eTopoi Journal for Ancient Studies

Volume 7 (2018), pp. 1–32
DOI: 10.17171/4-7-1

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Communicated by Michael Meyer

Received October 10, 2017
Revised January 22, 2018
Accepted January 26, 2018
Published February 28, 2018

Edited by Gerd Graßhoff and Michael Meyer,
Excellence Cluster Topoi, Berlin

eTopoi ISSN 2192-2608
http://journal.topoi.org

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In this article, we develop a mathematical model for the spreading of the wool-bearing sheep in a population of herders in the Near East and Southeast Europe between 6200 and 4200 BC. Herders are considered as agents moving diffusively in a suitability landscape, such that motion into regions attractive for sheep herding are more probable than to unattractive regions. Simultaneously agents interact socially with another and pass on the innovation with some probability. The parameters of the agent-based model are fitted to available archaeological information. A simulation tool is proposed for computing the evolution of the spreading process in time and space, offering a way to study qualitative effects of different aspects affecting speed and spatial evolution of the spreading process.

Agent-based model; mathematical modeling; simulation; innovation spreading; wool-bearing sheep; human mobility

In diesem Artikel wird ein mathematisches Modell entwickelt für die Ausbreitung des Wollschafs unter Hirten im Nahen Osten und in Südosteuropa zwischen 6200 und 4200 v. Chr. In unserem Modell werden Hirten als Agenten betrachtet, deren Bewegungen durch Zufallsprozesse gesteuert werden, sodass sich die Agenten mit größerer Wahrscheinlichkeit in Regionen aufhalten, die attraktiv für die Schafhaltung sind. Das Modell berücksichtigt außerdem soziale Interaktionen zwischen Agenten und erlaubt die Weitergabe der Innovation zwischen Agenten mit einer bestimmten Wahrscheinlichkeit. Die Parameter des agentenbasierten Modells werden an die verfügbaren archäologischen Daten angepasst. Ein Simulationsverfahren für die räumliche und zeitliche Entwicklung des Ausbreitungsprozesses soll es ermöglichen, qualitative Effekte von verschiedenen Aspekten zu studieren, die den Ausbreitungsprozess beeinflussen.

Agentenbasiertes Modell; Mathematische Modellierung; Simulation; Innovationsausbreitung; Wollschaf; menschliche Mobilität

1 Introduction

Modeling of spreading processes has gained a lot of attention in the last decades, since these processes play a crucial role in understanding a wide range of real-world systems, that span biological, technical, economical and social sciences.[1] Different spreading processes however, can have a very different influence on the observed system. For example when dealing with epidemic processes, which account for the spreading of infectious diseases, one is interested in developing accurate models that would allow finding necessary steps

The authors would like to thank Hans-Christian Küchelmann for providing the bone data. This research has been partially funded by the Excellence Cluster *Topoi – The Formation and Transformation of Space and Knowledge in Ancient Civilizations* and ECMath (Einstein Center for Mathematics Berlin).

1 Weng et al. 2013; Brockmann and Helbing 2013; Stehlé et al. 2011; Moreno, Nekovee, and Pacheco 2004.

for terminating the process.[2] For product placement on the other hand, fast and effective propagation is of crucial importance.[3]

We will study the spreading of innovations in ancient times by integrating physical landscape characteristics as these control the actions of the population. A widely used definition of the innovation process has been established by E. M. Rogers, who defines an innovation as "an idea, practice or object that is perceived as new by an individual or other unit of adoption."[4] The adoption process of such an innovation occurs in five stages: the knowledge stage, the persuasion stage, the decision stage, the implementation stage and the confirmation stage.

Following this paradigm, different approaches have been developed to model innovation spreading.[5] However, most of them focus on innovations appearing in modern times.[6] In comparison, spreading processes in ancient times differ in two important ways. Firstly, we cannot replicate or observe anymore a process that has happened in ancient times. Thus, we have to base our knowledge on available data which is usually limited, since archaeological evidences for prehistoric conditions are difficult to obtain and usually are spatially and temporally incomplete. This means that the data we are dealing with is often very sparse,[7] making the usual statistical approaches difficult to apply. Besides, different models might explain the same data, thus distinguishing between different hypotheses (model selection) is not always feasible. Secondly, social interactions between individuals and communities in ancient times have been influenced by spatial distances and the lack of their technological development. These factors had a great impact on the nature and speed of innovation spreading, which needs to be taken into account. Many existing mathematical models for the spreading of innovations assume perfect social mixing,[8] i.e. that everyone interacts with everyone else. In modern social systems using today's communication technologies, this assumption can be realistic in some cases, but not when modeling innovation diffusion in ancient times.

In this paper we present a novel mathematical approach for modeling the diffusion of the wool-bearing sheep, which took place during Copper and Early Bronze Age and which was the basis for the development of wool processing. Our model is based on using spatially differentiated information on physical landscape characteristics and parametrized by using existing material traces. We applied an agent-based modeling (ABM) approach,[9] a well-known technique often used for studying complex social and economic systems.[10] Main objects in this type of models are agents, representing for example people, social groups or other discrete entities like corporations. Usually, the behavior of each agent is given by a set of rules that can be formalized by mathematical expressions, i.e., equations.[11] ABM contributes to the heterogeneity of the observed system, since agents can behave differently and also change their behavior in the course of a simulation. The interplay of the behavior on the local agent scale where the agents act produces patterns on the global scale. Here, one of our main questions is how to model the behavior of the agents, given incomplete and limited data.

We are dealing with agents that are moving diffusively in the region, while simultaneously interacting socially with other agents that are nearby. The diffusive motion is biased

2 Hufnagel, Brockmann, and Geisel 2004.
3 Iyengar, Van den Bulte, and Valente 2011, 4–9.
4 Rogers 1995, 132.
5 Valente 1995; Lane et al. 2004.
6 Peres, Muller, and Mahajan 2010, 1–3; Kiesling et al. 2012, 1–3.
7 Burg, Peeters, and Lovis 2016.
8 Bass 1969.
9 Macy and Willer 2002.
10 Helbing 2012.
11 Macy and Willer 2002, 145.

towards parts of the overall region being attractive for herding and, thus, for the agents and for employing the innovation. Thus the agents move in a suitability landscape[12] where motion into suitable parts of the region occurs with higher probability than motion into less attractive parts. If agents are close to each other in space, they are able to communicate and they can pass on the innovation with a certain rate. Those connections between agents form a network that is changing in time as the connections between the agents are changing due to their movements. The diffusion of the innovation is thus happening by passing on the innovation among agents and by the migration of agents. We will show how one can model the spreading of innovations in these large, time-evolving networks.[13]

In particular, we will demonstrate the applicability of our approach to study the emergence and spreading of the wool-bearing sheep from South-West Asia towards Central Europe. Prior to the appearance of the wool-bearing sheep, herders in Central Europe and the Near East were already herding hairy sheep amongst other domestic animals.[14]

Direct evidence for prehistoric wool processing is scarce, since wool decays quickly under common environmental conditions. The oldest woolen textiles from the study area were found in Novorossiya and date back to around 3700–3300 BC[15] and it is assumed that the wool-bearing sheep and woolen textiles were well established in most of the study area by then.[16] The introduction of wool had an important influence on the growth of the textile production and had strongly affected the socioeconomic development of past societies.[17] Cuneiform texts from Uruk in Ancient Mesopotamia give detailed information on livestock farming and prove organized wool production around 3100 BC.[18] First evidences for the use of sheep wool come from Tell Sabi Abyad, located in modern day northern Syria (Fig. 1) and dating back to ca. 6200 BC.[19] This early indication of wool use is based on indirect evidences. Excavated animal bones give information on the exploited animal species and textile tools such as spindle whorls may indicate the processing of wool fibres.[20]

However, little is known about the spatio-temporal propagation of the wool-bearing sheep. We will apply our model to this problem and we will study and determine most probable paths of the spreading of the wool-bearing sheep.

The outline of the article is as follows, in Section 2 we will introduce our general model for the spreading of an innovation in the ancient world and the concepts behind it. In Section 3 we will demonstrate the method on a real world example and explain simulation details and discuss the results. Finally, in Section 4 we will conclude with discussing the plausibility of our model and possible extensions.

12 The term “suitability landscape” requires some explanation because it highlights interesting differences regarding scientific jargon: In the field of mathematical modeling and particle dynamics, the terms “energy landscape” or “property landscape” are used for the field in which the motion of the particles is happening and which determines the forces acting on the particles. Therefore, coining the term “suitability landscape” makes sense since the so-named object governs the motion of our agents and determines the forces acting on them. However, the term “landscape” as well stands for the landforms of a region and “landscape suitability” should be used in order to describe the suitability / attractivity of a region for agents (herders) in our case. The authors, torn between “suitability landscape” and “landscape suitability”, decide for the first option simply because subsequently the term is used more frequently in this meaning.

13 Holme and Saramäki 2012.

14 Arbuckle et al. 2014, 3.

15 Shishlina, Orfinskaya, and Golikov 2003, 331.

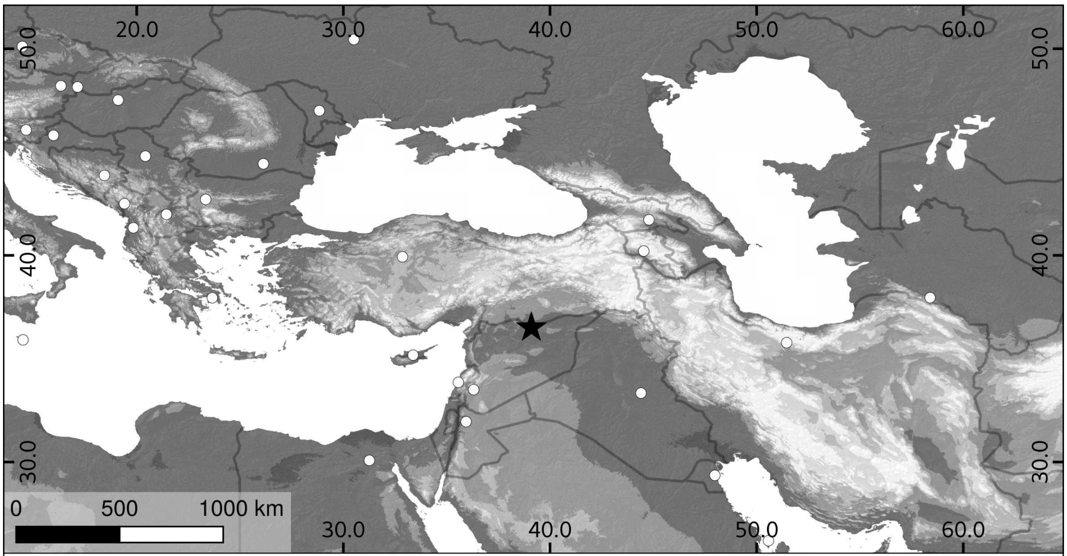
16 Becker et al. 2016, 112–117.

17 Becker et al. 2016, 117–122; McCorriston 1997.

18 Waetzoldt 1972, 4; McCorriston 1997, 527–528, 532, 534.

19 Rooijakkers 2012, 95.

20 Rooijakkers 2012, 103; Grabundžija and Russo 2016.



Reference system: WGS84; data: SRTM 500 (Jarvis et al. 2008).
★ Tell Sabi Abyad: wool is assumed to have been produced here for the first time, around 6200 BC (Rooijakkers 2012).
○ Modern day capital cities.

Elevation (m a.s.l.)
0, 500, 1000, 1500, 3000, 4000

Fig. 1 | A map of our study area, present-day borders and capitals are indicated.

2 Model description

2.1 Agent-based modeling for the spreading of innovations

Agent-based modeling is a modeling approach that captures the global behavior of a large number of interacting agents. It is a technique that is often used in social sciences and economics to model for example trading, opinion formation, traffic dynamics and migration.[21] ABM is a tool that can be used for understanding, prediction, formalization and discovery of complex phenomena.[22] It is a good choice for modeling the process of innovation spreading as it is a technique capable of incorporating data and modeling complex patterns that emerge from described interactions among individuals. Thereupon the model outcome can be compared to available data in order to verify the model assumptions.

Agents are discrete units that behave according to given rules and can learn and adapt their behavior in response to other agents and changes in their environment. The outcome of the agent-based simulation depends on the rules one specifies for the agents. One challenge is therefore to find a set of reasonable assumptions. More precisely, one has to find a balance between making too simplistic assumptions, which would lead to a model that is not plausible, and an unnecessarily complicated model, which would be hard to analyze,[23] and go against the principle of Occam's Razor. This can be achieved by aiming at a model that can be put into a mathematical formulation and thus laying the foundation for a thorough mathematical analysis.

21 Helbing 2012.
22 Gilbert and Troitzsch 2005, 172–198.
23 Conte and Paolucci 2014, 1–6.

2.2 Assumptions and rules for the agents

In order to set up an ABM for the diffusion of an innovation in prehistoric times we have to make some simplified assumptions and set rules for the agents.

- We will call the area of interest in which the agents live the domain D. We are modeling a closed system, so our agents will not be able to leave the domain. This assumption is not realistic for all historical examples, but it is in the case when the domain boundaries agree well with cultural or landscape borders and where the innovation was almost surely introduced inside the domain boundaries.
- An agent represents a small tribe or group of people that are able to adopt the innovation. We assume that the number of agents n is constant, i.e. we are not considering population growth and decline. The main reason for this assumption is due to the uncertainty and incompleteness of the available demographic data in ancient times. Further details on this type of data would offer a different setting.
- Each agent $i,\ i = 1 \dots n$, has a position in the domain and an innovation state that indicates whether the innovation is known and has been adopted by the agent.
- Each agent has limited knowledge about the surrounding area and other agents in the neighborhood. This is because in the ancient world, communication between people was restricted by the distances that could be traversed.
- Agents are able to move and change their position in the domain of our system and are attracted to regions that are suitable for them. This rule follows the historical knowledge, that human migration was often motivated by the search for a suitable environment and to acquire resources. It has also been shown that mobility in the ancient world is strongly related to technological change,[24] i.e. that in times of increased human mobility, there was an advance in technological developments.
- Agents like to group together in space and form communities but also keep some positional distance between each other. This type of agents' behavior reflects social behavior of humans in ancient times, i.e. that people were often grouped together in tribes.
- Agents can pass on the innovation if they are close to another in spatial distance. The spreading process is simplified, there is a fixed spreading rate at which agents pass on the innovation to another agent that is close.

2.3 Modeling the migration of agents

In our model, all n agents are following the same set of rules regarding spatial migration in the region of interest. We assume that the position changes of agents depend on the suitability of their physical environment and in reaction to the movements of the other agents. More formally, we define the suitability landscape that governs the migration patterns of our agents, see 2.3.1, and include pairwise attraction and repulsion forces, that account for social attractions between agents on the one hand and that avoid unrealistic crowding on the other hand, see 2.3.2. Finally, we introduce some randomness in the motion of the agents in order to account for influences that are not taken into account explicitly, see 2.3.3.

24 Loog et al. 2017.

2.3.1 Suitability landscape

The suitability landscape accounts for environmental factors (e.g. the climate) and the attractivity of parts of the study area for an agent and for employing the innovation (e.g. availability of resources such as water and pasture). Valleys in the suitability landscape correspond to attractive areas for agents, whereas peaks and divides correspond to regions of minor suitability. The choice of the landscape is problem-dependent and we will explain its construction for the wool-bearing sheep spreading process in Section 3. More general mathematical details on this construction can be found in the supplementary material.

2.3.2 Social interaction

Social interaction and proximity is another reason for agents to move. Our understanding of a social interaction in this context is the tendency for agents to find a balance between forming clusters of agents and between distributing in space such that communication and exchange with other agents is possible but conflict over resources is avoided. We will call those two tendencies attraction and repulsion. Attraction between agents occurs when agents at long distances are driven towards another, and repulsion appears when agents are forced apart at short ranges. We model the pairwise attraction and repulsion between agents using a model similar to the potential energy models used in physics for describing interactions between atoms or molecules.[25] The total attraction-repulsion potential is the sum of the pairwise interactions over all pairs of agents.

2.3.3 Stochastic effects

Not only the need for a suitable region and the rules of social interaction are reasons for agents to change their location. To account for all various individual motivations for an agent to change its position, we are also adding a random force that acts on the agents. As a consequence, this also leads to agents not being stuck once they reach a suitable region. Since regions offer only a finite amount of resources, the random force will give agents an incentive to leave those regions, i.e. it models the seemingly stochastic behavior of agents in their search for alternative resources. In the next Section, we will derive the random force from the Brownian motion.[26]

2.3.4 Resulting migration model

As we discussed above, the main idea of modeling the agents' migration is that the agents are moving diffusively in the suitability landscape while simultaneously interacting with other agents. The overall migration potential, given by the sum of the suitability landscape and all attraction-repulsion potentials, describes how favorable different positions in space are. The lower the value of this migration potential at a point, the more attractive the position is. Both the suitability landscape and our attraction-repulsion potentials are types of potentials from which one can derive a force (the negative gradient of the potential). The agent then moves in the direction of the overall force acting on it with a momentum of motion proportional to its magnitude.

25 Jones 1924; Buckingham 1938.
26 Pavliotis 2016, 55.

The random force acting on each agent is scaled with a pre-defined factor σ such that the forces originating from the overall potential are still the dominating influences in our migration model. The random force models the unknown influences causing migration of agents and is needed to enable rare transitions through less suitable regions.

Given the force derived from the suitability landscape, the pairwise attractive and repulsive forces and given the random force, we can write down the dynamical equation governing the motion of agents as

$$dX_t = -\nabla(U + V)dt + \sigma dB_t, \tag{1.1}$$

where X_t is denoting the (vector of all) positions of our agents over time t, V is the suitability landscape, U denotes the sum of all pairwise attraction-repulsion potentials, ∇ the gradient operator, and B_t the model for the random forces (standard Brownian process). This type of equation is known as a stochastic differential equation (SDE).[27] While equation (1.1) reads quite technical, it results in a simulation strategy that is easier to understand, see equation (1.2) below.

2.4 Modeling and analyzing the spreading of innovations

In the previous section we introduced a migration model that describes position changes of agents. Here, we will show how one can model the innovation spreading process among agents using networks. We will discuss two ways to construct networks from our agent-based model. On the one hand, we build a network on the micro-scale level, where agents correspond to nodes and there is an edge between two nodes if the spatial distance of the respective agents is smaller than some threshold value. On the other hand, in stark contrast, we consider a macro-scale network on a partition of our area of interest into regions. Nodes represent regions and edges exist between regions that share a border. Both types of networks, on the micro- and macro-scale, are used as tools for understanding and analyzing the spreading process. The micro-scale network will be used in the modeling and simulation of the spreading process, while the macro-scale network is a tool for the a-posteriori analysis of a simulation and will be used in Section 3.3. The macro-scale network between neighbouring regions is motivated by the need for a tool to compare outcomes of different simulations.

2.4.1 Micro-scale network

In the micro-scale network the set of nodes is given by the set of agents. An edge exists between two nodes if the distance between corresponding agents in the position space is smaller than a given interaction radius R. Since the positions of the agents are changing in time, this is a time-evolving network.

In our model each agent also has an innovation state. The innovation state space is 0 or 1, which corresponds to an agent not employing the innovation and an agent having adopted an innovation, respectively. Agents can spread the innovation to their neighbors in the network independently according to a fixed spreading rate r, i.e., if two agents are neighbors in the micro-network, one in state 1 having adopted the innovation, the other in state 0, then the second will accept the innovation and go to innovation state 1 with probability $1 - \exp(-r\tau)$ where τ is the timespan of contact. The network can have many disconnected components, but because the edges are changing in the time, the innovation can spread through the network. The spreading process is asymmetric: as soon as an agent adopts the innovation, it employs the innovation for all time.

27 Kloeden and Platen 1992; Pavliotis 2016, 55.

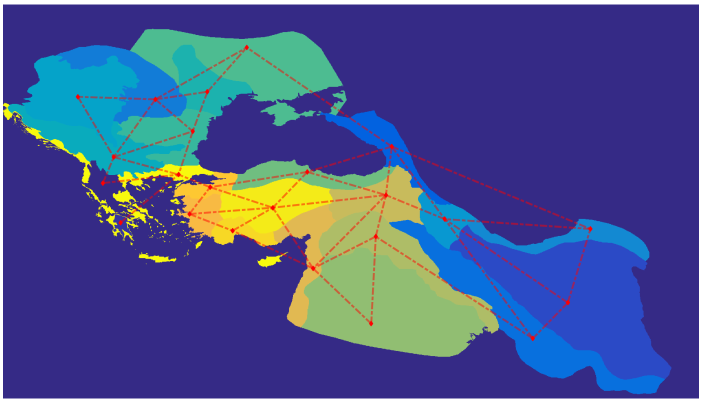


Fig. 2 | This is an example of a macro-scale network between regions. In particular it is the network constructed for our study area which can be divided into 23 regions, compare Fig. 3 for the partitioning of the area. Edges correspond to shared borders between regions.

2.4.2 Macro-scale network

For the a-posteriori analysis of a simulation of the spreading process, we partition the area of interest into a number of regions (see Fig. 3). Connecting adjacent regions by edges will lead to the macro-scale network of neighboring regions, see Fig. 2. This macro-scale network is not varying in time, as long as the borders of our regions are not changing in time. Given a simulation of the spreading it allows to compute quantities like the first arrival time T_k of the innovation in region k (which is the first time an agent having adopted the innovation is crossing the border of region k in the course of the simulation or the first time an agent presently located in region k adopts it). Since each simulation is different, each simulation will in general lead to a different value of T_k and, thus, a couple of simulations will generate a statistical distribution of possible values for T_k. Therefore, when asking a question like "How much time did it take for the innovation to spread to a certain region given it started in a specified starting location?", we will have to generate a couple of independent simulations and then analyze the resulting distribution of the first arrival times. While this is quite straightforward, questions like "How does the spreading path between regions look like?" are more demanding since each simulation may result in a different order of regions between which the innovation is communicated before it first arrives in the region of interest. From information about the statistics generated for such spreading paths, one can also infer the amount of interaction between neighboring regions, i.e. along the edges of the macro-scale network.

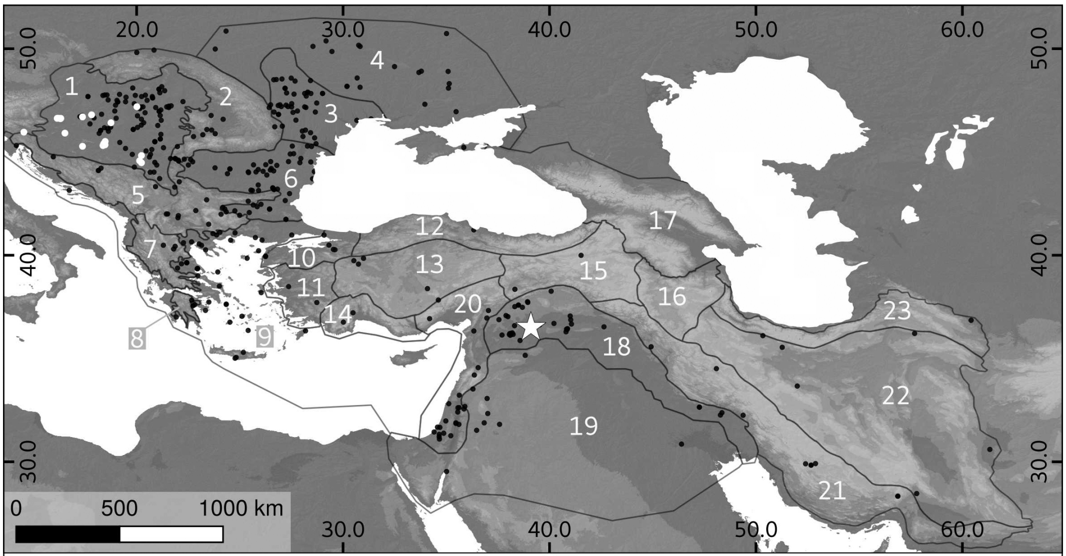
3 The spreading of the wool-bearing sheep

3.1 Setting the scene

In the following, we will use our modeling approach to trace the spreading of the wool-bearing sheep. The innovation will be the herding of wool-bearing sheep and each agent represents a group of people that herd sheep. Attractive areas in our domain correspond to suitable environmental conditions for herding sheep, while less suitable areas will be avoided by the agents.

The area of interest spans from the Zagros Mountains in the south-east to the Carpathian Basin in the north-west (see Fig. 1).

Based on topographical criteria the area can be structured into 23 natural regions (Fig. 3) which we will use to investigate the spreading paths of the innovation. The regions were not determined considering the enhancing or inhibiting effects of environmental features on human interactions, but based on mountainous regions and mountain chains which often form distinct regions next to lowlands only due to their topographical dissim-



Reference system: WGS 84; data: SRTM 500 (Jarvis et al. 2008).

Natural regions according to numbers:
1 Pannonian basin **2** Carpathian mountain range **3** Pontic steppe **4** Continental Pontic steppe **5** Northern Balkan mountain range **6** Lower Danube reach **7** Central Balkan mountain range **8** Peleponnes mountain range **9** Northern mediterranean coast region **10** Maramara region **11** Aegean region **12** Pontus region **13** Central Anatolia **14** Anatolian south coast **15** Eastern Anatolia **16** Azerbaijan **17** Caucasus mountains **18** Mesopotamia **19** Syrian desert **20** Levant **21** Zagros system **22** Central Iranian Highlands **23** Northern Iranian mountains.

☆ Tell Sabi Abyad: wool is assumed to have been produced here for the first time, around 6200 BC (Rooijakkers 2012).

• Archaeological sites with bone finds (n=401) (Küchelmann et al. in preparation).

○ Archaeological sites with finds of spindle whorls (n=23) (Grabundzija and Russo 2016).

Fig. 3 | Map of our study area with excavation sites, borders and a partition into major landscape units.

ilarity. Navigable rivers or existing land routes which may enhance human interactions, were most probably not stable over the study period and it is mostly unknown how such features were actually used in prehistoric times. Therefore, these features, as well as vegetation or regional climates were not considered in structuring the study area or in developing the suitability landscape. The time period covered by the simulation lasts from 6200 BC to 4200 BC, as this period is assumed to be covering the complete spreading in our area of interest.

3.1.1 Available data

One can distinguish between two types of data we collected and worked with: archaeological and environmental data.

Archaeological data indicating the use of wool-bearing sheep were used for validation of the agent-based model. This data comprises finds of animal bones available from 401 excavation sites.

Environmental data is used to model the landscape suitability for herding sheep. The data consists of topographical data and its derivatives slope, terrestrial landforms and topographic compound index (TCI). The TCI represents the tendency of water to accumulate at any point in the catchment and the tendency for gravitational forces to

move that water downslope. Topographical data are based on SRTM 90 data from the Shuttle Radar Topography Mission in 2000[28] with $90\,m{\times}90\,m$ ground resolution and a vertical error $< 16\ m$.[29] We used resampled data with $500\,m{\times}500\,m$ ground resolution (SRTM 500) to develop a digital elevation model (DEM). Based on the DEM the TCI[30] was computed using the GRASS GIS module r.watershed. The slope was computed using the module r.slope.aspect.
Terrestrial landforms were computed on the basis of SRTM 90 data using the r.geomorphon algorithm.[31] For the simulation the landforms data set was resampled to $500\ m{\times}500\ m$ ground resolution using the GRASS GIS module r.resamp.stats with method=mode for assigning the most abundant landform within a $500\ m{\times}500\ m$ grid cell of the input layer to the respective output cell.

3.1.2 Incorporating the data

If one could infer a reference spreading path from the archaeological data, one could use it to estimate the parameters of our model, such that the model output is as close as possible to the prehistorical evidences. However, in our case the archaeological data is very sparse and provides only indirect evidence, so it does not guarantee the existence of woolly sheep at all respective locations and points in time. Thus, we can not infer a reference spreading path directly from the data. Instead of fitting our result to the data, we will fit a quality of our model to the data. More precisely, we will use the available bone data to fit the parameters of the suitability landscape such that the distribution of agents is as close as possible to the distribution of the ovicaprid bone findings. The details of this process are described in Section 3.2.1 and the supplementary part.

3.2 Simulation

3.2.1 Simulating migration

The dynamical equation (1.1) gives rise to a simulation method via temporal discretization by means of the Euler-Maruyama scheme Kloeden and Platen 1992. The change of position $X_t^{(i)}$ of agent i in a small time step Δt is given by

$$X_{t+\Delta t}^{(i)} = X_t^{(i)} - (\nabla U_i(X_t^{(i)}) + \nabla V(X_t^{(i)}))\Delta t + \sigma\sqrt{\Delta t}\, y_i, \qquad (1.2)$$

where U_i denotes the sum of all pairwise attraction-repulsion potentials concerning agent i, V denotes again the suitability landscape and y_i is a random variable drawn from a standard normal distribution.

Each simulation of the migration model will iteratively repeat the calculation (1.2) for each agent i starting with $t = t_0$, the initial time of the simulation period and ending as soon as $t = t_1$, the final time of the period. Because of the random variable y_i, every new simulation of the migration process will be different from the previous ones.

We constructed the suitability landscape V by combining the different environmental influences that determine the suitability of a region for keeping sheep, see Fig. 5 and by taking into account the different types of landforms. The influences we incorporated are the elevation of our domain, the TCI and the slope of the area. As it is not clear how exactly

28 Van Zyl 2001.
29 Jarvis et al. 2008, 1–5.
30 After Quinn et al. 1991, 59–64.
31 Stepinski and Jasiewicz 2011; Jasiewicz and Stepinski 2013.

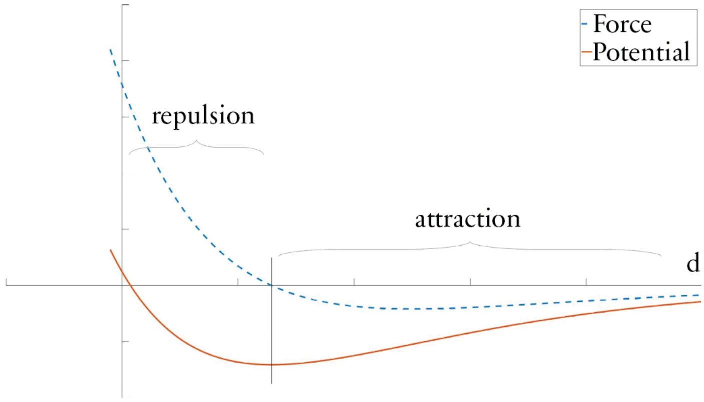


Fig. 4 | Attraction-repulsion potential (solid red line) and force (dashed blue line) plotted against the distance between agents *d*. At short distances the repulsion term dominates, at long distances the attraction term takes over.

these factors influence sheep farming, we assumed that the known ovicaprid bone sites are situated in attractive areas (i.e. valleys) in our suitability landscape. Then we can tune and optimize[32] the parameters of the suitability landscape in a way so that the distribution of ovicaprid bone sites agrees with the agents' position distribution in the area of interest. More details about this procedure can be found in the supplementary material.

The attraction-repulsion potential U we employed is of the form depicted in Fig. 4. It is a combination of a strong repulsion between each pair of agents at short distance and a weak attraction between each pair at long range. The potential and also the force become unimportant when agents are very far from another. There is some equilibrium distance between agents, where the attraction and repulsion between each pair of agents is in balance. Thus, if the distance between two agents is exactly this equilibrium distance, then there is no incentive to deviate from the positioning and the force is zero.

We start our simulation with $n = 4000$ agents that are distributed uniformly all over the domain D shown in Fig. 5. The dynamical equation is scaled, such that the average yearly travel distance is 12 km and the parameters for the attraction-repulsion potential are such that the equilibrium distance between agents is 500 m. The time period we are observing is $[t_0,\ t_1] = [6200\ \text{BC},\ 4200\ \text{BC}]$. We simulate the movement of the agents for some time without innovation spreading until at time t_0 the agents are distributed according to the suitability of the different areas and in accordance with the attraction-repulsion potential. Then, at t_0 we start with the spreading of the innovation see also Section 3.2.2.

Our domain D is bounded. In some parts the boundary of our domain coincides with natural borders like the coast, in other parts the boundary is a cultural border. We have to impose boundary conditions to specify what happens when agents reach this boundary. In our model agents can not cross the boundary, in other words there is no agent flux over the boundary of the domain. It is imporant to remark that for systems with open boundaries, where agents can leave and enter the domain, the resulting patterns could be quite different. In our setting, most of the domain boundaries coincide well with natural and cultural borders, such that we can expect that the agent flux is very small and thus should not change the outcome much. The attraction-repulsion potential term in the dynamical equation (1.1) ensures that agents do not cluster near the boundary of the domain.

32 Gelbart 2015, 6–8.

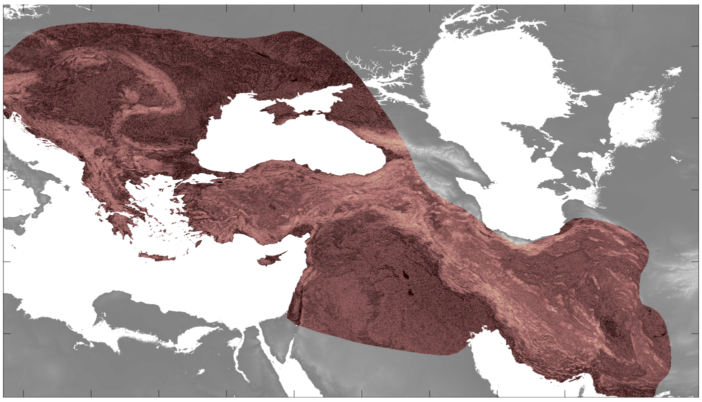


Fig. 5 | The suitability landscape V of our domain D. Areas marked in gray are outside of our domain, darker colors inside our domain correspond to more suitable areas.

3.2.2 Simulating innovation spreading

The innovation of herding woolly sheep is assumed to have started spreading from Tell Sabi Abyad (36.504°N, 39.093°E) at time t_0.[33] In the simulation at time t_0, we set the innovation state of all agents inside some chosen distance of the starting location to state 1, i.e. the usage of the innovation, while all other agents are in state 0.

In parallel to simulating the migration of the agents, we also simulate the innovation spreading process where the spreading probability p depends on the length of the time step Δt and the spreading rate r via the formula

$$p = 1 - \exp(-r\Delta t). \tag{1.3}$$

That is, after each migration step using (1.2) we first update the micro-scale network as described in Section 2.4 with interaction radius $R = 10$km. Then each agent in innovation state 1 can independently spread the innovation to connected agents with innovation state 0 with probability p. In our simulations for the choices of innovation rate r and length of the time step Δt the spreading probability is $p = 0.2$. Agents who adopt the innovation change their innovation state from 0 to 1.

The micro-scale network is changing in time as agents are changing their positions. The network has a very sparse structure, there are many isolated agents and only some small connected components in the network.

3.3 Results

To quantify the spreading paths for our simulations we are measuring the first arrival times of the innovation in specified regions and the proportion of adopters in the population of agents for all time steps. For this, we are using a partition of the domain into 23 regions corresponding to the major landscape units and the macro-scale network (see Fig. 2), constructed as explained in Section 2.4.2. The first arrival times for the regions indicate the spreading path of the innovation and when important first transition events between regions have happened. We define the spreading path as the ordering of the regions by increasing values of the first arrival times. When analyzing the outcome of a simulation, we consider the macro-scale network, giving us all possible paths for the interregional spreading, and the first arrival times, telling us the ordering in which the regions received the innovation. The change in the proportion of adopters also is an indicator for the important transition events, which are happening when an agent who adopted the innovation is transitioning from his current region to a region where

33 Rooijakkers 2012, 105.

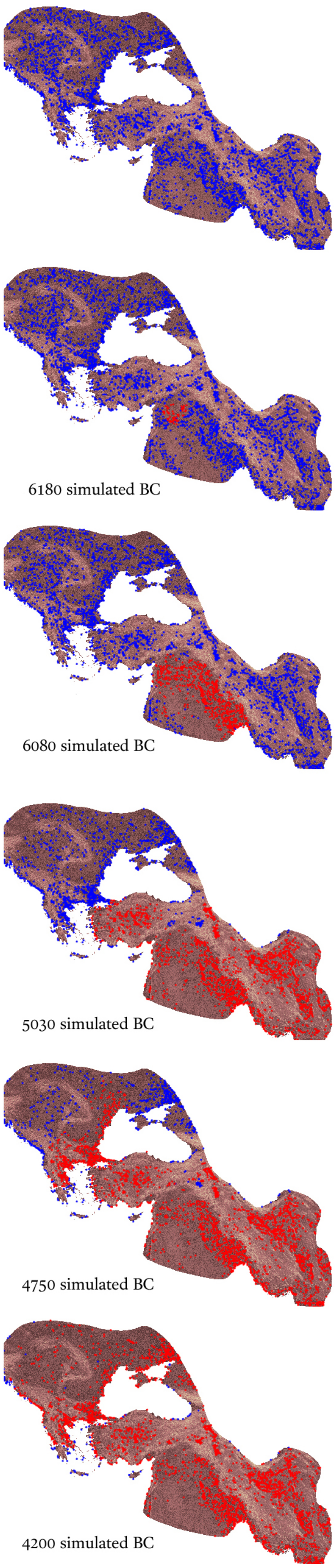


Fig. 6 | A time series from our sample simulation with spreading probability $p = 0.2$. The first subplot shows the stationary distribution of agents in the suitability landscape. Then in the second subplot the innovation is introduced to agents (red dots) near the starting location, all other agents don't know about the innovation yet (blue dots).

the innovation is unknown. When the suitability landscape barrier is high, then those rare transitions are influenced mainly by the random force of the dynamical equation. Thus they are happening at different times in each simulation even with the same set of parameters. So the simulated paths of one parameter set can be called consistent in the sense that the same transition events happen in the same order just at different arrival times and the variance of these arrival times being relatively small.

Region	Mean	SD	Region	Mean	SD
1	5067 BC	243 BC	12	5835 BC	164 BC
2	5111 BC	235 BC	13	5921 BC	138 BC
3	5153 BC	233 BC	14	5763 BC	203 BC
4	5090 BC	249 BC	15	6136 BC	35 BC
5	5227 BC	228 BC	16	5879 BC	120 BC
6	5241 BC	227 BC	17	5671 BC	224 BC
7	5167 BC	231 BC	18/19	6200 BC	0 BC
8	–	–	20	6183 BC	6 BC
9	5257 BC	227 BC	21	6151 BC	10 BC
10	5693 BC	183 BC	22	6026 BC	41 BC
11	5668 BC	182 BC	23	5876 BC	64 BC

Tab. 1 | The mean and standard deviation (SD) of the first arrival times (in simulated BC) calculated from 25 simulations and for each of the 23 regions are listed here. The innovation started spreading from region 18 and 19, in most simulations the innovation never reached region 8 (marked with –). See Fig. 3 for the partition of our area of interest into regions and compare Fig. 7 for a visualization of the mean of the first arrival times in different regions. The full results can be found in the supplementary material.

We are interested in understanding how consistent our simulation results are for the chosen set of parameters and for this we ran 25 simulations. In Section 3.4 we will argue about our particular choice of parameters and their influence.

We will perform the following analysis:

1. compare first arrival times in different regions and the spreading path of the innovation.

2. measure how the number of adopters of the innovation changes with time and whether there are any rare transitions.

3. visualize the relevant quantities for an example simulation.

The first arrival times for the different regions vary for each simulation, see Table 1 for the mean and standard deviation of the first arrival times for our 25 simulations. But the ordering in which regions are reached by the innovation is consistent for all simulations of the same parameter set. See Fig. 7 for a plot of the mean of the first arrival times in different regions.

In our simulations the spreading of the innovation starts in northern Syria, from there spreads to Levant, Albuorz Mountains and Caucasus. Then the innovation reaches southeast Europe via the Bosporus. After traversing the Bosporus the diffusion continued along two major paths: to the south along the coastal plains and lowlands of the Attic Peninsula and, to the west along the coastal plains of the Black sea and the Danube valley. At the gorge of the Iron Gates at the transition into the Dinarides the westward diffusion of the wool-bearing sheep got stuck. It can be observed that after a while the diffusion process

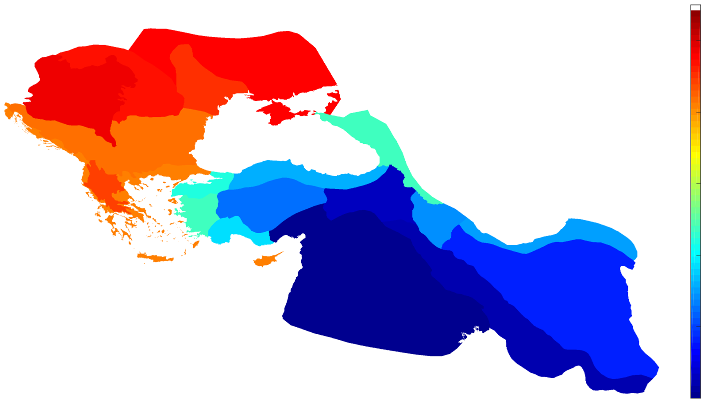


Fig. 7 | Results of running 25 simulations with the spreading probability $p = 0.2$. The mean of the first arrival time of the innovation in a region is depicted using different colors, for the exact values compare Table 1. First the innovation reached the regions in dark blue, then light blue, green and the regions in red are reached at last. The spreading path and also the crucial events for the spreading of the innovation can be analyzed from this figure.

continued first northward and then westward around the Carpatians following its footzone, reaching the Banat with some delay.

Looking at the development of the proportion of adopters among the agents (compare Fig. 8), it is clear that this proportion has to be increasing because of the asymmetric nature of our innovation spreading process. But there are several periods where the increase in the number of agents in state 1 stagnates. Those periods correspond to rare transitions of agents who adopted the innovation from one region to another region where the innovation is unknown. For example the transition of the innovation across the Bosporus takes a long time, this corresponds to a period of stagnation in the number of agents in state 1. Another way to look at this is via the first arrival times, there is a big time gap in the first arrival times of the regions adjacent to the Bosporus.

As our results are very consistent and the spreading path is similar for all simulations of the same parameter set, we are now going to look at one example simulation to analyze the spreading path in more detail. In Fig. 6 the time series for the spreading of the innovation across space and time is plotted for one realization of our model. In the first snapshot of the time series, the innovation is unknown in the whole domain but agents are distributed according to the suitability landscape and the attraction-repulsion potential. Blue dots denote agents that are in innovation state 0 whereas red dots indicate agents that have adopted the innovation. In the second snapshot, the innovation is introduced and all agents inside some radius have adopted the innovation. From there the innovation starts spreading via the time-evolving micro-scale network between agents and via migration of agents. The innovation spreads faster in areas of higher agent density. When most of the agents in Anatolia have adopted the innovation, it takes some time until an agent with the innovation manages to cross the Bosporus or to pass on the innovation to another agent on the other side of the Bosporus, and thereby the innovation is introduced to the Balkan area. This rare event is happening at different times in each simulation, which can be seen in Fig. 8. The innovation spreads further until nearly all agents in our domain have adopted the innovation. Some agents will never adopt the innovation because they are at no time connected to agents in innovation state 1 via the micro-scale network. In the next section we will also discuss underlying reasons for this to happen and introduce some possibilities to expand our model in a way that might prevent this situation.

Small changes in the choice of parameters such as the spreading probability p and the number of agents n, do not change the resulting spreading path, but the first arrival times. For higher values of n, the innovation spreads more quickly as the agent density is higher and also rare transitions occur more frequently. Varying p influences the speed of the spreading process inside regions, but has minor influence on the frequency of rare transition events. In this example, a lot of weight is given to the suitability landscape

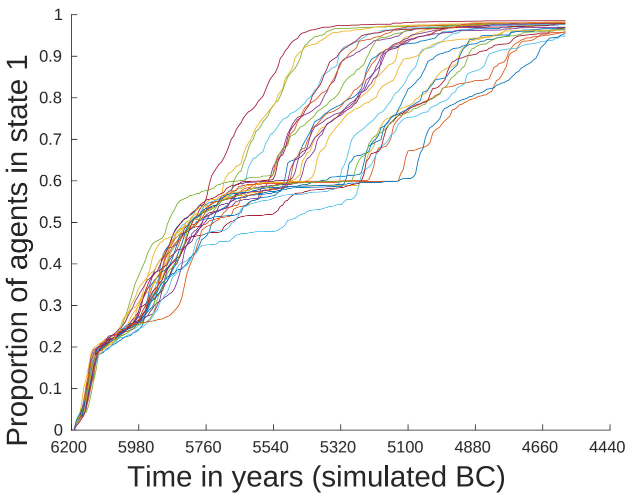


Fig. 8 | The proportion of agents in state 1 (i.e. of agents employing the innovation) is plotted versus simulation time for all 25 simulations with $p = 0.2$.

and thus it strongly determines the motion of agents. Since the innovation spreading is coupled to the migration, the suitability landscape describes also the barriers for the innovation spreading.

3.4 Discussion

The application of the proposed modeling approach to the wool-bearing sheep spreading process is not intended to reproduce the prehistorical spreading path itself in all its details. Instead, simulation of the model results in statistical distributions regarding objects like first arrival time and spreading paths that can be analyzed for a better understanding of the prehistorical spreading process. Moreover, simulations allow for sensitivity analysis, i.e., based on simulation results we can infer which parameters of the model have the highest influence on the (simulated) process. Comparison of simulation results, archaeological data (e.g. data on spindle whorls), expert knowledge and archaeological literature may allow for validation of (parts of) the assumptions and settings underlying the model. Although the subsequent discussion will provide some of the necessary steps in the direction of validation, the authors want to emphasize that the aim of this article cannot be the model validation but still is the proposal of a basic model that by its design can be extended and improved and may allow for validation in future research. Then, the model may allow for resilient conclusions about which qualities have been important for the spreading of the woolly sheep and how the process might have happened.

In order to reflect on the modeling assumptions retrospectively, we need to argue about our particular choice of assumptions and agents' rules for modeling the spreading process, as well as to discuss the reasoning behind setting parameters such as the number of agents and spreading probability as described in Section 3.2.1.

The main impact on the simulated spreading path comes from the suitability landscape and the social interactions. Mobility and migration in prehistoric times have indeed been shown to be strongly coupled to the spreading of new technologies.[34] The number of agents and to a smaller extend the spreading probability mainly influence the speed of the spreading, but to a much smaller extend the spatial path. So, it is reasonable to assume that the suitability of a region for herding sheep and the geographic borders that are insurmountable were very influential for the ancient spreading path as well. Because

34 Loog et al. 2017.

of the barriers in the suitability landscape that are hard to cross for agents, the diffusion of the innovation is not continuous (compare Fig. 8) but exhibits metastability, i.e., longer periods in which an innovation is known in some region before it arrives for the first time in another one. Checking the details of the impact of the suitability landscape on the spreading path is one of the most important issues for future research on improving the proposed model. Additionally, demographics of and communication structure between herders and agents should have been of influence for the spreading path and the speed of the spreading process.

Regarding our particular choice of parameters, we mainly have to discuss whether the speed of the simulated spreading process agrees with archaeological findings. We find that the simulation results presented above suggest that if the spreading of the wool-bearing sheep started in 6200 BC, it was known in the whole area of interest after on average 1200 years. The range from 700 to 1600 years for the complete simulated spreading in our study area allows for several possible explanations for the prehistoric development. The oldest spindle whorls with high probability for wool usage that were found in the Pannonian basin region, which is the region reached last in the simulated process, date back to 5500–4500 BC. This observation and the resulting spreading path agrees well with the generally accepted diffusion routes of Chalcolithic and Bronze Age agro-pastoral achievements.[35] In this sense, our specific choice of the spreading probability at least seems reasonable.

Furthermore, we comment on the choices for the specific value of the number of agents in our simulations. There are just a few estimates that can guide our choice: Each agent represents a small group of maybe 10–30 people[36] that herd sheep, that is, 4.000 agents represent a herding population of about 40.000 to 100.000 people in the entire study area. But not much is known about the population density and the proportion of people that herded sheep around 6000 to 4000 BC. Simulations suggest that the population density in today's Greece was 2–5 people per square kilometer during that time period,[37] so that the above number of herders seems also reasonable.

4 Conclusions and future outlook

We presented a mathematical modeling approach for the spreading of innovations on the example of the wool-bearing sheep spreading from the Near East to south-east Europe in the period 6200 and 4200 BC. Our method is using an agent-based modeling approach, where agents (herders) move diffusively in the suitability landscape formed from available archaeological and environmental data. Communication between agents that are close to each other in distance enables the innovation spread in our model. This is done by employing time-evolving networks, where the innovation is spreading diffusively along the edges of the network. Our approach produces simulation paths of the spreading process which can be used for better understanding of the ancient spreading process in both space and time.

Finally, we mention some open problems regarding our modeling approach that remain for future research. First, we are analyzing only a closed system and thereby exclude the possibility of innovation exchange with regions outside of our system like Northern Europe, Africa etc.. Second, at the current stage of our research we do not consider movements by ship which would allow for contact between agents on islands and the mainland. Thus the islands are left out of our area of interest despite their important role

35 Sherratt 1983, 93; Sherratt 1981, 261–306; Greenfield 2010, 35–37.
36 Bowles and Choi 2013.
37 Lemmen 2013, 52.

in that period.[38] Inclusion of sea travel does not pose a fundamental problem, the reason for the decision to exclude sea travel is that additional information on the sea traveling dynamics is needed for building a suitable model environment. Third, we do not address the problem of uncertainty in our data, which comes in the form of false assumptions and missing data. We assume that ovicaprid bone findings clearly indicate a herding area, but it is known that animals were often buried at specific places during traditional rituals. Also, we did not account for the missing data in the areas where there is no archaeological site or where there were no excavations. This is especially emphasized for the archaeological data used for fitting of the suitability landscape, which is available only for the Balkan area. However, this question opens a completely new topic of research and exceeds the scope of this manuscript. Fourth, in our model we consider a static suitability landscape. A more realistic picture would include a time-evolving suitability landscape, where the changes in the landscape could appear due to seasonal influences and (long-term) climate changes. Fifth, for the interaction between the agents, we implemented only very simple social interaction effects and did not take into account more complex social behavior often studied in the ABM literature.[39] In general, these effects could also be incorporated into our model, offering more detailed perspective of the social components in our study. However, one has to keep in mind that in the case of social systems from ancient times, very little is known about the nature of such interactions and including additional assumptions of this type and their effects should be studied in more detail. Sixth, another aspect one should take into account is the influence of other processes on our innovation spreading process. Here, one should consider competing and/or collaborative processes, such as the development of other occupations or early herding of other types of animals or economic factors. These six remarks are not meant to cover all of the requirements and possibilities but to be used as indicators for the direction of future research.

38 Forenbaher 2008, 236; Farr 2010.

39 Macy and Willer 2002, 152–161; An 2012, 26–32; Hedström and Wennberg 2017, 97–98.

Supplementary part

Here we will go into more details about the construction of the suitability landscape and attraction-repulsion potential for the woolly sheep example. We will also include the first arrival times in the different regions of all our simulations.

Constructing the suitability landscape

We constructed the suitability landscape V as a superposition of different factors that influence the attractiveness of an area for herders. More formally,

$$V = \sum_{j=1}^{3} \sum_{k=1}^{10} w_{jk}\, \phi_j\, \chi_k,$$

where ϕ_j for $j = 1, 2, 3$ are different environmental influence functions such as elevation, topographic compound index and the slope of the area. $\chi_k(x)$ denotes the indicator function of different landforms

$$\chi_k(x) = \begin{cases} 1, & \text{if } x \in \text{landform } k \\ 0, & \text{else.} \end{cases}$$

We consider 10 different landforms, i.e. areas that are similar in their properties, but not necessarily connected areas. The three environmental influences j are weighted differently for each kind of landform k with weights w_{jk}. The weights $w_{jk} > 0$ are constrained, such that $\sum_{j=1}^{3} w_{jk}$ for each landform k lies in some chosen interval and we fitted them with respect to our data by employing a Bayesian optimization scheme.[40] In other applications there could be used other influence functions and different methods to calculate the weights.

Optimizing parameters of the suitability landscape

Given the the known findings of ovicaprid bones, we assume that sheep and goats have been kept at the locations of the findings. Thus the bone sites can be seen as an indicator for the suitability of a location for keeping sheep and goat in ancient times. We assume that this suitability was constant for our time period of interest. Because we must suppose that we have imperfect knowledge, we can not infer from not having found bones in a region that there were no sheep and goats.

Using the Bayesian optimization scheme[41] we want to minimize the difference between the ovicaprid bone distribution B coming from the data (posterior) in our domain and the agent distribution A (prior) generated by our model. The objective function is the Kullback-Leibler divergence of the bone and agent distribution. The Kullback-Leibler divergence is a measure for the difference between two distributions. It is asymmetric regarding the two distributions that are compared and therefore a tool we could use here to include this information asymmetry. Thus, the Kullback-Leibler divergence can be understood as the information gain achieved by using the bone distribution instead of the agent distribution. It is defined as[42]

40 Gelbart 2015, 6–7.
41 Gelbart 2015, 6–7.
42 Kullback and Leibler 1951, 80.

$$D_{KL}(B||A) = \int_D b(x) \log \frac{b(x)}{a(x)} \, dx$$

with a and b being the density functions of A and B respectively. Since the agent distribution is induced by the suitability landscape, the objective function should be minimized over all possible suitability landscapes by varying the weights of the basis functions.

Attraction-repulsion potential

The attraction-repulsion potential we employ is of the form[43]

$$U_{pair}(d) = -C_{At} \exp\left(-\frac{d}{l_{At}}\right) + C_R \exp\left(-\frac{d}{l_R}\right)$$

where C_{At} is the attraction potential constant and C_R is the repulsion potential constant and $l_{At}, \; l_R$ are the respective decay rate constants. The constants are chosen s.t. $C_R > C_{At}$ and $l_{At} > l_R$. This leads to the repulsion dominating at short range while the attraction is the dominating term when the distance $d = \|x_i - x_j\|$ between two agents is larger.

Simulation results

For analysis of the full spectrum of spreading paths and to show how similar they are we refer to Table 2. For comparison the spreading path of each simulation can be read off the table.

43 Morse 1929, 57.

Region	Sim 1	Sim 2	Sim 3	Sim 4	Sim 5	Sim 6	Sim 7	Sim 8	Sim 9	Sim 10	Sim 11	Sim 12	Sim 13	Sim 14
1	5158 BC	4894 BC	5064 BC	4681 BC	5240 BC	4985 BC	4778 BC	5125 BC	5468 BC	4733 BC	4918 BC	4759 BC	4823 BC	5244 BC
2	5258 BC	5097 BC	5181 BC	4677 BC	5208 BC	5002 BC	4935 BC	5135 BC	5427 BC	4723 BC	4940 BC	4877 BC	4958 BC	5166 BC
3	5245 BC	5151 BC	5240 BC	4738 BC	5314 BC	5050 BC	4951 BC	5203 BC	5520 BC	4726 BC	4963 BC	4930 BC	4974 BC	5241 BC
4	5192 BC	5104 BC	5175 BC	4590 BC	5283 BC	4966 BC	4895 BC	5144 BC	5487 BC	4669 BC	4898 BC	4874 BC	4905 BC	5184 BC
5	5312 BC	5214 BC	5304 BC	4794 BC	5373 BC	5111 BC	5037 BC	5266 BC	5589 BC	4853 BC	5082 BC	5007 BC	5030 BC	5371 BC
6	5328 BC	5226 BC	5318 BC	4808 BC	5381 BC	5124 BC	5061 BC	5277 BC	5600 BC	4870 BC	5092 BC	5023 BC	5042 BC	5380 BC
7	5244 BC	5157 BC	5250 BC	4769 BC	5338 BC	5085 BC	4991 BC	5212 BC	5509 BC	4726 BC	4976 BC	4944 BC	4977 BC	5324 BC
8	–	–	–	–	–	–	–	–	–	–	–	–	–	–
9	5346 BC	5251 BC	5325 BC	4829 BC	5395 BC	5140 BC	5067 BC	5285 BC	5611 BC	4878 BC	5108 BC	5037 BC	5074 BC	5399 BC
10	5545 BC	5988 BC	5692 BC	5831 BC	5688 BC	5470 BC	5681 BC	5758 BC	5759 BC	5850 BC	5637 BC	5287 BC	5230 BC	5893 BC
11	5512 BC	5941 BC	5685 BC	5780 BC	5675 BC	5466 BC	5644 BC	5742 BC	5751 BC	5842 BC	5608 BC	5277 BC	5163 BC	5811 BC
12	5745 BC	6011 BC	6093 BC	5886 BC	5759 BC	5526 BC	5724 BC	5982 BC	5939 BC	5975 BC	5694 BC	5646 BC	5387 BC	5988 BC
13	5994 BC	6050 BC	6008 BC	5965 BC	5854 BC	5800 BC	5854 BC	6056 BC	5953 BC	6011 BC	5882 BC	5694 BC	5546 BC	6107 BC
14	5612 BC	5946 BC	5697 BC	5992 BC	5805 BC	5487 BC	5666 BC	5834 BC	6010 BC	5836 BC	5668 BC	5289 BC	5294 BC	5950 BC
15	6116 BC	6108 BC	6149 BC	6175 BC	6111 BC	6145 BC	6157 BC	6091 BC	6164 BC	6066 BC	6151 BC	6175 BC	6097 BC	617 BC6
16	5759 BC	5994 BC	5709 BC	5807 BC	6100 BC	5896 BC	5854 BC	5850 BC	6089 BC	6028 BC	5903 BC	5832 BC	5727 BC	6030 BC
17	5730 BC	5895 BC	5641 BC	5687 BC	5861 BC	5722 BC	5827 BC	5568 BC	5323 BC	5602 BC	5654 BC	5843 BC	4895 BC	5909 BC
18	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC
19	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC
20	6169 BC	6186 BC	6176 BC	6187 BC	6186 BC	6182 BC	6182 BC	6177 BC	6175 BC	6177 BC	6187 BC	6184 BC	6177 BC	6193 BC
21	6150 BC	6138 BC	6149 BC	6161 BC	6141 BC	6139 BC	6147 BC	6163 BC	6173 BC	6162 BC	6139 BC	6149 BC	6156 BC	6142 BC
22	6087 BC	6042 BC	5969 BC	5962 BC	6033 BC	6003 BC	6015 BC	5962 BC	6089 BC	6074 BC	6080 BC	6018 BC	6058 BC	6044 BC
23	5966 BC	5976 BC	5751 BC	5774 BC	5916 BC	5830 BC	5830 BC	5769 BC	5965 BC	5942 BC	5894 BC	5857 BC	5894 BC	5932 BC

Region	Sim 15	Sim 16	Sim 17	Sim 18	Sim 19	Sim 20	Sim 21	Sim 22	Sim 23	Sim 24	Sim 25	Mean	SD
1	5130 BC	5145 BC	5252 BC	4824 BC	5531 BC	5324 BC	5433 BC	5236 BC	4907 BC	4853 BC	5178 BC	5067 BC	243 BC
2	5220 BC	5179 BC	5335 BC	4852 BC	5513 BC	5372 BC	5483 BC	5293 BC	4935 BC	4773 BC	5237 BC	5111 BC	235 BC
3	5203 BC	5189 BC	5302 BC	4923 BC	5573 BC	5425 BC	5491 BC	5344 BC	4970 BC	4889 BC	5261 BC	5153 BC	233 BC
4	5116 BC	5097 BC	5264 BC	4878 BC	5518 BC	5377 BC	5447 BC	5308 BC	4892 BC	4783 BC	5195 BC	5090 BC	249 BC
5	5285 BC	5265 BC	5378 BC	5010 BC	5643 BC	5485 BC	5552 BC	5397 BC	5008 BC	4976 BC	5336 BC	5227 BC	228 BC
6	5302 BC	5280 BC	5394 BC	5022 BC	5660 BC	5500 BC	5562 BC	5408 BC	5025 BC	4987 BC	5347 BC	5241 BC	227 BC
7	5225 BC	5223 BC	5318 BC	4934 BC	5580 BC	5417 BC	5499 BC	5342 BC	4957 BC	4908 BC	5273 BC	5167 BC	231 BC
8	–	–	4450 BC	–	–	–	4270 BC	–	–	–	–	–	–
9	5320 BC	5300 BC	5427 BC	5037 BC	5671 BC	5515 BC	5576 BC	5427 BC	5036 BC	5004 BC	5363 BC	5257 BC	227 BC
10	5884 BC	5707 BC	5757 BC	5435 BC	5862 BC	5724 BC	5798 BC	5625 BC	5658 BC	5757 BC	5821 BC	5693 BC	183 BC
11	5844 BC	5655 BC	5743 BC	5403 BC	5822 BC	5736 BC	5814 BC	5633 BC	5682 BC	5729 BC	5737 BC	5668 BC	182 BC
12	6003 BC	5890 BC	5782 BC	5834 BC	5934 BC	5779 BC	5890 BC	5687 BC	5885 BC	5964 BC	5871 BC	5835 BC	164 BC
13	6051 BC	5920 BC	5870 BC	5633 BC	6043 BC	5985 BC	5955 BC	5833 BC	6004 BC	6012 BC	5935 BC	5921 BC	138 BC
14	5853 BC	5726 BC	5910 BC	5475 BC	5906 BC	5960 BC	5908 BC	5934 BC	5704 BC	5782 BC	5834 BC	5763 BC	203 BC
15	6172 BC	6183 BC	6158 BC	6168 BC	6147 BC	6115 BC	6075 BC	6134 BC	6077 BC	6142 BC	6144 BC	6136 BC	35 BC
16	5868 BC	5839 BC	5932 BC	5936 BC	5865 BC	5912 BC	5960 BC	5672 BC	5849 BC	5637 BC	5916 BC	5879 BC	120 BC
17	5626 BC	5642 BC	5892 BC	5785 BC	5843 BC	5915 BC	5622 BC	5447 BC	5679 BC	5449 BC	5715 BC	5671 BC	224 BC
18	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	0 BC
19	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	6200 BC	0 BC
20	6181 BC	6179 BC	6192 BC	6188 BC	6191 BC	6184 BC	6182 BC	6187 BC	6186 BC	6184 BC	6180 BC	6183 BC	6 BC
21	6159 BC	6158 BC	6146 BC	6162 BC	6148 BC	6151 BC	6131 BC	6165 BC	6150 BC	6145 BC	6163 BC	6151 BC	10 BC
22	6038 BC	6030 BC	6035 BC	6060 BC	5936 BC	6020 BC	5986 BC	6006 BC	6065 BC	6047 BC	6002 BC	6026 BC	41 BC
23	5916 BC	5862 BC	5922 BC	5878 BC	5820 BC	5864 BC	5861 BC	5814 BC	5966 BC	5870 BC	5843 BC	5876 BC	64 BC

Tab. 2 | The first arrival times (in simulated BC), their mean and standard deviation for each of the 23 regions from our 25 simulations with $p = 0.2$. Arrival times for simulations in which the innovation never reached a region are marked with – .

Illustration credits

1 Authors, information on wool production based on Rooijakkers 2012. **2** Authors. **3** Authors, information on wool production based on Rooijakkers 2012, on bone finds based on Hans Christian Küchelmann *in* Grabundzija et al. (forthcoming), and on spindle whorls based on Grabundžija and Russo 2016. **4–8** Authors.

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