Applications of Doubly Quasi-Periodic Boundary Value Problems in Elasticity Theory

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ABSTRACT

In the present thesis, we investigate the first and the second fundamental complete plane strain (CPS) problems of the three-dimensional nonhomogeneous elastic body with a doubly-periodic set of cracks and the mixed CPS problems of the three-dimensional nonhomogeneous elastic body with a doubly-periodic set of holes. At first, we resolve the complete plane strain state into two linearly independent plane elastic systems by the superposition principle of forces. Then, on the basis of that when the stress distributions are doubly-periodic in the elastic body, then the displacements, the complex stress function $\phi(z)$, the expression $z\overline{\phi'(z)} + \overline{\psi(z)}$ and the complex torsion function $F(z)$ are all doubly quasi-periodic, we construct Kolosov functions, and establish boundary value problems by using the complex potential method, furthermore, based on a suitable modification of Cauchy-type integrals, which is defined by the replacement of the Cauchy kernel $1/(t-z)$ by the Weierstrass zeta function $\zeta(t-z)$, the general representations for the solutions are constructed, under some general restrictions the boundary value problems are reduced to the normal type singular integral equations with Weierstrass zeta kernel, and the existences of the essentially unique solution are proved. In addtion, we pose three formulations of the modified doubly-periodic second fundamental CPS problem with relative displacements. It is proved that, for the unique existence of solution, the external resultant principal vectors and moments must be given in advance. At last, the general solutions are obtained in closed form for several specific cases. For some illustrating examples of practical interest, the exact solutions are obtained. moreover when we fix one of its periods, while the other tends to infinity, we get the exact solutions of the singly-periodic case, furthermore, when we let the two periods both tend to infinity, we have immediately the solutions of non-periodic case, which are identical with the classical ones.

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