Chapter 3

Methodology

3.1 Introduction

The aim in the present work is to interpret the maps derived from satellite data and to infer geological information from these maps. The approach followed here is not that of direct inversion of satellite data sets which may be erroneous mainly because of the non-uniqueness of the inverse problem and the field models still retaining the uncorrected ionospheric and magnetospheric signals and also some part of main field. I start with an initial VIS model about the susceptibility and depth distribution on the Earth's surface based on present geology and crustal structures constrained by other geophysical data and perform the forward computation using spherical harmonic transforms to compute the initial magnetic anomaly field. The next step is to locate specific regions on the map where the anomaly patterns or amplitude of the initial and the observed map are in disagreement. The next iteration would require modification in parameters value for the lower crust or even the shape of a geological unit to arrive at the *first iteration* model. This iteration is used to infer geological information for the region studied in detail.

The entire set of modelling steps described above requires first computing the magnetic potential at the satellite altitude for a given global initial VIS model and subsequently Gauss coefficients for this potential are derived using spherical harmonic expansion of the potential. The next step involves predicting the vertical component of the crustal field anomaly for degrees 16-80 using the Gauss coefficients derived above. For modelling, two distinct approaches have been followed. The first approach assumes a distribution of equivalent dipoles for grid cells on the Earth's surface and is called the 'Equivalent Dipole Method' (Langel & Hinze, 1998). The second approach follow the methodology to arrive at the potential derived by Nolte & Siebert (1987) and is referred to in the following as Nolte-Siebert method. Both the approaches yield the Gauss coefficients of the crustal field potential. The details of the above mentioned steps and the approaches are explained below.

3.2 Equivalent dipole method

The equivalent dipole method computes the crustal magnetic field of the earth from the crustal and geomagnetic main field data. The assumptions followed for the derivation of the necessary equations are as follows:

- Earth topography is neglected, while Earth ellipticity is taken into account.
- Induced magnetisation is considered as the only source for the anomalies.

The crust here is characterized by an vertically integrated susceptibility that depends upon the angular coordinates of the spherical surface, defined on a local geographic coordinate which coincide with the spherical coordinate system, by radius *r*, colatitude θ and longitude ϕ.

Considering a volume element $d\tau'$ of magnetizable material located at point $\vec{r}' = (r', \theta', \phi')$ within the earth's crust. The material attains an induced magnetisation due to the geomagnetic main field $\vec{B}$. The potential due to this magnetised volume element $d\tau'$ at a fixed external point $\vec{r} = (r, \theta, \phi)$ is defined as dipole potential dV and is written as

$$dV(\vec{r}, \vec{r}') = \frac{\mu_0}{4\pi} d\vec{m}(\vec{r}') \cdot \nabla' \frac{1}{R} \qquad (3.1)$$

where μ_0 is the permeability of the vacuum, $R = |\vec{r} - \vec{r}'|$ and the prime with the Nabla operator denotes differentiation with respect to source coordinates. $d\vec{m}(\vec{r}')$ is the dipole moment and is the product of $d\tau'$ with the magnetisation $\vec{M}$. The situation is shown in Figure (3.1).

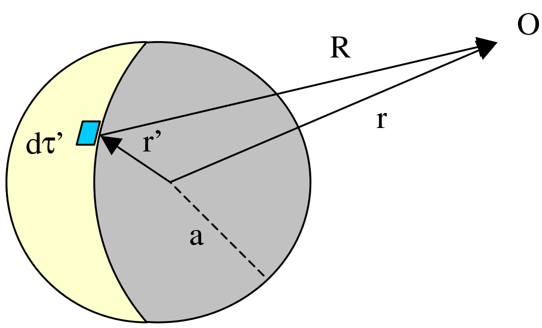


Fig. 3.1. Schematic diagram showing an elemental dipole moment producing a potential at point O.

As the crustal thickness *d* is assumed negligible, the integrated susceptibility of the crust can be defined as

$$\tilde{\chi}(\theta', \phi') = \lim_{\substack{d \to 0 \\ \chi \to \infty}} \int_{a-d}^{a} \chi(r') dr' \qquad (3.2)$$

where $\tilde{\chi}(\theta',\phi')$ is the magnetic susceptibility assumed to be a scalar quantity and a is the Earth's radius. The susceptibility is further assumed to be nonzero only in the region between Curie isotherm and the Earth's surface. Since the thickness of the magnetised layer is small compared with the altitude, the three-dimensional (3D) susceptibility distribution can be represented by the vertically integrated susceptibility of the thin shell. Thus we can compute vertically integrated magnetisation or susceptibility defined as the thickness of the magnetised crust multiplied with the average value of the susceptibility, at every point on the Earth's surface.

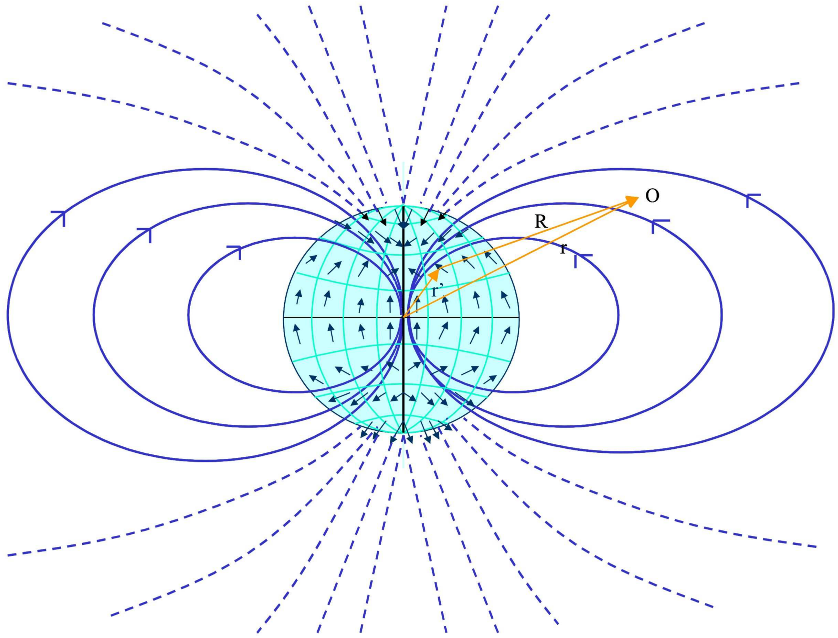


Fig. 3.2. The distribution of induced dipoles on the surface of Earth in an inducing dipole field. Each dipole is aligned with the direction of the inducing field.

Considering a global crustal model it is desirable to work in spherical coordinates. Dividing the spherical earth into 15' × 15' grid cells, integrated susceptibility, $\tilde{\chi}(\theta',\phi')$ can be computed in every cell by assuming a magnetic depth multiplied with the average susceptibility in that column. The situation is shown in Figure (3.2). Each cell is represented by a single dipole whose contribution to the magnetic potential can be written in spherical coordinates as

$$dV(\vec{r},\vec{r}') = \frac{\mu_0}{4\pi}\frac{d\vec{m}(\theta',\phi')\cdot\vec{R}}{|R|^3} \tag{3.3}$$

where $d\vec{m}(\theta',\phi') = \tilde{M}(\theta',\phi')ds' = \frac{1}{\mu_0}\tilde{\chi}(\theta',\phi')\vec{B}(\vec{r}')\mathrm{ds}'$.

$$dV(\vec{r},\vec{r}')=\frac{1}{4\pi}\tilde{\chi}(\theta',\phi')\frac{\vec{B}(\vec{r}')\cdot\vec{R}}{|R|^3}ds' \qquad (3.4)$$

where $ds' = \sin\theta' s_0$ and (θ',ϕ') are colatitude and longitude of the source dipole. Each pair of points (θ',ϕ') is defined at the centre of the grid cell. The area of the grid cell varies according to $\sin\theta' s_0$, where s_0 is the area of the cell at the equator and it decreases down to zero at the poles. Each dipole represents an area of ds', on the Earth's surface. Each $d\vec{m}(\theta',\phi')$ is assumed to have the direction of the present-day ambient field. The assumption that $d\vec{m}(\theta',\phi')$ is oriented along the ambient field ignores the possibility of remanent magnetisation in any other direction. Potential field is calculated at every point on a spherical grid at an altitude of 400 km from the surface of the Earth, by summing the contribution of each dipole. This potential is then expanded into a spherical harmonics to degree 80 to get the Gauss coefficients of the potential. The program performing the above steps is defined in the following.

A program in C language is developed to compute the potential and expand it into spherical harmonics to find Gauss coefficients of the potential for degree 1-80. The program takes the vertically integrated susceptibility for the entire Earth's surface as the input as a 15' × 15' grid. This input is generated with the help of GIS package Arc/Info 8.1. Each cell represents a single dipole and the direction of the dipole is aligned with the direction of the inducing main field. It is to be noted that the grid structure described above makes the data structure sparse at the equator and very closely spaced at the poles. Thus, it would be appropriate to redesign the grid structure so as to use grid cells having same effective area on the Earth's surface. This in turn reduces the number of grid cells at the poles. Each cell represents an equivalent dipole, representing an area of 2^0 X 2^0 called internal grid cells. The program uses an algorithm to compute spherical harmonic transformation of the potential field value at only Gauss-Legendre (G-L) positions. G-L positions are unequally spaced points on the latitude of the spherical shell where the field value is to be calculated. This means that the G-L positions at a particular latitude have more number of points at the equator than at the poles. Potential is computed for each dipole on the internal grid cells at every point on the G-L position at an altitude of 400 km. Total potential is calculated at every point of the G-L spherical shell as a summation of potential contribution from all the dipoles. This total potential is then expanded using spherical harmonic transform to get the Gauss coefficients for degrees 1-80. Subsequently, the vertical component of the crustal anomaly field is computed from these Gauss coefficients.

3.3 Nolte-Siebert method

The work of Nolte and Siebert (1987) introduced for the first time an analytical relation between the spherical harmonic coefficients of susceptibility and the potential of the inducing field. Unlike the equivalent dipole method, which works in space domain, Nolte-Siebert method works in wavenumber domain. The pertinent relations to solve the forward problem of the crustal field were originally derived by one of the authors, Siebert, and was further extended and applied to a crustal model field by his co-worker Nolte (1985). The method developed by Nolte and Siebert (1987) computes the Gauss coefficients of the potential of the crustal field of the Earth. The assumptions followed for the derivation of the necessary equations have been already defined in the previous section.

The integrated susceptibility of the crust is defined by the Equation (3.2). If the inducing field is replaced by the gradient of the scalar potential V outside the Earth's core, then the dipole moment becomes

$$d\vec{m}(\vec{r}\,') = \vec{M}(\vec{r}\,')d\tau' = -\frac{1}{\mu_0}\chi(\vec{r}\,')\nabla' V(\vec{r}\,')d\tau' \tag{3.5}$$

The VIS can be expanded into spherical surface harmonics to degree n as

$$\tilde{\chi}(\theta',\phi') = \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(p_n^m \cos m\phi' + q_n^m \sin m\phi'\right)P_n^m(\cos\theta') \tag{3.6}$$

The term for degree n=0 has not been taken into account as it represents a constant distribution of the integrated susceptibility over the entire surface of the Earth, which would not contribute to any measurable magnetic field outside the spherical Earth (Runcorn, 1975).

The potential of the geomagnetic main field is expanded into a series of spherical harmonics with reference radius a

$$V_c(\vec{r}\,') = a\sum_{n=1}^{\infty}\left(\frac{a}{r'}\right)^{n+1} Y_n(\theta',\phi') \tag{3.7}$$

where c stands for the core, and Y_n represents sum over one or more elementary surface harmonics of the same degree n but with orders varying between $m = 0$ and $m = n$.

$$Y_n = \sum_{m=0}^{n}\left(g_n^m \cos m\phi' + h_n^m \sin m\phi'\right)P_n^m(\theta') \tag{3.8}$$

Quasi-normalization according to Schmidt is used throughout for Legendre's associated functions P_n^m. Gauss coefficients g_n^m and h_n^m are given by a main field model. For the dipole inducing geomagnetic field (n=1), the terms in equation (3.8) can be written as

$$Y_1(\theta',\phi') = g_1^0 P_1(\theta') + \left(g_1^1 \cos\phi' + h_1^1 \sin\phi'\right)P_1^1(\theta') \tag{3.9}$$

The potential can be derived by allowing full expansions of the main field and the susceptibility and by combining all the equations [4-27] (Nolte and Siebert, 1987) to arrive at Eqn. [29] (Nolte and Siebert, 1987) for the expression of potential $V(\vec{r})$. If further G_N^M and H_N^M are the spherical harmonic expansion coefficients of the crustal potential $V(\vec{r})$, it can be written as

$$V(\vec{r}) = a\sum_{N=1}^{\infty}\sum_{M=0}^{N}\left(\frac{a}{r}\right)^{N+1}\left(G_N^M \cos M\phi + H_N^M \sin M\phi\right)P_N^M(\theta) \tag{3.10}$$

and can be compared with coefficients of eqn [38] (Nolte and Siebert, 1987) to get the following relation for the coefficients for crustal potential G_N^M and H_N^M for the axial and non-axial dipolar inducing field, respectively shown in eqn (3.11) and (3.12),

$$\begin{Bmatrix}\left(G_N^M\right)_{10}\\ \left(H_N^M\right)_{10}\end{Bmatrix} = \frac{1}{2a(2N+1)}\left[\frac{N-1}{2N-1}\left[g_{10}\sqrt{(N-M)(N+M)}p_{N-1,M}\right] + \frac{3N}{2N+3}\left[g_{10}\sqrt{(N-M+1)(N+M+1)}p_{N+1,M}\right]\right] \tag{3.11}$$

$$\begin{aligned}\begin{Bmatrix}\left(G_N^M\right)_{11}\\ \left(H_N^M\right)_{11}\end{Bmatrix} = \frac{\sqrt{g_{11}^2+h_{11}^2}}{2a(2N+1)}\Bigg[\frac{N-1}{2N-1}\Bigg\{&\sqrt{\frac{2-\delta_{M1}}{2}}\sqrt{(N+M-1)(N+M)}\begin{Bmatrix}p_{N-1,M-1}\cos\varepsilon_{11} - q_{N-1,M-1}\sin\varepsilon_{11}\\ q_{N-1,M-1}\cos\varepsilon_{11} + p_{N-1,M-1}\sin\varepsilon_{11}\end{Bmatrix}\\ &-\sqrt{\frac{2}{2-\delta_{M0}}}\sqrt{(N-M-1)(N-M)}\begin{Bmatrix}p_{N-1,M+1}\cos\varepsilon_{11} + q_{N-1,M+1}\sin\varepsilon_{11}\\ q_{N-1,M+1}\cos\varepsilon_{11} - p_{N-1,M+1}\sin\varepsilon_{11}\end{Bmatrix}\Bigg\}\\ -\frac{3N}{2N+3}\Bigg\{&\sqrt{\frac{2-\delta_{M1}}{2}}\sqrt{(N-M-1)(N-M+2)}\begin{Bmatrix}p_{N+1,M-1}\cos\varepsilon_{11} - q_{N+1,M-1}\sin\varepsilon_{11}\\ q_{N+1,M-1}\cos\varepsilon_{11} + p_{N+1,M-1}\sin\varepsilon_{11}\end{Bmatrix}\\ &-\sqrt{\frac{2}{2-\delta_{M0}}}\sqrt{(N+M-1)(N+M+2)}\begin{Bmatrix}p_{N+1,M+1}\cos\varepsilon_{11} + q_{N+1,M+1}\sin\varepsilon_{11}\\ q_{N+1,M+1}\cos\varepsilon_{11} - p_{N+1,M+1}\sin\varepsilon_{11}\end{Bmatrix}\Bigg\}\Bigg]\end{aligned} \tag{3.12}$$

where $p_{N+1,M-1} = 0$ when M=0 and $p_{N-1,M+1} = 0$ when M≥N-1 and the symbol δ_{MN} is the Kronecker delta. Nolte (1985) derived more complicated analytical expressions for coefficients of the crustal potential for the inducing quadrupole and octupole terms of the geomagnetic main field. We have used all the expressions derived for terms n=1,2,3 of geomagnetic main field, to compute the coefficient of potential on an external spherical grid for an integrated susceptibility distribution on the Earth's surface. Interested readers should refer to the work by Nolte, (1985) for a complete set of expressions used in the present analysis. The pertinent equations are used to compute the Gauss coefficients of the lithospheric field potential.

A program based on the equations derived by Nolte and Siebert (1987) is developed to compute the spherical harmonic coefficients (VIS) of the potential. The program takes the vertically integrated susceptibility global model as the input. Spherical harmonic expansion of the VIS is performed to obtain the coefficients p_n^m and q_n^m. Next, main field is considered starting with the dipole terms first, followed by the terms for quadrupole and octupole terms as the only inducing fields. Each of these terms is taken separately and then together to study the effects of induction to the magnetised layer. Gauss coefficients of the potential are computed using all the terms of inducing main field. The vertical component of the crustal anomaly field is computed from the Gauss coefficients in the same way as for the equivalent dipole method.

3.4 Comparison of the two methods

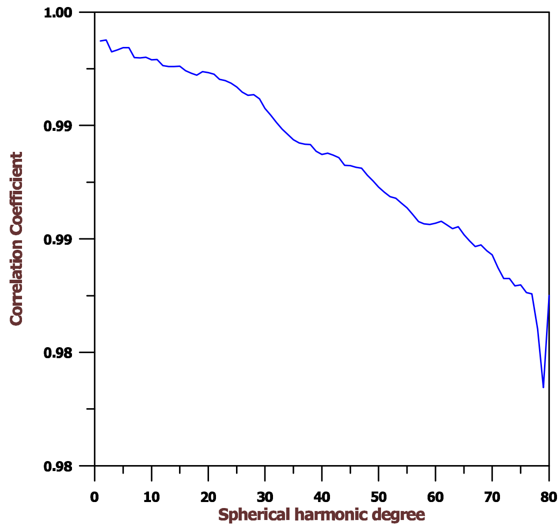
In the above two sections two forward modelling methods were described. The methods followed two different approaches to arrive at the Gauss coefficients of the potential. After implementing these two approaches in the form of computer programs, these programs were tested for their accuracy and speed.

In the process of testing, a simple VIS model was considered as input because the initial crustal model was built only at a later stage. This VIS model was derived by multiplying the ETOPO map with the 3SMAC seismic model. ETOPO map is an elevation map produced by NGDC (http://www.ngdc.noaa.gov/mgg/global/seltopo.html). This elevation map was separated in two parts as continents and oceans. Continents were denoted with fixed susceptibility value of 0.4 and oceans with 0.2. These values were then multiplied with 3SMAC seismic crustal thickness to derive the VIS model. This VIS model was now fed as input, which was in the form of 2^0 X 2^0 grid, in the two programs developed above. The Gauss coefficients for the potential were derived using the above two methods. Producing the vertical field anomaly maps and comparing the Gauss coefficients derived by the two programs did not reveal all the details and hence a correlation analysis was done using these coefficients. The correlation analysis was done following the definition by (Arkani-Hamed et al., 1994)

$$K(n) = \frac{\sum_{m=0}^{n}\left(G_n^m G_n^{m}{}' + H_n^m H_n^{m}{}'\right)}{\left[\sum_{m=0}^{n}\left(G_n^m G_n^m + H_n^m H_n^m\right)\sum_{m=0}^{n}\left(G_n^{m}{}' G_n^{m}{}' + H_n^{m}{}' H_n^{m}{}'\right)\right]^{1/2}} \tag{3.13}$$

The coefficients G_n^m and H_n^m are the Gauss coefficients for the potential derived after Nolte-Siebert method and $G_n^{m}{}'$ and $H_n^{m}{}'$ for the equivalent dipole method. The correlation coefficient curve is shown in Figure (3.3). The figure showed a very good correlation between the two sets of coefficients. This statistical analysis verified that nearly the same coefficients were produced by these methods. It is to be noted that the Nolte-Siebert method uses only degree 1-3 of the inducing main field while the equivalent dipole method uses all the degrees of the main field and hence, arguably the equivalent dipole method is more accurate than the Nolte-Siebert method. Later, the speed of the algorithms was also analysed. To produce the Gauss coefficients using the above programs, the equivalent dipole method took 8 hours while the Nolte-Siebert method took only 1 minute on a Dell Workstation (2.6 GHz Pentium 4) machine. On comparing the global magnetic anomaly maps produced by both the methods, a difference of 1.5 – 2 nT is observed. Over the poles, the amplitude of the anomaly computed using dipole method is less than computed using Nolte-Siebert method while it is greater over the equator. As the average magnetic crustal anomaly is approximately 2 – 4 nT, the difference in the two maps as mentioned above may be significant. Thus, even though all the initial computations were done using the Nolte-Siebert method, the final maps were prepared using the more accurate equivalent dipole method.

Fig. 3.3. Correlation coefficients between the Gauss coefficients derived using Nolte-Siebert and equivalent dipole method.



3.5 Computing vertical field anomaly

The spherical harmonic expansion of the magnetic crustal potential computed in the above sections can be written as

$$V(r,\theta,\phi) = a\sum_{n=1}^{n}\sum_{m=-n}^{n}\left(\frac{a}{r}\right)^{n+1} g_n^m Y_n^m(\theta,\phi) \tag{3.14}$$

where a is the mean earth radius (6371.2 km), and

$$Y_n^m(\theta,\phi) = \cos(m\phi)P_n^m(\cos\theta) \tag{3.15a}$$

$$Y_n^{-m}(\theta,\phi) = \sin(m\phi)P_n^m(\cos\theta) \tag{3.15b}$$

and g_n^m are the Gauss coefficients which are known from the above two methods, n is the degree and m is the order of the spherical harmonic expansion of the potential. The crustal magnetic field $\vec{B}$ is related to the potential by the expression

$$\vec{B}(r,\theta,\phi) = -\vec{\nabla}V(r,\theta,\phi) \tag{3.16}$$

then taking the gradients with respect to $r, \theta \text{ and } \phi$, we get the three components of the magnetic field as

$$B_r(r,\theta,\phi) = -\frac{\partial V}{\partial r} = \sum_{n=1}^{n} \sum_{m=-n}^{n} (n+1)\left(\frac{a}{r}\right)^{n+2} g_n^m Y_n^m(\theta,\phi) \tag{3.17}$$

$$B_\theta(r,\theta,\phi) = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\sum_{n=1}^{n} \sum_{m=-n}^{n} \left(\frac{a}{r}\right)^{n+2} g_n^m \cos(m\phi)\frac{\partial P_n^m(\cos\theta)}{\partial \theta} \tag{3.18}$$

$$B_\phi(r,\theta,\phi) = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} = \frac{1}{\sin\theta}\sum_{n=1}^{n} \sum_{m=-n}^{n} m\left(\frac{a}{r}\right)^{n+2} g_n^m Y_n^{-m}(\theta,\phi) \tag{3.19}$$

The z component of the crustal magnetic field is further related to the radial component of the crustal magnetic field as

$$B_z(r,\theta,\phi) = -B_r(r,\theta,\phi) = \frac{\partial V}{\partial r} = -\sum_{n=1}^{n} \sum_{m=-n}^{n} (n+1)\left(\frac{a}{r}\right)^{n+2} g_n^m Y_n^m(\theta,\phi) \tag{3.20}$$

Using the above equation and taking the Gauss coefficients derived above for the potential, we can get the z-component of the crustal magnetic anomaly field. The scalar anomaly field can be computed by taking the component of the residual field in the unit vector direction of the inducing main field.

Both the scalar anomaly and vertical field anomaly map is computed using the above methodology and is compared with the observed CHAMP magnetic field anomaly map. We have chosen to compare only the vertical component map for predicted and observed anomaly map. Unlike the scalar anomaly map, the vertical component on a spherical shell completely determines the magnetic potential of the field.

An algorithm is developed to compute the z component of the crustal anomaly field. The program takes the Gauss coefficients as the input and following the equations derived above in this section computes the vertical component (B_Z). To compute the vertical field anomaly of the VIS model, the magnetic potential is computed and spherical harmonic degrees 1-15 are set to zero because this long wavelength crustal field is masked by the main field and is not observable. Finally, only spherical harmonic degrees 16-80 of the model field are compared with the corresponding degrees 16-80 of the observed crustal field.