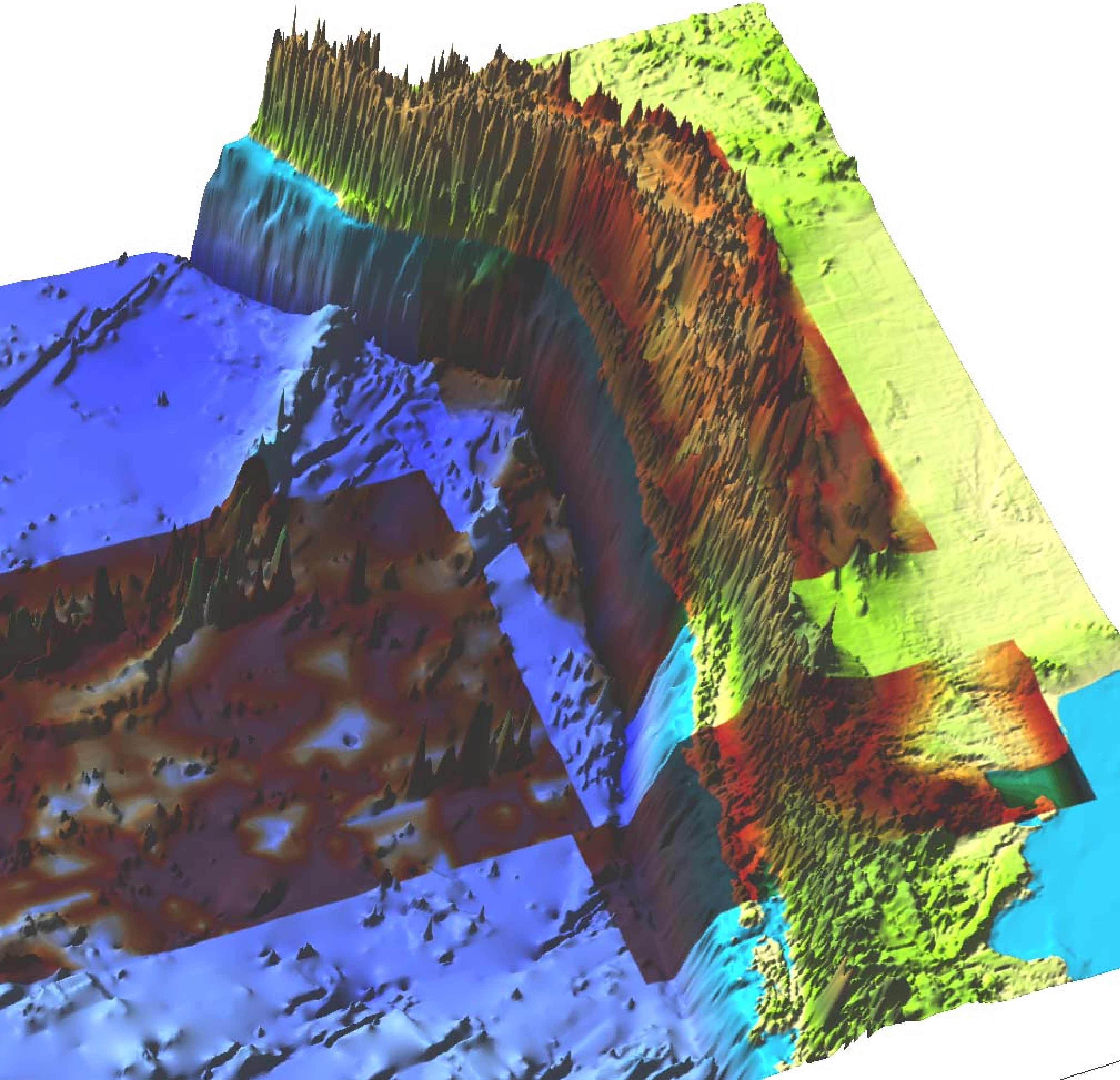
A NEW ANALYTICAL SOLUTION FOR THE CALCULATION OF FLEXURAL RIGIDITY:

SIGNIFICANCE AND APPLICATIONS



„Die Liebe zum Lernen ist der Weisheit verwandt.“ (Konfuzius)

A NEW ANALYTICAL SOLUTION FOR THE CALCULATION OF FLEXURAL RIGIDITY: SIGNIFICANCE AND APPLICATIONS

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ZUSAMMENFASSUNG

Im Jahre 1939 wurde von VENING-MEINESZ eine neue Theorie entwickelt, welche die Rigidität der Lithosphärenplatte innerhalb isostatischer Betrachtungen mit berücksichtigte. Dazu wurde eine Differentialgleichung 4. Ordnung verwendet, welche das Deformationsverhalten einer dünnen elastischen Platte beschreibt. In der Vergangenheit wurde das Problem in den Frequenz-Bereich verlagert, und die Gleichung wurde mittels der Spektralmethoden lösbar gemacht. Aber bezüglich der Anwendung der Kohärenz- und Admittanzmethode auf die Kontinente wurde ihre Nützlichkeit aufgrund der Nachteile, welche durch den Spektralansatz entstehen, in Frage gestellt.

Dieser Ansatz bedingt eine Durchschnittsbildung, welche im Falle einer sich räumlich variierenden Biegesteifigkeit/Rigidität dazu führen kann, dass jene Variation nur bis zu einem begrenzten Maβe aufgelöst werden kann. Darüber hinaus ist für das Untersuchungsgebiet eine Seitenlänge von mindestens $375 km$ erforderlich. Ein weiteres Problem tritt im Falle niedriger Topographie auf, da kleinere Spektralwerte der Topographie zu Instabilitaeten innerhalb der Anwendung führen können. Durch die Verwendung der Konvolutionsmethode (BRAITENBERG ET AL. 2002) und darüber hinaus mit dem Nutzen der neu entwickelten analytischen Lösung der obig eingeführten Differentialgleichung können diese Nachteile aber überwunden werden. Diese analytische Lösung wurde aus drei verschiedenen Lösungen nach HERTZ 1884 entwickelt und für die geologischen Wissenschaften anwendbar gemacht.

Die analytische Lösung wurde auf die ozeanische Lithosphäre im Bereich des Pazifik (Nazca-Platte) und auf die kontinentale Lithosphäre im Bereich der Zentral Anden und der Patagonischen Anden angewendet. Die Ergebnisse der Rigiditätsverteilung wurden mit den von den Mitgliedern der SFB267 Gemeinschaft entwickelten Ideen und Konzepten verglichen. Die Rigiditätsverteilung ist durch eine gute Korrelation mit den tektonischen Einheiten und Störungssystemen charakterisiert.

Bisher wurde die elastische Dicke und die flexurelle Rigidität synonym verwendet. Aber die analytische Lösung führte zu einem neuen Verständnis und einer neuen Interpretation der elastischen Dicke. In Anbetracht der Untersuchungen zur Signifikanz der Inputparameter ist es zulässig mit einem konstanten Wert für die Schwere und dem Poisson- Verhältnis zu arbeiten, denn dies wird nicht zu signifikanten Unterschieden im Ergebnis führen. Dies ist nicht für das Elastizitätsmodul gültig, denn dieser Parameter ist der entscheidende Faktor für das Deformationsverhalten. Daher kann die elastische Dicke auch als äquivalente Plattendicke für eine Platte konstanten Elastizitätsmoduls definiert werden. Zudem wurde herausgefunden, daß das Temperaturmoment in den Untersuchungen des Deformationsverhaltens mit berücksichtigt werden muss. Damit kann die beobachtete Variation der elastischen Dicke durch die Temperaturverteilung und die Veränderung des Elastizitätsmoduls erklärt werden.

Zusätzlich wurde gezeigt, daß die Berechnungen mittels der Differentialgleichung und der analytischen Lösung sowohl für die Krusten/Mantel Grenze als auch die Lithosphären/Asthenosphären Grenze gültig sind. Dabei ist entscheidend, an welcher Grenzfläche die Änderung des Elastizitätsmoduls stattfindet.

ABSTRACT

In 1939 a new concept was introduced by VENING-MEINESZ proposing that the flexural strength of the lithosphere must be considered for isostatic models. A 4th order differential equation describing the flexure of a thin plate was developed. In the past the equation has been solved in frequency space using spectral methods (coherence and admittance). However, the admittance and coherence techniques have been questioned when applied to continental lithosphere. Both methods require an averaging process; therefore the variation in rigidity may be retrieved only to a limited extent. A large spatial window with a side length of at least 375 km is required over the study area. And, in where the input topography is characterized by low topographic variation, the method becomes unstable.

These problems can be overcome by calculating the flexural rigidity with the convolution approach (BRAITENBERG ET AL. 2002) and furthermore with the use of a newly derived analytical solution of the differential equation mentioned above. This solution was developed out of three solutions from HERTZ 1884 and has been made applicable to geological science. The analytical solution has been applied to both oceanic lithosphere (Nazca plate) and continental lithosphere (Central and Patagonian Andes). The resulting flexural rigidity values and their variations have been compared with the ideas and concepts developed by the members of the SFB267 community, and correlate well with tectonic units and fault systems.

In the past the elastic thickness has been used synonymously for the flexural rigidity. However, the analytical solution leads to a new interpretation and meaning of the elastic thickness. It is shown that it is sufficient to operate with a constant value for both gravity and Poisson's ratio, as the variation of either parameter does not lead to a significant change in the distribution of flexural rigidity. Young's modulus is shown to be the driving factor for the flexural deformation. A temperature moment must also be taken into account in flexural investigations. Thus, the variation of the elastic thickness can be explained by temperature distribution and a change of the Young's modulus. A new definition of elastic thickness can be obtained: the value of the calculated elastic thickness is equivalent to the value of thickness of a corresponding plate described by a constant Young's modulus.

Computations using the differential equation are valid for the crust/mantle interface (Moho) as well as the lithosphere/ asthenosphere boundary. The calculated boundary surface can be shifted at the position of the boundary at which a significant change of Young's modulus takes place.

CONTENTS

MOTIVATION

Since early 1993, members of the collaborative research program SFB 267 “Deformation processes in the Andes” have been working on establishing a broad scientific basis of the Andes and its surroundings.

The research focus is on two main study areas. In the northern study area between 10° and 30°S, the so-called Central Andes, the formation of a non-collisional plateau has played the dominant role. This area is characterized by an erosive forearc, a significant topographic relief, and crustal thickening. In the southern study area, between the 37° and 42°S, the so-called Southern Andes, a 'standard type' of continental subduction orogen without plateau formation has developed in spite of similar plate-kinematical boundary conditions. This contrasting study area is characterized by an accretive margin, a thinned crust, a mostly extensional tectonic history, and a relatively subdued topography.

The different projects within the research program SFB 267 aim to evaluate the parameters, which are identified as the main factors controlling forearc and orogen evolution.

The present thesis is integrated in the SFB- subproject F1 (KUKOWSKI ET AL. 2002) and F4 (GÖTZE ET AL. 2002). The subproject F1 investigates the transmission of forces as well as related deformation at convergent margins and plans a quantitative investigation of the 3D stress patterns, strike-slip fault evolution and mass transfer modes with several simulation methods. These include e.g. sandbox analogue experiments and Finite Element modeling. The subproject F4 models regional potential field data and provides new insights into the large-scale distribution of controlling parameters. They aim to examine the rigidity and viscosity variations of the Andean lithosphere through the modeling of the isostatic gravity field and the geoid.

Dealing with rigidity calculations led to the idea of developing an analytical solution. Many numerical solutions exist in the term of Finite Element modeling, but the substantial characteristics cannot always be considered since the solutions are often approximated by the method of the weighted residuals. The motivation was to find a new analytical solution, which solves the differential equation describing the flexure of a thin elastic plate. A simple analytical solution exists only for the deflection of a thin plate due to line loads. However, the development of a new analytical solution would made possible to calculate analytically the deflection of a thin plate for any irregular shape of the topography.

A further motivation for conducting this thesis is to develop a software program, which effectively calculates the elastic parameters of the crust and lithosphere in order to interpret their mechanical behavior in a consistent way. Another motivation is to find general statements on the quantitative value of the results of rigidity distribution. Therefore, it is important to investigate the significance of the input parameters necessary for the rigidity calculation.

My personal motivation is to clarify the physical meaning of the elastic thickness. Additionally, I want to conduct methodical investigations to resolve the term of elastic deformation of the lithosphere and to find a mathematical description. My goal is to explain the scientific facts in a simple and comprehensive way.

CONCEPTS

This chapter presents a general explanation and simplification of the methods concerning the calculation of elastic thickness/flexural rigidity. The reader, familiar with the idea and concept of the elastic theory may skip this chapter.

This thesis considers the mechanics of the Earth's crust and the lithosphere. It is a part of classical mechanics. We aim to examine the characteristics of physical bodies as well as to find a mathematical description. Classical mechanics contain **universal** statements, which apply to all physical bodies as well as **individual** statements, which describe the characteristics of individual physical bodies. This description expresses the experience that outwardly identical bodies show a completely different behavior under the same conditions.
Only from the combination of these statements can mathematical models result, whose characteristics are comparable with the experimentally observable behavior of physical bodies (HAUPT 1977). In geophysics the individual characteristics of the Earth's crust and the upper mantle are investigated. These physical bodies are characterized by different material properties. Some physical bodies are easy to deform. Other physical bodies are more resistant to deformation. This individual characteristic can be described by the flexural rigidity. In addition the material parameters Young's modulus and Poisson's ratio are necessary for the description of the individual characteristics.

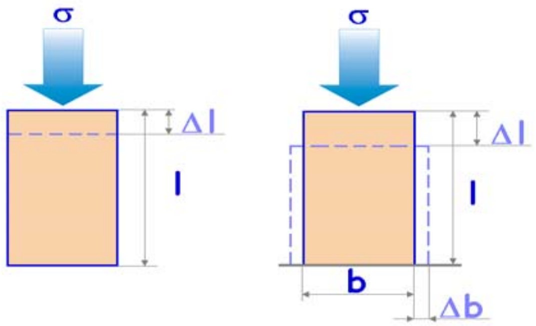

Figure 1.0.1) Different deformation of a physical body. The Young's modulus results from the ratio of length variation to acting stress . The ratio of length variation to change of width is called Poisson's ratio.

If stress affects a body, a deformation takes place (see Fig. 1.0.1). From the relationship of length variation $\frac{\Delta l}{l}$ to stress σ we achieve the Young's modulus E, if the material deforms elastically:

$$\frac{\Delta l}{l} = \frac{1}{E} \cdot \sigma \qquad \textbf{(1.0.1)}$$

Regarding the relationship of length variation $\frac{\Delta l}{l}$ to change of width $\frac{\Delta b}{b}$ we obtain the Poisson's ratio ν.

$$\frac{\Delta b}{b} = -\nu \cdot \frac{\Delta l}{l} \qquad \textbf{(1.0.2)}$$

Although physics of solids gives an answer to many questions, one could not find a continuous relation between the atomic and macroscopic behavior of a material. Therefore, a macroscopic description is generally used for mathematical expressions. Only average values of strain and stress are taken into account. The material is regarded as continuum and not as accumulation of discrete particles. Deformations are assumed to be small compared to the dimensions of the investigated body. We hypothesize a static equilibrium condition for the deformed body. The relation between deformation and internal forces caused by the load are described by material laws. The simplest of these is Hook's law. This law explains a linear relationship between stress and strain and provides the basis of elastic theory.
The realization of a mathematical description for the flexure of a plate due to a load is one goal of this work. In order to be able to provide a mathematical expression, one needs many idealizations of the observations. According to scientific research principle an idealization is permitted if it still considers the substantial features but leads to important simplification in application.

In the following a very simple example is given, in order to understand the way of computation of flexural rigidity. We imagine an elephant in the ring of a circus located on muddy ground. The elephant could be standing on a tarpaulin (example 1) or on a wooden floor (example 2). If we draw the analogue to geophysics, then the mud corresponds to the mantle of the Earth, the tarpaulin or wooden floor is the crust of the Earth and the elephant represents the topographic load. The tarpaulin floor represents a “soft crust”, whereas the wooden floor a rigid one. In the first example, the elephant would leave deep tracks in the mud, or -in other words- a local compensation would take place (see Fig. 1.0.2).

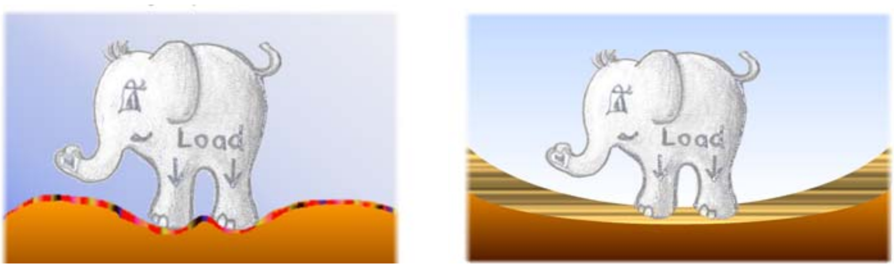

Figure 1.0.2) Simple analogue for describing flexural rigidity. An elephant is standing on a weak plane, e.g. a tarpaulin floor (see left side). He leaves deep tracks, because of the low rigidity of the plane. The load of the elephant easily deforms the tent plane. This leads to a local compensation. A situation of a more rigid plane is shown on the right side, e.g. wooden floor. Because of the higher rigidity, a global compensation takes place. For this reason, we can distinguish the rigidity of the planes if we compare the flexure of the planes with the tracks the elephant would leave.

In the second example, because the load of the elephant distributes over the surface, a global compensation would take place. The difference, between these two situations, is the mode of compensation due to the difference in the flexural behavior. We can therefore distinguish the rigidity of the crust if we look at the mode of isostatic compensation.

The tracks due the load of the elephant are demonstrated in the sketch 1.0.3. The geophysical analogues are the isolines of Moho undulation. For a soft plane (e.g. tarpaulin) we would be able to distinguish the 4 legs of the elephant. A corresponding example in the field of geophysics are the seamounts in Pacific Ocean, because of the lower rigidity we can see single "seamount traces" in the Moho surface.

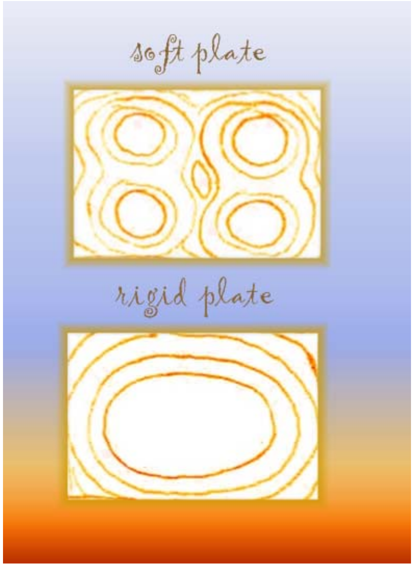

Figure 1.0.3) The sketch shows the tracks the elephant left in the ground (isolines for the depth). In the figure above the 4 legs of the elephant can been distinguish. The analogue is a seamount; because of lower rigidity around the seamount it is local compensated. The isolines in the lower figure are showing the global compensation. In case of a rigid plate no "tracks" would be left, and we wouldn't be able to distinguish the 4 legs of the elephant.

This means, if we obtain the pattern of the tracks or in the geophysics the Moho undulation, we can estimate the elastic properties of the plate for a known load. We can calculate the load from the topography. The two materials the elephant stood on have a different flexural rigidity, whereas the load is the same. (The flexural rigidity of the wooden floor is much higher). With this simple example it's easy understandable, how we can determine the flexural rigidity of the lithosphere, with information about the topographic load and the Moho undulation.

Since the long wavelength part of the gravity anomaly reflects in most cases the density contrast between crust and mantle, the long wavelength of the gravity anomaly corresponds to the "gravity Moho" undulation. In the following the Moho is called Crust-Mantel-Interface (short: CMI).
Therefore all methods of estimation of flexural rigidity/elastic thickness (see Chapter 1.2) can be described with this simple example. Either the relation between the topography and the CMI is investigated or the relation between the topography and measured gravity anomaly.

INTRODUCTION

State of the Art

The knowledge of the isostatic behavior of the lithosphere is important to provide statements concerning the conditions and processes of the mountain building. The spatial distribution of the flexural rigidity (D) and of the elastic thickness (T_e), as well as the rheology of the investigation area give information on the regional isostatic compensation. Furthergoing, the combination and correlation with results from the neighboring disciplines of geophysics, result in a differentiation of the lithosphere into significant structural units.
A differential equation of the 4th order describing the flexure of a thin elastic plate was developed, which recently had not been analytically solved for an irregularly shaped topography. A simple analytical solution only exists for the deflection by line loads of thin elastic beams that overlie a viscous substratum. A simple analytical solution (WATTS 2001) only exist for the deflection by line loads of thin elastic beams overlying a substratum. The deflection for more complex rectangularly and triangularly shaped loads could be evaluated only by integration. In the past a two-dimensional solution has been developed by WEDDELING (1996), which is based on the idea of TIMOSHENKO & WOINOWSKI-KRIEGER (1959); the equation was solved with fast Fourier transformation techniques (coherence and admittance). Only one value of T_e / D could be calculated for an area, which was required to have a side length of at least 340km (MCKENZIE & FAIRHEAD, 1997).
For the calculation process an input parameter is needed called “reference depth”, of questionable size and meaning. Some disadvantages of the spectral methods were overcome by the convolution approach developed by BRAITENBERG ET AL. (2001). This approach additionally requires a radius of convolution to be determined, in order to calculate the distribution of the T_e / D.

The utility of the elastic thickness of the lithosphere are based on the concept that the gravitational equilibrium of the lithosphere can be maintained over geological time and space scales and that the resulting static deformation is explicable as a flexure of a thin elastic plate overlying a fluid (BUROV & DIAMENT, 1995). By that, the lithosphere/asthenosphere boundary is calculated, which contradicts the fact, that by the differential equation describing the flexure of the thin plate, a density contrast is considered. Since the density contrast describes the restoring force of the underlying mantle. Therefore it is questionable which boundary is considered by the calculating methods, as a definition is needed if the crust-mantle interface or the lithosphere/asthenosphere boundary is modeled.

The physical meaning and significance of T_e are still a matter of debate (BUROV & DIAMENT, 1995). The estimated T_e values for the oceanic lithosphere follow approximately the depth of the isotherm of $600°C$, which mark the base of the mechanical lithosphere. However, for the continental lithosphere the results of T_e bear little relation to specific geological and physical boundaries: Although high values for cratons with $T_e = 70 \ldots 90km$ can be partly explained by the present day temperature gradients, the low values for volcanic arcs and mountain belts with $T_e = 10 \ldots 20 km$ can not be described by this relation (BUROV & DIAMENT 1995). It is evident that there is dependence between T_e and the composition of the plate, the geometry of the plate, external forces and the thermal structure (e.g. GOETZE & EVANS 1979, LYON-CAEN & MOLNAR 1983, BUROV & DIAMENT 1995, TASSARA 2005).

This leads to following open question, which are intended to answer in the present study.

1. Is it possible to find a formula in order to calculate the radius of convolution?
2. What is the physical meaning of the input parameter of the reference depth?
3. Is the calculation of flexure valid for the crust-mantle interface or the lithosphere/asthenosphere boundary?
4. Is it acceptable and sufficient to calculate T_e / D for an area of a side length lower than 340km?
5. How important is the influence of the Young's modulus on the results of T_e / D modeling?
6. Can the influence of the temperature be mathematically included in the T_e / D modeling?
7. To what extend a calculation of viscosities is permitted, if the dimension of time is not included in the modeling?
8. What is the physical meaning of the elastic thickness?

The fundamentals and the state of the art are explained in Chapter 1. In this study a new analytical solution for the computation of T_e / D has been developed. Consequently, in Chapter 2 the background of the analytical solution is explained. With the new analytical solution, it became possible to solve the differential equation of the 4^{th} order for any irregular shape of the topography. Therefore a solution from HERTZ (1884) was modified in order to apply it on the lithosphere. Additionally two self-designed computer programs are described. As examples for the application of the software three areas (Central Andes, Southern Andes and Pacific Ocean) are presented in Chapter 3. The possible error resulting from the initial assumptions are estimated and analyzed in Chapter 4. Therein the results are discussed in order to answer the open questions mentioned above. The significance of the input parameters for the calculation is considered, as well the importance of the Young's modulus is investigated. The present study culminates in a synthesis and closes with final comments and future directions.