5 In-Plane Magnetic Field

Very recently, the application of an additional in-plane magnetic field to a high-mobility 2DEG under microwave irradiation showing MIRO and/or ZRS has been investigated experimentally by Yang *et al.* [41]. The authors observe a pronounced suppression or even destruction of these phenomena for sufficiently strong parallel components of the magnetic field. For the ultraclean two-dimensional electron system in a AlGaAs/GaAs/AlGaAs square quantum well sample under microwave irradiation, the oscillatory photoconductivity and the ZRS are significantly suppressed already for moderate parallel components of the magnetic field, $B_{\parallel} \simeq 0.5$ T.

Fig. 5.1 shows several traces of the dark and microwave longitudinal resistivities at temperature $T \simeq 0.95$ K for different values of the in-plane field directed in x-direction of the sample. Initially, for $B_{\parallel} = 0$, up to four ZRS are observed at the minima of the oscillating resistivity. When increasing the in-plane field, the amplitude of the microwave-induced resistivity oscillations decreases quickly and, as a consequence, the width of the ZRS diminishes until they are suppressed at fields of the order of or higher than $B_{\parallel} \simeq 0.3$ T. In addition, when increasing the in-plane field further, the position of the oscillation maxima shifts to higher values of the perpendicular magnetic field $B_{\perp}$, while the position of the ZRS and the oscillation minima stays roughly constant. For in-plane fields of the order of $B_{\parallel} \simeq 0.75$ T, the oscillations in the resistivity are practically washed out and the behavior of the photoconductivity reverts to the behavior of the dark case. Qualitatively, these results prevail also for in-plane-fields in the y-direction as well as for different samples, in particular also for single-interface GaAs/AlGaAs heterostructures.

Until now, the spin degree of freedom of the electrons has been neglected in our considerations. Especially at lower magnetic fields, however, the electrons cannot be considered to be completely spin-polarized. In a finite magnetic field, each Landau level spin-splits into two levels separated by the Zeeman energy

$$\Delta E = g\mu_b|\mathbf{B}| \quad , \tag{5.1}$$

where g is the Landé g-factor and μ_b the Bohr magneton. In general, the filling factor ν therefore refers to the number of occupied spin-split LLs.[1]

The Zeeman splitting of the spin-resolved Landau levels depends linearly on the total magnetic field $B = \left(B_{\perp}^2 + B_{\parallel}^2\right)^{1/2}$ (see Eq. (5.1)). For fixed perpendicular magnetic field $B_{\perp}$, the application of an in-plane magnetic field therefore

[1] Interesting features can arise in 2D electron systems due to the spin degree of freedom. For instance at $\nu = 1$, the 2D electron system can be shown to assume a ferromagnetically ordered state with a spin-polarized ground state due to the Zeeman splitting of the lowest LL. The elementary excitations of this state are spin textures – the so-called Skyrmions [77] – rather than individual single-spin flips.

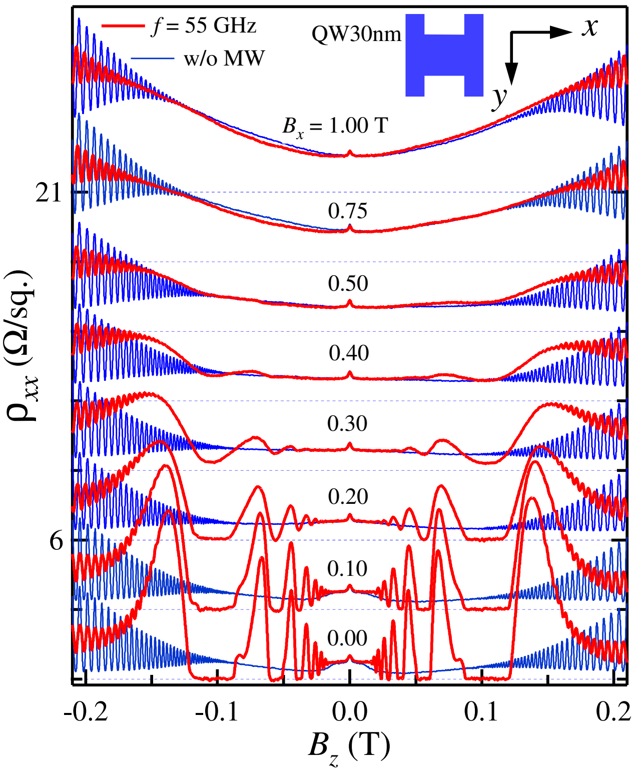

Figure 5.1: Experimental results for the diagonal resistivity ρ_{xx} as a function of the perpendicular magnetic field B_z for different in-plane magnetic fields B_x under irradiation with microwaves of frequency $f = 55$ GHz (thick lines) and, for comparison, without microwave irradiation (thin lines). This figure has been taken from Ref. [41].

increases the Zeeman splitting of the spin-resolved Landau levels. This fact could be responsible for the observed significant suppression of the microwave-induced effects with increasing amplitude of the in-plane magnetic field.

In what follows, we attempt to explain the experiment of Yang *et al.* [41] extending the approach of Ref. [50], based on an irradiation-induced change in the electronic distribution function, by including the spin degree of freedom and a phenomenological spin-relaxation time to account for the effect of a relative shift of the Landau bands for different spin orientations. Section 5.1 briefly recapitulates the approach of Ref. [50] without spin splitting, but is mainly devoted to our theory of the photoconductivity including the spin degree of freedom. We present the results for the photoconductivity in the presence of an in-plane field and discuss some limiting cases in Section 5.2. In Section 5.3, we comment on the experimental relevance of our results.

5.1 Theory

The energy of a 2D electron in a tilted magnetic field (i.e. a magnetic field having perpendicular *and* parallel components) is composed of the kinetic term,

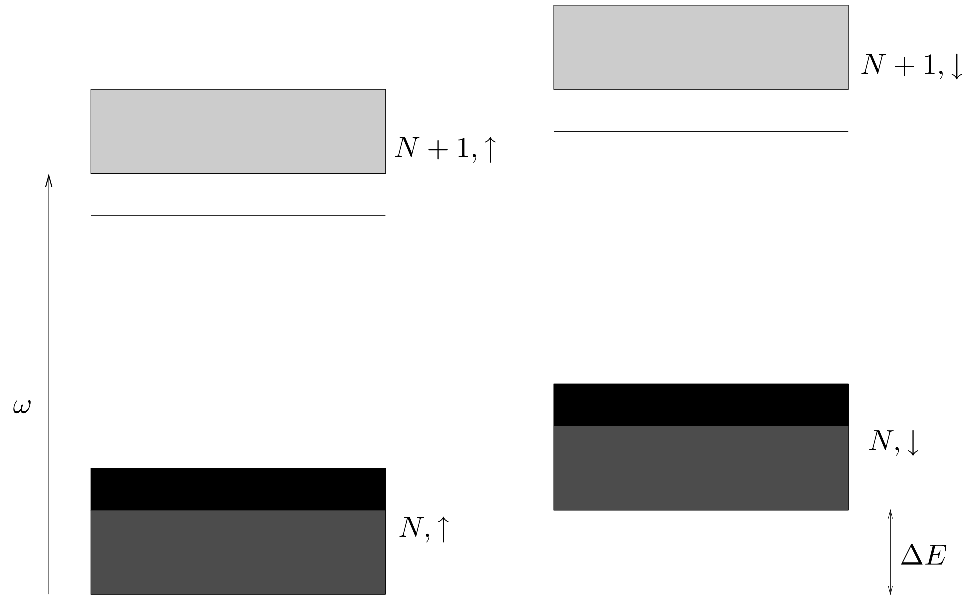

Figure 5.2: Spin-split landau levels in a driving microwave field. Due to the application of a sufficiently strong parallel magnetic field, the Zeeman splitting $\Delta E = g\mu_B B$ of the Landau bands for different spin indices can get comparable to the Landau level broadening (due to disorder or periodic modulation) and thus cannot be neglected. This figure schematically shows the behavior of the spin-split Landau bands of LL indices N and $N+1$ for spin up ($\uparrow$, left) and spin down ($\downarrow$, right) bands in a microwave field of frequency $\omega > \omega_c$. No relaxation between the bands has been included in the depicted case.

which is due to the perpendicular component $B_\perp$ leading to the cyclotron motion, and of the Zeeman term. Neglecting effects of finite layer thickness, the parallel component of the magnetic field, $B_\|$, only couples to the spin degree of freedom of the electrons, making it a powerful probe for spin-related phenomena in 2DEGs. The spin either aligns parallel or antiparallel with the total magnetic field $\mathbf{B}$, so that the energy of an electron of spin $\sigma = \pm 1/2$ is given by

$$E = \epsilon + \sigma g \mu_B B \quad , \tag{5.2}$$

where ϵ is the kinetic energy of the electron. Note that the spin gyromagnetic ratio g does not assume the the free-electron value $g \simeq 2$ but rather the value $g \simeq -0.44$ appropriate for the conduction electrons in GaAs. This value might be altered further by the application of the in-plane magnetic field, as was pointed out in Ref. [78].

Increasing the magnitude of the in-plane magnetic field increases the Zeeman splitting of the Landau bands. Since the perpendicular magnetic field is small ($B_\perp \lesssim 1$ T) in the regime of MIRO/ZRS, the Zeeman splitting is mostly due to the parallel field as long as $B_\| \gg B_\perp$. In general, spin splitting becomes important if the Zeeman energy is comparable to or larger than the Landau band broadening due to disorder or periodic modulation. A schematic view of spin-split Landau bands is provided in Fig. 5.2.

In the case of spin-resolved Landau levels, the total dc conductivity can be written as

$$\sigma = \int d\epsilon \left\{ \sigma_\uparrow(\epsilon) \left(-\frac{\partial f_\uparrow(\epsilon)}{\partial \epsilon} \right) + \sigma_\downarrow(\epsilon) \left(-\frac{\partial f_\downarrow(\epsilon)}{\partial \epsilon} \right) \right\} \quad , \tag{5.3}$$

where $f_\uparrow(\epsilon)$ ($f_\downarrow(\epsilon)$) is the distribution function for spin up (down) electrons and $\sigma_\uparrow(\epsilon)$ ($\sigma_\downarrow(\epsilon)$) determines the contribution of spin up (down) electrons of energy ϵ to dissipative transport. Although the irradiation-induced changes in the distribution functions can be small, the derivatives $\partial_\epsilon f_\sigma(\epsilon)$ appearing in Eq. (5.3) might be large if the irradiation-induced oscillations of the distribution function are fast enough.

In what follows, we generalize the calculation of Dmitriev and co-workers [50] to spin-split Landau bands including the in-plane magnetic field.

5.1.1 Without Spin Splitting

First, we quickly recapitulate the calculation without in-plane magnetic field, neglecting the Zeeman splitting due to the perpendicular magnetic field. Without dc field, one finds the kinetic equation

$$\gamma_\omega \sum_\pm \bar{\nu}(\epsilon \pm \omega) \left[f(\epsilon \pm \omega) - f(\epsilon) \right] = \frac{f(\epsilon) - f_T(\epsilon)}{\tau_{in}} \quad , \tag{5.4}$$

where $\gamma_\omega = \mathcal{P}_\omega / (4\tau_{in})$ is a measure of the strength of the ac field[2] and τ_{in} a phenomenological inelastic relaxation time [50]. The distribution function $f(\epsilon)$ in general deviates from the Fermi distribution $f_T(\epsilon)$ due to the driving microwave field. In presence of a dc field, there would be an additional term on the left-hand side of Eq (5.4) [50].

Assuming overlapping LLs, the normalized density of states $\bar{\nu}(\epsilon) = \nu(\epsilon)/\nu_0$ shows a weak cosine modulation

$$\bar{\nu}(\epsilon) = 1 - 2\eta \cos\left(\frac{2\pi\epsilon}{\omega_c} \right) \quad , \tag{5.5}$$

where

$$\eta = \exp\left(-\frac{\pi}{\omega_c \tau_s} \right) \ll 1$$

is a small parameter and τ_s is the zero-B single particle time.

The smallness of the parameter η enables us to search for a solution to the distribution function $f(\epsilon)$ in the form

$$f(\epsilon) = f_0(\epsilon) + f_{\text{osc.}}(\epsilon) + \mathcal{O}(\eta^2) \tag{5.6}$$

[2] The dimensionless measure of the strength of the microwave field is related to the amplitude $\mathcal{E}_\omega$ of the microwave field $E_\omega = \mathcal{E}_\omega \cos(\omega t)$ via

$$\mathcal{P}_\omega = \frac{\tau_{in}}{\tau_{tr}} \left(\frac{e\mathcal{E}_\omega v_F}{\omega} \right)^2 \frac{\omega_c^2 + \omega^2}{(\omega^2 - \omega_c^2)^2} \quad ,$$

where v_F is the Fermi velocity and ω the microwave frequency [50].

to first order in η (note that $f_{\rm osc.}$ is $\mathcal{O}(\eta^1)$). To zeroth order in η, the kinetic equation, Eq. (5.4), then reduces to

$$\gamma_\omega \sum_\pm [f_0(\epsilon \pm \omega) - f_0(\epsilon)] = \frac{f_0(\epsilon) - f_T(\epsilon)}{\tau_{in}} \quad . \tag{5.7}$$

If one now assumes that the microwave field is not too strong,

$$\mathcal{P}_\omega \left(\frac{\omega}{T}\right)^2 \ll 1 \quad ,$$

the difference $[f_0(\epsilon) - f_T(\epsilon)]$ is small, so that we can neglect the heating of electrons by the microwaves,

$$f_0(\epsilon) \simeq f_T(\epsilon) \quad , \tag{5.8}$$

i.e., to zeroth order in η, the distribution function is simply the Fermi distribution.

To first order in η, Eq. (5.4) yields

$$\begin{aligned} & \gamma_\omega \sum_\pm [f_{\rm osc.}(\epsilon \pm \omega) - f_{\rm osc.}(\epsilon)] - \frac{f_{\rm osc.}(\epsilon)}{\tau_{in}} \\ = \;& 2\eta\gamma_\omega \sum_\pm \cos\left(\frac{2\pi(\epsilon \pm \omega)}{\omega_c}\right) [f_0(\epsilon \pm \omega) - f_0(\epsilon)] \quad . \end{aligned} \tag{5.9}$$

Replacing $f_0(\epsilon)$ by the Fermi distribution $f_T(\epsilon)$ according to Eq. (5.8), we obtain

$$\begin{aligned} & \gamma_\omega \sum_\pm [f_{\rm osc.}(\epsilon \pm \omega) - f_{\rm osc.}(\epsilon)] - \frac{f_{\rm osc.}(\epsilon)}{\tau_{in}} \\ = \;& 2\eta\gamma_\omega \sum_\pm \cos\left(\frac{2\pi(\epsilon \pm \omega)}{\omega_c}\right) [f_T(\epsilon \pm \omega) - f_T(\epsilon)] \quad . \end{aligned} \tag{5.10}$$

In the limit $T \gg \omega_c$, the right-hand side of this expression can be simplified using

$$[f_T(\epsilon \pm \omega) - f_T(\epsilon)] \simeq (\pm\omega) \left(\frac{\partial f_T(\epsilon)}{\partial \epsilon}\right) \tag{5.11}$$

along with basic trigonometric identities. This yields

$$\begin{aligned} & 2\eta\gamma_\omega \sum_\pm \cos\left(\frac{2\pi(\epsilon \pm \omega)}{\omega_c}\right) [f_T(\epsilon \pm \omega) - f_T(\epsilon)] \\ = \;& 2\eta\gamma_\omega \sum_\pm \cos\left(\frac{2\pi(\epsilon \pm \omega)}{\omega_c}\right) (\pm\omega) \frac{\partial f_T(\epsilon)}{\partial \epsilon} \\ = \;& 2\eta\gamma_\omega\omega \frac{\partial f_T(\epsilon)}{\partial \epsilon} \left[\cos\left(\frac{2\pi(\epsilon + \omega)}{\omega_c}\right) - \cos\left(\frac{2\pi(\epsilon - \omega)}{\omega_c}\right)\right] \\ = \;& -4\eta\gamma_\omega\omega \frac{\partial f_T(\epsilon)}{\partial \epsilon} \sin\left(\frac{2\pi\epsilon}{\omega_c}\right) \sin\left(\frac{2\pi\omega}{\omega_c}\right) \quad . \end{aligned} \tag{5.12}$$

Thus, we obtain

$$
\begin{aligned}
&\gamma_\omega \sum_{\pm} \left[f_{\text{osc.}}(\epsilon \pm \omega) - f_{\text{osc.}}(\epsilon) \right] - \frac{f_{\text{osc.}}(\epsilon)}{\tau_{in}} \\
= \quad & -4\eta\gamma_\omega \omega \frac{\partial f_T(\epsilon)}{\partial \epsilon} \sin\left(\frac{2\pi\epsilon}{\omega_c}\right) \sin\left(\frac{2\pi\omega}{\omega_c}\right) \quad . \qquad (5.13)
\end{aligned}
$$

For $\mathcal{P}_\omega(\omega/T)^2 \ll 1$, the first term on the LHS is negligible, so that, in this limit,

$$
\begin{aligned}
f_{\text{osc.}}(\epsilon) \quad &= \quad 4\eta\gamma_\omega\omega\tau_{in} \frac{\partial f_T(\epsilon)}{\partial \epsilon} \sin\left(\frac{2\pi\epsilon}{\omega_c}\right) \sin\left(\frac{2\pi\omega}{\omega_c}\right) \\
&= \quad \eta\omega\mathcal{P}_\omega \frac{\partial f_T(\epsilon)}{\partial \epsilon} \sin\left(\frac{2\pi\epsilon}{\omega_c}\right) \sin\left(\frac{2\pi\omega}{\omega_c}\right) \quad . \qquad (5.14)
\end{aligned}
$$

In general, for $T \gg \omega$, the oscillatory contribution to the distribution function, $f_{\text{osc.}}(\epsilon)$ in Eq. (5.13), oscillates on the same scale as the density of states (i.e., with a period ω_c)

$$
f_{\text{osc.}}(\epsilon) = \eta \text{Re} \left[f_1(\epsilon) e^{i(2\pi\epsilon/\omega_c)} \right] \quad , \qquad (5.15)
$$

where $f_1(\epsilon)$ is a smoothly varying function which varies as a function of T and therefore much more slowly than the fast oscillations of period ω_c. Retaining the first term in the left-hand side of Eq. (5.13), we obtain the more general result [50]

$$
f_{\text{osc.}}(\epsilon) = \eta\mathcal{P}_\omega\omega \frac{\partial f_T(\epsilon)}{\partial \epsilon} \frac{\sin\left(\frac{2\pi\epsilon}{\omega_c}\right)\sin\left(\frac{2\pi\omega}{\omega_c}\right)}{1 + \mathcal{P}_\omega \sin^2 \frac{\pi\omega}{\omega_c}} \quad . \qquad (5.16)
$$

We now move on to a generalization of this calculation to the case where the Landau bands are spin-split due to the application of a parallel magnetic field.

5.1.2 With Spin Splitting

Including the spin splitting, the two equations for the two spin orientations in the absence of a dc field read

$$
\gamma_\omega \sum_{\sigma=\pm} \tilde{\nu}_\sigma(\epsilon \pm \omega) \left[f_\sigma(\epsilon \pm \omega) - f_\sigma(\epsilon) \right] = \frac{f_\sigma(\epsilon) - f_{\bar{\sigma}}(\epsilon)}{2\tau_{spin}} + \frac{f_\sigma(\epsilon) - f_T(\epsilon)}{\tau_{in}} \quad , \qquad (5.17)
$$

where $\sigma = \pm 1$ denotes the spin index[3] (and $\bar{\sigma} = -\sigma$) and τ_{in} a phenomenological inelastic relaxation time. The quantity τ_{spin} is a relaxation time describing the relaxation of electrons from a spin σ to a spin $\bar{\sigma}$ band. The factor of 1/2 in the first term of the right-hand side of Eq. (5.17) has been introduced for convenience. As above, the quantity γ_ω is a measure of the strength of the ac field.

The spin-dependent density of states and the spin-dependent distribution function can be written as

$$
\tilde{\nu}_\sigma(\epsilon) = \bar{\nu}(\epsilon) + \sigma\delta\tilde{\nu}(\epsilon) \quad , \qquad (5.18)
$$

[3]The actual spin projection of the electrons thereby being $\sigma/2$.

$$f_\sigma(\epsilon) = f(\epsilon) + \sigma \delta f(\epsilon) \quad . \tag{5.19}$$

By adding and subtracting the equations for spin up and spin down in Eq. (5.17) and using Eqs. (5.18-5.19), we arrive at

$$\begin{aligned} & \gamma_\omega \sum_\pm \Big\{ \bar{\nu}(\epsilon \pm \omega) \left[f(\epsilon \pm \omega) - f(\epsilon) \right] + \delta\tilde{\nu}(\epsilon \pm \omega) \left[\delta f(\epsilon \pm \omega) - \delta f(\epsilon) \right] \Big\} \\ = & \ \frac{f(\epsilon) - f_T(\epsilon)}{\tau_{in}} \quad , \end{aligned} \tag{5.20}$$

$$\begin{aligned} & \gamma_\omega \sum_\pm \Big\{ \bar{\nu}(\epsilon \pm \omega) \left[\delta f(\epsilon \pm \omega) - \delta f(\epsilon) \right] + \delta\tilde{\nu}(\epsilon \pm \omega) \left[f(\epsilon \pm \omega) - f(\epsilon) \right] \Big\} \\ = & \ \frac{\tau_{spin} + \tau_{in}}{\tau_{spin}\tau_{in}} \, \delta f(\epsilon) \quad . \end{aligned} \tag{5.21}$$

In order to be able to discuss Eqs. (5.20-5.21) order by order in η, we now express $\bar{\nu}(\epsilon)$ and $\delta\tilde{\nu}(\epsilon)$ from Eq. (5.18) as series in η

$$\bar{\nu}(\epsilon) = \bar{\nu}_0(\epsilon) + \bar{\nu}_{\text{osc.}}(\epsilon) + \mathcal{O}(\eta^2) \quad , \tag{5.22}$$

$$\delta\tilde{\nu}(\epsilon) = \delta\tilde{\nu}_0(\epsilon) + \delta\tilde{\nu}_{\text{osc.}}(\epsilon) + \mathcal{O}(\eta^2) \quad , \tag{5.23}$$

where $\bar{\nu}_{\text{osc.}}(\epsilon)$ and $\delta\tilde{\nu}_{\text{osc.}}(\epsilon)$ are $\mathcal{O}(\eta^1)$. We find

$$\begin{aligned} \bar{\nu}(\epsilon) &= \frac{1}{2} \left[\tilde{\nu} \left(\epsilon + \frac{1}{2} g\mu_B B \right) + \tilde{\nu} \left(\epsilon - \frac{1}{2} g\mu_B B \right) \right] \\ &= \frac{1}{2} \left[2 - 2\eta \cos \left(\frac{2\pi \left(\epsilon + \frac{1}{2} g\mu_B B \right)}{\omega_c} \right) - 2\eta \cos \left(\frac{2\pi \left(\epsilon - \frac{1}{2} g\mu_B B \right)}{\omega_c} \right) \right] \\ &= 1 - 2\eta \cos \left(\frac{2\pi\epsilon}{\omega_c} \right) \cos \left(\frac{\pi g \mu_B B}{\omega_c} \right) \quad , \end{aligned} \tag{5.24}$$

and

$$\begin{aligned} \delta\tilde{\nu}(\epsilon) &= \frac{1}{2} \left[\tilde{\nu} \left(\epsilon + \frac{1}{2} g\mu_B B \right) - \tilde{\nu} \left(\epsilon - \frac{1}{2} g\mu_B B \right) \right] \\ &= \eta \left[\cos \left(\frac{2\pi \left(\epsilon - \frac{1}{2} g\mu_B B \right)}{\omega_c} \right) - \cos \left(\frac{2\pi \left(\epsilon + \frac{1}{2} g\mu_B B \right)}{\omega_c} \right) \right] \\ &= 2\eta \sin \left(\frac{2\pi\epsilon}{\omega_c} \right) \sin \left(\frac{\pi g \mu_B B}{\omega_c} \right) \quad . \end{aligned} \tag{5.25}$$

From Eq. (5.25), one immediately concludes that there is no contribution to $\delta\tilde{\nu}(\epsilon)$ to first order in η, i.e. that $\delta\tilde{\nu}_0(\epsilon) = 0$

We first address Eq. (5.20), in which τ_{spin} drops out. Again, the density of states and the distribution function are expanded in the small parameter η as for the case without spin. To zeroth order, all terms containing $\delta\tilde{\nu}$ drop out due to the above-said and we find

$$\gamma_\omega \sum_\pm \left[f_0(\epsilon \pm \omega) - f_0(\epsilon) \right] = \frac{f_0(\epsilon) - f_T(\epsilon)}{\tau_{in}} \quad . \tag{5.26}$$

If the microwave field is sufficiently weak, this again leads to

$$f_0(\epsilon) \simeq f_T(\epsilon) \quad . \tag{5.27}$$

The second equation, Eq. (5.21), yields to zeroth order in η (again terms containing $\delta\tilde{\nu}$ drop out)

$$\gamma_\omega \sum_\pm [\delta f_0(\epsilon \pm \omega) - \delta f_0(\epsilon)] = \frac{\tau_{spin} + \tau_{in}}{\tau_{spin}\tau_{in}} \delta f_0(\epsilon) \quad , \tag{5.28}$$

so that for $T \gg \omega$,

$$\gamma_\omega \omega^2 \frac{\partial^2 \delta f_0(\epsilon)}{\partial \epsilon^2} = \frac{\tau_{spin} + \tau_{in}}{\tau_{spin}\tau_{in}} \delta f_0(\epsilon) \quad . \tag{5.29}$$

In keeping with the assumption of weak microwave fields, $\mathcal{P}_\omega(\omega/T)^2 \ll 1$, this leads to

$$\delta f_0(\epsilon) \simeq 0 \quad , \tag{5.30}$$

so that, to zeroth order in η, the distribution function is simply given by the Fermi distribution $f_T(\epsilon)$.

We now turn to the first order in the small parameter η. The two equations, Eq. (5.20) and Eq. (5.21), then yield

$$\begin{aligned} & \gamma_\omega \sum_\pm [f_{\text{osc.}}(\epsilon \pm \omega) - f_{\text{osc.}}(\epsilon)] \\ & -\gamma_\omega \sum_\pm 2\eta \cos\left(\frac{2\pi(\epsilon \pm \omega)}{\omega_c}\right) \cos\left(\frac{\pi g \mu_B B}{\omega_c}\right) [f_T(\epsilon \pm \omega) - f_T(\epsilon)] \\ = & \frac{f_{\text{osc.}}(\epsilon)}{\tau_{in}} \quad , \end{aligned} \tag{5.31}$$

and

$$\begin{aligned} & \gamma_\omega \sum_\pm [\delta f_{\text{osc.}}(\epsilon \pm \omega) - \delta f_{\text{osc.}}(\epsilon)] \\ & +\gamma_\omega \sum_\pm 2\eta \sin\left(\frac{2\pi(\epsilon \pm \omega)}{\omega_c}\right) \sin\left(\frac{\pi g \mu_B B}{\omega_c}\right) [f_T(\epsilon \pm \omega) - f_T(\epsilon)] \\ = & \frac{\tau_{spin} + \tau_{in}}{\tau_{spin}\tau_{in}} \delta f_{\text{osc.}}(\epsilon) \quad . \end{aligned} \tag{5.32}$$

Assuming again $\mathcal{P}_\omega(\omega/T)^2 \ll 1$, we now discuss the equation for the oscillatory contribution to the distribution function, $f_{\text{osc.}}(\epsilon)$ (Eq. (5.31)). Neglecting the first term on the left-hand side, it reads

$$-\gamma_\omega \sum_\pm 2\eta \cos\frac{2\pi(\epsilon \pm \omega)}{\omega_c} \cos\frac{2\pi g \mu_B B}{\omega_c} [f_T(\epsilon \pm \omega) - f_T(\epsilon)] = \frac{f_{\text{osc.}}(\epsilon)}{\tau_{in}} \quad . \tag{5.33}$$

Using Eq. (5.11), valid in the limit $T \gg \omega$, this yields

$$\begin{aligned} f_{\text{osc.}}(\epsilon) &= \frac{\eta\omega\mathcal{P}_\omega}{2} \frac{\partial f_T(\epsilon)}{\partial \epsilon} \cos\left(\frac{\pi g \mu_B B}{\omega_c}\right) \left[\cos\left(\frac{2\pi\,(\epsilon - \omega)}{\omega_c}\right) - \cos\left(\frac{2\pi\,(\epsilon + \omega)}{\omega_c}\right)\right] \\ &= \eta\omega\mathcal{P}_\omega \frac{\partial f_T(\epsilon)}{\partial \epsilon} \cos\left(\frac{\pi g \mu_B B}{\omega_c}\right) \sin\left(\frac{2\pi\epsilon}{\omega_c}\right) \sin\left(\frac{2\pi\omega}{\omega_c}\right) \quad . \end{aligned} \tag{5.34}$$

Comparing this result[4] to Eq. (5.14) for the case without spin splitting, we see that it differs by a factor $\cos(\pi g\mu_B B/\omega_c)$ oscillating with magnetic field. Therefore, the magnetic field modulates the strength of the oscillatory contribution to the electron distribution function. For

$$B = \left(\frac{2k+1}{2}\right)\frac{\omega_c}{g\mu_B} \qquad (k = 0, \pm 1, \pm 2, ...) \quad , \tag{5.35}$$

we find that $f_{\text{osc.}}(\epsilon) = 0$. For these values of magnetic field, there is no oscillatory contribution to the spin-independent part of the electronic distribution function. As we will see below, the photoconductivity is due to the oscillatory part of the distribution function. This suggests a suppression of the photoconductivity in the vicinity of magnetic fields fulfilling Eq. (5.35). However, there should be a reappearance of the photoconductivity due to the fact that the dependence of the distribution function on magnetic field is oscillatory. In addition, the behavior of the spin-dependent part of the oscillatory contribution to the distribution function is different from the behavior of the spin-independent part discussed above. Therefore, only a study of both contributions together will reveal the exact B-dependence of the photoconductivity.

Not neglecting the first term on the LHS of Eq. (5.31), we find that essentially the same happens as in Eq. (5.16) for the case without spin splitting. We obtain

$$f_{\text{osc.}}(\epsilon) \;=\; \eta\omega\mathcal{P}_\omega \frac{\partial f_T(\epsilon)}{\partial\epsilon}\cos\left(\frac{\pi g\mu_B B}{\omega_c}\right)\frac{\sin\left(\frac{2\pi\epsilon}{\omega_c}\right)\sin\left(\frac{2\pi\omega}{\omega_c}\right)}{1+\mathcal{P}_\omega\sin^2\left(\frac{\pi\omega}{\omega_c}\right)} \quad . \tag{5.36}$$

Thus, the periodicity in B persists also in this case and only the magnitude of the oscillations of the distribution function is changed through microwave intensity and frequency.

Finally, we examine Eq. (5.32). This equation contains the spin relaxation which couples the two spin-split Landau bands. Again assuming a weak microwave field, we neglect the first term on the left-hand side of Eq. (5.32), which then reads

$$\begin{aligned} &\gamma_\omega \sum_{\pm} 2\eta \sin\left(\frac{2\pi(\epsilon\pm\omega)}{\omega_c}\right)\sin\left(\frac{\pi g\mu_B B}{\omega_c}\right)[f_T(\epsilon\pm\omega) - f_T(\epsilon)] \\ =\; &\frac{\tau_{spin}+\tau_{in}}{\tau_{spin}\tau_{in}}\delta f_{\text{osc.}}(\epsilon) \quad . \end{aligned} \tag{5.37}$$

We again use Eq. (5.11), valid for $T \gg \omega_c$, and find

$$\begin{aligned} &\delta f_{\text{osc.}}(\epsilon) \\ =\; &\frac{\tau_{spin}}{\tau_{spin}+\tau_{in}}\eta\omega\mathcal{P}_\omega\frac{\partial f_T(\epsilon)}{\partial\epsilon}\sin\left(\frac{\pi g\mu_B B}{\omega_c}\right)\cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\sin\left(\frac{\pi\omega}{\omega_c}\right) \end{aligned} \tag{5.38}$$

[4] The factor $\cos(\pi g\mu_B B/\omega_c)$ can be understood as a correction to the factor $\sin\frac{2\pi\epsilon}{\omega_c}$ (which reflects the oscillations due to the DOS) since $2\sin x\cos y = \sin(x-y)+\sin(x+y)$, the two sines at the RHS representing the individual contributions of the two shifted bands which are merely added in the case under study since there is no coupling between the bands. The fact that the oscillatory component vanishes is thus simply the exact annihilation of the contribution of the spin up band by the contribution of the spin down band.

For the more general case, i.e. not neglecting the first term on the left-hand side of Eq. (5.32), we obtain

$$\delta f_{\text{osc.}}(\epsilon) = \frac{\tau_{spin}}{\tau_{spin}+\tau_{in}}\eta\omega\mathcal{P}_\omega \frac{\partial f_T(\epsilon)}{\partial\epsilon}\sin\left(\frac{\pi g\mu_B B}{\omega_c}\right)\frac{\cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\sin\left(\frac{\pi\omega}{\omega_c}\right)}{1+\mathcal{P}_\omega\sin^2\left(\frac{\pi\omega}{\omega_c}\right)}. \quad (5.39)$$

5.2 Results

5.2.1 Oscillatory Photoconductivity

As we have shown above, in the presence of an in-plane magnetic field, the relative Zeeman shift of the Landau bands for different spins induces a change in the density of states and the distribution function of the system under irradiation. The photoconductivity – defined here as the correction to the dark conductivity arising from the presence of the microwaves and not as the total conductivity under irradiation – is then due to the oscillatory part of the distribution function

$$f_\sigma(\epsilon) - f_T(\epsilon) = f_{\text{osc.}}(\epsilon) + \sigma\delta f_{\text{osc.}}(\epsilon) \quad . \quad (5.40)$$

Due to Eq. (5.30) there is no spin-dependent correction to the non-oscillatory part of the distribution function, which under reasonable assumptions, as shown above, is simply the Fermi distribution $f_T(\epsilon)$. The photoconductivity is then given by

$$\begin{aligned}\sigma_{ph} &= \int d\epsilon\left(-\frac{\partial f_{\text{osc.}}(\epsilon)}{\partial\epsilon}\right)[\sigma_\uparrow(\epsilon)+\sigma_\downarrow(\epsilon)] \\ &\quad +\int d\epsilon\left(-\frac{\partial\delta f_{\text{osc.}}(\epsilon)}{\partial\epsilon}\right)[\sigma_\uparrow(\epsilon)-\sigma_\downarrow(\epsilon)] \quad , \end{aligned} \quad (5.41)$$

where, for $\omega_c\tau_{tr} \gg 1$,

$$\sigma_{\uparrow,\downarrow}(\epsilon) = \sigma_D\tilde{\nu}^2_{\uparrow,\downarrow}(\epsilon) \quad , \quad (5.42)$$

σ_D being the Drude conductivity. The density of states $\tilde{\nu}_\sigma(\epsilon)$ is given by

$$\begin{aligned}\tilde{\nu}_\sigma(\epsilon) &= \bar{\nu}(\epsilon)+\sigma\delta\tilde{\nu}(\epsilon) \\ &= 1-2\eta\cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\cos\left(\frac{\pi g\mu_B B}{\omega_c}\right) \\ &\quad +2\sigma\eta\sin\left(\frac{2\pi\epsilon}{\omega_c}\right)\sin\left(\frac{\pi g\mu_B B}{\omega_c}\right)\end{aligned} \quad (5.43)$$

Motivated by experiment, we assume that the temperature T is much larger than the Dingle temperature

$$T \gg T_D = \frac{1}{2\pi\tau_s} \quad . \quad (5.44)$$

In this limit, the terms linear in η in the equation for the photoconductivity, Eq. (5.41), are exponentially suppressed,

$$\eta \int d\epsilon \cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\left[\partial_\epsilon f_T(\epsilon)\right] \propto \eta \exp\left(\frac{-2\pi^2 T}{\omega_c}\right) \ll \eta^2 \quad , \tag{5.45}$$

and can be neglected [50]. The leading ω-dependent contribution to σ_{ph} therefore is due to the $\mathcal{O}(\eta^2)$-term which is generated by the product of the $\mathcal{O}(\eta^1)$ oscillatory part of $\tilde{\nu}^2_{\uparrow,\downarrow}(\epsilon)$ and the $\mathcal{O}(\eta^1)$-contribution to $-\partial_\epsilon f_{\text{osc.}}(\epsilon)$ or $-\partial_\epsilon \delta f_{\text{osc.}}(\epsilon)$, respectively.

Neglecting terms of order $\mathcal{O}(\eta^0)$ due to the above said, we find to first order in η

$$\sigma_\uparrow(\epsilon) + \sigma_\downarrow(\epsilon) = -8\eta\sigma_D \cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\cos\left(\frac{\pi g \mu_B B}{\omega_c}\right) \tag{5.46}$$

$$\sigma_\uparrow(\epsilon) - \sigma_\downarrow(\epsilon) = +8\eta\sigma_D \sin\left(\frac{2\pi\epsilon}{\omega_c}\right)\sin\left(\frac{\pi g \mu_B B}{\omega_c}\right) \tag{5.47}$$

The oscillating part of the distribution function contains slow oscillations due to the term $\partial_\epsilon f_T(\epsilon)$ and fast oscillations due to the terms $\cos(2\pi\epsilon/\omega_c)$ or $\sin(2\pi\epsilon/\omega_c)$. The derivatives with respect to ϵ of $f_{\text{osc.}}(\epsilon)$ and $\delta f_{\text{osc.}}(\epsilon)$ are thus given to first order in η by

$$\begin{aligned}\partial_\epsilon f_{\text{osc.}}(\epsilon) \quad &\simeq \quad \eta\frac{2\pi\omega}{\omega_c}\mathcal{P}_\omega\left[\partial_\epsilon f_T(\epsilon)\right] \\ &\quad \times \cos\left(\frac{\pi g\mu_B B}{\omega_c}\right)\sin\left(\frac{2\pi\omega}{\omega_c}\right)\cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\end{aligned} \tag{5.48}$$

and

$$\begin{aligned}\partial_\epsilon \delta f_{\text{osc.}}(\epsilon) \quad &\simeq \quad -\eta\frac{\tau_{spin}}{\tau_{spin}+\tau_{in}}\frac{2\pi\omega}{\omega_c}\mathcal{P}_\omega\left[\partial_\epsilon f_T(\epsilon)\right] \\ &\quad \times \sin\left(\frac{\pi g\mu_B B}{\omega_c}\right)\sin\left(\frac{2\pi\omega}{\omega_c}\right)\sin\left(\frac{2\pi\epsilon}{\omega_c}\right) \quad .\end{aligned} \tag{5.49}$$

Inserting these dominant contributions into Eq. (5.41), we find

$$\begin{aligned}\frac{\sigma_{ph}}{\sigma_D} \quad &\simeq \quad 8\eta^2\mathcal{P}_\omega\frac{2\pi\omega}{\omega_c}\sin\left(\frac{2\pi\omega}{\omega_c}\right) \\ &\quad \times\left\{\cos^2\left(\frac{\pi g\mu_B B}{\omega_c}\right)\int d\epsilon\left[\partial_\epsilon f_T(\epsilon)\right]\cos^2\left(\frac{2\pi\epsilon}{\omega_c}\right)\right. \\ &\quad \left.+\frac{\tau_{spin}}{\tau_{spin}+\tau_{in}}\sin^2\left(\frac{\pi g\mu_B B}{\omega_c}\right)\int d\epsilon\left[\partial_\epsilon f_T(\epsilon)\right]\sin^2\left(\frac{2\pi\epsilon}{\omega_c}\right)\right\}.\end{aligned} \tag{5.50}$$

Finally performing the energy averaging

$$-\int d\epsilon \cos^2\left(\frac{2\pi\epsilon}{\omega_c}\right)\left[\partial_\epsilon f_T(\epsilon)\right] \simeq \frac{1}{2} \quad , \tag{5.51}$$

we find the following general expression for the photoconductivity

$$\begin{aligned}\frac{\sigma_{ph}}{\sigma_D} &\simeq -4\eta^2\mathcal{P}_\omega\frac{2\pi\omega}{\omega_c}\sin\left(\frac{2\pi\omega}{\omega_c}\right)\\ &\times\left[\cos^2\left(\frac{\pi g\mu_B B}{\omega_c}\right)+\frac{\tau_{spin}}{\tau_{spin}+\tau_{in}}\sin^2\left(\frac{\pi g\mu_B B}{\omega_c}\right)\right] \quad . \end{aligned}\tag{5.52}$$

5.2.2 Limiting Cases

In this section, we discuss some limiting cases of Eq. (5.52). In the absence of a parallel magnetic field, Eq. (5.52) reduces to

$$\frac{\sigma_{ph}}{\sigma_D}\simeq -4\eta^2\mathcal{P}_\omega\frac{2\pi\omega}{\omega_c}\sin\left(\frac{2\pi\omega}{\omega_c}\right) \quad , \tag{5.53}$$

which is in perfect agreement with the results of Ref. [50] for the case without spin-splitting.[5]

Next, we address the influence of the spin relaxation on the photoconductivity. In the absence of spin relaxation ($\tau_{spin}\to\infty$) and at finite B, there is no difference to the case without spin splitting

$$\left[\frac{\sigma_{ph}}{\sigma_D}\right]_{\tau_{spin}\to\infty}\simeq -4\eta^2\mathcal{P}_\omega\frac{2\pi\omega}{\omega_c}\sin\left(\frac{2\pi\omega}{\omega_c}\right) \quad . \tag{5.54}$$

On the other hand, for perfect spin relaxation ($\tau_{spin}\to 0$), we obtain at finite magnetic field

$$\left[\frac{\sigma_{ph}}{\sigma_D}\right]_{\tau_{spin}\to 0}\simeq -4\eta^2\mathcal{P}_\omega\frac{2\pi\omega}{\omega_c}\sin\left(\frac{2\pi\omega}{\omega_c}\right)\cos^2\left(\frac{\pi g\mu_B B}{\omega_c}\right) \quad . \tag{5.55}$$

This suggests that a dependence of the photoconductivity σ_{ph} on the (in-plane) magnetic field, as it is observed in experiment, can only be explained within our model if some mechanism assures at least some amount of spin relaxation. If we assume perfect spin relaxation, the B-dependence is governed by the factor $\cos^2(\pi g\mu_B B/\omega_c)$. The photoconductivity then vanishes at the magnetic fields given by Eq. (5.35) and is oscillatory in B, i.e. reappears at higher fields. This reappearance has not been observed in experiment. For finite spin relaxation, there can still be a considerable suppression of the photoconductivity, but no complete annihilation. The position of the minima in the photoconductivity are then also given by Eq. (5.35).

5.3 Discussion

If we include spin relaxation, our model predicts a periodic dependence of the photoconductivity on the total applied magnetic field whereas without inclusion

[5] Note that our result differs from the result obtained in Ref. [50] by a factor of two. This is due to the fact that we did not calculate the conductivity *per spin* but the total photoconductivity for both spin indices.

of spin relaxation, we are not able to derive such an effect. Experimentally, a microwave-driven 2DEG in the presence of an additional in-plane magnetic field does not show a periodic behavior of the photoconductivity but rather a suppression with increasing in-plane magnetic field and *no* reappearance of the photoconductivity at higher fields. This suggests that the spin-splitting due to the in-plane field does not explain the observations of Ref. [41].

Most likely, the suppression observed by Yang *et al.* is due to an additional effect: Due to the in-plane magnetic field, the electrons are pushed closer towards the interface, which leads to an increase of scattering by interface imperfections. This increase suppresses the zero resistance states due to the exponential sensitivity to the single-particle time τ_s. Experimentally, this is reflected in an increase of ρ_{xx} in the presence of a parallel magnetic field for the case without perpendicular magnetic field ($B_\perp = 0$).

Under more favorable conditions, the periodic effects predicted in this chapter as a consequence of the removal of spin degeneracy should nevertheless be observable in experiment. To this end, the influence of interface imperfections would have to be reduced considerably.