

Are Costs of Capital Necessarily Constant over Time and across States of Nature?

Some Remarks on the Debate on ‘WACC is not quite right’

Daniela Lorenz, Lutz Kruschwitz, and Andreas Löffler¹

Freie Universität Berlin, School of Economics and Business.

Abstract

Miller (2009a) opened a debate in this journal on the correct determination of weighted average costs of capital (WACC). So far Bade (2009), Pierru (2009a), Lobe (2009) as well as Keef et al. (2012) have contributed to this debate. Even though they discuss the same, rather simple valuation problem, the dispute cannot be considered resolved. Whilst they agree that Miller erroneously assumed constant leverage ratios, the center of discussion is now placed on the question whether or not cost of capital is constant over time when leverage changes and interest paid is not tax deductible. In particular, Keef et al. (2012) demand time-invariant WACC and criticize Bade (2009) and Pierru (2009a) for allowing WACC to change over time. The aim of this paper is twofold. Firstly, we show that the arguments of Keef et al. (2012) are flawed and their criticism of Bade (2009) and Pierru (2009a) is thus unfounded. Keef et al. (2012) are wrong to ignore that not only financial risk but also operational risk can change over time. Secondly, we provide evidence that cost of capital can also be dependent on the future state of nature. So far this fact has been neglected by all contributors to this debate and becomes obvious only if state-dependent cash flow realizations, not only their expected values, are considered as well.

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(Daniela Lorenz, Lutz Kruschwitz, and Andreas Löffler)

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1. Introduction

Recently this journal hosted a heated debate on how the weighted average cost of capital is to be interpreted and how it should advantageously be determined. Miller (2009a) argued that a simple linear interpolation of cost of equity and cost of debt leads to wrong valuation results. To prove his claim he presented an extensive example.

Bade (2009) as well as Pierru (2009a) object to Miller vehemently and point out that his valuation approach contains an error. Miller (2009a) wrongly assumed that the leverage ratio is constant over time. Based on the same example used by Miller (2009a) and disregarding taxes, Bade (2009) and Pierru (2009a) show that this leverage ratio does change over time and that hence Miller's result cannot be upheld. In particular, they illustrate that the weighted average cost of capital (WACC) must increase over time when cost of levered equity and of debt remain constant and Miller's repayment schedules apply, i.e. leverage declines. Thus, WACC should be time-dependent. Alternatively, Pierru (2009a) adjusts the repayment schedules assumed in Miller's example in such a manner that the debt ratio as well as WACC remain constant.¹ In his reply Miller (2009b) admits that his original repayment schedule is not compatible with a constant debt

¹Similarly, Lobe (2009) presents the identical two alternatives for proving the correctness of the traditional WACC. The latter alternative was heavily criticized by Tanha and Foroutan (2013) who base their argumentation on the distinction between the so-called *Total Cash Flow* approach that considers the tax shield in the numerator (see, e.g., (Kruschwitz and Löffler, 2006, section 2.4.2)) and the *WACC* approach that accounts for the tax shield in the denominator. They state that Pierru (2009a), Bade (2009), and Miller (2009a) "failed to answer Miller's question correctly because they did not consider this fact that they were assuming one description of the cash flow while using another formula for WACC which assumes the other description of the cash flow" (Tanha and Foroutan, 2013, page 2083). However, the distinction between both approaches (i.e. cash flow descriptions and WACC formulas) is by no means suitable to demonstrate weaknesses in the afore-mentioned papers

ratio and additionally proposes another three (non-self-amortizing) repayment
20 schedules.²

Recently Keef et al. (2012) have attempted to summarize the discussion and
place it in a wider context. From their point of view, Bade (2009) as well as
Pierru (2009a) have made another hitherto hidden mistake and therefore the
question raised by Miller remains unsolved. Whilst Keef et al. (2012) agree
25 that leverage indeed decreases in Miller’s example, they claim that Bade (2009)
and Pierru (2009a) “incorrectly conclude that the annual WACC increases over
time”.³ Instead they argue that in a world without taxes WACC is independent
of leverage and should thus be constant over time. They believe that Bade (2009)
and Pierru (2009a) erroneously “assume, for convenience, that the required rate
30 of return on levered equity is independent of leverage”.⁴ Analogously, this
critique applies to Lobe (2009) whose argumentation is in line with Bade (2009)
and Pierru (2009a).

The aim of our study is twofold. Firstly, we are convinced that the arguments
of Keef et al. (2012) supporting time-independency of WACC are flawed. Whilst
35 Keef et al. (2012) solely focus on changes in financial risk (resulting from changes
in leverage) they neglect the fact that operational risk can change over time,
too. In particular, we show that even regardless of taxes WACC can indeed
be time-dependent and that the criticism voiced by Keef et al. (2012) is thus
ill-founded.

that all assume a world without taxes where both approaches inevitably coincide.

²Also, in his reply Miller claims that the only relevant costs of capital and debt ratios are
those that exist at $t = 0$, the time the project is accepted and financed. However, the project’s
present value is generally calculated according to $\frac{E[\widetilde{CF}_1]}{1+WACC_0} + \frac{E[\widetilde{CF}_2]}{(1+WACC_0)(1+WACC_1)} + \dots +$
 $\frac{E[\widetilde{CF}_n]}{(1+WACC_0)\dots(1+WACC_{n-1})}$. Thus, apart from the special case of constant WACC it is indeed
indispensable for the capital budgeting decision in $t = 0$ to calculate $WACC_t$ for every future
period $t = 0, 1, \dots, n - 1$ for which future costs of equity, costs of debt and future debt ratios
apply. Similarly, Pierru (2009b) argues that Miller’s argumentation is unsubstantiated and
that the resulting discounting procedure might violate essential consistency properties.

³Keef et al. (2012), page 441.

⁴Keef et al. (2012), page 441.

40 Secondly, we want to draw the reader’s attention to another issue that has not been discussed in any of the previously cited papers with sufficient care. Remarkably, all contributors to this debate have ignored the way risk affects cash flows. If, however, one properly accounts for the fact that cash flows are state-contingent, we show that cost of capital does not necessarily have to be
45 independent of future states of nature. This, however, is the implicit assumption in all previously cited papers.

2. Time-dependency of Cost of Capital

The focus of this section is to analyze the time-dependency of WACC from a theoretical point of view. To illustrate our argumentation and improve its
50 comparability to prior papers in this field we refer to the same example introduced by Miller (2009a) and taken up by Bade (2009), Pierru (2009a), [Lobe \(2009\)](#) as well as Keef et al. (2012). Table 1 in the appendix summarizes the setting. Table 2 reflects the proposed cost of capital that differs according to whether or not WACC has to be constant over time. In line with the proposal
55 of Bade (2009), Pierru (2009a), [and Lobe \(2009\)](#), we show that WACC can indeed change over time and the criticism of Keef et al. (2012) is thus unfounded. This holds true even in absence of taxes – a setting in which WACC equals the cost of unlevered equity r_e^U and is thus independent of leverage. However, it is wrong to assume that r_e^U (a measure for the operational risk to which a firm is
60 exposed) has to be constant over time. [Note, that in line with all other papers contributing to this discussion we do not explicitly account for interest rate risk. However, the modeling of interest rate changes over time would affect costs of capital which again supports time-varying WACC.](#)

In contrast to Bade (2009), Pierru (2009a), [and Lobe \(2009\)](#) Keef et al.
65 (2012) support the idea of constant WACC over time and base their argumentation on the following relationship between the cost of levered equity r_e^L , cost of unlevered equity r_e^U , cost of debt r_b as well as leverage L

$$r_e^L = r_e^U + (r_e^U - r_b) \times L. \quad (1)$$

This relationship is derived from the combination of two theorems that we discuss separately in the following.

- 70 1. The first theorem necessary to derive Equation (1) is an adjustment formula that appropriately describes the relationship between weighted average cost of capital WACC and cost of unlevered equity r_e^U . In this respect Keef et al. (2012) refer to the seminal works of Modigliani and Miller (1963) as well as Miles and Ezzell (1980) from which such adjustment formulas can be inferred. In their most general form they read as follows:

$$WACC = (1 - \tau \times l_0) \times r_e^U \quad (2a)$$

$$WACC_t = (1 + r_{e,t}^U) \left(1 - \frac{\tau \times r_f}{1 + r_f} \times l_t \right) - 1 \quad (2b)$$

They are known as the Modigliani-Miller adjustment formula (2a) and the Miles-Ezzell adjustment formula (2b), respectively. In these equations l denotes the leverage ratio whereas L in Equation (1) stands for the debt equity ratio. Both measures of leverage can easily be converted into one another. For the risk-free rate we use the symbol r_f . Obviously, if we assume the absence of taxes, $\tau = 0$, both equations coincide and arrive at the result that WACC equals the cost of unlevered equity. Nevertheless, it is crucial to distinguish clearly between both formulas because they are based on different assumptions and only in case of the Miles-Ezzell adjustment formula (2b) do the cost of capital and the leverage ratio carry time subscripts and are thus time-dependent. In particular, the Modigliani-Miller adjustment formula (2a) can only be derived if one assumes that the amount of debt D_t does not change over time (the firm never redeems its debt) and that the firm's time horizon is either infinite or just one period. Both assumptions are clearly not met in the example at the center of the debate (see Table 1). By contrast, the Miles-Ezzell adjustment for-

95 mula assumes deterministic but not necessarily constant leverage ratios⁵
as well as deterministic but not necessarily constant cost of capital. Thus,
the Miles-Ezzell adjustment formula is applicable to the example in Table
1 without contradiction. In a world without taxes, $\tau = 0$, it simplifies to

$$WACC_t = r_{e,t}^U. \quad (3)$$

Obviously, weighted average costs of capital can indeed be time-dependent
– a fact that is neglected by Keef et al..

100 2. In order to derive Equation (1), one also has to use a formula that de-
termines WACC as the average of the cost of levered equity and debt
weighted by equity and debt ratio, respectively,

$$WACC_t = (1 - l_t) \times r_{e,t}^L + l_t \times (1 - \tau) \times r_{b,t}. \quad (4)$$

105 Equation (4) can be found in almost every finance textbook and is thus
known as the textbook formula. In order to apply this formula signifi-
cantly fewer assumptions are involved compared to the adjustment for-
mulas. Rather, it is possible to show that the textbook formula is a trivial
conclusion of the cost of capital definition and holds even if the expected
returns are random variables.⁶ Note, however, that it is possible that
these variables change over time.⁷

110 Against this background we do not understand why Keef et al. (2012) argue
that WACC should always be constant over time. According to our considera-
tions above, neither Equation (3) nor Equation (4) requires the assumption of

⁵Whilst Miles and Ezzell derive their formula only for the case of a constant leverage ratio, it has been proven that the assumption of a deterministic but time-variant leverage ratio is sufficient to derive the formula (2b). See, e.g., (Kruschwitz and Löffler, 2006, section 2.4.4). In this respect (Pierru, 2009a, p. 1220) is mistaken in believing that “a constant WACC implicitly requires the debt ratio to also remain constant”.

⁶See, e.g., (Kruschwitz and Löffler, 2006, section 2.4.3). The fact that cost of capital may be random variables, i.e., are state-dependent, is not acknowledged in any paper involved in the debate. We focus on this issue in the next section.

⁷See, e.g., Lobe (2009).

constant WACC. The claim of Keef et al. (2012) is thus unfounded and unsubstantiated by theory.

115 For $\tau = 0$ equalizing both equations and rearranging yields

$$r_{e,t}^L = r_{e,t}^U + (r_{e,t}^U - r_{b,t}) \times L_t. \quad (5)$$

A comparison to Equation (1) reveals that cost of levered equity r_e^L can also be time-variant and positively depend on the debt equity ratio L which itself can be time-dependent.⁸ This relates directly to another criticism leveled at Bade (2009) and Pierru (2009a) that is expressed by Keef et al. (2012) and
 120 that is, we believe, also unfounded. Specifically, Keef et al. (2012) claim “their error is to assume, for convenience, that the required rate of return on levered equity r_e^L is independent of leverage.” They refer to Equation (1) in order to underpin this statement. We agree that this equation clearly indicates that cost of equity are higher the more a firm is exposed to financial risk, i.e., the
 125 greater its leverage. Since the leverage ratio in Table 1 decreases over time, Keef et al. (2012) conclude that r_e^L should then also decrease which, as they claim, is ignored by Bade (2009) and Pierru (2009a). However, this line of argumentation neglects the fact that according to Equation (5) cost of levered equity r_e^L are not only driven by financial risk (leverage L_t) but also by operational risk (measured
 130 by cost of unlevered equity $r_{e,t}^U$) which itself can change over time, [see, e.g., Lobe \(2009\)](#). In the (albeit unrealistic but at least consistent) example of Bade (2009) and Pierru (2009a) the effect of an increasing operational risk and that of a decreasing financial risk perfectly compensate for each other, resulting in cost of levered equity r_e^L amounting to 12% at every future point of time. By taking
 135 a closer look at the numbers in Table 2 one must recognize that the relation (5) is fulfilled at every single future date in the example of Bade (2009) and Pierru (2009a) which was wrongly doubted by Keef et al. (2012).⁹

Finally, we wish to point out that the cost of capital proposed by Keef

⁸See, e.g., Buus (2015).

⁹At date $t = 4$, for example, $r_{e,4}^U$ amounts to 10.60%, costs of debt $r_{b,4}$ are 6% and debt equity ratio amounts to $L_4 = \frac{27.9}{91.714}$. Plugging these numbers into Equation (5) yields

et al. (2012) (see Table 2) – in contrast to those proposed by Bade (2009),
 140 Pierru (2009a), and Lobe (2009) – are inconsistent with the underlying setting
 described in Table 1. The market value of equity E_t must equal the sum of
 future dividends discounted by the cost of levered equity,

$$E_t = \sum_{s=t+1}^T \frac{E[\widetilde{CF}_{e,s}]}{(1 + r_{e,t}^L) \cdots (1 + r_{e,s-1}^L)}. \quad (6)$$

This basic relationship is however not fulfilled by Keef et al. (2012). Using
 the proposed cost of levered equity does not yield the given equity values and
 145 leverage ratios.

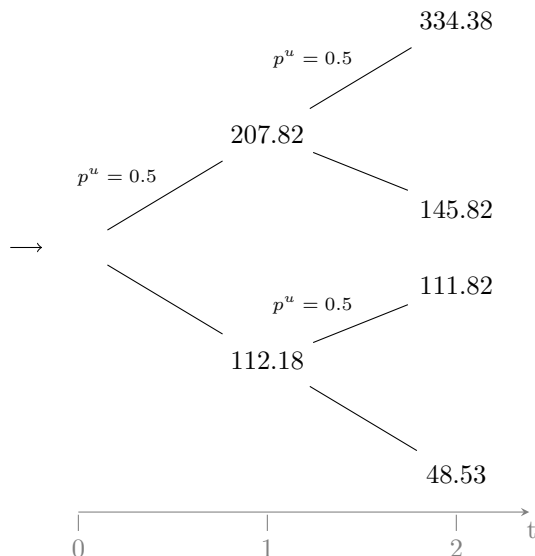
3. State-Dependency of Cost of Capital

Whilst the previous section focuses on time-dependency of weighted average
 cost of capital, we now turn to the question of whether they are necessarily
 independent of the future states of nature. This assumption is implicitly made
 150 by all contributions that are involved in the debate. For simplicity and comparability
 we take up another example that is already provided by Keef et al.
 (2012): we consider a firm with a lifespan of two periods that carries out a
 single investment project. The (expected) cash flows are -260 in $t = 0$, 160 in
 $t = 1$ and 160 in $t = 2$. In order to clarify what we want to discuss, we draw a
 155 binomial tree, see figure 1. This tree does not show the expected cash flows but
 their state-contingent realizations. In this regard, it goes beyond the examples
 of Miller, Bade, Pierru as well as Keef et al.. The (subjective) probability for
 any path of the tree is 50%. This makes it fairly easy to calculate the expected
 cash flows. They amount to 160 in $t = 1$ and $t = 2$. There are no taxes.

160 We assume that claims on said cash flows are traded in a market that is
 both complete and arbitrage-free. Such an assumption is far from unusual in
 the theory of finance. It is a standard assumption that, for example, is often

$r_{e,4}^L = 12\%$ which corresponds to the numbers presented by Bade (2009) and Pierru (2009a).
 See Table 2.

Figure 1: A first example with expected cash flows of 160 at $t = 1$ and 160 at $t = 2$.



used in option pricing theory. If the assumption is made as described, then the fundamental theorem of asset pricing applies.¹⁰ The theorem states that there exists a risk-neutral probability with which one can easily evaluate any risky asset. Let us assume that the risk-neutral probabilities for the up and down movements of the binomial tree are given with $q^u = 40\%$ and $q^d = 1 - q^u = 60\%$. Finally, we want to assume that the risk-free rate amounts to $r_f = 6\%$. Based on this information, the values of the company at $t = 0$ and $t = 1$ can easily be calculated. We obtain at $t = 1$ in the case of an upward movement

$$V_1^u = \frac{E_Q[\widetilde{CF}_2^u]}{1 + r_f} = \frac{0.4 \times 334.38 + 0.6 \times 145.82}{1.06} = 208.72, \quad (7)$$

and in the case of a downward movement

$$V_1^d = \frac{E_Q[\widetilde{CF}_2^d]}{1 + r_f} = \frac{0.4 \times 111.82 + 0.6 \times 48.53}{1.06} = 69.46, \quad (8)$$

¹⁰See Harrison and Kreps (1979).

and at $t = 0$

$$V_0 = \frac{0.4 \times (207.82 + 208.72) + 0.6 \times (112.18 + 69.46)}{1.06} = 260. \quad (9)$$

Using the subjective probability measure this corresponds to the following weighted average costs of capital:

$$WACC_1^u = \frac{0.5 \times 334.83 + 0.5 \times 145.82}{208.72} - 1 = 15.03\% \quad (10)$$

$$WACC_1^d = \frac{0.5 \times 111.28 + 0.5 \times 48.53}{69.46} - 1 = 15.03\% \quad (11)$$

$$WACC_0 = \frac{0.5 \times (207.82 + 208.72) + 0.5 \times (112.18 + 69.49)}{260} - 1 = 15.03\%. \quad (12)$$

As we can see, in this example weighted average costs of capital are constant not only over time but also across states. This corresponds to the numbers provided
 175 by Keef et al. (2012)

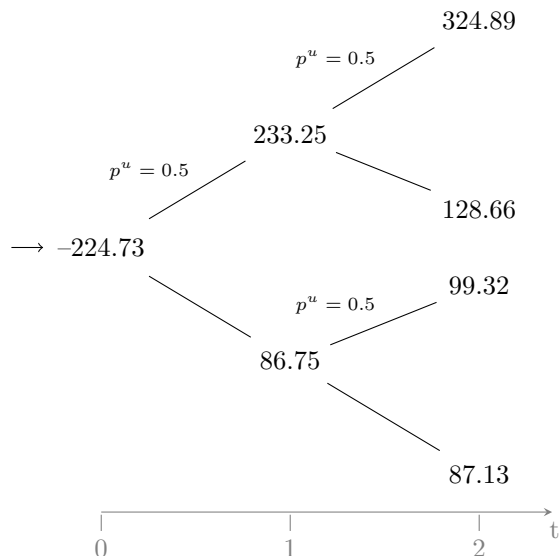
In a second step we now consider another binomial tree shown in figure 2. Again the expected cash flows amount to 160 at each future point of time. Using the same probabilities as before and again assuming a riskless rate of 6%, we obtain $V_0 = 260$ for the value of the company, a result which in no way
 180 differs from the earlier case. However, one obtains completely different weighted average costs of capital than before. By performing the required calculations we obtain

$$WACC_1^u = 16.04\% \quad WACC_1^d = 7.40\% \quad WACC_0 = 15.81\%. \quad (13)$$

In this example, weighted costs of capital are obviously not only dependent on time, they also vary across states. In this case fundamental theorems like the
 185 adjustment formulas described above are no longer applicable.

What can we learn from these two examples? Whilst the general setting of both examples is identical (particularly the expected cash flows), only the state-dependent realizations of the cash flows differ. None of the authors who have previously contributed to the subject in this journal have been able to
 190 distinguish the two cases described by figures 1 and 2, respectively. They only

Figure 2: A second example with expected cash flows of 160 at $t = 1$ and 160 at $t = 2$.



focus on expected cash flows and do not consider the underlying probability distribution of their realizations. But that is exactly what is needed when it comes to determining the appropriate cost of capital.

The cost of capital concept is surely essential to the theory of finance. Surprisingly, clear and unambiguous definitions of the term are few and far between in the relevant literature. In a multi-period model, the terms “(expected) return”, “cost of capital” and “discount factor” do not necessarily denote the same concept.¹¹ Our examples reveal that there may be not only one but several expected returns and that costs of capital are not necessarily state-independent. It is not sufficient to deal with expected cash flows. Rather, one must examine the stochastic structure of those payments.¹² He who ignores all this behaves like someone who keeps his ears closed while the orchestra is playing because he believes that he can appreciate the music by merely looking at the musicians.

¹¹See (Kruschwitz and Löffler, 2006, pp 22 ff.) for more details.

¹²For more details, see again (Kruschwitz and Löffler, 2006, pp 33 ff.).

4. Summary

205 In 2009 Miller initiated a debate in this journal on the correct determination
of costs of capital, to which Bade (2009), Pierru (2009a), [Lobe \(2009\)](#) as well as
Keef et al. (2012) have contributed. All these authors focus on a rather simple
problem, namely the valuation of a firm with a finite life-time in the absence
of taxes. One would expect this valuation problem to be an easy task, however
210 the debate on this remains unresolved.

In this paper we scrutinize the study of Keef et al. that criticizes the solu-
tions to the valuation problem provided by Bade (2009) and Pierru (2009a). In
contrast to the latter, Keef et al. assume that weighted average costs of capital
have to be constant over time and that the costs of levered equity necessarily
215 have to decrease if leverage declines. We show that this is a possible but not
mandatory assumption. In particular, if one takes into account that operational
risk may increase over time, we have two opposing effects on costs of levered
equity: a lower financial risk on the one hand and a higher operational risk on
the other. Which effect dominates depends on the specific valuation environ-
220 ment. It is even possible that both opposing effects perfectly balance each other
out, resulting in time-independent costs of levered equity, which is assumed by
Bade (2009) and Pierru (2009a). Thus, the criticism raised by Keef et al. is
unfounded.

Moreover, we discuss another issue that has not been studied with sufficient
225 care in any of the previously cited papers. Whilst all authors assume that future
cash flows are uncertain, no one specifies the cash flows' probability distribution.
Instead they only concentrate on the expected value of future cash flows. We
show that costs of capital may vary not only over time but also across the
future states of nature, if one takes a closer look at the state-dependent cash
230 flow realizations. To the best of our knowledge, remarkably, this fact is ignored
by many valuation theorists.

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Appendix

Table 1: Miller-Example

This table reports the market value of a firm V_t carrying out a single investment project with total cash flow amounting to CF_t . For the leverage ratio the symbol l_t is used. E_t and D_t denote market value of equity and debt, respectively. $CF_{e,t}$ and $CF_{b,t}$ are the amounts of cash that flow to equity and debt, respectively. Numbers are taken from (Bade, 2009, Table 1, page 1478), (Pierru, 2009a, Table 2, page 1222) and (Lobe, 2009, Table 2, page 48), respectively.

t	$V_t = E_t + D_t$	E_t	D_t	$l_t = D_t/V_t$	CF_t	$CF_{e,t}$	$CF_{b,t}$
0	200.000	150.000	50.000	25.00 %	-200.000	-150.000	-50.000
1	182.753	137.805	44.948	24.60 %	38.247	30.195	8.052
2	163.739	124.146	39.593	24.18 %	38.247	30.195	8.052
3	142.765	108.848	33.917	23.76 %	38.247	30.195	8.052
4	119.614	91.714	27.900	23.33 %	38.247	30.195	8.052
5	94.047	72.524	21.523	22.88 %	38.247	30.195	8.052
6	65.794	51.032	14.762	22.44 %	38.247	30.195	8.052
7	34.556	26.960	7.596	21.98 %	38.247	30.195	8.052
8	0.000	0.000	0.000		38.247	30.195	8.052

Table 2: Proposed Costs of Capital of the Miller Example

This table reports the costs of capital as proposed by Bade (2009)/Pierru (2009a)/Lobe (2009) and Keef et al. (2012). In case of Bade (2009)/Pierru (2009a)/Lobe (2009) the numbers for all points in time t can directly be taken from their tables. By contrast, Keef et al. (2012) perform their calculation only for $t = 4$ exemplarily, see (Keef et al., 2012, Table 4, Panel A and C, page 441). Applying the same rationale with regard to other dates yields the numbers in the last columns. Cost of levered equity is denoted by $r_{e,t}^L$, $r_{b,t}$ stand for costs of debt and $WACC_t$ are the weighted average costs of capital that equal the costs of unlevered equity $r_{e,t}^U$ assuming absent taxes.

t	Bade (2009)/Pierru (2009a)/Lobe (2009)			Keef et al. (2012)		
	$r_{e,t}^L$	$r_{b,t}$	$WACC_t = r_{e,t}^U$	$r_{e,t}^L$	$r_{b,t}$	$WACC_t = r_{e,t}^U$
0	12 %	6 %	10.50 %	12.07 %	6 %	10.55 %
1	12 %	6 %	10.52 %	12.04 %	6 %	10.55 %
2	12 %	6 %	10.55 %	12.01 %	6 %	10.55 %
3	12 %	6 %	10.57 %	11.97 %	6 %	10.55 %
4	12 %	6 %	10.60 %	11.94 %	6 %	10.55 %
5	12 %	6 %	10.63 %	11.90 %	6 %	10.55 %
6	12 %	6 %	10.65 %	11.87 %	6 %	10.55 %
7	12 %	6 %	10.68 %	11.84 %	6 %	10.55 %