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Abstract

We study road congestion as a mechanism design problem. In our basic model we analyze the allocation of a set of drivers among two roads, one of which may be congested. An additional driver on the congestible road imposes an externality on the other drivers by increasing their travel time. Each driver is privately informed about her value of time and asked to report that value to the mechanism designer, who assigns drivers to roads. With a finite number of drivers, there is aggregate uncertainty and the efficient allocation is ex ante unknown. Setting a single Pigouvian price is then not optimal. However, the efficient allocation is implementable by a Vickrey-Clarke-Groves price schedule that lets each driver pay the externality she imposes on other drivers. This allows drivers to pay to have other drivers use the slow road instead of the congestible road. As the number of drivers becomes large, there is a single optimal Pigouvian price that leads to an efficient allocation. However, finding this price requires the mechanism designer to either know the precise distribution of the value of time or the use of our mechanism. We analyze some extensions and apply our model to various congestion problems arising in other contexts.

JEL classification: D82, D62, R48, R41
Keywords: Mechanism Design, Congestion Pricing, VCG Mechanism, Externalities, Value of Time

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1 Introduction

Congestion is an ever-present nuisance to many people living in the world's urban areas. There is a substantial literature on the pricing of roads to reduce congestion. As more cars drive on a road, the lower the overall rate of flow will be. Each driver will slow down other drivers on the same road, but will not take this effect into account when deciding which road to use. Therefore in the absence of government intervention there will be excess use of some roads, leading to welfare losses. Economists have long recognized the externality inherent in road transport and have suggested charging road users a price for using the road. This price or congestion charge should be set at a level to ensure that each driver faces the marginal value of the increased travel time of other drivers on the same road.

The existing literature on congestion pricing is primarily concerned with finding the right level of such a congestion charge. This is often linked to the idea of Pigouvian taxation, in which problems resulting from externalities are solved by charging a price that reflects the externality. In practice the economists' suggestion requires the government to have an estimate of the value of time in travel. Note however that one would expect the value of travel time to differ substantially across different drivers. For example, an ambulance responding to an emergency will have a higher value for travel time than other road users. Small et al. (2005) as well as Steimetz and Brownstone (2005) have empirically demonstrated the existence of substantial heterogeneity in the value of time both from observed and unobserved sources.

Given the importance of the value of time for optimally setting congestion charges, the main issue for a successful implementation of a Pigouvian congestion charge is the availability of information about the value of time. To analyze the implications of the lack of detailed knowledge on the part of the government concerning the value of time, we reformulate this classical problem into a mechanism design problem. The theory of mechanism design is particularly well-suited to studying problems in which the government lacks important information, which is privately held by other agents and important in making a decision. Previous models of congestion pricing focused on cases in which the efficient allocation was known by the regulator. With finitely many drivers and privately known values of time, the efficient allocation is not known ex ante. Hence finding the optimal regulatory policy requires the use of a mechanism design approach. The literature on mechanism design has already provided many insights on problems of interest, such as the sale of items (Myerson, 1981) or the provision of public goods (Clarke, 1971; Montero, 2008), in which the agents have private information on their value for the item, their value of public good provision or about their cost of emissions reduction. Our paper introduces congestion pricing as a novel application of the theory of mechanism design.

In this paper each person using a road is assumed to have private information regarding their value of time. We consider a simple Origin-Destination model in which travelers can make use of one of two roads, one of which may potentially be congested. The mechanism designer, who can be seen as either a local government authority or a private company administering the roads, asks each driver to report their value of time and then allocates drivers to different roads to maximize her objective function, which we take to be either to maximize total surplus or revenues. If the mechanism is designed to elicit truthful reports of the value of time, then there is no need to estimate it. In the past this procedure might have seemed impractical as there was no way to easily communicate to a central decision-maker information concerning the value of time. There was also no way for the decision-maker to force drivers to take particular routes. Modern communication technology, i.e. smart phones and GPS, and the advent of self-driving cars imply that these practical problems may be overcome in the near future. When travelers use a self-driving car it becomes conceivable that travelers simply indicate their desired destination and their desire for reaching it on time. Provided with that information, a self-driving car would electronically transmit this information to a central authority, which calculates an efficient travel schedule for the self-driving cars.

We characterize the efficient allocation in this model. For a finite number of drivers, the optimal usage of each road depends on the realized values of time of each driver. If the value of time for one driver becomes very large, the efficient allocation will be to have only that driver use the congestible road. In practice, this could arise in the case of ambulances. If an ambulance reaches the site of an accident too late, there might be loss of life, while if other drivers are slightly delayed, their dinner might get cold.

To implement the efficient allocation, traditionally a single price that internalizes drivers' externality on each other is recommended. We show that this solution is not sufficient. Since the efficient allocation varies for different realizations of the values of travel time, a simple Pigouvian price cannot adjust allocations accordingly. Instead, we recommend a more flexible procedure: Given the reports the mechanism designer computes the efficient allocation of drivers to roads and sets payments in such a way that each driver pays his externality on the other drivers to ensure that drivers report their value of time truthfully. This is in line with the well-known Vickrey-Clarke-Groves (VCG) pivot mechanism. The externality of a driver on the others arises from replacing other drivers from the congested road and from changing the travel time on the congested road. The price schedule implementing the efficient allocation will feature transfer schedules for each driver that feature a finite number of upward jumps. At each jump the number of drivers on the congested road changes discontinuously. Holding others' reports fixed, a driver's allocated travel time is decreasing in the driver's value of time. The following example illustrates such a price scheme for a special case.

A Simple Example We consider a problem in which only the value of time of one driver is not known by the mechanism designer. There are five drivers $i = 0, 1, 2, 3, 4$ traveling from a common origin to a common destination at the same time. There are two roads that the drivers can take. It takes a driver half a minute to travel from the origin to the destination on road A if there are no other drivers. For each additional driver on road A, the average travel time for all drivers on that road increases by half a minute. On Road B it takes 2.5 minutes to travel from the origin to the destination irrespective of the number of drivers using it. It is common knowledge that the value of time is given by $\theta_i = 11 - i$ for $i \in \{1, 2, 3, 4\}$. The valuation of time of driver $i = 0$, given by $\theta_0 \in (0, \infty)$ is private information. The utility of a driver traveling from the origin to the destination in t minutes and paying a price of p is given by: $U_i = v - \theta_i t - p$. Let $\mathcal{A}^*(\theta_0)$ be the set of drivers on road A at the efficient allocation. This is given by:

$$\mathcal{A}^*(\theta_0) = \begin{cases} \{1,2\} & \theta_0 < 8.\overline{63} \\ \{0,1,2\} & 8.\overline{63} \leqslant \theta_0 < 9.8 \\ \{0,1\} & 9.8 \leqslant \theta_0 < 32 \\ \{0\} & 32 \leqslant \theta_0 \end{cases}$$

The efficient allocation depends on the value of θ_0. Therefore there is aggregate uncertainty over the efficient allocation ex ante. Knowing θ_0 the mechanism designer could set a single price as a function of θ_0 such that drivers use roads efficiently. When the mechanism designer does not know θ_0, a mechanism which sets the price as a function of the report of θ_0 gives driver 0 an incentive to lie about her value of time. However the efficient allocation can be implemented by letting driver 0 face the following payment schedule for a travel time $t \in \{0.5, 1, 1.5, 2.5\}$:

$$P^*(t) = \begin{cases} 0 & t = 2.5 \\ 9.5 & t = 1.5 \\ 14.4 & t = 1 \\ 30.4 & t = 0.5 \end{cases}$$

The difference of this payment schedule to setting a single price is that it allows to charge driver 0 different prices for different travel times, while a single price mechanism only charges for use of the fast road irrespective of the number of other drivers on the fast road. This payment schedule is constructed in a way that the prices faced by driver 0 capture the externalities this driver imposes on the other drivers. When the value of time is low, driver 0 is allocated to road B. For high enough values of θ_0, driver 0 is allocated to road A which will increase the travel time for drivers 1 and 2. The price of 9.5 reflects that increased travel time for those drivers. As the value of θ_0 increases it becomes efficient for only drivers 0 and 1 to be using road A. The presence of driver 0 thus makes driver 2 switch from the fast road A to the slower road B. The price of 14.4 reflects the cost of that change to driver 2. As θ_0 increases even more it becomes efficient for only driver 0 to use road A. To give driver 0 an incentive not to always claim to have such a high value of time, the payment is accordingly high.

In the absence of such a pricing schedule and a high realization of θ_0 it is possible that driver 0 could sign a contract with the other drivers and pay them not to use the road prone to congestion. This would be the classic Coasian solution to the externality problem (Coase, 1960). In practice, such contracts would not exist due to the high transaction cost for bilateral contractual agreements; however, our mechanisms implicitly implements these kinds of Coasian contracts.

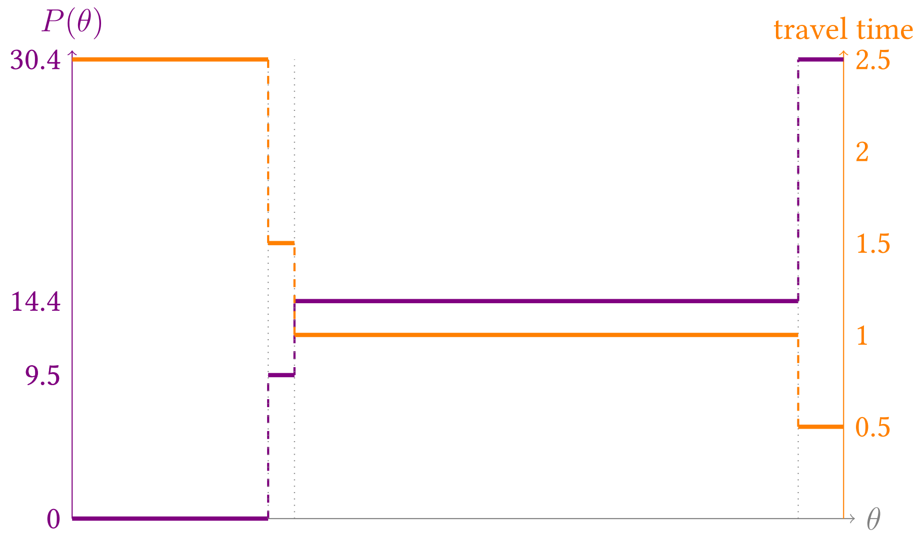

Figure 1.1: Efficient Allocation and Pricing

To bridge the gap between the traditional Pigouvian results and our approach, we study the limit case as the number of drivers approaches infinity. in this case aggregate uncertainty vanishes and there is an optimal Pigouvian congestion charge. Pricing in that limit case only needs to determine for each driver which road that driver uses, but not what the overall efficient level of road usage is.

Even if there is a large number of drivers, there may be aggregate uncertainty, for example when all drivers' value of time depends on an aggregate shock. In that case the optimal Pigouvian congestion charge will depend on the realization of the shock. Not adjusting the value of the Pigouvian congestion charge does not implement the efficient allocation.

When the objective of the mechanism designer is to maximize revenues, the problem is similar to the one of maximizing total surplus, except thats the true values of time are replaced by virtual values. Virtual values depend on the prior beliefs the mechanism designer has of the distribution of the value of time. We further consider a modified problem with two identical, congestible roads. The basic framework of our analysis continues to hold.

The following subsection 1.1 discusses previous papers on congestion pricing and mechanism design problems with externalities. Section 2 introduces our basic model and solves for the efficient allocation. Section 3 derives a payment schedule that implements the efficient allocation and considers the limit case when the number of drivers becomes large. Section 4 studies the case when the mechanism designer maximizes revenue rather than welfare. Section 5 extends our results to the case when there are two congestible roads. Section 6 discusses currently used congestion pricing schemes in the light of our results. Section 7 considers congestion problems arising in the internet, access to high-speed data links and online keyword auctions. Section 8 concludes.

1.1 Literature

The idea to use prices to implement efficient road usage dates back to Pigou (1920) and Knight (1924) and later gained popularity among economists. Vickrey and Sharp (Vickrey and Sharp, 1968; Vickrey, 1969) are still regarded as the founding fathers of transport economic theory (Verhoef, 2000). The unifying idea of this literature, the *Pigouvian approach*, is to implement efficient road usage by internalizing the social cost of congestion via a tax or price regulation: the price surcharge of road usage is set to equate the marginal social cost at the efficient level.

Subsequent empirical and theoretical research has identified several problems with this approach, namely *information requirements* and the users' *heterogeneity* in their value of travel time. It has been shown that the value of travel time varies in the course of the day and hence also the demand for road usage; moreover, there is considerable heterogeneity across users (for an overview, see Small, 2012). Small et al. (2005), for instance, estimate the distribution of the value of time, using data on the usage of so-called pay lanes. Commuters had the choice between using either a standard lane or a high occupancy tolled lane, which is available only for vehicles carrying more than one person or for drivers carrying a transponder and paying a toll. That study finds a median value of time of around \$23 with substantial heterogeneity unrelated to observable factors. Steimetz and Brownstone (2005) uses commuters' choices on the California Interstate 15 north of San Diego to characterize the heterogeneity in the value of time by observable characteristics. They find that while the mean value of time is \$30 per hour, this value ranges between \$7 and \$65 per hour.

Several theoretical papers analyze the congestion pricing problem assuming a commonly known value of travel time, identical across all drivers. For example, Bernstein and El Sanhouri (1994) and Verhoef et al. (1996) analyze the problem of optimally setting congestion charges in a network with two roads, in which only one of the roads can be tolled. The value of travel time is implicitly normalized to unity for all drivers. The heterogeneity present in those papers concerns mainly the overall value of a trip. In our paper, in contrast, the heterogeneity of drivers concerns the value of travel time and assumes that the value of a trip is such that using a road is efficient for all drivers. We thus focus on finding which road is used by the drivers, rather than whether to drive or not.

Another strand of the literature has studied optimal congestion pricing when there is heterogeneity in the value of time. Closest to our paper is Mayet and Hansen (2000), who also consider a model in which there are two roads, only one of which may be congested. Like in our model the heterogeneity of drivers concerns valuation of travel time, rather than the value of a trip. However they restrict the regulator to setting a single toll for using the congestible road. Their model is similar to the limit case of our model when there is no aggregate uncertainty. Therefore in their model the restriction to a single price does not harm welfare. Small and Yan (2001) consider a model in which there are only two types of drivers, one with a high value of time and another with a low value of time. They highlight that because of the heterogeneity, there is some welfare gain from having roads with different travel times, as drivers with a high value of time will be willing to pay more to reach their destination faster. Verhoef and Small (2004) also compare the social optimum to the congestion charges chosen by a private, profit-maximizing road operator. Arnott et al. (1994) analyze the choice of an optimal time-varying toll in a model with a heterogeneous value of travel time and random departure times. The number of drivers of each type in this model is known *ex ante*. Arnott and Kraus (1998) distinguish between anonymous and non-anonymous congestion charges and investigate under which conditions an anonymous congestion charge is optimal, when drivers can have varying values of time. Unlike our model, the departure times may vary across drivers.

One feature of those papers is that it is usually assumed that there is a continuum of drivers, with time valuations distributed according to a (continuous) cumulative distribution function. As a result, there is no aggregate uncertainty over the number of drivers on each road and over the optimal level of road usage. When setting a fixed congestion charge the mass of drivers using a road is perfectly determined, so determining the optimal number of drivers on each road is unproblematic in the previous papers. There is thus no role for the optimal pricing scheme in eliciting information on what the optimal level of road usage is. The mechanism used then only determines which driver uses which road. In contrast, in our mechanism the reports by the drivers will also determine the optimal number of drivers on each road. In the limit of our model, aggregate uncertainty disappears so that we also recover the optimality of a single congestion charge. In general this single congestion charge (for each observable type) is not optimal. This point has not been explicitly recognized in the earlier literature.

Methodically, we draw from the litertaure on mechanism design. Many of these papers focus on auctions in which buyers have private values for the items to be sold or look at the optimal provision of public goods such as Vickrey (1961), Clarke (1971), Groves (1973) or Myerson (1981). Jehiel et al. (1996) study a single unit auction in which a buyer is privately informed about the payoff received by other buyers when she is assigned the item. Jehiel et al. (1999) study a similar single unit auction in which a buyer has private information about her own payoff from owning the item as well as from others owning the item. The paper is therefore closer to ours, in the sense that a driver in our setting has private information about his valuation when another driver is added to a road. The main difference of our paper is that in our case private information is unidimensional, rather than multidimensional. This significantly reduces the difficulty of finding efficient and incentive compatible mechanisms. VCG-type mechansims have also been used to study efficient solutions to environmental externalities (Montero, 2008). He looks at the problem of emissions abatement where polluters are privately informed about their cost of abatement. Traditionally proposed solutions, such as a tax on emissions or an emissions trading scheme are not efficient mechanisms in this context. Montero (2008) proposes instead a VCG-type mechanism to give polluters an incentive to report their cost of abatement truthfully and to implement the efficient level of abatement.

2 Model

There are n drivers that simultaneously want to reach some destination D, starting from a common starting point O. To do so, they can take one of two roads $s \in \{A, B\}$. There is a congestion problem on road A but none on road B. The travel time for each driver on road B is $t^B = \bar{t}$ and $t^A = C(k)$ on road A, where k denotes the number of travelers on road A and $C(\cdot)$ is weakly increasing with weakly increasing differences, i.e. $C(k+1) \geqslant C(k)$ for all k and

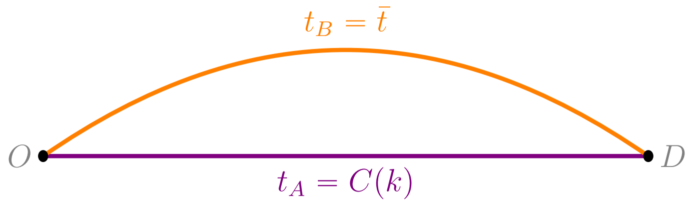

Figure 2.1: Case with a congestible and an uncongestible road

$C(k+1) - C(k) \geqslant C(k'+1) - C(k')$ for all $k > k'$. To make the congestion problem interesting, we assume that an uncongested road A is faster than road B: $C(1) < \bar{t}$.

We assume that drivers have private information on their value of time, represented by $\theta_i \in \Theta_i \subseteq \mathbb{R}_+ \backslash \{0\}$ for driver i. For some results we assume additionally that all θ_i are independently and identically distributed according to the well-behaved cumulative distribution function $F(\theta_i)$[1]. We let $\theta \in \Theta \equiv \times_i \Theta_i$ be the n-dimensional vector of all drivers' valuation of time. We assume that for all i, j, $\theta_i \neq \theta_j$. Given well-behaved distribution functions, this case is expected to occur with certainty. We will denote by θ_{-i} the vector of all valuations except that of driver i.[2] For most of our analysis the assumption on the distribution of the value of time $F(\cdot)$ is not necessary as the efficient mechanism induces the revelation of each drivers' value of time independent of distributional assumptions. However the distribution of drivers' value of time is needed when we consider limit cases and revenue maximization. Driver i's utility is $u_i(p, \theta_i, t) = v - p - \theta_i t$, where p is the transfer i has to pay, t is the amount of time it takes for i to travel from O to D and v is the valuation for reaching the destination. We assume that v is sufficiently large so that not traveling is not an option for the drivers. The allocation of agent i is given by $x_i \in \{0, 1\}$, where $x_i = 1$ means that i is allocated to road A, while $x_i = 0$ means that i is allocated to road B. We let $x \in X = \{0, 1\}^n$ denote the overall allocation. We can then write the time driver i will spend traveling from O to D as $t_i(x) = x_i C\left(\sum_{j=1}^{n} x_j\right) + (1 - x_i)\bar{t}$. Note that the travel time of an agent therefore will depend on the allocation of other agents, hence our problem is a specific instance of a resource allocation problem with externalities.

A mechanism designer, who can be thought of as the government authority or a private monopoly in charge of regulating traffic, is assumed to maximize total welfare, i.e. the sum of all drivers' well-being plus the total revenue collected. Hence the objective function of the mechanism designer is given by:

$$W = max_{[x_i(\theta), p(\theta)]} \sum_{i=1}^{n} [u_i(p_i, x; \theta_i) + p_i] \tag{2.1}$$

[1] Well-behaved means that $F(\cdot)$ is continuously differentiable with a strictly positive derivative $f(\cdot)$ and such that $\frac{1-F(\theta_i)}{f(\theta_i)}$ is weakly decreasing in θ_i.

[2] With a slight abuse of notation we will denote by $F(\theta)$ the joint distribution of the vector of valuations and $F(\theta_{-i})$ the joint distribution of all valuations except for that of driver i.

In section 4 we consider the case when the mechanism designer is only interested in maximizing total revenue collected.

While we have set up the model in terms of drivers being able to use one of two roads, there is an alternative interpretation. The model can also be interpreted as specifying the option to travel on a congested road and no travel. Drivers in this interpretation have a type-dependent outside option. The assumption that $\bar{t}$ is large then ensures that the overall utility of a driver is increasing in θ so that high value drivers will be using the road at efficient allocations. Alternatively one can reformulate that model as one in which there is an outside option with a fixed value of 0, but in which drivers with a high value of θ_i also have a high value for traveling overall.

In the absence of a mechanism, drivers would be free to choose either road without payments. In that case travel time on the fast road would need to be close to that on the slow road or all drivers would use the fast road. If it weren't, more drivers would start using the fast road, thereby increasing travel time on the fast road. Furthermore, there is a coordination problem as nearly identical travel times mean that there is no sorting according to the value of time in terms of road usage. In contrast, the efficient allocation both solves the coordination problem, as drivers with a high value of time are allocated to the fast road and ensures that the fast road is indeed faster than the slow road.

Before discussing the incentive problem associated with the fact that each driver's value of time is private knowledge, we will analyze the problem with perfect information.

First-Best Allocation In the first-best case we can cancel out the transfers and write the mechanism designer's problem as follows:

$$W = min_{[x_i(\theta)]} \sum_{i=1}^{n} \theta_i \left[x_i C \left(\sum_{j=1}^{n} x_j \right) + (1 - x_i)\bar{t} \right] \tag{2.2}$$

Hence maximizing social welfare is equivalent to minimizing the value of time spent on traveling from the origin O to the destination D. First, note that at an optimum we need that the travel time on road A has to be less than the travel time on road B. If it were differently, the total value of travel time could be reduced by increasing the number of drivers using road B. The driver moving from road A to road B not only will be able to take the faster route, but this driver will also no longer contribute to congestion on road A, thereby reducing the travel time for all other drivers using that road. From now on, we will therefore refer to road A as the fast road, while road B is referred to as the slow road. Second note that at the optimum drivers will be sorted according to their value of time, with high-value drivers using the faster road, while low value users taking the slower road. If this were not true, so that one driver using the slow route had a higher value of time than a driver using the fast road, then the total value of travel time could be reduced by changing the road used by each of those drivers.

Lemma 1. *Let $x^{FB}(\theta)$ be the allocation that solves Equation 2.2. Then it must satisfy:*

- $C\left(\sum_{i=1}^{n} x_i^{FB}(\theta)\right) < \bar{t}$.
- *If $\theta_i > \theta_j$ then $x_i^{FB}(\theta) \geqslant x_j^{FB}(\theta)$.*

Proof. For the first part suppose the travel time on road A were strictly greater than on road B. Then consider reassigning one driver from road A to road B. This driver would be strictly better off due to a decrease in travel time. Drivers that remain on road A, due to $C(\cdot)$ being weakly increasing, are weakly better off. Next suppose that travel time on road A were equal to that on road B, i.e. $\bar{t}$. Consider reassigning one driver from road A to road B. The reassigned driver is indifferent. Since $C(\cdot)$ is weakly increasing, this reassignment can either leave travel time on road A unchanged or lead to a reduction in travel time. In the latter case, drivers that remain on road A are strictly better off, while those on road B are indifferent. In the former case, drivers remaining on road A are also indifferent so reassigning one driver did not affect welfare. But then one can reassign another driver from road A to road B until the travel time on road A falls below $\bar{t}$, which will necessarily happen by the assumption that $C(1) < \bar{t}$. It follows that $C\left(\sum_{i=1}^n x_i^{FB}(\theta)\right) < \bar{t}$.

For the second part suppose otherwise. Then there is at least one pair of drivers $\{i, j\}$ such that $\theta_i > \theta_j$ and $x_i(\theta) < x_j(\theta)$. But then one could replace $x_i(\theta)$ and $x_j(\theta)$ by $x_i^*(\theta) := x_j(\theta)$ and $x_j^*(\theta) := x_i(\theta)$ which would lead to change in welfare of $(\theta_i - \theta_j)(\bar{t} - C\left(\sum_{l=1}^n x_l(\theta)\right))$. Note that because $\theta_i > \theta_j$ and because of the first part this term is strictly positive. □

Given Lemma 1 the first-best allocation has a simple structure: all drivers with a value of θ sufficiently high will be using the fast road while the remainder will use the slow road. To solve the problem it thus remains to find the optimal number of drivers using the fast road, as a function of θ. We define $\theta^{(k)}$ to be the k^{th} highest value of θ from among the n drivers. Suppose there are $k \in \{0, 1, ..., n-1\}$ drivers on the road and consider adding another driver to the fast road. The change in welfare resulting from this reallocation is given by:

$$\Delta_k(\theta) \equiv \theta^{(k+1)}\left(\bar{t} - C\left(k+1\right)\right) - \sum_{i=1}^{k} \theta^{(i)}\left(C(k+1) - C(k)\right)$$

The expression Δ_k characterizes the key trade-off in determining the efficient allocation. The first term appearing in Δ_k is the benefit of moving driver $k+1$ from the slow road to the fast road. The reduction in travel time for that driver is valued at that driver's valuation of time, $\theta^{(k+1)}$. The second term captures the cost of increased travel time on the first k drivers from another driver using the fast road. Note that the first term will generally be positive for efficient allocations, while the second term always enters negatively since $C(\cdot)$ is a strictly increasing function. The next Lemma characterizes some properties of $\Delta_k(\theta)$ that will be useful.

Lemma 2. *For all $\theta \in \Theta$ and all $k \in \{0, 1, ..., n-2\}$, we have that $\Delta_k(\theta) > \Delta_{k+1}(\theta)$. Furthermore $\Delta_0(\theta) > 0$.*

Proof. For the first result in Lemma 2 consider the difference in the first terms of $\Delta_k(\theta)$ and $\Delta_{k+1}(\theta)$ which is given by:

$$\theta^{(k+1)}\left(\bar{t} - C\left(k+1\right)\right) - \theta^{(k+2)}\left(\bar{t} - C\left(k+2\right)\right).$$

From the definition of $\theta^{(k)}$ we have that $\theta^{(k)} > \theta^{(k+1)}$. Furthermore we have that $C(k+2) \geqslant C(k+1)$, so that this difference is strictly positive. Next consider the difference in the second terms $\Delta_k(\theta)$ and $\Delta_{k+1}(\theta)$ which is given by:

$$\sum_{i=1}^{k+1} \theta^{(i)} \left(C(k+2) - C(k+1)\right) - \sum_{i=1}^{k} \theta^{(i)} \left(C(k+1) - C(k)\right).$$

This can be re-written as:

$$\sum_{i=1}^{k} \theta^{(i)} \left[\left(C(k+2) - C(k+1)\right) - \left(C(k+1) - C(k)\right)\right] + \theta^{(k+1)} \left(C(k+2) - C(k+1)\right).$$

The first term is weakly positive since each θ_i is positive and because of our assumption that $C(\cdot)$ has weakly increasing differences. The second term is weakly positive because $C(\cdot)$ is a weakly increasing function. Hence it follows that $\Delta_k(\theta) > \Delta_{k+1}(\theta)$.

The value of $\Delta_0(\theta)$ can be directly verified from the definition of $\Delta_k(\theta)$ by setting $k = 0$. $\square$

Lemma 2 shows how the trade-off from adding another driver evolves as more drivers use the fast road. First, the benefit of adding one driver gets lower, since the value of time of the marginal driver decreases. Second, the cost the additional driver imposes on the other drivers on the fast road increases, because more drivers are affected by increased congestion. Thus the two effects go in the same direction, so that the welfare gain of each additional driver falls. When there is no driver on the fast road, then there is no cost of adding another driver as no other drivers will be slowed down. The following proposition, which follows directly from Lemma 2, gives a necessary and sufficient condition for the first-best allocation.

Proposition 1. *There exists some k^* such that for all $i = \{1, ..., n\}$ at the optimum $x_i(\theta) = 1$ if and only if $\theta_i \geqslant \theta^{(k^*)}$. The value of k^* is given by:*

$$k^* = \max_{\Delta_k(\theta) \geqslant 0} k + 1 \tag{2.3}$$

The logic behind Proposition 1 follows from Lemma 2: as the number of drivers on the fast road increases, the benefit of adding another driver gets smaller. Welfare changes are strictly positive when the driver with the largest valuation enters the fast road, but become smaller as more drivers use the fast road. If n is large enough that $C(n) > \bar{t}$ then $\Delta_{n-1}(\theta)$ is negative, so that $k^* < n$. It may however happen that $\Delta_{n-1}(\theta) > 0$, which implies that $k^* = n$, i.e. all drivers use the fast road. Note however that this case is not particularly interesting, as even in the absence of a mechanism the efficient allocation would be chosen by drivers. Thus, there is a unique number of drivers, k^*, on the fast road for which both adding and subtracting a driver decreases welfare.

For subsequent results the effect of a change in a single θ_i on the optimal allocation needs to be determined holding θ_{-i} fixed. Let $\theta_{-i}^{(k)}$ be the k^{th} highest value of time among all drivers except driver i. Consider the auxiliary problem in which we set $\theta_i = 0$, but driver i still needs to be allocated to a road. In that case, it is clear that $x_i^{FB} = 0$. Denote the efficient allocation in this case by k_{-i}^*, which is a function of θ_{-i}. Refer to the driver with the k_{-i}^* highest value of time as driver $l((\theta_{-i}))$. Denote his associated value of time by $\theta_l(\theta_{-i}) = \theta_{-i}^{k^*_{-i}}$. We denote by θ' the value of θ_i such that adding driver i to the fast road when k^*_{-i} are allocated to the fast road results in no welfare change.

$$\theta'(\bar{t} - C(k^*_{-i}+1)) - \sum_{j=1}^{k^*_{-i}} \theta^{(j)} \left[C(k^*_{-i}+1) - C(k^*_{-i})\right] = 0$$

We consider two cases. First, suppose $\theta'_i \geqslant \theta_l(\theta_{-i})$. Then for $\theta_i \leqslant \theta_l(\theta_{-i})$ we have that k^* does not vary in θ_i and neither does the allocation x^{FB}. When $\theta_i > \theta_l(\theta_{-i})$ it follows that $x_i^{FB} = 1$ while $x_l^{FB} = 0$. As θ_i increases, k^* falls. This is because given that θ_i is allocated to the fast road, the cost of adding other drivers increases as the value of the congestion suffered by i increases. Therefore k^* is a decreasing function of θ_i.

Second, suppose $\theta'_i < \theta_l$. For $\theta_i < \theta'_i$ we have $k^* = k^*_{-i}$. For $\theta_i \in [\theta'_i, \theta_l(\theta_{-i})]$ we have that it is optimal to add driver i to the fast road without removing any other driver from it. Therefore we have $k^* = k^*_{-i} + 1$. For $\theta_i > \theta_l(\theta_{-i})$, k^* falls as θ_i increases. Since θ_i is allocated to the fast road, any increase in θ_i increases the cost of adding other drivers to the fast road, implying that k^* will fall. We summarize this discussion in the following Lemma:

Lemma 3.

1. *Comparative statics of k^*:*
   (a) *If $\theta'_i \geqslant \theta_l(\theta_{-i})$, then $k^*(\theta_i, \theta_{-i})$ is a weakly decreasing function in θ_i.*
   (b) *If $\theta'_i \geqslant \theta_l(\theta_{-i})$, then $k^*((\theta_i, \theta_{-i}))$ is constant for $\theta_i < \theta'_{-i}$, increases by one at $\theta_i = \theta'_i$ and for $\theta_i > \theta'_{-i}$ is weakly decreasing.*
2. *The efficient travel time $x_i^{FB}\left(C(\sum_{j=1}^n x_j^{FB})\right) + \left(1 - x_i^{FB}\right)\bar{t}$ of driver i is weakly decreasing in θ_i, $\forall i$.*

In short, if θ_i increases and becomes just large enough to be assigned to the fast road, she will be either added to the set of drivers on the fast road or she replaces someone else. Since k^* is only non-decreasing in θ_i at the point when driver i is added to the fast road, the travel time on the fast road is only non-decreasing in θ_i at that point. Nonetheless, the travel time for driver i is strictly lower due to Lemma 1, since for a lower θ_i that driver is assigned to the slow road. When driver i is assigned to the fast road and θ_i increases further, both, k^* and driver i‘s travel time decrease.

3 Implementing the First-Best Allocation

In this section we consider the problem of allocating drivers to roads as a mechanism design problem. We first derive an efficient and incentive compatible mechanism. We highlight this mechanism using a simple example. Last we consider the congestion pricing problem as the number of drivers increases and present simulation results.

An important consideration in the mechanism design literature is incentive compatibility, i.e. to design the mechanism in such a way that drivers always report their private information truthfully. As a first step, we apply the dominant strategy *revelation principle* of Gibbard (1973), which allows us to study a large class of mechanisms by focusing on a smaller subclass. By the revelation principle every complicated mechanism involving potentially very large message spaces may be replaced by a simpler mechanism that only asks drivers to directly report their type truthfully. Thus, instead of studying each of these complicated mechanisms where in equilibrium a type can be inferred from a message, it is w.l.o.g. to study a direct mechanism. In such a mechanism, each driver will be asked to report her private information, namely her valuation for travel time, to the mechanism designer who then allocates drivers to roads. More precisely, a (direct) mechanism is a function associating to each θ an allocation, x and a transfer function p, where p is the transfer paid by a driver to the mechanism designer[3]. In short, a mechanism is a mapping from reports of the drivers' valuation of time to an allocation and transfers, i.e. $[x(\theta), p(\theta)] : \theta \to X \times \mathbb{R}^n$.

We apply the concept of dominant strategy incentive compatibility. The mechanism designer requires each driver to prefer truth-telling for all possible valuations of the other drivers. Hence, the mechanism works regardless of what driver i thinks about driver j's valuation of travel time and the mechanism designer obtains the exact valuations of the n drivers without requiring precise ex-ante information.

Definition 1. *A direct mechanism $[x, p]$ is dominant strategy incentive compatible if for all $\theta_i, \hat{\theta}_i \in \Theta_i$ and $\theta_{-i} \in \Theta_{-i}$ it satisfies*

$$U_i(\theta_i; \theta_{-i}) \equiv v - p_i(\theta_i) - \theta_i t_i(x(\theta_i, \theta_{-i})) \geqslant v - p_i(\hat{\theta}_i) - \theta_i t_i(x(\hat{\theta}_i, \theta_{-i})) \equiv U_i(\hat{\theta}_i, \theta_i; \theta_{-i}) \quad \text{(DIC)}$$

A mechanism that satisfies DIC makes it optimal for a driver with type θ_i to report this value, rather than any other value $\hat{\theta}_i$ for all other possible reported values θ_{-i} of the other drivers. Under dominant strategy incentive compatibility, truth-telling is optimal regardless of a drivers' belief over other drivers' valuations. Alternatively one could consider truth-telling incentives of the drivers given their beliefs about others' types. However in our application it is unlikely that the mechanism designer knows these beliefs. Mechanisms that satisfy DIC are robust to incorrect beliefs of the mechanism designer and do not require detailed knowledge about the distribution of values by the mechanism designer. This is an attractive feature in our application.

The mechanism designer maximizes welfare given by equation 2.1 subject to the DIC constraints. We say that an allocation function $x(\theta)$ is **implemented in dominant strategies** by payment rules $p_i(\theta)$ if together they satisfy the incentive constraints. Notice that the objective function of the mechanism designer considers total welfare. There is thus no revenue-raising motive for the mechanism designer. Note also that the private information held by the drivers affects other drivers only indirectly through the travel time induced by the resulting allocation. Rather than seeing the allocation as specifying which driver uses which road, one might alternatively think about an allocation as simply specifying the travel time for each driver, subject to this allocation being consistent with the underlying model. When looking at allocations as travel times, there are thus no externalities present.

[3]In our notation we already assume that transfer functions are the same for all drivers. Hence the mechanisms we consider satisfy anonymity.

In the following discussion, we are going to make use of some extra notation that was introduced in the previous section. We denote by $\theta^{(k^*)}$ the k^*-highest value of time, where k^* is as defined in Proposition 1, where we have suppressed the dependence of k^* on θ for simplicity. We let $k^*_{-j}(\theta_{-j})$ to be the value of k^* as in Proposition 1 excluding driver j. As before we let $\theta^{(k)}_{-j}$ be the k^{th}-highest value of time in the problem excluding driver j among the $n-1$ remaining drivers, where we have again suppressed the dependence of k^*_{-j} on θ_{-j} for simplicity. We define for each $j \in \{1, ..., n\}$, the following two sets:

$$\Omega_j^+(\theta) \equiv \{i \neq j | \theta_i \geqslant \theta^{(k^*)}\}$$
$$\Omega_j^0(\theta) \equiv \left\{i \neq j | \theta_i \in \left(\theta_{-j}^{(k^*_{-j})}, \theta^{(k^*)}\right)\right\}$$

The set of drivers, excluding j, that use the fast road irrespective of driver j's allocation is denoted by Ω_j^+. Similarly the set of drivers that is only assigned to the fast road if driver j is allocated to the slow road is denoted by Ω_j^0. Note that both sets' dependence on θ has been suppressed for ease of notation. If Ω_j^0 is empty, then driver j's report does not affect other drivers' assigned road but may affect their travel time. Note that both Ω_j^+ and Ω_j^0 depend on the vector of the valuation of time, θ. Note that one consequence of Lemma 3 it holds that $k^* - k^*_{-j} \leqslant 1$.

Proposition 2. *The first-best allocation, $x^{FB}(\theta)$ is implemented by the following payment rule, which specifies for all $j \in \{1, ..., n\}$:*

$$p_j^{FB}(\theta) = \sum_{i \in \Omega_j^+} \theta_i \left[C(k^*) - C(k^*_{-j})\right] + \sum_{i \in \Omega_j^0} \theta_i \left[\bar{t} - C(k^*_{-j})\right] \tag{3.1}$$

Proof. The basic logic behind a Vickrey-Clarke-Groves mechanism is to let each driver internalize the mechanism designers problem by making her a residual claimant of total welfare. To this can be added a term which may depend on the reports of all the other drivers, as this does not affect truth-telling incentives. In our model, this implies that the VCG payment rule has the following form:

$$p_j^{VCG}(\theta) = \sum_{i=1; i \neq j}^{n} \theta_i \left(x_i^{FB}(\theta) C(k^*(\theta)) + (1 - x_i^{FB}(\theta))\bar{t}\right) + h_j(\theta_{-j})$$

Since $k^*(\theta)$ is by definition the function that maximizes welfare for each $\theta \in \Theta$, driver j maximizes her utility, given by:

$$\begin{aligned} U_j(\hat{\theta}_j, \theta_j; \theta_{-j}) = v &- \theta_j \left(x_j^{FB}(\theta) C(k^*(\theta)) + (1 - x_j^{FB}(\theta))\bar{t}\right) \\ &- \sum_{i=1; i \neq j}^{n} \theta_i \left(x_i^{FB}(\theta) C(k^*(\theta)) + (1 - x_i^{FB}(\theta))\bar{t}\right) - h_j(\theta_{-j}) \end{aligned}$$

This means that driver j faces the mechanism designer's problem 2.1, so that reporting $\hat{\theta}_j = \theta_j$ is optimal. The function $h_j(\theta_j)$ is a constant from j's point of view and therefore does not affect

j's incentives. For drivers whose presence does not affect the final allocation it can be normalized such that payments are zero. We thus choose:

$$h_j(\theta_{-j}) \equiv \sum_{i=1;i\neq j}^{n} \theta_i \left(x_{i,-j}^{FB}(\theta_{-j}) C(k_{-j}^*(\theta_{-j})) + (1 - x_{i,-j}^{FB}(\theta_{-j}))\bar{t} \right)$$

Since $k_{-j}^*(\theta_{-j})$ solves the allocation problem, effectively assuming the value of time of driver j is zero, driver j will in that case be allocated to the slow road. Hence the payment above is the surplus of the other drivers under the optimal allocation given that j is assigned to the slow road. Note that it may happen that $\theta_{-j}^{(k_{-j}^*)} > \theta^{(k^*)}$. In this case, the set Ω_j^0 is empty and only the first term in 3.1 remains. Given our choice of $h_j(\theta_{-j})$ and the definitions of Ω_j^+ and Ω_j^0 we can simplify the expression for the VCG payment to 3.1. □

The payment schedule 3.1 that drivers face consists of two components. The first component captures the effect that driver j's report has on other drivers via the travel time on the fast road.[4] This congestion effect may be either positive or negative. It will be negative (meaning that driver j has to pay less) under the condition in Part 1(a) of Lemma 3. In that case a higher report of θ_j reduces k^*, so that all driver on the fast road will benefit from a reduced travel time. The reduction in the price paid by driver j reflects the value of this reduced travel time of the other drivers. The first component will be positive (meaning driver j has to pay more) only if it holds that the valuations of time θ are such that $k^* - k_{-j}^* = 1$.[5] In that case the report of driver j increases the travel time on the fast road for the other drivers. It may also happen that the first component is zero, which occurs when $k^* = k_{-j}^*$. This happens when driver j replaces another driver on the fast road.

The second component of 3.1 captures the effect that driver j's report has on other drivers via reallocation of those driver from the fast road to the slow road. For high values of θ_j, it becomes efficient to reduce travel time on the fast road. This is accomplished by reallocating drivers with a lower value of time to the slow road. Those reallocated drivers due to Lemma 1 face a higher travel time, the cost of which is taken into account by the payment rule. Another feature of the payment schedule is that drivers allocated to the slow road pay nothing. By being allocated to the slow road, both the congestion and the reallocation effect are zero.[6]

Remark 1. *The payment schedule $p_j^{FB}(\theta)$ is a weakly increasing step function.*

The payment schedule 3.1 depends on θ_j only through its effect on the efficient allocation $k^*(\theta)$. From the definition of k^* it follows that k^* can only take a finite number of values. It is constant almost everywhere but there are jumps at a finite number of points, which is whenever the number of drivers allocated to the fast road changes. Therefore the payment schedule faced by driver j will also be constant almost everywhere and have a finite number of jumps. Furthermore by Lemma 3 the travel time of driver j is weakly decreasing in θ_j. This implies that the payment

[4]Note that this is similar, but different to the congestion effect that was present in the definition of $\Delta_k(\theta)$.

[5]This corresponds to case 1(b) in Lemma 3 at the point where k^* is increasing in θ_j.

[6]Note that this follows mainly from our assumption that there is no congestion on the slow road. This assumption will be relaxed in Section 5.

schedule of driver j needs to be weakly increasing. If it were not then there would be cases in which driver i by misreporting her valuation could obtain both a lower travel time and a lower payment to be made. This would violate incentive compatibility. Figure 1 in the Introduction features one such payment schedule.

3.1 Congestible Roads with a Known, Fixed Capacity

In this section we consider a special case of the congestion function $C(k)$, such that it has a fixed capacity of $\bar{k}$. This special case is of interest as it eliminates aggregate uncertainty over the efficient number of drivers to be allocated to the fast road. Travel time on the fast road is $\underline{t} < \bar{t}$ as long as fewer than $\bar{k}$ drivers use it. Above that capacity, travel time on the fast road increases to $\bar{t}$. Hence, $C(k)$ is given by:

$$C(k) = \begin{cases} \underline{t} & if \quad k \leqslant \bar{k} \\ \bar{t} & if \quad k > \bar{k} \end{cases}$$

Clearly, the efficient allocation irrespective of the realization of θ is for the $\bar{k}^{th}$-highest valuation drivers to travel on the fast road and all other drivers on the slow road. The function $C(k)$ satisfies the assumptions that we imposed in Section 2. However, since the optimal number of drivers on the road is known to the mechanism designer, the efficient allocation can be implemented by simply auctioning off $\bar{k}$ licenses to use the fast road among the n drivers. The reason that this procedure works is that the reports by the agents are no longer needed to determine the optimal number of drivers on the fast road.

3.2 Example with Two Drivers

To highlight some of the properties of our proposed mechanism, consider our model with two drivers. We consider the case in which $C(k) = k$. Parameters are $\bar{t} = 4$ and $n = 2$. In the efficient allocation, if $\theta_i > \theta_j$ both drivers use the fast road if and only if $-2(\theta_i + \theta_j) > -\theta_i - 4\theta_j$ which simplifies to $\theta_i < 2\theta_j$. Otherwise, only θ_i is efficiently allocated to the fast road. Overall, the efficient allocation is

$$(x_1, x_2)(\theta) = \begin{cases} \{1,1\} & if \quad \theta_1 \in [\frac{1}{2}\theta_2, 2\theta_2] \\ \{1,0\} & if \quad \theta_1 > 2\theta_2 \\ \{0,1\} & if \quad \theta_1 < \frac{1}{2}\theta_2 \end{cases}$$

In Figure 3.1 it can be seen that when the values of time of the two drivers are close to each other (i.e. we are in the violet area around the 45 degree line) then it is efficient for both of them to use the fast road. When the differences in the value of time are more extreme, it is however optimal to let only the driver with the higher value of time use the fast road.

We apply proposition 2 to compute the prices that implement this allocation for driver 1:

$$p_1(\theta) = \begin{cases} 0 & if \quad \theta_1 < \frac{1}{2}\theta_2 \\ \theta_2 & if \quad \theta_1 \in [\frac{1}{2}\theta_2, 2\theta_2] \\ 3\theta_2 & if \quad \theta_1 > 2\theta_2 \end{cases}$$

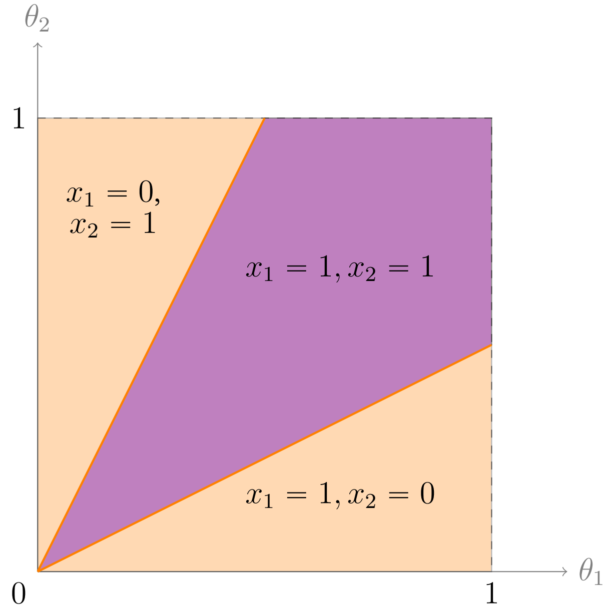

Figure 3.1: Optimal allocations with Two Drivers

Figure 3.2 plots the optimal price schedule faced by driver 1 for two values of θ_2. Note that the price paid by driver 1 is not monotone in θ_2.

The corresponding travel time for driver 1 is given in the following function:

$$t_1(\theta) = \begin{cases} 4 & if \quad \theta_1 < \frac{1}{2}\theta_2 \\ 2 & if \quad \theta_1 \in [\frac{1}{2}\theta_2, 2\theta_2] \\ 1 & if \quad \theta_1 > 2\theta_2 \end{cases}$$

Figure 3.3 plots the optimal travel time of driver 1 for two different values of θ_2 as a function of θ_1. Note that unlike the payment schedule, the travel time is a monotone function of the value of θ_2. Note that in this example we did not use information on the distribution of θ for the construction of the payment schedule.

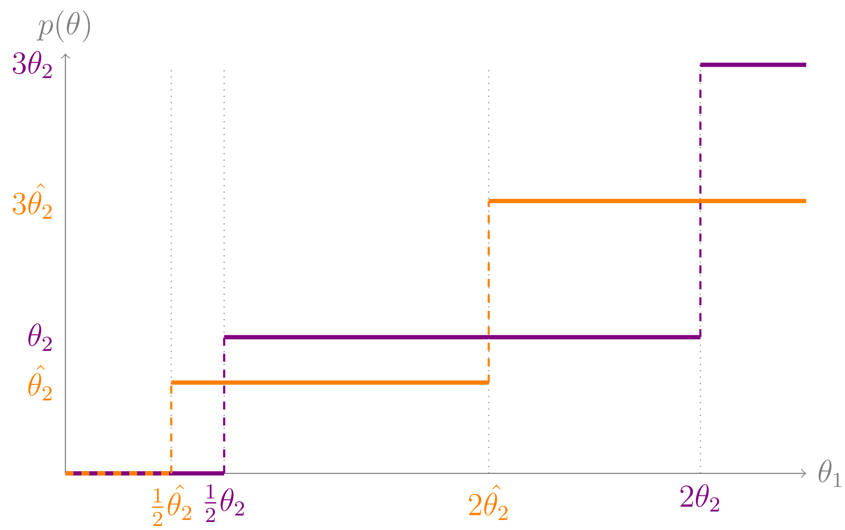

Figure 3.2: Payment schedule for two different values of θ_2

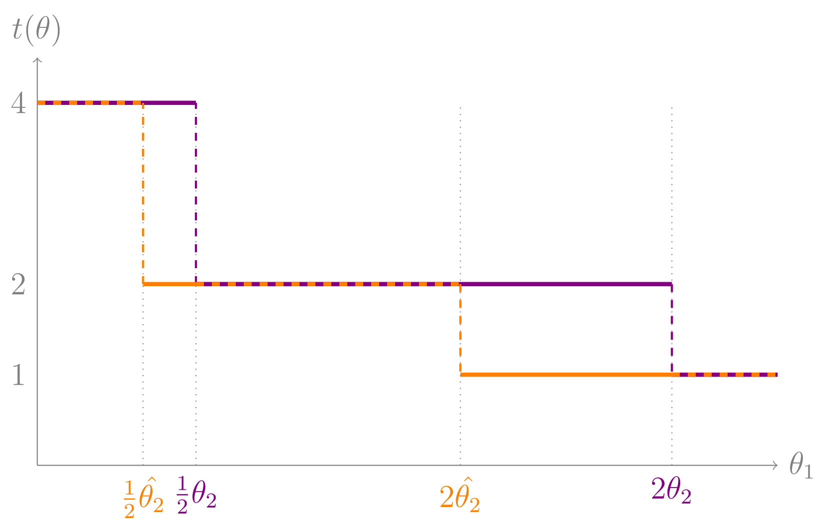

Figure 3.3: Travel time for two different values of θ_2

3.3 Congestion Pricing in the Limit

The previous discussion implies that each driver should face a menu of different transfers, depending on that driver's valuation and on all other driver's reported types. The previous literature has mainly focused on situations in which each driver faces a single transfer to be paid when using the fast road, but not a schedule of different prices. To reconcile our results with the existing literature, we will consider a variant of our model in which we let the number of drivers go to infinity. To ensure convergence, we need to change the congestion externality on the fast road. We assume that travel time on road A is $C_n(k) = \underline{t} + b_n k$, where $b_n = b/n$ and k is the number of drivers on the fast road. If the congestion each driver caused did not decline with the total number of drivers, then as n goes to infinity, the share of drivers using the fast road at the efficient allocation would converge towards zero. We assume that $\bar{t} - \underline{t} - b < 0$, to ensure that at the efficient allocation not all drivers will use the fast road in the limit. We also normalize the objective function by dividing through by the number of drivers n:

$$W = max_{[x_i(\theta),p(\theta)]} \frac{1}{n} \sum_{i=1}^{n} [U_i(p_i, x; \theta_i) + p_i] \tag{3.2}$$

For each n we can use the results of Lemmas 1 and 2 as well as Proposition 1 to find the optimal solution for each value of n. We consider now the probability limit of the value of $\Delta_k(\theta)$ as n converges to infinity, while letting k go to infinity, such that $\lim_{n\to\infty} \frac{k}{n} = q \in [0, 1]$.

$$\text{plim}_{n\to\infty} \Delta_k = \text{plim}_{n\to\infty} \theta^{(k+1)} \left(\bar{t} - \underline{t} - \frac{k+1}{n} b \right) - \frac{b}{n} \sum_{i=1}^{n} \theta_i \mathbb{1} \left(\theta_i > \theta^{(k+1)} \right) \tag{3.3}$$

Note that $\text{plim}_{n\to\infty} \theta^{(k+1)}$ is simply the $(1-q)^{th}$ quantile of the distribution function $F(\cdot)$, which we will denote by $\theta^q \equiv F^{-1}(1-q)$. The probability limit of the second term is:

$$\text{plim}_{n\to\infty} \frac{b}{n} \sum_{i=1}^{n} \theta_i \mathbb{1} \left(\theta_i > \theta^{(k+1)} \right) = b \int_{\theta^q}^{\bar{\theta}} \theta dF(\theta)$$

Therefore, we have that:

$$\text{plim}_{n\to\infty} \Delta_k = \theta^q \left(\bar{t} - \underline{t} - qb \right) - b \int_{\theta^q}^{\bar{\theta}} \theta dF(\theta) \equiv \Delta_q \tag{3.4}$$

We have that Δ_q is strictly decreasing in the value of q. To obtain the efficient level of q, we simply take the unique solution of $\Delta_q = 0$, denoted by q^*. Let θ^* be such that a fraction q^* of all drivers have a greater value of time. Then we have the following:

$$\theta^* \left(\bar{t} - \underline{t} - q^* b \right) = b \int_{\theta^*}^{\bar{\theta}} \theta dF(\theta) \tag{3.5}$$

Hence, like in the models of the earlier literature, in the limit there is a deterministic share of drivers efficiently using the fast road. Therefore, we can set a single congestion charge to implement the efficient allocation in the limit. The following Proposition summarizes the preceding discussion:

Proposition 3. *In the limit, the efficient allocation is implemented when the mechanism designer sets a unique Pigouvian price p^* for the use of the fast road. This price is given by:*

$$p^* = b \int_{\theta*}^{\overline{\theta}} \theta dF(\theta) \tag{3.6}$$

This efficient congestion charge equals the value of the marginal increase in travel time from a small increase in the number of drivers using the fast road. The Pigouvian approach of setting a price that includes the social cost of an externality is efficient, thus is found to hold only if there are many drivers, each of which has a negligible effect on the other drivers. Furthermore, the Pigouvian approach requires the mechanism designer to know the distribution of the value of time.

3.4 Simulations

The limit results of the previous section might suggest that an appropriately set Pigouvian price can ensure that maximal welfare is obtained. However, there are two impediments to setting the Pigouvian price optimally. First, the number of drivers in practice is finite. Given that we focus on a static problem in which all drivers are taken to be using the road simultaneously, the number of drivers in applications may be low depending on the situations. If this is the case, then limit results cannot be relied upon.

Second, setting the Pigouvian price optimally requires knowledge of the distribution of the value of time. As this is usually not known by the policy maker, the price will be set either as a function of the policy maker's prior or may need to be estimated, for example as in Small et al. (2005). When estimating the distribution, one however needs to allow for possible shifts in the distribution. For example, the estimation would need to allow for the distribution of the value of time to vary by time of day, weather and other, possibly irregular, factors. Given that some of these factors are unobservable and fluctuate randomly, it is unlikely that even a flexible estimation procedure allows to always set the price optimally.

To illustrate potential problems with mechanism designers' priors, suppose the common distribution of the value of time depends on an unobservable parameter α, so that each θ_i is distributed according to $F(\theta_i; \alpha)$ where α is distributed uniformly over the unit interval. Note that one implication of this assumption is that from the mechanism designer's view, the values of θ_i are not independent. The mechanism designer's prior is then $f_p(\theta_i) \equiv \int_0^1 f(\theta_i; \alpha) d\alpha$. When setting the Pigouvian price, based on the prior the mechanism designer will set:

$$p = b \int_{\theta*}^{\overline{\theta}} \theta dF_p(\theta)$$

This will not yield the optimal price $p^*(\alpha)$ that the mechanism designer would set if the value of α were known. In contrast, our proposed mechanism achieves an efficient allocation for each value of α. Note that in this case the mechanism designer's lack of knowledge over α reflects aggregate uncertainty other than that resulting from a finite number of drivers. So even with a continuum of drivers, aggregate uncertainty implies that setting a Pigouvian price based on priors is not efficient.

To investigate the consequences of both, a smaller number of drivers and errors in setting the Pigouvian price, we simulated the normalized welfare loss[7] from using a Pigouvian price as well as both a higher and a lower price. The results are summarized in Figure 3.4. The purple curve indicates the welfare loss when the Pigouvian price is set optimally. As can be seen the welfare loss from using the optimal Pigouvian price vanishes as the number of drivers increases. However for a small number of drivers there is still some welfare loss. The orange line indicates welfare loss when a price is set at 20% below the optimal Pigouvian price. It can clearly be seen that the welfare loss under this price converges to a strictly positive percentage. Note that when there are few drivers the lower price gives a lower welfare loss than the optimal Pigouvian price. This can be due to realizations of the valuations of time such that none of the drivers is willing to pay the price to use the fast road. In that case a lower price may induce at least one driver to use the fast road, which always dominates no driver using the fast road. This shows that with a finite number of drivers the Pigouvian price that maximizes welfare will in general depend on n. The blue curve shows the welfare loss when the price is set at 20% above the optimal Pigouvian price. Again, the welfare loss does not vanish as the number of drivers increases.

The simulation results highlight that even when the number of drivers is large our mechanism can achieve significant welfare gains by generating the information needed to set the right price. While the Pigouvian price is optimal in the limit, for each realization of the values of time with finitely many drivers, the Pigouvian price is not optimal in expectation.

[7] Welfare loss is calculated as follows: $Loss = \frac{W^* - W(p)}{W^* - W(0)}$, where W^* is maximum welfare, $W(p)$ gives welfare when a price of p is set and $W(0)$ is welfare resulting from a price of 0, so all drivers' travel time is given by $\bar{t}$. We compare the loss in welfare from setting a price p to the welfare loss from not having a mechanism. Thus welfare losses are normalized by the maximal welfare gain from a mechanism.

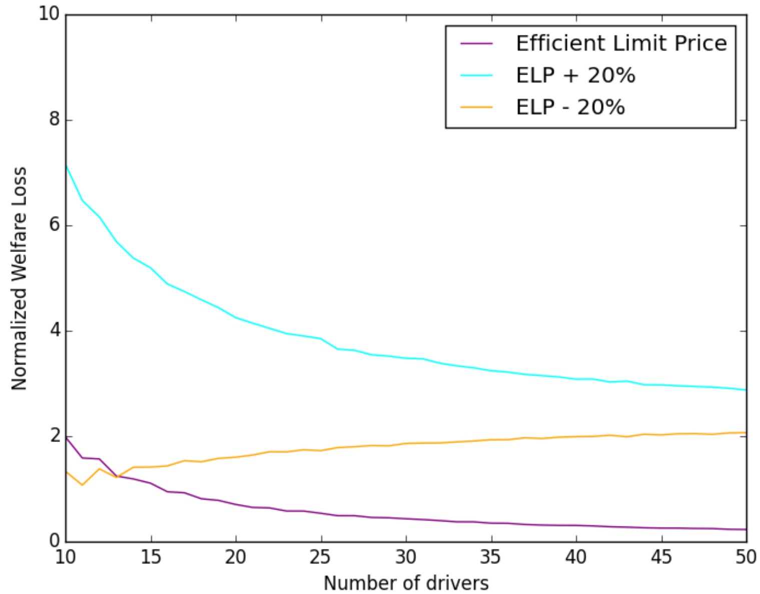

Figure 3.4: Welfare loss of single price mechanisms in the limit

Simulation results comparing welfare under the efficient limit price (Pigouvian price) $\pm 20\%$ to maximum welfare in %. For each n, 10,000 random draws were taken. Parameters were set as follows: $b = 15$, $\bar{t} = 14$, $\underline{t} = 0$. $F(\cdot)$ was chosen to be a lognormal distribution with a median of 21.46 \$/h and interquartile range of 10.47 \$/h, taken from Small et al. (2005), Table 3. Hence the natural logarithm of the value of time is distributed with a mean of 3.07 and a variance of 0.13. The efficient limit price under this set-up is given by: 186\$, implying that around 40.23% of drivers use the fast road in the limit.

4 Revenue Maximization

So far the focus of the mechanism designer is efficiency, in the sense of maximizing total surplus. In many applications, this may not be realistic. In this section the mechanism designer's objective is to maximize revenue given her beliefs about the distribution of θ, given by $F(\cdot)$. We assume that $F(\cdot)$ has positive mass only on the interval $\Theta = [\underline{\theta}, \bar{\theta}]$. We need to modify the interpretation of our set-up and allow for non-participation. If all drivers always participated, the mechanism designer could obtain unbounded profit. Now each driver has the option to travel on a congestible road or to not travel at all. The utility from not traveling is given by 0, while the utility of traveling on the congestible road is given by:

$$u_i(x_i, p_i) = \theta_i (\delta - bk) x_i - p_i \tag{4.1}$$

This can be derived from our earlier model where the value of the outside option is type-dependent and given by $v - \bar{t}\theta_i$ and letting $\delta = \bar{t} - \underline{t}$. The normalization of the outside option to a value of zero simplifies the model, but otherwise gives equivalent results. In this sense, the model with non-participation discussed here is equivalent to the model with a type-dependent outside option, i.e. a non-congestible road, discussed so far in the paper.

We denote by $U(\hat{\theta}_i, \theta_i; \theta_{-i})$ the utility to driver i when her true type is θ_i, she reports $\hat{\theta}_i$ and the value of time of the other drivers is given by the vector θ_{-i}. Hence we have:

$$U(\hat{\theta}_i, \theta_i; \theta_{-i}) \equiv \theta_i \left(\delta - b \left(\sum_{j=1}^{n} x_j(\hat{\theta}_i, \theta_{-i}) \right) \right) x_i(\hat{\theta}_i, \theta_{-i}) - p_i(\hat{\theta}_i, \theta_{-i})$$

The mechanism designer thus faces the following incentive constraints:

$$U_i(\theta_i; \theta_{-i}) \equiv U_i(\theta_i, \theta_i; \theta_{-i}) \geqslant U_i(\hat{\theta}_i, \theta_i; \theta_{-i}), \forall i \textit{ and } \hat{\theta}_i, \theta_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}. \tag{4.2}$$

and the participation constraint becomes

$$U_i(\theta_i; \theta_{-i}) \geqslant 0, \forall i \textit{ and } \theta_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}. \tag{4.3}$$

The objective function of the mechanism designer is to maximize total expected revenues subject to participation and incentive constraints:

$$W = max_{[x_i(\theta), p_i(\theta)]} \mathbb{E}_\theta \left[\sum_{i=1}^{n} p_i \right] = max_{[x_i(\theta), U_i(\theta_i; \theta_{-i})]} \sum_{i=1}^{n} \mathbb{E}_\theta \left[\theta_i (\delta - b \sum_{j=1}^{n} x_j(\theta)) x_i(\theta) - U_i(\theta_i, \theta_{-i}) \right] \tag{4.4}$$

subject to 4.2 and 4.3.

We solve this problem as in the classical optimal auction problem in Myerson (1981). The proof of the following Lemma is standard[8] and therefore omitted.

Lemma 4. *The incentive constraints in equation 4.2 are equivalent to:*

$$U_i(\theta_i; \theta_{-i}) = U_i(\underline{\theta}_i; \theta_{-i}) + \int_{\underline{\theta}_i}^{\theta_i} x_i(\tilde{\theta}_i, \theta_{-i}) \left(\delta - b \sum_{j=1}^{n} x_j(\tilde{\theta}_i, \theta_{-i}) \right) d\tilde{\theta}_i \tag{4.5}$$

and

$$x_i(\theta_i, \theta_{-i}) \left(\delta - b \sum_{j=1}^{n} x_j(\theta_i, \theta_{-i}) \right) \tag{4.6}$$

is weakly increasing in θ_i*.*

Profit maximization requires that the participation constraint is binding for the lowest type, i.e. $U_i(\underline{\theta}_i; \theta_{-i}) = 0$ for all $\theta_{-i} \in \Theta_{-i}$. We can then plug in the first expression of Lemma 4 into the

[8] See for example the proof of Proposition 23.D.2 in Mas-Colell et al. (1995).

mechanism designers' objective function and simplify in the usual way by using integration by parts to get the following expression:

$$max_{[x_i(\theta)]} \sum_{i=1}^{n} \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \right) \left[x_i(\theta)(\delta - b \sum_{j=1}^{n} x_j(\theta)) \right] f(\theta_i) d\theta. \tag{4.7}$$

If we substitute $v_i \equiv \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)}$, known as the *virtual value*, in the above equation, we obtain a maximization problem that corresponds to 2.2 except virtual valuations are used instead of true valuations and the optimization is subject to the additional monotonicity requirement of Lemma 4. This implies that all our results concerning the implementation can be applied to the problem of revenue maximization as well as long as the allocated value of travel is weakly increasing in θ_i for all $\theta_{-i} \in \Theta_{-i}$. One sufficient condition to ensure this holds true is that virtual valuations are weakly increasing in θ_i. This is implied for example if the distribution of the value of time is such that $\frac{1-F(\theta_i)}{f(\theta_i)}$ is weakly decreasing in θ_i.

To obtain the optimal number of drivers on the fast road, the mechanism designer trades off the additional virtual-valuation for travel time of adding another driver to the fast road with the reduced virtual valuation of all drivers already allocated to this road. Thus, the algorithm is the same as in Proposition 1 but using virtual valuations v_i instead of θ_i for all drivers i. Note that we have assumed that valuations are distributed identically across drivers. This implies that a driver with a higher virtual value also has the higher value of time so that a similar sorting of high value of time drivers occurs. Hence keeping the number of drivers fixed, the revenue maximizing mechanism efficiently selects drivers to drive on the road. If drivers' differed in their distribution of the value of time, this would no longer be true. The payment schedules that implement the revenue maximizing allocation can be obtained in the usual way by substituting the resulting allocation of drivers to roads into equation 4.5 and rearranging.

Despite the fact that this procedure is very similar to the one studied for efficient mechanisms, there is an important difference: knowledge of virtual valuations requires the mechanism designer to have some belief concerning the distribution $F(\cdot)$. As a result, the revenue maximizing mechanism will only maximize revenues if the mechanism designer knows the true distribution of the value of time, whereas the VCG mechanism will be efficient irrespective of the distribution of values of time.

4.1 Example with Two Drivers Continued

We highlight the differences between welfare maximization and revenue maximization using the simple example with two drivers from Subsection 3.2. In contrast to the results concerning welfare maximization, we need to make an assumption about the distribution of the value of time of the drivers. Specifically we assume that the distribution of the value of time is the uniform distribution over the unit interval. So $\theta_i \sim U(0, 1)$ for $i = 1, 2$. Substituting into equation 4.7 yields:

$$max_{[x_i(\theta)]} \sum_{i=1}^{n} \int_{0}^{1} (2\theta_i - 1) \left[x_i(\theta)(4 - \sum_{j=1}^{n} x_j(\theta)) \right] f(\theta_i) d\theta. \tag{4.8}$$

Clearly, a necessary condition for $x_i(\theta) = 1$ is that $\theta_i \geqslant 0.5$. If only one of the drivers satisfies that condition, that driver is allocated to driver on the road. If both drivers satisfy the condition, the driver with the higher value of time is allocated to the road. Let j be the driver with the lower value of time. The effect of adding driver j on the road on profit is given by $-(2\theta_{-j}-1)+2(2\theta_j-1)$. The first term captures the reduced virtual value of driving on the road for driver $-j$, while the second term represents the addition of the virtual value of driver j switching from not driving to driving. Hence the optimal allocation is given by:

$$(x_1^*, x_2^*)(\theta) = \begin{cases} \{0,0\} & if \quad \theta_1, \theta_2 < 0.5 \\ \{1,1\} & if \quad \theta_1, \theta_2 \geqslant 0.5 \;\&\; \theta_1 \in (0.5\theta_2 + 0.25, 2\theta_2 - 0.5) \\ \{1,0\} & if \quad \theta_1 \geqslant \max(0.5, 2\theta_2 - 0.5) \\ \{0,1\} & if \quad \theta_2 \geqslant \max(0.5, 2\theta_1 - 0.5) \end{cases}$$

The revenue maximal allocation is shown in the figure below:

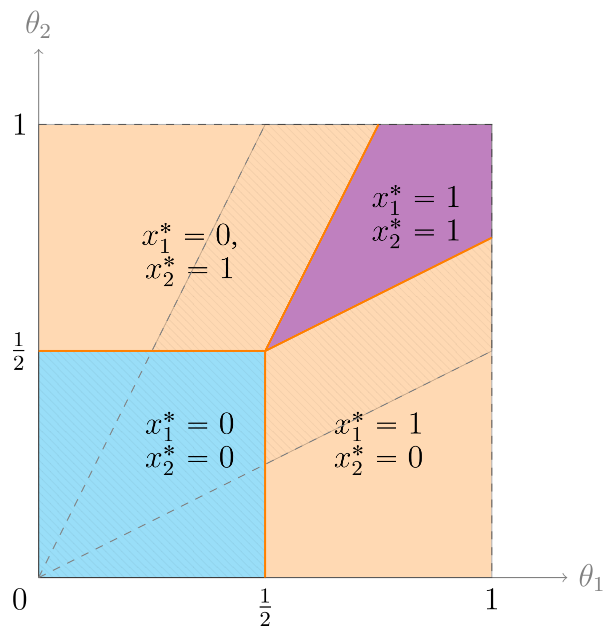

Figure 4.1: Revenue Maximizing Allocation with Two Drivers

As in the original example, the violet area shows values of (θ_1, θ_2) for which both drivers use the road. In orange areas only one driver uses the road, while in the blue area none of the drivers uses the road. The dashed lines in Figure 4.1 indicate the solution for the efficient allocation from Figure 3.1. Dashed areas highlight values of (θ_1, θ_2) in which the revenue maximizing allocation does not coincide with the efficient allocation. Notice that the number of drivers on the road is weakly lower in the revenue optimal allocation compared to the efficient allocation. Therefore the driving time on the road is also weakly lower in the revenue optimal allocation compared to the efficient allocation.

Given the revenue maximizing allocation we can use the drivers' incentive constraints to get the payment schedule that implements the revenue maximizing allocation:

$$p_i^*(\theta) = \begin{cases} 0 & if \quad \theta_i < \max\{0.5, 0.5\theta_{-i} + 0.25\} \\ \theta_{-i} + 0.5 & if \quad \theta_{-i} \geqslant 0.5 \ \& \ \theta_i \in [0.5\theta_{-i} + 0.25, 2\theta_{-i} - 0.5) \\ 3\max\{0.5, \theta_{-i}\} & if \quad \theta_{-i} \in [0, 0.75) \ \& \ \theta_i \geqslant \max\{0.5, 2\theta_{-i} - 0.5\} \end{cases}$$

The following graphs show the pricing function for driver i for different values of time of the other driver in violet. The payment schedule that implements the efficient allocation is shown for comparison purposes in orange.

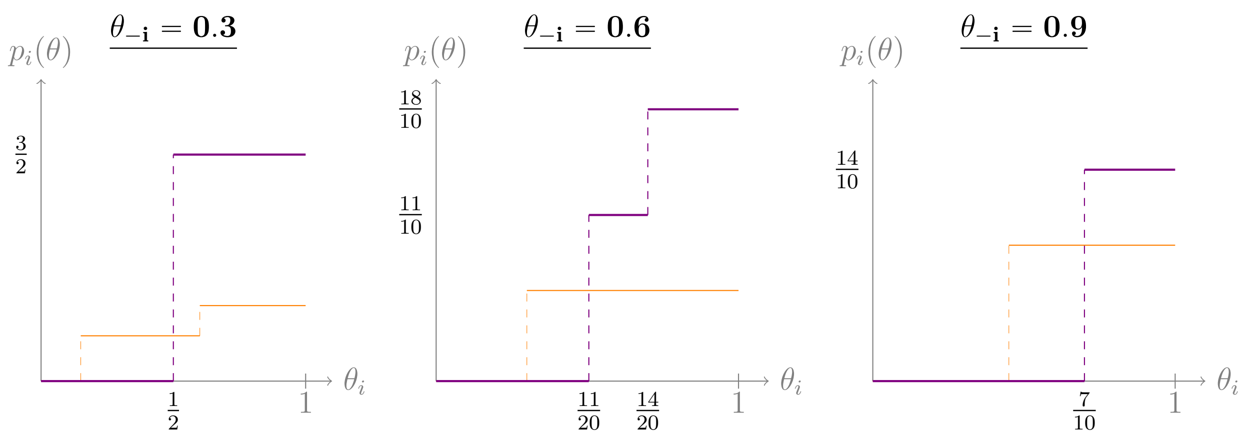

Figure 4.2: Payment schedules for Driver i given different values of θ_{-i}.

Figure 4.2 shows that the qualitative features of the revenue maximizing payment schedule are similar to the payment schedule that implements the efficient allocation. It can be seen that for values of θ_i below 0.5, the price charged by the efficient mechanism lies above that charged by the revenue maximizing one, which also occurs when comparing the optimal auction of Myerson (1981) with a standard auction without a reserve price. Not allowing drivers with a low value of time to use the road is optimal when maximizing revenues as this allows the mechanism designer to charge higher prices when the value of time is above 0.5. Note that payments are not monotone in the reported value of time of the other driver.

5 Two Identical Roads

The assumption that there is no congestion on one of the roads is not relevant for many applications. In the case in which the two roads are simply the two lanes of one road is not covered by that example. However this is a practically relevant case. For example, in California there are special high-occupancy vehicle (HOV) lanes, which differ from normal lanes (see for example Small et al. (2005)). In this section we extend our results to the case in which there are two identical roads. As before, there are two roads $s = A, B$. We focus throughout on the case of a linear congestion function for both roads, i.e. $C(k) = a + bk$, where $a \geqslant 0$ and $b > 0$. We assume that there is an odd number n of drivers, such that $n = 2m + 1$ and $m \in \mathbb{N}$. This is to ensure that at the efficient allocation there will necessarily be a fast and a slow road. We again refer to road A as the fast road. We assume that if there are q drivers on road s, then it takes those drivers on that road a time of $t^s = a + bq$ to go from the origin to the destination. When $x_i = 1$ driver i is allocated to road $s = 1$ and to road $s = 2$ when $x_i = 0$. Hence the travel time of a driver i when the allocation is x is given by:

$$a + x_i \left(b \sum_{j=1}^{n} x_j \right) + (1 - x_i) \left(b \sum_{j=1}^{n} (1 - x_j) \right)$$

Valuations and utility functions are otherwise as before. Similarly we only consider the allocation of drivers to one of the two roads assuming full participation.

First-Best Allocation with Two Identical Roads As before we begin by characterizing the efficient allocation when there are two roads that maximizes 2.1. We begin by noting that there will be a fast and a slow road at the efficient allocation. We will denote the efficient allocation as a function of the drivers' valuations by $x^{FB,2}(\theta)$, where the superscript 2 refers to the case of two identical roads.

Lemma 5. *Let $x^{FB,2}(\theta)$ be the efficient allocation. Then it must satisfy:*

- $a + b \left(\sum_i x_i^{FB,2}(\theta) \right) < a + b \left(n - \sum_i x_i^{FB,2}(\theta) \right)$.
- *For all i, j and $\theta \in \Theta$ such that $\theta_i > \theta_j$, we have $x_i^{FB,2}(\theta) \geqslant x_j^{FB,2}(\theta)$.*

The first point follows directly from assuming an odd number of drivers, full participation and having two identical roads. Since all drivers are allocated to one of the roads, one of the roads has to be faster. Since there is a slow road and a fast road, it continues to be optimal to put drivers with a high value of time on the fast road.

The proof for the second point is analogous to the proof of Lemma 1. Given the previous two results it remains to consider how many drivers will be using the fast road. To analyze this question, suppose there are $k \leqslant m$ drivers (i.e. those with the k highest valuations) using the fast road and we consider adding another driver to it. By Lemma 5 it can only be optimal to add the driver with the $(k+1)^{th}$ highest valuation to the fast road. The change in welfare that results is given by:

$$\Delta_k^2(\theta) = 2\theta^{(k+1)} b(m - k) - \sum_{i=1}^{k} b\theta^{(i)} + \sum_{i=k+1}^{n} b\theta^{(i)}$$

The first term is the benefit for the driver that was previously using the slow road and is now moved to the fast road. The second term is the loss to drivers on the fast road from having another driver added to the fast road. The trade-off between these two effects is also present in the base model without congestion on the slow road. The third term is new and represents the benefit to the drivers on the slow road from having one fewer driver on it. The next Lemma gives the properties of $\Delta_k^2(\theta)$.

Lemma 6. *For all $\theta \in \Theta$ and all $k \in 1, ..., m$, we have that $\Delta_k^2(\theta) > \Delta_{k+1}^2(\theta)$. Furthermore $\Delta_0^2(\theta) > 0$ and $\Delta_m^2(\theta) < 0$.*

The Proof of Lemma 6 is similar to that of Lemma 2.

Proof. We can write $\Delta_0^2(\theta) = 2\theta^{(1)}bm + \sum_{i=2}^n b\theta^{(i)}$. Clearly this expression is positive. For $k = m$ we have that $\Delta_m^2 = -\sum_{i=1}^m b\theta^{(i)} + \sum_{i=m+1}^n b\theta^{(i)}$, which is negative since the first sum contains the m largest values of θ, while the second term contains the m smallest values of θ. To see that $\Delta_k^2(\theta)$ is decreasing in k, consider the first difference, which is given by:

$$\Delta_{k+1}^2(\theta) - \Delta_k^2(\theta) = -\theta^{(k+2)}b - 2\left(\theta^{(k+1)} - \theta^{(k+2)}\right)(m-k) - \theta^{(k)}b$$

Since $k < m$, this expression is clearly negative. □

Using the results of Lemma 6 the efficient allocation is characterized as follows:

Proposition 4. *There exists some k^{**} such that for all $i = \{1, ..., n\}$ at the optimum $x_i(\theta) = 1$ if and only if $\theta_i \geqslant \theta^{(k^{**})}$. The value of k^{**} is given by:*

$$k^{**} = \max_{\Delta_k^2(\theta) \geqslant 0} k + 1 \tag{5.1}$$

The proof follows the same lines as the proof of Proposition 1 above making use of Lemmas 5 and 6.

Implementing the First Best Having characterized the first-best allocation, we again find a payment schedule for the drivers that ensures that truthfully reporting of drivers' types is a dominant strategy. Hence we maximize the mechanism designer's objective function 2.1 subject to DIC and taking into account that there will also be congestion effects on the slow road. We again need to define a few new terms, similarly to Proposition 2. We denote by k_{-j}^{**} the optimal number of drivers on the fast road in the auxiliary problem when driver j's value of time is not considered, i.e. set equal to zero. We denote by $\theta_{-j}^{(k)}$ the k^{th}-highest value of time among all drivers excluding driver j.

We define the following sets:

$$\Omega_j^+(\theta) \equiv \left\{i \neq j | \theta_i \geqslant \theta^{(k^{**})}\right\}$$
$$\Omega_j^0(\theta) \equiv \left\{i \neq j | \theta_i \in \left(\theta_{-j}^{(k_{-j}^{**})}, \theta^{(k^{**})}\right)\right\}$$
$$\Omega_j^-(\theta) \equiv \left\{i \neq j | \theta_i \leqslant \theta^{(k_{-j}^{**})}\right\}$$

The sets Ω_j^+ and Ω_j^0 are defined similarly as in the base model. The set Ω_j^- is the set of those drivers that are allocated to the slow road irrespective of the allocation of driver j.

Proposition 5. *The first-best allocation when there are two identical roads, $x^{FB,2}(\theta)$ is implemented by the following payment rule, which specifies for all $j \in 1, ..., n$:*

$$p_j^{FB,2}(\theta) = \sum_{i \in \Omega_j^+} b\theta_i(k^{**} - k_{-j}^{**}) + \sum_{i \in \Omega_j^0} \theta_i b\left(n - k^{**} - k_{-j}^{**}\right) + \sum_{i \in \Omega_j^-} \theta_i b\left(k_{-j}^{**} - k^{**}\right) \tag{5.2}$$

Proof. The proof follows along the same lines as the proof of Proposition 2. □

The main difference to the payments in Proposition 2 results from the fact that the slow road also becomes congested. The first two terms in equation 5.2 are analogous to the terms in equation 3.1. The first term captures the externality of driver i on those drivers which remain on the fast road when driver i is added to the optimization problem. The second term captures the externality of driver i on those drivers that were on the fast road when driver i was not considered in the optimization problem, but are assigned to the slow road when driver i is considered in the optimization problem. The third term, which was not present in equation 3.1 represents the externality on the drivers that remain on the slow road when driver i is added to the optimization problem.

When there is a slow road without congestion effects, moving driver i from the slow road to the fast road or adding more drivers to the slow road had no externality on those who were on the slow road. However when there are two identical roads the addition of more drivers to the slow road increases the travel time of those drivers who remain on the slow road. We have again normalized payments to be such that those drivers who do not affect the final allocation pay zero transfers. When there is congestion on both roads, this however means that drivers on the slow road may also pay a positive transfer. Intuitively drivers need to make a transfer payment whenever they affect the allocation, since in that case they impose an externality on others. In the case of two identical roads the number of drivers allocated to the slow road affects the travel time on the slow road.

More precisely, consider a driver i such that i is allocated to the slow road at the efficient allocation. When driver i is not considered in the maximization, the benefit of adding another driver to the fast road is lower than when the effect on driver i is also considered. This is because when driver i is not considered, she implicitly has a value of $\theta_i = 0$, meaning she does not care about congestion. This may lead to more drivers on the slow road when i is not considered, than when i is considered. Hence driver i may influence the final allocation with her report even though it does not affect the road to which she is allocated. Note however that while she uses the slow road in both circumstances, the travel time on the slow road will be lower when driver i is considered. As a consequence drivers do not just pay for faster travel on the fast road, but also for faster travel on the slow road.

Finally, notice that the payment schedule that implements the first best depends on θ_i only through its effect on k^{**} which implies that the payment schedule is again a weakly increasing step function. The result that a single Pigouvian price is not an efficient mechanism thus continues to hold.

Even Number of Drivers In this section we so far assumed that there was an odd number of drivers. This facilitated the search for an efficient allocation, as we could use the fact that there would be a slow and a fast road at the efficient allocation. We now show that in the case of an even number of drivers, the travel time on both roads may be the same at the efficient allocation.

Proposition 6. *Suppose n is even and $\theta_i = \hat{\theta}$ for all i and let $x^{FB,2}(\theta)$ be the efficient allocation. Then we have that $\sum_i x_i^{FB,2}(\theta) = \frac{n}{2}$.*

To see this, consider an allocation such that $\sum_i x_i^{FB,2}(\theta) = \frac{n}{2}$. Now consider changing one x_i from 0 to 1. The effect on those on road 1 is that the travel time increases by b which imposes a welfare loss of $(\frac{n}{2}+1)b\hat{\theta}$. The effect on those on road 2 is that the travel time decreases by b which leads to a welfare gain of $(\frac{n}{2}-1)b\hat{\theta}$. Hence there is a net loss of $2b\hat{\theta}$. For any further driver that is added to the first road there will be an additional net welfare loss of $2b\hat{\theta}$. The same applies when we consider moving drivers from the first road to the second road. As a result, the uniquely optimal allocation is to have the same number of drivers on both roads.

It should be noted that even with heterogeneous values of time it will continue to be optimal to have the same travel time on both roads, as long as the degree of heterogeneity is sufficiently small. However, if there is sufficient heterogeneity as indicated by the empirical literature, then also with an even number of drivers there will be a fast and a slow road at the efficient allocation. As we already analyzed this case for an odd number of drivers, we do not consider this for an even number of drivers as the analysis will be identical.

6 Congestion Pricing in Practice

Mechanism design is often criticized for being too complicated for real world implementation. In this section we will discuss currently used or proposed road and congestion pricing schemes around the world and explain how they relate to our mechanism. If applicable, we will suggest how to modify some of these schemes to approach our proposed mechanism and thereby achieve efficiency gains. Moreover, we will explore how our mechanism relates to incentives and special regulations regarding ride sharing and challenges of initial incomplete implementation.

Congestion pricing is a specific aspect of *road pricing* which attempts to (at least partially) internalize the marginal social cost of congestion. Even though the economic interest in road pricing emerged from the congestion problem (Pigou, 1920; Knight, 1924), road pricing is nowadays also concerned with local and global environmental externalities, as well as recovering the cost of infrastructure development (see for instance Morrison, 1986; May, 1992). In practice, the vast majority of road pricing schemes currently in place[9] do not contain an explicit congestion pricing element in the sense of price discrimination with respect to the level of congestion. These general schemes may only affect the congestion externality by reducing overall demand for car rides. Some systems however address the congestion externality in a more sophisticated way by charging higher prices during certain periods (e.g. the rush hour) and/or at congestion-prone areas.

To the best of our knowledge there is no congestion pricing system in place which asks drivers to report their value of travel time; however, part of the idea of our mechanism-design approach is frequently *implicitly* implemented without using monetary transfers. For instance, in case of accidents, ambulances have a high value of travel time since any delay might potentially cause a loss of life. Thus, regulation is usually in place to give ambulances privileged use of roads. Similarly, convoys transporting heads of state are often given privileged use of public roads to

[9]For instance per usage or per distance charges (Ordinary tollways, truck road pricing in Germany, highway fees in e.g. France and Italy, etc.) or a vignette to have the permission to use a road network (e.g. the Austrian or the Swiss highway system) for a certain period of time.

ensure timely arrival times, while increasing travel time for other drivers. In terms of our model, these regulations are efficient if the value of time for one driver (i.e. a convoy or an ambulance) is large enough such that the efficient allocation requires that driver to have a very low travel time.

Road pricing with a focus on alleviating congestion is currently only implemented in the urban context and only in a handful of cities around the world; however, in many cities, the implementation of such a system has been debated. A particularly simple form of congestion pricing is *cordon area congestion pricing*, i.e. a fee to enter a specific (congestion prone) area of the city. In many cases, price differ mainly for the purpose of achieving other objectives, for example environmental ones (e.g. in London or Milan).[10] If the fee structure of road prices is not flexible with respect to traffic conditions, the resulting level of congestion is unlikely to be efficient. Even though Leape (2006) argues that the introduction of the London congestion charge is "a triumph of economics" and led to time savings and better reliability of transport in general, he only finds small positive net benefits. Prud'homme and Bocarejo (2005) on the other hand argue that the economic benefits of the congestion charge represent less than 60% of the economic costs.

More complex systems are in place in the Swedish cities Gothenburg and Stockholm (The Swedish Transport Agency, 2015). These cities and the city state Singapore (Land Transport Authority, 2015) have more flexible systems, in which the toll is charged automatically and the prices vary over the course of the day and at different places. In Singapore, payments even vary with respect to traffic conditions[11]; hence, these systems are more adequate in charging the actual congestion externality, instead of giving an access ticket to a certain area of the city. Still, our mechanism could improve on such a system by not only taking into account the traffic level but also variations in the valuation of travel time. Moreover, our mechanism improves the existing systems by increasing the amount and the quality of the information. From a technical point of view, extending the Swedish or the Singaporean system would consist in offering a menu of travel times with respective prices to reveal the value of travel time, a dialog which could for instance be done by a simple smartphone app or on-board navigation systems.

Another concern regarding the practical implementation is a situation in which not all drivers participate in our mechanism right from the start. However, we do not believe this to be an insurmountable obstacle. For instance, if a mechanism designer considers to introduce our system by designating fast lanes only for people using our proposed mechanism, she could simply ban all drivers who are not using the mechanism from that road. Alternatively, cars could be charged based on the time it took them to reach their destination, assuming that they would have chosen the actual price and time combination offered by the mechanism designer. Charging without detailed notification about the cost is for instance currently done in Stockholm (The Swedish Transport Agency, 2015) and hence not an uncommon feature of road pricing.

Another area of transportation research which gained interest due to technological progress is ride sharing (see Furuhata et al., 2013 for a recent survey). Ride sharing refers to any action in which travelers share a vehicle to go from their origins to their destinations. Public transport is

[10]The London congestion charge for instance is a daily price (currently £11.50) for entering one zone in the inner city center which will be charged between 7:00 a.m. and 6:00 p.m.. However, there are various discounts for cars with low emissions, vans, residents, etc (Transport for London, 2015).

[11]Traffic conditions are estimated based on retrospective data and current traffic flows, similar to a real time weather forecast.

usually regarded as a cheap but inconvenient form due to the fixed routes and schedules. More flexible solutions could be obtained by so-called dynamic ride sharing, i.e. a real-time matching of travelers with similar itineraries, using cars or in the future autonomous vehicles. With the advent of new technologies such as GPS and smart-phones, dynamic ride sharing gained considerable commercial and academic interest.[12] Even though we ruled out the possibilities that drivers would like to engage in car pooling or ride sharing in our set-up; the introduction of any per vehicle fees generates a potentially large incentives to do so[13], which in practice would further reduce the extent of congestion problems.

Incentives to engage in car pooling and ride sharing are already implicitly implemented in many cities, e.g. the designation of special bus lanes, such as in Berlin, which can only be used by buses and taxis. This could be efficient in the context of our model, since a bus usually carries several people so that their value of time should be added, implying a large value for time of the bus overall. Similarly, the High-Occupancy Vehicle Lanes in California allow faster travel to cars which seat more passengers. Here the expectation is also that vehicles carrying more people implicitly have a higher value for time. The difference to an optimal mechanism is that the allocation is implemented through rigid rules which are based on implicit assumptions regarding the value of time of certain types of vehicles, while in our mechanism this information would be reported directly by different vehicles. In our congestion externality framework it is not clear from an ex ante perspective whether such a feature, namely the free access to the fast road for cars with a certain minimum number of drivers is desirable, since it increases the number of cars on the fast road and hence might hinder welfare gains from higher velocity for the drivers with a high valuation of travel time. Once our mechanism would be implemented and eliminating any discounts for cars with more than one driver, these types of inefficient allocations would disappear.

7 Further Applications

Congestion problems arise not only in the case of road pricing but in many other applications that are of interest to economists and policymakers. In this section we discuss a number of such additional applications in a preliminary manner.

7.1 Network Neutrality

The congestion problem studied so far is similar to the congestion problem that content providers (CP), similarly to drivers, face in the context of the recent net-neutrality debate (Economides and Hermalin, 2012; Economides and Tåg, 2012). Some CPs require a fast internet connection to offer their service, such as video-call services, compared to other CPs who offer a less time-sensitive

[12]Kleiner et al. (2011) for instance, evaluate the potential of a VCG mechanism for the efficient allocation of drivers in such a dynamic ride sharing system.

[13]When Singapore introduced congestion pricing in 1975, the number of cars entering the center decreased by 41.6% initially and 22.9% in the long-run, (Morrison, 1986). Interestingly, Leape (2006) does not report any effects of the London congestion charge on car pooling

service, e.g. video streaming. The sensitivity of their service to the connection speed is likely to be private information of the CPs since it depends mainly on the value consumers derive from those services. The Internet service provider (ISP) has a number of ways of delivering the content of CPs to users, each of which may lead to a different latency. The mechanism studied above offers an efficient solution to this congestion problem.

Note that in the context of data transmission, we can no longer assume that each CP is identical in terms of how much bandwidth capacitiy is required for service provision. Thus CPs can have heterogeneous effects on the overall level of congestion. This is not captured in our model so far. Nevertheless, the qualitative features of our mechanism should continue to hold. In particular, it is doubtful that a single Pigouvian price is an efficient mechanism when the number of CPs is finite.

To account for the particularities of congestion in data transmission, we adjust our model. There are n CPs, indexed by $i = 1, ..., n$ and a single ISP who offers access to a congestible high-speed data link that can be used by the CPs to send data to a consumer or a group of consumers and an alternative low speed data link, that is not congestible. CP i wishes to send a data package of size $\alpha_i > 0$ to consumers where the α_i are commonly known. This is because an ISP can observe the amount of data sent via its connections. The latency, i.e. the time it takes to send the data from a CP to consumers is given by $C\left(\sum_{i=1}^{n} \alpha_i x_i\right)$ where $x_i \in \{0, 1\}$ indicates whether or not CP i is using the ISP's high speed data link and $C(\cdot)$ is defined as in Section 2. This implies that different CPs have different effects on congestion. The value of using the high-speed data link to a CP is given by $u_i = v - \theta_i C\left(\sum_{i=1}^{n} \alpha_i x_i\right) - p$, while the value of using the low-speed data link is given by $u_i = v - \theta_i \bar{t} - p$, where θ_i is the private information of CP i concerning how critical latency is for the services offered by the CP and p is the payment made to the ISP. In the special cases when $\alpha_i = \bar{\alpha}$ for all i, then the results of our existing model can be directly applied. However in practice different CPs will wish to send data packages of different sizes. For example, some CP only needs to send an e-mail while another CP offers high-definition video conference calls. As a result it is no longer the case that CPs with a high value of θ_i will necessarily use the high-speed data link. This is shown in the following example.

Example Suppose $n = 2$, $C(x) = x$, $\alpha_1 = 0$, $\alpha_2 = 10$, $\bar{t} = 5$ and $\theta_1 = 1$, $\theta_2 = 10$. For example CP 1 could wish to send an email, while CP 2 tries to perform a video call in a rural region with insufficient bandwidth. Clearly, allocating CP 2 to the high-speed data link will imply a worse latency than allocating CP 2 to the low speed data link. Hence even though $\theta_1 < \theta_2$, CP 1 is optimally allocated to the high-speed data link.

The challenge arising when different CPs have different effects on congestion is that the second part of Lemma 1 might no longer hold. However if one is willing to make a few restrictions then parts of our existing results can be recovered. Irrespective of the restrictions one places on the model, a VCG-type mechanism could still be applied in this context. The main difficulty that arises lies in characterizing the resulting payment schedules and allocation rules.

Two Possible Package Sizes An alternative case to consider is one where for all i, $\alpha_i \in \{0, \alpha\}$. For example, there could be some services, such as sending an e-mail, that take up a negligible amount of bandwidth and other services that take some larger, but common, bandwidth such as streaming videos. In that case, it is clear that at an efficient allocation any CP for which $\alpha_i = 0$ would be allocated to the high-speed data link, irrespective of the vector of time-sensitivities. For those CPs for which $\alpha_i = \alpha$ the results of Sections 2 and 3 would continue to hold, taking account of the fact that congestion on the high-speed data link also affects CPs with $\alpha_i = 0$.

Negatively related Time-Sensitivity and Package Size Suppose it holds that for $i \neq j$ $\theta_i > \theta_j$ implies that $\alpha_i < \alpha_j$.[14] Then all the results of Lemma 1 continue to apply: CPs with a high value of θ_i will be allocated to the high-speed data link. The expression for the marginal effect of adding another CP's data package to the high-speed link would need to be adapted to take account of the heterogeneous congestion effects, but otherwise all of the logic of Sections 2 and 3.

7.2 High Frequency Trading

Our model may also be applied to some problems arising in financial markets. So-called high frequency trading (HFT) firms use speed advantages in order to benefit from highly transitory price changes lasting for tiny fractions of a second. The company "Spread Networks" built a tunnel connecting a stock exchange data center in New York to another in Chicago and sold access to this high-speed data link to several high frequency trading firms. Lewis (2014) writes that after hearing of the offer for access to the new data link one of the prospective customers asked whether it would be possible to *increase* the price. While such a request may appear puzzling, the model we have studied so far can lead to situations in which some agents are willing to pay to exclude other agents from a resource due to externalities that are present.

We again let θ_i denote the private information held by some HFT firm $i = 1, ..., n$. In this case we interpret θ_i to be the privately known skill of the HFT firm in benefiting from temporary price differences. This skill may depend on the type of code used by the firm or on the ability of the programmers working for the HFT firm. In our benchmark congestion pricing model, the value of being allocated to the fast road was only a function of the number of other drivers using the same road. In the context of HFT firms however, it appears more likely that the value of access to a high-speed data connection not only depends on the number of other HFT firms that have access, but also on their ability to exploit the same type of price differences as other firms. Hence we will write the utility to an HFT firm of having access to the high-speed data link as:

$$u_i = \theta_i \left(v - \sum_{j \neq i} \theta_j x_j \right) x_i - p$$

Note that this formulation is similar to the formulation we employed when analyzing network neutrality, where the "package size" is assumed to equal the privately held information fo the HFT firms. The difference is that package size was observable in the previous section, while the ability of an HFT firm is private information.

[14] Notice that in reality one would expect the opposite to hold, namely that those data packages that are the most time-sensitive are also those that take up the most bandwidth.

While it may be of interest to study the general problem of efficiently allocating access to the high-speed data link, we are going to focus on the case when there are just 2 HFT firms present, so $n = 2$. We focus on finding an efficient allocation.

Proposition 7. *It is efficient for both HFT firms to use the high-speed data link if and only if* $v \geqslant 2\max\{\theta_1, \theta_2\}$*. Otherwise the firm with the higher value of* θ_i *is the only one to use the high-speed data link.*

Proof. If both HFT firms are allocated to the data link, total welfare is $(v - \theta_1)\theta_2 + (v - \theta_2)\theta_1$. If only HFT firm i is allocated to the data link then total welfare is $v\theta_i$. Hence having both firms on the data link is optimal if and only if $v(\theta_1 + \theta_2) - 2\theta_1\theta_2 \geqslant v \max\{\theta_1, \theta_2\}$ which can be re-arranged to $v \geqslant 2\max\{\theta_1, \theta_2\}$. Clearly, if it is not optimal for one firm to use the data link, then the firm with the higher value of θ_i should use it. $\square$

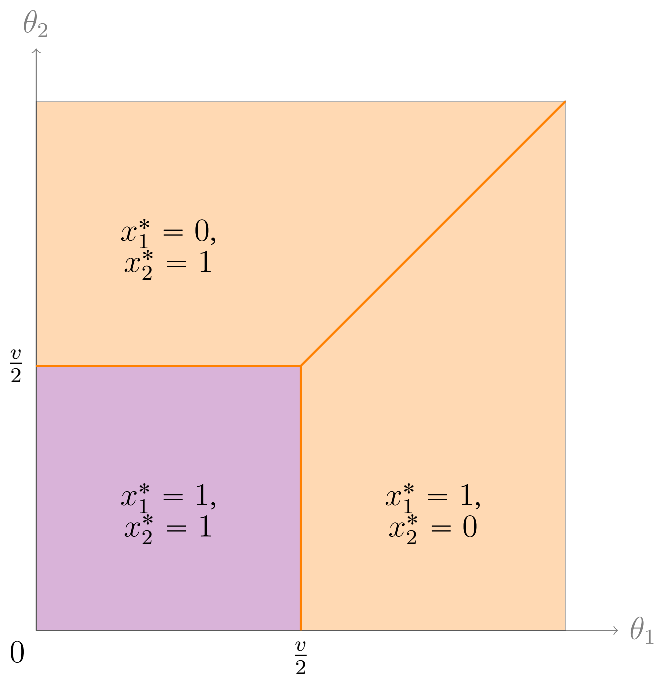

Figure 7.1: Efficient Allocation with Two HFT Firms

The set-up studied here is similar to mechanism design problems with interdependent values, as in Jehiel et al. (1999) and Jehiel et al. (2006). In the general set-up studied by these papers, it is not possible to implement efficient social choice functions. Our model differs from those papers in that the private information held by HFT firms is unidimensional, while the private information in those papers is multidimensional. We additionally put more structure on the problem by assuming particular functional forms.

We show that the strict impossibility of implementing any non-constant social choice function of Jehiel et al. (2006) does not apply in our context. Given the nature of the problem we no longer focus on dominant strategic incentive compatibility, but instead focus on the weaker notion of ex-post incentive compatibility, which requires agents not wishing to deviate from truth-telling from their report given that the other agents have reported their types truthfully.[15] The distinction arises because dominant strategy incentive compatibility would additionally require truth-telling from agents if others have lied, so that while the utility of agent i is given by $\theta x_i(v - \theta_j x_j)$, the other agent $j \neq i$ has reported some $\hat{\theta}_j \neq \theta_j$. In the previous models this did not matter as the private information of agent j did not directly affect the utility of agent i. Ex-post incentive compatibility requires that for all i, and all $\theta_i, \hat{\theta}_i \in \Theta_i$ and $\theta_{-i} \in \Theta_{-i}$ we have:

$$\theta_i x_i(\theta)\Big(v - \theta_{-i} x_{-i}(\theta)\Big) - p_i(\theta) \geqslant \theta_i x_i(\hat{\theta}_i, \theta_{-i})\Big(v - \theta_{-i} x_{-i}(\hat{\theta}_i, \theta_{-i})\Big) - p_i(\hat{\theta}_i, \theta_{-i}) \quad \text{(ExPost IC)}$$

Given our relaxed notion of incentive compatibility, we can show that the efficient allocation is implementable, implying that the results of Jehiel et al. (1999) are not applicable in our set-up despite featuring interdependent preferences.

Proposition 8. *The efficient allocation is implemented in the sense of ex-post incentive compatibility by the following payment schedule:*

$$p_1^*(\theta) = \begin{cases} 0 & \theta_1, \theta_2 \leqslant v/2 \\ 0 & \theta_1 < \theta_2,\ \theta_2 > v/2 \\ v\theta_2/2 & \theta_2 \leqslant v/2,\ \theta_1 > v/2 \\ v\theta_2 & \theta_2 > v/2,\ \theta_1 \geqslant \theta_2 \end{cases}$$

$$p_2^*(\theta) = \begin{cases} 0 & \theta_1, \theta_2 \leqslant v/2 \\ 0 & \theta_2 \leqslant \theta_1,\ \theta_1 > v/2 \\ v\theta_1/2 & \theta_1 \leqslant v/2,\ \theta_2 > v/2 \\ v\theta_1 & \theta_1 > v/2,\ \theta_2 > \theta_1 \end{cases}$$

Proof. Suppose $\theta_2 \leqslant v/2$. Then HFT firm 1 pays 0 when reporting $\hat{\theta}_1 \leqslant v/2$, giving a pay-off of $\theta_1(v - \theta_j)$, while paying $v\theta_2/2$ when reporting $\theta_1 > v/2$, giving a pay-off of $\theta_1 v$. Hence the difference in pay-off is given by $v\theta_2/2 - \theta_1\theta_2$, which is positive whenever $\theta_1 \leqslant v/2$, implying that HFT firm 1 optimally tells the truth.

Suppose $\theta_2 > v/2$. Then HFT firm 1 pays 0 when reporting $\hat{\theta}_1 \leqslant \theta_2$, giving a pay-off of 0, while paying $v\theta_2$ when reporting $\hat{\theta}_1 > \theta_2$, giving a pay-off of $(\theta_1 - \theta_2)v$. Hence the difference in pay-off is given by $v(\theta_1 - \theta_2)$, which is positive whenever $\theta_1 \geqslant \theta_2$, implying that HFT firm 1 optimally tells the truth.

A similar logic applies for HFT firm 2.

□

[15] In the case of our road congestion pricing set-up, dominant strategy compatibility and ex-post incentive compatibility are equivalent.

7.3 Keyword Search Auctions

Effects similar to congestion are also prevalent in keyword search auctions on the internet, which have first been analyzed by Varian (2007) and Edelman et al. (2007). These papers assumed that the total number of clicks of an ad depended solely on the position on which the ad is shown. However the attention and hence the number of clicks an ad receives on a website likely depends on the total number of other ads displayed in the same impression. Hence advertisers might be willing to pay to ensure that fewer other advertisers are shown on the same impression. In practice however the type of externalities that arise in this context are likely to be more complex than those arising in the application to traffic congestion pricing. Jeziorski and Segal (2012) empirically analyze clicking behavior of consumers. They find that the number of times an ad is clicked depends to a significant and economically meaningful extent on the identity of other ads shown both in higher and lower positions.

We expect that the following complexities would need to be added to our model to make it realistic. First, the attention diverted from an advertisement does not just depend on the number of other ads shown, but also on their identity and perceived similarity. For example, two ads for pizza delivery might draw attention away from each other to a larger extent than an ad for pizza delivery and one for sushi delivery food. Second, the attention received by an ad will also depend on the precise placement of the ad. For example, ads placed on top of the generic search results receive more clicks than those placed at the bottom. Overall, these complexities imply that the results of Lemma 1 might no longer apply. Note however that to the extent that the magnitude of these complexities is known to the mechanism designer, which appears realistic, a VCG-type mechanism is still applicable, but may be hard to characterize.

8 Conclusion

This paper shows that the traditional Pigouvian approach of internalizing social costs of congestion by setting a single congestion charge applies only when there are infinitely many drivers. The generally optimal solution involves charging drivers a variety of different prices depending on the speed on which they want to travel on a road, i.e. on the number of other drivers on that road.

One major advantage of applying mechanism design to congestion problems is that it obviates the need to conduct detailed econometric studies to estimate the distribution of the value of time. Moreover, other externality-related objectives, such as pollution-taxes could be easily implemented on top of our mechanism. The mechanism design approach requires, however, that each driver may communicate instantaneously with the mechanism designer. Given modern communication technology, we do not believe this is a major issue.

Requiring drivers to directly report their value of time may be impractical because such mechanisms might be hard to explain to people. Instead other solutions which that allow drivers to choose from a simplified menu of prices and arrival times may be easier to use by people. For example people could be offered a choice of three categories: fast, normal and slow. In that case they would be charged a premium for choosing faster options. The exact user interface, would need to be investigated further before such congestion pricing mechanisms are implemented.

Critics of congestion pricing schemes often point towards adverse effects on lower income drivers. In the case of revenue maximization, there is an incentive for the mechanism designer to treat drivers with a lower distribution (i.e. lower value of time on average) more favorably to extract more surplus from drivers with a relatively good distribution. In practice, this means that lower income drivers might optimally be favored relative to higher income drivers by a revenue maximizing congestion pricing scheme. Ultimately, distributive outcomes from congestion pricing also depend on how revenues generated in such a scheme are used.

One other potential concern is that congestion pricing mechanisms as envisioned here would provide too much information on citizens' travel behavior. However there are ways in which congestion pricing could be implemented without collecting detailed personal information. Charges for traveling in an autonomous vehicle could technically be depersonalized.

The main remaining theoretical question is how to extend our model and mechanism to allow for endogenous departure times. So far we assumed that all drivers traveled at the same time. In practice a high congestion charge at a particular time of the day is likely to lead to drivers switching their travel to other times of the day, when congestion charges are lower. So far our model does not capture this. We expect that an efficient mechanism that takes such dynamic issues into account will allow further efficiency gains. Congestion at a particular time of the day could then be alleviated not only be rerouting some traffic but also by postponing some trips. However information acquisition would become somewhat more complicated, since drivers also have private information regarding their preferred departure times. Therefore the mechanism designer would face a multi-dimensional screening problem. The decisions made by a mechanism would also depend on expected future traffic flows, for example as in Singapore now. We leave this for future research.

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