Chapter 4

Geometrical Modeling

The prerequisite for the adequate numerical simulation of soft tissue deformations in the surgical planning is the correct reflection of individual anatomy in geometrical models. In this chapter, we describe the major steps needed to create a useful volumetric model of the patient's anatomy from tomographic datasets. All artificial objects as well as 3D human models presented in this work have been created with the help of the multipurpose visualization and modeling system *Amira* [112].

4.1 Image Segmentation

The 3D digital models of human anatomy are available as a set of 2D images generated with the help of computer tomography (CT) or magnetic resonance imaging (MRI). The first step of the tomographic data processing is the import of medical image data. A typical CT slice is shown in Figure 4.1. The grey scale value of each pixel, the so-called *Hounsfield unit* (HU) of CT images is correlated with the intensity of X-ray scattering on different materials and differs from one tissue type to the other, which enables the identification of different anatomical structures and tissue regions, i.e. *image segmentation*. The task of medical image segmentation is the classification of different grey scale regions in accordance with their anatomical meaning. Since different morphological structures do not have a unique HU value, completely automatic segmentation of complex medical images is difficult and needs usually to be performed interactively with the help of computer graphic tools, which enable semi-automatic detection and marking of different HU subregions. In Table 4.1, the approximate range of HU values corresponding to the different materials and tissue types is shown. The precise values depend on

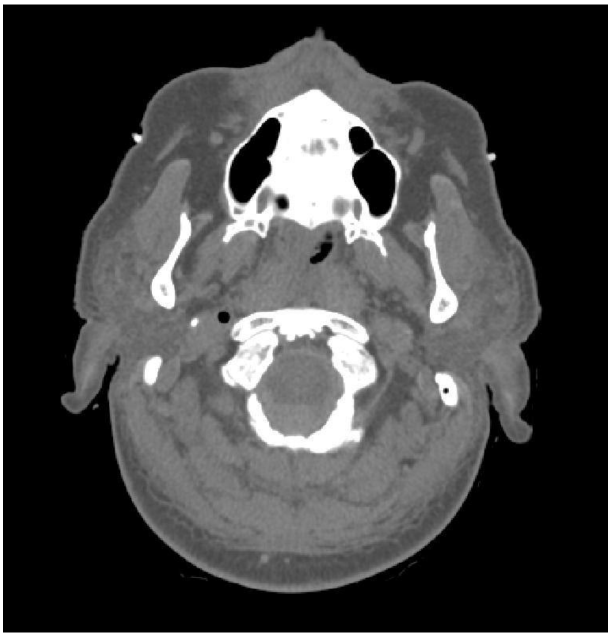

Figure 4.1: Typical CT slice.

Table 4.1: CT number range in HU.

Tissue	HU values
Air	-1000
Lungs	-1000 to -400
Fat	-100 to -50
Water	0
Brain	0 to 100
Muscle	10 to 600
Soft tissue	-100 to 300
Bone	$>$500

- the operating conditions of the CT scanner in terms of kVp and filtration,
- the concrete patient,
- noise in the image with a standard deviation in the range ± 3 to ± 30 HU,
- any artifacts within the CT image, e.g., beam hardening,

and require an individual analysis and justification. Once the range of HU values for different tissue types is defined, a preliminary automatic detection of substructures and their boundaries can be applied, see Figure 4.2(top). Morphological operators used for automatic segmentation are generally based on the recognition of grey scale differences along the boundaries between the different HU regions [60, 107]. Images containing thin or small structures, artifacts or simply diffuse images can be sometimes segmented only manually and require substantial timing expenses.

In order to model and to simulate the biomechanic behavior of complex living structures, a range of properties and attributes varying from tissue to tissue and from element to element has to be taken into consideration. Most of these characteristics assigned already during the segmentation stage are further reflected in the hierarchically structured geometrical model. A final geometrical model basically contains lists of nodes, surface and volumetric elements, as well as the additional lists of id's associated with these basic structures. Typically, the particular tissue type (or *material id* associated with tetrahedra) and the conectivity between the different tissue subregions (*boundary id* associated with triangles) are required for modeling of the multi-composite tissue. Optionally, tetrahedron-, triangle- or node-structures can be extended if further attributes are available such as the HU value on elements or nodes, for example.

4.2 Mesh Generation

With the digital tomographic data a natural discretization of continuous bodies is given, see Figure 4.2 (bottom,left). However, the representation of 3D objects as a set of 2D images consisting of *pixels* or corresponding 3D box-shaped elements, *voxels*, requires substantial computational resources and is not optimal for the visualization and the numerical simulation. On the one hand, natural volumetric voxel-based meshes can be easily parameterized and practically do not require any additional expenses for mesh generation. On the other hand, such volumetric grids consisting of billions of nodes represent the huge amount of data, which nowadays require the usage of massively parallel super-computers in order to perform finite element calculations, and thus are not suited for interactive simulations in an ordinary clinical environment.

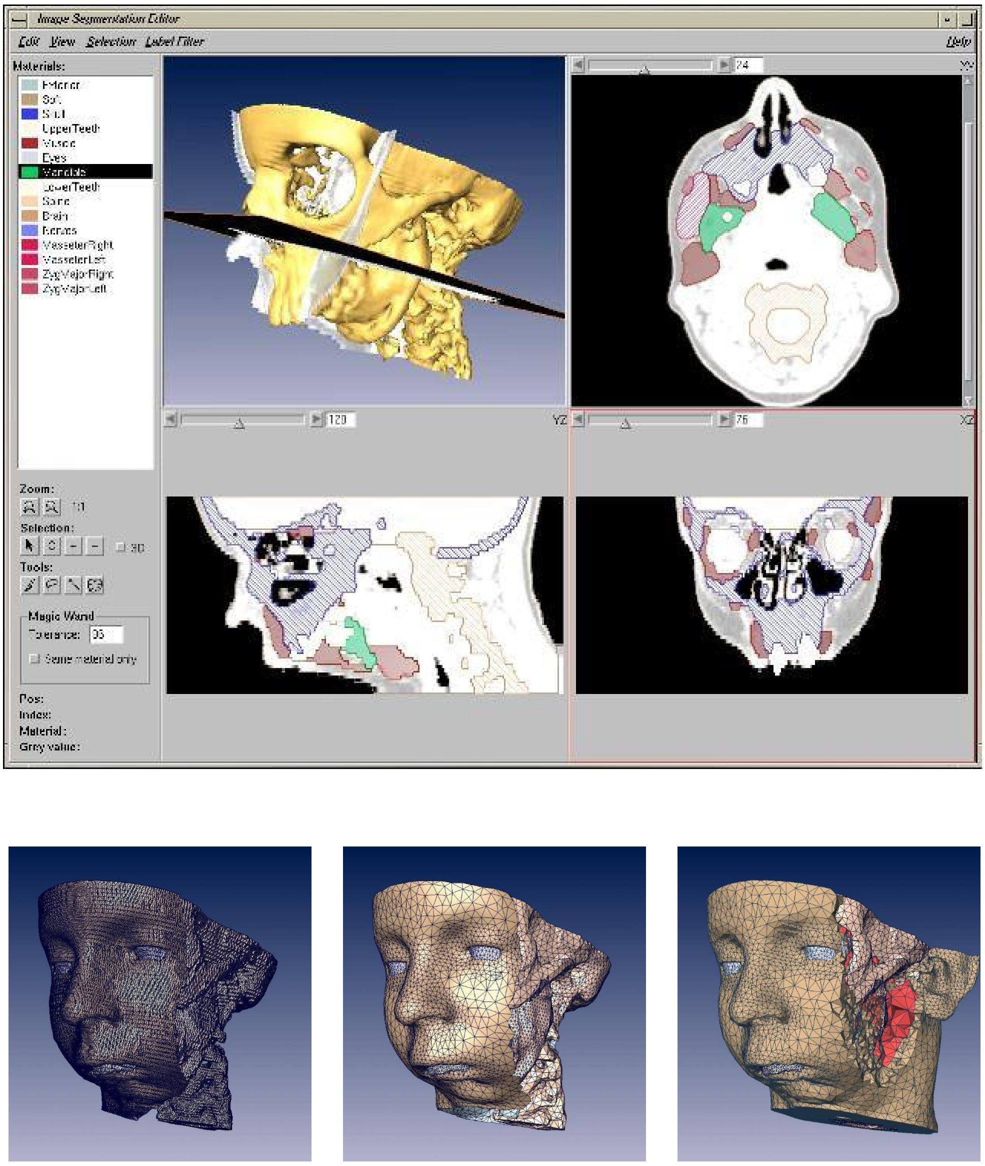

Figure 4.2: Top: image segmentation with Amira. Bottom: voxel-based discretization (left), simplified surface model (middle), resulting tetrahedral mesh (right) (from [119]).

A state of the art approach for the geometrical modeling is the generation of surface models and is known as *triangulation*. Triangulated surfaces represent the boundaries between the different tissue regions. Surface models substantially reduce the original data amount and enable a compact representation of complex geometrical structures, see Figure 4.2 (bottom,middle). In this work, the creation of surface models with correct topology and optimized triangular shape from the segmented tomographic data is carried out automatically with the help of the *marching cubes* related algorithm provided with *Amira* [55]. Adaptive refinement and coarsing of surface meshes is used to achieve an optimal resolution of curved regions (for example in mouth, eye or nose areas) and at the same time to keep the total number of elements as low as possible. For the numerical simulation of continuum mechanics problems via the finite element method, a volumetric mesh is needed. Meshes can be categorized as structured or unstructured [80]. Structured meshes exhibit uniform topological structure that unstructured meshes lack. In this work, an unstructured tetrahedral mesh generated on the basis of the non-manifold surface triangulation is used, see Figure 4.2 (bottom,right). The generation of the tetrahedral meshes for an arbitrary 3D domain of interest is generally a difficult task. A useful mesh has to satisfy constraints that sometimes seem almost contradictory. 3D mesh must conform to the object or domain being modeled, and ideally should meet constraints on both the size and shape of its elements. The most popular approaches for tetrahedral mesh generation can be divided into three groups: *Delaunay triangulation* [45, 111], *advancing front method* [43], and methods based on grids, quadtrees or octrees. Tetrahedral meshes used in this work are generated with the advancing front method. The starting point of advancing front algorithm is the triangulated surface. Triangles of the discretized boundaries form the initial front. Tetrahedrons are generated one-by-one, starting from the boundary edges or triangles and working toward the center of the domain. The exposed inner faces of these elements collectively form an advancing front. Advancing front methods typically create good tetrahedrons near the boundaries, but are less effective inside the domain, where the fronts collide. In Figure 4.3, stepwise advancing front triangulation of 2D domain is illustrated.

4.3 Mesh Quality Control

The accuracy of the finite element calculation depends on the quality of tetrahedral element. One of the reasons is that the derivatives of the basis functions needed for assembly of the elementary matrix are reciprocally proportional to the element volume $\nabla\varphi \sim V_t^{-1}$. Degenerated tetrahedrons with small volume in conjuction with large displacements of the associated nodes may lead to large local errors

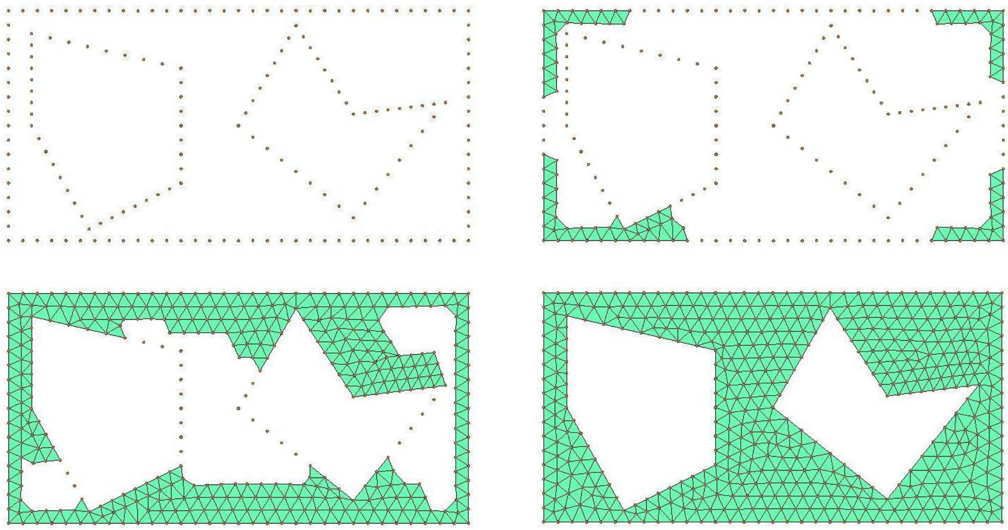

Figure 4.3: Advancing front triangulation of 2D domain. Top row: initial boundary, 100 elements mesh. Bottom row: 500 elements mesh, final triangulation: 790 elements mesh.

of the solution. In the worst case, the system of equations can even become unsolvable. This problem particularly concerns the linear basis functions, where the displacement gradient may become arbitrarily large, cf. (3.78). Even if all elements are approximately of the same size, another criteria of the element quality such as the maximal angle and the maximal edge length are important for the accuracy of the FE solution. Theoretically, the FE solution should approach the exact solution of the given BVP as the size of the largest element approaches zero. However, in [2] it has been shown that if dihedral angles approach π as the element size decreases, convergence to the exact solution may fail. In Figure 4.4, typical tetrahedral elements are shown, which have to be avoided in finite element calculations for the above mentioned reasons. For the indication of the poor quality of a tetrahedral element, different criteria are proposed. For the general characterization of the element shape, the ratio between the circumsphere and the insphere R/r for the characterization of the element shape can be used, see Figure 4.5 (a). The tetrahedrons with the large quotient R/r should be avoided within the grid generation or eleminated 'a posteriori'. In particular, the elements with the large dihedral angle are critical for FE calculations, see Figure 4.5 (b). Additionally, criteria for element quality also depend on the nature of the concrete problem. For physical phenomena that exhibit anisotropic behavior, the most suitable element shape may be the "needle". The effects of element shape on FE solutions are still being investigated.

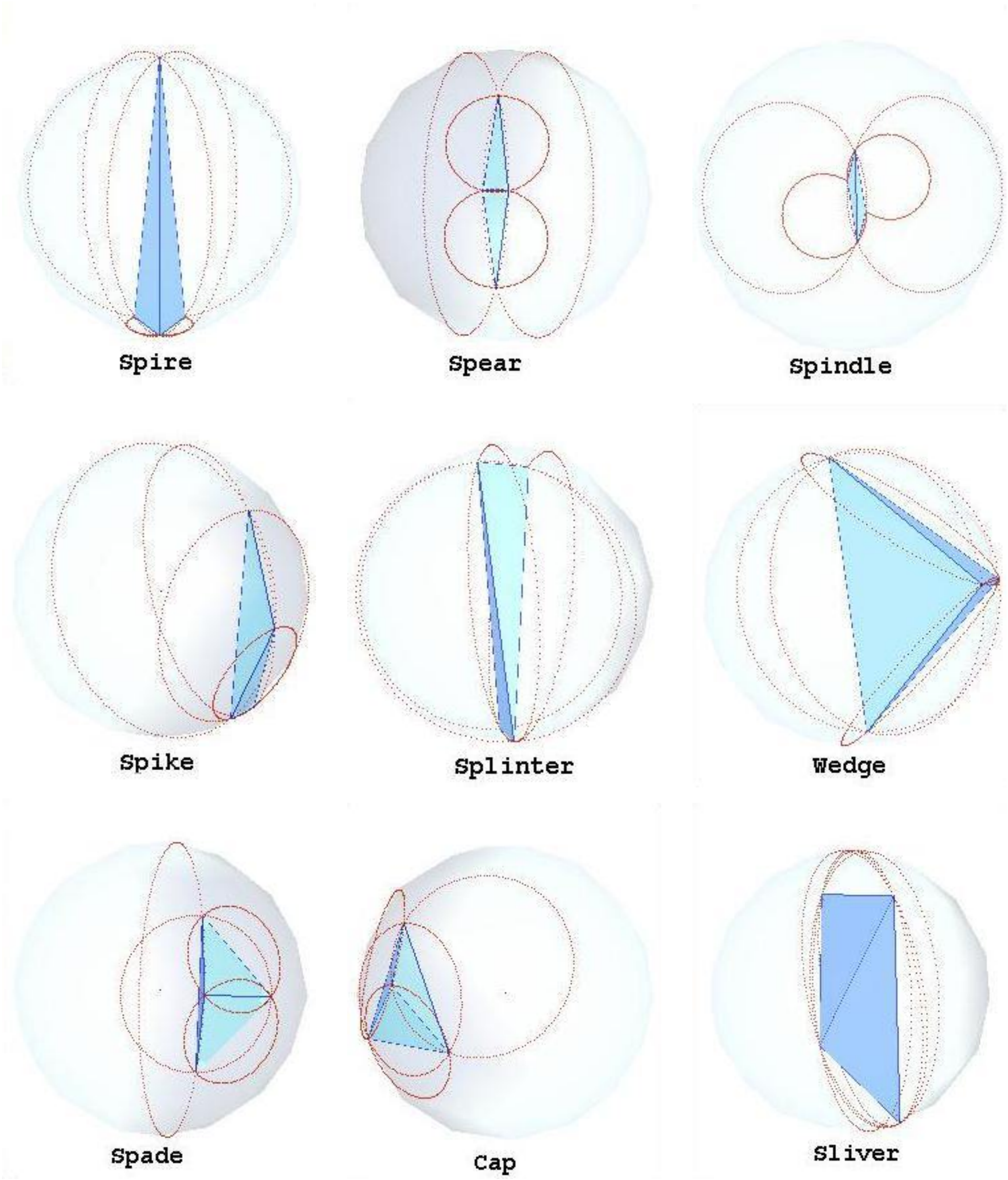

Figure 4.4: Bad tetrahedral elements (from [53]).

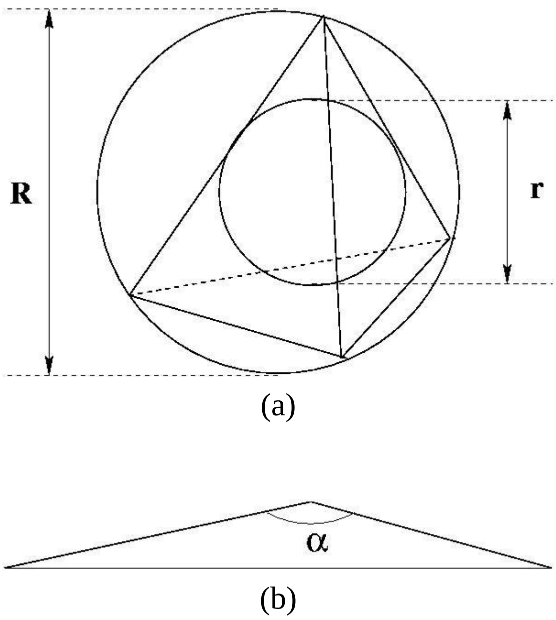

Figure 4.5: Tetrahedron quality criteria. Tetrahedrons with large aspect ratio R/r and, in particular, the elements with large dihedral angles α should be avoided.

4.4 Derivation of Boundary Conditions

In order to obtain a non-trivial solution of the BVP (3.50), the boundary conditions in the form of prescribed displacements or external forces have to be applied. In craniofacial surgery simulations, the forces acting during the surgical impact are usually unknown. Thus, the boundary conditions are given exclusively by the prescribed displacements, which are induced by the rearrangement of bone structures and are applied on soft tissue nodes and triangles belonging to the essential boundary, see Figure 4.6. In this work, the boundary conditions describing the particular surgical case are derived in a following straightforward manner

1. identifi cation and labeling of the bone structures to be removed,
2. simulation of the bone rearrangements and
3. application of the resulting displacements to boundary nodes and triangles.

These three steps are the essential part of the surgical planning simulation and are performed interactively with the help of the *Amira*-based interface.

A detailed description of craniofacial surgery simulations, including the soft tissue prediction, is given in Chapter 6. The derivation of the boundary conditions related to the modeling of contracting muscles and facial expressions will be discussed in Chapter 7.

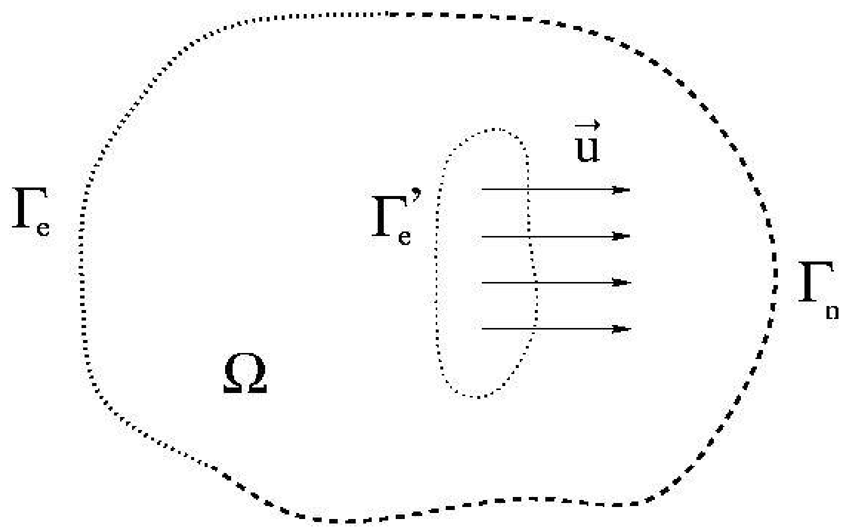

Figure 4.6: Boundary value problem typically arising in the craniofacial surgical planning: find the deformation of a physical body occupying the domain Ω for the boundary conditions given by the prescribed displacements of its partial boundary $\Gamma_e' \subset \Omega$.