The Ongoing Impact of Modular Localization on Particle Theory

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Abstract. Modular localization is the concise conceptual formulation of causal localization in the setting of local quantum physics. Unlike QM it does not refer to individual operators but rather to ensembles of observables which share the same localization region, as a result it explains the probabilistic aspects of QFT in terms of the impure KMS nature arising from the local restriction of the pure vacuum. Whereas it played no important role in the perturbation theory of low spin particles, it becomes indispensible for interactions which involve higher spin $s \geq 1$ fields, where is leads to the replacement of the operator (BRST) gauge theory setting in Krein space by a new formulation in terms of stringlocal fields in Hilbert space. The main purpose of this paper is to present new results which lead to a rethinking of important issues of the Standard Model concerning massive gauge theories and the Higgs mechanism. We place these new findings into the broader context of ongoing conceptual changes within QFT which already led to new nonperturbative constructions of models of integrable QFTs. It is also pointed out that modular localization does not support ideas coming from string theory, as extra dimensions and Kaluza–Klein dimensional reductions outside quasiclassical approximations. Apart from hologarphic projections on null-surfaces, holograhic relations between QFT in different spacetime dimensions violate the causal completeness property, this includes in particular the Maldacena conjecture. Last not least, modular localization sheds light onto unsolved problems from QFT's distant past since it reveals that the Einstein–Jordan conundrum is really an early harbinger of the Unruh effect.

Key words: modular localization; string-localization; integrable models

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To the memory of Hans-Jürgen Borchers (1926–2011)

1   Introduction

The course of quantum field theory (QFT) was to a large extend determined by four important conceptual conquests: its 1926 discovery by Pascual Jordan in the aftermath of what in recent times has been referred to as the *Einstein–Jordan conundrum* [22, 65] (a fascinating dispute between Einstein and Jordan), the discovery of renormalized perturbation in the context of quantum electrodynamics (QED) after world war II, the nonperturbative insights into the particle-field relation initiated in the Lehmann–Symanzik–Zimmermann (LSZ) work on scattering theory which subsequently was derived from first principles [27] and the extension of gauge theory leading up to the Standard Model and to the present research in particle physics.

Especially the nonperturbative derivation of time-dependent scattering theory from the foundational causal locality properties of QFT in conjunction with the difficult task to describe strong interactions led to the first solid insights into particle theory outside the range of perturbation theory. One of those nonperturbative results was the rigorous derivation of the particle analog of the Kramers–Kronig dispersion relations. This in turn led to their subsequent success-

ful experimental test which was very important for the continued trust in QFT's foundational causality principle at the new high energy scales. These results in turn encouraged a third project: the particle-based on-shell formulations known as the S-matrix bootstrap and Mandelstam's more analytic formulation in terms of auxiliary two-variable representations of elastic scattering amplitudes [47]. These on-shell projects as well as their dual model and string theory (ST) successors were less successful, to put it mildly. The later gauge theory of the Standard Model resulted from an extension of the QED quantization ideas. Despite its undeniable success it was not able to prevent the ascend of ST, partially because ST withdrew from problems of high energy particle theory with the (never fulfilled) promise to solve the foundational problem of quantum gravity at the length scale of the Planck distance.

One of the most remarkable innovative contributions of the 60s was Gell-Mann's idea of quark confinement and his later attempts to pose it as a conceptual challenge for QCD. Although its interpretative addition to QCD turned out to be remarkably consistent, its derivation as a mathematical consequence of that theory resisted all attempts undertaken during the 50 years of its existence. The reason it is mentioned in this introduction goes beyond historical completeness; the new stringlocal (SLF) Hilbert space setting of Yang–Mills couplings sheds new light on this old problem (Section 3).

Despite conceptual weaknesses, the Standard Model has remained the phenomenologically most inclusive and successful particle description. Its theoretical foundations date back to the early 70s and the experimental progress during more than 4 decades did not require any significant theoretical changes. In particular its central theoretical idea that masses of vector mesons and of particles with which they interact are generated by a spontaneous symmetry breaking (the Higgs mechanism), which led to last year's physics Nobel prize, remained in its original form in which it appeared first in the papers of Higgs and Englert. This is surprising since during its 40 years history several authors have cast valid doubts about its consistency with the principles of QFT.

The main point of the present work consists in the proposal of a new idea which extends *renormalized perturbation theory in a Hilbert space* setting to fields of higher spin $s \geq 1$. At this point it is important to remind the reader that the gauge theoretic formulation of interactions of vector mesons with matter fields (massless and massive abelian and nonabelian interactions) uses an indefinite metric Krein space and unphysical ghost operators. The loss of a Hilbert space description is the price one has to pay for maintaining the formalism of renormalized perturbation theory *in terms of pointlike fields* for interactions involving higher spin fields with $s \geq 1$. The new setting maintains the Hilbert space description but leaves it up to the causal localization principles to determine the tightest localization which is still consistent with the Hilbert space setting of quantum theory. The answer is that one never has to go beyond stringlocal fields. This clash between localization and the Hilbert space structure and pointlike localization of fields is a quantum phenomenon which has no counterpart in classical theory; it explains why Lagrangian quantization of $s \geq 1$ inevitably leads to Krein space formulations.

The reformulation of gauge theory in terms of interactions between stringlocal fields (SLF) in Hilbert space is much more than window-dressing: it extends the range of gauge theories beyond the construction of local observables to the inclusion of (necessarily stringlocal) physical matter fields and opens a realistic chance to understand confinement as a physical property of a model and not just an auxiliary metaphoric idea for exploring its physical consequences. The SLF inverts the relation between massless gauge theories and their massive counterparts; instead of considering models involving massless vector mesons as simpler than their massive counterparts, the SLF setting describes the massless models (QED, QCD) as massless limits of QFTs with a complete particle interpretation (validity of LSZ scattering theory) since the problem of scattering of stringlocal charged matter in QED remained on a level of a recipe (rather than of a spacetime explanation); not to mention this issue of gluon and quark confinement.

It is not surprising that such a paradigm shift also leads to a change of the "Higgs mechanism of spontaneous symmetry breaking" which in the new setting is simply the renormalizable coupling of a massive vector meson to real (Hermitian) scalar matter and the *postulated* Mexican hat potential (which served as the formal description of the symmetry breaking Higgs mechanism) is now the *result* of interaction terms which the implementation of the SLF locality principle *induces* from the iteration of the first-order interaction. In particular there is no generation of masses of vector mesons by a Higgs mechanism. Our findings show that interactions of massive vector mesons with matter can be consistently described within the Hilbert space setting of QT without referring to a mass-generating Higgs mechanism. Though fields of massive vector mesons are always accompanied by scalar fields, their inexorable presence ("intrinsic escorts") does not lead to additional degrees of freedom. Their presence is the result of by the positivity of Hilbert space which for interactions of massive $s \geq 1$ turns out to have very strong consequences; it does not only lead to stringlocal instead of pointlocal fields, but it also generates s additional escort fields of lower spin.

The scalar escort for $s = 1$ has many properties of a Higgs field except that it does not add degrees of freedom and therefore can only explain the LHC experimental result in terms of a bound state. On the other hand a Higgs coupling in the new setting is simply described by a coupling of a massive vector meson to a Hermitian field H ("chargeless QED"); but the principles of QFT certainly exclude the idea that massive vector mesons owe their mass to spontaneous symmetry breaking of gauge symmetry in scalar QED.

It is interesting that this is not the first time the Higgs mechanism came under critical scrutiny. In fact in the work of the Zürich group from the beginnings of the 90s [1, 56] based on the operator BRST formulation of (the simpler case) of massive vector mesons-matter interactions it was shown that the Mexican hat potential is not the defining interaction but rather the second order outcome from the implementation of the BRST gauge invariance on a first-order interaction which results from a transcription into the Krein space setting of a nonrenormalizable first-order $A^{\mathrm{P}} \cdot A^{\mathrm{P}} H$ coupling, where A_μ^{P} is the massive Proca potential of the vector meson. In all cases to which the new SLF Hilbert space formulation was applied, these earlier results from BRST gauge theory were confirmed, although the details of the SLF Hilbert space setting are different and the range of this method is larger. Results similar to those in Section 3 are contained in [67] and furthergoing results about nonabelian couplings will be contained in a forthcoming joint work with J. Mund [51].

The ongoing paradigmatic change also suggests to recall other critical ideas which were around at the time of the Higgs paper but whose content was lost in the maelstrom of time. On such idea is the Schwinger–Swieca charge screening which was suggested by Schwinger [69] and proven by Swieca [73]. It states that abelian massive vector meson couplings possess (in addition to the conserved current of complex fields which leads to the global counting charge) also an identically conserved Maxwell current (the divergence of the field strength) whose charge vanishes ("is screened"). For charge neutral matter fields, as in the Higgs model, this is the only current.

It would be possible to present these results directly without embedding them into their natural conceptual surroundings from which they emerged. But since these conceptual developments are only known to a very small circle of theoreticians, and also since the new emerging picture about what QFT can and should still achieve is as important as its ongoing impact on gauge theory and the Higgs mechanism, the special results on SLF will be placed into a larger context. To this enlarged setting also implies a foundational critique of ST, in particular because without a clear delimitations between the incorrect use of the word "string" in ST and its foundational deployment in SLF this could lead to misconceptions. In addition there is no better constructive use of an incorrect but widespread known theory than to use it for showing in what way a subtle concept as quantum causal localization has been misunderstood.

The starting point is what is nowadays referred to as Local Quantum Physics (LQP) [27]. This is a way of looking at QFT in which quantum fields are considered as generators of localized operator algebras; they "coordinatize" local nets of algebras in analogy to coordinates in geometry which coordinatize a given model geometry. This is quite different from the way one looks at classical field theories where, e.g., Maxwell's electromagnetic field strength has an intrinsic meaning. Such an "individuality" of fields gets lost in QFT beyond quasiclassical approximations. Experimentalists do not observe hadronic fields; what is being measured are hadronic particles entering or leaving a collision area. But unlike quantum mechanics (QM) particles have no direct relation to individual fields, rather a particle carrying a certain superselected charge is related to a whole field class which consists of fields carrying the same charge and belong to the same localization class (relative locality). The justification for this point of view results from the fact that these fields "interpolate" the same particle. For more details about the subtle field-particle relations see [27].

The first contact between the Tomita–Takesaki modular theory of operator algebras and quantum physics came from quantum statistical mechanics, to be more precise from the formulation of statistical mechanics directly in the thermodynamic infinite volume limit ("open systems") [27]. The important observation was that the prerequisites for the application of the T-T theory (an algebra $\mathcal{A}$ and a state vector Ω on which it acts cyclic and separating, see later) is always fulfilled in statistical mechanics. As a consequence the two "modular operators" $\Delta^{i\tau}$ and J have a physical interpretation in statistical mechanics where Δ is the so-called KMS operator (the thermodynamic limit of the Gibbs operator) and J is a anti-unitary reflection which maps the algebra $\mathcal{A}$ into its commutant (the thermal "shadow world"). The essential step which opened the use of the T-T theory in LQP was the realization of the validity of the Reeh–Schlieder theorem for the pair $(\mathcal{A}(\mathcal{O}), \Omega)$ where $\mathcal{A}(\mathcal{O})$ is an algebra localized in the spacetime region $\mathcal{O}$ and Ω is the vacuum state. The Reeh–Schlieder theorem is closely related to a very singular form of entanglement of the vacuum with respect to a subdivision of the global algebra $\mathcal{A}$ into $\mathcal{A}(\mathcal{O})$ of the region $\mathcal{O}$ and that of its causal complement $\mathcal{O}'$. This singular entanglement is related to the fact that although the algebra and its causal complement commute with each other, the global Hilbert space does not tensor-factorize. In contrast to the entanglement of quantum mechanical particle states which can be measured in terms of quantum-optical methods, the effects of the impurity of the $\mathcal{A}(\mathcal{O})$-restricted vacuum (Unruh effect, Hawking radiation) entanglement are numerically so tiny that they may remain always below what can be measured. Nevertheless the vacuum polarization through localization is behind almost most physical manifestations of QFT, from analytic on-shell behavior (as the particle crossing property of the S-matrix and formfactors) to the Unruh effect [70, 74] and the area law for localization entropy [64].

A historically particularly interesting manifestation of the statistical mechanics nature of the state resulting from the local restriction of the vacuum is the so-called Einstein–Jordan conundrum which similar to the Unruh effect shows that the subvolume fluctuations of a reduced vacuum state in the simplest QFT (the chiral abelian current model) are indistinguishable from those of in a thermal statistical mechanic state of the kind which Einstein used for his purely theoretical argument for the corpuscular nature of photons. If these facts would have been correctly identified, the history of the probability concept in quantum theory may have taken another turn. The algebra of local observables $\mathcal{A}(\mathcal{O})$ is an ensemble of observables to which the restriction of the pure vacuum state generates an impure KMS state. It is reasonable to use the name *physical states* only for finite energy states and to reserve the terminology *observable* to operators which are localized in some compact spacetime region and obey Einstein causality. Since the statistical mechanics-like KMS property holds not only for the vacuum but also for the *restriction of all physical states* to local observables, the probabilistic aspect resulting from the from the ensemble of observables localized in a spacetime region $\mathcal{O}$ is a generic intrinsic property of all physical states in QFT (which Einstein would have accepted). In contrast, for individual observables in QM

one needs to invoke Born's probability interpretation[1] which refers to a "Gedanken"-ensemble related to repeated measurements (to which Einstein had his philosophical objections). The best chance to obtain a deeper understanding of the QFT/QM relation is in the context of integrable models where actual particle creation (through collisions) is absent but vacuum polarization as the inexorable epiphenomenon of modular localization remains.

The SLF setting is an outgrowth of the solution of the problem of the QFT behind Wigner's 1939 third positive energy representation class (the massless infinite spin representations). In that case all fields associated to the representation are stringlocal, not just potentials of general field strengths. The resulting matter is "noncompact" in an intrinsic sense [52]. It has all the properties ascribed by astrophysicists to dark matter, i.e. it is inert and its arena of manifestations are galaxies and not earthly high energy laboratories [66].

The paper is organized as follows. The next section presents a foundational critique of ST in which already the terminology reveals the misconception of the meaning of quantum causal localization; part of this misunderstanding results from confusing Born's localization of wave functions based on the spectral decomposition of the (non-intrinsic) position operator and part is due to a "picture puzzle" resulting from the fact that the 10 component supersymmetric chiral current algebra is a representation of a corresponding irreducible C^*-algebra of oscillators on which there also exists a positive energy representation of the 10-dimensional highly reducible so-called superstring representation of the Poincaré group.

Having sharpened one's view on causal localization, the presentation then moves to *modular localization* which is the most appropriate conceptual as well as mathematical formulation of quantum causal localization. Its application to Wigner's positive energy representation theory of the Poincaré group led to the QFT of the infinite spin representation which is generated by irreducibly string-localized covariant fields. Irreducibly stringlocal interacting fields result from the interaction of reducibly stringlocal free fields. Section 3 and its subsections are the heart piece of a new SLF approach to perturbative QFT which includes higher spin interactions. Its relation to the existing BRST gauge setting is explained, and its already mentioned critical view of the Higgs mechanism is presented in detail. The SLF setting sheds new light on the confinement problem and reduces it to a computational problem involving perturbative resummations.

In the last section known results about existence proofs of integrable models are used to formulate conjectures about how modular theory may help to obtain a mathematical control of existence problems of QFT. The section also explains how the particle crossing property arises from modular wedge-localization.

Our findings support the title and the content of an important contribution by the late Hans-Jürgen Borchers in the millennium edition of Journal of Mathematical Physics [6] which reads: "Revolutionizing quantum field theory with Tomita–Takesaki's modular theory". With all reservations about misuses of the word "revolution" in particle physics, this paper is a comprehensive account of the role of modular operator theory in LQP, and its title is a premonition of the present progress which is driven by concepts coming from modular localization. LQP owns Borchers many of the concepts coming from modular operator theory; for this reason it is very appropriate to dedicate the present article to his memory.

2 Anomalous conformal dimensions, particle spectra and crossing properties

A large part of the conceptual derailment of string theory can be understood without invoking the subtleties of modular localization. This will be the subject of the following two subsections.

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[1]Here we use this terminology in the textbook sense of Born's localization probability density $|\psi(x)|^2$ which results from declaring a particular operator $\vec{x}_{op}$ to be a position operator.

The principle of *modular localization* becomes however essential for the correct foundational understanding of the particle crossing property. This is important for a new formulation of a constructive on-shell project based on the correct crossing property which replaces Mandelstam's attempt and is compatible with the principles of Haag's local quantum physics. This will be taken up in Section 4.

2.1 Quantum mechanical- versus causal-localization

Since part of the misunderstandings in connection with ST have to some extend their origin in confusing "Born localization" in QM with the causal localization in QFT, it may be helpful to review their significant differences [60].

It is well-known since Wigner's 1939 description of relativistic particles [27] in terms of irreducible positive energy representations of the Poincaré group that *there are no 4-component covariant operators* x^μ_{op}; in fact the impossibility to describe relativistic particles in terms of quantizing a classical relativistic particle action (or to achieve this in any other quantum mechanical setting) was one of the reasons which led to Wigner's representation theoretical construction of relativistic wave function spaces. The rather simple argument against covariant selfadjoint x^μ_{op} follows from the non-existence translationally covariant spectral projectors E which are consistent with the positive energy condition and fulfill with spacelike orthogonality

$$\vec{x}_{\mathrm{op}} = \int \vec{x}\, dE_{\vec{x}}, \qquad R \subset \mathbb{R}^3 \to E(R),$$
$$U(a)E(R)U(a)^{-1} = E(R+a), \qquad E(R)E(R') = 0 \quad \text{for} \quad R \times R',$$
$$(E(R)\psi, U(a)E(R)\psi) = (\psi, E(R)E(R+a)U(a)\psi) = 0,$$

where the second line expresses translational covariance and orthogonality of projections for spacelike separated regions. In the third line we assumed that the translation shifts $E(R)$ spacelike with respect to itself. But since $U(a)\psi$ is analytic in $\mathbb{R}^4 + iV^+$ (V^+ forward light cone) as a result of the spectrum condition, $\|E(R)\psi\|^2 = 0$ for all R and ψ which implies $E(R) \equiv 0$, i.e. covariant position operators do not exist.

The "Born probability" of QM results from Born's proposal to interpret the absolute square $|\psi(\vec{x}, t)|^2$ of the spectral decomposition $\psi(\vec{x}, t)$ of state vectors with respect to the spectral resolution of the position operator $\vec{x}_{\mathrm{op}}(t)$ as the probability density to find the particle at time t at the position $\vec{x}$. Its use as a localization probability density to find an individual particle in a pure state at a prescribed position became the beginning of one of longest lasting philosophical disputes in QM which Einstein entered through his famous saying: "God does not play dice".

In QFT in Haag's LQP formulation this problem does not exist since, as previously mentioned, its objects of interests are not global position operators in individual quantum mechanical systems, but rather ensembles of causally localized operators which share the same spacetime localization, i.e., which belong to the spacetime-indexed algebras $\mathcal{A}(\mathcal{O})$ of Haag's LQP (next section). As pointed out before this leads to a completely intrinsic notion of probability.

Traditionally the causal localization of QFT enters the theory with the (graded) spacelike commutation (Einstein causality) of pointlike localized covariant fields in Minkowski spacetime. There are very good reasons to pass to another slightly more general, but in a subtle sense also more specific formulation of QFT, namely to Haag's *local quantum physics* (LQP) in which the fields play a more auxiliary role of (necessary singular) generators of local algebras[2]. In analogy to coordinates in geometry as there are infinitely many such generators which generate the same local net of algebras as different coordinates which describe the same geometry. As

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[2]To be more precise they are operator-valued Schwartz distributions whose smearing with $\mathcal{O}$-supported test functions are (generally unbounded) operators affiliated with a weakly closed operator algebra $\mathcal{A}(\mathcal{O})$.

in Minkowski spacetime geometry these "field coordinates" can be chosen globally, i.e. for the generation of the full net of local algebras.

In this more conceptual LQP setting it is easier to express the *full* content of causal localization in a precise operational setting. It includes not only the Einstein causality for spacelike separated local observables, but also a timelike aspect of causal localization, namely the equality of an $\mathcal{O}$-localized operator algebra $\mathcal{A}(\mathcal{O})$ with that of its causal completion $\mathcal{O}''$

$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}''), \qquad \text{causal completeness,}$$
$$\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})', \qquad \text{Haag duality.}$$

Here $\mathcal{O}'$ denotes the causal complement (consisting of all points which are spacelike with respect to $\mathcal{O}$) and $\mathcal{O}'' = (\mathcal{O}')'$ is the causal completion. Haag duality is stronger than Einstein causality (which results from replacing $=$ by $\subset$). The notation $\mathcal{A}'$ for the commutant of $\mathcal{A}$ is standard in the theory of operator algebras. The causal completeness requirement corresponds to the classical causal propagation property. The advantage of the LQP formulation over the use of pointlike fields should be obvious. A more specific picture of a failure of causal completeness due to a mismatch of degrees of freedom results if one compares the definition of a local algebra localized in a convex spacetime region $\mathcal{O}$ obtained in two different ways, on the one hand as an intersection of wedge algebras (outer approximation defining the causal completion) and on the other hand as a union of arbitrary small double cones (inner approximation). In case the region is not causally complete the inner approximation is smaller that the outer one $\mathcal{A}(\mathcal{O}) := \mathcal{A}_{\text{in}}(\mathcal{O}) \subsetneqq \mathcal{A}_{\text{out}}(\mathcal{O}) =: \mathcal{A}(\mathcal{O}'')$. In this case there is a serious physical problem since there are degrees of freedom which have entered the causal completion from "nowhere" ("poltergeist" degrees of freedom).

Whereas both causality requirements are *formal* attributes of Lagrangian quantization (hyperbolic propagation), they have to be added in "axiomatic" settings based on *mathematically controlled* (and hence neither Lagrangian nor functional) formulations [28]. Their violations for subalgebras $\mathcal{A}(\mathcal{O})$ as a result of too many phase space degrees[3] of freedom leads to physically undesirable effects, which among other things prevents the mathematical AdS-CFT correspondence (last subsection) to admit a physical interpretation on both sides of the correspondence (i.e. one side is always unphysical).

Violations of Haag duality for disconnected or multiply connected regions have interesting physical consequences in connection with either the superselection sectors associated with observable algebras, or with the QFT Aharonov–Bohm effect for doubly connected spacetime algebras for the free quantum Maxwell field with possible generalizations to multiply connected spacetime regions in higher spin ($m = 0$, $s \geq 1$) representations [61, 62].

The LQP formulation of QFT is naturally related to the Tomita–Takesaki modular theory of operator algebras; its general validity for spacetime localized algebras in QFT is a direct result of the Reeh–Schlieder property [27] for localized algebras $\mathcal{A}(\mathcal{O})$, $\mathcal{O}'' \subset \mathbb{R}^4$ (next section).

It is important to understand that *quantum mechanical localization is not cogently related with spacetime*. A linear chain of oscillators simply does not care about the dimension of space in which it is pictured; in fact it does not even care if it is related to spacetime at all, or whether it refers to some internal space to which spacetime causality concepts are not applicable. The modular localization on the other hand is *imprinted* on causally local quantum matter, it is a totally *holistic* property of such matter. As life cannot be explained in terms of the chemical composition of a living body, localization does not follow from the mathematical description of the global oscillators (annihilation/creation operators) in a global algebra. These oscillators are the same in QM and QFT; free field oscillator variables $a(p)$, $a^*(p)$ which obey the oscillator commutation relations do not know whether they will be used in order to define Schrödinger fields or free covariant local quantum fields.

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[3]For the notion of phase space degree of freedoms see [14, 19, 29].

It is the holistic *modular localization principle* which imprints the causal properties of Minkowski spacetime (including the spacetime dimension) on operator algebras and thus determines in which way the irreducible system of oscillators will be used in the process of localization [30]; in QFT there is no abstract quantum matter as there is in QM; rather localization becomes an inseparable part of it. Contrary to a popular belief, this holistic aspect of QFT (in contrast to classical theory and Born's localization in QM) *does not permit an embedding of a lower-dimensional theory into a higher-dimensional* one, neither is its inversion (Kaluza–Klein reduction, branes) possible. To be more specific, the price for compressing a QFT onto a timelike hypersurface [5] is the loss of physical content namely one looses the important timelike causal completeness property due to an abundance of degrees of freedom. One may study such restrictions as laboratories for testing problems of mathematical physics, but they have no relevance for particle physics. This does however not include projections onto null-surfaces which *reduce the cardinality of degrees of freedom* (unlike the K-K reductions and AdS-CFT holographic isomorphisms[4] which maintain it). We will return to this issue in later parts of the paper. There has been an attempt by Mack [41, 42] to encode the overpopulation of degrees of freedom into a generalization of internal symmetries, but this does not seem to make the situation acceptable. If one only uses such situations as a mathematical trick (e.g. for doing calculations of an AdS_5 QFT on the side of the overpopulated CFT_4 theory before returning again) and not in the sense of Maldacena (allegedly relating two physical theories) this generates no harm.

One problem in reading articles or books on ST is that it is sometimes difficult to decide which localization they have in mind. When e.g. Polchinski [54] uses the relativistic particle action $\sqrt{ds^2}$ as a trailer for the introduction of the Nambu-Goto minimal surface action $\sqrt{A}$ (with A being the quadratic surface analog of the line element ds^2) for a description of ST, it is not clear why he does this. These Lagrangians lead to relativistic classical equation of motion but the classical particle Lagrangian is known to possess no associated relativistic quantum theory.

The Polyakov action A can be formally written in terms of the potential of an n-component chiral current

$$\int d\sigma d\tau \sum_{\xi=\sigma,\tau} \partial_\xi X_\mu(\sigma,\tau) g^{\mu\nu} \partial^\xi X_\mu(\sigma,\tau), \qquad X = \text{potential of conformal current } j.$$

However the quantum theory related to the Nambu–Goto action has nothing to do with its square (see later). The widespread use of the letter X for the potential of the multicomponent chiral current is very treacherous since it suggests that Polchinski's incorrect quantum mechanical reading of the classical $\sqrt{ds^2}$ has led to the incorrect idea that the action of a multi-component $d = 1 + 1$ massless field describes in some way a covariant string embedded into a higher-dimensional Minkowski spacetime (a kind of relativistic analog of a linear chain of oscillators into a higher-dimensional QM).

If the quantized X of the Polyakov action would really describe a covariant spacetime string, one could forget about the N-G square root action and take the Polyakov action for the construction of an embedded string. But this cannot work since the principle of modular localization simply contradicts the idea that a lower-dimensional QFT can be embedded into a higher-dimensional one. In particular an n-component chiral conformal QFT cannot be *embedded* as a "source" theory into a QFT which is associated with a representation of the Poincaré group acting on the n-component inner symmetry space (the "target" space) of a conformal field theory. The C^*-algebra generated by the oscillators contained in a $d = 10$ supersymmetric chiral current model carries a representation of the $d = 1 + 1$ Moebius group and possesses a (unitarily inequivalent) representation which carries a positive energy representation of the

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[4]A concise mathematical description of this phenomenon (but without a presentation of the physical consequences) can be found in [55].

Poincaré group; but from this one cannot infer the existence of a spacetime "embedding" of a 1-dimensional chiral theory localized on the compactified lightray into a 10-dimensional QFT.

String theorists gave a correct proof of this group theoretic fact [10], but in order to construct an S-matrix it takes more than group theory. *In fact the global oscillator algebra admits at least two inequivalent representations*: one on which the Möbius group acts and in which it is possible to construct pointlike Möbius covariant fields, and the other on which the mentioned unique 10-dimensional highly reducible representation of the Poincaré group acts. The easiest way to see that the representations are different is to notice that the multi-component charge spectrum is continuous whereas the corresponding Poincaré momentum spectrum has mass gaps. In addition the embedding picture would incorrectly suggest that the object is a spacetime string and not an infinite component pointlike wave function or quantum field as required by a finite spin/helicity positive energy representation[5]. The group theoretic theorem cannot be used in an on-shell S-matrix approach. To construct an S-matrix one needs more than just group representation theory of the Poincaré group. Admittedly the mentioned group theoretic theorem is somewhat surprising since *it is the only known irreducible algebra which leads to a discrete mass/spin tower* (no admixture of a continuous energy-momentum spectrum coming from multiparticle states).

Often a better conceptual understanding is obtained by generalizing a special situation. Instead of an irreducible algebra associated with a chiral current theory one may ask whether an *internal* symmetry space of a finite component quantum field can (i.e. not indices referring to spinor/tensor components of fields) carry the representation of a noncompact group. In classical theories this is always possible, whereas in QFT one would certainly not expect this in $d > 1+1$ models. For theories with mass gaps this is the result of a deep theorem about the possible superselection structure of observable LQP algebras [27]; there are good reasons to believe that this continues to hold for the charge structure in theories containing massless fields [13]. A necessary prerequisite is the existence of continuously many superselected charges as in the case of abelian current models. By definition this is the class of non-rational chiral models. Apart from the multicomponent abelian current model almost nothing is known about this class; so the problem of whether the "target spaces" of such models can accommodate unitary representations of non-compact groups (i.e. the question whether the above theorem about unitary representations on multicomponent current algebras is a special case of a more general phenomenon) remains open.

A rather trivial illustration of a classical theory on whose index space a Poincaré group acts without the existence of a quantum counterpart is the afore-mentioned relativistic classical mechanics. As covariant classical theories may not possess a quantum counterpart, there are also strong indications about the existence of QFTs which cannot be pictured as the quantized version of covariant classical fields[6]. The best way of presenting the group theoretical theorem of the string theorists is to view it in a historical context as the (presently only known) solution of the 1932 Majorana project [43]. Majorana was led to his idea about the possible natural existence of infinite component relativistic fields by the $O(4,2)$ group theoretical description of the nonrelativistic hydrogen spectrum. We take the liberty to formulate it here in a more modern terminology.

Problem 2.1 (Majorana [43]). *Find an irreducible algebraic structure which carries a infinite-component positive energy one-particle representation of the Poincaré group (an "infinite component wave equation").*

Majorana's own search, as well as that for the so-called "dynamic infinite component field equation" by a group in the 60s (Fronsdal, Barut, Kleinert, . . . ; see appendix of [4]) consisted in

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[5]Only the zero mass infinite spin representation leads to string-localization [52].

[6]This goes also in the opposite direction: there are many known $d = 1 + 1$ integrable models which have no Lagrangian description.

looking for irreducible group algebras of noncompact extensions of the Lorentz group ("dynamical groups"). No acceptable solution was ever found within such a setting. The only known solution is the above superstring representation which results from an irreducible oscillator algebra of the $n = 10$ supersymmetric Polyakov model. The positive energy property of its particle content (and the absence of components of Wigner's "infinite spin" components) secures the pointlike localizability of this "superstring representation" (too late to change this unfortunate terminology).

Sometimes the confusions about localization did not directly enter the calculations of string theorists but rather remained in the interpretation. A poignant illustration is the calculation of the (graded) commutator of string fields in [38, 48]. Apart from the technical problem that infinite component fields can not be tempered distribution (since the piling up of free fields over one point with ever increasing masses and spins leads to a diverging short distance scaling behavior which requires to project onto finite mass subspaces), the graded commutator is pointlike. This was precisely the result of that calculation; but the authors presented their result as "the (center) point on a string". Certainly this uncommon distributional behavior has no relation with the idea of spacetime strings; at most one may speak about a quantum mechanical chain of oscillators in "inner space" (over a localization point). The memory of the origin of ST from an *irreducible* oscillator algebra is imprinted in the fact that the degrees of freedoms used for the representation of the Poincaré group do not exhaust the oscillator degrees of freedom, there remain degrees of freedom which interconnect the representations in the (m, s) tower, i.e. which prevent that the oscillator algebra representation is only a direct sum of wave function spaces. But the localization properties reside fully in these wave function spaces and, as a result of the absence of Wigner's infinite spin representations, the localization is pointlike. This is precisely what the above-mentioned authors found, but why did they not state this clearly.

ST led to the bizarre suggestion that we are living in an (dimensionally reduced) target space of an (almost) unique[7] 10-dimensional chiral conformal theory. A related but at first sight more appearing idea is the dimensional reduction which was proposed in the early days of quantum theory by Kaluza and Klein. Both authors illustrated their idea in classical/semiclassical field theory[8]; nobody ever established its validity in a full-fledged QFT (e.g. on the level of its correlation functions) was never established. There is a good reason for this since the idea is in conflict with the foundational causal localization property. Unlike Born localization in QM, modular localization is an intrinsic property; the concept of matter in LQP cannot be separated from spacetime, it is rather coupled to its dimensionality through the spacetime dependent notion of "degrees of freedom". As explained in the previous section this is closely related to causality (the "causal completenes property") where it was pointed out that e.g. the mathematical AdS-CFT algebraic isomorphism converts a physical QFT on one side of the correspondence into a physically unacceptable model on the "wrong" side. This does however not exclude the possibility that it may be easier to do computations on the other side of the isomorphism and transform the computed result back to the physical side.

The idea of the use of variable spacetime dimensions in QFT (the "epsilon expansion") goes back to Ken Wilson who used it as a method (a technical trick) for computing anomalous dimensions (critical indices) of scalar fields. But whereas the method gave reasonable results for critical indices of scalar fields, this is certainly not the case for $s > 1$ matter; as already Wigner's classification of particles and their related free fields show, the appearance of changing "little groups" prevents an analytic dependence.

Our criticism of the dual model and ST is two-fold, on the one hand the reader will be reminded that the meromorphic crossing properties of the dual model, although not related to particle theory, represent a rigorous property of conformal correlations after passing to their Mellin transform. The poles in these variables occur at the scale dimensions of composites which

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[7]Up to a finite number of M-theoretic modifications.

[8]Also "branes" were only explained in the context of quasiclassical approximations.

appear in global operator expansions of two conformal covariant fields. In this formal game of producing crossing symmetric functions through Mellin transforms the spacetime dimensionality does not play any role; any conformal QFT leads to such a dual model function and that found by Veneziano belongs to a chiral current model. A special distinguished spectrum appears if one performs a Mellin-transforms on the correlations of the 10-component current model whose oscillator algebra carries the unitary positive energy "superstring representation" of the Poincaré group (the previously mentioned only known solution of the Majorana problem). In this case the (m, s) Poincaré spectrum is proportional to the dimensional spectrum (d, s) of composites which appear in the global operator expansion of the anomalous dimension-carrying sigma fields which are associated to the chiral current model.

The second criticism of the dual model/ST is that scattering amplitudes cannot be meromorphic in the Mandelstam variables; in integrable models they are meromorphic in the rapidities. The best way to understand the physical content of particle crossing is to derive it from the analytic formulation of the KMS property for modular wedge localization. This does not only reveal the difference to dual model crossing, but also suggests a new on-shell construction methods based on the S-matrix which may be capable to replace Mandelstam's approach (Section 3).

2.2 The picture puzzle of chiral models and particle spectra

There are two ways to see the correct mathematical-conceptual meaning of the dual model and ST.

One uses the "Mack machine" [41, 42] for the construction of dual models (including the dual model which Veneziano constructed "by hand"). It starts from a 4-point function of *any conformal QFT in any spacetime dimension.* To maintain simplicity we take the vacuum expectation of four not necessarily equal scalar fields

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle.$$

It is one of the specialities of interacting conformal theories that fields have no associated particles with a discrete mass, instead they carry (generally a non-canonical, anomalous, discrete) scale dimensions which are connected with the nontrivial *center of the conformal covering group* [63]. It is well known from the pre BPZ [3] conformal research in the 70s [40, 68] that conformal theories have converging operator expansions of the type

$$A_3(x_3)A_4(x_4)\Omega = \sum_k \int d^4z \Delta_{A_3,A_4,C_k}(x_1,x_2,y)C_k(z)\Omega,$$

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \ \rightarrow \ 3 \text{ different expansions.} \tag{2.1}$$

In distinction to the Wilson–Zimmermann short distance expansions, which only converge in an asymptotic sense, these expansions converge in the sense of state-vector valued Schwartz distributions. The form of the global 3-point-like expansion coefficients is completely fixed in terms of the anomalous scale dimension spectrum of the participating conformal fields.

It is clear that there are exactly three ways of applying global operator expansions to pairs of operators inside a 4-point-function (2.1); they are analogous to the three possible particle pairings in the elastic S-matrix which correspond to the s, t and u in Mandelstam's formulation of crossing. But beware, this dual model crossing arising from the Mellin transform of conformal correlation has nothing to do with S-matrix particle crossing of Mandelstam's on-shell project. If duality would have arisen in this context probably nobody would have connected them with the particle crossing in S-matrices and on-shell formfactors. Veneziano found these relations [20, 75] by using mathematical properties of Euler beta function Euler beta function; his construction did not reveal its conformal origin. Since particle crossing and its conceptual origin in the

principles of QFT remained ill-understood (for a recent account of its origin from modular localization see [63, 64]), the incorrect identification of crossing with Veneziano's duality met little resistance.

The Mellin transform of the 4-point-function is a meromorphic function in s, t, u which has first-order poles at the numerical values of the anomalous dimensions of those conformal composites which appear in the three different decompositions of products of conformal fields; they are related by analytic continuation [41, 42]. To enforce an interpretation of particle masses, one may rescale these dimensionless numbers by the same dimensionfull number. However this formal step of calling the scale dimensions of composites particle masses does not change the physical reality. Structural analogies in particle physics are worthless without an independent support concerning their physical origin.

The Mack machine to produce dual models (crossing symmetric analytic functions of 3 variables) has no definite relation to spacetime dimensions; one may start from a *conformal theory in any spacetime dimension* and end with a meromorphic crossing function in Mellin variables. Calling them Mandelstam variables does not change the conceptual-mathematical reality deletion; one is dealing with two quantum objects whose position in Hilbert space can hardly be more different than that of scattering amplitudes and conformal correlations.

However, and here we come to the picture-puzzle aspect of ST, one can ask the more modest question whether one can view the *dimensional spectrum of composites* in global operator expansions (after multiplication with a common dimensionfull $[m^2]$ parameter) *as arising from a positive energy representation of the Poincaré group*. The only such possibility which was found is the previously mentioned 10 component superymmetric chiral current theory which leads to the well-known superstring representation of the Poincaré group and constitutes the only known solution of the Majorana project. In this way the analogy of the anomalous composite dimensions of the poles in the dual model from the Mack machine to a (m, s) mass spectrum is extended to a genuine particle representation of the Poincaré group. But even this lucky circumstance which leads to the superstring representation remains on the level of group theory and by its very construction cannot be viewed as containing dynamic informations about a scattering amplitude.

There exists a presentation which exposes this "picture-puzzle" aspect between conformal chiral current models and Wigner's particle representation properties in an even stronger way: the so-called sigma-model representation. Schematically it can be described in terms of the following manipulation on abelian chiral currents ($x =$ lightray coordinate)

$$\partial \Phi_k(x) = j_k(x), \qquad \Phi_k(x) = \int_{-\infty}^{x} j_k(x), \qquad \langle j_k(x) j_l(x') \rangle \sim \delta_{k,l} \left(x - x' - i\varepsilon \right)^{-2}, \qquad (2.2)$$

$$Q_k = \Phi_k(\infty), \qquad \Psi(x, \vec{q}) =: e^{i\vec{q}\vec{\Phi}(x)} :, \qquad \text{carries } \vec{q}\text{-charge},$$

$$Q_k \simeq P_k, \qquad \dim(e^{i\vec{q}\vec{\Phi}(x)}) \sim \vec{q} \cdot \vec{q} \simeq p_\mu p^\mu, \qquad (d_{\mathrm{sd}}, s) \sim (m, s).$$

The first line defines the *potentials of the current*; it is formally infrared-divergent. The vacuum sector is instead created by applying the polynomial algebra generated by the infrared convergent current. In contrast *the exponential sigma field Ψ is the formal expression for a covariant superselected charge-carrying field.* Its symbolic exponential way of writing leads to the *correct correlation functions in total charge zero correlations* where the correlation functions agree with those computed from Wick-reordering of products of sigma model fields Ψ, all other correlations of the sigma-model field vanish (the quotation mark is meant to indicate this limitation of the Wick ordering).

The interesting line is the third in (2.2), since it expresses a "mock relation" with particle physics; the multi-component continuous charge spectrum of the conformal currents resemble a continuous momentum spectrum of a representation of the Poincaré group, whereas the spectrum of anomalous scale dimensions (being quadratic in the charges) is reminiscent the quadratic

relation between momenta and particle masses. The above analogy amounts to a genuine positive energy representation of the Poincaré group only in the special case of a supersymmetric 10-component chiral current model; it is the before-mentioned solution of the Majorana project. Its appearance in the Mellin transform of a conformal correlation bears no relation with an S-matrix. As also mentioned, the shared irreducible abstract oscillator algebra leads to different representations in its conformal use from that for a positive energy representation of the Poincaré group[9]. The difference between the representation leading to the conformal chiral theory and that of the Poincaré group on the target space (the superstring representation) prevents the (structurally anyhow impossible) interpretation in terms of an embedding of QFTs; although there remains a certain proximity as a result of the shared oscillator algebra.

The multicomponent Q_μ charge spectrum covers the *full* $\mathbb{R}^{10}$ whereas the P_μ spectrum of the superstring representation is concentrated on *positive mass hyperboloids*. The Hilbert space representation of the oscillator algebra from the Fourier decomposition of the compactified conformal current model on which one obtains a realization of the Möbius group is not the same as that which leads to the superstring representation of the Poincaré group. Hence presenting the result as an embedding of the chiral "source theory" into the 10 component "target theory" is a misunderstanding caused by the "picture-puzzle" aspect of the sigma field formulation of the chiral current model. The representation theoretical differences express the different holistic character of the two different localizations (the target localization being a direct consequence of the intrinsic localization of positive energy representations of the Poincaré group). This picture-puzzle situation leads to two mathematical questions which will not be further pursued: why does the positive energy representation of the Poincaré group only occur when the chiral realization has a vanishing Virasoro algebra parameter? And are there other non-rational (continuous set of superselection sectors) chiral models which solve the Majorana project?

It should be added that it would be totally misleading to reduce the mathematical/conceptual use of chiral abelian current models to their role in the solution of the Majorana project of constructing infinite component wave equations. The chiral n-component current models played an important conceptual role in mathematical physics; the so-called *maximal extensions* of these observable algebras can be classified by integer lattices, and the possible superselection sectors of these so extended algebras are classified in terms of their dual lattices [16, 21, 34, 71]. Interestingly the *selfdual lattices* and their known relation with exceptional final groups correspond precisely to the *absence of non-vacuum superselection sectors* (no nontrivial superselected charges) which in turn is equivalent to the validity of *full* Haag duality (Haag duality also for all multiply-connected algebras [61, 62]). They constitute the most explicitly constructed nontrivial chiral models. They shed light on the interplay of discrete group theory and Haag duality (and also on its violation for localization on disconnected intervals).

3   Wigner representations and their covariantization

Historically the use of the new setting of modular localization started with a challenge since the days of Wigner's particle classification: *find the causal localization of the third Wigner class* (the massless infinite spin class) of positive energy representations of the Poincaré group. Whereas the massive class as well as the zero-mass finite helicity class are pointlike generated, it is not possible to find covariant pointlike generating wave functions for this third Wigner class. The first representation theoretical argument showing the impossibility of a pointlike

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[9]The 26 component model does not appear here because we are interested in localizable representation; only positive energy representations are localizable.

generation dates back to [78]. Decades later new ideas about the use of modular localization in connection with integrable models emerged [58]. This was followed by the concept of modular localization of wave functions in the setting of Wigner's positive energy representation of the Poincaré group [11] which led to the introduction of spacelike string-generated fields in [52]. These are covariant fields $\Psi(x,e)$, e spacelike unit vector, which are localized on $x + \mathbb{R}_+ e$ in the sense that the (graded) commutator vanishes if the full semiinfinite strings (and not only their starting points x) are spacelike separated [52]

$$\left[\Psi(x,e), \Phi(x',e')\right]_{\mathrm{grad}} = 0, \qquad x + \mathbb{R}_+ e \rangle\langle\ x' + \mathbb{R}_+ e'. \tag{3.1}$$

Unlike decomposable stringlike fields (pointlike fields integrated along spacelike halflines) such *elementary stringlike fields* lead to serious problems with respect to the activation of (compactly localized) particle counters. The decomposable free strings of higher spin potentials (see next subsection) are in an appropriate sense "milder". As pointlike localized fields, free string-localized fields have Fourier transforms which are on-shell (mass-shell).

In the old days [76], infinite spin representations were rejected on the ground that nature does not make use of them. But whether in times of dark matter one would uphold such dismissals is questionable, in particular since it turn out that they have the desired inert/invisibility properties [66] which one attributes to dark matter.

Different from pointlike fields, string-localized quantum fields fluctuate both in x as well as in e;[10] this spread of fluctuations accounts for the reduction of the short distance scaling dimension, e.g. instead of $d_{\mathrm{sd}} = 2$ for the Proca field one arrives at $d_{\mathrm{sd}} = 1$ for its stringlocal partner. Whereas the d_{sd} for pointlike potentials increase with spin, their stringlike counterparts can always be constructed in such a way that their effective short distance dimension is the lowest one allowed by positivity, namely $d_{\mathrm{sd}} = 1$ *for all spins*. It is not possible to construct the covariant "infinite spin" fields by the group theoretic intertwiner method used by Weinberg [76]; in [52, 53] the more powerful setting of modular localization was used. In this way also the higher spin string-localized fields were constructed.

For finite spins the unique Wigner representation always has many covariant pointlike realizations; the associated quantum fields define linear covariant generators of the system of localized operator algebras whereas their Wick powers are nonlinear composite fields. In the following we will explain the reasons why even in case of pointlike generation one is interested in stringlike generating fields [52].

For pointlike generating covariant fields $\Psi^{(A,\dot{B})}(x)$ one finds the following possibilities which link the physical spin s to the (undotted, dotted) spinorial indices

$$|A - \dot{B}| \leqslant s \leqslant A + \dot{B}, \qquad m > 0, \tag{3.2}$$

$$h = A - \dot{B}, \qquad m = 0. \tag{3.3}$$

In the massive case all possibilities for the angular decomposition of two spinorial indices are allowed, whereas in the massless case the values of the helicities h are severely restricted (3.3). For $(m = 0, h = 1)$ the formula conveys the impossibility of reconciling pointlike vector potentials with the Hilbert space positivity. This clash holds for all $(m = 0, s \geq 0)$: pointlike localized "field strengths" (for $h = 2$, the linearized Riemann tensor) have no pointlike quantum "potentials" (got $h = 2$, the $g_{\mu\nu}$, ...) and similar statement holds for half-integer spins in case of $s > 1/2$. Allowing stringlike generators the possibilities of massless spinoral A, $\dot{B}$ realizations cover the same range as those in (3.2).

Since the classical theory does not care about positivity, (Lagrangian) quantization does not guaranty that the expected quantum objects are consistent with the Hilbert space positivity; in

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[10]These long distance (infrared) fluctuations are short distance fluctuation in the sense of the asymptotically associated $d = 1 + 2$ de Sitter spacetime.

fact it is well-known that the gauge theoretic description necessitates the use of indefinite metric Krein spaces (the Gupta–Bleuler or BRST formalism). The intrinsic Wigner representation-theoretical approach on the other hand keeps the Hilbert space and lifts the restriction to pointlike generators in favor of semiinfinite stringlike generating fields.

It is worthwhile to point out that contrary to popular belief perturbation theory does not require the validity of Lagrangian/functional quantization. Euler–Lagrange quantization limits the covariant realizations of (m, s) Wigner representations to a few spinorial/tensorial fields with low $(A, \dot{B})$ but as Weinberg already emphasized for setting up perturbation theory one does not need Euler–Lagrange equations to formulate Feynman rules; they are only necessary if one uses formulation in which the interaction-free part of the Lagrangian enters as it does in the Lagrangian/functional quantization. The only "classical" input into causal perturbation as the E-G approach is a (Wick-ordered) Lorentz-invariant field polynomial which implements the classical pointlike coupling, all subsequent inductive steps use quantum causality [24].

3.1 Modular localization and stringlocal quantum fields

An abstract modular S-operator is a closed antilinear involutive operator in Hilbert space H with a dense domain of definition

Def. S : antilin, densely def., closed, involutive $S^2 \subseteq \mathbf{1}$,

polar decomp. $S = J\Delta^{1/2}$, J modular reflection, Δ^{it} mod. group.

Such operators have been first introduced in the context of the Tomita–Takesaki theory of (von Neumann) operator algebras and are therefore referred to as "Tomita S-operator" within the setting of operator algebras $\mathcal{A}$ by Tomita and Takesaki

$SA\Omega = A^*\Omega$, $A \in \mathcal{A}$,   action of $\mathcal{A}$ on Ω is standard,

$S = J\Delta^{1/2}$, J modular reflection, $\Delta^{i\tau} = e^{-i\tau H_{\mathrm{mod}}}$ mod. group.

Here *standardness* of the pair $(\mathcal{A}, \Omega)$ means that the action is cyclic, i.e. $\overline{\mathcal{A}\Omega} = H$ and separating, i.e. $A\Omega = 0$, $A \in \mathcal{A}$ implies $A = 0$, where the separating property is needed for the uniqueness of S. In quantum physics one meets such operators in equilibrium statistical mechanics and QFT. According to the Reeh–Schlieder theorem each local subalgebra $\mathcal{A}(\mathcal{O})$ is standard with respect to the vacuum Ω (in fact with respect to every finite energy state) [27]. In case of the wedge region $\mathcal{O} = W$, the operators which appear in the polar decomposition are well known in QFT: J is the reflection along the edge of the wedge (the TCP operator up to a π-rotation within the edge of the wedge) whereas $\Delta^{i\tau} = U(\Lambda_W(\chi = -2\pi\tau))$ is the unitary representation of the W-preserving one-parametric Lorentz-boost group.

The modular localization theory has an interesting application within Wigner's positive energy representations of the connected (proper, orthochronous) part of the Poincaré group $\mathcal{P}_+^\uparrow$ as explained in the following. It has been realized, first in a special case [58], and then in the general setting [11] (see also [26, 52]), that there exists a *natural localization structure* on the Wigner representation space for any positive energy representation of the proper Poincaré group.

Let W_0 be a reference wedge region $W_0 = \{z > |t|; \mathbf{x} = (x, y) \in \mathbb{R}^2\}$. Such a region is naturally related with two commuting transformations: the W_0-preserving Lorentz-boost subgroup $\Lambda_{W_0}(\chi)$ and the $\mathbf{x}$-preserving reflection on the edge of the wedge r_{W_0} which maps the wedge into its causal complement W_0'. The product of r_{W_0} with the total reflection $x \to -x$ is a transformation in $\mathcal{P}_+^\uparrow$, namely a π-rotation in x-y edge. On the other hand the total reflection is the famous TCP transformation which, in order to preserve the energy positivity, has to be represented by an anti-unitary operator. With the only exception of zero mass finite helicity representations where one needs a helicity doubling (well known from the photon representation),

the total reflection and hence r_{W_0} is anti-unitarily represented on the irreducible Wigner representation. The resulting operator[11] is J_{W_0} together with the commuting Lorentz boost $\Delta_{W_0}^{i\tau}$. Its analytic continuation $\Delta_{W_0}^z$ is an unbounded operator whose dense domain in the one-particle space decreases with increasing $|\operatorname{Re} z|$. The anti-unitarity of J_{W_0} converts the commutativity with $\Delta^{i\tau}$ into the relation $J_{W_0}\Delta_{W_0}^a = \Delta_{W_0}^{-a}J_{W_0}$ on a dense set with the result that

$$S_{W_0} = J_{W_0}\Delta_{W_0}^{1/2}, \qquad S_{W_0}^2 \subset 1, \qquad \text{i.e.} \quad \operatorname{Range}(S_{W_0}) = \operatorname{Dom}(S_{W_0})$$

is the polar decomposition of a Tomita S-operator.

With a general W defined by covariance $W = gW_0$, where g is defined up to Poincaré transformations which leave W_0 invariant, we define

$$\Delta_W^{i\tau} = g\Delta_{W_0}^{i\tau}g^{-1},$$

Involutivity implies that the S-operator has ± 1 eigenspaces; since it is antilinear, the $+$ space multiplied with i changes the sign and becomes the $-$ space; hence it suffices to introduce a notation for just one real eigenspace

$$K(W) = \{\text{domain of } \Delta_W^{\frac{1}{2}},\ S_W\psi = \psi\},$$
$$J_W K(W) = K(W') = K(W)', \qquad \text{duality},$$
$$\overline{K(W) + iK(W)} = H_1, \qquad K(W) \cap iK(W) = 0.$$

It is important to be aware that one is dealing here with *real* (closed) subspaces K of the complex one-particle Wigner representation space H_1. An alternative is to directly work with the complex dense subspaces $K(W) + iK(W)$ as in the third line. Introducing the *graph norm* in terms of the positive operator Δ, the dense complex subspace becomes a Hilbert space $H_{1,\Delta}$ in its own right. The upper dash on regions denotes the causal disjoint (the opposite wedge), whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $\operatorname{Im}(\cdot,\cdot)$ on H. All the definitions work for arbitrary positive energy representations of the Poincaré group [11].

The two properties in the third line are the defining relations of what is called the *standardness property* of a real subspace[12]; any abstract standard subspace K of an arbitrary real Hilbert space permits to define an abstract S-operator in its complexified Hilbert space

$$S(\psi + i\varphi) = \psi - i\varphi, \qquad S = J\Delta^{\frac{1}{2}}, \tag{3.4}$$
$$\operatorname{dom} S = \operatorname{dom} \Delta^{\frac{1}{2}} = K + iK,$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group Δ^{it} and an antiunitary reflection which generally have however no geometric interpretation in terms of localization. The domain of the Tomita S-operator is the same as the domain of $\Delta^{\frac{1}{2}}$, namely the real sum of the K space and its imaginary multiple. Note that for the physical case at hand, this domain is intrinsically determined solely in terms of the Wigner group representation theory, showing the close relation between localization and covariance.

The K-spaces are the real parts of these complex $\operatorname{dom} S$, and in contrast to the complex domain spaces they are closed as real subspaces of the Hilbert space (corresponding to the one-particle projection of the real subspaces generated by Hermitian field operators). Their

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[11] We keep the same notation as in the Tomita–Takesaki operator setting since the difference between the algebraic and the representation theoretic S is always clear from the context.

[12] According to the Reeh–Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum, i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh–Schlieder property.

symplectic complement can be written in terms of the action of the J operator and leads to the K-space of the causal disjoint wedge W' (Haag duality)

$$K'_W := \{\chi \mid \mathrm{Im}(\chi, \varphi) = 0, \text{ all } \varphi \in K_W\} = J_W K_W = K_{W'}.$$

The extension of W-localization to general convex causally complete spacetime regions $\mathcal{O} = \mathcal{O}''$ is done by representing the causally closed $\mathcal{O}$ as an intersection of wedges and defining $K_\mathcal{O}$ as the corresponding intersection of wedge spaces

$$K_{\mathcal{O}''} \equiv \bigcap_{W \supset \mathcal{O}''} K_W, \qquad \mathcal{O}'' = \text{causal completion of } \mathcal{O} \tag{3.5}$$

These K-spaces lead via (3.4) and (3.5) to the modular operators associated with $K_\mathcal{O}$. For arbitrary spacetime regions one defines the K-spaces by "exhaustion from the inside"

$$K_\mathcal{O} = \bigcup_{W \subset \mathcal{O}} K_W.$$

For irreducible Wigner representations the two spaces are equal but it is easy to construct QFTs in which this causal completeness property is violated for simply connected convex region. As explained before QFT models which violate causal completeness for simply connected convex spacetime regions $\mathcal{O}$, i.e. $\mathcal{A}(\mathcal{O}) \subsetneq \mathcal{A}(\mathcal{O}'')$ are unphysical; this problem occurs in the context of isomorphism between QFT in different spacetime dimensions (the AdS-CFT correspondence, next section). It also limits Kaluza–Klein ideas of dimensional reductions to quasiclassical approximations.

Modular theory encodes localization properties of particle states into domain properties of Tomita S-operators; re-expressing the K-space properties in terms of Tomita S-operators the causal disjoint property between regions $\mathcal{O}_1 \rangle \langle \mathcal{O}_2$ reads for integer spin representations [52]

$$S_{\mathcal{O}_1} \subset S_{\mathcal{O}_2}^*. \tag{3.6}$$

Modular theory is the only known theory in which the operator content is fully encoded into domains. Defining field operators for $\mathcal{O}$-localized Wigner wave functions as

$$\Phi(\psi) = a^*(\psi) + a(S_\mathcal{O}\psi), \qquad S_\mathcal{O}\Phi(\psi)\Omega = \Phi(\psi)^*\Omega.$$

S acts as in the second line, independent of $\mathcal{O}$; the only difference is the $\mathcal{O}$-dependent domain. The commutator of two Φ equals

$$[\Phi(\psi_1), \Phi(\psi_2)] = (S_{\mathcal{O}_1}\psi_1, \psi_2) - (S_{\mathcal{O}_2}\psi_2, \psi_1) = 0 \qquad \text{in case of (3.6)}.$$

These operators implement a functorial relation between (localized) Wigner K-subspaces and interaction-free (localized) operator subalgebras of $B(H)$ where H is the Hilbert space which is generated by the successive action of $\Phi's$ on the vacuum Ω. The functorial map is

$$K_\mathcal{O} \to \mathcal{A}(\mathcal{O}) = \mathrm{Alg}\{e^{i(\Phi(\psi))} | \psi \in K_\mathcal{O}\}.$$

For half-integer (Fermion) representation there is a corresponding graded functor.

In the presence of interactions this functorial relation between particle subspaces and localized algebras is lost. What remains is a rather weak relation between wedge-local particle states and their "emulation" in terms of applying interacting operators affiliated to a wedge-local interacting algebra to the vacuum.

In order to make contact with the notion of generating covariant fields one needs intertwiners which map covariant $\mathcal{O}$-supported testfunctions into $\mathcal{O}$-localized Wigner wave functions.

For those who are familiar with Weinberg's intertwiner formalism [76] relating the (m, s) Wigner representation to the dotted/undotted spinor formalism, it may be helpful to recall the resulting "master formula"

$$\Psi^{(A,\dot{B})}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx} u^{(A,\dot{B})}(p) \cdot a(p) + e^{ipx} v^{(A,\dot{B})}(p) \cdot b^*(p) \right) \frac{d^3p}{2\omega}, \qquad (3.7)$$

$$u^{(A,\dot{B})}(p) \cdot a(p) := \begin{cases} \displaystyle\sum_{s_3=-s}^{s} u^{(A,\dot{B})}(p, s_3) a(p, s_3), & m > 0, \\ \displaystyle\sum_{h=\pm|h|} u^{(A,\dot{B})}(p, h) a(p, h), & m = 0, \end{cases}$$

where the a, b creation operators correspond to the Wigner momentum space wave functions of particles/antiparticles and the u, v represent the intertwiner (between the unitary Wigner representation and its covariant description) and its charge conjugate. In other words the covariant pointlike Wigner wave function becomes the positive frequency part of the field operator, i.e. covariant wave functions and pointlike covariant fields are functorially related.

For the third class (infinite spin, last line), the sum over spin components has to be replaced by an inner product between a p, e-dependent infinite component intertwiner u and an infinite component $a(p)$ since in this case Wigner's "little space" is infinite-dimensional. The $\Psi(x)$ respectively $\Psi(x, e)$ are "generating wave functions", i.e. they are wavefunction-valued Schwartz distributions which by smearing with $\mathcal{O}$-supported test functions become $\mathcal{O}$-localized wave functions. Adding the opposite frequency antiparticle contribution one obtains the above formula which, by re-interpreting the $a^\#$, $b^\#$ as creation/annihilation operators (second quantization functor), describes point-respectively string-local free fields. The resulting operator-valued Schwartz distributions are "global" generators in the sense that they generate $\mathcal{O}$-localized operators $\Psi(f)$ for all $\mathcal{O}$ by "smearing" them with $\mathcal{O}$-supported test functions.

Only in the massive case the full spectrum of spinorial indices A, $\dot{B}$ is exhausted (3.2), whereas the massless situation leads to the restrictions (3.3) which are the defining property of (generalized) field strengths; pointlike potentials violate the positivity of Wigner's representation spaces. This awareness about the possible conceptual clash between pointlike localization and the Hilbert space is important for the introduction of string-localization.

Whereas Weinberg [76] constructs the u, v-intertwiner functions of pointlike covariant fields (3.7) with the help of group theoretic methods, the modular localization approach is based on the direct construction of localized Wigner subspaces[13] and their generating stringlocal wave functions and associated fields. In that case the intertwiners depend on the spacelike direction e which is not a parameter but, similar to the point x, a variable in terms of which the field fluctuates [52]. Its presence allows the short distance fluctuations in x to be more temperate than in case of pointlike fields.

The short-distance reducing property of the generating stringlike fields is indispensable in the implementation of renormalizable perturbation theory in Hilbert space for interactions involving spins $s \geq 1$ fields[14]. Whereas pointlike fields are the mediators between classical and quantum localization, the stringlike fields are outside the Lagrangian or functional quantization setting since they are not solutions of Euler–Lagrange equations; enforcing the latter one arrives at pointlike fields in Krein space. String-localization lowers the power-counting limit, but requires a nontrivial extension [50, 67] of the iterative Epstein–Glaser renormalization machinery [24]. In

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[13]Modular localized subspaces of positive energy Wigner representations were first constructed in [11]. Before such concepts were used for the solution of integrable models [57, 58]. The construction of stringlocal covariant fields can be found in [52].

[14]These are also prcisely those interactions in which the absence of mass gaps does not lead to problems with the particle structure.

the next section it will be shown that modular localization is essential for *generalizing Wigner's intrinsic representation theoretical approach to the* (non-perturbative) *realm of interacting localized observable algebras.*

In order to arrive at Haag's algebraic setting of local quantum physics in the absence of interactions one may avoid "field coordinatizations" and apply the Weyl functor Γ (or its fermionic counterpart) directly to *wave function subspaces* where upon they are functorially passing directly to operator algebras, symbolically indicated by the functorial relation

$$K_{\mathcal{O}} \xrightarrow{\Gamma} \mathcal{A}(\mathcal{O}).$$

The functorial map Γ *also* relates the modular operators S, J, Δ from the Wigner wave function setting directly with their "second quantized" counterparts S_{Fock}, J_{Fock}, Δ_{Fock} in Wigner–Fock space; it is then straightforward to check that they are precisely the modular operators of the Tomita–Takesaki modular theory applied to causally localized operator algebras (using from now on the shorter S, J, Δ notation for modular objects in operator algebras)

$$\sigma_t(\mathcal{A}(\mathcal{O})) \equiv \Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it} = \mathcal{A}(\mathcal{O}),$$
$$J\mathcal{A}(\mathcal{O})J = \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}').$$

In the absence of interactions these operator relation are consequences of the modular relations for Wigner representations. The *Tomita–Takesaki theory secures their general existence for standard pairs* (A, Ω), i.e. an operator algebras $\mathcal{A}$ and a state vector $\Omega \in H$ on which $\mathcal{A}$ acts cyclic and separating (no annihilators of Ω in $\mathcal{A}$). The polar decomposition of the antilinear closed Tomita S-operator leads to the unitary modular automorphism group Δ^{it} associated with the subalgebra $\mathcal{A}(\mathcal{O}) \subset B(H)$ and the vacuum state vector Ω, i.e. with the pair $(\mathcal{A}(\mathcal{O}), \Omega)$.

Although $B(H)$ is generated from the two commuting algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O})'$, they do not form a tensor product in $B(H)$. Hence the standard quantum-information concepts concerning entanglement and density matrices are not applicable; the QFT realization of entanglement is stronger[15] since the vacuum state (in fact any finite energy state) restricted to a local algebra $\mathcal{A}(\mathcal{O})$ is an impure state which cannot be represented by a density matrix. In fact the "monad" (the unique hyperfinite type III_1 von Neumann factor algebra) $\mathcal{A}(\mathcal{O})$ has neither pure states nor density matrices.

Just in order to avoid confusions, modular localization of operators is more restrictive than modular localization of states. A state vector generated by applying an algebraically indecomposable stringlike localized field to the vacuum may have components to different irreducible pointlike Wigner representations. Apart from the infinite spin representations the distinction between point- and string-like is limited to fields and has no relevance for particles. When one calls an electron an "infraparticle" one refers to the fact that its interacting physical field is necessarily stringlocal and that its application to the vacuum, which generates a highly reducible Wigner representation state, does not contain a discrete component at $p^2 = m_e^2$.

The only case for which the modular localization theory (i.e. the adaptation of the Tomita–Takesaki modular theory to the causal localization principle of QFT) has a geometric interpretation (independent of whether interactions are present or not and independent of the type of quantum matter), is the wedge region, i.e. the Lorentz transforms of the standard wedge $W = \{x_0 < x_3 \,|\, \mathbf{x}_{\text{tr}} \in \mathbb{R}^2\}$. In that case the modular group is the wedge-preserving Lorentz boost and the J represents a reflection on the edge of the wedge, i.e. it is up to a π-rotation equal to the antiunitary TCP operator. The derivation of the TCP invariance as derived by Jost [32], together with scattering theory (the TCP transformation of the S-matrix) leads to the relation

$$J = S_{\text{scat}}J_{\text{in}},$$

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[15]The localization entropy of the vacuum entanglement for $\mathcal{A}(\mathcal{O})/\mathcal{A}(\mathcal{O}')$ is infinite.

which in [57, 58] has been applied to constructive problems of integrable QFTs. This is a relation which goes much beyond scattering theory; in fact it only holds in local quantum physics since it attributes the new role of a relative modular invariant of causal localization to the S-matrix which it does not have in QM.

This opens an unexpected possibility of a new access to QFT in which the first step is the construction of generators for the wedge-localized algebra $\mathcal{A}(W)$ with the aim to obtain spacelike cone-localized (with strings as a core) or double cone-localized algebras (with a point as core) from intersecting wedge algebras. In this top-to-bottom approach (which is based on the intuitive idea that the larger the localization region, the better the chance to construct generators with milder vacuum polarization) fields or compact localized operators would only appear at the end. In fact according to the underlying philosophy that all relevant physical data can be obtained from localized algebras the use of individual operators within such an algebra may be avoided; the *relative positioning of the localized algebras should account for all physical phenomena in particle phenomena.* The next section presents the first step in such a construction.

The only prerequisite for the general (abstract) case is the "standardness" of the pair $(\mathcal{A}, \Omega)$ where "standard" in the theory of operator algebras means that Ω is a cyclic and separating vector with respect to $\mathcal{A}$, a property *which in QFT is always fulfilled* for localized $\mathcal{A}(\mathcal{O})$ (thanks to the validity of the Reeh–Schlieder theorem [27]). These local operator algebras of QFT are what I referred to in previous publications as a *monad* [63]; their properties are remarkably different from the algebra of all bounded operators $B(H)$ which one encounters for Born-localized algebras in QM [60]. For general localization regions the one-parametric modular unitaries have no direct geometric interpretation since they describe a kind of fuzzy algebraic automorphism (which only near the boundary inside $\mathcal{O}$ permits a possible geometric visualization). But they are uniquely determined in terms of modular intersections of their geometric W-counterparts and are expected to become important in any top-to-bottom construction of models of QFT. Even in the simpler context of localized subspaces $K_{\mathcal{O}}$ related to Wigner's positive energy representation theory for the Poincaré group and its functorial relation to free fields these concepts have shown to be useful [11].

The most important conceptual contribution of modular localization theory in the context of the present work is the assertion that the reduction of the global vacuum (and more generally that of all physical (finite energy) states) to a local operator algebra $\mathcal{A}(\mathcal{O})$ leads to a thermal state for which the "thermal Hamiltonian" H_{mod} is the generator of the modular unitary group

$$e^{-i\tau H_{\mathrm{mod}}} := \Delta^{i\tau},$$
$$\langle AB \rangle = \langle B e^{-H_{\mathrm{mod}}} A \rangle,$$

where the second line has the form of the KMS property known from thermal states in the thermodynamic limit in which the Gibbs trace formula for a box-enclosed systems passes to the GNS state state formulation for open systems [27]. Whereas the trace formulation breaks down in the thermodynamic limit, this analytic KMS formula (asserting analyticity in $-1 < \mathrm{Im}\,\tau < 0$) remains. It is in this and only in this limit, that QM produces a global monad algebra (different from $B(H)$) whereas in QFT this situation is generic since it arises for any $\mathcal{A}(\mathcal{O})$-restricted finite energy state.

As mentioned in the introduction, the intrinsic thermal aspect of localization is the reason why the probability issue in QFT is conceptually radically different from QM for which one has to add the Born probability requirement.

Closely related to a modular localization is the "GPS characterization" of a QFT (including its Poincaré spacetime symmetry, as well as the internal symmetries of its quantum matter content) in terms of modular positioning of a finite number of monads in a shared Hilbert space. For $d = 1 + 1$ chiral models the minimal number of copies is 2, whereas in $d = 1 + 3$ the

smallest number for a GPS construction is 7 [33]. This way of looking at QFT is an extreme relational point of view in terms of objects which have no internal structure by themselves; this explains the terminology "monad" (a realization of Leibnitz's point of view about reality in the context of abstract quantum matter) [33, 60]. As life is an holistic phenomenon, since it cannot be explained from its chemical ingredients, so is QFT, which cannot be understood in terms of properties of a monad. This philosophical view of QFT exposes its radically holistic structure in the most forceful way; in praxis one starts with one monad and assumes that one knows the action of the Poincaré group on it [57, 58]; this was precisely the way in which the existence of factorizing models was shown [36].

In order to show the power of this new viewpoint for ongoing experimentally accessible physics, the following last subsection of this section presents some different viewpoint about some open problems in Standard Model physics.

3.2   Stringlocal vector mesons and their local equivalence classes

Modular localization theory shows how the conflict between pointlike quantization and the Hilbert space positivity structure, which appears in the Lagrangian or functional quantization of causally propagating classical field theory involving higher spin $s \geq 1$ fields, can be avoided. Instead of extending the quantum mechanical quantization rules to fields, one should notice that the Hilbert space *quantum* causal locality places limits on the "tightness" of localization. One may consider this as a sharpening of the radical different nature of quantum fields from their classical counterparts; whereas the latter are simply ordinary functions, causal quantum fields are operator-valued Schwartz distributions. It took a long time to get used to the fact that quantum localization leads to the formation of singular vacuum polarization clouds at the boundary of the localization region which only can be controlled by smearing pointlike fields with test functions with smoothly approach zero (surface roughening)[16]. In fact before these aspects were understood, QFT was even suspected to be inconsistent as a result of its "ultraviolet catastrophe".

The general principles of QFT lead to an interesting connection between the mass gap hypothesis and localization. QFT with mass gaps are generated by operators which are localized in arbitrarily narrow spacelike cones, i.e. regions whose core is a semiinfinite spacelike string (3.1). This theorem does not say anything about in which case mass gap QFT stringlocal generating fields are really needed in order to describe the physical content of the model, but at least one knows that one does not need operators localized on higher-dimensional spacelike subregions. In theories without a mass gap as QED one knows that operators carrying a Maxwell charge cannot be generated by the pointlike fields used in QED. On the other hand massive QED can be formulated as an interaction between a pointlike $s = 1$ Proca field coupled to charged spinor or scalar matter; but it is well known that as a consequence of $d_{\mathrm{sd}}(A_\mu^{\mathrm{P}}) = 2$ instead of 1 it turns out to be non-renormalizable. Using the fact that the short distance behavior can be improved by changing the problem to one in indefinite metric (Krein) space and setting up an elaborate ghost formalism, the BRST gauge formulation can be formulated as pointlike interaction in Krein space.

This poses the question whether the non-renormalizability of the Hilbert space matter fields is the way in which the model indicates that its full content cannot be described in terms of pointlike fields so that it illustrates a non-trivial realization of the above theorem. In the following we will show that this is indeed the case, i.e. behind the BRST gauge formalism there exists a stringlocal physical theory whose pointlike observables agree with the gauge invariant operators of the BRST gauge description. Pointlike physical matter fields still exist but only in the form of very singular

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[16]In the algebraic formulation in terms of localized operator algebras of bounded operators this leads to the area proportional *localization entropy* (which diverges in the limit of vanishing roughness).

(nonrenormalizable) Jaffe fields with a very restricted testfunction smearing (and a questionable role as generating fields for localized operator algebras) [31]. Whereas the localization problem in $s \geq 1$ zero mass interaction of the Wigner creation/annihilation operators $a^{\#}(p.s_3)$ already shows up in the nonexistence of pointlike interaction-free potentials, its appearance in the case of interacting massive higher spin field interactions is more discreet and happens through the connection of localization with renormalizability.

The unsolved problems which one encounters in trying to pass to physical operators in a gauge theory formulation are well known. Formal expressions for physical matter fields as stringlike composites in terms of gauge dependent pointlike fields in Krein space

$$\varphi(x, e) = \varphi^{K}(x) e^{ig \int_x^{\infty} A_\mu^{K}(x + \lambda e) e^\mu d\lambda}, \qquad e^\mu e_\mu = -1 \tag{3.8}$$

appeared already in publications of Jordan and Dirac during the 30s. But anybody who, apart from playing formal games, tried to obtain a computational control of such composite stringlocal expressions, knows that this is an impossible task. The new SLF setting inverts this problem from its head to its feet; instead of trying to represent physical charge-carrying fields in terms of pointlike fields, it bases renormalized perturbation theory direct on stringlocal fields. In this way one overcomes the clash between pointlike field in Krein space with the Hilbert space structure [57, 62].

Although modular localization plays an important conceptual role in the physical resolution of the clash between pointlike localization and the Hilbert space, most of the model calculations within the SLF setting can be done directly. An important role is played by the fact that pointlike massive free fields and their stringlike siblings are linearly related members of the same local equivalence class. For $s = 1$ the pointlike Proca field

$$\langle A_\mu^{P}(x) A_\nu^{P}(x') \rangle = \frac{1}{(2\pi)^{3/2}} \int e^{-ipx} M_{\mu\nu}^{P}(p) \frac{d^3 p}{2p_0}, \qquad M_{\mu\nu}^{P}(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

is related to its stringlike counterpart as

$$A_\mu(x, e) = A_\mu^{P}(x) + \partial_\mu \phi(x, e), \qquad e^\mu A_\mu = 0, \qquad \partial^\mu A_\mu = -m^2 \phi. \tag{3.9}$$

This relation is a direct consequence of the definition of $A_\mu(x, e)$ and $\phi(x, e)$

$$F_{\mu\nu}(x) := \partial_\mu A_\nu^{P}(x) - \partial_\nu A_\mu^{P}(x), \qquad \partial^\mu F_{\mu\nu} = m^2 A_\mu^{P}, \tag{3.10}$$

$$A_\mu(x, e) := \int_0^{\infty} F_{\mu\nu}(x + \lambda e) e^\nu d\lambda, \qquad \phi(x, e) := \int_0^{\infty} A_\mu^{P}(x + \lambda e) e^\mu d\lambda.$$

All three fields are linear combinations of the same $s = 1$ Wigner creation/annihilation operators $a_{s_2}^{\#}(p)$, $s_3 = -1, 0, +1$ with different linearly related intertwiner functions and the relation (3.9) follows from (3.10) and $e^\mu A_\mu(x, e) = 0$.[17] The scalar stringlocal field $\phi(x, e)$ will be referred to as the Stückelberg field; but in contrast to the Krein space Stückelberg field of the BRST gauge setting (see later) ϕ is physical in the sense of acting in Hilbert space.

It has been shown in [52] that *massive* scalar stringlocal fields can interpolate any integer spin; in the present case it creates $s = 1$ particles from the vacuum. This is not possible for massless particles; they can only be described by tensor potentials. In fact in the massless limit the linear relation to pointlike potentials is lost and the only surviving field is the $A_\mu(x, e)$. This can be explicitly seen by looking at the 2-point functions of the above fields

$$M_{\mu\nu}^{AA}(p; e, e') = -g_{\mu\nu} - \frac{p_\mu p_\nu (e \cdot e')}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)} + \frac{p_\mu e_\nu}{p \cdot e - i\varepsilon} + \frac{p_\nu e_\mu'}{p \cdot e' + i\varepsilon}, \tag{3.11}$$

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[17]The massless stringlocal fields look like vector potentials in the axial gauge apart from a significant conceptual difference: what led to their rejection in gauge theory (the fixed e and the incurable singularity at $pe = 0$), is here encoded into the essential SLF property: variable directional fluctuations (distributions in e).

$$M^{\phi\phi}(p; e, e') = \frac{1}{m^2} - \frac{e \cdot e'}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)}, \qquad M^{A\phi}_\mu = \dots, \qquad \text{etc.}$$

The appearance of mixed correlation between A^{P} and ϕ is (different from BRST) due to the fact that the degrees of freedom in terms of $a^{\#}_{s_2}(p)$ has been maintained.

The best way to interpret this situation is in terms of extending Borchers' concept of local equivalence classes of fields to stringlocal fields. The class of relative pointlike fields (Wick polynomials in the case of free fields) is known to play a similar role as coordinates in geometry: they describe the same model of QFT. The aforementioned theorem guaranties that this stays this way (at least in the presence of mass gaps) for local equivalence classes of stringlocal fields (with pointlike fields being considered as e-independent fields) since the S-matrix is independent on what stringlocal field coordinatization from the local equivalence class one uses. A similar statement holds for half-integer spins. The gain in using $A_\mu(x, e)$ instead of $A^{\mathrm{P}}_\mu(x)$ is the lowering of the short distance dimension $d_{\mathrm{sd}}(A^{\mathrm{P}}) = 2 \to d_{\mathrm{sd}}(A) = 1$. The derivative of the Stückelberg field compensates the leading short distance term of A^{P} at the price of string-localization: one unit of the nonrenormalizable interaction creating pointlike A^{P} has moved into directional e-fluctuations leaving the minimal possible value $d = 1$ for the end point fluctuations in x.

It is interesting to note that the local equivalence class picture permits a generalization in which the linear relation between $s = 1$ free fields is a special case of a more general relation for integer spin $s > 1$ fields

$$A_{\mu_1 \dots \mu_n}(x, e) = A^{\mathrm{P}}_{\mu_1 \dots \mu_n}(x) + \partial_{\mu_1}\phi_{\mu_2 \dots \mu_n} + \partial_{\mu_1}\partial_{\mu_2}\phi_{\mu_3 \dots \mu_n} + \cdots + \partial_{\mu_1} \cdots \partial_{\mu_{n_n}}\phi.$$

The left-hand side represents a stringlocal $s = n$ tensor potential associated to a pointlike tensor potential with the same spin. The $\phi's$ $s = n - i$, $i = 1, \dots, n$ tensorial Stückelberg fields of dimension $d = n - i + 1$. Each ϕ "peels off" a unit of dimension so that at the end one is left with the desired spin s stringlocal $d = 1$ counterpart of the tensor potential analog of the Proca field. The main problem of using such generalizations is the identification of those couplings which guaranty the existence of sufficiently many observables generated by pointlike Wightman fields (operator-valued Schwartz distributions). This may be important in attempts to generalize the idea of gauge theories in terms of SLF couplings involving massive $s > 1$ fields. It turns out that the stringlocal ϕ fields are inexorable "escorts" of stringlocal spin s tensor fields. Their presence in the interaction is the price to pay for converting nonrenormalizable spin s tensor fields of dimension $d = s + 1$ into their stringlocal $d = 1$ counterparts. This will be explicitly illustrated for $s = 1$ in the last subsection of this section.

The idea of SLF consists in starting from local zero-order equivalence class relations relations as (3.9) and show that they are either maintained in every order perturbation theory or replaced by other coupling dependent relations. In the case of massive QED the result is that (3.9) can be maintained in every order (plausible since the arguments in (3.10) survive perturbation theory) but have to be complemented by an equivalence class relation for the g-coupled matter fields

$$\psi(x, e) = e^{ig\phi(x, e)}\psi(x).$$

In the present work we will be satisfied with the simpler established property that the S-matrix is e-independent, i.e. $S_{\mathrm{scat}}(e) = S^{\mathrm{P}}_{\mathrm{scat}}$. In that case the calculation only involves time-ordered products of free fields (see below).

The relations (3.9) and (3.10) have the appearance of gauge transformations in the BRST gauge setting. But their conceptual content is quite different: instead of describing gauge transformations between pointlike gauge fields, their role in SLF is to *relate string- with point-localized quantum fields within the same local equivalence class.* Unlike the BRST gauge setting which maintains the quantization parallelism to classical gauge theory, the SLF relations are simply consequences of the foundational modular localization property of QFT.

Before passing to the calculation of the second-order S-matrix, it is instructive to point at some formal similarities with the BRST formalism. In the Krein setting the relation corresponding to (3.9)

$$\partial^\mu A_\mu^{\mathrm{K}} + m^2 \phi^{\mathrm{K}} \sim 0,$$

where the equivalence sign is meant to indicate that it cannot hold as operator relation on Krein space[18], one expects to find it in the cohomological descent to the Hilbert space. In fact in order to work with true operator relations one has to introduce in addition to the negative metric Stückelberg field two fermionic scalar ghost fields u and $\hat{u}$. In that formulation the cohomological equivalence relations are replaced by operator relation involving the nilpotent s-operation, e.g. $s\hat{u} = -i(\partial^\mu A_\mu^{\mathrm{K}} + m^2 \phi^{\mathrm{K}}) = 0$ on H_{phys}. To tighten the formal similarity with the BRST formalism, it is helpful to rewrite the relation (3.10) in terms of a differential form calculus in which d_e acts on a zero form

$$d_e(A_\mu(x, e) - \partial^\mu \phi(x, e)) = 0, \tag{3.12}$$

which follows from (3.9) and the e-independence of the Proca field. In contrast to the abstract algebraic s-operation the SLF localization setting uses the differential form calculus.

This differential form calculus can be used in order to express the string independence of interactions. Assume that we start from a pointlike nonrenormalizable massive QED $j^\mu A_\mu^{\mathrm{P}}$ interaction. Using the current conservation it is easy to convert this into a renormalizable stringlike interaction

$$\mathcal{L}^{\mathrm{P}} = j^\mu(x) A_\mu^{\mathrm{P}}(x) = j^\mu(x) A_\mu(x, e) - \partial_\mu V^\mu(x, e), \qquad V^\mu = j^\mu(x)\phi(x, e), \tag{3.13}$$
$$\text{or} \qquad d_e(\mathcal{L} - \partial_\mu V^\mu(x, e)) = 0, \qquad \mathcal{L} := j^\mu(x) A_\mu(x, e).$$

Here $\mathcal{L}$ is the renormalizable ($d_{\mathrm{sd}}(\mathcal{L}) = 4$) interaction density and the derivative part disposes (peels off) the $d_{\mathrm{sd}} = 5$ contribution as a boundary term at infinity, so that the pointlike first order $S = \lim_{g(x) \to g} \mathcal{L}^{\mathrm{P}}(g) = g \int \mathcal{L}^{\mathrm{P}}(x) d^4 x$ is the same as that of the stringlike interaction. We will say that the two interactions are asymptotically equivalent

$$\mathcal{L}^{\mathrm{P}} \overset{\mathrm{AE}}{\sim} \mathcal{L}. \tag{3.14}$$

The problem of showing the e-independence of the second-order renormalizable S-matrix defined in terms of is $\mathcal{L}$ is e-independent is obviously a renormalization problem since the treatment of the singularities in the "pointlike time ordering" $T(\mathcal{L}^{\mathrm{P}}(x)\mathcal{L}^{\mathrm{P}}(x'))$ has to be defined in such a way that it is AE equivalent its stringlike counterpart $T(\mathcal{L}(x, e)\mathcal{L}(x', e'))$.

3.3 Second-order calculations for abelian vector meson interactions

The strategy of the implementation of adiabatic equivalence starts with the zero-order relation (3.13) which is used in the Bogoliubov formula for the perturbative physical S-matrix and the physical fields. For massive QED the interaction density $\mathcal{L}$

$$\mathcal{L}(x, f) = \int de f(e)\mathcal{L}(x, e), \qquad \mathcal{L}(x, e) = A_\mu(x, e)j^\mu(x),$$
$$\mathcal{L} \equiv \mathcal{L}(g, f) = \int g(x)\mathcal{L}(x, f)dx,$$

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[18]This is analogous to the Gupta–Bleuler formalism in QED where relations between gauge dependent operators hold only on subspaces.

$$\psi_{\text{int}}(x,f) := \frac{\delta}{i\delta h(x)} S(\mathcal{L})^{-1} S(\mathcal{L} + h\psi)\big|_{h=0}, \qquad S(\mathcal{L}) = T e^{i \int g(x)\mathcal{L}(x,f)dx}$$

leads, according to the formal Bogoliubov prescription, to the perturbative S-matrix as well as to fields indicated for the simplest case in the second line for the interacting Dirac spinor; time-ordered products of interacting products originate from higher functional derivatives[19]. The physical S-matrix results from the Bogoliubov S-functional in the adiabatic limit $g(x) \equiv 1$. The existence of this limit is only guarantied in the presence of mass gaps. The interacting fields $\psi_{\text{int}}^{\text{phys}}(x,f)$ also require this adiabatic limit; but as a result of the appearance of the inverse S-functional, the requirement for their existence are less stringent. They are localized in a spacelike cone with apex x and require the same renormalization treatment as a pointlike $d = 1$ field.

The smearing function in the string direction can be fixed. The resulting physical $\psi(x,f)$-field depends nonlinearly on f and is localized in a spacelike cone with apex at x[20]. The e-independence of the scattering matrix is equivalent to f-independence for $f's$ normalized to $\int f = 1$. The f-independence of S_{scat} is expected since there exists a structural theorem stating that the S-matrix in models with mass gaps is independent of spacelike cone in which the interpolating operator (the operator used in the LSZ large time scattering limit) was localized [27]. The adaptation of the Stückelberg–Bogoliubov–Epstein–Glaser (SBEP) iterative formalism [24] to stringlocal fields requires to include string-crossing in addition to point-crossing. For the second-order calculation it is not necessary to study the full systematics of this new phenomenon which the reader will find in a forthcoming publication by Mund [50].

In the following we will present two second-order S-matrix calculations in which the problem of point- and string-crossings can be dealt with using pedestrian methods. Both models describe couplings of massive vector mesons to scalar fields; in the first case the matter field is complex ("scalar massive QED") whereas the second model describes a coupling to a Hermitian field. Whereas the application of the new SLF Hilbert space setting to massive QED shows the expected induction of the second-order quadratic A_μ dependence from the model defining first-order interaction, the induction[21] of terms in the second-order Hermitian coupling comes with some surprises. In that case there is no correspondence to a classical field theory since an interaction in the massless case does not exist and a massive vector meson-H coupling has no classical guidance which is the best prerequisite for encountering surprises.

The proof of e-independece (3.14) of the tree contribution in massive scalar QED (3.6) involves a renormalization problem for the two-point function

$$\langle T \partial_\mu \varphi^* \partial_\nu' \varphi' \rangle = \langle T_0 \partial_\mu \varphi^* \partial_\nu' \varphi' \rangle + g_{\mu\nu} c\delta(x - x'), \qquad \langle T_0 \partial_\mu \varphi^* \partial_\nu' \varphi' \rangle \equiv \partial_\mu \partial_\nu' \langle T_0 \varphi \varphi' \rangle, \ (3.15)$$

where the T_0 denotes the usual free field propagator without derivatives, c is a free renormalization parameter and the upper dash is used to avoid writing $\varphi(x', e')$ in order to shorten the notation; this notation will also be used in all subsequent equations. As a result of the two derivatives, the two-pointfunctions on the left hand side involves fields of scaling dimension 2 and hence has a scaling degree 4, which accounts for the presence of a delta function renormalization term in $T\mathcal{L}\mathcal{L}'|_{\text{1-contr}}$.

We define the one-contraction component (tree-component) of the second-order "anomaly" $\mathfrak{A}$ as

$$-\mathfrak{A}_e \equiv d_e(T_0\mathcal{L}\mathcal{L}' - \partial_\mu T_0 V^\mu \mathcal{L}')_1 = -d_e N + \partial^\mu N_\mu. \tag{3.16}$$

It is a measure of the violation of string-independence of the formal second-order extension of the first-order e-independence (3.13). The one-contraction component is a Wick-ordered product of

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[19]In order to include field strengths one needs another source term, i.e. $S(L + h\psi + kF)$.

[20]The apex is also the point which is relevant for the Epstein–Glaser distributional continuation.

[21]Different from counterterms which come with new parameters, induced terms depend only on the model-defining first-order couplings and the masses of the participating free fields.

4 fields which determines the second-order S-matrix. If we are able to represent it in the form of two normalization terms N, N_μ we could absorb these two terms into a renormalization of the second-order expression. In this case N can be used to define a renormalized second-order tree component

$$T_0\mathcal{L}\mathcal{L}'\big|_1 \to T_0\mathcal{L}\mathcal{L}'\big|_1 + N$$

and the N_μ can be likewise absorbed into local change of the derivative contribution. In case the differentiation acts inside the time-ordering one has

$$d_e(T_0\mathcal{L}\mathcal{L}' - \partial_\mu T_0 V^\mu \mathcal{L}') = 0.$$

Hence the delta contributions come from the singular part in

$$\partial_\mu T_0 V^\mu \mathcal{L}'|_1 = -i\big(\partial_\mu \partial^\mu \langle T_0 \varphi^* \varphi'\rangle \varphi \overleftrightarrow{\partial'_\nu} \varphi^{*\prime}\phi A^{\nu\prime} + \text{h.c.}\big)_{\text{sing}} + \text{reg.}, \tag{3.17}$$

$$\partial_\nu \delta(x - x')\varphi\varphi^{*\prime}\phi A^{\nu\prime} = \partial_\nu(\delta(x - x')\varphi\varphi^{*\prime}\phi A^{\nu\prime}) - \delta(x - x')\partial_\nu(\varphi\varphi^{*\prime}\phi A^{\nu\prime}), \tag{3.18}$$

where the delta in (3.18) results from the action of the wave operator on the T_0 propagator. The ∂'_ν derivative acting on the delta function can be replaced by a $-\partial_\nu$ derivative which in turn can be rewritten as in (3.18). The first term in (3.18) leads to $\partial_\nu N^\nu$ in (3.16).

Adding up all terms in (3.17) which contribute to the second term in (3.18) one finds that the contributions which involve derivatives ∂_ν of the φ-fields cancel, so that only the term for which the derivative acts on ϕ remains. Taking into account the relation $d_e\partial_\nu\phi = d_e A_\nu$ we obtain

$$d_e\delta(x - x')\varphi\varphi^*\partial_\nu\phi A^{\nu\prime} = d_e\delta(x - x')\varphi\varphi^* A_\nu A^{\nu\prime}.$$

The derivation of the e' contribution to the anomaly

$$-\mathfrak{A}_{e'} = d_{e'}(T_0\mathcal{L}\mathcal{L}' - \partial_\mu T_0 \mathcal{L} V'^\mu)_1$$

follows the same steps, so that the final result (including the correct numerical coefficients) is

$$(d_e + d_{e'})(T_0\mathcal{L}\mathcal{L}' + 2\delta(x - x')\varphi^*\varphi A_\nu A'^\nu)_1 = \text{derivative terms.} \tag{3.19}$$

Since derivative terms do not contribute to the adiabatic limit, our result implies that the second-order S-matrix is a string-independent global observable, i.e. the first-order adiabatic equivalence (3.14) has been extended to second order. The renormalized interaction is of the same form (apart from the fact that vector potential at the same point have different directional variables e) as that obtained from imposing the gauge formalism in the form

$$\partial_\mu \ \to \ D_\mu = \partial_\mu - igA_\mu.$$

In the present case the only requirement was the Hilbert space setting (which for $s \geq 1$ implies a weakening of localization from point- to string-like).

The result may be conveniently written in the form

$$T_0\mathcal{L}\mathcal{L}' \to T\mathcal{L}\mathcal{L}', \qquad \langle T\partial_\mu\varphi^*\partial_\nu\varphi\rangle = \langle T_0\partial_\mu\varphi^*\partial_\nu\varphi\rangle - g_{\mu\nu}\delta(x - x'),$$

i.e. the renormalization can be encoded into a change of the two derivative propagator where all propagators with a lesser number of derivatives remain unchanged. In this form it holds for tree contributions in any order.

Encouraged by this success one may ask the question whether this formalism can be generalized to define string-independent interaction densities. This is indeed possible. The respective relation is more restrictive and has the form [67]

$$d\big(T\mathcal{L}\mathcal{L}' - \partial^\mu TV_\mu\mathcal{L}' - \partial'^\nu T\mathcal{L}V'_\nu + \partial^\mu\partial'^\nu TV_\mu V'_\nu\big) = 0, \qquad d = d_e + d_{e'},$$

$$(T\mathcal{L}\mathcal{L}')^{\mathrm{P}} \equiv T\mathcal{L}\mathcal{L}' - \partial^{\mu}TV_{\mu}\mathcal{L}' - \partial^{\prime\nu}T\mathcal{L}V_{\nu}' + \partial^{\mu}\partial^{\prime\nu}TV_{\mu}V_{\nu}', \tag{3.20}$$

where T stands for a renormalized T_0. A renormalized pointlike second-order interaction density can then be defined as in (3.20).

The delta function term (3.19) corresponds to the second-order quadratic gauge contribution which in classical gauge theory is subsumed in the substitution of ∂_{μ} by $\partial_{\mu} \to D_{\mu} = \partial_{\mu} - ieA_{\mu}$. In our operator setting the second-order contribution (3.19) has no intrinsic significance since it is part of the renormalized TLL' which leads to the e-independent S-matrix. In fact the value of the normalization constant c depends on the choice T_0. The correct physical picture is that of second-order "induction" from the first order; with other words certain counterterms, which in $s < 1$ pointlike setting would introduce new parameters, are fixed by the causal localization principle which requires the e-independence of S_{scat}. This induction mechanism is of particular importance for the coupling to neutral matter to which we will move now.

Hermitian scalar fields H coupled to massive vector meson (the charge-neutral counterpart of massive scalar QED) represents the Higgs field in the work of the BRST operator gauge treatment by the Zürich group (see [56] and references therein) and more recently in [23]. The terminology "Higgs" in their presentation had no relation to any symmetry breaking (the "Higgs mechanism"); they showed that the imposition of the gauge formalism on massive vector meson interactions with neutral particles *induces* second-order terms which together with the model-defining first-order interaction takes the shape of a "Mexican hat" potential. There is some irony in this result because the Higgs mechanism had problems with the classical picture of gauge variance; a problem which was pointed out by several authors. On the other hand the correct induction mechanism of the Mexican hat potential was solely based on the implementation of operator BRST gauge formalism [56], or directly on the causal localization principle in the new Hilbert space SLF setting. It reveals that the Higgs mechanism is the result of a formal manipulation which has no relation to the correct interpretation in terms of a renormalizable interaction of massive vector mesons with Hermitian matter.

The formal similarity of the nilpotent s-operation of the BRST gauge formalism with the d_e differential form calculus of the SLF Hilbert space approach extends to higher order calculations. The one-contraction component of the second order anomaly in terms of the nilpotent BRST s-operation reads [56]

$$-\mathfrak{A}_s = \left(sT_0\mathcal{L}\mathcal{L}' - \partial^{\mu}T_0Q_{\mu}\mathcal{L}' - \partial^{\prime\,\mu}T_0\mathcal{L}Q'\right)_1.$$

It is a measure of the violation of gauge invariance which results from the use of the "unrenormalized" time ordering of the product of first-order interactions and its computation is the first step in the construction of a gauge invariant second order S-matrix. Since the action of s is always defined in a Krein space, we may economize on notation and omit the superscripts K on the operators. For the special case of vanishing selfinteraction H^3 ($c = 0$) in $\mathcal{L}$ the anomaly turns out to be of the form [56]

$$-\mathfrak{A}_s = sN + sR + \partial^{\mu}N_{\mu},$$
$$N = i\delta A \cdot A(H^2 + \phi^2), \qquad R = i\delta\frac{m_H^2}{m^2}\left(H^2\phi^2 - \frac{1}{4}\phi^4\right).$$

The expression for N_{μ} has been omitted since it renormalizes $T_0Q_{\mu}L' + T_0LQ'_{\mu}$ but does not contribute to the second order interaction density T_0LL'.

The N-term can be absorbed into a redefinition of the time ordered product

$$T\partial_{\mu}H(x)\partial_{\nu}'H(x') = T_0\partial_{\mu}H(x)\partial_{\nu}'H(x') + \alpha g_{\mu\nu}\delta(x - x'),$$
$$T\partial_{\mu}\phi(x)\partial_{\nu}'\phi(x') = T_0\partial_{\mu}\phi(x)\partial_{\nu}'\phi(x') + \beta g_{\mu\nu}\delta(x - x')$$

by appropriate choice of α, β, so that the second order result is of the form

$$\frac{g^2}{2}(T\mathcal{L}\mathcal{L}' + R).$$

Scharf [56] then shows that a nontrivial trilinear H selfinteraction $c \neq 0$ would lead to a third order anomaly unless one also introduces a quadrilinear self-coupling $c'H^4$. Whereas $c \sim g$, the latter is of second order $c' \sim g^2$ (with well-defined numerical coefficients). The resulting $\hat{R}$ is a 4th degree polynomial in H, ϕ. The final step consists in defining a "potential" V by combining the first- and second-order H, ϕ contributions from $\mathcal{L}$ and $\hat{R}$ to a "Mexican hat" potential

$$V = V_1 + \frac{1}{2}V_2 = g^2\frac{m_H}{8m^2}\left(H^2 + \phi^2 + \frac{2m}{g}H\right)^2 - \frac{m_H^2}{2}H^2,$$

$$S^{(2)} = 1 + gi\int \mathcal{L}_{\text{tot}}d^4x + i^2\frac{g^2}{2}\int d^4x\int d^4x'T\mathcal{L}\mathcal{L}', \qquad g\mathcal{L}_{\text{tot}} = g\mathcal{L} + V.$$

By construction the second order S-matrix is BRST gauge invariant $sS^{(2)} = 0$, although the Mexican hat potential by itself is not.

Note that this way of writing V only serves the purpose of facilitating a formal comparison with the alleged symmetry-breaking potential in which the two physical masses are replaced by the parameters of the "Higgs mechanism" (a coupling strength and a field shift). But the correct description is in terms of a coupling of a massive vector meson to a massive Hermitian field. In fact renormalized perturbation theory always starts with the model-defining free fields (including their masses) and a first-order polynomial interaction between these fields.

This result implies a radical conceptual change as compared to the Higgs mechanism. The Mexican hat potential is not the symmetry-breaking part of the defining A-H interaction, but it is *induced* in second order from the defining A-H interaction. The most surprising aspect of this induction mechanism is the fact that the coupling strengths of the most general renormalizable model of interaction of a massive vector meson A with a Hermitian field H (which includes H^3, H^4 selfinteractions) is *uniquely determined in terms of the A-H coupling g*. This kind of induction of additional couplings from BRST gauge invariance of the S-matrix continues to hold in the SLF Hilbert space setting. It also shows that the second order induced H^4 interaction is different from a possible higher order loop contributions ("box-graphs") of a renormalization-caused selfinteraction as known from scalar QED.

The implementation of the e-independence of S in the Hilbert space setting confirms these findings of gauge theory, but it also leads to the appearance of additional terms which contain A-fields and derivative of ϕ which cannot be incorporated into a Mexican hat potential. This is a consequence of the Hilbert space positivity which replaces the Krein space Stückelberg ϕ^K of the above BRST gauge setting (re-introducing the omitted K) by a physical stringlocal scalar intrinsic escort field ϕ. The Hilbert space approach has the additional advantage to relate the physical particle states directly with Wigner creation/annihilation operators whereas in the BRST gauge setting additional cohomological steps are necessary in order to recover physical states.

This places the construction of string-independent S-matrix in the Hilbert space setting on safer conceptual grounds than that of gauge invariant S-matrices in the BRST setting. Whereas there is no doubt about the correctness of the BRST description of *local observables in terms of gauge invariant pointlike fields*, it is less clear to what extend this can be extended to particle states beyond the vacuum sector. It remains questionable whether the proven field-particle relations (as, e.g., the LSZ scattering theory on which the connection between fields and the S-matrix is based) can be upheld in a Krein space setting of BRST gauge theory. The construction of e-independent S-matrices and a more detailed comparison with its $sS = 0$ characterization in the gauge setting will be the subject of forthcoming work of a collaboration with Jens Mund.

This raises the question why the inconsistency of the Higgs mechanism was not seen during the 40 year of its existence. Well, it was seen by some people. There have been several attempts to point at its metaphoric aspects, but all of them were eventually lost in the maelstrom of time. Swieca together with Ezawa [25] proved that behind Goldstone's Lagrangian model observation there was a structural theorem[22] which relates spontaneous symmetry breaking with the appearance of a zero mass Boson (the Goldstone Boson). This is the mechanism by which conserved currents to nonexisting (long-distance-diverging) charges. It is helpful to recall what was known at that time about conserved currents and charges in the form of a schematic table:

$$\text{screening: } Q = \int j_0(x)d^3x = 0, \qquad \partial^\mu j_\mu = 0,$$

$$\text{spont. symm.-breaking: } \int j_0(x)d^3x = \infty,$$

$$\text{symmetry: } \int j_0(x)d^3x = \text{finite} \neq 0.$$

Of special interest for theories involving massive vector mesons is the screening of the Maxwell charge, since it is the characterizing property of interacting massive vector mesons. This goes back to a conjecture by Schwinger and was later established as a structural theorem by Swieca [59]. Whereas in massive QED there exists besides the identically conserved Maxwell current also the standard charge counting current whose global charge counts the difference of charge and anticharge, there is also a *charge neutral model in which the Maxwell current* (*leading to the screened Maxwell charge*) is the only current. In the limit of vanishing vector meson mass, the chargeless model approaches a free massless vector meson. In the massive case the screening appears already in zero-order since the Maxwell current $j_\mu^M(x) \sim m^2 A_\mu^P$ and the charge of the identically conserved Proca "current" vanishes. The screening theorem [73] can be explicitly verified in low-order perturbation theory. It shows that the Maxwell charge in the abelian Higgs model remains screened (zero) and does not diverge as in case of spontaneous symmetry breaking.

Schwinger invented the two-dimensional *Schwinger model* in order to illustrate charge screening in a mathematically controlled situation, and Lowenstein and Swieca presented its full solution [39]. Some years later Swieca, after presenting a general structural proof of the Goldstone conjecture [25], succeeded to prove a theorem (the "Schwinger–Swieca screening theorem" [73]) which showed that charge screening is a structural consequence of massive Maxwell charges (an identically conserved current associated to a $F_{\mu\nu}$ field strength). In order to save the Higgs model from its incorrect interpretation in terms of the "Higgs mechanism" (mass creation through spontaneous symmetry breaking) and to direct attention to the fact that the physical content of the Higgs model is a realization of Schwinger's screening with a Hermitian instead of a complex matter field, he referred to it in all his publications the *Schwinger Higgs screening*. Here the name Higgs stands for the statement that Schwinger's screening can also be realized with neutral matter (coupling of massive vector mesons with Hermitian fields). His attempts to direct people away from a misunderstanding towards genuine intrinsic physical properties failed; his ideas succumbed to the maelstrom of time.

This poses the question of how an era, which was as rich in innovative foundational ideas as the three decades after wwII, could develop into situation in which a misunderstanding on a particular but important problem dominated particle theory for more than 40 years. The final answer will be left to historians, but for the author it seems that the unfortunate concurrence of two causes contributed to the present situation. On the scientific side it was the idea (coming from gauge theory) that massless $s = 1$ interactions (QED, QCD) are simpler than their massive counterparts. The Higgs mechanism of mass creation through spontaneous symmetry breaking

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[22]Based on the Jost–Lehmann–Dyson representation, which in turn was derived from locality in a Hilbert space setting.

imposed on scalar QED is the result of that Zeitgeist. We know nowadays, last but not least through the new SLF Hilbert space setting applied to interacting vector mesons, that the opposite situation prevails: models with mass gaps fall well within the standard particle-field setting, whereas the difficult problems of gluon/quark confinement and a spacetime understanding of QED collision theory involving electrically charged particles only occur in their massless limits.

Even in the case of Goldstone's spontaneous symmetry breaking the important point is not the 'manipulation of a Lagrangian by applying a constant shift in field space, but rather the existence of a conserved current whose charge diverges as a result of its coupling to a zero mass boson. In this case one may still tolerate the shift in field space as an anthropomorphic presentation of an intrinsic structural statement, namely the connection of the broken symmetry (its diverging charge which fails to generate the symmetry) with the existence of a Goldstone boson. To extend such manipulation to unphysical (gauge-dependent) charged fields (there are no pointlike physical charged fields) in scalar QED is meaningless since gauge "symmetry" is not a symmetry but a prescription which permits to extract certain physical observables from a Krein space formalism.

The price to pay is that one completely misses the renormalizable interaction of a massive vector meson with a charge-neutral (Hermitian) matter field which hides behind the "symmetry breaking and mass creating Higgs mechanism". Whereas one may always add to interacting massive vector mesons additional couplings to Hermitian matter fields, their presence is not needed to generate or sustain the mass of vector mesons. There exists however no renormalization theory of massive vector mesons in Hilbert space without the presence of intrinsic stringlocal scalar escort fields ϕ. As in the Ginsberg–Landau or BCS theory of superconductivity where one does not have to introduce outside degrees of freedom in order to obtain short-ranged vector potentials, there is presently no indication that interacting massive vector meons in QFT require the presence of additional degrees of freedom in the form of H fields.

This raises the question whether the observed LHC events could be bound states of the intrincic escort field ϕ fields. This is supported by the observation that in interactions of stringlocal massive vector mesons with charge matter or among themselves the ϕ appear in the interaction densities at the places where the H-terms in the BRST formalism appears. This should not be surprising since H states are indistinguishable from massive gluonium states. Could a Higgs model, after the loss of the foundational Higgs mechanism, be a phenomenological description of ϕ-states? The new Hilbert space description for $s \geq 1$ interactions requires a reinvestigation of the gauge-theoretic Krein space description. Particular attention is required with respect to results which were obtained on high-energy behavior from Feynman graphs since their use is limited to the pointlike Krein space formalism.

Recently several books by several authors (Unzicker, Lopez Corredoira, . . .) appeared in which the present situation in science and in particular in particle physics was critically analyzed. The line of attack is predominantly against "Big Science" as represented by CERN and their handling of the Higgs issue. In spite of their sometimes polemic arguments these authors are far from being crackpots; one of them (Unzicker) actually discloses a considerable amount of insider knowledge covering the activities of CERN during the last 3 decades. He accuses Big Science to use Peter Higgs, a very modest and shy individual who first exemplified the Higgs mechanism, to present the discovery of a new scalar chargeless particle as the centuries greatest contribution to particle physics in order to justify the enormous amount of resources and manpower of present High Energy Physics.

Whatever conceptual changes the Standard Model and in particular gauge theories will undergo in the years ahead, the clarification of the "Higgs mechanism" in terms of a new Hilbert space setting of $s \geq 1$ renormalization theory will certainly play an important role in the extension of QFT to interactions involving higher spin fields. According to the best of my knowledge it is the first time that ongoing foundational changes in local quantum physics come into direct

contact with observational aspects of Standard Model particle physics. Hopefully this will also lead to a reduction of the deep schism between the large community of users of QFT and a small group of researchers exploring its still largely unknown terrain which is the main concern of the present paper.

3.4 Nonabelian couplings and the SLF view of confinement

In the previous section it was shown that the Higgs mechanism is the result of a conceptual misunderstanding of QFT. The physical content of the abelian Higgs model, which remains after removing the meaningless idea of a spontaneous mass creation by postulating a Mexican hat potential, is that of a renormalizable coupling of a massive vector potential to a Hermitian scalar matter field. The Mexican hat potential is not an input for a spontaneous mass creation, but rather describes the terms which the requirement of string independence of the S-matrix induces within the renormalizable SLF Hilbert space setting. This result confirms previous findings within the BRST gauge setting (see a recent review [23]). These results invalidate the claim that massive vector mesons owe their mass to a Higgs breaking mechanism; instead they lead to the presence of scalar intrinsic escort fields ϕ of massive vector mesons, which is a new epiphenomenon of the SLF Hilbert space setting for renormalizable interactions involving higher spin $s \geq 1$ fields.

There is presently no indications that the Hilbert space positivity and locality of fields lead to further impositions on the renormalizable field content beyond the existence of the intrinsic escorts, but it would certainly be helpful to have a better understanding of the more elaborate self-interactions in terms of explicit second-order calculations [51]. The BRST gauge formulation leads to restrictions on the form of vector meson self-couplings to that which is known from classical gauge symmetry formulated in the mathematical setting of fibre-bundles; this is hardly surprising since this ghost formulation results from the adaptation of the classical gauge symmetry. The new SLF Hilbert space formulation on the other hand has no connection to gauge ideas, it has only the perturbative adaptation of the foundational locality principle (modular localization) as its disposal. The reduction of an Ansatz of the most general self-interaction between massive vector mesons with equal masses to the Yang–Mills form in which the self-couplings are connected by a Lie-algebraic structure is not the result of the imposition of a symmetry but rather follows from the consistency of perturbative renormalization with locality and the Hilbert space positivity for self-interactions between vector mesons. This phenomenon has no counterpart for $s < 1$ pointlike interactions and this explains why it hitherto remained overlooked. It is the nonabelian analog of the second-order induced $A \cdot A$ contribution in the previous subsection.

In more concrete terms, the SLF Hilbert space reformulation of a pointlike self-interaction of vector mesons with arbitrary real f_{abc} couplings and identical masses should lead to the expected Lie-algebraic restriction of f

$$\mathcal{L}^{\mathrm{P}} = \sum_{abc} f_{abc} F_a^{\mu\nu} A_{b,\mu}^{\mathrm{P}} A_{c,\nu}^{\mathrm{P}}, \qquad \mathcal{L}^{\mathrm{P}} = \mathcal{L} + \partial^\mu V_\mu.$$

In other words the symmetric form, which in the standard gauge setting is the result of the differential geometric properties of gauge symmetry and which the operator BRST setting incorporates through its ghost charge formalism, should follow solely from foundational Hilbert space setting of the causal localization principle of QFT. The requirement that the general pointlocal form can be rewritten in the form of the second line, with $\mathcal{L}$ being the stringlocal counterpart of $\mathcal{L}^{\mathrm{P}}$, leads to the total antisymmetry of f. Since derivative terms do not contribute to the S-matrix[23], the physical interpretation of that relation is that it guaranties the e-independence

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[23]The perturbative S-matrix is obtained from g-integrated time-ordered products of interaction densities in the adiabatic limit $g(x) \to g$. Boundary terms from derivative contributions vanish in massive theories (Section 3.3).

of the S-matrix. The formulation of this requirement in second order imposes the desired Lie algebraic restrictions on f [51].

More explicitly one has

$$\mathcal{L} = \sum f_{abc} \left\{ F_a^{\mu\nu} A_{b,\mu} A_{c,\nu} + m^2 A_{a,\mu}^{\mathrm{P}} A_b^\mu \phi^c \right\}, \qquad V_\mu = \sum f_{abc} F_a^{\mu\nu} (A_{b,\nu} + A_{b,v}^{\mathrm{P}}) \phi^c,$$

$$d_e(L - \partial^\mu V_\mu) = 0 \qquad \text{if } f_{abc} \text{ are totally antisymmetric.}$$

The validity of the Jacobi identity and hence their Lie-algebraic nature follows from the formulation of e independence in second order [51]. This is similar to Scharf's use of gauge invariance [56], except that in the SLF Hilbert space setting there is no reference to a gauge symmetry.

In agreement with our underlying philosophy which emphasizes the physical simplicity of massive models as compared to the incompletely understood subtleties of their massless counterpart, we consider the massless Yang–Mills models as limits of massive Yang–Mills couplings. In other words our viewpoint is opposite to that of the Higgs mechanism. Interestingly the implementation of our strategy leads to the appearance of a kind of substitute of the Higges field: the intrinsic escort fields ϕ_a which appear already in first-order perturbation.

The appearance of Lie-algebraic restrictions from the $s \geq 1$ implementation of QFT principles should be seen in connection with the Doplicher–Roberts result [27]. The latter states that the superselection structure following from the locality property of local observables can be encoded into a field-net extension of the observable net which is symmetric under the local action of a compact group. In perturbation theory one has to impose the specific symmetry in terms of relations between in principle independent renormalization terms. In the case of the symmetric appearance of induced interactions for $s \geq 1$ in our new setting there simply exists no less symmetric theory on whose renormalization theory we can impose symmetry conditions. What has been hitherto considered as a consequence of imposing local gauge symmetry is really the result of the general principles of QFT. Whether this perturbative observation can be backed up by a structural theorem, as it was possible in the Doplicher–Roberts superselection theory, remains to be seen.

An important difference of the new setting compared to pointlike perturbation theory is that the connection between off-shell properties and high energy behavior on-shell restrictions in terms of Feynman diagrams break down. The presence of stringlocal propagators and stringlocal vertices (from string-crossings) invalidate phenomenological arguments in favor of Higgs particles based on high-energy improvements of perturbative on-shell unitarity. The SLF Hilbert space formalism leads to a very subtle connection between the bad off-shell behavior and its on-shell improvements. The new perturbation theory with its string-crossings cannot be encoded into Feynman diagrams.

Coming to the relation with the LHC experiment one should point out that these results cannot distinguish between a "gluonium" bound state of the intrinsic escort ϕ, and an added H-coupling. Unfortunately the impossibility of understanding bound states within perturbation theory impedes reliable quantitative predictions, but this is not different from the description of hadrons in terms of bound states of quarks. Since the induced couplings of the intrinsic escort ϕ to the vector potentials is indistinguishable from a Higgs–Kibble coupling, the latter could be a phenomenological description of the SLF Hilbert space situation.

An even more important problem for the future path of QFT and the Standard Model is the question whether it is possible to show that confinement has a clear physical meaning in Yang–Mills theories, i.e. that it can be derived on the basis of the infrared behavior of massless limits in expectation values of stringlocal physical (Hilbert space[24]) massive gluon/quark fields. The new setting strongly suggests that confinement means that correlation functions, which

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[24]The positivity of Hilbert space is expected to play an important role in order to obtain the physical infrared behavior of stringlike gluons/quarks as opposed to that of their unphysical counterparts in the gauge setting.

besides pointlike observable composite fields (gluonium, hadrons) also contain stringlocal gluon and quark fields, should vanish. The only exception should be spacelike separated q-$\bar{q}$ pairs whose string direction is parallel to the direction of their spacelike separation.

Free zero mass string fields, with the exception of those belonging to the third Wigner positive energy class (massless infinite spin fields), are reducible strings, i.e. they can be written as semi-infinite integrals over pointlike field strengths. This is certainly not the case for *interacting* massless gluon fields since the lowest pointlike composites are of polynomial degree 4. As the free infinite spin strings [66], their are their noncompact localization is irreducible. Both noncompact types of strings have problems with causality which forbids their emergence from collisions of ordinary particles (i.e. particles localized in compact regions). Abelian zero mass theories are somewhere in the middle; the vector potential strings are reducible, but this is not the case for the strings of charged matter.

In the case of interacting gluon strings one mechanism (perhaps the only one) of avoiding contradictions with causality which arise from their appearance in collisions of compact matter is that correlations which, besides pointlike composites (gluonium fields) also contain gluon fields, must vanish. The SLF setting presents a realistic scenario to check such a situation because, different from the BRST gauge setting, there is a natural physical covariant stringlocal massive gluon field which for $m \to 0$ passes to its massless physical counterpart; so a proof would consist in showing that a partial resummation of the leading logarithmic contributions to the infrared divergences leads to a zero result after interchanging the limit with the summation of the leading terms. The infraparticle situation of charged particles is less radical, since in that case one expects the correlation functions of physical stringlocal charged fields to remain infrared-finite and to represent the SLF counterpart of (3.8). The Yennie–Frautschi–Suura argument [77] (generalizing previous model calculations by Bloch and Nordsiek) is based on the logarithmic divergencies in an infrared cut-off parameter $\lambda \to 0$ which appear in the mass shell restriction of the stringlocal physical charged matter. From low-order logarithmic divergencies one reads off the systematics of the leading contributions from the higher terms and finds a coupling-dependent power behavior $\lambda^{f(g)}$.[25] One then concludes that the $\lambda \to 0$ limes (the scattering amplitude for charged particle with a fixed finite number of outgoing photons) vanishes and that the perturbative logarithmic divergencies only appeared because of the illegitimate inversion of the perturbative expansion with the $\lambda \to 0$ limit. The authors [77] then show that a nontrivial scattering information resides in photon inclusive cross sections rather than scattering amplitudes; such a construction has no off-shell counterpart.

The SLF setting suggests an interesting improvement of the YFS argument which consists in replacing the ad hoc noncovariant infrared regulator λ by the covariant physical vector meson mass m. The limit should reproduce the YFS result of vanishing scattering amplitudes for charged particle scattering with a finite number of outgoing photons. Another a possible proof which is of a more structural kind is to first show that the infrared properties replace the mass-shell pole by a less singular cut. The resulting milder singularity cannot compensate the dissipation of wave packets which enter the LSZ formalism; this leads the vanishing of the $t \to \infty$ LSZ limit.

In the Yang–Mills case one expects that the off-shell correlation of massive gluons, which for $m \to 0$ are logarithmically divergent, vanish after interchanging the massless limit with the summation of leading logarithmic divergencies. Correlations which only contain pointlike composites are expected to stay finite in this limit. This would resolve the causality violating emergence of noncompact localized objects from compact spacetime collision regions. In a certain sense the this causality problem which is avoided through confinement, is opposite to that of the irreducible free strings of the infinite spin Wigner positive energy representations class; in

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[25]In the spacetime LSZ scattering setting of infraparticles the mass shell singularities have been softened; so that it cannot be compensated by the large time wave packet behavior.

that case one expects that apart, from its coupling to gravitation (any kind of positive energy matter couples to gravitation), noncompact matter cannot change into ordinary matter; this kind of inertness makes such matter an excellent candidate for dark matter [66].

The SLF setting also presents a rigorous perturbative way to check the asymptotic freedom property based on the beta function in well-defined Callen–Symanzik equations for well-defined correlation functions. The existing derivation is only a consistency argument[26] and not a proof; it shows that the educated guess of a massless Yang–Mills beta function is consistent with the computable short distance behavior. A rigorous calculation would first establish the Callen–Symanzik equations for the renormalizable stringlocal massive Yang–Mills coupling and then appeal to the mass-independence of the beta function.

It is appropriate to end this section with two remarks which relate the present results to other ideas which arose in the history of particle physics.

The SLF Hilbert space approach is vaguely reminiscent of Mandelstam's idea to formulate QED solely in terms of field strengths. It turns out that precisely the directional fluctuation of the $x + \mathbb{R}_+ e$ localized $A_\mu(x, e)$ in e (a point in $d = 1 + 2$ de Sitter spacetime) attenuate the strength of the x-fluctuations and renders the interaction renormalizable in the sense of power-counting. The picture is that the nonvanishing commutators for string crossing are *necessary for lowering the singularity for coalescent x*. Mandelstam's approach failed because in his setting it seems to be difficult to take care of this advantage [45, 46]. In both, the massless as well as the massive case, there always exists a string-localized description in which the e-fluctations lower the strength of the x-fluctuation of the pointlike description in such a way that the resulting short distance scale dimension is $d = 1$, independent of spin.

As mentioned before, there is also a formal relation to the "axial gauge". Although it was seen that this gauge is formally compatible with a Hilbert space structure, the interpretation of e as a fixed gauge parameter (not participating in Poincaré transformations) misses its role in the formulation of stringlocal renormalizable interactions which avoids the use of unphysical Krein spaces. The special status which gauge theory attributes to interactions of vectormesons has been removed and $s = 1$ interactions have been united with $s < 1$ interactions under the common roof of the localization principle in a Hilbert space setting. In this way SLF leads to a democratization of low and high spin QFT under the shared conceptual roof of its foundational quantum causal (modular) localization principle. This "democratization" on the level of fields parallels that of particles in that the new setting also removes the hierarchical role of Higgs particle (the "God particle") and re-establishes "nuclear democracy" between particles.

Stringlike localization also appears in the axiomatic approach as the tightest localization which can be generically derived from a theory with local observables and a mass gap (the existence of pointlike generators is viewed as a special case of stringlike localization). This was the result of a structural theorem by Fredenhagen and Buchholz in the 80s [27]. It is natural to think of the strings of matter fields in massive gauge theories (which unlike the vector meson strings cannot be removed by differentiations) as Buchholz–Fredenhagen [15] spacelike-cone-localized objects whose singular generators are strings. Their description remains somewhat abstract and does not reveal the connection of stringlike fields with the perturbative nonrenormalizability and the singular Jaffe (in contrast to Wightman) type structure. As a curious historical side remark it should be added that it was the improvement of Swieca's screening theorem by Buchholz and Fredenhagen which led to their derivation of string-localization from the mass gap assumption.

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[26]The coefficients of the Callen–Symanzik equations are global quantities and as such cannot be computed solely on the basis of the known perturbative short distance behavior.

4 Generators of wedge algebras, extension of Wigner representation theory in the presence of interactions

Theoretical physics is one of the few areas of human endeavor in which the identification of an error may be as important as the discovery of a new theory. This is especially the case if the error is related to a lack of understanding or a misunderstanding of the causal localization principle which is the basis of QFT. The more remote the properties of interest are related to the defining causal localization properties of QFT, the more speculative becomes the research and the larger is the probability to run into misunderstandings.

A perfect illustration of this point is the *on-shell approach to particle theory* in connection with the study of the S-matrix and formfactors in the aftermath of the successful application of dispersion relations to high energy nuclear reactions in the late 50s. Leaning on this limited but important success particle theorists in the 60s begun to use analytic on-shell properties for general on-shell constructions as the *S-matrix bootstrap* and Mandelstam's subsequent attempts to use crossing symmetric two variable spectral representations for the actual construction of high energy nuclear elastic scattering amplitudes.

Whereas off-shell analytic properties of correlation function were systematically analyzed in the pathbreaking work of Bargmann, Hall and Wightman [72], it was already clear at the time of the dispersion relations that on-shell analytic properties are of a different conceptual caliber. The analytic properties coming from the causal structure of correlation function could not account for the analytic on-shell properties. In particular the foundational origin of the important particle crossing property, one of the most subtle particle-field connections, remained outside the range of at that time known methods, apart from some very special cases which were solved with the help of the (still) intricate mathematics of several complex variables [8]. The unfavorable relation between mathematical effort and meager physical result led to an end of these attempts.

Only after the arrival of modular localization and its role in the construction of $d = 1 + 1$ integrable models [58] for the spacetime localization aspects of the Zamolodchikov algebra structure, the situation began to improve. The crucial step was the realization that the S-matrix was not only an operator resulting from time dependent scattering theory (which it is in every QT), but also represented a relative[27] modular invariant of wedge-localized algebras. This led to the idea that the analytic particle aspects of the crossing property could be a consequence of the analytic KMS identity for wedge-localized algebras (after rewriting its field content into particle properties). The resulting derivation of the particle crossing relation from the same modular localization principle which solves the E-J conundrum and explains the Unruh effect [17, 74] is somewhat surprising; this and the closely related proposal for a general on-shell construction [63] which extends the successful approach of integrable models from the structure of their generators of wedge algebras [36] will be the theme of this section.

In this way the original aim of Mandelstam's on-shell project for finding a route to particle theory which is different to quantization and perturbation theory (and stays closer to directly observational accessible objects) will be recovered, and the errors which led to the dual model and ST will be avoided. The new on-shell project is a "top-to-bottom" approach in which the aims and concepts are laid down before their mathematical and computational implementation starts. This is opposite to the perturbative SLF setting which starts from interaction densities $\mathcal{L}$ in terms of (string- or point-local) free fields and tries to construct a QFT of interacting fields. What binds them together in this paper is that both of them are realizations of the quantum causal localization principle. The on-shell approach starts from the algebraic structure of generators of the wedge algebra (the Zamolodchikov–Faddeev algebra in the integrable case)

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[27]It connects the modular wedge localization of the incoming fields with that of the interacting wedge-local algebra.

and sharpens the localization by constructing compact localized algebras as intersections of wedge algebras. Point- or string-local generating fields of such algebras only appear, if at all, only at the very end[28].

In order to motivate the reader to enter a journey which takes him far away from text-book QFT, it is helpful to start with a theorem which shows that the familiar particle-field relations breaks down in the presence of *any* interaction. The following theorem shows that the separation between particles and *interacting* localized fields and their algebras is very drastic indeed [63]:

Theorem 4.1 (Mund's algebraic extension [49] of the J-S theorem [72]). *A Poincaré-covariant QFT in $d \geq 1 + 2$ fulfilling the mass-gap hypothesis and containing (a sufficiently large set of) "temperate" wedge-like localized vacuum polarization-free one-particle generators (PFGs) is unitarily equivalent to a free field theory.*

It will be shown below that the requirement of temperateness of generators (Schwartz distributions, equivalent to the existence of a translation covariant domain [7]) is a very strong restriction; it only allows integrable models, and integrability in QFT can only be realized in $d = 1+1$. Note that Wightman fields are assumed to be operator-valued temperate distributions. Hence the theorem says that even in case of a weak localization requirement as wedge-localization one cannot find interacting operators with reasonable domain properties which, as in Wightman QFT, allow their subsequent application. However any QFT permits wedge-localized nontemperate generators [7]. The theorem has a rich history which dates back to Furry and Oppenheimer's observation (shortly after Heisenberg's discovery of localization-cause vacuum polarization) that Lagrangian interactions always lead to fields which, if applied to the vacuum, inevitably create a particle-antiparticle polarization cloud in addition to the desired one-particle state.

The only remaining possibility to maintain a relation between a polarization-free generator (PFG) leading to a pure one-particle state and a localized operator (representing the field side) has to go through the bottleneck of *nontemperate PFG generators of wedge-localized algebras*; this is what remains of the non-interacting particle-field relation in the presence of interactions.

For the on-shell construction one needs also a relation between *multiparticle states* and (naturally nontemperate) operators affiliated to wedge algebra. The idea is to construct a kind of *"emulation" of free incoming fields* (particles) restricted to a wedge regions *inside the interacting wedge algebra* as *a replacement for the nonexisting second quantization functor*. As the construction of one-particle PFGs, this is achieved with the help of modular localization theory.

The starting point is a *bijection* between wedge-localized incoming field operators representing the particle aspects and interacting wedge-local operators. This bijection is based on the equality of the dense subspace which these operators of the two different algebras create from the vacuum. Since the domain of the Tomita S operators for two algebras which share the same modular unitary Δ^{it} is the same, a vector $\eta \in \mathrm{dom}\, S \equiv \mathrm{dom}\, S_{\mathcal{A}(W)} = \mathrm{dom}\, \Delta^{\frac{1}{2}}$ is also in $\mathrm{dom}\, S_{\mathcal{A}_{\mathrm{in}}(W)} = \Delta^{\frac{1}{2}}$ (in [7] it was used for one-particle states). In more explicit notation, which emphasizes the bijective nature, one has

$$A|0\rangle = A_{\mathcal{A}(W)}|0\rangle, \qquad A \in \mathcal{A}_{\mathrm{in}}(W), \qquad A_{\mathcal{A}(W)} \in \mathcal{A}(W),$$

$$S(A)_{\mathcal{A}(W)}|0\rangle = (A_{\mathcal{A}(W)})^*|0\rangle = S_{\mathrm{scat}} A^* S_{\mathrm{scat}}^{-1}|0\rangle, \qquad S = S_{\mathrm{scat}} S_{\mathrm{in}},$$

$$S_{\mathrm{scat}} A^* S_{\mathrm{scat}}^{-1} \in \mathcal{A}_{\mathrm{out}}(W).$$

Here A is either an operator from the wedge localized free field operator algebra $\mathcal{A}_{\mathrm{in}}(W)$ or an (unbounded) operator affiliated with this algebra (e.g. products of incoming free fields $A(f)$ smeared with f, supp $f \in W$); S denotes the Tomita operator of the interacting algebra $\mathcal{A}(W)$

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[28]In the LQP formulation one does not need them since all physical informations can be directly derived from the net of local algebras [27].

whereas S_{in} denotes that associated with the interaction-free incoming algebra. Under the assumption that the dense set generated by the dual wedge algebra $\mathcal{A}(W)'\,|0\rangle$ is in the domain of definition of the bijective defined "emulats" (of the wedge-localized free field operators inside its interacting counterpart) the $A_{\mathcal{A}(W)}$ are uniquely defined; in order to be able to use them for the reconstruction of $\mathcal{A}(W)$ the domain should be a core for the emulats. Unlike smeared Wightman fields, the emulats $A_{\mathcal{A}(W)}$ do not define a polynomial algebra, since their unique existence does not allow to impose additional properties; in fact they only form a vector space and the associated algebras have to be constructed by spectral theory or by other means to extract an algebra from a vector space of closed operators (as Connes reconstruction of an operator algebra from its positive cone state structure).

Having settled the problem of uniqueness, the remaining task is to determine the action of emulats on wedge-localized multi-particle vectors and to obtain explicit formulas for their particle formfactors. These problems have been solved in case the domains of emulats are invariant under translations; in that case they possess a Fourier transform [7]. This requirement is extremely restrictive and is only compatible with $d = 1 + 1$ elastic two-particle scattering matrices of integrable models[29]; in fact it should be considered as the *foundational definition of integrability of QFT in terms of properties of wedge-localized generator* [63].

Since the action of emulats on particle states is quite complicated and still in a conjectural stage, we will return to this problem after explaining some more notation which is useful for formulating and proving the crossing identity in connection with its KMS counterpart. It will be helpful to the reader to recall how these properties have been derived in the integrable case.

For integrable models the wedge duality requirement leads to a unique solution (the Zamolodchikov–Faddeev algebra), whereas for the general non-integrable case we will present arguments which, together with some hindsight from the integrable case, determine the action of emulates on particle states. The main additional assumption is that the only way in which the interaction enters this construction of bijections is through the S-matrix[30]. With this assumption the form of the action of the operators $A_{\mathcal{A}(W)}$ on multiparticle states is fixed. The ultimate check of its correctness through the verification of wedge duality is left to future investigations.

Whereas domains of emulats in the integrable case are translation invariant [7], the only domain property which is *always* preserved in the general case is the invariance under the subgroup of those Poincaré transformations which leave W invariant. In contrast to QM, for which integrability occurs in any dimension, integrability in QFT is restricted to $d = 1 + 1$ factorizing models [63].

A basic fact used in the derivation of the crossing identity, including its analytic properties which are necessary in order to return to the physical boundary, is the *cyclic KMS property*. For three operators connected with the interacting algebra $\mathcal{A}(W)$, one being from the algebra and two being emulates of incoming operators[31], it reads

$$\langle 0|BA^{(1)}_{\mathcal{A}(W)}A^{(2)}_{\mathcal{A}(W)}|0\rangle \overset{\text{KMS}(\mathcal{A}(W))}{=} \langle 0|A^{(2)}_{\mathcal{A}(W)}\Delta B A^{(1)}_{\mathcal{A}(W)}|0\rangle \tag{4.1}$$

$$A^{(1)} \equiv\, :A(f_1)\cdots A(f_k):, \qquad A^{(2)}_{\text{in}} \equiv\, :A(f_{k+1})\cdots A(f_n):, \qquad \text{supp}\, f_i \in W,$$

where in the second line the operators were specialized to Wick-ordered products of smeared free fields $A(f)$ which are then emulated within $\mathcal{A}(W)$. Their use is necessary in order to convert

[29] This statement, which I owe to Michael Karowski, is slightly stronger than that in [7] in that that higher elastic amplitudes are combinatorial products of two-particle scattering functions, i.e. the only solutions are the factorizing models.

[30] A very reasonable assumption indeed because this is the only interaction-dependent object which enters as a relative modular invariant the modular theory for wedge localization.

[31] There exists also a "free" KMS identity in which B is replaced by $(B)_{\mathcal{A}_{\text{in}}(W)}$ so everything refers to the algebra $\mathcal{A}_{\text{in}}(W)$. The derivation of the corresponding crossing identity is rather simple [63] and its use is limited to problems of writing iterating fields as a series of Wick-ordered product of free fields.

the KMS relation for *fields* affiliated with $\mathcal{A}(W)$ into an identity of *particle formfactors* of the operator $B \in \mathcal{A}(W)$. If the bijective image acts on the vacuum, the subscript $\mathcal{A}(W)$ for the emulats can be omitted and the resulting Wick-ordered product applied to the vacuum describe a multi-particle state in $\hat{f}_i$ momentum space wave functions. The roof on top of f denotes the wave function which results from the forward mass shell restriction of the Fourier transform of W-supported test function. The result are wave functions in a Hilbert space of the graph norm $(\hat{f}, (\Delta + 1)\hat{f})$ which forces them to be analytic in the strip $0 < \operatorname{Im}\theta < \pi$.

The derivation of the crossing relation requires to compute the formfactor of the emulate $A_{\mathcal{A}(W)}^{(1)}$ between W-localized particle states and a general W-localized state. For simplicity of notation we specialize to $d = 1 + 1$ in which case neither the wedge has a transverse extension nor the mass-shell momenta have a transverse component; particles are then characterized by their rapidity. Using the analytic properties of the wave functions which connect the complex conjugate of the antiparticle wave function with the $i\pi$ boundary value of the particle wave function, one obtains

$$\int \cdots \int \hat{f}_1(\theta_1) \cdots \hat{f}_1(\theta_n) F^{(k)}(\theta_1, \ldots, \theta_n) d\theta_1 \cdots d\theta_n = 0 \tag{4.2}$$

with

$$F^{(k)}(\theta_1, \ldots, \theta_n) = \langle 0 | B A_{\mathcal{A}(W)}^{(1)}(\theta_1, \ldots, \theta_k) | \theta_{k+1}, \ldots, \theta_n \rangle_{\text{in}}$$
$$- {}_{\text{out}}\langle \bar{\theta}_{k+1}, \ldots, \bar{\theta}_n | \Delta^{\frac{1}{2}} B | \theta_1, \ldots, \theta_k \rangle_{\text{in}}.$$

Here $\Delta^{\frac{1}{2}}$ of Δ was used to re-convert the antiparticle wave functions in the outgoing bra vector back into the original particle wave functions. The vanishing of $F^{(k)}$ is a crossing relation which is certainly sufficient for the validity of (4.1), but it does not have the expected standard form which would result if we could omit the emulation subscript (in which case one obtains the vacuum to n-particle matrix element of B). This is not allowed in the presence of interactions. In the following we will show how integrable models solve this problem before we return to the general case.

In the integrable case [2] the matrix-elements $\langle 0 | B | \theta_1, \ldots, \theta_n \rangle$ are meromorphic functions in the rapidities (not in the invariant Mandelstam variables!). In that case there exists besides the degeneracy under statistics exchange of θs also the possibility of a *nontrivial exchange via analytic continuation*. In that case an analytic transposition of adjacent θs produces an $S(\theta_i - \theta_{i+1})$ factor, where S is the scattering function of the model (the two-particle S-matrix from which all higher elastic S-matrices are given in terms of a product formula) [2]. In order to distinguish between the analytic and the statistics ordering change let us introduce the following notation

$$\langle 0 | B | \theta_1, \ldots, \theta_n \rangle = \langle 0 | B | \theta_1, \ldots, \theta_n \rangle_{\text{in}} \equiv \langle 0 | B | \theta_{p_1}, \ldots, \theta_{p_n} \rangle_{\text{in}}, \qquad \theta_1 > \theta_2 > \cdots > \theta_n,$$

where the formfactors with the subscript in obey the rules of statistics degeneracy whereas the natural order on the left hand side requires analytic continuations of the formfactor.

It has been observed in [58] and extended in [36] that the analytic exchange can be encoded into the Zamolodchikov–Faddeev algebra

$$Z^*(\theta_1) \cdots Z^*(\theta_n) | 0 \rangle = | \theta_1, \ldots, \theta_n \rangle_{\text{in}}, \qquad \theta_1 > \cdots > \theta_n,$$
$$Z(\theta) Z^*(\theta') = \delta(\theta - \theta') + S(\theta - \theta' + i\pi) Z^*(\theta') Z(\theta),$$
$$Z^*(\theta) Z^*(\theta') = S(\theta - \theta') Z^*(\theta') Z^*(\theta),$$

where the $Z's$ are related to the emulate operators as

$$A_{\text{in}}(f)_{\mathcal{A}(W)} = \int_{\partial C} Z^*(\theta) e^{ip(\theta)x} \hat{f}(\theta) d\theta, \qquad C = (0, i\pi) \text{ strip}, \qquad Z(\theta) = Z^*(\theta + i\pi).$$

This leads in particular to

$$\langle 0|B|\theta_2,\ldots,\theta_k,\theta_1,\theta_{k+1},\ldots,\theta_n\rangle = S_{\text{gs}}\langle 0|B|\theta_1,\ldots,\theta_n\rangle_{\text{in}},$$

$$S_{\text{gs}} = \prod_{l=2}^{k} S(\theta_l - \theta_1), \qquad \theta_1 > \cdots > \theta_n.$$

We will refer to S_{gs} as the "grazing shot S-matrix", the reason being that it describes a scattering process in which scattering θ_1 through the θ-cluster $\theta_2,\ldots,\theta_k$ does not cause any change within the cluster.

We now look for an analog of this construction in the generals case. The main complication results from the presence of all inelastic threshold singularities of multiparticle scattering which enter all analytic order changes. So the first question is whether there exists an analog of the above grazing shot S-matrix in the general case. For this purpose it is helpful to rewrite the above integrable S_{gs} into an expression which only involves the full S-matrices. It is clear that in the above example this leads

$$S_{\text{gr}}(\theta_1;\theta_1,\ldots,\theta_k) = S(\theta_2,\ldots,\theta_k)^* S(\theta_1,\ldots,\theta_k)$$

with the S being the full S-matrices of k respectively $k-1$ particles does the job. In case the two-particle scattering matrix is not just a scattering function but rather a matrix of scattering functions, one has to use the Yang–Baxter relation in order to cancel all interactions within the $k-1$ cluster $\theta_2,\ldots,\theta_k$.

This idea suggest to define a general grazing shot S-matrix as

$$S_{\text{gs}}^{(m,n)}(\chi|\theta_1;\theta) \equiv \sum_l \int \cdots \int d\vartheta_1 \cdots d\vartheta_m \langle\chi_1\cdots\chi_m|S^*|\vartheta_1,\ldots,\vartheta_l\rangle$$
$$\times \langle\theta_1,\vartheta_1,\ldots,\vartheta_l|S|\theta_1,\theta_2,\ldots,\theta_k\rangle. \tag{4.3}$$

The χ represents the $\chi = \chi_1,\ldots,\chi_m$ component of a scattering process in which the grazing shot "bullet" θ_1 impinges on a $k-1$ particle θ-cluster consisting of $\theta_2,\ldots,\theta_k$ particles. Here the sum extends over all intermediate particles with energetically accessible thresholds, i.e. the number of intermediate open l-channels increase with the initial energy. The matrix elements of the creation part of an emulate sandwiched between two multi-particle states can directly be written in terms of the grazing shot S-matrix as

$$_{\text{in}}\langle\chi_1,\ldots,\chi_m|Z^*(\theta)_{\mathcal{A}(W)}|\theta_1;\theta_2,\ldots,\theta_n\rangle_{\text{in}} = S_{\text{gs}}^{(m,n)}(\chi,\theta_1;\theta),$$

where the Z^* denotes the previously defined creation part of the emulat $A_{\text{in}}(x)_{\mathcal{A}(W)}$. Once the annihilation operator has been commuted through to its natural position, it annihilats the next particle on the right and contributes a delta contraction. This procedure may be interpreted as a generalization of Wick ordering to interacting emulats.

The indicated idea to generalize the algebraic structure of integrable models by extending the concept of a grazing shot S-matrix is very speculative. But without knowing more about the structure of emulats it is not possible to generalize the S-matrix based ideas which finally led to the mathematical existence proof for integrable models of QFT.

Note that the previous arguments which led to the crossing relation

$$\langle 0|B|\theta_1,\ldots,\theta_k,\theta_{k+1},\ldots,\theta_n\rangle_{\text{in}} = {}_{\text{out}}\langle\bar\theta_{k+1},\ldots,\bar\theta_n|U(\Lambda_{W_{(0,1)}}(\pi i))B|\theta_1,\ldots,\theta_k\rangle_{\text{in}},$$

$$\langle 0|B|p_1,\ldots,p_k,p_{k+1},\ldots,p_n\rangle_{\text{in}} = {}_{\text{out}}\langle-\bar p_{k+1},\ldots,-\bar p_n|U(\Lambda_{W_{(0,1)}}(\pi i))B|p_1,\ldots,p_k\rangle_{\text{in}},$$

$$B \in \mathcal{A}(\mathcal{O}), \qquad \mathcal{O} \subseteq W_{(0,1)}, \qquad \bar\theta = \text{antiparticle of } \theta, \qquad \theta_1 > \cdots > \theta_n \tag{4.4}$$

are only valid in case of the natural order, in particular they do not depend on the form of the emulat operators. The form of the crossing relation for other orderings has to be computed in terms of analytic continuation which leads to interaction-dependent modifications whose form is only known for integrable models. The second line in (4.4) is the usual momentum space form of the crossing relation; the θ-ordering passes to the p-ordering.

The ordering dependence of the crossing relation receives additional support from the Haag–Ruelle derivation of the LSZ reduction formalism [18]. There are indeed threshold modifications from overlapping wave functions which invalidate the derivation LSZ reduction formalism to such cases [12].

The connections between the restriction of the LSZ scattering theory with the idea of analytic θ-changes looks very interesting and should be pursued further. They indicate the possible existence of deep unexplored connections between analytic threshold singularities and algebraic emulats. In this context the concept of emulation should be seen (as indicated in the title of this section) as a generalization of the functorial relation between the Wigner representation theoretical particle setting and the net structure of interaction-free QFTs

$$\text{functorial relation} \stackrel{\text{interaction}}{\longrightarrow} \text{emulation}.$$

In both cases the important role is played by modular localization theory. Its use in the presence of (any) interactions is however a double-edged sword. Without it QFT would loose its foundational character expressed in its many structural theorems which have no counterpart in QM. But it is precisely this fundamental aspect which leads to the coupling of all states with the same superselected charges which renders quantum mechanical approximation methods rather inutil. Looked upon from the side of QM and its operator methods (establishing selfadjointness, spectral resolutions, . . .) QFT appears like a realization of "Murphy's law": *everything which is not forbidden to couple (subject to the validity of the superselection rules) actually does couple.* Only if one learns the appropriate operator algebraic methods this curse becomes a blessing. It is precisely the modular localization property in the presence of (any) interaction which is behind the derivation of all structural properties of QFT.

The relative simplicity of integrable models results from the rather easy algebraic structure of its wedge-localized generators which in turn results from the simplicity of the (possibly matrix-valued) elastic scattering functions. This makes it possible to describe the wedge-generators in terms of deformed free fields [37]. In this case Murphy's law only applies to off-shell correlation functions or compact localized operators; they continue to couple to all states to which the superselection principle allows them to couple. For this reason the proof of the nontriviality of compact localized double cone algebras from intersection of wedge algebras is quite an important achievement; the proof is based on the use of "modular nuclearity" [36].

In the integrable case it leads to a representation of the permutation group [36] and the possibility to construct wedge generators for given scattering function by "deformations" of free fields [37]. In general the analytic exchange is path-dependent (reflecting the influence of the inelastic threshold cuts) and the action of emulats on particle states becomes much more complicated. This situation is vaguely reminiscent of a $d = 1 + 2$ Wightman theory with braid group statistics [9] for which the Bargman–Wightman–Hall analyticity domain [72] is not "schlicht" but contains cuts in the physical spacetime region. It is an interesting question whether the path-dependence of analytic ordering changes can be encoded into a group structure which resembles that of an infinite braid group representation.

5   Resumé and concluding remarks

The main point of the present work is to introduce a new Hilbert space setting for higher spin interaction, including those which hitherto have been treated in the Krein space setting of

gauge theory. The idea came out of modular localization, a concept which the author already introduced in the 90s and which a decade ago became the starting point of a new project for rigorous constructions of integrable models. The third subject of this work is the critique of string theory from the viewpoint of causal localization. Although these two subjects were already treated in previous publications by the author, there are new interesting observations about their relation to modular localization.

Modular localization theory helps to recognize and analyze past failures. Looking back at the S-matrix based on-shell construction attempts of the 60s with present hindsight, one realizes that there was not much of a chance at that time for understanding the subtle role of the particle crossing property in such project.

The predominant trial and error correcting computational oriented conduct of research was amazingly successful in connection with the post wwII renormalized perturbation theory; but its success begun to wane in the S-matrix based on-shell construction project as formulated by Stanley Mandelstam; in particular the conceptual origin of the particle crossing property remained outside its range.

In Section 2 it was shown that the recognition of some of conceptual errors in the dual model and ST leads to profitable new insights. The most intriguing misunderstanding which led to the dual model and ST was referred to in Section 2 as the picture puzzle situation. It is based in the curious observation that there exists an irreducible operator algebra which carries a positive energy representation with a discrete (m, s) particle spectrum in $d = 10$ spacetime dimensions; the famous superstring representation. With a little more forensic work one notices that it is the only known solution of a problem formulated 1932 by Majorana (in analogy to the $O(4, 2)$ description of the hydrogen spectrum): construct an infinite component purely discrete positive energy representation of the Poincaré group (infinite component field equation) from an irreducible operator algebra. Neither Majorana nor the group of physicists who during the 60s studied "dynamical groups" tried to embed the Lorentz group into a larger noncompact group (the "dynamical group" project of Barut, Fronsdal, Kleinert, ..., [4]) found a solution. The connection between the $d = 10$ component supersymmetric chiral model with the positive energy supersting representation of the Poincaré group provided the only known solution of this group theoretic problem. Its misreading as a solution of an S-matrix problem of a stringlocal object in spacetime is a result of a misunderstanding which in this paper was referred to as a "picture puzzle". Brower's theorem [10] is a pure group theoretical kinematical conclusion, it has no bearing on the scattering theory of particles.

In Section 2 it was also shown that such misreading of mathematical facts is not limited to string theory but also affects surrounding areas. The AdS-CFT correspondence is certainly a mathematical fact but, its *physical* use by Maldacena is the result of overlooking the causality issue [44]. Relations between QFTs in different spacetime dimensions (with the exception of the holographic projection onto null-surfaces) violate the causal completion property which is an indispensable part of causality (the timelike counterpart of Einstein's spacelike causal independence). One may use such isomorphic relations between local nets in different spacetimes for calculational purposes (certain calculations on the unphysical side may be simpler) but the interpretation has to be done on the physical side. In terms of the modular localization property this refers to the possible mismatch between the inner approximation $\mathcal{A}(\mathcal{O}) = \cup_{\mathcal{D} \subset \mathcal{O}} \mathcal{A}(\mathcal{D})$ by unions of small double cones D and the outer approximation in terms of wedges $\mathcal{A}(\mathcal{O}'') = \cap_{W \supset \mathcal{O}} \mathcal{A}(W)$. This also affects the alleged validity of the quasiclassical Kaluza–Klein dimensional reduction in full QFT and other popular games with extra dimensions.

In most cases the incorrect conclusions result from the belief that quantum degree of freedom issues can be dealt with in quasiclassical approximations. Any attempt to prove such incorrect ideas of QFT in terms of correlation functions (instead of manipulating Lagrangians) would have failed.

The once very successful approach to particle physics, which consisted in moving ahead on a pure computational track by trial and error without precise conceptual investments and guidance, seem to have lost its momentum with the discovery of the Standard Model which us the first successful project to describe electroweak interactions together with the strong nuclear forces within a framework of nonabelian gauge theory. In this setting the Higgs mechanism played the role of relating massive vector mesons to their massless counterparts. Early criticism of this idea disappeared in the maelstrom of time and gave way to a complete stagnation which is manifested in the fact that despite theoretical shortcomings this mechanism remained unchanged for more than 4 decades.

Instead of pursuing the serious objections of the first years after the appearance of the "Higgs mechanism", Big Science has used it for its justification by declaring the Higgs mechanism to be this most important discovery of this century. The situation is aggravated by the fact that the small community of theoreticians dedicated to foundational research (which shares most of the critical view of this paper) has resigned and turned away from ongoing problems of particle theory. This led to a deep schism which makes it even more difficult to get out of the present situation.

The SLF setting of $s \geq 1$ renormalizable perturbation theory in Hilbert space does not only shed a quite different light on the issue of the "Higgs mechanism", but also suggests a precise definition of confinement in terms of the vanishing of all correlations in which stringlocal zero mass fields (gluons) or stringlocal quarks appear (except q-$\bar{q}$ pairs with a finite connecting string) so that apart from such pairs only pointlike generated observables survive. It also suggests a perturbative proof based on generatlzations of Yennie–Frautschi–Suura type perturbative calculations.

As in earlier times, progress in particle theory is not possible without removing incorrect ideas of the past and seeing problems in a in a new light. What is however different is that in earlier times (the times of Pauli, Feynman, Landau, Lehmann, Jost, . . .) the influence of "Big Science" on fundamental theoretical research was much smaller. There was a well developed "Streikultur" in which the formation of globalized monocultures had no place.

I do not have an answer to this problem, but I think that it is necessary to find one in order to preserve the important role which particle physics played in the past.

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