APPENDICES

APPENDIX I.1 **STUDY 1, STUDY 2**

Tasks 1 – 4 of Study 1, Probability and Natural Frequency Versions of Each Task
(Tasks 1 and 3 were used in Study 2)

Task 1: Three Cue Values (positive, negative and unclear test result)

Probability Version

The probability of breast cancer is 13.8% for a woman with a dominant gene mutation
If a woman with a dominant gene mutation actually has breast cancer, the probability is
- 53.7% that she will receive a positive mammogram.
- 24.6% that she will receive an unclear result.
- 21.7% that she will receive a false negative mammogram.

If a woman with a dominant gene mutation does not have breast cancer, the probability is
- 7.7% that she will receive a positive mammogram.
- 7.7 % that she will receive an unclear result.
- 84.6% that she will receive a false negative mammogram.

What is the probability that a woman with a dominant gene mutation actually has breast cancer, given that she has a positive mammogram?

Natural Frequency Version

138 out of every 1,000 women with a dominant gene mutation have breast cancer.
Out of every 138 women with a dominant gene mutation who actually have breast cancer
- 80 will receive a positive mammogram.
- 34 will receive an unclear result.
- 30 will receive a false negative mammogram.

Out of every 862 women with a dominant gene mutation who do not have breast cancer
- 66 will receive a positive mammogram.
- 66 will receive an unclear result.
- 730 will receive a false negative mammogram.

How many of the women with a dominant gene mutation who receive a positive mammogram do you expect to actually have breast cancer?

Task 2: Three Criterion Values (disease A, disease B or healthy)

Probability Version

The probability of having disease A is 0.4% for a person without any symptoms.
The probability of having disease B is 0.1% for a person without any symptoms.
(Diseases A and B never occur at the same time)
There is a medical test which can detect both diseases. But it is unable to discriminate between disease A and B and, furthermore, it is not safe.
- If a person has disease A, the probability is 90% that she or he will receive a positive result.
- If a person has disease B, the probability is 80% that she or he will receive a positive result.
- If a person has neither disease A nor disease B, the probability is 10% that she or he will nevertheless receive a positive result.

What is the probability that a person without any symptoms obtains a positive test result, if the person actually suffers from
- disease A?
- disease B?
- any of the diseases (A or B)?

Natural Frequency Version

40 out of every 10,000 persons without any symptons have disease A and 10 have disease B (diseases A and B never occur at the same time).
There is a medical which that can detect both diseases, but it is unable to discriminate between disease A and B and, furthermore, it is not safe.
36 out of every 40 persons who have disease A receive a positive result.
8 out of every 10 persons who have disease B receive a positive result.
995 out of every 9,950 persons who do not have disease A nor B receive a positive result.
How many of the persons without any symptoms who receive a positive test result suffer from
- disease A?
- disease B?
- any of the diseases (A or B)?

<u>Task 3</u>: Two Cues (2 medical tests)
Probability Version and Natural Frequency Version: Both wordings can be found on page 20.

<u>Task 4</u>: Three Cues (3 unnamed medical tests)

Probability Version

The probability of having a special complaint is 2%.
If a person has the complaint, the probability is 80% that she or he will receive a positive test result in test 1.
If a person has the complaint, the probability is 95% that she or he will receive a positive test result in test 2.
If a person has the complaint, the probability is 75% that she or he will receive a positive test result in test 3.
If a person does not have the complaint, the probability is 25% that she or he will nevertheless receive a positive test result in test 1.
If a person does not have the complaint, the probability is 10% that she or he will nevertheless receive a positive test result in test 2.
If a person does not have the complaint, the probability is 20% that she or he will nevertheless receive a positive test result in test 3.
What is the probablity that a person actually has the complaint if all 3 tests are positive?

Frequency Version

200 out of every 10,000 persons have a complaint.
160 out of every 200 persons who have the complaint receive a positive result in test 1.
152 out of every 160 persons who have the complaint receive a positive result in test 2.
114 out of every 152 persons who have the complaint receive a positive result in test 3.
2,450 out of every 9,800 persons who do not have the complaint nevertheless receive a positive result in test 1.

245 out of every 2,450 persons who do not have the complaint nevertheless receive a positive result in test 2.
49 out of every 245 persons who do not have the complaint nevertheless receive a positive result in test 3.
How many of the persons who receive a positive result in all 3 tests actually have the complaint?

APPENDIX I.2	**STUDY 2**
Two-page instruction sheets for Group 1, Group 2 and Group 3	

Group 1:

Here we present a medical diagnosis task and elucidate its mathematical solution. We are interested in whether it is possible to generalize this explanation to more complex tasks. Thus, you will receive two tasks which are similarily structured like the elucidated one but slightly more complex. Your task is to try to solve both complex tasks by extending the provided explanation.

Task:

> *The probability of breast cancer is 1% for a woman at age 40 who participates in routine screening. If a woman has breast cancer, the probability is 80% that she will receive a positive mammography. If a woman does not have breast cancer, the probability is 10% that she will also receive a positive mammography. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?*

Solution:

- The first number (1%) describes the prevalence, which is the probability that a woman of the sample actually has breast cancer (B). Mathematically we write this as $p(\mathrm{B}) = 1\%$.
- The second number (80%) is the sensitivity of the mammography (M). The sensitivity is the probability that the mammography detects the disease *if it is present*. Mathematically we write $p(\mathrm{M+} \mid \mathrm{B})$.
- The last number (10%) is the false-alarm rate. This is the probability that the mammography wrongly indicates breast cancer, although it is not present. Mathematically the false-alarm rate is $p(\mathrm{M+} \mid \overline{\mathrm{B}})$.

To assess the probability that a woman has cancer when provided with a positive mammogram, i.e. $p(\mathrm{B} \mid \mathrm{M+})$, one can insert these probabilities into Bayes' rule:

$$p(\mathrm{B} \mid \mathrm{M+}) = \frac{p(\mathrm{M+} \mid \mathrm{B})\, p(\mathrm{B})}{p(\mathrm{M+} \mid \mathrm{B})\mathrm{p}(\mathrm{B}) + p(\mathrm{M+} \mid \overline{\mathrm{B}})\, p(\overline{\mathrm{B}})}$$

The probability $\mathrm{p}(\overline{\mathrm{B}})$, that is the probability that a woman of the sample actually does *not* have breast cancer, cannot be read from the wording. This is the complement of p(B) and thus it is: $\mathrm{p}(\overline{\mathrm{B}}) = 1 - \mathrm{p}(\mathrm{B}) = 99\%$. Together we have:

$$p(\mathrm{B} \mid \mathrm{M+}) = \frac{80\% \cdot 1\%}{80\% \cdot 1\% + 10\% \cdot 99\%} = 7.5\%$$

This means: If a woman of this sample receives a positive mammogram, the probability that she indeed has breast cancer is only 7.5%.

Because we want to know how you can transfer what you learned, you will now receive two – slightly more difficult – new problems.

Participants now were confronted with Task 1 and Task 3 (Appendix I.1) in terms of probabilities.

Group 2:

Here we present a medical diagnosis task and elucidate its mathematical solution. We are interested in whether it is possible to generalize this explanation to more complex tasks. Thus, you will receive two tasks which are similarily structured like the elucidated one but slightly more complex. Your task is to try to solve both complex tasks by extending the provided explanation.

Task:

> *The probability of breast cancer is 1% for a woman at age 40 who participates in routine screening. If a woman has breast cancer, the probability is 80% that she will receive a positive mammography. If a woman does not have breast cancer, the probability is 10% that she will also receive a positive mammography. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?*

Solution:

- The first number (1%) describes the prevalence, which is the probability that a woman of the sample actually has breast cancer (B).
- The second number (80%) is the sensitivity of the mammography (M). The sensitivity is the probability that the mammography detects the disease *if it is present*.
- The last number (10%) is the false-alarm rate. This is the probability that the mammography wrongly indicates breast cancer, although it is not present.

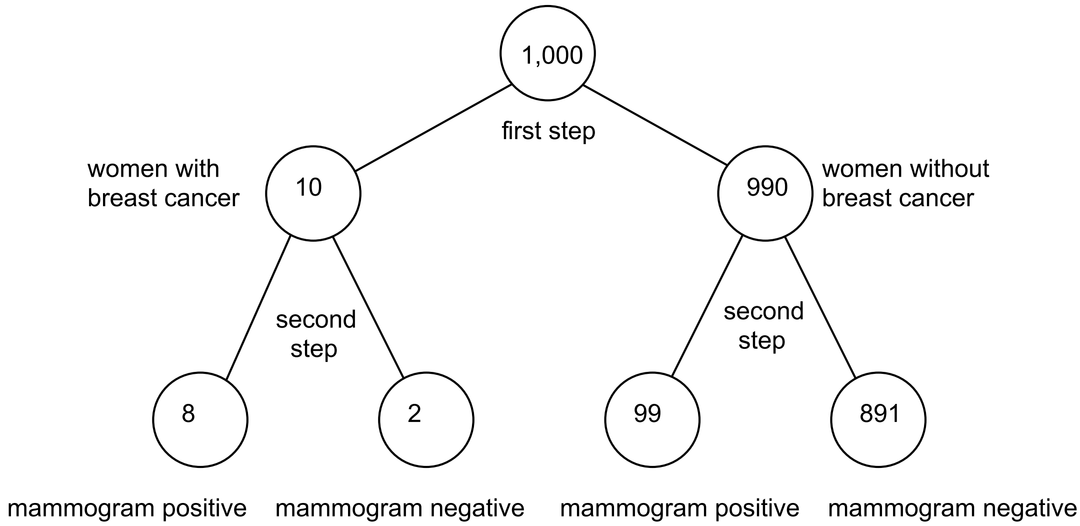
To assess the probability that a woman has cancer when provided with a positive mammogram, it is convenient to translate the given information into frequencies and then depict these frequencies into a tree diagram.

Imagine a population of women, say, 1,000. To construct a frequency tree in a first step, the population has to be divided into ill and healthy women. The *prevalence* says that out of every 1,000 women 1% have breast cancer, which is 10. Consequently, 990 women are healthy.

In a second step, both the healthy and the ill women have to be divided again according to the *sensitivity* (which makes a statement about the ill women) and the *false-alarm rate* (which makes a statement about the healthy women).

The sensitivity of 80% indicates that out of the 10 women with breast cancer 8 will be detected by a mammography and 2 will be missed by this test. The false-alarm rate of 10%

indicates that out of the 990 healthy women 99 will be wrongly classified as having breast cancer.
Now we have all numbers to yield the following tree diagram:



From the tree diagram, the answer to the question "How many of the women with a positive mammogram actually have breast cancer?" can be seen. Overall, 107 = 8 + 99 women obtained a positive mammogram. Out of these, only 8 indeed have breast cancer. Therefore, the probability of having breast cancer provided with a positive mammogram is

$$\frac{8}{8+99} = \frac{8}{107} \approx 7.5\%$$

Because we want to know how you can transfer what you have learned, you now will receive two – slightly more difficult – new problems.

Participants now were confronted with Task 1 and Task 3 (Appendix I.1) in terms of probabilities.

Group 3:

Here we present a medical diagnosis task and elucidate its mathematical solution. We are interested in whether it is possible to generalize this explanation to more complex tasks. Thus, you will receive two tasks which are similarly structured like the elucidated one but slightly more complex. Your task is to try to solve both complex tasks by extending the provided explanation.

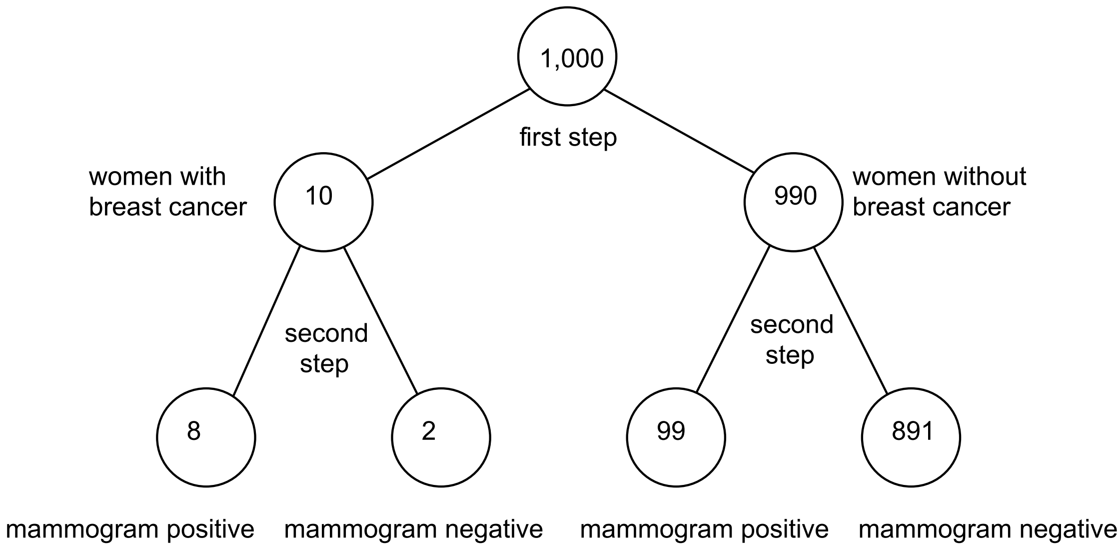
Task:

> *10 out of every 1,000 women at age forty who participate in routine screening have breast cancer. 8 out of every 10 women with breast cancer will receive a positive mammography. 99 out of every 990 women without breast cancer will also receive a positive mammography. Here is a new representative sample of women at age forty who got a positive mammography in a routine screening. How many of these women do you expect to actually have breast cancer?*

Solution:

- The first frequency (10 out of 1,000) is the prevalence, which is the proportion of women with breast cancer in the sample (of 1,000 women).
- The second frequency (8 out of 10) is the sensitivity of the mammography. It indicates how many of a population of women with breast cancer the mammography will detect.
- The last frequency (99 out of 990) is the false-alarm rate. It indicates how many of a population of healthy women by the mammography wrongly will be classified as having breast cancer.

To "see" the solution of the task it is convenient to depict these frequencies into a tree diagram, which should be developed step by step beginning at the top.
In a first step, the population has to be divided into ill and healthy women. The way how to do this is given by the *prevalence*:
In a second step, both the healthy and the ill women have to be divided again according to the *sensitivity* (which makes a statement about the ill women) and the *specificity* (which makes a statement about the healthy women).
Conducting both steps yields the following tree diagram:



From the tree diagram, the answer to the question "How many of the women with a positive mammogram actually have breast cancer?" can be seen. Overall, 107 = 8 + 99 women got a positive mammogram. Out of these, only 8 indeed have breast cancer. Therefore, 8 out of 107 women (ca. 7.5%) with a positive mammogram have breast cancer.

Because we want to know how you can transfer what you have learned, you now will receive two – slightly more difficult – new problems.

Participants now were confronted with Task 1 and Task 3 (Appendix I.1) in terms of natural frequencies.

APPENDIX II.1 **STUDY 3, STUDY 4**

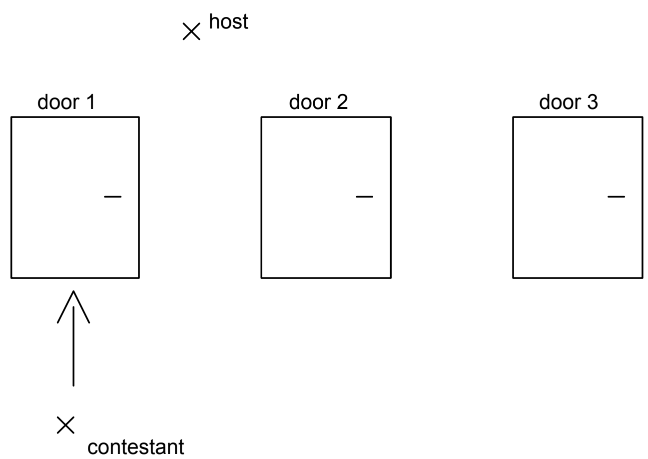
Versions of the Monty Hall Problem
(Versions of groups 1, 3 and 4 appear in the text)

Group 2:

LET´S MAKE A DEAL

In America there is a game show called “Let´s Make a Deal”. The contestant is allowed to choose one out of three closed doors. Behind one door is the first prize, a car. Behind the other two doors are goats. Monty Hall (the host of the game show) asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the car is) first opens one of the other two doors and shows a goat to the contestant. Now the contestant can again decide whether she wants to stay with her first choice or whether she wants to switch to the last remaining door.

<u>Task:</u>
Imagine you are the contestant and you don`t know behind which door the car is. You chose a door, say number one.



Afterwards Monty opens another door according to the rules and shows you a goat. Now he asks you whether you want to stay with your first choice (door 1) or to switch to the last remaining door.

What therefore you should do? ___ stay ___ switch

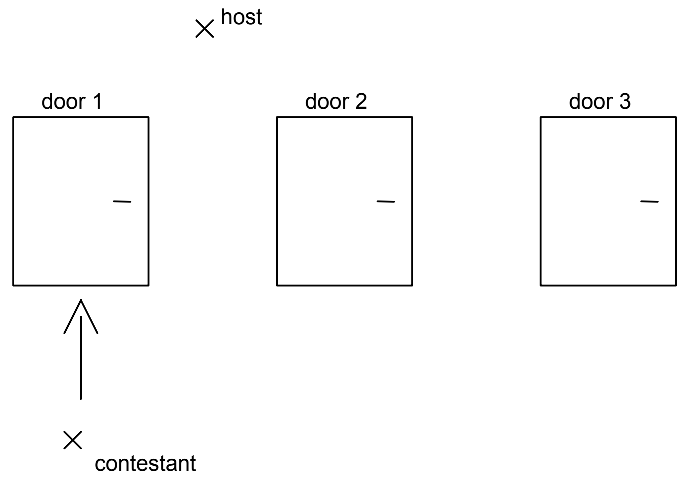
<u>Important:</u>
Please tell us in writing what went on in your head when you thought of your answers. In explaining your answers, you may make use of things like sketches, etc.
Please also tell us if you learned this game before ___ (Yes) ____ (No) and knew what the correct answer should be ____ (Yes) ____ (No).

Group 5:

LET´S MAKE A DEAL

In America there is a game show called "Let´s Make a Deal". The contestant is allowed to choose one out of three closed doors. Behind one door is the first prize, a car. Behind the other two doors are goats. Monty Hall (the host of the game show) asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the car is) first *opens one of the other two doors and shows a goat to the contestant.* Now the contestant can again decide whether she wants to stay with her first choice or whether she wants to switch to the last remaining door.

<u>**Task:**</u>
Imagine you are the contestant and you don`t know behind which door the car is. You pick a door, say number one.



Afterwards Monty opens another door according to the rules and shows you a goat. Now he asks you whether you want to stay with your first choice (door 1) or to switch to the last remaining door.

There are *three doors* behind which the car can be.

In how many of these three possible cases would the you win the car after Monty`s opening of a "goat door",
- if you stay with your first choice (door 1)? in___out of 3
- it you switch to the last remaining door? in___out of 3

What therefore you should do? ___ stay ___ switch

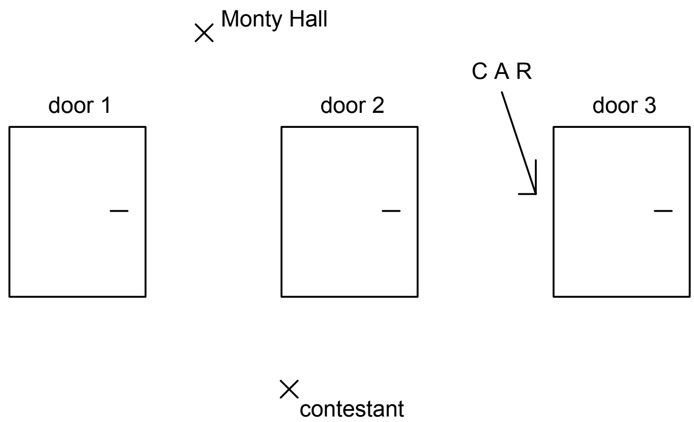
<u>Important:</u>
Please tell us in writing what went on in your head when you thought of your answers. In explaining your answers, you may make use of things like sketches, etc.
Please also tell us if you learned this game before ___ (Yes) ____ (No) and knew what the correct answer should be ____ (Yes) ____ (No).

Group 6:

LET´S MAKE A DEAL

In America there is a game show called "Let´s Make a Deal". The contestant is allowed to choose one out of three closed doors. Behind one door is the first prize, a car. Behind the other two doors are goats. Monty Hall (the host of the game show) asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the car is) first *opens one of the other two doors and shows a goat to the contestant.* Now the contestant can again decide whether she wants to stay with her first choice or whether she wants to switch to the last remaining door.

Task:
Imagine you are Monty Hall, the host of this game show, and you know that the car is behind door 3.



The contestant, who doesn`t know where the car is, now chooses a door.
Afterwards you open another door according to the rules and show the contestant a goat. Now you ask her whether she wants to stay with her first choice or to switch to the last remaining door.

What the contestant therefore should do? ___ stay ___ switch

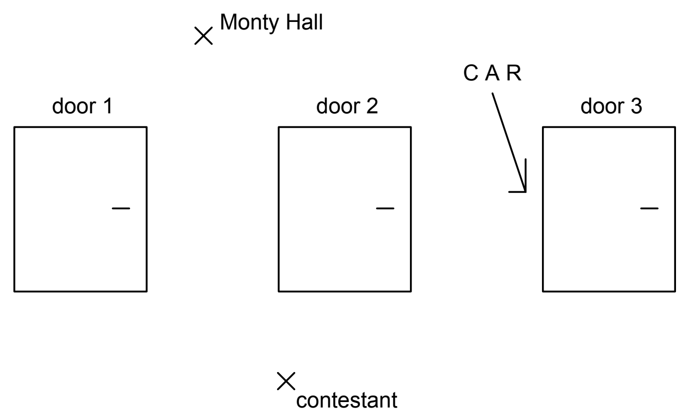
Important:
Please tell us in writing what went on in your head when you thought of your answers. In explaining your answers, you may make use of things like sketches, etc.
Please also tell us if you learned this game before ___ (Yes) ____ (No) and knew what the correct answer should be ____ (Yes) ____ (No).

Group 7

LET´S MAKE A DEAL

In America there is a game show called “Let´s Make a Deal”. The contestant is allowed to choose one out of three closed doors. Behind one door is the first prize, a car. Behind the other two doors are goats. Monty Hall (the host of the game show) asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the car is) first *opens one of the other two doors and shows a goat to the contestant.* Now the contestant can again decide whether she wants to stay with her first choice or whether she wants to switch to the last remaining door.

<u>Task:</u>
Imagine you are Monty Hall, the host of this game show, and you know that the car is behind door 3.



The contestant, who doesn`t know where the car is, now chooses a door.
Afterwards you open another door according to the rules and show the contestant a goat. Now you ask her whether she wants to stay with her first choice or to switch to the last remaining door.

There are *three doors* which the contestant could choose first.

In how many of these three possible cases would the contestant win the car after your opening of a “goat door”,

- if she stays with her first choice? in___out of 3
- it she switches to the last remaining door? in___out of 3

What the contestant therefore should do? ___ stay ___ switch

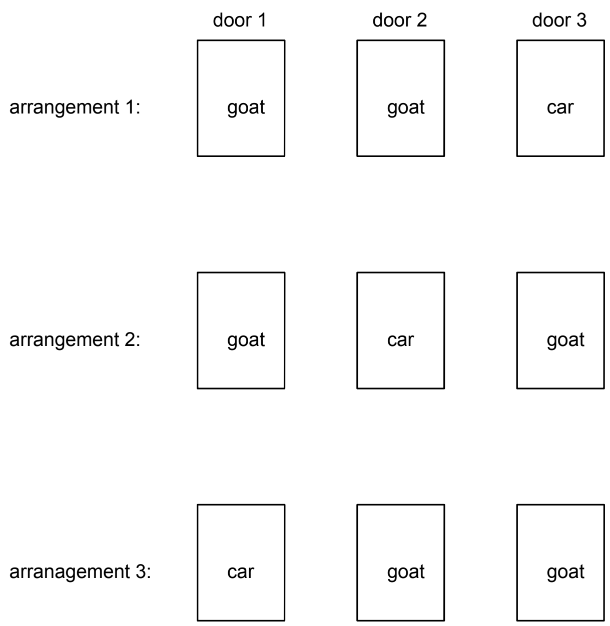
<u>Important:</u>
Please tell us in writing what went on in your head when you thought of your answers. In explaining your answers, you may make use of things like sketches, etc.
Please also tell us if you learned this game before ___ (Yes) ____ (No) and knew what the correct answer should be ____ (Yes) ____ (No).

APPENDIX II.2 STUDY 5

Explanations of the Monty Hall Problem

Explanation 1, Frequency Simulation ("FS"):

It is possible to solve the Monty Hall problem the following way:
Generally there are three possibilities, as to behind which door the car can be. Let us consider these three car-goat arrangements and let us furthermore assume that the contestant has chosen door 1. The car might be behind door 1, door 2, or door 3.



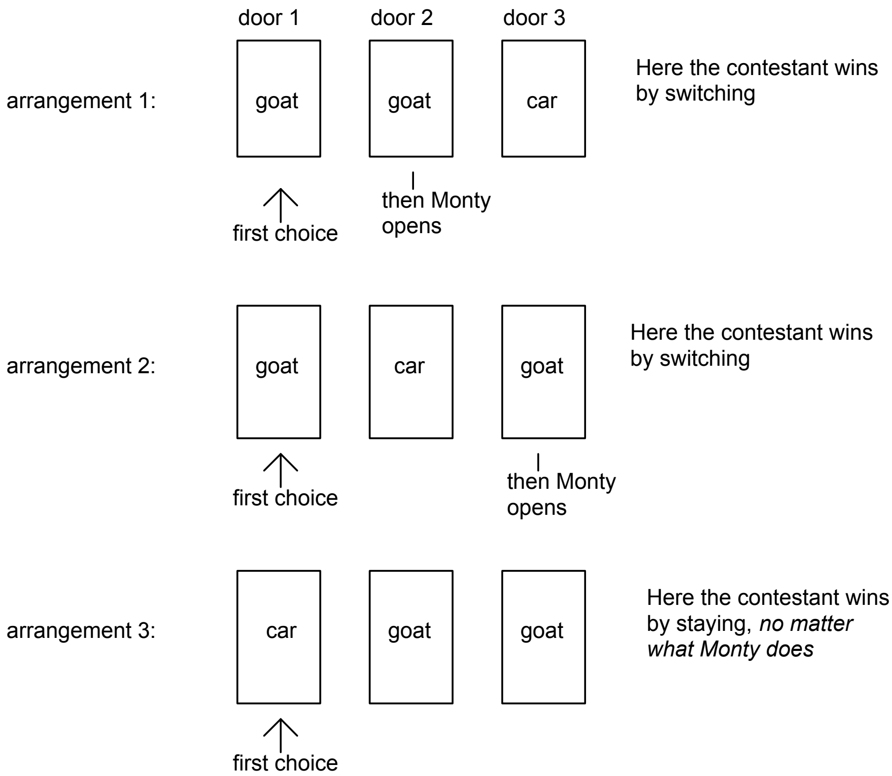
The contestant's choosing door 1 first has the following implications for each arrangement:

Arrangement 1
According to the rules the host has to open door 2. In this arrangement the contestant would win the car by switching to door 3.

Arrangement 2
According to the rules the host has to open door 3. In this arrangement the contestant would win the car by switching to door 2.

Arrangement 3
In this arrangement the contestant would have hit the car, thus she should stay with this choice, regardless of what Monty Hall does.

The following diagram summarizes all “simulated arrangements”:



Considering all arrangements it becomes clear that the contestant would win the car in two out of three arrangements if she switches away from door 1, namely in arrangements 1 and 2. Only in arrangement 3 should she stay.
Conclusion: The probability that she will win the car by switching is therefore $2/3$ and thus she should switch.

Explanation 2, Six Mental Models (“6MM”):

It is possible to solve the Monty Hall problem the following way:
Generally there are three possibilities, as to behind which door the car can be. Let us split these three car-goat arrangements each into two cases and let us furthermore assume that the contestant has chosen door 1.

<u>Arrangement 1</u> (including cases 1 and 2):

The car is behind door 1.
The host can now open door 2 or door 3 according to the rules (this is the reason why we have to consider two cases in this arrangement). The following diagram exhibits the two cases:

Cases	Door 1 (Chosen Door)	Door 2	Door 3
1	car	open	
2	car		open

Arrangement 2 (including cases 3 and 4):

The car is behind door 2.
The host now has to open door 3. Although the host in this arrangement can only open door 3, this arrangement also has to be listed twice, because all arrangements are equiprobable and thus must be represented equally often. This yields the following diagram:

Cases	Door 1 (Chosen Door)	Door 2	Door 3
3		car	open
4		car	open

Arrangement 3 (including cases 5 and 6):

The car is behind door 3.
The host now has to open door 2. Although the host in this arrangement can only open door 2, this arrangement – like arrangements 1 and 2 - has to be listed twice, because all arrangements are equiprobable and thus must be represented equally often. The following diagram clarifies the situation:

Cases	Door 1 (Chosen Door)	Door 2	Door 3
5		open	car
6		open	car

Let us now summarize all possible arrangements including the possible 6 cases:

Cases	Door 1 (Chosen Door)	Door 2	Door 3
1	car	open	
2	car		open
3		car	open
4		car	open
5		open	car
6		open	car

Considering all cases it becomes clear that the contestant would win the car in 4 out of 6 cases if she switches away from door 1 (namely in cases 3, 4, 5, and 6). Only in cases 1 and 2 she should stay.

Conclusion: The probability that she will win the car by switching is 4/6 (or 2/3) and thus she should switch.

Explanation 3, Bayes' Rule, ("BR"):

It is possible to solve the Monty Hall problem the following way:
First we have to clarify the notations of possible events in the Monty Hall problem.
Let us label
"C_1" the event "car is behind door 1",
"C_2" the event "car is behind door 2", and
"C_3" the event "car is behind door 3".

At the beginning the probabilities of these events – written as $p(C_1)$, $p(C_2)$, and $p(C_3)$ – are all equal to one third:

$$p(C_1) = p(C_2) = p(C_3) = 1/3.$$

Furthermore, let us assume that the contestant has chosen door 1 and thereupon Monty Hall has opened door 3 (the event "Monty Hall opens door 3" we will label "M_3").

In the following we need the concept of "conditional probabilities", therefore we will briefly explain it (using two arbitrary events X and Y):
The conditional probability $p(X \mid Y)$ means the "probability of X, if Y is given". Sometimes $p(X \mid Y)$ is given, and $p(Y \mid X)$ (the probability of Y, if X is already given) is asked for. Bayes' rule allows us to derive the conditional probability $p(Y \mid X)$ from $p(X \mid Y)$.

To apply Bayes' rule in the Monty Hall problem, all possible car-goat arrangements have to be considered:

The probability that Monty opens door 3, given that the car is behind door 1, is
$p(M_3 \mid C_1) = 1/2$, because in this case Monty Hall can open either door 2 or door 3.
The probability that Monty opens door 3, given that the car is behind door 2, is
$p(M_3 \mid C_2) = 1$, because in this case, according to the rules, Monty Hall has to open door 3.
The probability that Monty opens door 3, given that the car is behind door 3, is
$p(M_3 \mid C_3) = 0$, because Monty Hall cannot open the door that conceals the car.

Whether it is better to stay with door 1 or to switch to door 2 can now be calculated with Bayes' rule. If we want to assess the probability of winning by switching, we actually have to calculate $p(C_2 \mid M_3)$, which is the probability that the car is behind door 2 (C_2) when Monty Hall has revealed door 3 (M_3). For the Monty Hall problem Bayes' rule looks like this:

$$p(C_2 | M_3) = \frac{p(M_3 | C_2) \cdot p(C_2)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)}$$

We have already assessed all probabilities in the right side of this equation. Just inserting the numbers yields

$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} = \frac{2}{3}$$

This means that the car is behind door 2 has a probability of 2/3 and thus the probability of winning by switching is 2/3. The probability that one wins by staying, namely $p(C_1 | M_3)$, respectively is

$$p(C_1 | M_3) = \frac{p(M_3 | C_1) \cdot p(C_1)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)} = \frac{1}{3}$$

Conclusion: Hence, in the given situation one should switch to door 2, because the probability of finding the car behind door 2 is twice as high as the probability of finding it behind the initial chosen door 1.

APPENDIX II.3 **Study 5**

Problems A, B, C and D

Immediately after the explanation participants got the following variant of the Monty Hall problem:

Problem A:

Imagine that behind the doors now are two cars and one goat. After the contestant has chosen a door, the door remains closed for the time being. The game show host now has to open one of the two unchosen doors and reveal a car. After Monty Hall shows a car to the contestant, he asks the contestant to make a decision of whether she wants to stay with the first choice or switch to the last remaining door. (The car behind the open door cannot be chosen.)

After 10 weeks participants had to solve the following three tasks:

Problem B:

A contestant in a game show is allowed to choose one of four doors. Behind one door is the first prize, a car. Behind the other three doors are three goats. Monty Hall asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the car is) first opens two of the other three doors and reveal two goats to the contestant. Now the contestant can again decide whether she wants to stay with the first choice or switch to the last remaining door.

Problem C:

A contestant in a game show is allowed to choose one of four doors. Behind two doors are cars. Behind the other two doors are goats. Monty Hall asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the cars are) first opens two of the other three doors and reveal a goat and a car to the contestant. Now the contestant can again decide whether he wants to stay with his first choice or switch to the last remaining door.

Problem D:

Imagine three prisoners (A, B, and C) sitting in three different cells. They know that one of them will be set free; the other two will be executed. The only one who already knows who the lucky one is, is the guard. He is not allowed to inform the prisoners about their fate. Therefore one of the prisoners (A) asks him: ”Please name at least one of the others who will be executed.” The guard thinks for a while and says: “Prisoner C will be executed”. Did this change the chances of prisoner A being set free?