8 Conclusions

In this thesis, we investigated two classes of magnetotransport measurements in quantum Hall systems. Both occur within the regime of high Landau levels, i.e. at low magnetic fields. The first part of the present work (Chapters 2-5) is devoted to the study of microwave-irradiated high-mobility two-dimensional electron gases, while the second part (Chapters 6-7) deals with the interactions between the two parallel 2DEG layers of a bilayer system, more precisely the phonon-mediated frictional drag between electrons of both layers.

The remarkable discovery of zero resistance states (ZRS) in extremely clean GaAs/AlGaAs heterostructures subjected to microwave irradiation still remains only partially understood. While the underlying principle of negative conductivity and the basic microscopic mechanisms leading to this type of instability have been singled out in the literature on the subject, more recent experiments continue to yield new insights into the rich physics of this class of systems. In this thesis, we examined three such experiments:

- bichromatic microwave irradiation
- intra-LL effects
- application of an in-plane magnetic field

Within a classical model, we were able to predict interesting novel effects under bichromatic irradiation. In addition, we suggested a way to parametrically excite the cyclotron mode using bichromatic irradiation. Within the microscopic theory for ZRS, we then studied the effect of microwaves in the so-called intra-LL regime, i.e. of frequency smaller than the cyclotron frequency. We were able to explain (i) why no ZRS are observed experimentally in this regime and (ii) why the Shubnikov-deHaas oscillations are strongly suppressed in this regime. We also studied the effects of the application of an in-plane magnetic field, which, in experiment, was shown to suppress or even destroy the ZRS. We estimated the importance of Zeeman splitting for this suppression within a kinetic approach and predicted an interesting oscillatory dependence of the longitudinal conductivity with respect to magnetic field in extremely-high-purity systems.

The second part of this thesis was devoted to the study of frictional drag between parallel 2DEGs in high Landau levels. We extended the well-established linear response theory for Coulomb drag to the case of phonon drag and calculated the leading contribution to the phonon drag conductivity for the regime of well-separated Landau levels and low temperatures analytically. Unlike in the well-studied case of Coulomb drag, the phonon-mediated interaction between the two layers of the 2DEG does not only involve small momentum exchange but momentum exchange over the full range of momenta $0 \leq q \leq 2k_F$. This difference in the momentum dependence of the phonon-mediated interaction led us to expect a different scaling with temperature of the phonon contribution to the drag conductivity. We were able to show analytically that the temperature dependence of the phonon contribution to frictional drag deviates from the T^2-behavior observed and theoretically predicted for Coulomb drag. This renders a separation of both effects in actual experiments possible. Such experiments remain to be carried out in high Landau levels. The importance of the inclusion of phonon drag into a consistent explanation of the total measured drag resistivity was further supported by numerical calculations of the phonon drag resistivity going well beyond the analytically studied low-temperature regime. We studied the temperature dependence of the phonon drag resistivity for finite phonon mean free paths and examined its dependence on interlayer separation, Landau level broadening and layer filling. All numerical results were in perfect agreement with our expectations from the analytical calculations performed in the simplified picture of infinite phonon mean free path.

Certainly, both classes of 2DEG systems require further experimental and theoretical investigations. For the ZRS, the first experiments on bichromatic irradiation have been conducted very recently, yielding interesting new results [38]. A very interesting question is the topology of the current domains produced by the negative local conductivity. Also the dependence on microwave power remains to be understood.

For bilayer systems, a thorough analytical investigation would be highly desirable, since it is in principle feasible with the methods presented in this work. A systematic study of the different regimes and the scaling laws of the drag conductivity within these regimes remains to be undertaken. In addition, analytical studies of the filling-factor or electron-density dependence of the phonon contribution to drag remain to be carried out. Unlike in the case of Coulomb drag, however, we do not expect an oscillating sign of drag for the phonon contribution.

More generally, 2DEG systems can be expected to continue yielding insights about fundamental properties of the solid state. Very recently, for example, the isolated fabrication and subsequent examination of graphene – monolayers of graphite – has attracted much attention [102]. These hexagonal carbon layers exhibit remarkable properties: Due to its band structure, electrons in graphene behave as massless Dirac fermions. Roughly speaking, electrons in graphene thus act like relativistic particles that have no rest mass. This leads to a relativistic analog of the integer quantum Hall effect when graphene is subjected to a magnetic field.