Gini decompositions and Gini elasticities:
On measuring the importance of income sources and population subgroups for income inequality

Simon Jurkatis
Wolfgang Strehl

Gini decompositions and Gini elasticities:
On measuring the importance of income sources and population subgroups for income inequality

Simon Jurkatis[*,a,c] and Wolfgang Strehl[b,c]

[a]*Humboldt-Universität zu Berlin*
[b]*Freie Universität Berlin*
[c]*Berlin Doctoral Program in Economics & Management Science*

September 30, 2014

Abstract

This paper points to flaws in Gini decompositions by income sources and population subgroups and to common pitfalls in the interpretation of decomposition results, focusing on methods within the framework of Rao (1969). We argue that within this framework Gini elasticities may provide the only meaningful way to examine the relevance of income sources or population subgroups for total income inequality. Moreover, we show that existing methods are unsuitable to decompose the *trend* in the Gini coefficient and provide a coherent method to decompose the Gini trend by income sources. We add to the recent trend of multi-decompositions by deriving Gini elasticities from a simultaneous decomposition by income sources and population subgroups.

Keywords: Income inequality; Gini decomposition; Gini elasticity; Income sources; Population subgroups; Multi-Decomposition

JEL classification: C43, D31, D33, D63, O15

1 Introduction

> "... the disaggregation of the Gini coefficient is probably the most misused and misunderstood concept in the income inequality literature."
>
> (Podder and Chatterjee, 2002, p. 3)

Gini decompositions have been proposed early on to analyze the role of different income sources (e.g. capital income or government transfers) or population subgroups (e.g. different ethnical, geographical or generational groups) for overall income inequality (Bhattacharya and Mahalanobis, 1967; Rao, 1969). Yet, despite the extensive use of Gini decompositions in the income inequality literature, mistakes continue to be made when it comes to the interpretation of decomposition results, and misleading methods of decomposition do not stand corrected. The consequences of misinterpretations or misleading methods cannot be overstated, as Gini decomposition results may be used by policymakers to understand underlying trends in the distribution of income and, most relevantly, to assess different tax and transfer policies in terms of their effectiveness to reduce overall income inequality.[1]

This paper critically assesses Gini decompositions by income sources and population subgroups within the framework of Rao (1969)—a framework which has been widely used in the income inequality literature (for recent applications, see e.g. Podder and Chatterjee, 2002; Chatterjee and Podder, 2007; Davis et al., 2010; Mussini, 2013). In this framework the Gini is disaggregated into the income sources' (or subgroups') so-called concentration coefficients[2], each coefficient being weighted by the share of the respective income source (or subgroup income)

---

[1]To give one example, falsely attributing an increase in inequality, as measured by the Gini coefficient, to changes in the distribution of capital income, as opposed to changes in wage income, may lead to wrong conclusions about redistributional measures enacted in the past and/or to misdirecting policy recommendations to counteract the increase in inequality.

[2]Roughly speaking, a concentration coefficient measures the relation of an income source (or the income of a population subgroup) with the rank of its recipients in total income, i.e. it indicates whether an income source (or the income of a subgroup) accrues mainly to relatively poor or rich households.

in total income. Generally speaking, such decompositions aim at providing an understanding of the importance of an income source or population subgroup for total income inequality. In this paper we show which questions can and cannot be addressed by the Gini decompositions proposed within Rao's (1969) framework. In addition to this assessment of the existing methods, we provide a new method to decompose the change in the Gini coefficient by income sources and derive Gini elasticities from a simultaneous decomposition by income sources and population subgroups, adding to the recent trend of multi-decompositions (Mussard, 2004; Mussard and Richard, 2012; Mussard and Savard, 2012; Mussini, 2013).

First, we examine the Gini decomposition by income sources. Specifically, we point to mistakes in the interpretation of the decomposition results obtained from the method of Podder (1993b). Podder (1993b) proposes a transformation of Rao's (1969) traditional decomposition by income sources to circumvent what is known as the violation of the property of uniform additions (Morduch and Sicular, 2002).[3] According to Podder (1993b), the transformation allows assessing whether the presence of an income source increases or decreases total income inequality. We show that the method, however, is generally not able to provide such an understanding. Instead, we find that the results obtained from this method are to be interpreted as the semi-elasticities of the Gini coefficient with respect to changes in the income sources.

Turning to the Gini trend decomposition by income sources, which examines the role of *changes* in income sources for the *change* in the Gini coefficient over time, we show that the method proposed by Podder and Chatterjee (2002) can lead to erroneous conclusions. For example, using Podder and Chatterjee's (2002) trend decomposition, one may conclude that a change in an income source caused total income inequality to increase, when in fact the change contributed to a more equally distributed total income. We are able to provide a method that does not admit such unwanted conclusions. In particular, we show that changes in an in-

---

[3]This violation states that an equally distributed income source will have a zero contribution to total income inequality, though adding a constant income source to the existing income distribution would lower total income inequality.

come source over time can contribute to an increase in the Gini in two different ways: first, if the distribution of that income source changes in favor of relatively rich individuals (or households); second, if the share of that income source in total income increases while the distribution of the income source is more in favor of relatively rich individuals than that of total income (or, conversely, if the income share of that source decreases while its distribution is less in favor of relatively rich individuals than that of total income). It is that latter comparison of the distribution of an income source with the distribution of total income that is missing in the approach of Podder and Chatterjee (2002). The trend decomposition of Fei et al. (1978) provides a similar intuition as to the consequences of changes in income sources as our approach. Their method, however, is less practical when more than two income sources are considered.

Discussing the Gini decomposition by population subgroups, we focus on the approach of Podder (1993a). Equivalent to Podder (1993b), Podder (1993a) proposes a transformation of the Gini subgroup decomposition of Rao (1969), thus aiming at assessing whether the presence of the income of a subgroup increases or decreases overall income inequality. Again we show that the transformation does not allow drawing conclusions on the (dis)equalizing effect of the presence of the income of population subgroups.

We next discuss the Gini trend decomposition by population subgroups that emerged from the decomposition method of Rao (1969). The trend decomposition was put forth by Chatterjee and Podder (2007) and examines the role of the change in the income of the different subgroups for the change in the Gini coefficient. We show that their method, however, can lead to highly misleading conclusions as changes in the concentration coefficients cannot be mapped unambiguously to changes in the population subgroups.[4] In contrast to the trend decomposition by income sources, the ambiguous interpretation of changes in the subgroup concentration coefficients, unfortunately, does not allow for an insightful Gini trend decomposition within Rao's (1969) framework, as changes in the Gini

---

[4]For example, one may prematurely conclude that the relative income position of a population subgroup has worsened, although the opposite is the case.

would, inter alia, be explained by changes in the concentration coefficients.

Throughout the paper, we highlight the importance of Gini elasticities to analyze the role of income sources and population subgroups for overall income inequality within Rao's (1969) decomposition framework (see Lerman and Yitzhaki, 1985; Podder, 1993a). Gini elasticities measure the impact of marginal changes in income sources (or in the income of population subgroups) on the Gini coefficient. Even though Gini elasticities thus aim at a quite different understanding than the contributions of income sources to total income inequality traditionally derived from the decomposition methods, in light of the non-interpretability result of Shorrocks (1988) and our assessment of Podder's (1993b) decomposition by income sources, we stress that Gini elasticities provide a valuable tool—or are arguably even to be preferred—to assess the importance of income sources for overall income inequality in a Gini decomposition framework.[5] Given our assessment of Podder's (1993a) Gini subgroup decomposition, we derive the same conclusion for the use of Gini elasticities within the subgroup decomposition framework of Rao (1969).[6]

Recently proposed multi-decompositions combine decompositions by income sources and population subgroups and are thus inevitable subject to Shorrocks'

---

[5]The non-interpretability result of Shorrocks (1988) states that the contribution of an income source to overall income inequality derived from decompositions of a general class of inequality indices (including the Gini coefficient) does not admit an interpretation of what could be reasonably understood under the term "contribution" (see below). Gini elasticities and elasticities of other inequality indices (see Paul, 2004), on the other hand, are by definition straightforward interpretable. We may thus extend our statement to elasticities and decompositions of this general class of inequality indices.

[6]Note that numerous other decompositions of the Gini coefficient by population subgroup have been proposed (among others Yitzhaki, 1994; Dagum, 1997) and that we view the Gini elasticity derived from Rao's (1969) decomposition not as a substitute, but as a complement to these methods, depending on the specific research question. This said, the reader may be reminded of the discussion regarding the decomposability of the Gini by population subgroups (Mookherjee and Shorrocks, 1982; Cowell, 1988; Silber, 1989; Yitzhaki and Lerman, 1991; Lambert and Aronson, 1993) due to the failure of *subgroup consistency* (see Cowell, 2000, p. 123). Although this discussion is beyond the scope of this paper, we want to note that whatever caveat may be put forth the Gini elasticity provides a clear-cut interpretation, which is enhanced by the intuitive appeal of the Gini itself, and that deriving this elasticity from the decomposition framework of Rao (1969) is straightforward.

(1988) critique. Due to the unambiguous interpretability of Gini elasticities, they provide a useful extension to multi-decomposition frameworks. We add to the literature by deriving the Gini elasticity from a multi-decomposition within Rao's (1969) framework. This elasticity gives the percentage change in the Gini coefficient due to a marginal, percentage change in the mean of an income source of a particular population subgroup. The elasticity is thus particularly suitable for analyzing how changes in income sources differentiated across different subgroups (e.g. changes in government transfers targeted at specific regions of a country) affect total income inequality. Gini elasticities in a multi-decomposition framework have also been derived by Mussard and Richard (2012). Unlike our approach, however, their decomposition is only valid for non-overlapping subgroup populations.

The remainder of the paper is structured as follows. Section 2 discusses the Gini decomposition and the corresponding trend decomposition by income sources. Section 3 discusses the Gini decomposition and Gini trend decomposition by population subgroups. Section 4 presents a simultaneous decomposition by income sources and population subgroups based on Rao (1969) and derives the Gini elasticity from this decomposition. Section 5 concludes.

2 Gini Decomposition by Income Sources

2.1 Explaining Income Inequality in Terms of Income Sources

The decomposition of the Gini coefficient by income sources was early developed by Rao (1969), followed by the contributions of Fei et al. (1978), Pyatt et al. (1980) and Lerman and Yitzhaki (1985). The objective of the decomposition is to explain total income inequality in terms of the underlying income sources. The contribution of an income source to total income inequality has been of particular interest.

Assume that individuals' (or households') total income Y is made up of $I \in \mathbb{N}$ number of components, such that $Y = \sum_i Y_i$ where Y_i is the income from source

i. The Gini can then be expressed as

$$G = \sum_i^I S_i\, C_i, \tag{1}$$

where $S_i := \mu_i/\mu$ is the mean of income source i divided by the mean of total income and C_i is the concentration coefficient (also referred to as the 'pseudo Gini') associated with income source i.[7] The concentration coefficient is defined as one minus twice the area under the concentration curve, which plots the cumulative proportions of income source i against the cumulative proportions of the population ordered ascendingly according to their total income. That is, the concentration curve makes statements like: the poorest $p\%$ of the population receive $q\%$ of income source i. Hence, it should be obvious that $C_i \in [-1, 1]$, as the concentration curve may very well lie above the diagonal of the unit square, for example, if an income source is mostly received by relatively poor households.

Regarding the contribution of an income source to total income inequality, Shorrocks (1988) establishes a very unsatisfactory impossibility result that relates to the question of how to interpret the term "contribution". He names four different concepts that can be principally understood as the contribution of an income source i to total income inequality: (a) the inequality due to income source i alone, (b) the reduction in inequality that would result if income source i would be eliminated, (c) the observed inequality if income source i would be the only source not distributed equally, and (d) the reduction in inequality that would result from eliminating the inequality in the distribution of income source i. He shows that in general no reasonable inequality index that can be expressed as $I(Y) = \sum_i \alpha_i$ (as in equation (1)) admits an interpretation of α_i in the sense of (a)-(d).[8]

---

[7]The concentration coefficient can be further decomposed into a "Gini correlation" and the Gini coefficient of income source i (see Pyatt et al., 1980; Lerman and Yitzhaki, 1985).

[8]Shorrocks (1988) provides four criteria that should be fulfilled by any reasonable inequality index, which are symmetry, the principal of transfers, the normalization restriction, and the continuity assumption (see Shorrocks (1988) for details).

Abandoning the wish of an interpretive assignment in terms of (a)-(d) to a decomposition method, one can divide equation (1) by the Gini coefficient to get

$$1 = \sum_i \frac{S_i \, C_i}{G} = \sum_i s_i, \tag{2}$$

and then to attribute the term $s_i := S_i C_i / G$ to income source i as its proportional contribution to total inequality (see e.g. Fields, 1979; Shorrocks, 1982; Silber, 1989; Achdut, 1996; Davis et al., 2010).

Shorrocks (1982) already suggested that s_i may not be a desirable measure of the proportional contribution of income source i, which was again forcefully pointed out by Podder (1993b) and Podder and Chatterjee (2002). Consider, for example, an income source which is equally distributed across households. The concentration coefficient of such an income source is zero and so its (proportional) contribution to total income inequality according to s_i. We know, however, that adding a constant to each household's income lowers total income inequality. The contribution of such an income source should thus be negative.[9] The failure of the Gini decomposition—as stated in equation (2)—in this respect is known as the violation of the *property of uniform additions* (Morduch and Sicular, 2002).

Motivated by this failure, Podder (1993b) suggests to transform equation (1) in a simple manner to get

$$0 = \sum_i S_i(C_i - G) = \sum_i \tilde{s}_i. \tag{3}$$

---

[9]One may want to argue that whether the contribution of such an income source should indeed be negative depends on the baseline of the analysis. That an equally distributed income source should have a negative contribution to total income inequality implies that the baseline is given by the status quo (with positive income inequality). In a world of equally distributed income, on the other hand, an equally distributed income source would not contribute in any direction to total income inequality. Therefore, equation (2) simply takes such a hypothetical world as the baseline of the analysis. This argument, however, is self-contradictory. To see this, note that in a world with equally distributed income adding *any* income source that is not distributed equally will increase total income inequality. Yet, an income source that is (in the status quo) mostly received by relatively poor households has a negative contribution to total income inequality according to equation (2)—a contradiction.

Although it is not possible to interpret the term $\tilde{s}_i := S_i(C_i - G)$ as the proportional contribution of source i to total income inequality, according to Podder (1993b), the sign of $C_i - G$ tells us whether the ith income source has an inequality decreasing or increasing effect on total inequality. Or, in the words of Podder (1993b): "the sign indicates if the presence of the k-th [here ith] component increases or decreases total inequality"(p. 53). That is, $C_i - G > 0$ "means that the presence of income from the ith source makes the total inequality higher than what it would be in the absence ... from that source" (Podder and Chatterjee, 2002, p. 7).

We will show that such an interpretation of equation (3) is misleading by means of a simple example. Consider a population with $n = 1, \ldots, N$ individuals, sorted ascendingly in their income, y_n. Let the richest individual N receive only, say, capital income. The rest of the population earns labor income only. Clearly, $C_i - G$ is positive for capital income suggesting—according to the above interpretation—that in the absence of capital income the Gini coefficient should be smaller. However, it can be shown that if the initial (capital) income of the richest individual satisfies

$$y_N < \frac{(\sum_{n=1}^{N-1} y_n)^2}{\sum_{n=1}^{N-1}(N-1-n)y_n}, \tag{4}$$

the absence of this income would lead to an increase in the Gini coefficient contradicting the above interpretation of equation (3).[10] The intuition is clear: eliminating an income source i which is disproportionately received by relatively rich

---

[10]Recalling the definition of the Gini coefficient

$$G = \frac{2 \sum_n n \, y_n}{N \sum_n y_n} - \frac{N+1}{N},$$

we derive this result by solving the inequality

$$\frac{\sum_n^N n y_n}{\sum_n^N y_n} < \frac{\sum_n^{N-1}(n+1)y_n}{\sum_n^{N-1} y_n}$$

for y_N.

9

individuals, i.e. where $C_i - G > 0$, leads to an increase in total inequality when the worsening in the relative position of these individuals outweighs the improvement in the relative position of the remaining population.

So far we have reminded the reader that an intuitive interpretation of the (proportional) contribution of an income source is not possible in the Gini decomposition framework and showed that the qualitative approach of Podder (1993b) is no feasible alternative—a most unsatisfactory conjuncture.[11] Yet, another possibility to assess the importance of an income source for total income inequality is given by the elasticity of an inequality index with respect to the mean of an income source, also called marginal effects. Lerman and Yitzhaki (1985) were the first to derive this expression within the Gini decomposition framework considered here. They show that the Gini elasticity is given by $\eta_i = S_i(C_i - G)/G$, which is the percentage change in the Gini coefficient due to a marginal, percentage increase in the mean of income source i (for an extension to other inequality indices see Paul, 2004)[12]. Given the non-interpretability of (proportional) contributions and given that the elasticities provide, by definition, a clear-cut assessment we are inclined to conclude that these elasticities are to be preferred when examining the role of income sources for total income inequality.[13]

Before proceeding with the next section, two last remarks regarding the decomposition approach taken by Podder (1993b) are in order. First, the mistake of Podder (1993b) and Podder and Chatterjee (2002) is to interpret—based on the sign of $C_i - G$— the importance of an income source i for total income in-

---

[11]Recent research circumvents the non-interpretability problem by deriving the *expected* contribution of an income source to total income inequality similar to the Shapley value known from cooperative game theory (see Chantreuil and Trannoy, 2011; Shorrocks, 2013).

[12]Note that the sign of η_i is determined by the sign of $C_i - G$. The Gini elasticity is thus consistent with the property of uniform additions.

[13]The importance of examining marginal effects has also been stressed by Paul (2004) and Kimhi (2011). Paul (2004) argues that policy makers can affect income sources only at the margin and that, therefore, it is more important to know how marginal changes in income sources affect total income inequality than to know the proportional contributions of income sources. Reviewing decompositions of different inequality indices by income sources, Kimhi (2011) argues that marginal effects are more robust across decompositions of different inequality indices than proportional contributions.

equality in absolute terms. The sign of $C_i - G$ remains informative about the (dis)equalizing character of an income source when considering marginal changes since $\text{sgn}(\eta_i) = \text{sgn}(C_i - G)$.[14] Second, observe that $\tilde{s}_i = \eta_i G$. Hence, Podder's (1993b) transformation (3) yields the term $\tilde{s}_i$ as the semi-elasticity of the Gini with respect to the mean of income source i. That is, $\tilde{s}_i$ is the absolute change in the Gini due to a marginal, percentage increase in the mean of income source i.

2.2 Explaining Income Inequality Trends in Terms of Income Sources

We now turn to the decomposition of inequality *trends*. That is, we want to attribute the *change* in the Gini coefficient over time to *changes* in income sources. We again restrict our attention to the framework proposed by Rao (1969).

Fei et al. (1978) are the first to study how the change in the Gini can be traced back to changes in the shares and concentration coefficients of income sources. They start out with two income sources, labor and capital income, and show that any increase in the concentration coefficients increases the Gini. Further, they show that an increase in the share of an income source has a negative effect on the Gini if the concentration coefficient is smaller than that of the other income source.

Hence, to determine the effect of a change in the share of an income source on total income inequality Fei et al. (1978) highlight the importance of comparing the relative inequality of the two sources. Clearly, such a comparison becomes less tractable when splitting income into more than two sources. In fact, with a third income source, agricultural income, they summarize wage and labor income to non-agricultural income so that the analysis of the change in the Gini can be carried out as before. Yet, changes in capital and labor income become convoluted in the change in non-agricultural income, making the analysis less and less instructive the more income sources are added.

A different approach is taken by Podder and Chatterjee (2002). They analyze changes in the Gini coefficient by differentiating equation (1) with respect to time

---

[14]In fact, reducing capital income in the above example only slightly would reduce total income inequality.

t, yielding

$$\dot{G} = \sum_i C_i \dot{S}_i + \sum_i S_i \dot{C}_i, \tag{5}$$

with $\dot{x} := \partial x / \partial t$. According to Podder and Chatterjee (2002), the term $C_i \dot{S}_i + S_i \dot{C}_i$ describes the contribution of income source i to the change in the Gini coefficient of total income, i.e. the change in the Gini that is due to the changes in the share and the concentration coefficient of the ith income source.[15] Specifically, such an interpretation implies that any increase in the share of an income source raises total inequality whenever its concentration coefficient is positive. However, this understanding contradicts the Gini elasticity η_i: a marginal increase in the share of an income source that has an equalizing effect according to the sign of $C_i - G$ should lower total income inequality.

Instead of using equation (1), we propose to base the decomposition of the change in the Gini coefficient on equation (3). This approach allows for an interpretation that is consistent with the Gini elasticity and is still instructive when more than two income sources are considered.

Differentiating equation (3) with respect to time and rearranging terms, we obtain[16]

$$\dot{G} = \sum_i (C_i - G)\dot{S}_i + \sum_i S_i \dot{C}_i, \tag{6}$$

which yields $(C_i - G)\dot{S}_i + S_i \dot{C}_i$ as the change in the Gini that is due to the change in the concentration coefficient and the income share of income source i. We see that an increase in the concentration coefficient of an income source always increases the Gini coefficient. More importantly, according to this decomposition a ceteris paribus increase in the share of an income source increases the Gini only

---

[15] See the remarks referring to equation (16), Table 5 and Table 9 in Podder and Chatterjee (2002).

[16] Note that $\dot{C}_i = \dot{R}_i G_i + R_i \dot{G}_i$ if C_i would have been decomposed into the "Gini correlation", R_i, and the Gini coefficient, G_i, of income source i, where $C_i = R_i G_i$.

if this income source has a disequalizing effect on total income inequality by the sign of $C_i - G$. Our approach is thus consistent with the Gini elasticity.[17]

3 Gini Decomposition by Population Subgroups

3.1 Explaining Income Inequality in Terms of Population Subgroups

Gini decompositions by population subgroups aim at explaining income inequality in terms of the income of different population subgroups. In this section, we draw the attention to the decomposition proposed by Podder (1993a), recently reasserted by Chatterjee and Podder (2007). Building on Rao (1969), the method is in a similar spirit as the decomposition by income sources presented in the previous section. Its focus lies on the assessment of whether the presence of income of a particular subgroup increases or decreases total income inequality. We will briefly introduce the proposed method before showing that such an assessment cannot be drawn from the decomposition.

Imagine an economy of N individuals (or households). We can collect their income in ascending order in a vector y, such that $y_1 \leq y_2 \leq \cdots \leq y_N$. Imagine further that each individual can be assigned to one and only one of $\mathcal{G} \in \mathbb{N}$ groups. We can then construct $\mathcal{G}$ vectors x_g, $g = 1, \ldots, \mathcal{G}$, with elements

$$x_{gn} = \begin{cases} y_n & \text{if and only if individual } n \text{ is a member of group } g, \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

such that $y = \sum_g x_g$. Let us denote Y as the total income of the population and X_g as the total income of group g. Equivalent to equation (1) we can write the Gini coefficient of total income as

$$G = \sum_g^{\mathcal{G}} \frac{X_g}{Y} \, C_g, \tag{8}$$

where C_g is again the concentration coefficient, but here of the gth population

---

[17]A discrete time formulation of equation (6) would be used for an empirical application.

subgroup vector x_g.[18] That is, here the concentration curve plots the cumulative proportions of vector x_g against the cumulative proportions of the total population ordered ascendingly according to their income. Again, it should be clear that $C_g \in [-1, 1]$, as the concentration curve may very well lie above the diagonal of the unit-square.

Equivalent to the decomposition by income sources, Podder (1993a) and Chatterjee and Podder (2007) infer from the sign of $C_g - G$ whether the presence of the income of group g increases or decreases total inequality: $C_g - G > 0$ (< 0) would imply that the presence of the income of group g increases (decreases) total income inequality. For the same argument as above, however, such a conclusion cannot be deduced from the sign of $C_g - G$. For example, eliminating the income of the richest group, for which $C_g - G > 0$, may very well increase the Gini by the shift of the subgroup to the bottom of the income distribution.

Again, we want to stress that, analogously to the decomposition by income sources, a straightforward assessment of the (dis)equalizing effect of the income of a specific subgroup on total income inequality is given by the Gini elasticity with respect to the mean of the population subgroup income vector.[19] Here, the elasticity is defined as the percentage change in the Gini due to a marginal, percentage increase in the mean of income vector x_g. For the decomposition of Podder (1993a) this elasticity can be derived equivalently to the Gini elasticity with respect to the mean of an income source and is given by $\eta_g = X_g(C_g - G)/YG$

---

[18] Note that a further decomposition of the concentration coefficient into a "Gini correlation" and a Gini of subgroup income vector x_g—as is often done in the case of a decomposition by income sources (see e.g. Lerman and Yitzhaki, 1985)—is not meaningful here. The Gini of a subgroup income vector should not be misinterpreted as the *within* Gini, i.e. the Gini of a subgroup. Recall that $x_{gm} = 0$ if $m \notin \mathcal{G}$, where $\mathcal{G}$ is the set of individuals belonging to subgroup g. Therefore, the Gini of income vector x_g, $G(x_g)$, will be different from zero if $\exists n \in \mathcal{G} : x_{gn} > 0$ and $\exists m \notin \mathcal{G}$. Consequently, even when income within subgroup g is equally distributed, $G(x_g)$ can be different from 0. Put differently, the Gini of the subgroup income vector x_g depends not only on the distribution of the income of subgroup g, but also on the population share of that subgroup and is thus difficult to interpret.

[19] Note that the decomposition in (8) can be viewed as a relabeling of the decomposition in (1). It follows that by rephrasing the interpretations of a contribution offered in (a)-(d) in terms of the income of a subgroup, the contributions derived from the decomposition in (8) lack the same interpretive content as their counterpart contributions from the income source decomposition.

(see Podder, 1993a; Chatterjee and Podder, 2007).[20]

We would also like to stress that—in this respect—the approach of Podder (1993a) may provide a particular advantage over decompositions of alternative inequality indices. For the decomposition offered by Podder (1993a) the elasticity is readily computed and does not depend on derivatives of, e.g., "between-group" terms or "transvariation" terms, as they would arise if one would want to derive the elasticity for indices belonging to the Generalized Entropy family[21] or for Dagum's (1997) Gini decomposition.[22]

3.2 Explaining Income Inequality Trends in Terms of Population Subgroups

Next, we want to turn to the decomposition of inequality *trends* in the context of the Gini decomposition by population subgroups of Podder (1993a). The trend decomposition attributes *changes* in the Gini coefficient to *changes* in the population subgroups.

More precisely, using equation (8), Chatterjee and Podder (2007) decompose the change in the Gini into changes in the concentration coefficients, as well as changes in population and income shares of the different subgroups. In what follows, we show that this trend decomposition by population subgroups, contrary to its counterpart decomposition by income sources from section 2.2, however, is limited in its ability to provide insightful results. In particular, we argue that this limitation is due to the inability to derive precise conclusions from changes

---

[20]Note that by transforming equation (8), analogously to the transformation in the case of the decomposition by income sources, into
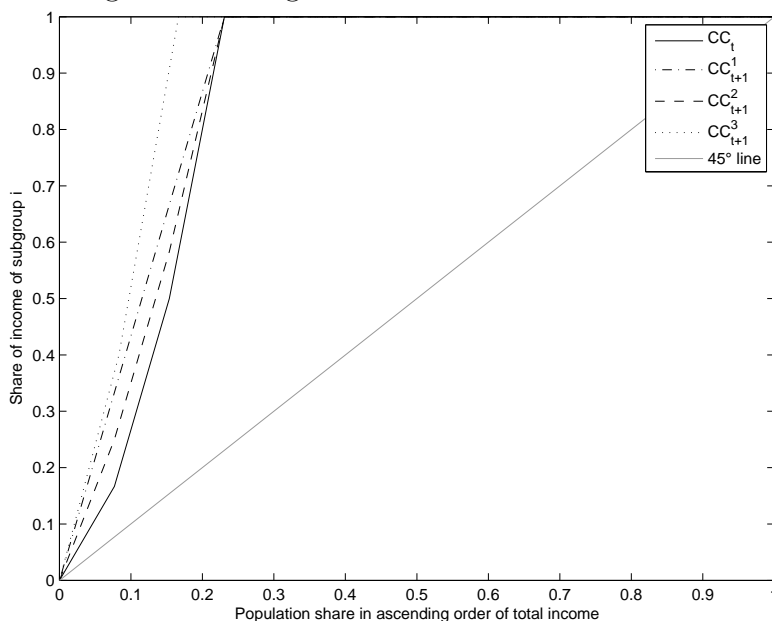
$$0 = \sum_g \frac{X_g}{Y} \left(C_g - G \right) = \sum_g \hat{s}_g \tag{9}$$

one obtains $\hat{s}_g$ as the semi-elasticity of the Gini coefficient with respect to the mean of the income of subgroup g.

[21]For a decomposition of these indices by population subgroups see Shorrocks (1984).

[22]Of course, these derivatives do not pose a disadvantage if an analysis of the interplay between different subgroups following a marginal change in the income of a particular subgroup is of interest. Yet, we are not aware of a subgroup income elasticity derived for inequality indices other than the Gini coefficient.

Figure 1: Changes in the Concentration Curve

Notes: This figure depicts Examples 1 to 3. In period t the economy's income vector is $y = (1\ 2\ 3\ 10\ \cdots\ 10)'$ with number of individuals without loss of generality set equal to $\dim(y) = 13$. The poorest three individuals belong to subgroup g, the remaining 10 individuals belong to another subgroup. CC_t plots the concentration curve of subgroup income vector x_g in period t. CC_{t+1}^1 plots the concentration curve in period $t + 1$ as described in Example 1, CC_{t+1}^2 as described in Example 2, and CC_{t+1}^3 as described in Example 3. The shift of the concentration curve to the left indicates a fall in the concentration coefficient.

in the concentration coefficients of the population subgroups. We will show this by means of three illustrative examples.

In the following analysis, let us focus on a negative concentration coefficient of an arbitrary subgroup g, which decreases from one period to the next, i.e. $C_{g,t+1} - C_{gt} < 0$ where t is a time index. At first sight it might be appealing to follow Chatterjee and Podder (2007), who interpret such a change as "indicating that the within-group distribution shifted towards the lower-income population" (p. 282), suggesting "that the incomes of more [of group g] ... were concentrated in the lower half of the income distribution for the sample as a whole" (p. 282), or simply that "the distribution worsened" (p. 282) for subgroup g. Yet, the examples show that these interpretations of a negative change in the concentration coefficient may be misguided.

16

Consider an economy where the poorest three individuals belong to subgroup g receiving income of 1, 2 and 3 units, respectively. The rest of the population belongs to a different subgroup $j \neq g$ and receives income of, say, 10 units each. Clearly, $C_g < 0$.

Example 1. Imagine that from period t to $t+1$ income within subgroup g is redistributed such that each of the three individuals now receives income of 2 units. It is apparent from Figure 1 that such a redistribution induces a fall in C_g. However, one can hardly interpret such a redistribution as a worsening in the distribution of the income of subgroup g.
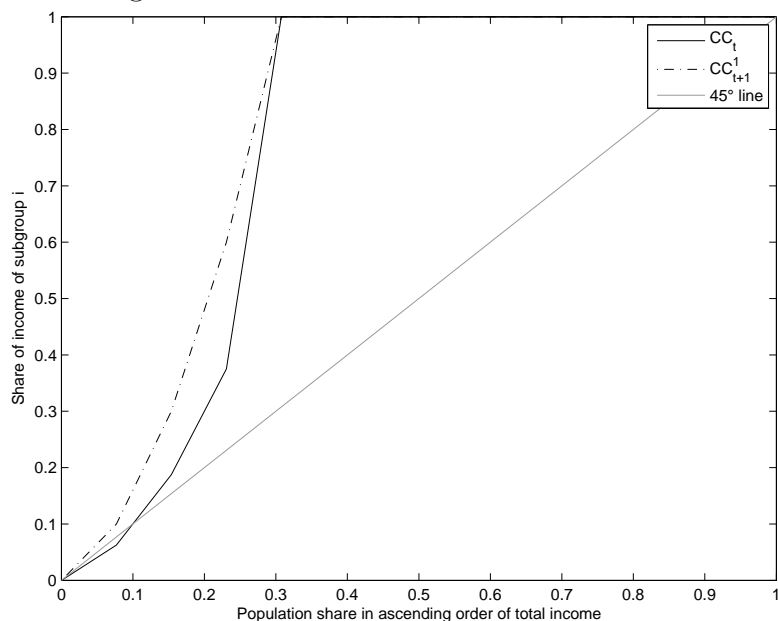
Example 2. Now imagine instead that each of the individuals of subgroup g receives 2 additional units of income. Figure 1 illustrates that such a change leads to a decrease in the concentration coefficient. Yet again, this decrease in the concentration coefficient does not admit any of the interpretations offered by Chatterjee and Podder (2007).

Example 3. Imagine that the poorest individual dies between t and $t+1$. Subgroup g, thus, reduces in size to two individuals. Figure 1 shows that such a change in demography again leads to a decrease in the concentration coefficient of subgroup g. However, it would be mistaken to state that the incomes of more of subgroup g were concentrated in the lower half of the income distribution.

It is, of course, correct that a change that leads to more individuals of a subgroup being concentrated in the lower half of the income distribution, as described by Chatterjee and Podder (2007), implies a decrease in the concentration coefficient. To see this, consider again the economy described above, but imagine that a fourth individual receiving an income of 10 units belongs to subgroup g. Since the rest of the population is larger than subgroup g and since each of the individuals not belonging to group g receives an income of 10 units, the median income is 10. Figure 2 shows that reducing the income of the fourth member of group g from 10 to 4 units—which implies that more of the individuals in group g receive an income left of the median—reduces the concentration coefficient.

However, our illustrative examples made it abundantly clear that there are

17

Figure 2: Change in the Concentration Curve due to a Fall in Income

Notes: In period t the economy's income vector is $y = (1\ 2\ 3\ 10\ \cdots\ 10)'$ with number of individuals without loss of generality set equal to $\dim(y) = 13$. The poorest four individuals belong to subgroup g, the remaining 9 individuals belong to another subgroup. CC_t plots the concentration curve of subgroup income vector x_g in period t. CC_{t+1} plots the concentration curve in period $t+1$ where the income of the 4th individual falls from 10 to 4 units. The shift of the concentration curve to the left indicates a fall in the concentration coefficient.

several other possible changes within the subgroup that can account for the same effect, which, however, are incompatible with the interpretation of Chatterjee and Podder (2007). It is easy to think of many other, more complex, changes within a subgroup g, and even of changes in other subgroups $j \neq g$, that can account for the same change in the concentration coefficient C_g. It is thus difficult to relate changes in the Gini coefficient to underlying changes in the population subgroups and, therefore, to derive policy relevant conclusions from the Gini *trend* decomposition of Chatterjee and Podder (2007). We want to stress, however, that the usefulness of the Gini elasticity referred to above is not affected by this result as it is obtained by increasing the income of the respective population subgroup proportionally such that the concentration coefficients do not change.

18

4 Multi-dimensional Gini elasticity

Recent contributions in the inequality literature focus on a combination of decompositions by income sources and population subgroups, called multi-decomposition (Mussard, 2004; Mussard and Richard, 2012; Mussard and Savard, 2012; Mussini, 2013). By merging a decomposition by population subgroups with a decomposition by income sources, however, any such approach is inevitable subject to Shorrocks' (1988) non-interpretability critique.[23]

Considering the clear interpretation of the Gini elasticity with respect to income sources or population subgroups presented in the previous sections, the purpose of this section is to extend the single-dimension Gini elasticities to the framework of multi-decompositions. Such an extension provides a tool for assessing the effect of a marginal, proportional change in an income source of a specific population subgroup on the Gini coefficient, e.g., for analyzing the distributional effect of a tax reform in different regions of a country.

We start from Rao's (1969) decomposition by population subgroups which we restate here,[24]

$$G = \sum_g^{\mathcal{G}} \frac{X_g}{Y} \, C_g.$$

The same way total income can be expressed as a sum of income sources, each subgroup income vector x_g can be rewritten as $x_g = \sum_i x_g^{(i)}$, where $x_g^{(i)}$ denotes subgroup g's income vector of income source i. Using the covariance definition of the concentration coefficient, we can rewrite equation (8) as

$$G = \sum_g^{\mathcal{G}} \frac{X_g}{Y} \frac{2 \operatorname{cov}(\sum_i^I x_g^{(i)}, F(y))}{X_g/N}, \tag{10}$$

where $F(\cdot)$ denotes the cumulative density function over total income y. Rear-

---

[23]For example, Mussini (2013) merges Dagum's (1997) Gini decomposition by population subgroups with Rao's (1969) decomposition by income sources.

[24]All variables follow their definitions from the previous sections.

ranging terms, we obtain

$$G = \sum_{g,i} \frac{N_g}{N} \frac{\mu_g^{(i)}}{\mu} C_g^{(i)}, \tag{11}$$

where $\sum_{g,i}$ denotes summation over all ordered pairs $\langle g, i \rangle \in \{1, \ldots, \mathcal{G}\} \times \{1, \ldots, I\}$, N_g denotes the number units (e.g. households) belonging to subgroup g, $\mu_g^{(i)}$ denotes the mean of income source i in subgroup g, and $C_g^{(i)}$ denotes the concentration coefficient of income source i in subgroup g.[25]

From the multi-dimensional decomposition in (11) we can easily derive the Gini elasticity with respect to the mean of income source i in subgroup g defined as

$$\eta_g^{(i)} := \frac{\partial G}{\partial \mu_g^{(i)}} \frac{\mu_g^{(i)}}{G}, \tag{12}$$

which gives the percentage change in the Gini due to a marginal, percentage increase in $\mu_g^{(i)}$.

We first derive the partial derivative of G with respect to $\mu_g^{(i)}$. It is important to note that the change in $\mu_g^{(i)}$ is a proportional change in the income vector $x_g^{(i)}$ such that the concentration coefficient $C_g^{(i)}$ is unaltered $\forall\, g$. The derivative is then given by

$$\frac{\partial G}{\partial \mu_g^{(i)}} = -\sum_{-g,-i} \frac{N_g}{N} \frac{\mu_{-g}^{(-i)}}{\mu^2} \frac{\partial \mu}{\partial \mu_g^{(i)}} C_{-g}^{(-i)} + \frac{N_g}{N} \frac{\mu - \frac{\partial \mu}{\partial \mu_g^{(i)}} \mu_g^{(i)}}{\mu^2} C_g^{(i)}, \tag{13}$$

where $\sum_{-g,-i}$ denotes summation over all pairs in $(\{1, \ldots, \mathcal{G}\} \times \{1, \ldots, I\}) \setminus \{\langle g, i \rangle\}$. Rewriting μ as $\mu = \sum_{g,i} N_g \mu_g^{(i)}/N$, the derivative $\partial \mu / \partial \mu_g^{(i)}$ is given by

$$\frac{\partial \mu}{\partial \mu_g^{(i)}} = \frac{N_g}{N}. \tag{14}$$

---

[25]Note that for (11) to be defined we require that $\forall\, g, i \quad \exists\, n : x_{gn}^{(i)} > 0$.

20

Inserting (14) into (13) and rearranging terms, we get

$$\frac{\partial G}{\partial \mu_g^{(i)}} = \frac{N_g}{N} \frac{1}{\mu} (C_g^{(i)} - G).$$ (15)

Multiplying this expression by $\mu_g^{(i)}/G$, yields the Gini elasticity (12)

$$\eta_g^{(i)} = \frac{N_g}{N} \frac{\mu_g^{(i)}}{\mu} \frac{(C_g^{(i)} - G)}{G}.$$ (16)

We see that combining Rao's (1969) Gini decomposition by income sources and population subgroups provides a straightforward approach to analyze the effect of a marginal, percentage change in the income of a particular source in a specific subgroup on total income inequality.[26] Similar to the single-dimensional elasticities, a marginal increase in the mean of income source i in subgroup g decreases the Gini if this income source is more favorably distributed for that subgroup than total income.

5 Conclusion

This paper closely examined Gini decompositions by income sources and population subgroups within the well-known framework of Rao (1969). We showed that the methods put forth by Podder (1993b) and Podder (1993a) to analyse the role of income sources and (the income of) population subgroups, respectively, for total income inequality do not admit the interpretation intended by

---

[26]Integrating the multi-elasticity over either subgroups or income sources will bring us back to the single dimension Gini elasticities from the previous sections,

$$\sum_g \eta_g^{(i)} = \eta_i$$

$$\sum_i \eta_g^{(i)} = \eta_g.$$

Equivalent to the single-dimension Gini decompositions, manipulating the multi-decomposition in equation (11) in the same manner as equation (3) yields a summation over $\eta_g^{(i)} G$, i.e. over multi-dimensional semi-elasticities.

the authors. Furthermore, we showed that the method of Podder and Chatterjee (2002) to decompose the change in the Gini by income sources is at odds with the Gini elasticity, thus leading to erroneous conclusions. We were able to provide a trend decomposition consistent with the Gini elasticity. With respect to the contribution by Chatterjee and Podder (2007), we showed that the ambiguous interpretation of changes in the concentration coefficients does not allow for an insightful Gini trend decomposition by population subgroups within the framework of Rao (1969).

Throughout the paper, we highlighted the importance of Gini elasticities as a valuable tool for analyzing the (dis)equalizing character of income sources or (the income of) population subgroups, and in particular for evaluating the effectiveness of different tax and transfer policies to affect overall income inequality. We contributed to the recent trend of multi-decompositions by deriving the Gini elasticity with respect to an income source of a population subgroup from a simultaneous decomposition of the Gini coefficient by income sources and population subgroups. This facilitates the analysis of the distributional effect of changes in income sources in different population subgroups induced by, e.g., tax reforms aimed at different regions of a country.

References

Achdut, L., 1996. Income Inequality, Income Composition and Macroeconomic Trends: Israel, 1979-93. Economica 63 (250), S1–S27.

Bhattacharya, N., Mahalanobis, B., 1967. Regional Disparities in Household Consumption in India. Journal of the American Statistical Association 62 (317), 143–161.

Chantreuil, F., Trannoy, A., 2011. Inequality decomposition values. Annals of Economics and Statistics/Annales d'Économie et de Statistique 101/102, 13–36.

Chatterjee, S., Podder, N., 2007. Some Ethnic Dimensions of Income Distribution from Pre- to Post-reform New Zealand, 1984-1998*. Economic Record 83 (262), 275–286.

Cowell, F. A., 1988. Inequality decomposition: Three bad measures. Bulletin of Economic Research 40 (4), 309–312.

Cowell, F. A., 2000. Measurement of Inequality. In A. B. Atkinson and F. Bourguignon (Eds.), Handbook of Income Distribution, Chapter 2, 87–166, Amsterdam: North Holland.

Dagum, C., 1997. A New Approach to the Decomposition of the Gini Income Inequality Ratio. Empirical Economics 22 (4), 515–531.

Davis, B., Winters, P., Carletto, G., Covarrubias, K., Quiñones, E. J., Zezza, A., Stamoulis, K., Azzarri, C., DiGiuseppe, S., 2010. A cross-country comparison of rural income generating activities. World Development 38 (1), 48–63.

Fei, J. C. H., Ranis, G., Kuo, S. W. Y., 1978. Growth and the family distribution of income by factor components. The Quarterly Journal of Economics 92 (1), 17–53.

Fields, G. S., 1979. Income inequality in urban colombia: A decomposition analysis. Review of Income and Wealth 25 (3), 327–341.

Kimhi, A., 2011. Comment: On the Interpretation (and Misinterpretation) of Inequality Decompositions by Income Sources. World Development 39 (10), 1888–1890.

Lambert, P. J., Aronson, J. R., 1993. Inequality Decomposition Analysis and the Gini Coefficient Revisited. The Economic Journal 103 (420), 1221–1227.

Lerman, R. I., Yitzhaki, S., 1985. Income Inequality Effects by Income Source: A New Approach and Applications to the United States. The Review of Economics and Statistics 67 (1), 151–156.

Mookherjee, D., Shorrocks, A., 1982. A Decomposition Analysis of the Trend in UK Income Inequality. The Economic Journal 92 (368), 886–902.

Morduch, J., Sicular, T., 2002. Rethinking inequality decomposition, with evidence from rural China. The Economic Journal 112 (476), 93–106.

Mussard, S., 2004. The bidimensional decomposition of the Gini ratio. A case study: Italy. Applied Economics Letters 11 (8), 503–505.

Mussard, S., Richard, P., 2012. Linking Yitzhaki's and Dagum's Gini decompositions. Applied Economics 44 (23), 2997–3010.

Mussard, S., Savard, L., 2012. The Gini multi-decomposition and the role of Gini's transvariation: application to partial trade liberalization in the Philippines. Applied Economics 44 (10), 1235–1249.

Mussini, M., 2013. A matrix approach to the Gini index decomposition by subgroup and by income source. Applied Economics 45 (17), 2457–2468.

Paul, S., 2004. Income sources effects on inequality. Journal of Development Economics 73 (1), 435–451.

Podder, N., 1993a. A New Decomposition of the Gini Coefficient among Groups and Its Interpretations with Applications to Australia. Sankhyā: The Indian Journal of Statistics, Series B (1960-2002) 55 (2), 262–271.

Podder, N., 1993b. The disaggregation of the gini coefficient by factor components and its applications to Australia. Review of Income and Wealth 39 (1), 51–61.

Podder, N., Chatterjee, S., 2002. Sharing the national cake in post reform New Zealand: income inequality trends in terms of income sources. Journal of Public Economics 86 (1), 1–27.

Pyatt, G., Chen, C., Fei, J., 1980. The Distribution of Income by Factor Components. The Quarterly Journal of Economics 95 (3), 451–473.

Rao, V. M., 1969. Two Decompositions of Concentration Ratio. Journal of the Royal Statistical Society. Series A (General) 132 (3), 418–425.

Shorrocks, A. F., 1982. Inequality Decomposition by Factor Components. Econometrica 50 (1), 193–211.

Shorrocks, A. F., 1984. Inequality decomposition by population subgroups. Econometrica 52 (6), 1369–1385.

Shorrocks, A. F., 1988. Aggregation issues in inequality measures. in Eichhorn, W. (ed.), Measurement in Economics Physica-Verlag, 429–451.

Shorrocks, A. F., 2013. Decomposition procedures for distributional analysis: a unified framework based on the Shapley value. The Journal of Economic Inequality 11 (1), 99–126.

Silber, J., 1989. Factor Components, Population Subgroups and the Computation of the Gini Index of Inequality. The Review of Economics and Statistics 71 (1), 107–115.

Yitzhaki, S., 1994. Economic distance and overlapping of distributions. Journal of Econometrics 61 (1), 147–159.

Yitzhaki, S., Lerman, R. I., 1991. Income stratification and income inequality. Review of income and wealth 37 (3), 313–329.

Diskussionsbeiträge - Fachbereich Wirtschaftswissenschaft - Freie Universität Berlin
Discussion Paper - School of Business and Economics - Freie Universität Berlin

2014 bereits erschienen:

2014/1   FOSSEN, Frank M.; Ronny FREIER und Thorsten MARTIN
        Race to the debt trap? : Spatial econometric evidence on debt in German
        municipalities
        Economics

2014/2   SCHREIBER, Sven
        Anticipating business-cycle turning points in real time using density forecasts
        from a VAR
        Economics

2014/3   FOSSEN, Frank M. und Daniela GLOCKER
        Stated and revealed heterogeneous risk preferences in educational choice
        Economics

2014/4   KNABE, Andreas, Ronnie SCHÖB und Marcel THUM
        Der flächendeckende Mindestlohn
        Economics

2014/5   QUINT, Dominic und Pau RABANAL
        Monetary and Macroprudential Policy in an Estimated DSGE Model of the Euro Area
        Economics

2014/6   HUGHES HALLETT, Andrew; Ansgar RANNENBERG und Sven Schreiber
        New Keynesian versus old Keynesian government spending multipliers:
        a comment
        Economics

2014/7   DOBUSCH, Leonhard
        Digitale Zivilgesellschaft in Deutschland : Stand und Perspektiven 2014
        Management

2014/8   BEZNOSKA, Martin
        Estimating a Consumer Demand System of Energy, Mobility and Leisure :
        a Microdata Approach for Germany
        Economics

2014/9   GÖRLITZ, Katja und Sylvi RZEPKA
        Does Regional Training Supply Determine Employees' Training Participation?
        Economics

2014/10   BRANDSTETTER, Laura
        Do Corporate Tax Cuts Reduce International Profit Shifting?
        FACTS

2014/11   HETSCHKO, Clemens; Andreas KNABE und Ronnie SCHÖB
           Looking Back in Anger? Retirement and Unemployment Scarring
           Economics

2014/12   HEIMSTÄDT, Maximilian; Frederic SAUNDERSON und Tom HEATH
           Conceptualizing Open Data Ecosystems: a timeline analysis of Open Data
           development in the UK
           Management

2014/13   QUINT, Dominic
           Is it Really More Dispersed?: Measuring and Comparing the Stress from the
           Common Monetary Policy in the Euro Area
           Economics

2014/14   SCHREYÖGG, Georg und Simone M. OSTERMANN
           Managing uncertainty in Intensive Care Units: Exploring formal and informal coping
           practices in a university hospital
           Management

2014/15   LÜTHEN, Holger
           Rates of Return and Early Retirement Disincentives: Evidence from a German
           Pension Reform
           Economics

2014/16   MEISSNER, Thomas und Davud ROSTAM-AFSCHAR
           Do Tax Cuts Increase Consumption? An Experimental Test of Ricardian Equivalence
           Economics

2014/17   EICHFELDER, Sebastian und Mona LAU
           Capital Gains Taces and Asset Prices: The Impact of Tax Awareness and
           Procrastination
           FACTS

2014/18   BLAUFUS, Kay; Matthias BRAUNE; Jochen HUNDSDOERFER und Martin JACOB
           Self-Serving Bias and Tax Morale
           FACTS

2014/19   AMBROSIUS, Christian und Alfredo Cuecuecha
           Do Remittances Increase Borrowing?
           Economics

2014/20   EVERS, Andrea und Jochen Hundsdoerfer
           Country-by-Country Reporting: Eine neue Rechnungslegung über länderspezifische
           Wertschöpfung und Ertragsteuern?
           FACTS

2014/21   BÖNKE, Timm und Guido NEIDHÖFER
           Parental background matters: Intergenerational mobility and assimilation of Italian
           immigrants in Germany
           Economics