | 0.07 (0.01) | 0.04 (0.00) | 0.01 (0.00) | 0.03 (0.01) | 0.02 (0.01) | 0.08 (0.01) | 0.03 (0.00) | 0.04 (0.01) | 0.04 (0.01) |

*Note.* Correlations are printed below, and covariances above the main diagonal. Values in parentheses are standard errors. All correlations between state and method factors were fixed to zero a priori. Shaded cells indicate estimates derived from the neighbor change version.

## 5.6 Summary and Discussion of the Applications

The application of both the state and the latent difference CS-C(*M*–1) models revealed that these models is appropriate to analyze the complex structure of a MTMM-MO matrix and that they provide interesting insights into the data. The CS-C(*M*–1) approach is useful to study a variety of research questions with regard to the psychometric properties of measures (i.e., convergent validity, method-specificity, indicator-specificity, and reliability) as well as the discriminant validity and stability of constructs and method effects in a longitudinal setting. For example, in the first application, it turned out that the reference method (self-report) showed strict measurement invariance over time, whereas the non-reference methods (parent and teacher ratings) exhibited weaker forms of invariance[11]. The analysis thus revealed a change in the psychometric properties of the measures belonging to the non-reference methods.

Another substantively important finding was that parent and teacher ratings of depression and anxiety consistently showed poor convergent validity with respect to self-reports. This finding was consistent across both applications. The convergent validity of teacher ratings was slightly higher at the end of a school year than at the beginning, possibly reflecting the fact that the teachers' ability to judge the mental state of their students increased as they got to know the children better.

Poor convergent validity was also found with regard to the depression and anxiety means and regarding mean change. Parent ratings appeared to produce underestimates of the mean depression and anxiety levels relative to the self-report and were rather insensitive to mean changes. High correlations between depression and anxiety were found on the level of the state factors (reference method), on the level of the method factors (non-reference methods), and on the level of the latent difference factors indicating a rather low degree of discriminant validity. However, there was evidence for discriminant validity of mean changes over time (a relatively strong mean decrease in anxiety versus a rather weak mean decrease in depression). Furthermore, inter-individual differences with regard to the constructs and rater biases turned out to be quite stable over time.

The CS-C(*M*–1) change model allows quantifying the convergent and discriminant validity of observed and latent change scores by means of variance components. Covariates of the latent state and change factors can be included in both model versions. In this way,

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[11] In the second application, for both the self- and parent report scales, strict factorial invariance was not rejected. This may be due to the fact that in this application, only three time points were considered.

researchers can test potential predictors or outcomes of states and change. Furthermore, attempts can be made to explain method-specific deviations of non-reference methods by external variables. For instance, in the present reanalysis, I identified sex as a significant predictor of anxiety states as measured by self-report (there was a tendency for girls to have higher latent anxiety scores), but not of any of the parent report states or of any change scores. Moreover, sex was slightly correlated with the specific view of parents regarding children's depression at T1. I will discuss the most important advantages and limitations of the CS-C($M$–1) approach in the final discussion.

# 6 Monte Carlo Simulation Study

## 6.1 Rational and Aims

In order to study the applicability of the CS-C($M$–1) approach in more detail, a Monte Carlo (MC) simulation study was carried out for both the state and change versions using the Mplus program. Simulation studies are useful to determine the minimum sample size (and other conditions) required for valid results, as these can not be determined analytically. Studying the limits of applicability is particularly important for models that are rather complex [like the CS-C($M$–1) model].

The basic principle of MC studies is that a researcher predetermines the "true" population parameters for one or several theoretical models of interest and then draws random samples of the desired size from this "population". The model (or a different model) is then fit to each of these "Monte Carlo samples" and the parameters and fit statistics are estimated and recorded for each MC sample. By comparing the (averaged) parameters and fit statistics from the MC samples to the known population values or theoretical distributions (e.g., the $\chi^2$ distribution), one can estimate bias in parameter estimates, standard errors, fit statistics, and so on. By relating bias to different MC conditions (e.g., different sample sizes, model misspecification etc.), one may identify favorable and unfavorable conditions for proper estimation of parameters, standard errors, fit statistics etc. (e.g., too small $N$). For a more detailed introduction into the rational and implementation of MC studies in SEM, see Bandalos (2006) as well as Muthén and Muthén (2002).

Several simulation studies examining the performance of SEM under various conditions have already been conducted (e.g., Boomsma, 1982; Gerbing & Anderson, 1985; Jackson, 2001; Marsh, Hau, Balla, & Grayson, 1998). In general, it has been found that SEM results become more reliable as the sample size and the number of unidimensional indicators per factor increase (e.g., Anderson & Gerbing, 1984; Boomsma, 1982; Marsh et al., 1998). The performance of cross-sectional MTMM models has also been investigated in a number of simulation studies (e.g., Conway, Lievens, Scullen, & Lance, 2004; Marsh & Bailey, 1991; Nussbeck, Eid, & Lischetzke, 2006; Tomás, Hontangas, & Oliver, 2000). However, the performance of complex longitudinal SEM-MTMM models with multiple indicators per CMOU has not yet been thoroughly studied. An exception is Crayen's (2008) simulation study, in which she investigated the performance of the CS-C($M$–1) state model under various conditions. The principal findings of Crayen's (2008) study were that the parameters of the CS-C($M$–1) state model are generally well recovered, even for relatively small sample sizes

($N$ = 125). Significant biases occurred for the standard errors of some parameters, but only when relatively small samples were used ($N$ = 125). The inclusion of indicator-specific factors across time tended to cause estimation problems when a model for only two time points was simulated. Furthermore, it was found that the $\chi^2$ distribution was not well approximated for complex models (with many variables and degrees of freedom) unless the sample size was extremely large.

In the present work, I conducted a small application-oriented simulation study in which I studied both the CS-C($M$–1) state and change models. In contrast to Crayen (2008), I used actual parameter estimates obtained from applications to the MTMM-MO data set of Cole et al. (see Chapter 5.1) as "true" population values for the simulation. This made it possible to study the performance of the models under realistic conditions. I was particularly interested in the adequacy of the $\chi^2$ test statistic for evaluating goodness-of-fit, the parameter estimates, and the standard errors. Furthermore, as outlined in Chapter 1.3.1.1, cross-sectional MTMM models are often prone to convergence problems and improper solutions (Kenny & Kashy, 1992; Marsh & Bailey, 1991). The longitudinal MTMM models presented in this work are even more complex models, given that more factors and an additional facet (indicator-specificity over time) are considered. Hence, I was interested in determining the frequency of convergence problems and improper solutions for the CS-C($M$–1) model. It is well-known that problems of nonconvergence and improper solutions are more likely for complex structural equation models, especially in small samples (Anderson & Gerbing, 1984). Researchers often do not have time and money to collect large samples, particularly when employing a MTMM longitudinal design. It is thus very important to determine to which degree the results obtained from the CS-C($M$–1) state and change models are trustworthy also for relatively small samples (e.g., for $N$ = 125 or 250).

## 6.2 Method

### *6.2.1 Population Models*

In line with a "classical" MTMM design, I fit a state, baseline change, and neighbor change CS-C($M$–1) model with three constructs (depression, anxiety, and competence), three methods (self-, parent-, and teacher report), three occasions of measurement, and two indicators per CMOU (3 x 3 x 3 x 2 version) to the data set of Cole et al. (1996). The models assumed general state factors, invariance of factor loadings for all indicators over time, and included indicator-specific factors over time for all methods in line with the model variant introduced in Section 3.2. All indicator-specific factor loadings were fixed to unity. Each model was rather complex, comprising 837 *df* and 702 freely estimated parameters. [The models fit the actual data well, $\chi^2$ (837, $N = 906$) = 1,095.01, $p < .01$; $CFI = 0.99$; *RMSEA* = .02.] The parameter estimates obtained for the models were used as true population values for the MC simulations[12]. Five different sample sizes were employed for each model variant (*N* = 125, 250, 500, 750, and 1000), leading to a 3 (model variant) x 5 (sample size) MC design. I used 500 replications (MC samples) for each condition and ML estimation, thereby assuming complete data (no missing values) for each condition.

### 6.2.2 Criteria for Evaluating the Performance of the Models

The following six criteria were used to evaluate the performance of the ML estimator for the models and a given sample size (see also Muthén & Muthén, 2002; as well as Nussbeck et al., 2006):

#### 6.2.2.1 Non-Convergence

*Non-convergence* refers to the inability of SEM software to find unique solutions for the parameters of a model after a certain number of iterations. I recorded the number of replications for which the estimation process did not converge to a solution after 1000 iterations. The criterion in the present study was that the rate of non-convergence should be below 1%.

#### 6.2.2.2 Improper Solutions

The number of replications in which non-positive definite residual covariance matrices ("Heywood cases") occurred was recorded. No more than 5% of the replications should produce such improper solutions.

#### 6.2.2.3 $\chi^2$ Test

I investigated the adequacy of the $\chi^2$ statistic for evaluating goodness-of-fit by comparing the observed MC $\chi^2$ distribution with the theoretical $\chi^2$ distribution. Large discrepancies between the distributions indicate that the $\chi^2$ approximation does not work well for a given sample size. (It is expected that both distributions become more and more similar with increasing sample size, implying that the $\chi^2$ test is generally more reliable in larger samples.) The present criterion was that the proportion of models that would be rejected at the 5% level based on the theoretical $\chi^2$ distribution should not be larger than .10 according to the MC $\chi^2$ distribution.

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[12] Sample Mplus input files used for the simulations are provided in the appendix (see Sections 13.2 to 13.2.3).

6.2.2.4 Parameter Estimation Bias

The *parameter estimation bias* (*peb*) is a measure of the reliability of the model parameters estimates and is given by:

$$peb = \frac{M_p - e_p}{e_p}, \tag{219}$$

where $M_p$ is the mean of the parameter estimates over all replications and $e_p$ is the true population value. The smaller the bias, the closer are the estimates to the true population values. It is commonly accepted that *peb* should not exceed .10 (i.e., 10%).

6.2.2.5 Standard Error Bias

The *standard error bias* (*seb*) is useful as a measure of the appropriateness of tests of significance of the model parameters. Biased standard errors lead to inaccurate significance testing (i.e., increased type I or type II error rates). *seb* is calculated by comparing the mean of the estimated standard errors over all replications ($M_{SE}$) with the standard deviation of the estimated model parameters over the MC replications ($SD_p$) and is given by:

$$seb = \frac{M_{SE} - SD_p}{SD_p}. \tag{220}$$

The criterion in the present simulation study was that *seb* should not exceed .10.

6.2.2.6 Coverage

*Coverage* refers to the proportion of replications for which the 95% confidence interval for an estimate contains the true population parameter value. Coverage should be between .91 and .98.

## 6.3 Results

### *6.3.1 CS-C(M–1) State Model*

6.3.1.1 Non-Convergence

Problems of non-convergence did not occur for any sample size.

#### 6.3.1.2 Improper Solutions

Although all replications converged to a solution, the number of replications in which a Heywood case (improper solution) occurred was unacceptably high for smaller sample sizes. For $N$ = 125, 218 out of 500 replications (43.6%) showed a non-positive definite residual covariance matrix (i.e., a solution with at least one negative residual variance). For $N$ = 250, there were still 59 replications (11.8%) with an improper solution, and for $N$ = 500, 11 replications (2.2%) with an improper solution were encountered. For the larger sample sizes, there were almost no Heywood cases: One improper solution (0.2%) was encountered for $N$ = 750 and none for $N$ = 1000.

#### 6.3.1.3 $\chi^2$ Test

Table 18 shows the expected and observed proportions of the $\chi^2$ distribution for the different sample sizes (shown under the header *Large model* in Table 18). It can be seen that the $\chi^2$ approximation was not appropriate for any of the sample sizes considered, although it seemed to work better for larger sample sizes. (I added $N$ = 1500 and 2000 as sample sizes in order to check whether the approximation would be better in still larger samples.) The observed proportions were larger than the theoretical proportions in all cases, implying a strongly increased type I error rate. That is, too many correctly specified models would be rejected according to the usual ML $\chi^2$ test statistic. The $\chi^2$ approximation was especially bad for $N$ = 125, 250, and 500. Even for a relatively large sample size of 500, there was a type I error rate of 25% for a nominal alpha level of 5%.)

Consequently, when fitting a 3 x 3 x 3 x 2 version of the CS-C($M$–1) model, the conventional ML-$\chi^2$ test cannot be trusted unless the sample size is extremely large. This finding can be explained by the fact that the 3 x 3 x 3 x 2 version represents a very complex model. In many practical applications, researchers use simpler models with fewer constructs or methods. It is likely that the $\chi^2$ approximation works better for less complicated variants of the CS-C($M$–1) model.

To further investigate this issue, I extended the simulations to a smaller model version with only one construct (depression), two methods (self and parent report), two occasions of measurement, and two indicators per CMOU. This 1 x 2 x 2 x 2 version of the model is useful for many applications in psychology since research designs with one construct and two methods are very common in psychological intervention and evaluation studies (e.g., in clinical psychology, when the goal is to evaluate the effectiveness of a therapy for a specific disorder in a pre-post design including multiple raters).

The simulation results for the small model were encouraging in terms of both, the number of improper solutions and the $\chi^2$ approximation. In fact, all runs converged properly and only a very small number of Heywood cases were encountered for this model (four Heywood cases [0.8%] were encountered for $N$ = 125, and none for the remaining sample sizes). The results for the smaller model are shown in the left part of Table 18 (under the header *Small model*). It can be seen that the $\chi^2$ approximation works much better for the smaller model, even for smaller sample sizes. Especially for the critical proportions (i.e., .05, .02, and .01), the observed values approximate the theoretical values reasonably well.

Table 18

*Expected and Observed Proportions of the $\chi^2$ Statistic for Different Sample Sizes*

| | Observed proportions | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Small model | | | | | Large model | | | | | | |
| Expected proportions | 125 | 250 | 500 | 750 | 1000 | 125 | 250 | 500 | 750 | 1000 | 1500 | 2000 |
| .99 | .99 | .99 | .99 | .99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .99 | .99 |
| .98 | .99 | .98 | .99 | .98 | .98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .99 | .99 |
| .95 | .95 | .96 | .97 | .94 | .95 | 1.00 | 1.00 | 1.00 | .99 | .99 | .98 | .97 |
| .90 | .89 | .93 | .91 | .89 | .90 | 1.00 | 1.00 | .99 | .97 | .98 | .95 | .93 |
| .80 | .81 | .84 | .79 | .78 | .79 | 1.00 | .99 | .96 | .91 | .90 | .87 | .87 |
| .70 | .73 | .74 | .69 | .70 | .70 | 1.00 | .99 | .92 | .86 | .83 | .79 | .77 |
| .50 | .55 | .52 | .49 | .49 | .48 | 1.00 | .98 | .83 | .73 | .67 | .61 | .59 |
| .30 | .34 | .32 | .28 | .28 | .28 | 1.00 | .91 | .66 | .51 | .46 | .42 | .41 |
| .20 | .23 | .22 | .17 | .19 | .17 | 1.00 | .85 | .56 | .39 | .35 | .31 | .29 |
| .10 | .14 | .12 | .08 | .09 | .08 | 1.00 | .76 | .39 | .23 | .19 | .16 | .16 |
| .05 | .08 | .07 | .04 | .04 | .04 | .99 | .63 | .25 | .13 | .12 | .10 | .08 |
| .02 | .03 | .04 | .02 | .02 | .02 | .99 | .50 | .14 | .07 | .06 | .04 | .04 |
| .01 | .02 | .03 | .01 | .01 | .01 | .99 | .40 | .07 | .05 | .03 | .03 | .02 |

*Note.* Small model = one construct, two methods, two occasions of measurement, and two indicators per CMOU (1 x 2 x 2 x 2 version); Large model = three constructs, three methods, three occasions of measurement, and two indicators per CMOU (3 x 3 x 3 x 2 version); Expected proportions = proportions based on the theoretical chi-square distribution; 125, 250, 500, 750, 1000, 1500, and 2000 indicate the sample size.

#### 6.3.1.4 Parameter Bias, Standard Error Bias, and Coverage

I assessed parameter bias, standard error bias and coverage separately for different types of model parameters (e.g., loadings, variances, covariances etc.) given that possible bias might not affect all parameters in the same way. A detailed list of the simulation results for the 3 x 3 x 3 x 2 model version is provided in Table 19. The *peb* and *seb* values presented in Table 19 represent averages of the absolute *peb* and *seb* values for each type of parameter.

(There were too many parameters to consider every single parameter separately.) Furthermore, mean coverage values are provided. Note that simulation results for theoretically meaningless covariances (see Chapter 3) have been omitted from Table 19 to save space. It can be seen that mean parameter bias is negligible regardless of the sample size (all mean *peb* values are $\leq .02$). Mean standard error bias is also generally low (below .10), except for the state factor loadings (mean *seb* = .14), method factor loadings (mean *seb* = .19), and error variances (mean *seb* = .23) for the smallest sample size ($N$ = 125). Also, for $N$ = 250, the error variances show a non-negligible mean *seb* of .11. Coverage is close to the optimal value of .95 for most parameters and sample sizes. Values outside the desired range of .91–.98 were found only for the method factor loadings and error variances for $N$ = 125. These parameters also showed problematic values in terms of *seb* for $N$ = 125. In sum, the *peb*, *seb*, and coverage values are rather encouraging and show that the ML model parameters and standard errors of complex versions of the CS-C($M$–1) model can reliably be estimated, even in samples of moderate size.

Table 19

*Parameter Estimate Bias, Standard Error Bias, and Coverage for the Parameters of the CS-C(M–1) State Model*

| | Sample size | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 125 | | | 250 | | | 500 | | | 750 | | | 1000 | | |
| Type of Parameter | *peb* | *seb* | Coverage | *peb* | *seb* | Coverage | *peb* | *seb* | Coverage | *peb* | *seb* | Coverage | *peb* | *seb* | Coverage |
| Intercepts | .02 | .08 | .93 | .02 | .05 | .94 | .01 | .04 | .94 | .01 | .02 | .95 | .01 | .02 | .95 |
| State factor loadings | .01 | .14 | .91 | .01 | .07 | .93 | .01 | .04 | .94 | .01 | .02 | .94 | .00 | .02 | .95 |
| Method factor loadings | .00 | .19 | .88 | .00 | .09 | .93 | .00 | .05 | .93 | .00 | .04 | .94 | .00 | .03 | .94 |
| Error variances | .02 | .23 | .87 | .01 | .11 | .92 | .01 | .05 | .94 | .01 | .04 | .94 | .00 | .03 | .94 |
| State factor means | .00 | .03 | .95 | .00 | .03 | .94 | .00 | .03 | .95 | .00 | .01 | .95 | .00 | .01 | .95 |
| State factor variances | .01 | .07 | .92 | .00 | .02 | .94 | .00 | .02 | .94 | .00 | .01 | .95 | .00 | .02 | .95 |
| Method factor variances | .01 | .05 | .93 | .01 | .02 | .94 | .00 | .02 | .94 | .00 | .01 | .95 | .00 | .02 | .95 |
| Indicator-specific variances | .01 | .05 | .94 | .01 | .03 | .94 | .00 | .02 | .95 | .00 | .01 | .94 | .00 | .01 | .95 |
| Inter-State covariances | .02 | .06 | .92 | .01 | .02 | .95 | .00 | .02 | .94 | .00 | .03 | .95 | .00 | .03 | .95 |
| Inter-method covariances | .02 | .04 | .94 | .01 | .02 | .95 | .01 | .03 | .95 | .01 | .02 | .95 | .01 | .02 | .95 |

*Note.* *peb* = mean absolute parameter bias; *seb* = mean absolute standard error bias; coverage = mean proportion of replications for which the 95% confidence interval contained the true population value.

### 6.3.2 CS-C(M–1) Change Models

#### 6.3.2.1 Non-Convergence, Improper Solutions, and $\chi^2$ Test

Given that the baseline and neighbor change versions are just reformulations of the CS-C(*M*–1) state model (cp. Chapter 4), the simulation results with respect to convergence, improper solutions, and $\chi^2$ approximation are identical to the results obtained for the state version (identical seeds were used in the simulations of the state and change versions). Hence, these findings do not need to be repeated here. In the following, only the *peb*, *seb*, and coverage values for the change versions are discussed.

#### 6.3.2.2 Parameter Bias, Standard Error Bias, and Coverage

The mean *peb*, *seb*, and coverage results for parameters not available in the CS-C(*M*–1) state model (i.e., the means, variances, and covariances of the latent difference variables) are shown in Table 20. The results are similar to the results obtained for the state version. As expected, the values become generally better as *N* increases. For $N \geq 500$, *peb* and *seb* values are negligible and coverage is close to .95. There is one exception: For the neighbor difference version, the mean *peb* values are large for the state difference factor means for all sample sizes ($.18 \leq peb \leq .46$). These high values are caused by the very small population mean of the difference factor for the construct *competence* (competence 3 – competence 2) which equals –0.001. This mean shows a large bias in all samples. However, as this mean practically equals zero, it is not of great theoretical interest so that this result can be neglected.

## 6.4 Summary and Recommendations

Although the generalizability of the findings of this small simulation study is limited to the specific model versions and parameter sets used here, some general trends are obvious. It turned out that the parameters of the CS-C(*M*–1) model can reliably be estimated for an MTMM-MO design with three constructs, three methods, three time points, and two indicators per CMOU, even if the sample size is as small as *N* = 125. For most parameters, negligible mean parameter and standard error biases were found, and coverage differed only marginally from .95 for most cases. Standard error bias was only found for small sample sizes (i.e., *N* = 125 and 250). These small samples were also found to be problematic in terms of Heywood cases. The large number of improper solutions that were encountered for sample sizes ≤ 250, as well as large standard error biases for some parameters for *N* ≤ 250 indicates that the 3 x 3 x 3 x 2 model version should be fit only with great caution and as many parameter-reducing restrictions as possible if $N$ is ≤ 250. In this respect, it is interesting that in Marsh et al.'s (1998) MC study on the performance of conventional CFA models, increasing the number of appropriate indicators per factor had a positive effect in terms of fewer improper solutions. It would be worthwhile to investigate whether the use of more indicators per CMOU would lead to fewer improper solutions if $N$ is small in the case of the CS-C($M$–1) model as well.

With respect to the ML $\chi^2$ test of model fit, it was found that the $\chi^2$ statistic may not approximate the theoretical $\chi^2$ distribution well if a 3 x 3 x 3 x 2 model version is used. This is true even when the sample size is very large. Strongly inflated type I error rates were found even for $N = 1000$, and the approximation was still bad for $N$ as large as 2000. Sample sizes greater than $N = 2000$ seem to be needed to obtain reliable $\chi^2$ values that approximate the theoretical $\chi^2$ distribution well if the remaining assumptions (e.g., random sample, correct specification, multivariate normality) are met. Hence, an important finding of the present simulation study is that researchers who fit a 3 x 3 x 3 x 2 version of the CS-C($M$–1) model and use the conventional $\chi^2$ test to evaluate the fit of the model take a large risk of incorrectly rejecting a proper model even if they use multivariate normal data and a very large sample size.

An extension of the simulations revealed that for a smaller model, the $\chi^2$ approximation was quite accurate. These findings are in agreement with the simulation studies of other researchers (e.g., Crayen, 2008; Kenny & McCoach, 2003; Marsh et al., 1998), who also reported inflated type I error rates for the $\chi^2$ statistic when SEMs with a large number of *df* were analyzed.

Crayen's (2008) simulation study of a small and a complex version of the CS-C($M$–1) state model yielded very similar results as did the present simulation: The $\chi^2$ approximation worked well for the small model version, but lead to strongly increased type I error rates for the complex model version, even when large samples were considered. Herzog, Boomsma, and Reinecke (2007) recommend that a corrected version of the $\chi^2$ statistic as proposed by Swain (1975) be used for the evaluation of SEMs with a large number of *df*. This appears to be a promising way to avoid an incorrect rejection of too many proper models. Another possible solution for the problem could be specific bootstrapping methods (e.g., Bollen & Stine, 1992). Further research investigating the adequacy of such methods for the case of the CS-C($M$–1) model would be worthwhile.

In addition, more research is needed to determine under which special conditions reliable $\chi^2$ results are obtained also with larger model variants. For example, it might be that for a highly constrained model, more constructs and methods could be used. In the present case, I only assumed invariance of factor loadings. Furthermore, many latent variable covariances were admitted that were not theoretically meaningful, and that turned out to be very close to zero in the application. It is likely that the $\chi^2$ approximation will be better for larger model variants if the number of free parameters can be reduced by imposing additional meaningful (and tenable) restrictions (e.g., equal loadings also within occasions of measurement, time-invariant intercepts and error variances, time-invariant factor variances). This seems possible at least for some data sets if one carefully selects appropriate indicators that show strict measurement invariance over time and constrains all theoretically meaningless factor covariances to zero. In sum, the results of the simulation are rather encouraging, although researchers should not select too small samples if they plan to estimate very complex model versions.

Table 20

*Parameter Estimate Bias, Standard Error Bias, and Coverage for the Parameters of the CS-C(M–1) Change Models*

| | Sample size | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 125 | | | 250 | | | 500 | | | 750 | | | 1000 | | |
| Type of parameter | *peb* | *seb* | coverage | *peb* | *seb* | coverage | *peb* | *seb* | coverage | *peb* | *seb* | coverage | *peb* | *seb* | coverage |
| Baseline difference version | | | | | | | | | | | | | | | |
| State difference factor means | .03 | .02 | .95 | .04 | .02 | .94 | .02 | .03 | .96 | .02 | .02 | .95 | .02 | .02 | .95 |
| State difference factor variances | .01 | .09 | .92 | .00 | .05 | .94 | .00 | .03 | .95 | .00 | .02 | .95 | .00 | .03 | .94 |
| State difference factor covariances | .03 | .11 | .92 | .01 | .04 | .94 | .01 | .03 | .94 | .01 | .02 | .95 | .01 | .03 | .94 |
| Method difference factor variances | .01 | .11 | .92 | .00 | .04 | .93 | .00 | .04 | .94 | .00 | .02 | .94 | .00 | .03 | .95 |
| Method difference factor covariances | .05 | .07 | .93 | .04 | .04 | .94 | .03 | .03 | .95 | .02 | .03 | .95 | .01 | .03 | .95 |
| Neighbor difference version | | | | | | | | | | | | | | | |
| State difference factor means | .29 | .02 | .95 | .46 | .03 | .95 | .20 | .03 | .95 | .22 | .03 | .95 | .18 | .03 | .95 |
| State difference factor variances | .01 | .08 | .92 | .00 | .05 | .94 | .00 | .03 | .94 | .00 | .03 | .95 | .00 | .02 | .95 |
| State difference factor covariances | .03 | .11 | .92 | .02 | .04 | .94 | .01 | .04 | .94 | .01 | .02 | .95 | .01 | .03 | .95 |
| Method difference factor variances | .01 | .10 | .92 | .01 | .05 | .94 | .00 | .04 | .94 | .00 | .02 | .94 | .00 | .03 | .94 |
| Method difference factor covariances | .06 | .07 | .93 | .05 | .04 | .94 | .02 | .03 | .95 | .02 | .03 | .95 | .02 | .03 | .95 |

*Note.* *peb* = mean absolute parameter bias; *seb* = mean absolute standard error bias; coverage = mean proportion of replications for which the 95% confidence interval contained the true population value.

# 7 Final Discussion

In this work, I presented new SEMs for analyzing longitudinal MTMM data, the CS-C(*M*–1) state and change models. My aim was to develop models that are general and flexible and that allow for an assessment of latent change in a MTMM context. It is for this reason that the models were developed as multiple indicator models that allow analyzing construct-specific, occasion-specific method effects. Furthermore, I presented two different model variants that take different forms of generalizing indicator-specific effects into account. In the final discussion, I summarize the most important advantages and limitations of the models, provide guidelines and tips for applied researchers, discuss links to other modeling approaches, and outline directions for future research.

## 7.1 Advantages

### *7.1.1 Simultaneous Analysis of MTMM-MO Data*

First of all, the CS-C(*M*–1) model overcomes a number of serious limitations of other approaches to analyzing MTMM-MO data by allowing for a simultaneous analysis of an entire MTMM-MO matrix in a single model. The first consequence is that it is no longer necessary to analyze separate models for each wave, rendering the analysis more practical and comprehensive. The second consequence is that there is no loss of information. Both the information at each time point and the (longitudinal) information about stability and change are taken into account. The model allows scrutinizing relationships among latent variables within and across time. For example, the discriminant validity as well as stability of constructs and method effects can be analyzed through latent correlations. (Note that in order to study the discriminant validity of constructs and the generalizability of method effects across constructs, at least two constructs are needed.) The model can also be used to study mean change over time. Hence, a complete longitudinal MTMM model can in principle be tested in a single step. (Note, however, that for practical reasons, I recommend an analysis strategy that makes use of several steps; see Section 5.2 and discussion below.)

### *7.1.2 Separation of Variance Components*

Another important advantage of the models presented here is that they explicitly take measurement error into account. A detailed variance decomposition allows separating true score variance from random error variance, and the true variance can be further partitioned into up to three different sources (occasion-specific consistency, method-specificity, and indicator-specificity). Hence, the models allow for a very fine-grained analysis of the psychometric properties of the measures at each time point.

### *7.1.3 Determination of the Degree of Indicator-Specificity*

Indicator-specific effects that generalize across time frequently cause problems in longitudinal investigations if these are not properly represented in the model used to analyze the data. In MTMM(-MO) designs, issues of indicator-specificity are particularly complicated, especially when equivalent scales are used across methods. It might then be necessary to deal with specific variance that generalizes across methods.

In the past, approaches to handle these types of indicator-specificity have often focused on correlated uniqueness (CU) models (models with correlated error variables; e.g., the multi-occasion CU approach presented by Cole & Maxwell, 2003, see Chapter 1.3.2.1). Although CU models are relatively straightforward, one disadvantage of these models is that they do not model indicator-specificity directly by latent factors, but indirectly through correlated uniquenesses. Hence, random error is confounded with reliable specific variance in CU models, leading to an underestimation of the reliabilities of the indicators (see discussion in Chapter 1.3.1.1). The present models allow modeling different forms of indicator-specificity appropriately. They make it possible to disentangle reliable specific and error components of variance. Different CS-C($M$–1) model variants have been presented in Chapter 3, and additional strategies for detecting and handling different forms of indicator-specificity have been discussed in Chapter 5.2.

### *7.1.4 Measurement Invariance Testing*

By analyzing a series of nested models, fine-grained tests of measurement invariance over time can be conducted. Measurement invariance is crucial for many types of research questions in longitudinal research. In the CS-C($M$–1) model, different degrees of measurement invariance over time are represented by different sets of parameter equality constraints. Different nested models can be statistically tested against each other to determine the degree of (non)invariance for different methods. In this way, it is possible to detect changes in the psychometric properties of the indicators over time (e.g., changes in the degree of convergent validity or method-specificity). Detailed guidelines for a useful sequential modeling strategy were provided in Chapter 5.2.

### 7.1.5 *Analysis of Latent Change*

Both the CS-C($M$–1) state and change models allow for an analysis of inter-individual differences in intra-individual change and of mean change over time. In the state model, change is indicated by correlations < 1.0 between state factors over time. Mean differences between state variables can also be analyzed. In the change versions, inter-individual differences in intra-individual change are directly represented by latent difference variables. The advantage of latent difference variables is that they are corrected for measurement error, whereas observed change scores are often greatly distorted by measurement error (cp. Chapter 4.7). Although the CS-C($M$–1) change versions represent just reparameterizations of the CS-C($M$–1) state models, they are very useful as they make the information about change more directly available. Thus, the change models are useful if one seeks to analyze individual differences in change more thoroughly, for instance by relating latent change scores to external variables.

Furthermore, the latent difference models offer a variance decomposition of the change score into consistency, method-specificity, and reliability. In contrast to growth curve models, no specific functional form of change is assumed in the change models, making these models rather general and unrestrictive. Note, however, that the CS-C($M$–1) state model can easily be extended to a second-order growth model, in which specific hypotheses with respect to the form of change (i.e., linear vs. curvilinear) can also be tested on the level of the latent state factors.

### 7.1.6 *Inclusion of Covariates*

An important advantage of either CS-C($M$–1) model version (state vs. change) is that additional external variables can be added to the model. Such variables can be correlates, predictors, or outcomes of the latent factors in the model. For example, in the application presented in this work, I found that the factors representing change in anxiety were positively related to the factors representing change in depression. One might also attempt to explain method-specific deviations (the deviation from the reference method; lack of convergent validity) by external variables. Method factors can be regressed on such explanatory variables. In this way, a researcher can try to answer the question of why different methods diverge in the assessment of states and change. In the present application, I found that sex was correlated with parent's rater bias with respect to the depression state at T1—although the effect was rather small.

## 7.2 Limitations

### *7.2.1 Model Complexity and Required Sample Size*

An important limitation of the CS-C(*M*–1) models concerns their complexity. Even with a moderate number of indicators, constructs, methods, and time points, the number of estimated parameters is rather large. Therefore, the models are not appropriate for small samples. This was shown in the application-oriented MC simulation study presented in Chapter 6. Although convergence problems did not occur at all, the likelihood of improper solutions, parameter bias, and standard error bias increased significantly for *N*'s smaller than 250, at least for a complex model version with three constructs, three methods, and three time points. With larger *N*, these problems were less common, but the $\chi^2$ approximation of the $\chi^2$ goodness-of-fit test was still bad for the complex model version considered. These results were in line with Crayen's (2008) extended simulation study of different CS-C(*M*–1) state models.

It is a practical problem that large sample sizes are often unavailable, especially in a MTMM-MO context, where data collection is very costly in the first place. A possible solution for this problem is to reduce model complexity (i.e., the number of freely estimated parameters) by adding additional constraints (such as fixing theoretically meaningless factor correlations to zero, setting intercepts, loadings, and variances equal where this is appropriate etc.). This has the additional benefit of making (1) the model more parsimonious and (2) the model test even more stringent as additional hypotheses are tested. Furthermore, indicators should be selected that are as homogeneous as possible to avoid additional model complexity due to indicator-specific factors.

Note, however, that greater model complexity does not always lead to more problems in SEM. More information (e.g., a larger number of appropriate indicators per factor) can also have beneficial effects as has been shown by Marsh et al. (1998). Further research is needed to determine the specific conditions under which complex versions of the CS-C(*M*–1) model can be used even with small *N*. Until then, given potential estimation problems with complex models in small samples, I recommend that researchers who are in doubt as to whether their sample is large enough conduct an application-oriented simulation study to determine the minimum sample size required for valid results (e.g., Bandalos, 2006). Muthén and Muthén (2002) discuss how such a simulation can easily be implemented in the Mplus program. Furthermore, I provide sample Mplus scripts that were used for the present simulation study in the appendix (see Section 13.2).

### 7.2.2 Types of Indicators

The models presented in this work assume continuous indicators (e.g., test sum scores). In practice, one often has only categorical (ordinal) items. In principle, the models presented here can also be applied to ordinal or dichotomous items. However, in this case, special CFA models and appropriate estimators for ordinal/dichotomous variables should be employed, especially if the indicators have few response categories and are skewed (Finney & DiStefano, 2006). The Mplus program offers a special adjusted weighted least squares estimator (so-called WLSMV estimator, Muthén & Muthén, 1998-2007) for such type of data. The basis for a WLSMV analysis is the matrix of polychoric correlations (tetrachoric correlations for dichotomous outcomes), which is estimated by the program. The WLSMV estimator has been found to perform well for SEMs with ordinal latent variable indicators (Beauducel & Herzberg, 2006; Muthén, du Toit, & Spisic, 1997; Nussbeck et al., 2006).

If the indicators are continuous but non-normal, robust versions of the ML estimator (e.g., Satorra & Bentler, 1994) or bootstrap methods (Bollen & Stine, 1992) are available to obtain robust standard errors and test statistics.

### 7.2.3 Reference Method

Another aspect that may be conceived as a limitation is related to the choice of a reference method. Recall that the CS-C($M$–1) approach requires that one method be selected as reference method. The choice of the reference method has important consequences for the interpretation of the model results, as the meaning of the latent state, method, and indicator-specific factors depends on the choice of the reference method. Therefore, I recommend that researchers make an informed choice based on substantive considerations and ease of interpretation. Which method is most outstanding? Which comparison or contrast between methods is most interesting from a theoretical or substantive point of view? Also, the guidelines provided in Geiser et al. (2008) for the cross-sectional CT-C($M$–1) model should be considered when applying the CS-C($M$–1) approach.

Furthermore, it should be noted that the CS-C($M$–1) model is not symmetric across different reference methods. That is, the fit of the model may (and in general will) change if an alternative reference method is selected. However, this issue can be solved by specifying a slightly more restricted variant of the model that is nested within the general CS-C($M$–1) model. In the restricted version, specific constraints are imposed on loading parameters, leading to a model variant whose fit is invariant across different reference methods. This restricted specification has been described in detail for the cross-sectional CT-C($M$–1) approach (Geiser et al., 2008). This specification was not discussed here in detail to simplify the presentation and to keep the focus on the main goals of this work. However, readers may consult Geiser et al. (2008) and transfer the restricted specification to the CS-C(*M*–1) model.

### 7.2.4 Types of Methods

Given that a reference method needs to be selected, the CS-C(*M*–1) model is most appropriate for MTMM-MO designs that employ structurally different methods (Eid et al., 2003; Geiser et al., 2008). *Structurally different* means that the methods (raters) considered are not a random sample drawn from a common set of raters (Eid et al., in press). For example, in the application presented in this thesis, depression and anxiety were assessed by self-, parent, and teacher ratings, representing three different types of raters. It is obvious that self-, parent, and teacher reports cannot be conceived of as interchangeable methods, given that, for instance, a given self-report is fixed and cannot be replaced by another self-report drawn from a set of equivalent self-reports. In the case of structurally different methods, it makes sense to select one method (e.g., the self-rating) as the reference against which the remaining methods are contrasted. Yet the choice of a reference method may be difficult (and not really meaningful) if one uses interchangeable methods. Methods are considered interchangeable if they are randomly drawn from a set of structurally equivalent methods (Eid et al., 2003, in press). As an example, consider the case of university teachers who are evaluated by a random sample of their students. Given that all students have more or less the same access to the teacher's behavior, it does not really matter *which* students are chosen. The different "methods" can therefore be regarded as interchangeable which implies that one could equally well select another sample of raters. (For a detailed discussion of different types of methods see Eid et al., in press.) In the present work, I only considered the case of structurally different methods, given that structurally different methods are most often used in MTMM research designs. An important task for future research is to develop appropriate MTMM-MO models for interchangeable methods.

### 7.2.5 Measurement Invariance

I return to the issue of measurement invariance here once again as it is a very important issue in longitudinal data analysis. As I pointed out before, a requirement for the proper interpretation of latent difference variables as considered in the CS-C(*M*–1) change models is that the measurement structure remains invariant over time. This means that at least intercept and factor loading invariance is required. As I illustrated in Chapter 5.5, this assumption is testable, and in the present application, it was not rejected by the goodness-of-fit criteria.

However, measurement invariance might not always be tenable. If it is not tenable, a meaningful interpretation of the latent change factors might not be possible. However, as discussed above, in many cases, researchers will be able to establish at least *partial* measurement invariance (Byrne et al., 1989), meaning that invariance is tenable for at least some indicators of a construct. Partial invariance might under certain circumstances be sufficient to the warrant proper interpretation of the latent change factors. However, future research is needed to clearly determine invariance conditions that are necessary and sufficient for a sound interpretation of the results.

### *7.2.6 Detection of Invariance and Changes in Method Effects*

An interesting question is whether all kinds of non-invariance and changes in method effects over time are detectable by the CS-C(*M*–1) model under various research designs. In principal, changes in parameter values over time can be detected whenever a parameter can be freely estimated. Whether a parameter is freely estimable or not depends on the identification status of the model. For example, in a 2 x 1 x 2 x 2 design (one construct measured by two methods on two occasions of measurement; two indicators per CMOU), loading invariance over time for the method factors can only be detected if the method factor at T1 is correlated with the method factor at T2 (given that both factors are measured by only two indicators in this design). Otherwise, the model with free method factor loadings for the second indicator would be underidentified (unless the method factors are correlated with some kind of external variable that is also included in the model). Likewise, under this design, it may not be possible to detect changes in the indicator-specific factor loadings over time, unless the indicator-specific factor for the second indicator of the reference method is substantially correlated with the indicator-specific factor for the second indicator of the non-reference method (or the factors are correlated with an external variable).

In this respect, it is noteworthy that Crayen (2008) found the 2 x 1 x 2 x 2 case to be prone to improper solutions if indicator-specific factors were included. I therefore recommend that at least three homogeneous indicators per CMOU be used in this case to avoid indicator-specific factors and to ensure the proper identification of the method factors. Another recommendation is that it is better to have at least three occasions of measurement, as this enhances the identification status of the indicator-specific factors.

### *7.2.7 Indicator-Specific Effects*

The parameters of models with $i - 1$ indicator-specific factors over time (see Section 3.2) may be difficult to interpret if the indicator-specific factors account for a large portion (say more than 20–30%) of the variance of the indicators. High indicator-specificity implies that the indicators are heterogeneous and mirror distinct facets of a construct. (This problem is neither specific to the case of only two time points nor is it specific to the models presented here.)

The choice of the reference indicators for which no indicator-specific factors are included has consequences for the interpretation of the state factors. This is especially true if the proportion of indicator-specific variance is large. The results obtained from the structural model (e.g., the latent variable correlation matrix and correlations with covariates) might then strongly depend on the choice of the reference indicators for which no indicator-specific factors are specified. (This is essentially the same issue as with the selection of a reference method, see discussion above.)

Given these potentially problematic issues, I recommend that researchers make an effort to identify and use indicators that are as homogeneous as possible to avoid potential estimation and interpretation problems associated with indicator-specific effects. If this is not possible, researchers should carefully check the degree of indicator-specificity and make a theoretically sound choice of the reference indicator for each latent state factor. The reference indicator should be a "gold standard" measure that allows for an unambiguous interpretation of the latent state factors. Furthermore, a sensitivity analysis in which different reference indicators are used can help clarifying the consequences of choosing a particular reference indicator. If the results differ strongly for different reference indicators, researchers should be very careful in deciding which model version to use.

### *7.2.8 State-Trait Distinction*

Finally, it should be noted that the CS-C(*M*–1) model does not allow for a separation of occasion-specific influences from stable (trait-like) components of variance (e.g., Courvoisier et al., in press; Steyer et al., 1992, 1999). The state and method factors in the CS-C(*M*–1) model comprise both, error-free variance due to momentary states and variance due to stable trait influences. The model is therefore most appropriate for studies in which researchers are interested in trait change over time.

Nonetheless, the strengths of occasion-specific influences can be evaluated indirectly in the CS-C(*M*–1) model by assessing the correlations between the same state and method factors over time. Correlations close to one indicate that a construct is trait-like, rather than strongly influenced by occasion-specific influences. However, if the goal is not to assess trait change, but to separate variance components due to trait and occasion-specific influences in a multimethod context, the multi-method LST model should be applied (Courvoisier et al., in press; see Chapter 1.3.2.3 and Figure 7).

## 7.3 Summary of Guidelines and Tips for Applications

Given the limitations discussed in the previous section, some general guidelines and tips for the use of the CS-C($M$–1) approach in practice can be summarized as follows:

1. Use the CS-C($M$–1) model only for structurally different methods.
2. Conduct an application-oriented simulation study if you are not sure whether your (anticipated) sample size is large enough to obtain valid results from a CS-C($M$–1) analysis.
3. Be careful in interpreting the conventional $\chi^2$ test for complex model versions unless the sample size is very large.
4. Introduce as many parameter-reducing constraints as possible as long as these are tenable (do not lead to a distortion in model fit) and theoretically meaningful.
5. Use appropriate estimation methods (e.g., WLSMV) if ordinal items are selected as latent variable indicators.
6. Select the reference method based on careful theoretical considerations and ease of interpretation.
7. Select indicators that are as homogeneous as possible and carefully check the degree of indicator-specificity (see also Point 8 and Point 9). If you use the CS-C($M$–1) model with indicator-specific factors across time, conduct sensitivity analyses using different reference indicators to determine the consequences of changing the reference indicator.
8. If the proportion of indicator-specific variance over time is strong, possibly select a “gold standard” measure as reference indicator for each latent state factor.
9. Use the 3-Step procedure described in Chapter 5.2 to test for indicator-specificity and measurement invariance across time.
10. If measurement invariance is not tenable for all indicators, try to establish partial measurement invariance.

11. Collect data on more than two time points if possible and possibly use more than two indicators per factor to ensure proper identification and to minimize the likelihood of improper solutions.
12. Use Courvoisier et al.'s (in press) multi-method LST model if the goal is to separate variance components due to stable trait influences from momentary states.

## 7.4 Comparison With Other Approaches

To my knowledge, Burns et al. (2003; see also Burns & Haynes, 2006) were the first who explicitly proposed a CFA model for analyzing MTMM-MO data. As mentioned above, their model represents an extension of the single occasion CT-CM model (Marsh, 1989; Marsh & Bailey, 1991; Marsh & Grayson, 1995). The difference between Burns et al.'s multioccasion CT-CM model and the CS-C(*M*–1) model is that by defining one method as reference, the CS-C(*M*–1) model needs one method factor less than methods considered. In this way, an overfactorization that often leads to estimation problems in applications of the CT-CM model is avoided. Furthermore, the CS-C(*M*–1) model is in some sense less restrictive than Burns et al.'s model as it makes use of multiple indicators per CMOU. In this way, method effects can be conceptualized as being construct-specific and are not forced to generalize perfectly across different constructs. The use of multiple indicators per CMOU makes it possible to define construct-specific change factors also on the level of the method factors as illustrated in this paper. This would not be possible in Burns et al.'s model unless one would extend this model to a multiple indicator model.

Scherpenzeel and Saris (2007) presented a similar model, but with uncorrelated method factors and specific equality assumptions with regard to method effects. Their model represents an extension of the single indicator CT-UM model (see Chapter 1.3.1.1) to a longitudinal model. Scherpenzeel and Saris (2007) also discussed how their model can be extended to a multi-method LST model.

Vautier et al. (in press) recently presented a true change model with method effects. Their model is useful to deal with heterogeneous indicators in longitudinal studies. However, it is not specifically designed to analyze MTMM-MO data. Vautier et al.'s model includes only *one* indicator per CMOU, whereas the models discussed here are multiple indicator models. As a consequence, Vautier et al.'s model assumes perfect temporal stability of method effects and thus is not suitable for analyzing the convergent and discriminant validity of change. In contrast, the models presented here are specifically designed to deal with *changes* in all methods. Note, however, that Vautier et al.'s model could be extended to a multiple indicator model in which the assumption of perfect stability of method effects could be tested. Yet the extended model would still be different from the CS-C($M$–1) change model in that the method factors in Vautier et al.'s model are defined as *difference factors* (Vautier et al., in press), whereas the method factors in the CS-C($M$–1) model are defined as *residual factors* (Geiser et al., 2007). The detailed variance decomposition into consistency, method-specificity, indicator-specificity, and reliability available in the CS-C($M$–1) model is not available in Vautier et al.'s model.

As mentioned above, some research questions are less concerned with trait change (as assessed by the models presented here), but rather with situation-specific fluctuations around a stable trait value. For example, a researcher might be less interested in inter-individual differences in intra-individual change with respect to anxiety, but rather in the degree of situation-specific influences on the measurement of anxiety (e.g., Vautier, 2004). If the goal is to separate stable from situation-specific components of variance (in addition to the separation of method and error variance) in a MTMM-MO study, the multi-method LST model proposed by Courvoisier et al. (in press) can be applied. A related approach has been presented by Vautier (2004) who showed how method effects caused by bipolar items can be studied in an extended LST model. Scherpenzeel and Saris (2007) also presented a multi-method LST approach, but based on single indicators per CMOU.

## 7.5 Directions for Future Research

Finally, I think that it is useful to point out some aspects of MTMM-MO modeling that deserve attention in future studies. First of all, as was obvious from the simulation study presented in Chapter 6, further research is needed to identify optimal conditions for the applicability of the CS-C($M$–1) approach and for MTMM-MO models in general. Although my preliminary findings were rather encouraging, more detailed analyses of the limits of SEMs for MTMM-MO data are necessary.

Second, I already mentioned that MTMM-MO models specifically designed for interchangeable methods have not yet been developed. Interchangeable methods are quite common in psychology, so that the development of appropriate models for MTMM-MO data obtained from interchangeable methods is an important task for future research.

Third, given the omnipresence of categorical data in psychology and other social science disciplines, it would be worthwhile to consider MTMM-MO modeling approaches for categorical data in more detail. Which kind of specific problems are associated with the analysis of ordered categorical (ordinal) and dichotomous MTMM-MO data (e.g., regarding item-specificity and measurement invariance over time)? How can latent variable MTMM-MO models for non-ordered categorical (nominal) data (e.g., multimethod latent transition models) be defined?

Fourth, as I showed in my discussion of the indicator-specificity issue, this is an important practical modeling problem in facetted data structures such as MTMM-MO data. Although I presented two approaches to dealing with indicator specificity (one model for indicator-specific effects that generalize across methods but not across time, and one model for indicator-specific effects that generalize across time but not across methods), these two models might not cover all possible constellations of indicator-specificity that might occur in real data. Although this was not the case in the application presented in this work, it might be possible that indicator-specific effects generalize across both, different methods *and* across time. Future research should define and explore models that are able to handle both types of generalizing indicator-specificity simultaneously, as homogeneous indicators are rare in the social sciences. Moreover, the models presented here assume that indicator-specific effects are unidimensional. This might not always be a reasonable assumption. For example, indicator-specific effects may change in the course of a longitudinal investigation and the residual covariance structure might not be in line with a unidimensional model. Hence, it seems worthwhile to explore possibilities for modeling heterogeneous indicator-specific effects appropriately in the future.

Furthermore, it would be interesting to consider further extensions of the CS-C($M$–1) approach. For instance, I mentioned that the CS-C($M$–1) model can easily be extended to a latent autoregressive model or to a second-order latent growth curve model by imposing a second order growth structure on the latent state factors. Second-order growth models are very useful to test various hypotheses about change (Ferrer, Balluerka, & Widaman, 2008; Hancock, Kuo, & Lawrence, 2001; McArdle, 1988; Sayer & Cumsille, 2001). Another useful extension would be a multiple groups CS-C($M$–1) model that could for example be used to analyze data obtained from multi-method intervention or evaluation studies.

# 8 Conclusion

Longitudinal multi-method research designs have become increasingly popular in psychology over the past years. MTMM-MO data offer exciting new insights into psychological phenomena. In the present work, my goal was to show how sophisticated statistical models can be defined and applied to properly analyze MTMM-MO data and to extract as much information as possible from such kind of data. Although MTMM-MO data sets are very complex, with a theoretically sound and well-structured step-by-step analysis strategy, there is no need for researchers to be afraid of SEM analyses of such data.

# 9 Summary

In the present work, new structural equation models (SEMs) for the analysis of multitrait-multimethod-multioccasion (MTMM-MO) data are presented. The definition and psychometric analysis of the models is based on stochastic measurement theory (Steyer, 1989; Suppes & Zinnes, 1963). The applicability of the new models is evaluated through a reanalysis of a real MTMM-MO data set and a Monte Carlo simulation study.

In the introduction, an overview of existing SEMs for cross-sectional MTMM data is provided. The *Correlated Trait-Correlated Uniqueness* (CT-CU; Marsh, 1989), *Correlated Trait-Correlated Method* (CT-CM; Widaman, 1985), *Correlated Trait-Uncorrelated Method-* (CT-UM), and *Correlated Trait-Correlated (Method Minus One)-* [CT-C($M$–1); Eid, 2000] models are briefly reviewed and compared. It is concluded that the CT-C($M$–1) model for multiple indicators per trait-method unit (Eid et al., 2003) is one of the most useful models currently available for cross-sectional MTMM data. Subsequently, three different SEM approaches to the analysis of longitudinal MTMM data are discussed: Cole and Maxwell's (2003) multi-occasion CU model; Burns, Walsh, and Gomez' (2003) *Correlated State-Correlated Method model* (Burn & Haynes, 2006); and Courvoisier's (2006) multi-method latent state trait model (Courvoisier, Nussbeck, Eid, Geiser, & Cole, 2007). It is shown that a general measurement model for analyzing MTMM data and for analyzing change in MTMM-MO studies has not yet been developed.

Subsequently, basic principles of classical test theory (Steyer, 1989, Steyer & Eid, 2001) and latent state theory (Steyer, 1988; Steyer, Ferring, & Schmitt, 1992) are reviewed. These concepts are used in the formulation of the new MTMM-MO models. Afterwards, two versions of the *Correlated State-Correlated (Method Minus One)* [CS-C($M$–1)] model are introduced. These models represent a combination of Eid et al.'s (2003) multiple indicator CT-C($M$–1) model and the correlated state model for mono-method data (Steyer et al., 1992). A detailed psychometric analysis of the CS-C($M$–1) models is provided. It is then shown how CS-C($M$–1) models can be extended to latent difference models to study inter-individual differences in intra-individual change over time. The so-called CS-C($M$–1) change model represents a multimethod extension of Steyer, Eid, and Schwenkmezger's (1997) true change model (Steyer, Partchev, & Shanahan, 2000). The CS-C($M$–1) change model can be used to study change in different methods simultaneously and to determine the degree of convergent validity and method-specificity of observed and latent change scores.

In the empirical part, a 3-Step procedure for analyzing, testing and selecting an appropriate CS-C($M$–1) model is presented and the applicability of the new models is investigated in a reanalysis of a MTMM-MO data set and a Monte Carlo simulation study. The results show that the new models are useful to analyze the complex structure of a MTMM-MO matrix obtained from multiple indicators per construct-method-occasion unit. In the final part, advantages and limitations of the models as well as detailed guidelines for potential users are discussed. Furthermore, the new models are compared with already established methods for analyzing MTMM-MO data and directions for future research are pointed out.

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# 13 Appendix

## 13.1 Mplus Scripts for Estimating the CS-C(*M*–1) Model

### *13.1.1 CS-C(M–1) State Model*

```
TITLE:     Correlated State-Correlated (Methods-1) Model
           State version with general state factors
           and indicator-specific factors over time (see Figure 17)
           Constructs: Depression, Anxiety
           Methods: Self-report (reference method), parent report,
           teacher report
           4 measurement occasions (T1-T4)
           Model variant 3D_2 reported in Chapter 5.4
           Lines beginning with an exclamation mark (!) represent
           comments

! Name of the ASCII file containing the data to be analyzed:
DATA:      FILE = mtmmmo.dat;

VARIABLE:

! Names of the variables to be analyzed:
NAMES =  class
        ds11 ds21 ds12 ds22 ds13 ds23 ds14 ds24
        dp11 dp21 dp12 dp22 dp13 dp23 dp14 dp24
        dt11 dt21 dt12 dt22 dt13 dt23 dt14 dt24
        as11 as21 as12 as22 as13 as23 as14 as24
        ap11 ap21 ap12 ap22 ap13 ap23 ap14 ap24
        at11 at21 at12 at22 at13 at23 at14 at24;

! class is the cluster variable indicating the
! hierarchical (multilevel) structure of the data
! (children nested within school classes)
! ds = depression self-report (reference method)
! dp = depression parent report
! dt = depression teacher report
! as = anxiety self-report (reference method)
! ap = anxiety parent report
! at = anxiety teacher report
! The first number refers to the indicator
! The second number indicates the occasion of measurement

! Variables to be used in the model:
USEVAR = ds11 ds21 ds12 ds22 ds13 ds23 ds14 ds24
         dp11 dp21 dp12 dp22 dp13 dp23 dp14 dp24
         dt11 dt21 dt12 dt22 dt13 dt23 dt14 dt24
         as11 as21 as12 as22 as13 as23 as14 as24
         ap11 ap21 ap12 ap22 ap13 ap23 ap14 ap24
         at11 at21 at12 at22 at13 at23 at14 at24;

! Missing value flag:
! Missing values are coded with “99” for all variables
MISSING = ALL(99);

! Grouping variable indicating in which way
! the observations are clustered (school class number)
CLUSTER = class;
```

```
! Multilevel structure is accounted for by using a robust
! Maximum Likelihood (ML) estimator (TYPE = COMPLEX)
! In Mplus 4, Full information ML (FIML) estimation with missing data
! is requested by choosing "MISSING H1"
! (as of Mplus version 5, FIML estimation is the default)
ANALYSIS:  TYPE = COMPLEX MISSING H1;

! Model to be estimated:
MODEL:
! Depression: Latent state factors T1-T4
! factor loadings are set equal over time
! for the self- and parent report indicators
! the teacher report loadings are unconstrained
! (note: the loading of the first indicator following
! the BY statement is set equal to 1 for identification as the default)
! dep1 by ds11
        ds21 (1)
        dp11 (2)
        dp21 (3)
        dt11
        dt21;

dep2 by ds12
        ds22 (1)
        dp12 (2)
        dp22 (3)
        dt12
        dt22;

dep3 by ds13
        ds23 (1)
        dp13 (2)
        dp23 (3)
        dt13
        dt23;

dep4 by ds14
        ds24 (1)
        dp14 (2)
        dp24 (3)
        dt14
        dt24;

! Depression: Method factors parent rating T1-T4
! The loadings are constrained to be time-invariant
mpd1 by dp11
        dp21 (4);

mpd2 by dp12
        dp22 (4);

mpd3 by dp13
        dp23 (4);

mpd4 by dp14
        dp24 (4);

! Depression: Method factors teacher rating T1-T4
! The loadings are constrained to be time-invariant
mtd1 by dt11
        dt21 (5);
```

```
mtd2 by dt12
        dt22 (5);

mtd3 by dt13
        dt23 (5);

mtd4 by dt14
        dt24 (5);

! Depression: Indicator-specific factors for the second indicator
! Self-report T1-T4
! The loadings are fixed to one on all occasions of measurement
isd by ds21 ds22@1 ds23@1 ds24@1;

! Parent report T1-T4
! The first loading is fixed to one by default
! The loadings at T2, T3, and T4 are freely estimated
ipd by dp21 dp22 dp23 dp24;

! Teacher report T1-T2
! The loadings are fixed to one on both occasions of measurement
itd1 by dt21 dt22@1;

! Teacher report T3-T4
! The loadings are fixed to one on both occasions of measurement
itd2 by dt23 dt24@1;

! Depression: Factor covariances that are constrained to zero
mpd1 with dep1@0 dep2@0 dep3@0 dep4@0;
mpd2 with dep1@0 dep2@0 dep3@0 dep4@0;
mpd3 with dep1@0 dep2@0 dep3@0 dep4@0;
mpd4 with dep1@0 dep2@0 dep3@0 dep4@0;
mtd1 with dep1@0 dep2@0 dep3@0 dep4@0;
mtd2 with dep1@0 dep2@0 dep3@0 dep4@0;
mtd3 with dep1@0 dep2@0 dep3@0 dep4@0;
mtd4 with dep1@0 dep2@0 dep3@0 dep4@0;
isd with dep1@0 dep2@0 dep3@0 dep4@0;
ipd with dep1@0 dep2@0 dep3@0 dep4@0 mpd1@0 mpd2@0 mpd3@0 mpd4@0;
itd1 with dep1@0 dep2@0 mtd1@0 mtd2@0;
itd2 with dep3@0 dep4@0 mtd3@0 mtd4@0;

! Depression: Variances
! Factor variances are constrained to be time-invariant
! for the state factors and method factors
! pertaining to the parent rating
dep1-dep4 (6);

mpd1-mpd4 (7);

! Depression: Residual variances
! Residual variances are constrained to be
! time-invariant for all indicators
ds11 (8);
ds12 (8);
ds13 (8);
ds14 (8);
ds21 (9);
ds22 (9);
ds23 (9);
ds24 (9);
```

```
dp11 (10);
dp12 (10);
dp13 (10);
dp14 (10);
dp21 (11);
dp22 (11);
dp23 (11);
dp24 (11);

dt11 (12);
dt12 (12);
dt13 (12);
dt14 (12);
dt21 (13);
dt22 (13);
dt23 (13);
dt24 (13);

! Depression: Intercepts and latent means
! Measurement intercepts for first indicator of the reference
! method are constrained to zero to identify
! the depression state factor means
! on each occasion of measurement
[ds11@0];
[ds12@0];
[ds13@0];
[ds14@0];

! Measurement intercepts constrained to be time-invariant
! for the second indicator of the reference method
[ds21] (14);
[ds22] (14);
[ds23] (14);
[ds24] (14);

! Latent state factor means for depression are freely estimated
[dep1];
[dep2];
[dep3];
[dep4];

! The means of the method factors and
! indicator-specific factors are fixed to zero.
! This is done because these factors are residual factors
[mpd1@0];
[mpd2@0];
[mpd3@0];
[mpd4@0];
[mtd1@0];
[mtd2@0];
[mtd3@0];
[mtd4@0];
[isd@0];
[ipd@0];
[itd1@0];
[itd2@0];


! Anxiety: Latent state factors T1-T4
! factor loadings are held equal over time
! for the self- and parent report indicators
! the teacher report loadings are unconstrained
```

```
anx1 by as11
        as21 (15)
        ap11 (16)
        ap21 (17)
        at11
        at21;

anx2 by as12
        as22 (15)
        ap12 (16)
        ap22 (17)
        at12
        at22;

anx3 by as13
        as23 (15)
        ap13 (16)
        ap23 (17)
        at13
        at23;

anx4 by as14
        as24 (15)
        ap14 (16)
        ap24 (17)
        at14
        at24;

! Anxiety: Method factors parent rating T1-T4
! The loadings are constrained to be time-invariant
mpa1 by ap11
        ap21 (18);

mpa2 by ap12
        ap22 (18);

mpa3 by ap13
        ap23 (18);

mpa4 by ap14
        ap24 (18);

! Anxiety: Method factors teacher rating T1-T4
! The loadings are constrained to be time-invariant
mta1 by at11
        at21 (19);

mta2 by at12
        at22 (19);

mta3 by at13
        at23 (19);

mta4 by at14
        at24 (19);

! Depression: Indicator-specific factors for the second indicator
! Self-report T1-T4
! The loadings are fixed to one on all occasions of measurement

isa by as21 as22@1 as23@1 as24@1;
```

```
!Parent report T1-T4
!The first loading is fixed to one by default
!The loadings at T2, T3, and T4 are freely estimated
ipa by ap21 ap22 ap23 ap24;

!Teacher report T1-T2
!The loadings are fixed to one on both occasions of measurement
ita1 by at21 at22@1;

!Teacher report T3-T4
!The loadings are fixed to one on both occasions of measurement
ita2 by at23 at24@1;

!Anxiety: Covariances constrained to zero
mpa1 with anx1@0 anx2@0 anx3@0 anx4@0;
mpa2 with anx1@0 anx2@0 anx3@0 anx4@0;
mpa3 with anx1@0 anx2@0 anx3@0 anx4@0;
mpa4 with anx1@0 anx2@0 anx3@0 anx4@0;
mta1 with anx1@0 anx2@0 anx3@0 anx4@0;
mta2 with anx1@0 anx2@0 anx3@0 anx4@0;
mta3 with anx1@0 anx2@0 anx3@0 anx4@0;
mta4 with anx1@0 anx2@0 anx3@0 anx4@0;
isa with anx1@0 anx2@0 anx3@0 anx4@0;
ipa with anx1@0 anx2@0 anx3@0 anx4@0 mpa1@0 mpa2@0 mpa3@0 mpa4@0;
ita1 with anx1@0 anx2@0 mta1@0 mta2@0;
ita2 with anx3@0 anx4@0 mta3@0 mta4@0;

!Anxiety: Variances
!State factor variances and method factor variances
!pertaining to the parent rating are constrained to be time-invariant
anx1-anx4 (20);

mpa1-mpa4 (21);

!Anxiety: Residual variances
!Residual variances are constrained to be
!time-invariant for all indicators
as11 (22);
as12 (22);
as13 (22);
as14 (22);
as21 (23);
as22 (23);
as23 (23);
as24 (23);

ap11 (24);
ap12 (24);
ap13 (24);
ap14 (24);
ap21 (25);
ap22 (25);
ap23 (25);
ap24 (25);

at11 (26);
at12 (26);
at13 (26);
at14 (26);
at21 (27);
at22 (27);
at23 (27);
```

```
at24 (27);

! Anxiety: Intercepts and latent means
! Measurement intercepts for first indicator of the reference
! method are constrained to zero to identify the
! anxiety state factor means
! on each occasion of measurement
[as11@0];
[as12@0];
[as13@0];
[as14@0];

! Measurement intercepts constrained to be time-invariant
! for the second indicator of the reference method
[as21] (28);
[as22] (28);
[as23] (28);
[as24] (28);

! Latent state factor means for anxiety are freely estimated
[anx1];
[anx2];
[anx3];
[anx4];

! The means of the method factors and
! indicator-specific factors are fixed to zero.
! This is done because these factors are residual factors
[mpa1@0];
[mpa2@0];
[mpa3@0];
[mpa4@0];
[mta1@0];
[mta2@0];
[mta3@0];
[mta4@0];
[isa@0];
[ipa@0];
[ita1@0];
[ita2@0];

! The following commands request additional output (sample statistics,
! the completely standardized solution, and
! the observed missing data patterns)
OUTPUT:  SAMPSTAT STANDARDIZED STDYX PATTERNS;
```

### *13.1.2 CS-C(M–1) Baseline Change Model*

```
TITLE:     Correlated State-Correlated (Methods-1) Model
           with general state factors,
           indicator-specific factors over time, and strict factorial
           invariance
           Baseline change version with sex as a covariate
           Constructs: Depression, Anxiety
           Methods: Self-report (reference method), parent report
           4 measurement occasions (T1-T4)
           Model reported in Chapter 5.5
           Lines beginning with an exclamation mark (!) represent
           comments

! Name of the ASCII file containing the data to be analyzed:
DATA:      FILE = mtmmmo.dat;

VARIABLE:

! Definition of the names of the observed variables
! in the file "mtmmmo.dat"
! class = cluster variable indicating the
! hierarchical (multilevel) structure of the data
! (children nested within school classes)
! ds = depression self-report
! dp = depression parent report
! as = anxiety self-report
! ap = anxiety parent report
! the first number refers to the indicator
! the second number indicates the occasion of measurement
NAMES =  class sex
        ds11 ds21 ds12 ds22 ds13 ds23
        dp11 dp21 dp12 dp22 dp13 dp23
        as11 as21 as12 as22 as13 as23
        ap11 ap21 ap12 ap22 ap13 ap23;

! Variables to be used in the model:
USEVAR = ds11 ds21 ds12 ds22 ds13 ds23
         dp11 dp21 dp12 dp22 dp13 dp23
         as11 as21 as12 as22 as13 as23
         ap11 ap21 ap12 ap22 ap13 ap23 sex;

! Missing value flag:
! Missing values are coded with "99" for all variables
MISSING = ALL(99);

! Grouping variable indicating in which way
! the observations are clustered (school class number)
CLUSTER = class;

! Multilevel structure is accounted for by using a robust
! Maximum Likelihood (ML) estimator (TYPE = COMPLEX)
! In Mplus 4, Full information ML (FIML) estimation with missing data
! is requested by choosing "TYPE = MISSING H1"
! (as of Mplus version 5, FIML estimation is the default)
ANALYSIS:  TYPE = COMPLEX MISSING H1;

! Model to be estimated
! Note: the loading of the first indicator following
! the BY statement is always fixed to 1 for identification as the default.
! For example, the loading of ds11 on dep1 is fixed to 1 automatically.
```

```
MODEL:
! Depression state factors
! Loadings are constrained to be time-invariant

        dep1 by ds11
                ds21 (1)
                dp11 (2)
                dp21 (3);

        dep2 by ds12
                ds22 (1)
                dp12 (2)
                dp22 (3);

        dep3 by ds13
                ds23 (1)
                dp13 (2)
                dp23 (3);

! Depression method factors for the parent report indicators
! Loadings are constrained to be time-invariant
        md1 by dp11
               dp21 (4);

        md2 by dp12
               dp22 (4);

        md3 by dp13
               dp23 (4);

! Specification of depression state change factors (self report)
! Introducing names for the latent difference variables
! Latent difference dep2 minus dep1
        dep21 by ds11@0;

! Latent difference dep3 minus dep1
        dep31 by ds11@0;

! Latent “regressions” defining the change scores
        dep2 on dep1@1 dep21@1;
        dep3 on dep1@1 dep31@1;

! The latent residual variances of the states
! are constrained to zero as the states at T2 and T3
! are completely determined by
! the initial status (dep1) and change (dep21, dep31)
        dep2@0;
        dep3@0;

! Specification of depression method change factors (parent report)
! Introducing names for the latent difference variables
! Latent difference md2 minus md1
        md21 by ds11@0;

! Latent difference md3 minus md1
        md31 by ds11@0;

! Latent “regressions” defining the change scores
        md2 on md1@1 md21@1;
        md3 on md1@1 md31@1;

! The latent residual variances of the method factors
```

```
! are constrained to zero as the method factors at T2 and T3
! are completely determined by
! the initial status (md1) and change (md21, md31)
        md2@0;
        md3@0;

! Indicator-specific factors
! Loadings are constrained to be time-invariant

! Self-report
        isd by ds21 ds22@1 ds23@1;
! Parent report
        ipd by dp21 dp22@1 dp23@1;

! Correlations between method (change) factors
! and state (change) factors are fixed to zero
        md1 with dep1@0 dep21@0 dep31@0;
        md21 with dep1@0 dep21@0 dep31@0;
        md31 with dep1@0 dep21@0 dep31@0;

! Correlations between indicator-specific factors and
! method (change) factors / state (change) factors are fixed to zero
        isd with dep1@0 dep21@0 dep31@0 md1@0 md21@0 md31@0;
        ipd with dep1@0 dep21@0 dep31@0 md1@0 md21@0 md31@0;

! Measurement intercepts
! The intercepts of the marker indicators are fixed to zero
! in order to identify the means of the state/change factors.
! The remaining intercepts are constrained to be time-invariant
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [ds21] (5);
        [ds22] (5);
        [ds23] (5);
        [dp11] (6);
        [dp12] (6);
        [dp13] (6);
        [dp21] (7);
        [dp22] (7);
        [dp23] (7);

! Estimation of the latent state/change factor means
        [dep1];
        [dep21];
        [dep31];

! The means of the method (change) factors and indicator-specific factors
! are fixed to zero
        [md1@0];
        [md21@0];
        [md31@0];
        [isd@0];
        [ipd@0];

! Variances of all error variables are constrained to be time-invariant
        ds11 (15);
        ds12 (15);
        ds13 (15);
        ds21 (16);
        ds22 (16);
        ds23 (16);
```

```
        dp11 (17);
        dp12 (17);
        dp13 (17);
        dp21 (18);
        dp22 (18);
        dp23 (18);

! Anxiety state factors
! Loadings are constrained to be time-invariant
        anx1 by as11
                as21 (8)
                ap11 (9)
                ap21 (10);

        anx2 by as12
                as22 (8)
                ap12 (9)
                ap22 (10);

        anx3 by as13
                as23 (8)
                ap13 (9)
                ap23 (10);

! Anxiety method factors for the parent report indicators
! Loadings are constrained to be time-invariant
        ma1 by ap11
               ap21 (11);

        ma2 by ap12
               ap22 (11);

        ma3 by ap13
               ap23 (11);

! Specification of anxiety state change factors (self report)
! Introducing names for the latent difference variables
! Latent difference anx2 minus anx1
        anx21 by ds11@0;

! Latent difference anx3 minus anx1
        anx31 by ds11@0;

! Latent "regressions" defining the change scores
        anx2 on anx1@1 anx21@1;
        anx3 on anx1@1 anx31@1;

! The latent residual variances of the states
! are constrained to zero as the states at T2 and T3
! are completely determined by
! the initial status (anx1) and change (anx21, anx31)
        anx2@0;
        anx3@0;

! Specification of anxiety method change factors (parent report)
! Introducing names for the latent difference variables
! Latent difference ma2 minus ma1
        ma21 by ds11@0;

! Latent difference ma3 minus ma1
        ma31 by ds11@0;
```

```
! Latent "regressions" defining the change scores
        ma2 on ma1@1 ma21@1;
        ma3 on ma1@1 ma31@1;

! The latent residual variances of the method factors
! are constrained to zero as the method factors at T2 and T3
! are completely determined by
! the initial status (ma1) and change (ma21, ma31)
        ma2@0;
        ma3@0;

! Indicator-specific factors
! Loadings are constrained to be time-invariant

! Self-report
        isa by as21 as22@1 as23@1;
! Parent report
        ipa by ap21 ap22@1 ap23@1;

! Correlations between method (change) factors
! and state (change) factors are fixed to zero
        ma1 with anx1@0 anx21@0 anx31@0;
        ma21 with anx1@0 anx21@0 anx31@0;
        ma31 with anx1@0 anx21@0 anx31@0;

! Correlations between indicator-specific factors and
! method (change) factors / state (change) factors are fixed to zero
        isa with anx1@0 anx21@0 anx31@0 ma1@0 ma21@0 ma31@0;
        ipa with anx1@0 anx21@0 anx31@0 ma1@0 ma21@0 ma31@0;

! Measurement intercepts
! The intercepts of the marker indicators are fixed to zero
! in order to identify the means of the state/change factors.
! The remaining intercepts are constrained to be time-invariant
        [as11@0];
        [as12@0];
        [as13@0];
        [as21] (12);
        [as22] (12);
        [as23] (12);
        [ap11] (13);
        [ap12] (13);
        [ap13] (13);
        [ap21] (14);
        [ap22] (14);
        [ap23] (14);

! Estimation of the latent state/change factor means
        [anx1];
        [anx21];
        [anx31];

! The means of the method (change) factors and indicator-specific factors
! are fixed to zero
        [ma1@0];
        [ma21@0];
        [ma31@0];
        [isa@0];
        [ipa@0];

! Variances of all error variables are constrained to be time-invariant
        as11 (19);
```

```
        as12 (19);
        as13 (19);
        as21 (20);
        as22 (20);
        as23 (20);
        ag11 (21);
        ag12 (21);
        ag13 (21);
        ag21 (22);
        ag22 (22);
        ag23 (22);

! Correlations between T2 and T3 state and method factors
! and other variables must be fixed to zero
        dep2 with sex@0;
        dep3 with sex@0 dep2@0;

        md2 with sex@0 dep2@0 dep3@0;
        md3 with sex@0 dep2@0 dep3@0 md2@0;

        anx2 with sex@0 dep2@0 dep3@0 md2@0 md3@0;
        anx3 with sex@0 dep2@0 dep3@0 md2@0 md3@0 anx2@0;

        ma2 with sex@0 dep2@0 dep3@0 md2@0 md3@0 anx2@0 anx3@0;
        ma3 with sex@0 dep2@0 dep3@0 md2@0 md3@0 anx2@0 anx3@0 ma2@0;

! Estimation of the admissible correlations between sex
! and latent state/method/change/indicator-specific factors
        sex with dep1 dep21 dep31 md1 md21 md31
                 anx1 anx21 anx31 ma1 ma21 ma31
                 isd ipd isa ipa;

! The following commands request additional output (sample statistics,
! the completely standardized solution (STANDARDIZED STDYX), and
! the observed missing data patterns
OUTPUT:  SAMPSTAT STANDARDIZED STDYX PATTERNS;
```

### 13.1.3 CS-C(M–1) Neighbor Change Model

```
TITLE:     Correlated State-Correlated (Methods-1) Model
           with general state factors,
           indicator-specific factors over time, and strict factorial
           invariance
           Neighbor change version with sex as a covariate
           Constructs: Depression, Anxiety
           Methods: Self-report (reference method), parent report
           4 measurement occasions (T1-T4)
           Model reported in Chapter 5.5
           Lines beginning with an exclamation mark (!) represent
           comments

! Name of the ASCII file containing the data to be analyzed:
DATA:      FILE = mtmmmo.dat;

VARIABLE:

! Definition of the names of the observed variables
! in the file "mtmmmo.dat"
! class = cluster variable indicating the
! hierarchical (multilevel) structure of the data
! (children nested within school classes)
! ds = depression self-report
! dp = depression parent report
! as = anxiety self-report
! ap = anxiety parent report
! the first number refers to the indicator
! the second number indicates the occasion of measurement
NAMES =  class sex
        ds11 ds21 ds12 ds22 ds13 ds23
        dp11 dp21 dp12 dp22 dp13 dp23
        as11 as21 as12 as22 as13 as23
        ap11 ap21 ap12 ap22 ap13 ap23;

! Variables to be used in the model:
USEVAR = ds11 ds21 ds12 ds22 ds13 ds23
         dp11 dp21 dp12 dp22 dp13 dp23
         as11 as21 as12 as22 as13 as23
         ap11 ap21 ap12 ap22 ap13 ap23 sex;

! Missing value flag:
! Missing values are coded with "99" for all variables
MISSING = ALL(99);

! Grouping variable indicating in which way
! the observations are clustered (school class number)
CLUSTER = class;

! Multilevel structure is accounted for by using a robust
! Maximum Likelihood (ML) estimator (TYPE = COMPLEX)
! In Mplus 4, Full information ML (FIML) estimation with missing data
! is requested by choosing "TYPE = MISSING H1"
! (as of Mplus version 5, FIML estimation is the default)
ANALYSIS:  TYPE = COMPLEX MISSING H1;

! Model to be estimated
! Note: the loading of the first indicator following
! the BY statement is always fixed to 1 for identification as the default.
! For example, the loading of ds11 on dep1 is fixed to 1 automatically.
```

```
MODEL:
! Depression state factors
! Loadings are constrained to be time-invariant

        dep1 by ds11
                ds21 (1)
                dp11 (2)
                dp21 (3);

        dep2 by ds12
                ds22 (1)
                dp12 (2)
                dp22 (3);

        dep3 by ds13
                ds23 (1)
                dp13 (2)
                dp23 (3);

! Depression method factors for the parent report indicators
! Loadings are constrained to be time-invariant
        md1 by dp11
               dp21 (4);

        md2 by dp12
               dp22 (4);

        md3 by dp13
               dp23 (4);

! Specification of depression state change factors (self report)
! Introducing names for the latent difference variables
! Latent difference dep2 minus dep1
        dep21 by ds11@0;

! Latent difference dep3 minus dep2
        dep32 by ds11@0;

! Latent "regressions" defining the change scores
        dep2 on dep1@1 dep21@1;
        dep3 on dep2@1 dep32@1;

! The latent residual variances of the states
! are constrained to zero as the states at T2 and T3
! are completely determined by
! the initial status (dep1) and change (dep21, dep32)
        dep2@0;
        dep3@0;

! Specification of depression method change factors (parent report)
! Introducing names for the latent difference variables
! Latent difference md2 minus md1
        md21 by ds11@0;

! Latent difference md3 minus md2
        md32 by ds11@0;

! Latent "regressions" defining the change scores
        md2 on md1@1 md21@1;
        md3 on md2@1 md32@1;

! The latent residual variances of the method factors
```

```
! are constrained to zero as the method factors at T2 and T3
! are completely determined by
! the initial status (md1) and change (md21, md32)
        md2@0;
        md3@0;

! Indicator-specific factors
! Loadings are constrained to be time-invariant

! Self-report
        isd by ds21 ds22@1 ds23@1;
! Parent report
        ipd by dp21 dp22@1 dp23@1;

! Correlations between method (change) factors
! and state (change) factors are fixed to zero
        md1 with dep1@0 dep21@0 dep32@0;
        md21 with dep1@0 dep21@0 dep32@0;
        md32 with dep1@0 dep21@0 dep32@0;

! Correlations between indicator-specific factors and
! method (change) factors / state (change) factors are fixed to zero
        isd with dep1@0 dep21@0 dep32@0 md1@0 md21@0 md32@0;
        ipd with dep1@0 dep21@0 dep32@0 md1@0 md21@0 md32@0;

! Measurement intercepts
! The intercepts of the marker indicators are fixed to zero
! in order to identify the means of the state/change factors.
! The remaining intercepts are constrained to be time-invariant
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [ds21] (5);
        [ds22] (5);
        [ds23] (5);
        [dp11] (6);
        [dp12] (6);
        [dp13] (6);
        [dp21] (7);
        [dp22] (7);
        [dp23] (7);

! Estimation of the latent state/change factor means
        [dep1];
        [dep21];
        [dep32];

! The means of the method (change) factors and indicator-specific factors
! are fixed to zero
        [md1@0];
        [md21@0];
        [md32@0];
        [isd@0];
        [ipd@0];

! Variances of all error variables are constrained to be time-invariant
        ds11 (15);
        ds12 (15);
        ds13 (15);
        ds21 (16);
        ds22 (16);
        ds23 (16);
```

```
        dp11 (17);
        dp12 (17);
        dp13 (17);
        dp21 (18);
        dp22 (18);
        dp23 (18);

! Anxiety state factors
! Loadings are constrained to be time-invariant
        anx1 by as11
                as21 (8)
                ap11 (9)
                ap21 (10);

        anx2 by as12
                as22 (8)
                ap12 (9)
                ap22 (10);

        anx3 by as13
                as23 (8)
                ap13 (9)
                ap23 (10);

! Anxiety method factors for the parent report indicators
! Loadings are constrained to be time-invariant
        ma1 by ap11
               ap21 (11);

        ma2 by ap12
               ap22 (11);

        ma3 by ap13
               ap23 (11);

! Specification of anxiety state change factors (self report)
! Introducing names for the latent difference variables
! Latent difference anx2 minus anx1
        anx21 by ds11@0;

! Latent difference anx3 minus anx2
        anx32 by ds11@0;

! Latent “regressions” defining the change scores
        anx2 on anx1@1 anx21@1;
        anx3 on anx2@1 anx32@1;

! The latent residual variances of the states
! are constrained to zero as the states at T2 and T3
! are completely determined by
! the initial status (anx1) and change (anx21, anx32)
        anx2@0;
        anx3@0;

! Specification of anxiety method change factors (parent report)
! Introducing names for the latent difference variables
! Latent difference ma2 minus ma1
        ma21 by ds11@0;

! Latent difference ma3 minus ma2
        ma32 by ds11@0;
```

```
! Latent "regressions" defining the change scores
        ma2 on ma1@1 ma21@1;
        ma3 on ma2@1 ma32@1;

! The latent residual variances of the method factors
! are constrained to zero as the method factors at T2 and T3
! are completely determined by
! the initial status (ma1) and change (ma21, ma32)
        ma2@0;
        ma3@0;

! Indicator-specific factors
! Loadings are constrained to be time-invariant

! Self-report
        isa by as21 as22@1 as23@1;
! Parent report
        ipa by ap21 ap22@1 ap23@1;

! Correlations between method (change) factors
! and state (change) factors are fixed to zero
        ma1 with anx1@0 anx21@0 anx32@0;
        ma21 with anx1@0 anx21@0 anx32@0;
        ma32 with anx1@0 anx21@0 anx32@0;

! Correlations between indicator-specific factors and
! method (change) factors / state (change) factors are fixed to zero
        isa with anx1@0 anx21@0 anx32@0 ma1@0 ma21@0 ma32@0;
        ipa with anx1@0 anx21@0 anx32@0 ma1@0 ma21@0 ma32@0;

! Measurement intercepts
! The intercepts of the marker indicators are fixed to zero
! in order to identify the means of the state/change factors.
! The remaining intercepts are constrained to be time-invariant
        [as11@0];
        [as12@0];
        [as13@0];
        [as21] (12);
        [as22] (12);
        [as23] (12);
        [ap11] (13);
        [ap12] (13);
        [ap13] (13);
        [ap21] (14);
        [ap22] (14);
        [ap23] (14);

! Estimation of the latent state/change factor means
        [anx1];
        [anx21];
        [anx32];

! The means of the method (change) factors and indicator-specific factors
! are fixed to zero
        [ma1@0];
        [ma21@0];
        [ma32@0];
        [isa@0];
        [ipa@0];

! Variances of all error variables are constrained to be time-invariant
        as11 (19);
```

```
        as12 (19);
        as13 (19);
        as21 (20);
        as22 (20);
        as23 (20);
        ag11 (21);
        ag12 (21);
        ag13 (21);
        ag21 (22);
        ag22 (22);
        ag23 (22);

! Correlations between T2 and T3 state and method factors
! and other variables must be fixed to zero
        dep2 with sex@0;
        dep3 with sex@0 dep2@0;

        md2 with sex@0 dep2@0 dep3@0;
        md3 with sex@0 dep2@0 dep3@0 md2@0;

        anx2 with sex@0 dep2@0 dep3@0 md2@0 md3@0;
        anx3 with sex@0 dep2@0 dep3@0 md2@0 md3@0 anx2@0;

        ma2 with sex@0 dep2@0 dep3@0 md2@0 md3@0 anx2@0 anx3@0;
        ma3 with sex@0 dep2@0 dep3@0 md2@0 md3@0 anx2@0 anx3@0 ma2@0;

! Estimation of the admissible correlations between sex
! and latent state/method/change/indicator-specific factors
        sex with dep1 dep21 dep32 md1 md21 md32
                 anx1 anx21 anx32 ma1 ma21 ma32
                 isd ipd isa ipa;

! The following commands request additional output (sample statistics,
! the completely standardized solution (STANDARDIZED STDYX), and
! the observed missing data patterns
OUTPUT:  SAMPSTAT STANDARDIZED STDYX PATTERNS;
```

## 13.2 Mplus Scripts for the Monte Carlo Simulation Study

### *13.2.1 CS-C(M–1) State Model*

```
TITLE:  Monte Carlo Simulation of the CS-C(M-1) State Model
        Simulations reported in Chapter 6
        State version with general state factors
        fit to the data set of Prof. David Cole
        3 Constructs (Depression, Anxiety, Competence)
        3 Methods (Self, Parent, Teacher)
        3 Measurement Occasions
        2 Indicators per CMOU
        Estimates based on FIML analysis (file: state_complex_FIML.out)
        Sample size condition N = 125

MONTECARLO:

!Names of the observed variables
!d = depression, a = anxiety, c = competence
!s = self report, g = guardian (parent) report, t = teacher report
!the first number refers to the indicator,
!the second number refers to the measurement occasion
  NAMES = ds11 ds21
          ds12 ds22
          ds13 ds23
          dg11 dg21
          dg12 dg22
          dg13 dg23
          dt11 dt21
          dt12 dt22
          dt13 dt23
          as11 as21
          as12 as22
          as13 as23
          ag11 ag21
          ag12 ag22
          ag13 ag23
          at11 at21
          at12 at22
          at13 at23
          cs11 cs21
          cs12 cs22
          cs13 cs23
          cg11 cg21
          cg12 cg22
          cg13 cg23
          ct11 ct21
          ct12 ct22
          ct13 ct23;

!Sample size for the MC samples (here: N = 125)
NOBSERVATIONS = 125;

!Number of replications (here: 500)
NREPS = 500;

!Seed used for the simulation
SEED = 111111;

!File containing the actual parameter estimates
```

```
!obtained for Prof. Cole's data.
!These estimates are used as population values for the simulation,
!for determining coverage, and as starting values.
!Note: The population values are available from the author
!upon request
Population = state_3traits_FIML.dat;
Coverage = state_3traits_FIML.dat;
starting = state_3traits_FIML.dat;

!Definition of the population model
MODEL POPULATION:
!Depression state factors
        dep1 by ds11
                ds21 (1)
                dg11 (2)
                dg21 (3)
                dt11 (4)
                dt21 (5);

        dep2 by ds12
                ds22 (1)
                dg12 (2)
                dg22 (3)
                dt12 (4)
                dt22 (5);

        dep3 by ds13
                ds23 (1)
                dg13 (2)
                dg23 (3)
                dt13 (4)
                dt23 (5);

!Method factors parent (guardian) rating
        mgd1 by dg11
                dg21 (6);

        mgd2 by dg12
                dg22 (6);

        mgd3 by dg13
                dg23 (6);

!Method factors teacher rating
        mtd1 by dt11
                dt21 (7);

        mtd2 by dt12
                dt22 (7);

        mtd3 by dt13
                dt23 (7);

!Indicator-specific factors
        isd by ds21 ds22@1 ds23@1;
        igd by dg21 dg22@1 dg23@1;
        itd by dt21 dt22@1;

!Non-admissible latent correlations constrained to zero
        mgd1 with dep1@0 dep2@0 dep3@0;
        mgd2 with dep1@0 dep2@0 dep3@0;
        mgd3 with dep1@0 dep2@0 dep3@0;
```

```
        mtd1 with dep1@0 dep2@0 dep3@0;
        mtd2 with dep1@0 dep2@0 dep3@0;
        mtd3 with dep1@0 dep2@0 dep3@0;
        isd with dep1@0 dep2@0 dep3@0;
        igd with dep1@0 dep2@0 dep3@0;
        igd with mgd1@0 mgd2@0 mgd3@0;
        itd with dep1@0 dep2@0;
        itd with mtd1@0 mtd2@0;

! Intercepts and latent means
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [dep1];
        [dep2];
        [dep3];
        [mgd1@0];
        [mgd2@0];
        [mgd3@0];
        [mtd1@0];
        [mtd2@0];
        [mtd3@0];
        [isd@0];
        [igd@0];
        [itd@0];

! Anxiety state factors
        anx1 by as11
                as21 (8)
                ag11 (9)
                ag21 (10)
                at11 (11)
                at21 (12);

        anx2 by as12
                as22 (8)
                ag12 (9)
                ag22 (10)
                at12 (11)
                at22 (12);

        anx3 by as13
                as23 (8)
                ag13 (9)
                ag23 (10)
                at13 (11)
                at23 (12);

! Method factors guardian rating
        mga1 by ag11
                ag21 (13);

        mga2 by ag12
                ag22 (13);

        mga3 by ag13
                ag23 (13);

! Method factors teacher rating
        mta1 by at11
                at21 (14);
```

```
        mta2 by at12
                at22 (14);

        mta3 by at13
                at23 (14);

! Indicator-specific factors
        isa by as21 as22@1 as23@1;
        iga by ag21 ag22@1 ag23@1;
        ita by at21 at22@1;

! Non-admissible latent correlations constrained to zero
        mga1 with anx1@0 anx2@0 anx3@0;
        mga2 with anx1@0 anx2@0 anx3@0;
        mga3 with anx1@0 anx2@0 anx3@0;
        mta1 with anx1@0 anx2@0 anx3@0;
        mta2 with anx1@0 anx2@0 anx3@0;
        mta3 with anx1@0 anx2@0 anx3@0;
        isa with anx1@0 anx2@0 anx3@0;
        iga with anx1@0 anx2@0 anx3@0;
        iga with mga1@0 mga2@0 mga3@0;
        ita with anx1@0 anx2@0;
        ita with mta1@0 mta2@0;

! Intercepts and latent means
        [as11@0];
        [as12@0];
        [as13@0];
        [anx1];
        [anx2];
        [anx3];
        [mga1@0];
        [mga2@0];
        [mga3@0];
        [mta1@0];
        [mta2@0];
        [mta3@0];
        [isa@0];
        [iga@0];
        [ita@0];

! Competence state factors
        com1 by cs11
                cs21 (15)
                cg11 (16)
                cg21 (17)
                ct11 (18)
                ct21 (19);

        com2 by cs12
                cs22 (15)
                cg12 (16)
                cg22 (17)
                ct12 (18)
                ct22 (19);

        com3 by cs13
                cs23 (15)
                cg13 (16)
                cg23 (17)
                ct13 (18)
                ct23 (19);
```

```
!Method factors guardian rating
        mgc1 by cg11
                cg21 (20);

        mgc2 by cg12
                cg22 (20);

        mgc3 by cg13
                cg23 (20);

!Method factors teacher rating
        mtc1 by ct11
                ct21 (21);

        mtc2 by ct12
                ct22 (21);

        mtc3 by ct13
                ct23 (21);

!Indicator-specific factors
        isc by cs21 cs22@1 cs23@1;
        igc by cg21 cg22@1 cg23@1;
        itc by ct21 ct22@1;

!Non-admissible latent correlations constrained to zero
        mgc1 with com1@0 com2@0 com3@0;
        mgc2 with com1@0 com2@0 com3@0;
        mgc3 with com1@0 com2@0 com3@0;
        mtc1 with com1@0 com2@0 com3@0;
        mtc2 with com1@0 com2@0 com3@0;
        mtc3 with com1@0 com2@0 com3@0;
        isc with com1@0 com2@0 com3@0;
        igc with com1@0 com2@0 com3@0;
        igc with mgc1@0 mgc2@0 mgc3@0;
        itc with com1@0 com2@0;
        itc with mtc1@0 mtc2@0;

!Intercepts and latent means
        [cs11@0];
        [cs12@0];
        [cs13@0];
        [com1];
        [com2];
        [com3];
        [mgc1@0];
        [mgc2@0];
        [mgc3@0];
        [mtc1@0];
        [mtc2@0];
        [mtc3@0];
        [isc@0];
        [igc@0];
        [itc@0];

!Definition of the analysis type
ANALYSIS:
  Type = MEANSTRUCTURE;
  Estimator = ML;

!Model to be fit to each MC sample
```

```
 MODEL:
! Depression state factors
        dep1 by ds11
                ds21 (1)
                dg11 (2)
                dg21 (3)
                dt11 (4)
                dt21 (5);

        dep2 by ds12
                ds22 (1)
                dg12 (2)
                dg22 (3)
                dt12 (4)
                dt22 (5);

        dep3 by ds13
                ds23 (1)
                dg13 (2)
                dg23 (3)
                dt13 (4)
                dt23 (5);

! Method factors parent (guardian) rating
        mgd1 by dg11
                dg21 (6);

        mgd2 by dg12
                dg22 (6);

        mgd3 by dg13
                dg23 (6);

! Method factors teacher rating
        mtd1 by dt11
                dt21 (7);

        mtd2 by dt12
                dt22 (7);

        mtd3 by dt13
                dt23 (7);

! Indicator-specific factors
        isd by ds21 ds22@1 ds23@1;
        igd by dg21 dg22@1 dg23@1;
        itd by dt21 dt22@1;

! Non-admissible latent correlations constrained to zero
        mgd1 with dep1@0 dep2@0 dep3@0;
        mgd2 with dep1@0 dep2@0 dep3@0;
        mgd3 with dep1@0 dep2@0 dep3@0;
        mtd1 with dep1@0 dep2@0 dep3@0;
        mtd2 with dep1@0 dep2@0 dep3@0;
        mtd3 with dep1@0 dep2@0 dep3@0;
        isd with dep1@0 dep2@0 dep3@0;
        igd with dep1@0 dep2@0 dep3@0;
        igd with mgd1@0 mgd2@0 mgd3@0;
        itd with dep1@0 dep2@0;
        itd with mtd1@0 mtd2@0;

! Intercepts and latent means
```

```
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [dep1];
        [dep2];
        [dep3];
        [mgd1@0];
        [mgd2@0];
        [mgd3@0];
        [mtd1@0];
        [mtd2@0];
        [mtd3@0];
        [isd@0];
        [igd@0];
        [itd@0];

! Anxiety state factors
        anx1 by as11
                as21 (8)
                ag11 (9)
                ag21 (10)
                at11 (11)
                at21 (12);

        anx2 by as12
                as22 (8)
                ag12 (9)
                ag22 (10)
                at12 (11)
                at22 (12);

        anx3 by as13
                as23 (8)
                ag13 (9)
                ag23 (10)
                at13 (11)
                at23 (12);

! Method factors guardian rating
        mga1 by ag11
                ag21 (13);

        mga2 by ag12
                ag22 (13);

        mga3 by ag13
                ag23 (13);

! Method factors teacher rating
        mta1 by at11
                at21 (14);

        mta2 by at12
                at22 (14);

        mta3 by at13
                at23 (14);

! Indicator-specific factors
        isa by as21 as22@1 as23@1;
        iga by ag21 ag22@1 ag23@1;
        ita by at21 at22@1;
```

```
! Non-admissible latent correlations constrained to zero
        mga1 with anx1@0 anx2@0 anx3@0;
        mga2 with anx1@0 anx2@0 anx3@0;
        mga3 with anx1@0 anx2@0 anx3@0;
        mta1 with anx1@0 anx2@0 anx3@0;
        mta2 with anx1@0 anx2@0 anx3@0;
        mta3 with anx1@0 anx2@0 anx3@0;
        isa with anx1@0 anx2@0 anx3@0;
        iga with anx1@0 anx2@0 anx3@0;
        iga with mga1@0 mga2@0 mga3@0;
        ita with anx1@0 anx2@0;
        ita with mta1@0 mta2@0;

! Intercepts and latent means
        [as11@0];
        [as12@0];
        [as13@0];
        [anx1];
        [anx2];
        [anx3];
        [mga1@0];
        [mga2@0];
        [mga3@0];
        [mta1@0];
        [mta2@0];
        [mta3@0];
        [isa@0];
        [iga@0];
        [ita@0];

! Competence state factors
        com1 by cs11
                cs21 (15)
                cg11 (16)
                cg21 (17)
                ct11 (18)
                ct21 (19);

        com2 by cs12
                cs22 (15)
                cg12 (16)
                cg22 (17)
                ct12 (18)
                ct22 (19);

        com3 by cs13
                cs23 (15)
                cg13 (16)
                cg23 (17)
                ct13 (18)
                ct23 (19);

! Method factors guardian rating
        mgc1 by cg11
                cg21 (20);

        mgc2 by cg12
                cg22 (20);

        mgc3 by cg13
                cg23 (20);
```

```
! Method factors teacher rating
        mtc1 by ct11
                ct21 (21);

        mtc2 by ct12
                ct22 (21);

        mtc3 by ct13
                ct23 (21);

! Indicator-specific factors
        isc by cs21 cs22@1 cs23@1;
        igc by cg21 cg22@1 cg23@1;
        itc by ct21 ct22@1;

! Non-admissible latent correlations constrained to zero
        mgc1 with com1@0 com2@0 com3@0;
        mgc2 with com1@0 com2@0 com3@0;
        mgc3 with com1@0 com2@0 com3@0;
        mtc1 with com1@0 com2@0 com3@0;
        mtc2 with com1@0 com2@0 com3@0;
        mtc3 with com1@0 com2@0 com3@0;
        isc with com1@0 com2@0 com3@0;
        igc with com1@0 com2@0 com3@0;
        igc with mgc1@0 mgc2@0 mgc3@0;
        itc with com1@0 com2@0;
        itc with mtc1@0 mtc2@0;

! Intercepts and latent means
        [cs11@0];
        [cs12@0];
        [cs13@0];
        [com1];
        [com2];
        [com3];
        [mgc1@0];
        [mgc2@0];
        [mgc3@0];
        [mtc1@0];
        [mtc2@0];
        [mtc3@0];
        [isc@0];
        [igc@0];
        [itc@0];

! TECH9 output provides possible error messages (e.g. for Heywood cases)
! for each replication
OUTPUT:  TECH9;
```

## *13.2.2 CS-C(M–1) Baseline Change Model*

```
TITLE:  Monte Carlo Simulation of the CS-C(M-1) Baseline Change Model
        Simulations reported in Chapter 6
        Model version with general state factors
        fit to the data set of Prof. David Cole
        3 Constructs (Depression, Anxiety, Competence)
        3 Methods (Self, Parent, Teacher)
        3 Measurement Occasions
        2 Indicators per CMOU
        Estimates based on FIML analysis
        (file: baseline_complex_FIML.out)
        Sample size condition N = 125

MONTECARLO:

!Names of the observed variables
!d = depression, a = anxiety, c = competence
!s = self report, g = guardian (parent) report, t = teacher report
!the first number refers to the indicator,
!the second number refers to the measurement occasion
  NAMES = ds11 ds21
          ds12 ds22
          ds13 ds23
          dg11 dg21
          dg12 dg22
          dg13 dg23
          dt11 dt21
          dt12 dt22
          dt13 dt23
          as11 as21
          as12 as22
          as13 as23
          ag11 ag21
          ag12 ag22
          ag13 ag23
          at11 at21
          at12 at22
          at13 at23
          cs11 cs21
          cs12 cs22
          cs13 cs23
          cg11 cg21
          cg12 cg22
          cg13 cg23
          ct11 ct21
          ct12 ct22
          ct13 ct23;

!Sample size for the MC samples (here: N = 125)
NOBSERVATIONS = 125;

!Number of replications (here: 500)
NREPS = 500;

!Seed used for the simulation
SEED = 111111;

!File containing the actual parameter estimates
!obtained for Prof. Cole's data.
!These estimates are used as population values for the simulation,
```

```
! for determining coverage, and as starting values.
! Note: The population values are available from the author
! upon request
Population = baseline_3traits_FIML.dat;
Coverage = baseline_3traits_FIML.dat;
Starting = baseline_3traits_FIML.dat;

! Definition of the population model
MODEL POPULATION:
! Depression state and difference factors
        dep1 by ds11
                ds21 (1)
                dg11 (2)
                dg21 (3)
                dt11 (4)
                dt21 (5)
                ds12@1
                ds22 (1)
                dg12 (2)
                dg22 (3)
                dt12 (4)
                dt22 (5)
                ds13@1
                ds23 (1)
                dg13 (2)
                dg23 (3)
                dt13 (4)
                dt23 (5);

        dep21 by ds12
                 ds22 (1)
                 dg12 (2)
                 dg22 (3)
                 dt12 (4)
                 dt22 (5);

        dep31 by ds13
                 ds23 (1)
                 dg13 (2)
                 dg23 (3)
                 dt13 (4)
                 dt23 (5);

! Method (difference) factors parent (guardian) rating
        mgd1 by dg11
                dg21 (6)
                dg12@1
                dg22 (6)
                dg13@1
                dg23 (6);

        mgd21 by dg12
                 dg22 (6);

        mgd31 by dg13
                 dg23 (6);

! Method (difference) factors teacher rating
        mtd1 by dt11
                dt21 (7)
                dt12@1
                dt22 (7)
```

```
                dt13@1
                dt23 (7);

        mtd21 by dt12
                 dt22 (7);

        mtd31 by dt13
                 dt23 (7);

! Indicator-specific factors
        isd by ds21 ds22@1 ds23@1;
        igd by dg21 dg22@1 dg23@1;
        itd by dt21 dt22@1;

! Non-admissible latent correlations constrained to zero
        mgd1 with dep1@0 dep21@0 dep31@0;
        mgd21 with dep1@0 dep21@0 dep31@0;
        mgd31 with dep1@0 dep21@0 dep31@0;
        mtd1 with dep1@0 dep21@0 dep31@0;
        mtd21 with dep1@0 dep21@0 dep31@0;
        mtd31 with dep1@0 dep21@0 dep31@0;
        isd with dep1@0 dep21@0 dep31@0;
        igd with dep1@0 dep21@0 dep31@0;
        igd with mgd1@0 mgd21@0 mgd31@0;
        itd with dep1@0 dep21@0;
        itd with mtd1@0 mtd21@0;

! Intercepts and latent means
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [dep1];
        [dep21];
        [dep31];
        [mgd1@0];
        [mgd21@0];
        [mgd31@0];
        [mtd1@0];
        [mtd21@0];
        [mtd31@0];
        [isd@0];
        [igd@0];
        [itd@0];

! Anxiety state and difference factors
        anx1 by as11
                as21 (8)
                ag11 (9)
                ag21 (10)
                at11 (11)
                at21 (12)
                as12@1
                as22 (8)
                ag12 (9)
                ag22 (10)
                at12 (11)
                at22 (12)
                as13@1
                as23 (8)
                ag13 (9)
                ag23 (10)
                at13 (11)
```

```
                at23 (12);

        anx21 by as12
                 as22 (8)
                 ag12 (9)
                 ag22 (10)
                 at12 (11)
                 at22 (12);

        anx31 by as13
                 as23 (8)
                 ag13 (9)
                 ag23 (10)
                 at13 (11)
                 at23 (12);

! Method (difference) factors parent (guardian) rating
        mga1 by ag11
                ag21 (13)
                ag12@1
                ag22 (13)
                ag13@1
                ag23 (13);

        mga21 by ag12
                 ag22 (13);

        mga31 by ag13
                 ag23 (13);

! Method (difference) factors teacher rating
        mta1 by at11
                at21 (14)
                at12@1
                at22 (14)
                at13@1
                at23 (14);

        mta21 by at12
                 at22 (14);

        mta31 by at13
                 at23 (14);

! Indicator-specific factors
        isa by as21 as22@1 as23@1;
        iga by ag21 ag22@1 ag23@1;
        ita by at21 at22@1;

! Non-admissible latent correlations constrained to zero
        mga1 with anx1@0 anx21@0 anx31@0;
        mga21 with anx1@0 anx21@0 anx31@0;
        mga31 with anx1@0 anx21@0 anx31@0;
        mta1 with anx1@0 anx21@0 anx31@0;
        mta21 with anx1@0 anx21@0 anx31@0;
        mta31 with anx1@0 anx21@0 anx31@0;
        isa with anx1@0 anx21@0 anx31@0;
        iga with anx1@0 anx21@0 anx31@0;
        iga with mga1@0 mga21@0 mga31@0;
        ita with anx1@0 anx21@0;
        ita with mta1@0 mta21@0;
```

```
! Intercepts and latent means
        [as11@0];
        [as12@0];
        [as13@0];
        [anx1];
        [anx21];
        [anx31];
        [mga1@0];
        [mga21@0];
        [mga31@0];
        [mta1@0];
        [mta21@0];
        [mta31@0];
        [isa@0];
        [iga@0];
        [ita@0];

! Competence state and difference factors
        com1 by cs11
                cs21 (15)
                cg11 (16)
                cg21 (17)
                ct11 (18)
                ct21 (19)
                cs12@1
                cs22 (15)
                cg12 (16)
                cg22 (17)
                ct12 (18)
                ct22 (19)
                cs13@1
                cs23 (15)
                cg13 (16)
                cg23 (17)
                ct13 (18)
                ct23 (19);

        com21 by cs12
                 cs22 (15)
                 cg12 (16)
                 cg22 (17)
                 ct12 (18)
                 ct22 (19);

        com31 by cs13
                 cs23 (15)
                 cg13 (16)
                 cg23 (17)
                 ct13 (18)
                 ct23 (19);

! Method (difference) factors parent (guardian) rating
        mgc1 by cg11
                cg21 (20)
                cg12@1
                cg22 (20)
                cg13@1
                cg23 (20);

        mgc21 by cg12
                 cg22 (20);
```

```
        mgc31 by cg13
                 cg23 (20);

! Method (difference) factors teacher rating
        mtc1 by ct11
                ct21 (21)
                ct12@1
                ct22 (21)
                ct13@1
                ct23 (21);

        mtc21 by ct12
                 ct22 (21);

        mtc31 by ct13
                 ct23 (21);

! Indicator-specific factors
        isc by cs21 cs22@1 cs23@1;
        igc by cg21 cg22@1 cg23@1;
        itc by ct21 ct22@1;

! Non-admissible latent correlations constrained to zero
        mgc1 with com1@0 com21@0 com31@0;
        mgc21 with com1@0 com21@0 com31@0;
        mgc31 with com1@0 com21@0 com31@0;
        mtc1 with com1@0 com21@0 com31@0;
        mtc21 with com1@0 com21@0 com31@0;
        mtc31 with com1@0 com21@0 com31@0;
        isc with com1@0 com21@0 com31@0;
        igc with com1@0 com21@0 com31@0;
        igc with mgc1@0 mgc21@0 mgc31@0;
        itc with com1@0 com21@0;
        itc with mtc1@0 mtc21@0;

! Intercepts and latent means
        [cs11@0];
        [cs12@0];
        [cs13@0];
        [com1];
        [com21];
        [com31];
        [mgc1@0];
        [mgc21@0];
        [mgc31@0];
        [mtc1@0];
        [mtc21@0];
        [mtc31@0];
        [isc@0];
        [igc@0];
        [itc@0];

! Definition of the analysis type
ANALYSIS:
  Type = MEANSTRUCTURE;
  Estimator = ML;

! Model to be fit to each MC sample
 MODEL:
! Depression state and difference factors
        dep1 by ds11
                ds21 (1)
```

```
               dg11 (2)
               dg21 (3)
               dt11 (4)
               dt21 (5)
               ds12@1
               ds22 (1)
               dg12 (2)
               dg22 (3)
               dt12 (4)
               dt22 (5)
               ds13@1
               ds23 (1)
               dg13 (2)
               dg23 (3)
               dt13 (4)
               dt23 (5);

        dep21 by ds12
                ds22 (1)
                dg12 (2)
                dg22 (3)
                dt12 (4)
                dt22 (5);

        dep31 by ds13
                ds23 (1)
                dg13 (2)
                dg23 (3)
                dt13 (4)
                dt23 (5);

! Method (difference) factors parent (guardian) rating
        mgd1 by dg11
               dg21 (6)
               dg12@1
               dg22 (6)
               dg13@1
               dg23 (6);

        mgd21 by dg12
                dg22 (6);

        mgd31 by dg13
                dg23 (6);

! Method (difference) factors teacher rating
        mtd1 by dt11
               dt21 (7)
               dt12@1
               dt22 (7)
               dt13@1
               dt23 (7);

        mtd21 by dt12
                dt22 (7);

        mtd31 by dt13
                dt23 (7);

! Indicator-specific factors
        isd by ds21 ds22@1 ds23@1;
        igd by dg21 dg22@1 dg23@1;
```

```
        itd by dt21 dt22@1;

! Non-admissible latent correlations constrained to zero
        mgd1 with dep1@0 dep21@0 dep31@0;
        mgd21 with dep1@0 dep21@0 dep31@0;
        mgd31 with dep1@0 dep21@0 dep31@0;
        mtd1 with dep1@0 dep21@0 dep31@0;
        mtd21 with dep1@0 dep21@0 dep31@0;
        mtd31 with dep1@0 dep21@0 dep31@0;
        isd with dep1@0 dep21@0 dep31@0;
        igd with dep1@0 dep21@0 dep31@0;
        igd with mgd1@0 mgd21@0 mgd31@0;
        itd with dep1@0 dep21@0;
        itd with mtd1@0 mtd21@0;

! Intercepts and latent means
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [dep1];
        [dep21];
        [dep31];
        [mgd1@0];
        [mgd21@0];
        [mgd31@0];
        [mtd1@0];
        [mtd21@0];
        [mtd31@0];
        [isd@0];
        [igd@0];
        [itd@0];

! Anxiety state and difference factors
        anx1 by as11
                as21 (8)
                ag11 (9)
                ag21 (10)
                at11 (11)
                at21 (12)
                as12@1
                as22 (8)
                ag12 (9)
                ag22 (10)
                at12 (11)
                at22 (12)
                as13@1
                as23 (8)
                ag13 (9)
                ag23 (10)
                at13 (11)
                at23 (12);

        anx21 by as12
                 as22 (8)
                 ag12 (9)
                 ag22 (10)
                 at12 (11)
                 at22 (12);

        anx31 by as13
                 as23 (8)
                 ag13 (9)
```

```
                 ag23 (10)
                 at13 (11)
                 at23 (12);

! Method (difference) factors parent (guardian) rating
        mga1 by ag11
                ag21 (13)
                ag12@1
                ag22 (13)
                ag13@1
                ag23 (13);

        mga21 by ag12
                 ag22 (13);

        mga31 by ag13
                 ag23 (13);

! Method (difference) factors teacher rating
        mta1 by at11
                at21 (14)
                at12@1
                at22 (14)
                at13@1
                at23 (14);

        mta21 by at12
                 at22 (14);

        mta31 by at13
                 at23 (14);

! Indicator-specific factors
        isa by as21 as22@1 as23@1;
        iga by ag21 ag22@1 ag23@1;
        ita by at21 at22@1;

! Non-admissible latent correlations constrained to zero
        mga1 with anx1@0 anx21@0 anx31@0;
        mga21 with anx1@0 anx21@0 anx31@0;
        mga31 with anx1@0 anx21@0 anx31@0;
        mta1 with anx1@0 anx21@0 anx31@0;
        mta21 with anx1@0 anx21@0 anx31@0;
        mta31 with anx1@0 anx21@0 anx31@0;
        isa with anx1@0 anx21@0 anx31@0;
        iga with anx1@0 anx21@0 anx31@0;
        iga with mga1@0 mga21@0 mga31@0;
        ita with anx1@0 anx21@0;
        ita with mta1@0 mta21@0;

! Intercepts and latent means
        [as11@0];
        [as12@0];
        [as13@0];
        [anx1];
        [anx21];
        [anx31];
        [mga1@0];
        [mga21@0];
        [mga31@0];
        [mta1@0];
        [mta21@0];
```

```
        [mta31@0];
        [isa@0];
        [iga@0];
        [ita@0];

! Competence state and difference factors
        com1 by cs11
                cs21 (15)
                cg11 (16)
                cg21 (17)
                ct11 (18)
                ct21 (19)
                cs12@1
                cs22 (15)
                cg12 (16)
                cg22 (17)
                ct12 (18)
                ct22 (19)
                cs13@1
                cs23 (15)
                cg13 (16)
                cg23 (17)
                ct13 (18)
                ct23 (19);

        com21 by cs12
                 cs22 (15)
                 cg12 (16)
                 cg22 (17)
                 ct12 (18)
                 ct22 (19);

        com31 by cs13
                 cs23 (15)
                 cg13 (16)
                 cg23 (17)
                 ct13 (18)
                 ct23 (19);

! Method (difference) factors parent (guardian) rating
        mgc1 by cg11
                cg21 (20)
                cg12@1
                cg22 (20)
                cg13@1
                cg23 (20);

        mgc21 by cg12
                 cg22 (20);

        mgc31 by cg13
                 cg23 (20);

! Method (difference) factors teacher rating
        mtc1 by ct11
                ct21 (21)
                ct12@1
                ct22 (21)
                ct13@1
                ct23 (21);

        mtc21 by ct12
```

```
                  ct22 (21);

        mtc31 by ct13
                  ct23 (21);

! Indicator-specific factors
        isc by cs21 cs22@1 cs23@1;
        igc by cg21 cg22@1 cg23@1;
        itc by ct21 ct22@1;

! Non-admissible latent correlations constrained to zero
        mgc1 with com1@0 com21@0 com31@0;
        mgc21 with com1@0 com21@0 com31@0;
        mgc31 with com1@0 com21@0 com31@0;
        mtc1 with com1@0 com21@0 com31@0;
        mtc21 with com1@0 com21@0 com31@0;
        mtc31 with com1@0 com21@0 com31@0;
        isc with com1@0 com21@0 com31@0;
        igc with com1@0 com21@0 com31@0;
        igc with mgc1@0 mgc21@0 mgc31@0;
        itc with com1@0 com21@0;
        itc with mtc1@0 mtc21@0;

! Intercepts and latent means
        [cs11@0];
        [cs12@0];
        [cs13@0];
        [com1];
        [com21];
        [com31];
        [mgc1@0];
        [mgc21@0];
        [mgc31@0];
        [mtc1@0];
        [mtc21@0];
        [mtc31@0];
        [isc@0];
        [igc@0];
        [itc@0];

! TECH9 output provides possible error messages (e.g. for Heywood cases)
! for each replication
OUTPUT:  TECH9;
```

### *13.2.3 CS-C(M–1) Neighbor Change Model*

```
TITLE:  Monte Carlo Simulation of the CS-C(M-1) Neighbor Change Model
        Simulations reported in Chapter 6
        Model version with general state factors
        fit to the data set of Prof. David Cole
        3 Constructs (Depression, Anxiety, Competence)
        3 Methods (Self, Parent, Teacher)
        3 Measurement Occasions
        2 Indicators per CMOU
        Estimates based on FIML analysis
        (file: neighbor_complex_FIML.out)
        Sample size condition N = 125

MONTECARLO:

!Names of the observed variables
!d = depression, a = anxiety, c = competence
!s = self report, g = guardian (parent) report, t = teacher report
!the first number refers to the indicator,
!the second number refers to the measurement occasion
  NAMES = ds11 ds21
          ds12 ds22
          ds13 ds23
          dg11 dg21
          dg12 dg22
          dg13 dg23
          dt11 dt21
          dt12 dt22
          dt13 dt23
          as11 as21
          as12 as22
          as13 as23
          ag11 ag21
          ag12 ag22
          ag13 ag23
          at11 at21
          at12 at22
          at13 at23
          cs11 cs21
          cs12 cs22
          cs13 cs23
          cg11 cg21
          cg12 cg22
          cg13 cg23
          ct11 ct21
          ct12 ct22
          ct13 ct23;

!Sample size for the MC samples (here: N = 125)
NOBSERVATIONS = 125;

!Number of replications (here: 500)
NREPS = 500;

!Seed used for the simulation
SEED = 111111;

!File containing the actual parameter estimates
!obtained for Prof. Cole's data.
!These estimates are used as population values for the simulation,
```

```
! for determining coverage, and as starting values.
! Note: The population values are available from the author
! upon request
Population = neighbor_3traits_FIML.dat;
Coverage = neighbor_3traits_FIML.dat;
Starting = neighbor_3traits_FIML.dat;

! Definition of the population model
MODEL POPULATION:
! Depression state and difference factors
        dep1 by ds11
                ds21 (1)
                dg11 (2)
                dg21 (3)
                dt11 (4)
                dt21 (5)
                ds12@1
                ds22 (1)
                dg12 (2)
                dg22 (3)
                dt12 (4)
                dt22 (5)
                ds13@1
                ds23 (1)
                dg13 (2)
                dg23 (3)
                dt13 (4)
                dt23 (5);

        dep21 by ds12
                 ds22 (1)
                 dg12 (2)
                 dg22 (3)
                 dt12 (4)
                 dt22 (5)
                 ds13@1
                 ds23 (1)
                 dg13 (2)
                 dg23 (3)
                 dt13 (4)
                 dt23 (5);

        dep32 by ds13
                 ds23 (1)
                 dg13 (2)
                 dg23 (3)
                 dt13 (4)
                 dt23 (5);

! Method (difference) factors parent (guardian) rating
        mgd1 by dg11
                dg21 (6)
                dg12@1
                dg22 (6)
                dg13@1
                dg23 (6);

        mgd21 by dg12
                 dg22 (6)
                 dg13@1
                 dg23 (6);
```

```
        mgd32 by dg13
                 dg23 (6);

! Method (difference) factors teacher rating
        mtd1 by dt11
                dt21 (7)
                dt12@1
                dt22 (7)
                dt13@1
                dt23 (7);

        mtd21 by dt12
                 dt22 (7)
                 dt13@1
                 dt23 (7);

        mtd32 by dt13
                 dt23 (7);

! Indicator-specific factors
        isd by ds21 ds22@1 ds23@1;
        igd by dg21 dg22@1 dg23@1;
        itd by dt21 dt22@1;

! Non-admissible latent correlations constrained to zero
        mgd1 with dep1@0 dep21@0 dep32@0;
        mgd21 with dep1@0 dep21@0 dep32@0;
        mgd32 with dep1@0 dep21@0 dep32@0;
        mtd1 with dep1@0 dep21@0 dep32@0;
        mtd21 with dep1@0 dep21@0 dep32@0;
        mtd32 with dep1@0 dep21@0 dep32@0;
        isd with dep1@0 dep21@0 dep32@0;
        igd with dep1@0 dep21@0 dep32@0;
        igd with mgd1@0 mgd21@0 mgd32@0;
        itd with dep1@0 dep21@0;
        itd with mtd1@0 mtd21@0;

! Intercepts and latent means
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [dep1];
        [dep21];
        [dep32];
        [mgd1@0];
        [mgd21@0];
        [mgd32@0];
        [mtd1@0];
        [mtd21@0];
        [mtd32@0];
        [isd@0];
        [igd@0];
        [itd@0];

! Anxiety state and difference factors
        anx1 by as11
                as21 (8)
                ag11 (9)
                ag21 (10)
                at11 (11)
                at21 (12)
                as12@1
```

```
                 as22 (8)
                 ag12 (9)
                 ag22 (10)
                 at12 (11)
                 at22 (12)
                 as13@1
                 as23 (8)
                 ag13 (9)
                 ag23 (10)
                 at13 (11)
                 at23 (12);

        anx21 by as12
                  as22 (8)
                  ag12 (9)
                  ag22 (10)
                  at12 (11)
                  at22 (12)
                  as13@1
                  as23 (8)
                  ag13 (9)
                  ag23 (10)
                  at13 (11)
                  at23 (12);

        anx32 by as13
                  as23 (8)
                  ag13 (9)
                  ag23 (10)
                  at13 (11)
                  at23 (12);

! Method (difference) factors parent (guardian) rating
        mga1 by ag11
                 ag21 (13)
                 ag12@1
                 ag22 (13)
                 ag13@1
                 ag23 (13);

        mga21 by ag12
                  ag22 (13)
                  ag13@1
                  ag23 (13);

        mga32 by ag13
                  ag23 (13);

! Method (difference) factors teacher rating
        mta1 by at11
                 at21 (14)
                 at12@1
                 at22 (14)
                 at13@1
                 at23 (14);

        mta21 by at12
                  at22 (14)
                  at13@1
                  at23 (14);

        mta32 by at13
```

```
                  at23 (14);

! Indicator-specific factors
        isa by as21 as22@1 as23@1;
        iga by ag21 ag22@1 ag23@1;
        ita by at21 at22@1;

! Non-admissible latent correlations constrained to zero
        mga1 with anx1@0 anx21@0 anx32@0;
        mga21 with anx1@0 anx21@0 anx32@0;
        mga32 with anx1@0 anx21@0 anx32@0;
        mta1 with anx1@0 anx21@0 anx32@0;
        mta21 with anx1@0 anx21@0 anx32@0;
        mta32 with anx1@0 anx21@0 anx32@0;
        isa with anx1@0 anx21@0 anx32@0;
        iga with anx1@0 anx21@0 anx32@0;
        iga with mga1@0 mga21@0 mga32@0;
        ita with anx1@0 anx21@0;
        ita with mta1@0 mta21@0;

! Intercepts and latent means
        [as11@0];
        [as12@0];
        [as13@0];
        [anx1];
        [anx21];
        [anx32];
        [mga1@0];
        [mga21@0];
        [mga32@0];
        [mta1@0];
        [mta21@0];
        [mta32@0];
        [isa@0];
        [iga@0];
        [ita@0];

! Competence state and difference factors
        com1 by cs11
                cs21 (15)
                cg11 (16)
                cg21 (17)
                ct11 (18)
                ct21 (19)
                cs12@1
                cs22 (15)
                cg12 (16)
                cg22 (17)
                ct12 (18)
                ct22 (19)
                cs13@1
                cs23 (15)
                cg13 (16)
                cg23 (17)
                ct13 (18)
                ct23 (19);

        com21 by cs12
                 cs22 (15)
                 cg12 (16)
                 cg22 (17)
                 ct12 (18)
```

```
                  ct22 (19)
                  cs13@1
                  cs23 (15)
                  cg13 (16)
                  cg23 (17)
                  ct13 (18)
                  ct23 (19);

         com32 by cs13
                  cs23 (15)
                  cg13 (16)
                  cg23 (17)
                  ct13 (18)
                  ct23 (19);

! Method (difference) factors parent (guardian) rating
         mgc1 by cg11
                 cg21 (20)
                 cg12@1
                 cg22 (20)
                 cg13@1
                 cg23 (20);

         mgc21 by cg12
                  cg22 (20)
                  cg13@1
                  cg23 (20);

         mgc32 by cg13
                  cg23 (20);

! Method (difference) factors teacher rating
         mtc1 by ct11
                 ct21 (21)
                 ct12@1
                 ct22 (21)
                 ct13@1
                 ct23 (21);

         mtc21 by ct12
                  ct22 (21)
                  ct13@1
                  ct23 (21);

         mtc32 by ct13
                  ct23 (21);

! Indicator-specific factors
         isc by cs21 cs22@1 cs23@1;
         igc by cg21 cg22@1 cg23@1;
         itc by ct21 ct22@1;

! Non-admissible latent correlations constrained to zero
         mgc1 with com1@0 com21@0 com32@0;
         mgc21 with com1@0 com21@0 com32@0;
         mgc32 with com1@0 com21@0 com32@0;
         mtc1 with com1@0 com21@0 com32@0;
         mtc21 with com1@0 com21@0 com32@0;
         mtc32 with com1@0 com21@0 com32@0;
         isc with com1@0 com21@0 com32@0;
         igc with com1@0 com21@0 com32@0;
         igc with mgc1@0 mgc21@0 mgc32@0;
```

```
        itc with com1@0 com21@0;
        itc with mtc1@0 mtc21@0;

! Intercepts and latent means
        [cs11@0];
        [cs12@0];
        [cs13@0];
        [com1];
        [com21];
        [com32];
        [mgc1@0];
        [mgc21@0];
        [mgc32@0];
        [mtc1@0];
        [mtc21@0];
        [mtc32@0];
        [isc@0];
        [igc@0];
        [itc@0];

! Definition of the analysis type
ANALYSIS:
  Type = MEANSTRUCTURE;
  Estimator = ML;

! Model to be fit to each MC sample
MODEL:
! Depression state and difference factors
        dep1 by ds11
                ds21 (1)
                dg11 (2)
                dg21 (3)
                dt11 (4)
                dt21 (5)
                ds12@1
                ds22 (1)
                dg12 (2)
                dg22 (3)
                dt12 (4)
                dt22 (5)
                ds13@1
                ds23 (1)
                dg13 (2)
                dg23 (3)
                dt13 (4)
                dt23 (5);

        dep21 by ds12
                 ds22 (1)
                 dg12 (2)
                 dg22 (3)
                 dt12 (4)
                 dt22 (5)
                 ds13@1
                 ds23 (1)
                 dg13 (2)
                 dg23 (3)
                 dt13 (4)
                 dt23 (5);

        dep32 by ds13
                 ds23 (1)
```

```
                  dg13 (2)
                  dg23 (3)
                  dt13 (4)
                  dt23 (5);

! Method (difference) factors parent (guardian) rating
        mgd1 by dg11
                dg21 (6)
                dg12@1
                dg22 (6)
                dg13@1
                dg23 (6);

        mgd21 by dg12
                 dg22 (6)
                 dg13@1
                 dg23 (6);

        mgd32 by dg13
                 dg23 (6);

! Method factors teacher rating
        mtd1 by dt11
                dt21 (7)
                dt12@1
                dt22 (7)
                dt13@1
                dt23 (7);

        mtd21 by dt12
                 dt22 (7)
                 dt13@1
                 dt23 (7);

        mtd32 by dt13
                 dt23 (7);

! Indicator-specific factors
        isd by ds21 ds22@1 ds23@1;
        igd by dg21 dg22@1 dg23@1;
        itd by dt21 dt22@1;

! Non-admissible latent correlations constrained to zero
        mgd1 with dep1@0 dep21@0 dep32@0;
        mgd21 with dep1@0 dep21@0 dep32@0;
        mgd32 with dep1@0 dep21@0 dep32@0;
        mtd1 with dep1@0 dep21@0 dep32@0;
        mtd21 with dep1@0 dep21@0 dep32@0;
        mtd32 with dep1@0 dep21@0 dep32@0;
        isd with dep1@0 dep21@0 dep32@0;
        igd with dep1@0 dep21@0 dep32@0;
        igd with mgd1@0 mgd21@0 mgd32@0;
        itd with dep1@0 dep21@0;
        itd with mtd1@0 mtd21@0;

! Intercepts and latent means
        [ds11@0];
        [ds12@0];
        [ds13@0];
        [dep1];
        [dep21];
        [dep32];
```

```
        [mgd1@0];
        [mgd21@0];
        [mgd32@0];
        [mtd1@0];
        [mtd21@0];
        [mtd32@0];
        [isd@0];
        [igd@0];
        [itd@0];

! Anxiety state and difference factors
        anx1 by as11
                as21 (8)
                ag11 (9)
                ag21 (10)
                at11 (11)
                at21 (12)
                as12@1
                as22 (8)
                ag12 (9)
                ag22 (10)
                at12 (11)
                at22 (12)
                as13@1
                as23 (8)
                ag13 (9)
                ag23 (10)
                at13 (11)
                at23 (12);

        anx21 by as12
                 as22 (8)
                 ag12 (9)
                 ag22 (10)
                 at12 (11)
                 at22 (12)
                 as13@1
                 as23 (8)
                 ag13 (9)
                 ag23 (10)
                 at13 (11)
                 at23 (12);

        anx32 by as13
                 as23 (8)
                 ag13 (9)
                 ag23 (10)
                 at13 (11)
                 at23 (12);

! Method (difference) factors parent (guardian) rating
        mga1 by ag11
                ag21 (13)
                ag12@1
                ag22 (13)
                ag13@1
                ag23 (13);

        mga21 by ag12
                 ag22 (13)
                 ag13@1
                 ag23 (13);
```

```
        mga32 by ag13
                 ag23 (13);

! Method (difference) factors teacher rating
        mta1 by at11
                at21 (14)
                at12@1
                at22 (14)
                at13@1
                at23 (14);

        mta21 by at12
                 at22 (14)
                 at13@1
                 at23 (14);

        mta32 by at13
                 at23 (14);

! Indicator-specific factors
        isa by as21 as22@1 as23@1;
        iga by ag21 ag22@1 ag23@1;
        ita by at21 at22@1;

! Non-admissible latent correlations constrained to zero
        mga1 with anx1@0 anx21@0 anx32@0;
        mga21 with anx1@0 anx21@0 anx32@0;
        mga32 with anx1@0 anx21@0 anx32@0;
        mta1 with anx1@0 anx21@0 anx32@0;
        mta21 with anx1@0 anx21@0 anx32@0;
        mta32 with anx1@0 anx21@0 anx32@0;
        isa with anx1@0 anx21@0 anx32@0;
        iga with anx1@0 anx21@0 anx32@0;
        iga with mga1@0 mga21@0 mga32@0;
        ita with anx1@0 anx21@0;
        ita with mta1@0 mta21@0;

! Intercepts and latent means
        [as11@0];
        [as12@0];
        [as13@0];
        [anx1];
        [anx21];
        [anx32];
        [mga1@0];
        [mga21@0];
        [mga32@0];
        [mta1@0];
        [mta21@0];
        [mta32@0];
        [isa@0];
        [iga@0];
        [ita@0];

! Competence state and difference factors
        com1 by cs11
                cs21 (15)
                cg11 (16)
                cg21 (17)
                ct11 (18)
                ct21 (19)
```

```
                cs12@1
                cs22 (15)
                cg12 (16)
                cg22 (17)
                ct12 (18)
                ct22 (19)
                cs13@1
                cs23 (15)
                cg13 (16)
                cg23 (17)
                ct13 (18)
                ct23 (19);

        com21 by cs12
                 cs22 (15)
                 cg12 (16)
                 cg22 (17)
                 ct12 (18)
                 ct22 (19)
                 cs13@1
                 cs23 (15)
                 cg13 (16)
                 cg23 (17)
                 ct13 (18)
                 ct23 (19);

        com32 by cs13
                 cs23 (15)
                 cg13 (16)
                 cg23 (17)
                 ct13 (18)
                 ct23 (19);

! Method (difference) factors parent (guardian) rating
        mgc1 by cg11
                cg21 (20)
                cg12@1
                cg22 (20)
                cg13@1
                cg23 (20);

        mgc21 by cg12
                 cg22 (20)
                 cg13@1
                 cg23 (20);

        mgc32 by cg13
                 cg23 (20);

! Method (difference) factors teacher rating
        mtc1 by ct11
                ct21 (21)
                ct12@1
                ct22 (21)
                ct13@1
                ct23 (21);

        mtc21 by ct12
                 ct22 (21)
                 ct13@1
                 ct23 (21);
```

```
        mtc32 by ct13
                 ct23 (21);

! Indicator-specific factors
        isc by cs21 cs22@1 cs23@1;
        igc by cg21 cg22@1 cg23@1;
        itc by ct21 ct22@1;

! Non-admissible latent correlations constrained to zero
        mgc1 with com1@0 com21@0 com32@0;
        mgc21 with com1@0 com21@0 com32@0;
        mgc32 with com1@0 com21@0 com32@0;
        mtc1 with com1@0 com21@0 com32@0;
        mtc21 with com1@0 com21@0 com32@0;
        mtc32 with com1@0 com21@0 com32@0;
        isc with com1@0 com21@0 com32@0;
        igc with com1@0 com21@0 com32@0;
        igc with mgc1@0 mgc21@0 mgc32@0;
        itc with com1@0 com21@0;
        itc with mtc1@0 mtc21@0;

! Intercepts and latent means
        [cs11@0];
        [cs12@0];
        [cs13@0];
        [com1];
        [com21];
        [com32];
        [mgc1@0];
        [mgc21@0];
        [mgc32@0];
        [mtc1@0];
        [mtc21@0];
        [mtc32@0];
        [isc@0];
        [igc@0];
        [itc@0];

! TECH9 output provides possible error messages (e.g. for Heywood cases)
! for each replication
OUTPUT:  TECH9;
```

# 14 German Appendix (Anhang in deutscher Sprache)

## Zusammenfassung in deutscher Sprache

In der vorliegenden Arbeit werden Strukturgleichungsmodelle zur Analyse von längsschnittlich erhobenen Multitrait-Multimethod-(MTMM) Daten präsentiert, messtheoretisch analysiert und auf ihre praktische Nützlichkeit hin überprüft. Die Definition der Modelle erfolgt auf der Basis der stochastischen Messtheorie (Steyer, 1989; Suppes & Zinnes, 1963). Die Überprüfung der praktischen Anwendbarkeit der Modelle wird anhand einer Reanalyse von empirischen Daten sowie einer Monte-Carlo-Simulationsstudie vorgenommen.

In der Einleitung werden zunächst mit dem *Correlated Trait-Correlated Uniqueness-* (CT-CU; Marsh, 1989), *Correlated Trait-Correlated Method-* (CT-CM; Widaman, 1985), *Correlated Trait-Uncorrelated Method-* (CT-UM) und dem *Correlated Trait-Correlated (Method Minus One)-* [CT-C(*M*–1); Eid, 2000] Modell die bekanntesten Strukturgleichungsmodelle zur Analyse von querschnittlichen MTMM-Daten diskutiert (siehe auch Eid, Lischetzke, & Nussbeck, 2006, Eid, Nussbeck, & Lischetzke, 2006; Geiser, Eid, Nussbeck, & Lischetzke, im Druck). Im Vergleich erweist sich dabei das CT-C(*M*–1)-Modell für multiple Indikatoren (Eid, Lischetzke, Nussbeck, & Trierweiler, 2003) als eines der leistungsfähigsten derzeit verfügbaren MTMM-Modelle. Anschließend werden verschiedene bereits etablierte Ansätze zur Analyse längsschnittlicher MTMM-Daten präsentiert. Dazu zählen das Multi-Occasion-CU-Modell (Cole & Maxwell, 2003), das Multi-Occasion-CT-CM-Modell (Burns, Walsh, & Gomez, 2003, Burns & Haynes, 2006) und das Multimethod-Latent-State-Trait-Modell (Courvoisier, 2006; Courvoisier, Nussbeck, Eid, Geiser, & Cole, 2007). Es wird gezeigt, dass ein allgemeines längsschnittliches MTMM-Messmodell für multiple Indikatoren und für die Analyse latenter Veränderung über die Zeit bislang noch fehlt.

In einem weiteren Einleitungskapitel werden die für die Entwicklung der neuen Modelle benötigten messtheoretischen Grundlagen der Klassischen Testtheorie (Steyer, 1989, Steyer & Eid, 2001) und der Latent-State-Theorie (Steyer, 1988; Steyer, Ferring, & Schmitt, 1992) besprochen. Anschließend werden zwei Versionen des *Correlated State-Correlated (Method Minus One)-* [CS-C(*M*–1)] Modells eingeführt, welche Kombinationen aus dem CT-C(*M*–1)-Modell für multiple Indikatoren (Eid et al., 2003) und dem Correlated-State-Modell (Steyer et al., 1992) darstellen. Nach einer messtheoretischen Analyse der CS-C(*M*–1)-Modelle wird die Erweiterung zu einem Modell mit latenten Differenzvariablen zur Untersuchung von interindividuellen Unterschieden in intraindividuellen Veränderungen über die Zeit vorgestellt. Dieses sogenannte CS-C(*M*–1)-Change-Modell stellt eine multimethodale Erweiterung des True-Change-Ansatzes von Steyer, Eid und Schwenkmezger (1997; Steyer, Partchev, & Shanahan, 2000) dar. Mit Hilfe des CS-C(*M*–1)-Change-Modells kann latente Veränderung simultan für mehrere Methoden untersucht werden. Zudem können die konvergente Validität und Methodenspezifität von beobachteten und latenten Differenzenscores bestimmt werden.

Nach der theoretischen Analyse der CS-C(*M*–1)-State- und Change-Modelle wird die Anwendbarkeit der Modelle auf reale Daten anhand einer umfangreichen Reanalyse eines längsschnittlichen MTMM-Datensatzes und einer anwendungsbezogenen Simulationsstudie überprüft. In der Anwendung wird ein 3-stufiger Ansatz zur Analyse, Testung und Selektion von Modellvarianten vorgeschlagen. Die Ergebnisse beider Studien zeigen, dass sich die Modelle gewinnbringend zur Analyse von MTMM-MO-Daten einsetzen lassen. Im letzten Teil der Arbeit werden Vorteile und Einschränkungen der Modelle diskutiert, detaillierte Hinweise und Tipps für potentielle Anwender gegeben, Vergleiche zu anderen Ansätzen zur Analyse von längsschnittlichen MTMM-Daten gezogen sowie Aufgaben und Ziele für die zukünftige Forschung aufgezeigt.

## Erklärung

Hiermit versichere ich, dass ich die vorgelegte Arbeit selbständig verfasst habe. Andere als die angegebenen Hilfsmittel habe ich nicht verwendet. Die Arbeit ist in keinem früheren Promotionsverfahren angenommen oder abgelehnt worden.

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