out using the classical approach. Beside some other facts, it gave relatively simple proofs that the dimension of the space of Vassiliev invariants of degree $\leq n$ on certain knot classes is finite (arborescent knots), and in some cases even exponential upper bounds in n for this dimension (e. g., rational knots, closed 3 braids), something, which was not yet achieved by chord diagrams.

Moreover, while the Kontsevich-Drinfel'd approach (used in [CD]) works only over zero (field) characteristic, our arguments with braiding sequences hold for any zero divisor free ring, in particular the fields $\mathbb{Z}_p$, p prime.

Definition 1.6 *For some odd $k \in \mathbb{Z}$, a k-braiding of a crossing p in a diagram D is a replacement of (a neighborhood of) p by the braid σ_1^k (see figure 1). A braiding sequence (associated to a numbered set P of crossings in a diagram D; all crossings by default) is a family of diagrams, parametrized by $|P|$ odd numbers $x_1, \ldots, x_{|P|}$, each one indicating that at crossing number i an x_i-braiding is done.*

Any Vassiliev invariant v of degree at most k behaves on a braiding sequence as a polynomial of degree at most k in $x_1, \ldots, x_{|P|}$ (see [St4] and [Tr]), and this polynomial is called the braiding polynomial of v on this braiding sequence.

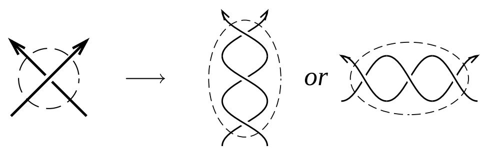

Figure 1. Two ways to do a -3-braiding at a crossing.

2 The results of this thesis

The subject of the present thesis are combinatorics of chord diagrams and asymptotics of Vassiliev invariants.

In sections 3 and 4 we will derive some (purely) enumerative results on special kinds of chord diagrams. Although not directly related to Vassiliev invariants, these results provide a glimpse of the combinatorial complexity of chord diagrams – already for chord diagrams with properties, which are easy to define, the enumeration is rather hard and requires additional ideas.

We show consecutively how to count in a non-brute force way all chord diagrams of given degree, all chord diagrams up to mirroring, all chord diagrams with an isolated chord (the ones sent to zero by the FI relation), all chord diagrams with an (isolated) chord of length one, chord diagrams, whose intersection graph is connected and those for which it is a tree.

In section 5 we will use combinatorial techniques to relate the enumeration of special chord diagrams to the enumeration of Vassiliev invariants and will prove the asymptotical upper bound $D!/1.1^D$ for the number of Vassiliev invariants in degree D.

The basic idea for this improvement is to work with linearized chord diagrams (LCD's) and the order of chord basepoints from left to right.

In section 6 we will use the techniques of section 5 and the result of Chmutov and Duzhin [CD2] to deduce a lower bound for the number of all Vassiliev invariants and discuss the relation between the asymptotics of primitive and all Vassiliev invariants. At the same time, we give a summary on what we know about the asymptotics of Vassiliev invariants.

Finally, in section 7 we use the rather different approach of braiding sequences to prove exponential upper bounds for the number of Vassiliev invariants on knots with bounded braid index and arborescent knots.

Parts of this work can be found in several papers of mine [St2, St6, St8, St9, St10].

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