Chapter 2

State of the Art

In this chapter, we briefly review the existing techniques for the modeling of deformable objects, which have been developed within the last three decades for different computer graphics and medical imaging applications.

2.1 Deformable Modeling

Deformable modeling of physical objects has a long history. Since computers become an indispensable tool in modeling, sophisticated simulation of complex physical scenes becomes a major everlasting trend in computer graphics and many other applications dealing with the computer assisted modeling of physical reality.

The simulation of deformable objects is essential for many applications. Historically, deformable models appeared in computer graphics and were used to create and edit complex curves, surfaces and solids. Computer aided design uses deformable models to simulate the deformation of industrial materials and tissues. In image analysis, deformable models are used for fitting curved surfaces, boundary smoothing, registration and image segmentation. Later, deformable models are used in character animation and computer graphics for the realistic simulation of skin, clothing and human or animal characters [66, 86, 59, 47]. The modeling of deformable soft tissue is, in particular, of great interest for a wide range of medical imaging applications, where the realistic interaction with virtual objects is required. Especially, computer assisted surgery (CAS) applications demand the physically realistic modeling of complex tissue biomechanics.

Generally, existing modeling approaches can be ranged into two major groups. Models based on solving continuum mechanics problems under consideration of material properties and other environmental constraints are called *physical mod-*

els. All other modeling techniques, even if they are somehow related to mathematical physics, are known as *non-physical models*. A comprehensive review of deformable modeling for medical applications can be found in [87].

2.2 Non-physical Modeling

Non-physical methods for modeling of deformable objects are usually based on pure heuristic geometric techniques or use a sort of simplified physical principles to achieve the reality-like effect. These techniques are very popular in computer graphics and sometimes used in real time applications, since they are computationally efficient in comparison with expensive physical approaches.

Spline techniques. Many early approaches for modeling deformable objects were developed in the field of computer aided geometric design (CAGD), where flexible tools for creation of interpolating curves and surfaces as well as the intuitive ways to modify and refine these objects were needed. From this need came Bezier-curves and subsequently many other methods of compact description of warped curves and surfaces by a small vector of numbers, including B-splines, non-uniform rational B-splines (NURBS) and other types of spline techniques.

The spline technique is based on the representation of both planar and 3D curves and surfaces by a set of control points, also called landmarks. The main idea of spline based methods is to modify the shape of complex objects by varying the position of few control points. Also the number of landmarks as well as their weights can be used for adjustment of the object deformation. Such parameter-based object representation is computationally efficient and supports interactive modification. A comprehensive introduction in curve and surface modeling with splines can be found in [6].

A particular group of landmark-based techniques represent methods, which are used in the elastic image registration and based on radial basis functions derived from some special closed-form solutions of elasticity theory. In [9], a spline technique based on the radial basis function $r \log(r)$ derived from the linear elastic solution of the thin-plate deformation problem is proposed. Such *thin-plate splines* (TPS), globally defined in the image domain, are used for interpolation of the deformation given by the prescribed displacements of control points. Extended TPS-techniques are described in [102, 107]. In [26], an analogous landmark-based approach is proposed, where *elastic body spline* (EBS) derived from the special solution of 3D elasticity is used as an interpolating radial basis function.

Free-form deformation. Free-form deformation (FFD) became popular in computer assisted geometric design and animation in the last decade. The main idea of FFD is to deform the shape of an object by deforming the space in which it is embedded. In early work [5], a general method based on the geometric mappings of 3D space was proposed. This deformation technique uses a set of hierarchical transformations for deforming an object, including rigid motion, stretching, bending, twisting and other operators. The elementary space-warpings are obtained by using the surface normal vector of the undeformed surface and a transformation matrix to calculate the normal vector of an arbitrarily deformed smooth surface. Complex objects can be created from simpler ones, since the deformations are easily combined in a hierarchical structure. The position vector and normal vector in more complex objects are calculated from the position vector and normal vector in simpler objects. Each level in the deformation hierarchy requires an additional matrix multiply for the normal vector calculation.

The term free-form deformation has been introduced in a later work [110], where a more generalized approach based on the embedding an object in a grid of mesh points of some standard geometry, such as a cube or cylinder, has been proposed.

The basic FFD method has been extended by several others [22, 17]. In [84], a modally-controlled FFD technique based on a combination of the FFD method and the modal analysis [97] for the non-rigid registration in image-guided surgery is presented.

2.3 Physical Modeling

In the applications, which demand the realistic simulation of deformable physical bodies, there is no alternative to consistent physical modeling, i.e., numerical solving partial differential equations (PDEs) of elasticity theory. The major problem of physical modeling is that

- the observed physical phenomena can be very complex and
- solution of underlying PDEs requires substantial computational expenses.

The answers to these two questions consist in

- finding an adequate simplified model of the given problem covering the essential observations and
- applying efficient numerical techniques for solving the PDEs.

A variety of approaches for deformable modeling, which have been developed in the past, were bound to give their particular answers to these two questions.

It is difficult to trace who first proposed a working physical model of deformable living tissue. The list of names and research groups, which made their contributions to this topic, is quite long. The study of biomechanical properties of living tissues and their numerical modeling was triggered by single research programs of car-, space- and military-industry beginning from the 50s and later substantially boosted in the early 80s with the development of computer tomography [15, 3]. Further physically motivated techniques for elastic registration and segmentation of medical images are in [67, 4, 19]. At the same time, first fundamental theoretical and experimental investigations of tissue biomechanics appear. In the last decade, a plethora of various approaches and applications related to biomechanical modeling is developed. These methods can be classified by different criteria. One of such classifications is based on the type of the numerical technique used in the modeling approach. There are four common numerical methods for physically based modeling of deformable objects. These are

- mass-spring-damper systems,
- the finite difference method,
- the boundary element method,
- the finite element method.

Mass-spring-damper systems. In the early approaches to soft tissue modeling, an approximation of mechanical continuum by a mass-spring-damper (MSD) system was used. The physical body is represented by a set of mass-points connected by springs exerting forces on neighbor points when a mass is displaced from its rest positions. MSD systems can be seen as a simplified model of particle interaction, since physical bodies in fact consist of discrete sub-elements, atoms and molecules. The spring forces $\mathbf{F}_{\text{s}}$ are usually considered to be linear (Hookean)

$$\mathbf{F}_{\text{s}} = -k\,\mathbf{u}, \tag{2.1}$$

where $\mathbf{u}$ is the displacement of mass-point and k denotes the spring constant corresponding to the material stiffness. The Newton equations of motion for the entire system of N mass-points under the external forces $\mathbf{F}_{\text{ex}}$ are given by

$$M\,\frac{d^2\mathbf{u}}{dt^2} + C\,\frac{d\mathbf{u}}{dt} + K\,\mathbf{u} = \mathbf{F}_{\text{ex}}, \tag{2.2}$$

where M, C and K are the $3N \times 3N$ mass, damping and stiffness matrices, respectively. The solution of (2.2) respectively the displacements $\mathbf{u}$ yields the linear elastic deformation of a physical body discretized by N mass-points. In one of the first works on the field of facial animation [101], a muscle model based on MSD systems, which essentially solve the static system

$$K\,\mathbf{u} = \mathbf{F}_{\text{ex}}, \tag{2.3}$$

is presented. The face is modeled as a two-dimensional mesh of points connected by linear springs. Muscle actions are represented by forces applied to the corresponding region of mesh nodes. This approach was expanded in the later works, where a more sophisticated MSD model of muscles was developed. In [115, 93], muscles directly displace nodes within zones of influence, which are parameterized by radius, fall-off coefficients and other parameters. In [113], dynamic mass-spring systems for facial modeling are described. In this approach, a multi-layer mesh of mass points representing three anatomically distinct facial tissue layers: the dermis, the subcutaneous fat layer and the muscle layer is used. This approach has been extended in [79], where a mesh adaptation algorithm is used that tailors a generic mesh to the individual features by locating these features in a laser-scanned image. For improved realism, this formulation also includes constraint forces to prevent muscles and fascia nodes from penetrating the skull.

In [69], a mass spring model of facial tissue for the soft tissue prediction in craniofacial surgery simulations is proposed. Alternatively to (2.1), non-linear springs $\mathbf{F}_{\text{s}}(\mathbf{u}) \sim \mathbf{u}^n$ can be used to model soft tissue, which generally exhibits non-linear elastic behavior [114].

The major drawback of MSD systems is their insufficient approximation of true material properties. Being a very simplified model of mechanical continuum, particle systems do not provide the required accuracy for the realistic simulation of complex composite materials such as soft tissue. MSD systems are also weak, if complex, arbitrary shaped objects such as thin surfaces, which are resistant to bending, are to be modeled.

Finite difference method. The finite difference method (FDM) is historically the first true discretization technique for solving partial differential equations. The general approach of the FDM is to replace the continuous derivatives within the given boundary value problem with finite difference approximations on a grid of mesh points that spans the domain of interest. Consequently, the differential operator is approximated by an algebraic operator as for instance

$$
\begin{aligned}
\frac{df(x)}{dx} &\approx \frac{f(x+h)-f(x)}{h}\,, \\
\frac{d^2f(x)}{dx^2} &\approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}\,,
\end{aligned}
\tag{2.4}
$$

where h is the characteristic dimension of the discretization. The resulting system of equations can then be solved by a variety of standard techniques. A general algorithm for the finite difference discretization of linear boundary value problems is as follows:

1. Convert continuous variables to discrete variables.

2. Approximate the derivatives at each point using formulae derived from a Taylor series expansion using the most accurate approximation available that is consistent with the given problem.

3. Assemble the linear system of equations respectively to the nodal values.

4. Apply boundary conditions on the boundary points separately.

5. Solve the resulting set of coupled equations using either direct or iterative schemes as appropriate for the given problem.

The FDM achieves efficiency and accuracy when the geometry of the problem is regular. The FDM is usually applied on cubic grids, which are naturally given by pixels or voxels of 2D or 3D digital images, respectively. However, the discretization of objects with the irregular geometry becomes extremely dense, which requires extensive computational resources for data storage and system solving.

In [105], the FD approach for the linear elastic prediction of facial tissue in craniofacial surgery planning is applied. Massively parallel super-computers are used to compute the deformation of $120 \times 120 \times 150$ voxel-grids derived directly from 3D tomographic datasets.

Boundary element method. A general principle of solving the boundary value problem given by the partial differential equation (PDE) and the boundary conditions consists in bringing the differential problem into an integral form. For a certain class of problems, the resulting integration over the whole domain of interest Ω can be substituted by the integration over the boundary $\Gamma \subset \Omega$. Consequently, only the boundary of the domain has to be discretized, which in turn means that

- the dimension of the resulting system of equations is significantly smaller than in the case of total volume discretization,
- the difficult problem of volumetric mesh generation becomes redundant.

For the differential operator of elasticity theory, such boundary integral formulation can be obtained. In [12, 8], the boundary element method (BEM) for static and dynamic problems of continuum mechanics is described. Unfortunately, the volume integrals in the BEM can be completely eliminated only if

- the material is homogeneous and
- no volumetric forces are given.

This is generally not the case in soft tissue modeling. Furthermore, the system matrix when using BEM is fully occupied, which makes the application of efficient iterative solving techniques difficult or even impossible. The investigation carried out in [48] shows that the condition of the BEM system matrix essentially depends on the smoothness of the domain boundary, which possibly requires additional boundary smoothing to achieve the required accuracy of the solution. For elastic registration of medical images, the BEM is, in general, not that robust and flexible as the finite element method [49].

Examples of the application of the boundary element method for the modeling of deformable objects are given in [88, 65, 64].

Finite element method. The finite element method (FEM) becomes the ultimate ”state of the art” technique in physically based modeling and simulation. The FEM is superior to all previously discussed methods when accurate solution of continuum mechanics problems with the complex geometry has to be found. It also provides the most flexible modeling platform free of all limitations with respect to the material type and the boundary conditions.

More accurate physical models treat deformable objects as a mechanical continuum: solid bodies with mass and energies distributed throughout the three-dimensional domain they occupy. Unlike the discrete MSD systems, the FEM is derived directly from the equations of continuum mechanics. In a difference to the FDM, the differential operators are not approximated by simple algebraic expressions, but applied ”as they are” on the subspaces of those admissible solution fields. The difference to the BEM consists in the volume integration, which enables a more general approach to the continuum modeling.

In elasticity theory (Section 3.2), the deformation of a physical body is described as the equilibrium of external forces and internal stresses. The static equilibrium for an infinitesimal volume is given by the partial differential equations, which implies the relationship between the deformation variables such as stresses, strains or displacements and the applied force density, and also contains the constants describing the object material properties. To compute the object deformation, the PDEs of elasticity theory have to be integrated over the domain occupied by a body. Since it is usually impossible to find a closed-form analytical solution for an arbitrary domain, numerical methods are used to approximate the object deformation for a discrete number of points (mesh nodes). MSD or FD methods approximate objects as a finite mesh of nodes and discretize the equilibrium equation at the mesh nodes. The FEM divides the object into a set of elements and approximate the continuous equilibrium equation over each element. The main advantage of the FEM over the node-based discretization techniques is the more flexible node placement and the substantial reduction of the total number of degrees of freedom needed to achieve the required accuracy of the solution.

The main idea of continuum based deformable modeling consists in the minimization of the stored deformation energy, since the object reaches equilibrium when its potential energy is at a minimum. The basic steps of the FEM approach to compute the object deformations are the following:

1. Derive an equilibrium equation for a continuum with given material properties.

2. Select the appropriate finite elements and corresponding interpolation functions for the problem.

3. Subdivide the object into the elements.

4. All relevant variables on each element have to be interpolated by interpolation functions.

5. Assemble the set of equilibrium equations for all of the elements into a single system.

6. Implement the given boundary constrains.

7. Solve the system of equations for the vector of unknowns.

A detailed description of the linear and non-linear elastic finite element approach is in Section 3.3.

Finite element methods enable the most realistic simulation of deformable living objects. However, even this sophisticated approach has its limitations. The material properties of living tissues are highly complex and usually have to be estimated empirically. Living objects are composite materials with a very complex geometrical structure. Various contact and obstacle problems are associated with the modeling of such multi-body systems. A general problem concerns the modeling of large deformations. A widely used linear elastic approach can only be applied under the assumption of small deformations, which often does not hold for soft tissue rearrangements in craniofacial surgery interventions. All these and many other problems make the consistent FE based modeling of soft tissue a very challenging task.

The FE analysis is widely used for modeling deformable living tissues in medical imaging and CAS applications [23, 14, 13, 27, 40]. The most advanced FE based approach for modeling of facial tissue within the scope of the craniofacial surgery planning is in [73, 103]. Throughout all these and other early works, the linear elastic approximation of soft tissue behavior is typically usually used. In [99, 117], the application of the non-linear elastic FEM for real-time simulations of surgical interventions is reported. Till now, no investigations of non-linear FE-based models of facial tissue are known.