Fachbereich Erziehungswissenschaften und Psychologie
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NUMBERS *DO* COUNT.
Neurocognitive INVESTIGATIONS INTO TYPICAL AND ATYPICAL DEVELOPMENT OF NUMERICAL ABILITIES

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Und schließlich: Lea, mein gutes Kind, bald gehen wir Eis essen und dann fängt der Spaß erst richtig an. Ha!!

[*] In alphabetical order! ☺

ELEWAITORR OPERRATORR

Dramatis personæ:
Count von Count
Kermit the Frog

The Count: Ha, ha, hotsy-potsy. My first day on the job as elevator operator. I can not wait to take someone upstairs on the elevator so I can count the numbers on the different floors. Ah, ha. Here comes someone now.

(Enters Kermit the Frog.)

Kermit: Oh, hi there. Uh, could you take me up to ... Count!? Is ... is it you?

The Count: Yes!

Kermit: Well, what are you doing here?

The Count: I am the new elevator operator, ah, ah!

Kermit: Ooh.

The Count: I got the job this morning.

Kermit: Wonderful.

The Count: Ah, yes. And I'm going to love it because I get to count the floors.

Kermit: Ah, well, listen, could you take me up to the seventh floor?

The Count: Ah, at last! Walk this way.

Kermit: Okay. Into the elevator.

The Count: Watch your step, please. Going up.

(The elevator door closes.)

The Count: Starting at one and going up. That's two, two floors! Three, three floors! Four, four floors! Five, five floors! Six, six floors! Seven floors! Eight floors!

Kermit: Um, um. Wait a second, Count! I ... I wanted to get off on the seventh floor.

The Count: Nine floors! Ten, that's ten floors! I love it! Ah, ha, ha, ha!!!

(Thunder and lightning.)

Kermit: Uh, Count! I wanted to get off on the seventh floor!

The Count: I'm sorry, Kermit. Oh, but I could not stop till I reached the top, ah, ah, ah.

Kermit: Well, would you please take me back down to the seventh floor,
please?

The Count: Well, of course. That's my job.

Kermit: Yes, it is, your job.

The Count: Starting at ten and going down. Nine! Eight! Seven!

Kermit: Uh, this is it.

The Count: Six!

Kermit: No, no, no! You went to the wrong floor again! Listen, Count! Will you stop this please!? Count! Oh, I'll run this elevator myself!

(Kermit tries to get to the controller but The Count won't let him.)

The Count: Five, four, three, two, one! Ah, ah, ah. ah! I love it! Wait, I can't! Oh!

(Both get off the elevator.)

The Count: But wait, Kermit. You're angry with me.

Kermit: Yes, I'm angry!

The Count: But why?

Kermit: Because you were supposed to stop at the seventh floor!!!

The Count: Why?

Kermit: Well, because that's what elevator operators do! They're supposed to take people to whatever floor they want to go in the building!

The Count: They are?

Kermit: Yes! And I wanted to go to the seventh floor!!!

The Count: Ah, but that is no problem.

Kermit: Yeah?

The Count: I know how to take you to whatever floor you want to go to in the building, and I can still count all the floors without stopping, ah, ah.

Kermit: Oh yeah? How?

The Count: I will take the elevator, and you can hop up the stairs.

Kermit: The ... the stairs?

(The elevator door closes.)

The Count: Bye-bye!

(http://www.youtube.com/watch?v=Fup_IpYtDHQ)

CONTENTS

ZUSAMMENFASSUNG

Etwa 3 bis 6% der Bevölkerung sind nicht in der Lage wenigstens basale numerische und arithmetische Fertigkeiten zu entwickeln (Shalev, 2007). Dies lässt die Ergebnisse (neuro-)kognitiver Studien, die sich den Ursachen entwicklungsbedingter Störungen im Bereich mathematischer Verarbeitung, d.h. der Dyskalkulie, widmen, eher dürftig erscheinen und zudem nicht ausreichend eindeutig. Die vorliegende Dissertation setzt sich aus vier experimentellen Studien zusammen, die neue Erkenntnisse über die Entwicklung normaler und gestörter numerischer Verarbeitungsprozesse generieren sollen. Behaviorale, Blickbewegungs- und elektrophysiologische Parameter wurden eingesetzt, um die Repräsentation numerischer Größe sowie die entsprechenden Verarbeitungsprozesse bei Kindern zu untersuchen.

Die erste Studie widmet sich der Frage, ob bzw. inwiefern sich die Verarbeitung numerischer Größeninformation bei Kindern in Abhängigkeit von der Menge der zu erfassenden Objekte unterscheidet. Dazu wurden EEG-Daten erhoben, während Kinder im Grundschulalter Aufgaben lösten, die einen numerischen Größenvergleich non-symbolischer Stimuli erforderten. Die zweite Studie untersuchte elektrophysiologische Korrelate basaler numerischer Verarbeitung bei Kindern mit Rechenstörung im Vergleich zu einer Kontrollgruppe. Auch hier wurde ein klassischer numerischer Größenvergleich non-symbolischer Stimuli als Experimentalanordnung eingesetzt, um eine Manipulation der numerischen Distanz zwischen zu vergleichenden Mengen von Objekten zu ermöglichen. Die dritte Studie nutzte Blickbewegungsmessung, um die Entwicklung basaler Repräsentation numerischer Größe im Grundschulalter zu untersuchen. Dazu wurden die teilnehmenden Kinder mit zwei parallelen Implementierungen einer klassischen Zahlenstrahl-Schätzaufgabe konfrontiert, bei der sich eine auf die Erhebung behavioraler Daten beschränkte, während die zweite zusätzlich eine Erhebung von Blickbewegungsdaten beinhaltete. Für die vierte Studie wurde ein numerischer Stroop-Test eingesetzt, bei der Kinder einerseits einen numerischen Größenvergleich und andererseits einen Vergleich der Schriftgröße von Ziffernpaaren durchführten. Effekte der Manipulation von Kongruenz zwischen numerischer und Schriftgröße sowie Distanzeffekte wurden für eine Gruppe von Grundschülern ermittelt, von denen ein Teil unterdurchschnittliche Leistung in einem standardisierten Mathematikleistungstest zeigte, während die Vergleichsgruppen im mittleren Leistungsbereich bzw. überdurchschnittlich abschnitten.

Die Ergebnisse der Studien deuten darauf hin, dass (a) Repräsentation numerischer Größe bedeutsame qualitative Veränderungen während der ersten Schuljahre durchläuft (vgl. Studie 3), dass (b) bei expliziten numerischen Entscheidungsprozessen die Rekrutierung domänenspezifischer Ressourcen, die in parietalen Hirnarealen lokalisiert werden, nicht abhängig von der Anzahl der zu

verarbeitenden Objekte ist (vgl. Studie 1), dass (c) gestörte numerische Verarbeitung nicht auf ein Automatisierungsdefizit im Bereich der Zugriffs auf numerische Information zurückzuführen ist (vgl. Studie 4), sondern (d) eher auf Unterschiede in der Funktionalität domänenspezifischer Verarbeitungssysteme, die vor allem in rechtshemisphärischen inferioren parietalen Kortexarealen verankert sind (vgl. Studie 2). Es ist jedoch festzustellen, dass diese Studien zwar einerseits gewisse Rückschlüsse über behaviorale und neurophysiologische Charakteristiken normaler und gestörter Verarbeitung numerischer Information bei Schulkindern zulassen, dass andererseits aber experimentelle Zugänge gefunden werden sollten, die eine Entwicklungsperspektive implizieren. Nur so werden tiefere Einblicke in typische und abweichende Entwicklungsverläufe basaler und höherer numerischer Fertigkeiten ermöglicht.

SUMMARY

Considering the fact that about 3 to 6% of the population fail to develop even the most basic numerical and arithmetic skills (Shalev, 2007), the outcome of (neuro-) cognitive research into the causes of developmental impairments in the mathematical domain, i.e. developmental dyscalculia, seems rather sparse and, at the same time, controversial. This thesis consists of four experimental studies that aim to gain further insight into the development of normal and impaired numerical processing. Behavioral, eye-movement and electrophysiological measures were used to tap into children's representation and processing of numerical magnitude information.

In order to shed light on the question whether and in what way enumeration of small and large sets of objects is functionally different, the first study focused on basic numerical magnitude processing in normally developing children. EEG data were collected from sixty primary schoolers performing a non-symbolic numerical comparison task. The second study investigated electrophysiological correlates of basic numerical processing in children with mathematical learning disabilities compared to a matched group of normally developing children. Again, children were tested with a standard non-symbolic numerical comparison paradigm that allowed for a manipulation of numerical distances between stimulus arrays for different quantity ranges. Study three used eye movement measurement to investigate the development of basic knowledge about numerical magnitude in primary school children. Sixty-six children from grades one to three (i.e. 6 to 9 years) were presented with parallel versions of a classic number line estimation task. The fourth study adopted a numerical Stroop paradigm, where children were asked to make numerical and physical size comparisons on digit pairs. The effects of congruity and numerical distance were determined for primary school children, of which a subgroup scored low in a standardized math achievement test, while others were normal or high achievers.

The results suggest that (a) numerical magnitude representations undergo relevant qualitative changes during the first years of formal mathematical training (cf. study 3), that (b) for explicit numerical decisions the involvement of domain-specific processing resources in parietal regions does not depend on quantity features of the input, i.e. numerical range (cf. study 1), that (c) impaired numerical processing may not be caused by a lack of automaticity in accessing numerical magnitude representations (cf. study 4), but rather (d) by differential recruitment of domain-specific processing resources in predominantly right parietal regions by low math achievers compared to their normally developing peers (cf. study 2). However, even though these studies allow for certain insights into the behavioral and neurophysiological characteristics of normal and impaired numerical processing in school-aged children, future studies should implement truly developmental approaches so as to provide more fine-grained information about typical and atypical developmental trajectories of basic and higher-level number-related skills.

INTRODUCTION

> Mind is a leaky organ, forever escaping its "natural" confines and mingling shamelessly with body and with world.*

Literacy is commonly used as a standard measure of a society's degree of sophistication. However, it is almost impossible to imagine a society that was able to read and write and, yet, had no grasp of numerical concepts. It is my strong conviction that numeracy – the ability to understand and deal with numbers – is as important for describing, predicting and manipulating our complex environments as is our use of written language.

It is thus hardly surprising that there is a growing scientific interest not only in describing the manifold behavioral manifestations, but also in decoding the neural correlates of basic number processing and arithmetic. Ever since Dehaene (1992) wrote his first review article on "Varieties of Numerical Abilities", cognitive neuroscience made significant advances in identifying and locating numerical processing functions in the adult brain. And, whereas research on the development of numeracy is still lagging behind research on normal and impaired written language acquisition, there is meanwhile a substantial body of studies available that address issues of typical and atypical development of numerical abilities. All the more reason to start reflecting upon a certain theoretical framework that pervades the emerging field of the developmental cognitive neuroscience of numerical cognition: Namely, the more or less implicit assumption that, as Johnson (2011a) puts it, "…functional brain development is the reverse of adult neuropsychology, with the difference that specific brain regions are added-in instead of being damaged" (p. 14).

In the case of numerical processing, there is a continued research tradition that probably started with Henschen's (1919) treatise on selective impairments of certain higher cognitive processes, and culminates in Butterworth's (1999) conception of an innate "number module". Ever since Henschen observed "…dass die Zifferblindheit durch Herde in der Angularwindung entsteht, die sich nach vorn (und oben?) auf die Rinde der Fissura intraparietalis ausdehnen." (p. 288), researchers set out to identify the specific neural circuits specialized for all kinds of quantity processing in healthy adults. As a result of these concerted efforts, the commonly held assumption is that basic and higher numerical abilities are the result of the workings of a genetically predetermined modular system that is assumed to be localized in bilateral inferior parietal lobes. Neuro-

* Andy Clark, *Being There.* Cambridge, MA: The MIT Press, 1997.
(Chapter 3: *Mind and world: The Plastic Frontier.*)

psychological studies added further evidence in support of this approach to number processing by demonstrating that focal damage to these brain regions typically results in impaired numerical abilities.

0.1 The Development of Numerical Processing

Not wholly surprising is that, in what I would call a scientific knee-jerk reaction, the general findings from studies on adult numerical cognition in healthy populations and in patients with neurological disorders were twisted into an approach to typical and atypical numerical development. The development of numerical abilities is, thus, assumed to be directly related to the genetically triggered coming into function of certain parietal cortex areas, i.e., the number module is "turned on" at a certain point in ontogeny. Consequently, developmental impairments of numerical and mathematical processing are immediately reducible to disruptions in the maturation of these very same cortical regions.

Karmiloff-Smith (1998) challenges the validity of such "non-developmental" (p. 389) scientific endeavors by pointing out that developmental disorders may not be so much the result of impairments at the level of genetically predetermined domain-specific processing systems, but rather the outcome of developmental pathways that involve complex interactions of more general genetic constraints with the child's physical and social environment. She explicitly calls for longitudinal approaches to the study of developmental disorders instead of focusing on deviant end-state processing systems (Ansari & Karmiloff-Smith, 2002).

Before addressing the issue of what I take to be an appropriate theoretical framework for neurocognitive research on developmental impairments of numerical cognition – or any developmental disorder, for that matter –, I will first describe a stage model of basic and numerical and arithmetic development. Subsequently, I will briefly outline the classic theoretic approaches to cognitive development, namely the nativist and the empiricist frame of reference, both of which will be contrasted with the interactive specialization framework (Johnson, 2011b).

0.1.1 A Model of Numerical Development

Even though human beings seem to be born with certain pre-numerical capacities (Feigenson, Dehaene, & Spelke, 2004) which are commonly taken to be phylogenetically advanced adaptive systems serving as precursors for the development of numerical processing functions, it typically takes at least until the end of the first years of formal schooling until children can be assumed to be equipped with the basic numerical abilities that allow for the formation of higher, i.e. formal and fully abstract mathematical concepts. In the following paragraphs, I will briefly describe the four-step model of number development suggested by von Aster and colleagues (von Aster, 2005; von Aster & Shalev, 2007) which summarizes concisely the current state of research into numerical development (see Figure 0.1).

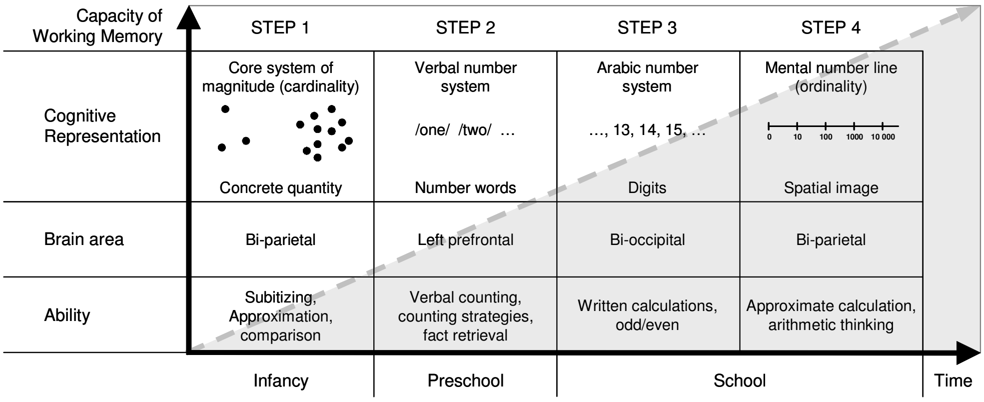

Figure 0.1: von Aster's four-step model of number development (cf. von Aster & Shalev, 2007)

0.1.1.1 Infants' Numerical Capacities

Whereas Piaget (Piaget & Szeminska, 1972), who was one of the first developmental psychologists to investigate children's numerical abilities systematically, started from the assumption that children are not able to carry out even the simplest forms of arithmetic before the age of six, when they finally have mastered basic concepts such as object permanence, more recent studies show that this picture is way to pessimistic. Only a few months after birth, human infants are already able to discriminate the cardinality of small sets of objects, i.e. numerosities within the subitizing range of 1 to 3 (Wynn, 1992). A second cognitive function that becomes apparent well before the infants' first birthday, is the ability to discriminate larger sets of items when their ratio is about 1:2 or 1:3 (Xu & Spelke, 2000). However, with smaller numerical distances between the to-be-compared sets of items, infants inevitably fail. The demonstrations of early capacities in handling non-symbolic quantities are interpreted as reflections of innate number-specific representation systems that kick in working long before verbal and other symbolic capacities develop. The assumption that human infants are endowed with certain cognitive tools apparently independent from individual learning or cultural transmission, are corroborated by comparative studies that found similar capacities in other species (Brannon & Terrace, 1998).

0.1.1.2 Children's Early Numerical Development

Children's numerical development during their first years of life is characterized mainly by a growth in language functions which are the basis for the development of early counting skills (see Figure 1.1 in Chapter 1, which illustrates the five main counting principles that children have to master in the course of their pre-school development). At the same time, children start to get a grip on simple arithmetic principles such as 'adding to' or 'subtracting from' by manipulating collections of physical objects. However, much of the child's early arithmetic capacities remain very concrete in that they are inextricably tied to the use of fingers, i.e. finger counting (Butterworth, 1999), a strategy which is typically abandoned in the course of the first years of formal mathematical training.

0.1.1.3 Starting School

Among the first things that children are confronted with in school is the arabic notation system and, only slightly later, the place-value system. One of the most challenging tasks that, for example, German children have to master, is to cope with irregularities of the number syntax, e.g. the inversion of tens and units in number words such as 'Dreizehn' or irregularities in number conversion such as 'Elf'. By comparing US-American and Chinese primary schoolers, Geary and colleagues (1993) demonstrated that such language-specific obstacles may delay children's formal math development considerably.

0.1.1.4 The First Years of Formal Training

When children have finally managed these basic steps, more complex numerical skills develop. Apart from the formation of ordinal number representations that allow for a sophisticated mental manipulation of numerical magnitudes, children start to build up numerical rote memory, e.g. mental storage of addition and multiplication tables. The latter is, on the one hand, tied to the development of verbal working memory functions at that age, and, on the other hand, crucial for the formation of higher mathematical abilities that rely on the availability of domain-general cognitive resources. When children finally finish primary school, they typically possess considerable conceptual, e.g. understanding of the base-10 system, and procedural knowledge, e.g. strategies such as columnar trading, which, in combination with an increased automaticity in the manipulation of numerical content, provides for their entering the realm of higher, abstract mathematics.

0.1.2 Theoretical Approaches to Cognitive Development

According to the maturational model, cognitive functions are immediately linked to the genetically predetermined maturation of certain brain regions. As soon as these mainly cortical areas become functional the new cognitive function becomes manifest on the behavioral level. The central assumption here is what Gottlieb (2007) referred to as *deterministic epigenesis*, i.e. an unidirectional relation between *genetic activity → structure → function*. The other extreme of the theoretical spectrum is the skill learning approach that assumes that new cognitive functions are the result of the child's exposure to and interaction with certain environmental stimuli. The empiricist basically takes as a given that the processes and functions effective in the infant's acquisition of a new cognitive, sensory or motor ability are the same that adult skill learning is based upon.

Both theoretical approaches, i.e. in Johnson's (2011b) terminology, the maturational and the skill learning perspectives, share the underlying assumption of static and localized mappings between certain brain structures and respective cognitive functions, i.e. the notion that the same discrete brain areas do – or in the case of impaired functioning, do not – subserve the same cognitive capacities throughout lifetime. However, there is meanwhile enough empirical evidence to refute this assumed functional stability over time where basic or higher numerical functions are concerned (see e.g. Rivera, Reiss, Eckert, & Menon, 2005). This means that it is essential for a developmental cognitive neuroscience of numerical cognition to find more adequate theoretical approaches so as to provide for an appropriate explanatory framework to tackle issues related to normal and impaired trajectories of numerical development.

An alternative model of cognitive development was put forward by Johnson et al. (2001, 2011b; Sirois et al., 2008). The interactive specialization framework is intended to provide a domain-general theoretical model for the field of developmental cognitive neuroscience. According to this view, developmental changes in a certain cortical region's response to intrinsic or extrinsic stimulation are the result of an interaction and competition between neighboring and other regions in a functional network. So, it is not so much intra-regional but inter-regional functional modification that is relevant for emerging competencies on the behavioral level. As a result of these processes of change, a certain brain area becomes more specific or specialized in its response behavior. This approach crucially depends on Gottlieb's (2007) concept of probabilistic epigenesis which holds that bidirectional influences exist between the levels of *genetic activity ↔ structure ↔ function*. Furthermore, the mappings between cognitive capacities and their neural substrates are assumed to be dynamic in that there is no one-to-one relation between behavioral performance and underlying neurocognitive mechanisms. Depending on age and group, a similar behavioral output may rely on rather different neural processing systems. Finally, the approach abandons the concept of brain development as the result of an encapsulated process that follows a certain genetic blue-print. Cognitive development is at least partially influenced by specific input from the brain's immediate physical milieu, as well the organism's social and cultural environment. So, to use Clark's metaphor, it is not only the mind itself that is leaky, but also its molding in the course of development.

Interestingly, while most research on typical and atypical development of numerical abilities actually starts from a nativist background, an interactive specialization model of higher-level cognitive development actually provides a considerably better fit to the available empirical data (for a review, see Ansari, 2008). This obvious inconsistency may actually have a rather trivial cause. While the maturational model may actually not be the most suitable theoretical basis for a developmental cognitive neuroscience, it is the by all means the most convenient theoretical approach. Convenient, in that it allows for the implementation of rather focused and straight-forward experimental studies that deal with certain local hypotheses. Incorporating more complex frameworks such as the interactive specialization approach entails considerably more sophisticated study designs.

This said, I have to concede that my own scientific undertakings in the domain of numerical development did not necessarily start from the realization of a set of well-defined theoretical assumptions that were to guide and pervade all steps of the research process from the planning of an experiment to the publication of the results. Even though I am actually firmly rooted in what was called developmental connectionism some years ago (Elman et al., 1996), when I started making up my mind about language in general and, specifically, the ontogenesis of syntactic categories, I may have cut a corner or two when it comes to the theoretical consolidation and integration of my own hands-on research activity into a broader framework. This may be the reason that the four experimental studies that make up the core of this doctoral thesis are reminiscent of what Johnson (2011b) called "isolated islands of data" (p. 8), symptomatic for a still prevalent neglect in terms of defining adequate theoretical foundations for the emerging field of developmental cognitive neuroscience. However, there is no better time to reflect on the broader issues of research in developmental cognitive neuroscience than now that this important period in my scientific training comes to an end. While I will not try to pretend something that wasn't, I will still use the opportunity to review my past research activities, and try to define some basic concepts that shall guide my future research.

0.2 Philosophical Background

Before starting this foray into the subject matter of philosophy of mathematics, it may be appropriate to first point out some central concepts, namely the dimensions of the ontology and epistemology of mathematical entities or objects. Discussions of mathematical ontology are concerned with the question of objectivity and truth of mathematical objects such as numbers, points, sets, functions. The issue here is, where those entities actually reside, and whether they exist independently of the human mind. Epistemology of mathematics, in turn, covers questions of how mathematical knowledge is possible, or, to put it in another way, what has to happen for a human knower to gain access to any mathematical object. That is, any philosophical approach to the subject matter of mathematics has to tackle not only the issue of what it is that is known, but also how it is known; its epistemology should dovetail its ontology.

The idea that mathematical entities exist objectively, i.e. independent from the human knower, is called *realism* in ontology. So, realism in the philosophy of mathematics refers to the assumption that mathematical objects are mind-independent, i.e. have an existence on their own. There are at least two main versions of realism in ontology, one of which is *Platonism*, according to which numbers are non-physical entities belonging to another realm than the physical world, and *empiricism*, which relates numbers to our physical world (cf. Balaguer, 2009). *Anti-realists* in ontology do either completely deny the existence of mathematical objects, i.e. mathematical objects are void of meaning, or, alternatively, hold that mathematical objects do exists, however, not objectively but as constructions of the individual or an ideal human mind, and are, thus, mind-dependent entities. The former school of thought in the philosophy of mathematics is called the nominalism, the latter constructivism or intuitionism. Realism in epistemology refers to the assumption that it is the nature of the object that determines the form of our knowledge about it, while epistemological anti-realism holds that we simply cannot know how nature really is.

0.2.1 Platonism in the Philosophy of Mathematics

With reference to the conversations between the neuroscientist Pierre Changeux and the mathematician Alain Connes, Dehane argues against what he calls an "ethereal [emphasis added] conception of mathematics" (1997; p. 117), with which he alludes to Connes' assumed Platonist stance. Interestingly, Connes himself rejects such a reading in stating his "belief in the existence of raw mathematical reality, as an inexhaustible source of information, [which] is the result of long personal experience, not of reading Plato – whose ideas [he doesn't] necessarily agree with" (Changeux & Connes, 1995; p. 233).

Since it is an allegedly Platonist version of realism in ontology, that the cognitive scientist guns for, its theory of Forms will be briefly elaborated (cf. Balaguer, 1998; Balaguer, 2008). Plato based his ontology on the assumption of two separate worlds, i.e. the physical world and the realm of what he calls Forms. While in our physical world everything is inevitably flawed and ultimately non-perfect, the realm of Forms only contains perfects entities, such as e.g. Beauty itself or Justice itself, and so on. Physical objects are beautiful only to the extent that they resemble these Forms to a certain degree. While objects of the physical world are subject to change, i.e. a just person can become unjust, or the beauty of objects can vanish over time, the Forms are eternal and unchanging.

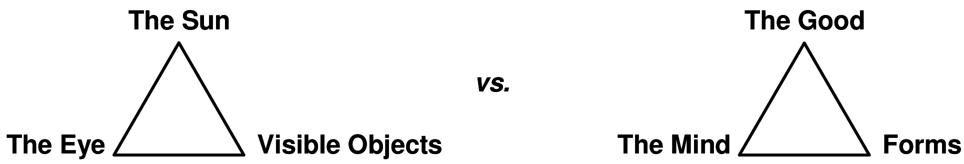

Figure 0.2: The physical and the realm of Forms, by Fogelin (1971; p. 371)

It is, however, through the inevitably non-perfect objects in our physical world, that we humans are able to gain a limited understanding of the ideal Forms. In Plato's view, the visible objects our physical world is made of are mere images or reflections of the originals, i.e. the objects of the realm of Forms. While we access the objects of the physical world through sense perception, the original Forms are intelligible only, i.e. accessible to a certain degree through mental reflections (see Figure 0.2). While we are able to see the beautiful things in our physical environment, e.g. a beautiful flower, we have to think in order to get in contact with Beauty itself. Plato also elaborates on the idea that it is through our souls, which are in contact with both realms, that we can do the trick. In this version, no experience with the worldly reflections of eternal Forms is necessary, knowledge is a priori.

> And do you not know that, although they [the mathematicians] make use of their visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter, and so on the forms which they draw or make, and which have shadows and reflections in water of their own, are converted by them into images, but they are really seeking to behold the things themselves, which can only be seen with the eye of the mind? (Plato, Republic, as cited by Kline, 1985; p. 46)

According to Plato, mathematics and its objects are entities of the realm of eternal Forms. Physical objects are reflections of mathematical objects, which, in turn, are ultimately reflections of eternal Forms. So, while some round objects such as, say, a saucer or a circle drawn by a pair of compasses approximate the a geometric circle, those physical entities are only imperfect reflections of the original mathematical object. The same applies for arithmetic properties, which are about the abstract objects of numbers, which, being part of mathematics, are independent from the mathematician. Numerosities of collections of objects are to be distinguished from numbers themselves which are not instantiated by the object collections but only reflected by them.

For a cognitive scientist, the main problem with Platonism is, of course, that it leaves us with the realization of an insurmountable epistemic gap between the world of physical objects, which the human knower belongs to, and the objects of mathematics proper, which are part of the inaccessible realm of the Forms. Since we can hardly content with mystical ideas of souls that mediate between these two worlds, the cognitive scientist is forced to turn his or her back to this version of realism in ontology.

However, it is as soon as through the contemplations of Aristotle's, Plato's own disciple, that something which resembles an empiricist view was introduced to the philosophy of mathematics. For Aristotle, the eternal Forms do not belong to another realm, but are not to be separated from the objects in our physical world that instantiate them. Universal Forms such as circles are abstractions over classes of objects sharing a certain property such as being round. And in this sense, the number four is grasped by abstractions over sets of four physical objects. So, instead of throwing out the baby with the bathwater by focusing on Plato's hard-to-swallow ideas about mathematics, it seems to be more appropriate to find out whether after more that two thousand years of philosophy of mathematics other approaches are available that are more in line with what we, or, at least what I think we are dealing with in a cognitive psychology of mathematics. Trying to avoid what Putnam called "the intellectual dishonesty of denying the existence of what one daily presupposes" (Putnam, 1971; p. 57), I am definitely siding with Clark here.

> I deliberately avoid this [i.e. an anti-realist view of the world], which runs the risk of obscuring the scientific value of an embodied, embedded approach [to cognition] by linking it to the problematic idea that objects are not independent of mind. My claim, in contrast, is simply that the aspects of real-world structure which biological brains represent will often be tightly geared to specific needs and sensorimotor capacities. (Clark, 1997; p. 173)

0.2.2 Outline of Contemporary Approaches to Realism in the Philosophy of Mathematics

In the following, I will briefly describe some contemporary realist positions in philosophy of mathematics that are well in line with the explicit or implicit assumptions that my scientific approach is based upon, i.e. the views of Quine, Gödel and, finally, Penelope Maddy.

0.2.2.1 Quine's Web of Belief

I will start with the naturalist-empiricist position of Quine which culminates in the posit that any science, which in Quine's view includes mathematics, is ultimately nothing but a tool "for predicting future experience in the light of past experience" (Quine, 1998; p. 49). For Quine, there is no principled boundary between mathematics and other sciences, be it natural sciences such as physics or more applied ones such as economics. The whole scientific enterprise forms a continuum from mere experimental sciences to abstract mathematics.

> I am inclined to lighten somewhat the emphatic contrast usually drawn between mathematics and natural science. I already equated the roles of mathematical laws with laws of nature. (Quine, 1995; p. 53)

Quine's entire thinking with respect to philosophy of mathematics is based on a metaphor that describes all kinds of knowledge, which he calls "beliefs", to form a "man-made fabric", i.e. Quine's web of belief (Resnik, 2007), which not only encompasses the empirical sciences but also mathematical knowledge. Within this web the atoms of our vast knowledge are represented and linked together to form a web.

> The totality of our so-called knowledge or belief, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experiences only along the edges. Or, to change the figure, total science is like a field of force whose boundary conditions are experience. (Quine, 1998; p. 47)

Empirical evidence, gathered e.g. by direct observation, enters the web only "along the edges"; all other knowledge arises from within. In case of new knowledge that is not immediately compatible with what is already there, the whole system - this reflects Quine's holism - is changed by readjustment of the links within the web until an equilibrium is restored.

> A conflict with experience at the periphery occasions readjustments in the interior of the field. Truth values have to be redistributed over some of our statements. Re-evaluation of some statements entails re-evaluation of others, because of their logical interconnections. (…) The edge of the system must kept squared with experience; the rest, with all its elaborate myths of fictions, has as its objective the simplicity of laws. (Quine, 1998; pp. 47-50)

The fact that mathematical knowledge appears to us as more stable and less prone to refutation or reinterpretation than, say, biological theories, is explained with reference to the fact that mathematical knowledge pervades the web more entirely than other types of knowledge. It is, thus, more parsimonious to seek equilibrium by first adjusting all other "chunks" (Quine, 1981; p. 71) of knowledge within the web before changing the mathematical basis. Even though we have the "natural tendency to disturb the total system as little as possible" (Quine, 1998; p. 49), there is, however, no categorical difference between falsification in natural sciences and the falsification of mathematical axioms. Since "no statement is immune to revision"(Quine, 1998; p. 48), all entities within our web of belief, from simple arithmetic facts to the most abstract mathematical theories, is prone to be rejected when proven false.

Quine obviously argues for a certain ontological realism, when he states that insofar as we belief in *any* object to be existent, there is no reason not to belief in the reality of *mathematical* objects.

> Now I suggest that experience is analogous to the rational numbers and that the physical objects, in analogy to the rational numbers are posits which serve merely to simplify our treatment of experience. (Quine, 1998; p. 49)

The assumption of the existence of mathematical objects is, thus, essentially the same as the assumption of, for example, dark-matter axions (Asztalos et al., 2010), hypothetical particles that were posited in astrophysics to explain certain phenomena. However, contrary to the Platonist view, there is no a priori knowledge, but the entire web of knowledge is based on empirical or sensory "experience".

Quinean realism is justified mainly by the so-called Quine/Putnam Indispensability Argument which starts from the observation that mathematics seems to be essential for the pursuit of science (Resnik, 2007). And, when we accept any given scientific claim as true and referring to real entities in our physical environment, then we must also accept as ontologically true, i.e. as referring to actually existing objects, the mathematical presuppositions that are tied to it – in for a penny, in for a pound, so to speak. According to Quine, the best explanation for the predictive power of any scientific theory is that it is more or less true, and so are its mathematical foundations.

> So far I have been developing an argument for realism along roughly the following lines: quantification over mathematical entities is indispensable for science, both formal and physical: therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question. This type of argument stems, of course, from Quine, who has for years stressed (…) the indispensability of quantification over mathematical entities. (Putnam, 1971; p. 57)

Quine assumes the scientist is not only using the tools the mathematician offers him or her to use for their scientific endeavors, but he or she actually presupposes the truth of the implied mathematical objects and principles. And it is ultimately scientific application and applicability that decides about the existential fate of mathematical objects, i.e. it is science that decides on mathematical truth and existence.

> So much of mathematics as is wanted for use in empirical science is for me on par with the rest of science. Transfinite ramifications are on the same footing insofar as they come of a simplicatory rounding out, but anything further is on par with uninterpreted systems. (Quine, 1984; p. 788)

Apart from the question of whether mathematics is indeed indispensable for science to work (cf. Field, 1980; chapter 6: A nominalistic treatment of Newtonian space-time, where Field presumably demonstrates how certain branches of physics, i.e. gravitational theory, can be approached without relying on mathematic descriptions*),* one obvious problem with Quine's naturalist position is, of course, that when he accepts the truth and existence of only those parts of mathematics – with some "rounding out" – that are 'applied', his theory covers only a relatively small part of mathematics. This does not go well with what mathematicians do and feel, and is also in conflict with the experiences of natural scientists such as Wigner, who described the "uncanny usefulness of mathematical concepts" (Wigner, 1995; 535) by pointing out that oftentimes the scientific applicability of a certain mathematical theory becomes apparent only much later after its initial discovery.

> It is true, of course, that physics chooses certain mathematical concepts for the formulation of the laws of nature, and surely only a fraction of all mathematical concepts is used in physics. It is true also that the concepts which were chosen were not selected arbitrarily from a listing of mathematical terms but were developed, in many if not most cases, independently by the physicist and recognized then as having been conceived before by the mathematician. (Wigner, 1995; 541)

Another objection that the Quinean has to face, was expressed by Parsons, who mentions the fact that some mathematical knowledge seems to come to us immediately without any obvious inductive reasoning, i.e. "mathematical intuition has a certain de re character"(Parsons, 1979; p. 146)

> The empiricist view, even in the subtle and complex form it takes in the work of Professor Quine, seems subject to the objection that it leaves unaccounted for precisely the obviousness of elementary mathematics. (Parsons, 1979; p. 151)

0.2.2.2 Gödel on Intuition

This point, i.e. the alleged obviousness of elementary numeric and geometric truths is nicely taken up by Gödel's theoretical views regarding the ontology and epistemology of mathematics. For him basic set-theoretical objects have some kind of 'cognitive obtrusiveness' in that everybody seems to immediately recognize the truth related, for example, to the concept of union in set theory which seems to be reflected in our physical world when we deal, for example, with collections of physical objects.

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* "Presumably" here means only that judging the feasibility of Field's approach is way over my head. There are others who certified this anti-realist theory to be a "major intellectual achievement" (Shapiro, 2000; p. 237).

> Despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves on us as being true. (Gödel, 1983; pp. 483-484)

So, even though the 'two-ness' of a set of two and the 'four-ness', that results when we unite two sets of two distinct elements, is nothing we grasp via simple sensory processes, we nevertheless have an immediate experience of what it means to unite two sets of two items, given that they are not identical. In this sense the basic algebra of sets, such as commutative properties related to set union, seems kind of obvious.

Explicitly relating mathematical intuition to sense perception, Gödel alludes to the fact that in both cases it is, of course, possible to be mistaken by relying on intuition.

> I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them (…). The set-theoretical paradoxes are hardly any more troublesome for mathematics than deceptions of the senses are for physics. (Gödel, 1983; p. 484)

So, similar to Quine, Gödel parallels the basic epistemological and ontological assumptions that guide mathematics with those of other sciences such as physics. Mathematical objects are introduced to explain certain mathematical experiences in the same vein, as the physicist introduces particles such as dark-matter axions to explain certain cosmological phenomena.

Interestingly, in contrast to Quine's view, Gödel assumes two epistemic levels. On the one hand, there is the lower tier of intuitively graspable mathematical truths, on top of which rests the considerable amount of higher and abstract mathematics which is, of course, beyond simple intuition.

> However, even disregarding the [intuitive immediacy] of some new axiom, and even in case it has no [intuitive immediacy] at all [emphasis added], a probable decision about its truth is possible also in another way, namely, inductively by studying its "success". Success here means fruitfulness in consequences, in particular in "verifiable" consequences (…). There might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole field, and yielding such powerful methods for solving problems (…) that, no matter whether or not they are [intuitive], they would have to be accepted at least in the same sense as any well-established physical theory. (Gödel, 1983; pp. 476-477)

In the higher realms of mathematics, on the other hand, truth is justified primarily from within the system of mathematics itself. Assumptions of higher mathematics are justified mainly by their consequences, i.e. their explanatory "success". Gödel's claim here is very similar to Quine's, in that both equate mathematics with any other sciences and, in that, in both cases, it is its explanatory power that renders a theory "true". However, while Quine identifies the natural sciences as the arbiter that decides on the truths of mathematical objects, Gödel allows for purely mathematical justification.

But, from the position of a cognitive science of mathematics, more interesting than the questions of proof for higher mathematical posits is Gödel's conception of our basic capacity to intuit certain mathematical concepts. He ties this intuition, as shown above, to basic set-theoretic objects and axioms, which he describes as experienced immediately in a way analogous to sensory perception. For a cognitive scientist, such an assumption seems very appealing since it offers an epistemological way in. Unfortunately, Gödel himself remains rather opaque in his description of this low-level access to mathematical objects and truths. Gödel offers only the fuzzy description of mathematical intuition – which Kitcher refers to as "one of the most overworked terms in the philosophy of mathematics" (Kitcher, 1984; p. 49) – as being somehow analogue to the natural scientist's sensory perception.

This analogy, however, is difficult to maintain, at least prima facie, since mathematical objects such as "natural numbers, real numbers, complex numbers, sets, geometric points, functions, topological spaces, groups, rings, and fields" (Shapiro, 2008; p. 158) are wholly abstract, which renders obvious the fact that the mathematician's access cannot be simple sensory perception. This is the crucial point of attack launched against Gödel's version of realism.

> For X to know that S is true requires some causal relation to obtain between X and the referents of the names, predicates, and quantifiers of S. (…) For Hermione to know that the black object she is holding is a truffle, [requires] that 'the black object she is holding is a truffle' must figure in a suitable way in a causal explanation of her belief that the black object she is holding is a truffle. (…) It [i.e. X's knowledge about S] will involve, causally, some direct reference to the facts known, and, through that, reference to these objects themselves. (Benacerraf, 1983; p. 412-413)

Since the Platonist – and Gödel's view is a version of Platonism in assuming "a 'given' underlying mathematics [that] may represent an aspect of objective reality [the] presence [of which] in us may be due to another kind of relationship between ourselves and reality [than sense perception]" (Gödel, 1983; p. 484) – assumes mathematical objects to be non-spatiotemporal entities, i.e. acausal objects, no causal relation between the knower and the known is possible. Yet, I obviously know something about mathematics – and there are others who know even more – which means that something is off with the Platonist's account.

0.2.2.3 Maddy's Way out of the Dilemma

For a cognitive science of mathematics, I believe Maddy's (1990) set-theoretic realism offers a way out of the dilemma without having to relinquish one's realistic stance, by relating Gödel's intuition to the actual perception of collections of medium-sized objects, i.e. entities in our physical environment.

Maddy meets the epistemological challenges the realist has to face by actually attributing physical properties to certain mathematical objects. Furthermore, she assumes that the higher tiers of set theory are rooted in basic knowledge gained in interaction with so-called impure sets, i.e. sets of physical objects such as a pair of shoes, or sets of sets of physical objects such as a cupboard full of pairs of shoes, and so on. So, we have access to knowledge about sets via causal connections to them, e.g. via observational interaction with collections of physical items. To use her own words, Maddy intends to "bring [mathematical objects such as sets] into the world we know" (Maddy, 1990; p. 48).

Maddy's theory of how we actually perceive sets is a compromise between the approaches to philosophy of mathematics proposed by Quine and by Gödel. With Quine, she assumes that the success of mathematical applications in other sciences justifies mathematical practice, i.e. the indispensability of mathematics for other sciences is reason enough to believe in the existence and truth of mathematical objects. At the same time, she builds her theory on Gödel's two-level approach with the lower-level, i.e. Gödel's intuition, providing the foundations for the whole edifice of mathematical reasoning, from the most basic to the most abstract. However, through her naturalistic view of how we actually interact with certain mathematical objects, she avoids Benacerraf's objections regarding the question of how to be able to interact with mathematical objects.

Referring to Quine's indispensability argument at this point would be rather dangerous ground for justifying the assumption that "mathematics is a science [and] that much of it at least approximates truth" (Maddy, 1990; p. 34), since it does not fully answer Benacerraf's objection. And this is exactly, where Maddy appeals to Gödel's intuition as a way to gain a kind of hands-on mathematical experience. Essentially, Maddy's set-theoretic realism translates Gödel's ideas by proposing a basic form of perception-like mathematical experience gained in (causal) interaction with physical objects, i.e collections of things in our environment. This offers an explanation of how we come to have mathematical knowledge. Perception of impure sets is a crucial aspect in Maddy's two-level epistemology of mathematics, which obviously follows Gödel's approach. In that, Maddy's ideas provide justificatory arguments for an ontological realism.

Maddy's proposition of quasi-perceptual access to certain kinds of sets rests on Hebb 's (Hebb, 1980) theory of cell-assemblies as the neurophysiological basis for any perceptual processes. In appealing to Hebb's theory, she provides a physiologically reasonable and at least partially realistic approach to a person's development of mathematical knowledge. Drawing heavily on Hebb's (1980; cf. pp. 88-89) model of how infants come to know about entities such as triangular figures, Maddy describes perceptual processes as based on certain neurophysiological mechanisms, i.e. Hebbian learning.

> The ability to perceive physical objects is not unlike the ability to perceive triangular figures, though it is more complex. The trick is to see a series of patterns as constituting views of a single thing. Just as the ability to see triangles develops over time, through the painstaking process of seeking out corners and comparing one triangle with another, the ability to see continuing physical objects develops over a period of experience with watching and manipulating them. (Maddy, 1990; p. 57)

Referring to Hebb's description of the formation of superordinate cell-assemblies (Hebb, 1980; p. 107), Maddy tackles Benacerraf's problem.

> The question is what bridges the gap between what is causally interacted with and what is perceived, and the hope is that something like what does the bridging in the case of physical object perception can be seen to do the same job in the case of set perception. Notice that this way of putting the problem already assumes that we do in fact perceive physical objects. (Maddy, 1990; p. 50).

On this basis, Maddy claims that the ability to perceive sets, i.e. mathematical objects, is just the same as our ability to perceive Gestalt-like entities or physical objects, in that it draws on similar perceptual mechanisms. As long as middle-sized objects are located close to another, we are endowed with the capacity to perceive them as sets.

> Steve needs two eggs for a certain recipe. The egg carton he takes from the refrigerator feels ominously light. He opens the carton and sees, to his relief, three eggs there. My claim is that Steve has perceived a set of three eggs. By the account of perception just canvassed, this requires that there be a set of three eggs in the carton, that Steve acquire perceptual beliefs about it, and that the set of eggs participate in the generation of these perceptual beliefs in the same way that my hand participates in the generation of my belief that there is a hand before me when I look at it in good light. (Maddy, 1990; p. 58)

She assumes that the ability to know of the actual set of eggs, instead of merely perceiving three individual eggs, is a specific perceptual capacity acquired through experience. So, according to Maddy it seems perfectly reasonable to infer that we do perceive sets in the same way we perceive triangles or more complex objects. This set perception mechanism is, according to Maddys view, the basis of our ability to have intuitive access to higher-level mathematical knowledge. The centrality of the capacity of 'set-perception' is why her approach is referred to as "set-theoretic realism". According to Maddy, "physical object detectors" (Maddy, 1990; p. 58), namely certain higher-level cell assemblies, allow for the perception of physical objects. In the same vein, "set detectors" allow for the discrimination of collections of objects from the environment. It goes without saying that Maddy does not claim the perception of impure sets to explain the realm of higher, i.e. truly abstract knowledge of set-theory. Maddy merely provides an 'inner-worldly', or, to use her own words, "down-to-earth" model (Maddy, 2000; p. 108) of the lower-level of Gödel's two-tiered epistemological theory, namely intuition, which provides the simple basis from which pure and formalized mathematics arises.

By suggesting that natural numbers are ultimately properties of sets, Maddy also provides a model of how basic numerical abilities are ultimately grounded in the perception of set.

> I agree that numbers (…) are not objects of some other sort. They are properties of sets [emphasis added], and number theory is that part of set theory which deals with number properties of finite sets. (Maddy, 2002; p. 352)

Interpreting the perception and cognitive manipulation of the numerical properties of sets as the basis of a development of complex faculties such as symbolic number processing, reverberates nicely with current cognitive science of mathematics, i.e. with my own approaches to the study of number processing and the development thereof.

The set-theoretic realist's assumption is, thus, that the basic numerical knowledge is knowledge about impure sets, and that the perception of those sets of physical objects which have certain numerical properties, i.e. numerosity, bridges the epistemic gap between the human knower and abstract mathematical objects.

> The physical stuff by itself cannot be three [because there is no definite way to split the eggs up, i.e. they can be divided into eggs, molecules, atoms, and so on]. If not the physical stuff making up the eggs, then what is the subject of a number property? (…) the set-theoretic realist opts for the set of eggs in the carton. (…) What I'm getting at is this: the amount we know about things by perception is very limited. About physical objects, for example, we no little more than that they are, in Hebb's words, "space-occupying and sense-stimulation somethings". Beyond that, the bulk of our knowledge abut them is theoretical (…). The same goes for sets. What we perceive is simply something with a number property (…) Nailing down this number-bearer's more esoteric properties is a theoretical matter. (Maddy, 1990; p. 61)

In contrast to the classical Platonist view, set-theoretic realism poses that not all sets are non-spatiotemporal, eternal mathematical objects, but some are rather physical and non-abstract, and therefore not causally inert. Given this translation of Gödel's intuition, Maddy offers a rather attractive approach to the questions regarding the ontology and epistemology of mathematical objects. Interestingly for the cognitive scientist, she explicitly ties the quasi-perceptual belief about numerical properties of small sets of objects to the subitizing capacity (Kaufman, Lord, Reese, & Volkmann, 1949), which is one of the central concepts in cognitive science of mathematics (see chapters 1 to 3).

> I content that the numerical belief – there are three eggs in the carton – is perceptual (…) There is empirical evidence, based on reaction times, that such beliefs about small numbers are non-inferential. (Maddy, 1990; p. 60)

Maddy's model, which basically links Quine's naturalist and empiricist view with Gödel's two-tiered epistemological approach, strikes me as a very attractive candidate for a philosophical foundation a cognitive scientist can work with, i.e. a plausible theory of what mathematical objects are, and how it is possible to come to know about them.

> The elementariness of the notion of set, its ease of manipulation, and the immense success of set theory, both as a foundation for other branches of mathematics and as a mathematical theory in its own right, all help to make the set of eggs the most attractive candidate for the role of number-bearer. (…) They are the best mathematical entities for the mathematical theory this particular world – with its continuous phenomena – requires. (Maddy, 1990; pp. 62-63)

Maddy's approach to a philosophy of mathematics is appealing to me, in that it provides an escape from the most obvious problems mathematical realism is confronted with by invoking originally psychological concepts such as *attentional* processes, on which it depends whether I perceive two shoes or a pair, *subitizing*, as a mechanism to extract numerical information from small sets, figure-ground discrimination, that allows for the perception of sets of objects, or, more general concepts such as *cognitive development*.

> Just as the concept of an independent and continuing [in the sense of object permanence] physical object is acquired in stages, the concept of a set with inclusions and a constant number property is (…) gained over time, and depends on experience with groups of objects. (Maddy, 1990; p. 64)

Returning to Dehaene, who seems to be torn between a rejection of classic Platonist views and a rather obvious, yet somewhat implicit realist stance, I think that Maddy's approach may offer a nice way out for him, too. His approach is easily compatible with a set-theoretic realism.

> Mathematics consists of the formalization and progressive refinement of our fundamental intuitions (Dehaene, 1997; 245)

So, in lieu of having to yield the decision of what "provides the most coherent and productive pathway for research [to the] mathematical community" (Dehaene, 1997; p. 245), I rather opt for a serious interaction with and discussion of current approaches to philosophy of mathematics within the field of cognitive science of mathematics.

In the course of the next few chapters first recapitulate the output of my research on typical and atypical numerical development: The first part was written as a stand-alone introduction into the neurocognitive basis of developmental disabilities in number processing. Even though it was written for a broader audience, it will provide the necessary background information for the four empirical studies that follow. To provide an outline of what is to come, they will be summarized briefly in the next few paragraphs.

Study 1

Whether and in what way enumeration processes differ for small and large sets of objects is still a matter of debate. In order to shed light on this issue, EEG data were obtained from sixty normally developing primary school children. Adopting a standard non-symbolic numerical comparison paradigm allowed us to manipulate numerical distance between stimulus arrays for different quantity ranges, i.e. the subitizing, counting and estimation ranges. In line with the existing literature, the amplitudes of parietal positive going ERP components showed systematic effects of numerical distance, which did not depend on set size. In contrast to the similarities in surface distribution of electrophysiological activity across all number ranges, applying source localization we found distance related current density effects in inferior parietal processing systems to be similar for all numerical ranges, there was, however, considerable variation in the involvement of medial parietal and lateral occipital regions. The precuneus, which is known to be involved in visual imagery, showed distance effects exclusively for numerical comparisons on large set sizes. In contrast, the processing of small quantities and stimulus arrays arranged into canonical patterns relied on lateral occipital areas that are linked to higher-level shape recognition. These findings suggest, on the one hand, that for explicit numerical decisions an involvement of domain-specific resources does not depend on quantity features of the visual input. On the other hand, it seems that the recruitment of mediating perceptual systems differs between the apprehension of small quantities and the enumeration of large sets of objects.

Study 2

The aim of this study was to probe electrophysiological effects of nonsymbolic numerical processing in 20 children with mathematical learning disabilities compared to a group of 20 typically developing matched controls. EEG data were obtained while children were tested with a standard non-symbolic numerical comparison paradigm that allowed us to investigate the effects of numerical distance manipulations for different set sizes, i.e. the subitizing, counting and estimation ranges. Effects of numerical distance manipulations on ERP amplitudes as well as activation patterns of underlying current sources were analyzed. In typically developing children, the amplitudes of a late parietal positive-going ERP component showed systematic numerical distance effects that did not depend on set size. For the group of children with mathematical learning disabilities, ERP distance effects were found only for stimuli within the subitizing range. Current source density analysis of distance-related group effects suggested that areas in right inferior parietal regions are involved in the generation of the parietal ERP amplitude differences. The results suggest that right inferior parietal regions are recruited differentially by controls compared to children with mathematical learning disabilities in response to non-symbolic numerical magnitude processing tasks, but only for stimuli with set sizes that exceed the subitizing range.

Study 3

To date, a number of studies have demonstrated the existence of mismatches between children's implicit and explicit knowledge at certain points in development that become manifest by their gestures and gaze orientation in different problem solving contexts. Stimulated by this research, we used eye movement measurement to investigate the development of basic knowledge about numerical magnitude in primary school children. Sixty-six children from grades one to three (i.e. 6 to 9 years) were presented with two parallel versions of a number line estimation task of which one was restricted to behavioral measures, while the other included the recording of eye movement data. The results of the eye movement experiment indicate a quantitative increase as well as a qualitative change in children's implicit knowledge about numerical magnitudes in this age group that precedes the overt, i.e. behavioral, demonstration of explicit numerical knowledge. The finding that children's eye movements reveal substantially more about the presence of implicit precursors of later explicit knowledge in the numerical domain than do classical diagnostic and experimental approaches, suggests further exploration of eye movement measurement as a potential early assessment tool of individual achievement levels in numerical processing.

Study 4

In the context of a numerical Stroop experiment, children were asked to make numerical and physical size comparisons on digit pairs. 66 primary school children were selected for this study, of which 21 scored low in a standardized math achievement test, 23 were normal and 22 high achievers. The effects of congruity and numerical distance were determined. All children exhibited congruity and distance effects in the numerical comparison. In the physical comparison, children of all performance groups showed Stroop effects when the numerical distance between the digits was large, but failed to show them when the distance was small. Numerical distance effects depended on the congruity condition, with a typical effect of distance in the congruent, and a reversed distance effect in the incongruent condition. Our results are hard to reconcile with theories that suggest that deficits in the automaticity of numerical processing can be related to differential math achievement levels. Immaturity in the precision of mappings between numbers and their numerical magnitudes might be better suited to explain the Stroop effects in children. However, as the results for the high achievers demonstrate, in addition to numerical processing capacity per se, domain-general functions might play a crucial role in Stroop performance, too.

The final part of this thesis will discuss the findings of the experimental studies with respect to the broader issues raised in the introductory section and will provide an outline of what should be the next steps.

1. RECHENSTÖRUNG[1]

[1] Book chapter in: Heine, A., Engl, V., Thaler, V., Fussenegger, B., & Jacobs, A. M. (in press). *Neuropsychologie von Entwicklungsstörungen schulischer Fertigkeiten*. Göttingen: Hogrefe.

2. Electrophysiological Correlates of Non-symbolic Numerical Magnitude Processing in Children: Joining the Dots[2]

[2] Published as: Heine, A., Tamm, S., Anders, J., & Jacobs, A. M. (2011). Electrophysiological correlates of non-symbolic numerical magnitude processing in children: Joining the dots. *Neuropsychologia*, *49,* 3238-3246. DOI: 10.1016/j.neuropsychologia.2011.07.028.

3. An Electrophysiological Investigation of Non-Symbolic Magnitude Processing: Numerical Distance Effects in Children with and without Mathematical Learning Disablities[3]

[3] MS submitted to *Cortex*.

4. What the Eyes Already 'Know': Using Eye Movement Measurement to Tap into Children's Implicit Numerical Magnitude Representations[4]

[4] Published as: Heine, A., Thaler, V., Tamm, S., Hawelka, S., Schneider, M., Torbeyns, J., De Smedt, B., Verschaffel, L., Stern, E., Jacobs, A. M. (2010). What the eyes already 'know': Using eye measurement to tap into children's implicit numerical magnitude representations. *Infant and Child Development*, *19,* 175-186. DOI: 10.1002/icd.640.

5. The Numerical Stroop Effect in Primary School Children: A Comparison of Low, Normal and High Maths Achievers[5]

[5] Published as: Heine, A., Tamm, S., De Smedt, B., Schneider, M., Thaler, V., Torbeyns, J., Stern, E., Verschaffel, L., & Jacobs, A. M. (2010). The numerical Stroop effect in primary school children: A comparison of low, normal, and high maths achievers. *Child Neuropsychology, 16*, 461-477. DOI: 10.1080/09297041003689780.

6. General Discussion and Outlook

The main purpose of the research studies that constitute the present doctoral thesis was to investigate the development of mental representations of numerical magnitude in normally developing, and in children suffering from mathematical learning disabilities.

Contrary to theories that assume numerical processing of small sets of items to be fundamentally different from enumeration of large set sizes, the results of the first study suggest that domain-specific systems in inferior parietal regions are recruited for magnitude processing across the whole numerical range. We found, however, that mediating domain-general perceptual and memory functions were recruited differentially depending on set size.

On the basis of these findings, the second study was designed to investigate behavioral and electrophysiological indices of basic numerical processing in children with mathematical learning disabilities compared to normally developing children. In contrast to similarities on the level of behavioral performance measures, the analysis of electrophysiological data revealed obvious differences between the two achievement groups. For large arrays of dots, no late parietal numerical distance effects were found for the group of children with mathematical disabilities. Current source analysis of the distance-related group effects demonstrated current density differences in right inferior parietal cortices, which can be assumed to reflect differential recruitment of format-independent numerical magnitude representations. However, for dot arrays in the subitizing range, the ERP data yielded no differences between the low and the normal achievers. Both groups showed similar distance effects, and no group differences in current source density in parietal regions were found.

The results of the third study demonstrate that children's representation of numerical magnitude develops substantially during their first years in primary school. Behavioral data from two number line estimation tasks showed that children's performance in this task improves considerably in the course of their first years of formal mathematical training. The behavioral data confirm the assumption that with the transition from grade one to grade two, children's numerical magnitude representations change from an immature logarithmic pattern of representation to the more appropriate linear model. Interestingly, the data demonstrate that even in cases where no evidence of representational change can be found in children's overt behavioral responses, eye movement parameters, which can be assumed to tap into a more implicit level of knowledge, may reveal early manifestations of new kinds of knowledge at work.

The fourth study used a numerical Stroop paradigm to test whether developmental learning abilities in the mathematical domain are related to deficits in the automatization of access to numerical magnitude representations. However, when the influence of numerical distance was taken into consideration, children of different achievement groups, i.e. low, normal and high achievers in the domain of mathematics, show similar congruity effects in the number Stroop tasks. In particular, the finding of considerably large reversed numerical distance effects not only for the group of normal achievers, but also for the children with mathematical disabilities, contradicts theories that propose automatization deficits to be the cause of developmental impairments of number processing.

6.1 Numerical Development

Classical domain-specific models suggest mathematical learning disabilities to be related to deficits in the mental representation of numerical magnitude (Butterworth, 1999; Dehaene, 1997). Ultimately, the present results corroborate this assumption. However, at the same time the eye-movement data suggest that representations of numerical magnitude are not stable over time, but subject to fundamental changes in the course of development. Such a finding entails that only research approaches that incorporate a dynamic perspective allow for an in-depth understanding of how impairments of number processing unfold over time.

In my opinion, the prevailing focus on the description of deviant "end-state processing systems" (Ansari & Karmiloff-Smith, 2002) obscures the fact that what is required is deeper insight into the developmental trajectories that lead to their formation. For instance, only longitudinal studies that incorporate behavioral and neurophysiological data on low-level number-relevant processes and representational functions can actually answer one rather important yet still open basic question: Namely, whether differential functioning of inferior parietal brain regions is primary, in the sense of an impaired maturation of phylogenetically specified localized modules for numerical processing, or rather secondary to basic functional impairments of more fundamental and probably widespread processing systems. The adequate way to study developmental disorders is to identify the most basic level of functional deviation and find out how, in interaction with the child's environment, it finally leads to impaired higher-level cognition.

With regard to the explanatory scope of the findings presented here, this means that while for an arbitrary point in developmental time certain differences in neurocognitive measures were demonstrated between groups of children – while others were not –, there is no way to explain the original causes of these deviations. In that, the four studies merely emphasize the fact that it is essential to actually start studying the *developmental pathways* of normal and impaired numerical abilities. Or, as Karmiloff-Smith (1998) put it so aptly, "development is the key to understanding developmental disorders".

Knowing this, a current research project at the FU Berlin on the neurocognitive basis of normal and impaired numerical abilities focuses on both, domain-general and domain-specific factors that may play a causal role. Using a longitudinal study design, this project focuses on the collection of more fine-grained data on the development of lower-level and higher-level functions. And, hopefully, future studies will provide a more comprehensive view on how behavioral, neurophysiological and even environmental factors interact in the development of number processing capacities.

6.2 Mathematical Knowledge

In the introduction to the present thesis, I raised the question of what it is that we in we, the cognitive scientists, who are interested in human number processing capacities, are actually thinking about when we refer to certain mathematical concepts such as numbers. And, while most research in cognitive science of mathematics is done without explicit reference to the questions of ontology and epistemology of the mathematical objects under scrutiny, I think that it is necessary to reflect upon at least once in a while what the premises are upon which one's own research is rooted in.

> All science, from physics to physiology, is a function of its philosophic presuppositions, but psychology is more vulnerable than others to the effect of misconceptions in fundamental matters because the object of its study is after all the human mind and the nature of human thought. As long as the ideas are implicit they are dangerous, make them explicit and perhaps they can be defused. (Hebb, 1980; p. 2)

Before linking my own studies to what I think is an approach to philosophy of mathematics that I can relate to, I will first examine how some prominent members of my scientific community, namely Dehaene (1997) Lakoff and Núñez (2000) dealt with the question of mathematical ontology and epistemology.

6.2.1 Dehaene's Approach to the What and How of Mathematical Knowledge

Interestingly, what we get from Dehaene are somewhat mixed signals. On the one hand, he writes that it is his conviction that "mathematics is a *human construction* [emphasis added]" (Dehaene, 1997; p. 247) and that "among the available theories on the nature of mathematics, *intuitionism* [emphasis added] seems to (…) to provide the best account of the relations between arithmetic and the human brain" (Dehaene, 1997; 244). This means that he assumes that human mathematicians, equipped with human brains, come up with mathematical concepts which fully depend on the evolved structure of our minds. On the other hand, he drops his anti-realistic stance when he writes that "Platonism hits upon an undeniable element of truth when it stresses that physical reality is organized according to *structures that predate the human mind* [emphasis added]" (Dehaene, 1997; p. 251) or that "it is rather remarkable that *nature founded the bases of arithmetic* [emphasis added] on the most fundamental laws of physics" (Dehaene, 1997; p. 60).

I think that his problems in committing himself to the one or the other is based on an obvious confusion of the dimensions of ontology and epistemology. The question here is, whether an anti-realism in epistemology – Dehaene closes his book with the unequivocal statement that "[mathematics] is the only language with which we can read [nature]" (Dehaene, 1997; p. 252) – necessarily entails ontological anti-realism? This is clearly not the case, i.e. "one may be a realist about some things without being a realist about others." (Resnik, 1999; p. 10). To clarify this point: realism in ontology *and* epistemology is the view that the world is independent of our minds and that we can know it as it is independently of the specifics of our cognitive predispositions. The thoroughbred anti-realist thinks that both the world and our knowledge of it depend on our minds, conceptual schemata, intellectual habits, social practices, and so on. In this view, mathematics is something we make up, a construct. But there is kind of a middle-ground between the two extreme versions, i.e. ontological realists, who are epistemological anti-realists, assume that the, or better *some* world exists independent of us, but that we cannot know it independently of our minds.

Dehaene's theory of number processing is originally only about how we come to know. In this sense, it is mainly an epistemological approach and here his constructivism comes into play.

> Number appears as one of the fundamental dimensions according to which our nervous system parses the external world [emphasis added]. Just as we cannot avoid seeing objects in color (…) and at definite locations in space (…), in the same way numerical quantities are imposed on us effortlessly through the specialized circuits of our inferior parietal lobe. The structure of our brain defines the categories according to which we apprehend the world. (Dehaene, 1997; p. 245)

For the nativist Dehaene, numerical concepts are cognitive primitives, i.e. in contrast to e.g. Piaget (Piaget & Szeminska, 1972) human mathematical abilities do not derive from logical reasoning or other domain-general precursors. Instead, we are endowed with a domain-specific mechanism that is shaped by natural selection to enable us and other animals to discriminate numerosities-cues in our environment. However, Dehaene does not deny the role of the external world in shaping our "number sense", to use his own terminology.

> Throughout phylogenetic evolution, as well as during cerebral development in childhood, selection has acted to ensure that the brain constructs internal representations that are adapted to the external world [emphasis added]. Arithmetic is such an adaptation. At our scale, the world is mostly made up of separable

> objects that combine into sets according to the familiar equation 1 + 1 = 2 [emphasis added]. This is why evolution has anchored this rule in our genes. Perhaps our arithmetic would have been radically different, if, like cherubs, we had evolved in the heavens where one cloud plus another cloud is still one cloud. (Dehaene, 1997, p. 249)

In this light, Dehaene's position has affinities with both intuitionism and realism. He views numerosity as a real and mind-independent property of the world, yet it is our mind that parses the world in accordance with our specific, i.e. innate cognitive predispositions. There is, in fact, no reason, why Dehaene's posit of an innate "number sense" should not be compatible with realism in ontology. Twisting Katz's statement that "the realist is only committed to there being some version of nativism that does the job" (Katz, 2002; p. 133), I would say that the nativist Dehane is only committed to there being some version of realism that does the job. In my reading of Dehaene, I would say that his approach is well in line with Maddy's (1990) set-theoretic realism. In this sense, Dehaene's view that subitizing and approximate estimation are the cognitive primitives that higher mathematical reasoning is based upon, is intuitively compatible with Maddy's two-level approach. Interpreting numbers as properties of sets is also a concept that Dehaene seems to be comfortable with, when he writes that "the maxim 'number is a property of sets of discrete physical objects' is deeply embedded in their brains" (Dehaene, 1997; p. 61).

Just for the sake of completeness, I will briefly summarize the only other approach from within cognitive science that explicitly tackles the more fundamental questions related to human mathematical knowledge.

6.2.2 The Constructivism of Lakoff and Núñez

Lakoff and Núñez (2000) propose a theory of how conceptual metaphors, i.e. basic grounding metaphors and higher-level linking metaphors, give rise to the emergence of mathematical concepts. Via processes of inference-preserving conceptual mapping, knowledge about mathematics is grounded in knowledge human beings necessarily gain by interaction with their respective physical environment.

> Conceptual mapping and other mechanisms for human imagination are universally available, but they are not genetically determined, allowing for cultural and historical development and variation. (Núñez, 2011; p. 663)

Complex mathematical thought is grounded in basic cognitive capacities through conceptual metaphor and conceptual blending.

The mapping of conceptual metaphors from a source domain such as "Object Collection" to the target domain "Arithmetic" would look like that:

SOURCE→TARGET

size of a object collection→size of a number or

putting collections together→Addition (cf. Lakoff & Núñez, 2000; p. 55).

According to Lakoff & Núñez, arithmetic objects and principles are, thus, the products of an inference-preserving mapping across domains in which the conceptual structure of the source domain renders a reasoning about the target domain possible.

> What we normally call 'laws' of arithmetic are in fact metaphorical entailments of the conceptual mapping we are operating with. (…) The Arithmetic is Object Collection metaphor is a precise conceptual mapping from the domain of physical objects to the domain of numbers. (Núñez, 2009; p. 79)

This means that when mathematicians are doing their work, they are guided by certain conceptual metaphors they not necessarily share with others. Consequently what remains unclear, of course, is how people are able to share higher mathematical ideas that presuppose rather complex conceptual mappings. And what about the fact that metaphorical ideas are sometimes misleading? In this sense, we should probably not speak of mathematics, but of 'the diversity of human mathematical constructions'.

> Mathematics, and number systems and arithmetic in particular – even in their simplest forms – are not hardwired but, but rather they emerge as culturally shaped sophisticated forms of sense-making. They are the product of the interaction of certain communities of individuals with the appropriate culturally and historically shaped phenotype supported by language, writing systems, artifacts, education, and specific forms of environmental dynamics. (Núñez, 2009; p. 69)

For the reader it is very annoying, on the one hand, that Lakoff and Núñez (2000) do not confine themselves to describing their approach to a cognitive theory of numerical capacities. Instead, they start a polemic about what I would call a kind of a scapegoat-Platonism which the authors dubbed "The Romance of Mathematics".

> A great many of those who have serious knowledge of mathematics not only tend to believe the mythology we call the Romance of Mathematics but tend to believe it fiercely. (…) It is a story that many people want to be true [since] the Romance serves the purposes of the mathematical community. It helps to maintain an elite (…) it is contributing to the social and economic stratification of society (…) and doing social harm. (Lakoff & Núñez, 2000; pp. 339-241)

Lakoff and Núñez are, thus, evading a constructive discussion of any form of mathematical realism, which is especially irritating since the authors, on the other hand, completely ignore what the philosophy of mathematics since Plato had and has to say about the subject matter.

However, apart from these shortcomings, I think that this constructivist approach to a cognitive science of mathematics is easily reconcilable with Maddy's version of realism. With Dehaene, Lakoff and Núñez assume that humans are born with certain innate capacities, which higher-level mathematical reasoning is rooted in.

> We are born with a minimal innate arithmetic, part of which we share with other animals. (…) Innate arithmetic includes at least two capacities: (1) a capacity for subitizing – instant recognizing small numbers of items – and (2) a capacity for the simplest forms of adding and subtracting small numbers. (By number here we mean cardinal number, a number that specifies how many objects there are in a collection.). (Lakoff & Núñez, 2000; p. 51)

Here, again, is seems obvious to relate this "minimal arithmetic" to the lower-tier in Maddy's (1990) epistomology, i.e. to what she calls mathematical intuition. The questions of how human beings develop the complex structure of abstract mathematics on the basis of their lower-level capacities, or how far these lower-level capacities carry us, and whether it is innate domain-specific cognitive functions (Dehaene, Piazza, Pinel, & Cohen, 2003; p. 498) or innate domain-general capacities such as the ability to form metaphors (Núñez, 2009; p. 81) that help us to go beyond basic mathematics, are linked to Maddy's realm of axiomatic mathematics in general, and set theory in particular. At this higher tier, mathematical truth is justified extrinsically, i.e. by the explanatory success (Gödel, 1983).

6.2.3 Fitting Experimental Approaches into the Bigger Picture

Being an experimental psychologist, I have difficulties to uphold an anti-realist stance of any shade. In designing the stimuli for my experiments, I have to assume some systematics between input and output, i.e. I start from the assumption that some real properties of real input are able to excite certain reactions. So, instead of paying lip-service to some admittedly smart ideas about everything being relative, and so on, I confess to a certain ontological realism – be it weekday or Sunday.

> Most (…) seem to agree that the typical working mathematician is a Platonist on weekdays and a formalist on Sundays. That is, when he is doing mathematics, he is convinced that he is dealing with an objective reality whose properties he is attempting to determine. But then, when challenged to give a philosophical account of this reality, he finds it easiest to pretend that he does not believe in it after all. (Hersh, 1979; p. 31)

On the other hand, I agree with Dehaene that no cognitive scientists can agree with full-blown Platonism, since we are investigating brains, not souls or other entities that are supposedly granting us access to other realms. So, for an experimental cognitive science of mathematics which is intrinsically incompatible with both, a Platonist version of mathematical realism and with mathematical anti-realism, Maddy's moderate philosophy of mathematics seems to be a plausible approach to the questions of how we come to know about mathematical objects and what they are, in the first place.

Reflecting on my own research in the domain of cognitive science of mathematics which basically amounts to this thesis, there seems to be a divide between the basic questions tackled by studies one and two, on the one hand, and studies three and four, on the other. In primarily targeting the developmental course of internalization of certain external number representations systems such as visuo-spatial and symbolic representations of numerical magnitude (Núñez, 2011), the last two studies are obviously not dealing with any form of mathematical processing to be linked to the lower tier of Maddy's approach. These two studies are actually more about domain-general capacities such as metaphorical mapping in the case of the number-line study, and symbolic functions in the case of the number-Stroop study. In that, they are not dealing with the fundamental questions of numerical development.

However, the two studies on non-symbolic magnitude processing in typically developing and children with mathematical disabilities, seem to aim exactly at the lower-level capacity of mathematical intuition as the basis for development of higher mathematical abilities, and, thus, at how the lower-level capacities lead to higher numerical skills. If I had to translate these studies into the Maddyan model, I would say that originally, the processing of numerosity of collections of dots is part of the lower-tier capacities, which I would interpret in agreement with Dehaene as not necessarily "mathematical in nature" (Dehaene, 1997; p. 251). This originally pre-numerical non-symbolic *numerosity* processing may be the reason that the finding of distance effects, which reflect the workings of end-state *numerical* representation systems, depend on the design of the task. If the task design is explicitly numeric in character, higher-level functions kick in, i.e. we can observe numerical distance effects. However, if the task targets only basic level capacities, such reflections of higher-level processes are not observable (Hyde & Spelke, 2009).

In line with that, I think it would have been interesting to investigate the non-symbolic numerical processing capacities of children with mathematical disabilities under different study designs, i.e. for example comparing children's performance in task designs that require explicit numerical decisions versus in habituation paradigms. Doing that would allow for deeper insights into the question of where children's problems originate from, be it the lower level capacities, as assumed e.g. by Butterworth (2010) or Landerl and colleagues (Landerl, Bevan, & Butterworth, 2004; Schleifer & Landerl, 2011), or at the transition to higher levels, which would be more in line with the results of my own studies.

Furthermore, it seems worthwhile to look into whether and when the assumed processes of abstraction, that Maddy proposes to be the basis of the transition from perceiving a number of individual objects to perceiving sets of objects, actually take place and what their manifestations may be. Such a focus would lead immediately to Hannula's studies on Spontaneous Focusing on Numerosity (Hannula, Lepola, & Lehtinen, 2010) which may be the cognitive processes at the basis of Maddyan abstraction.

In conclusion, I would want to point out that it not only seems appropriate from a theoretical point of view for a cognitive science of mathematics to find out what models and views neighboring disciplines such as the philosophy of mathematics have to offer, but it may actually help researchers to broaden their view on their specific subject matter and, thus, even help with the formulation of future research questions.

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Lebenslauf

Angela Heine

Der Lebenslauf ist in der Online-Version aus Gründen des Datenschutzes nicht enthalten

ERKLÄRUNG

Ich versichere hiermit, die vorliegende Dissertation selbständig und ohne Verwendung anderer als der angegebenen Hilfsmittel verfasst zu haben. Diese Arbeit in Gänze oder einzelne Teile waren nicht Gegenstand eines früheren Promotionsvorhabens. Die Kapitel wurden bereits veröffentlicht (Kap. 2, 4, 5), befinden sich im Druck (Kap. 1) oder im Begutachtungsprozess (Kap. 3).

Kapitel 1 ist ein von mir als Einzelautor verfasstes Kapitel eines Buches, das sich gegenwärtig im Druck befindet:

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Kapitel 2 ist ein Zeitschriftenartikel, der veröffentlich wurde als:

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Kapitel 3 ist ein Manuskript für einen Zeitschriftenartikel, das bei der Zeitschrift *Cortex* eingereicht wurde und sich im Begutachtungsprozess befindet:

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Kapitel 4 ist ein Zeitschriftenartikel, der veröffentlich wurde als:

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Alle Koautoren können bestätigen, dass die Planung und Implementierung aller Studien durch mich allein erfolgte, dass ich die Erhebung aller Daten als Studienleiter überwacht habe, dass sämtliche Daten von mir ausgewertet und alle Manuskripte, aus denen sich diese Arbeit zusammensetzt, von mir allein verfasst wurden.

Berlin, den 24. Oktober 2011
Angela Heine