Appendix

A. Temperature dependence of the crystalline anisotropy

The anisotropy constants K_l^m include contributions from single-ion (crystal field) and two-ion (exchange-type) interactions. In the rare earths the single-ion contribution dominate over the two-ion contributions due to the deep lying and well localized $4f$ moments. The theory of the temperature dependence of the single-ion crystalline anisotropy has been formulated by Callen and Callen in 1966 [66].

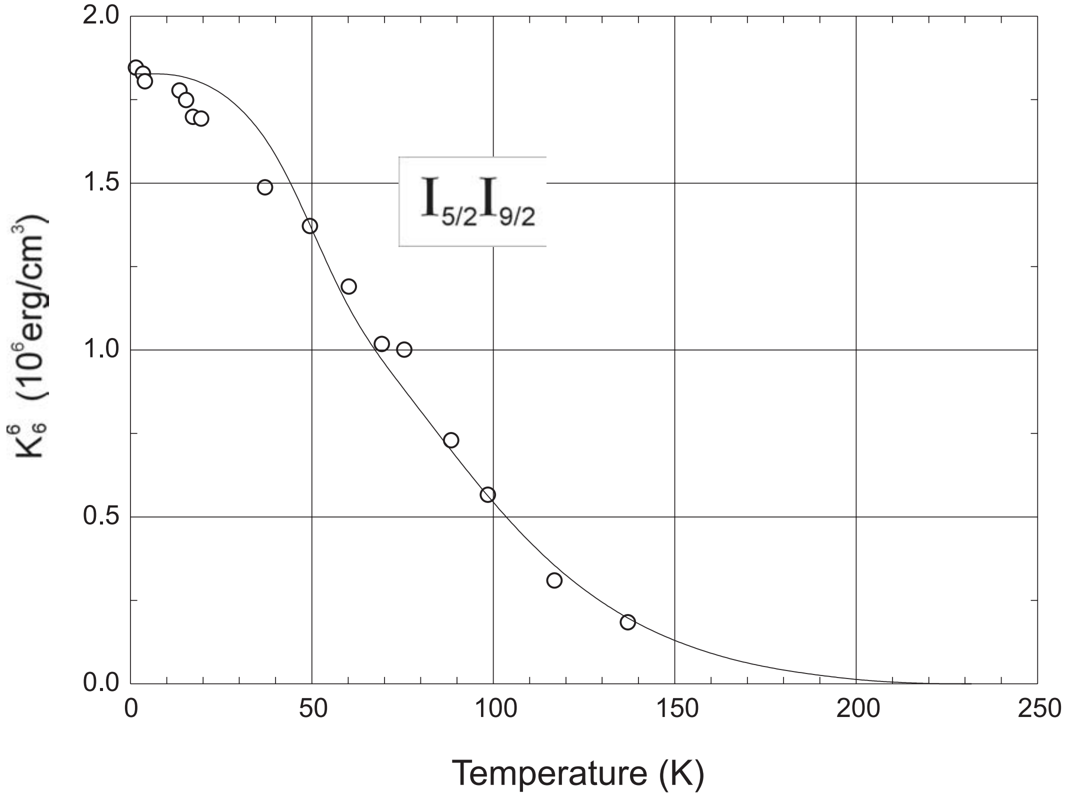

Figure 7.1. Experimental points and theoretical plot of the sixfold in-plane anisotropy K_6^6 versus temperature in Tb bulk. The circles represent the experimental data of Feron et al. (1970), the full line gives the theoretical curve $1.85 \times 10^6 \hat{I}_{l+1/2}(X)[I_{3/2}^{-1}(m)]$.

The Lth-order anisotropy constant is related to the averaged magnetization by

$$k_l(T) = k_l(0)\hat{I}_{l+1/2}[L^{-1}(m(T,H))], \tag{7.1}$$

where $\hat{I}_{l+1/2}(X)$ is a normalized hyperbolic Bessel function of the lth degree. The internal magnetic field energy appearing in the statistical averaging process is represented by the inverse of the Langevin function of the reduced magnetization m(T,H) [1]. The sixfold in-plane anisotropy of an unstrained crystal follows an $\hat{I}_{13/2}$ dependence which can be applied when the magnetostriction is low (e.g. in Gd). Experimental data of the K_6^6 anisotropy in Tb at higher temperatures deviate significantly from an $\hat{I}_{13/2}$ behavior. A better agreement is obtained by an $\hat{I}_{5/2}\hat{I}_{9/2}$ fit, where $\hat{I}_{5/2}$ describes the contribution of the in-plane magnetostriction [57]. In order to estimate how the K_6^6 anisotropy influences the magnetization close to T_N the experimental points of Feron et al. [57] where extrapolated using the $\hat{I}_{5/2}\hat{I}_{9/2}$ expression. As a proper adjustment of the theoretical curve to the experiment requires a description of the magnetization over a wide temperature range, the data-set of Hegland et al. was used which includes temperatures close to 0 K [71]. In the vicinity of the ordering temperature the effects of the external field on the magnetization are huge. Therefore we used at high temperatures the data taken at 1 kOe external field. At low temperatures we used the curve measured at 3 kOe external field, where effects of local anisotropy imperfections are better suppressed. The interpolation between the two curves was done at the steepest slope of the magnetization. The results are shown in Fig. 7.1.

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[1] In the case of an antiferromagnet m is taken as the sublattice magnetization.

B. Stability of the helical phase in Tb

The helical phase in Tb can be completely suppressed by sufficiently strong external magnetic fields converting the helix into a fan structure at $H = \frac{1}{2}H_C$ or aligning the magnetic moments ferromagnetically at $H = H_C$ respectively. A heli-fan structure which is a helix with periodic change in the sense of rotation is only partially stable and the its occurrence depends mainly on the history of the magnetization [63]. Under the assumption that the first-order phase transition from helical to ferromagnetic order is a unique function of temperature dependent exchange, an estimation of the field necessary to suppress the helix can be obtained by equating the field energy with the difference in exchange energy between helical and ferromagnetic order [51, 53, 62]:

$$\mu H_C = \frac{1}{2} J^2 \sigma^2 [\mathcal{J}(Q) - \mathcal{J}(0)]. \tag{7.2}$$

The thermal average of the total moment is described by the reduced magnetization σ [51, 62]. A plot of $\mathcal{J}(q) - \mathcal{J}(0)$ versus the reduced wave vector of the helical periodicity for different rare earths can be found in [51].

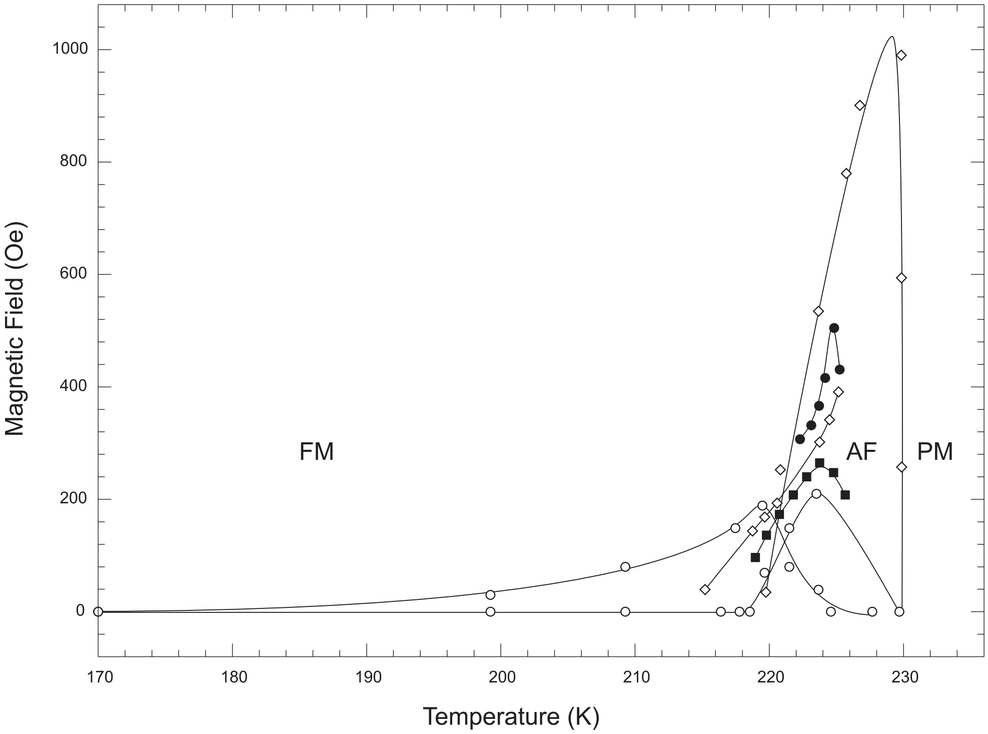

Figure 7.2. Compilation of literature data on the temperature dependence of the critical field required to suppress the helical order in Tb metal: Feron (1969; ■), Belov (1972; ●), Jiles (1980 and 1984: ◇), Dufour (1999; ○).

C. Accuracy of temperature measurements

The temperature measurement is based on a calibrated W/Re thermocouple [178]. Due to frequent flashing procedures, the stoechiometry of the thermo couple is likely to alter with time, and the absolute error has to be checked from time to time. Provided that two independent magnetization measurements are done within a reasonably short time, the relative error for a comparison of two independent measurements consists mainly of the voltage fluctuations (ΔU) times the calibration factor.

$$\Delta T = \left(\frac{\partial U}{\partial T}\right)_T \Delta U \tag{7.3}$$

The voltage stability depends on the quality of the regulator and was determined experimentally. Differential thermovoltage, voltage stability, and resulting error are shown in Fig. 7.3.

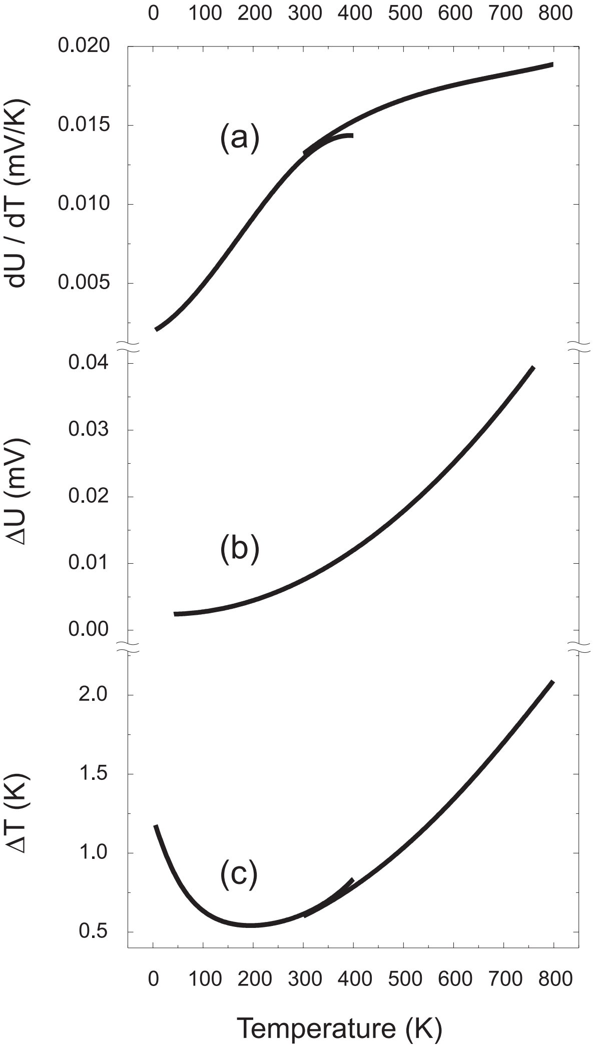

Figure 7.3. The relative temperature accuracy: (a) Differential thermovoltage of the W/Re tempermocouple in the range from 2 K to 800 K, (b) the stability of the temperature control, and (c) the resulting error in temperature $\Delta T = \partial U/\partial T \Delta U$.

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11. J. E. Prieto, F. Heigl, O. Krupin, G. Kaindl and K. Starke:
Quantitative magnetooptics of Gd and Tb in the soft x-ray region
Preprint (November 2002)

10. F. Heigl, J. E. Prieto, O. Krupin, K. Starke, G. Kaindl and M. Bode:
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9. O. Krupin, J. E. Prieto, S. Gorovikov, F. Heigl, K. Starke and G. Kaindl:
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8. J. E. Prieto, F. Heigl, O. Krupin, G. Kaindl and K. Starke:
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