

CHAPTER II

The Psychology of the Monty Hall Problem: Discovering Psychological Mechanisms for Solving a Tenacious Brain Teaser

SUMMARY

The *Monty Hall problem* (or *three door problem*) is one of the most famous examples of a “cognitive illusion”, often used by psychologists, economists, and even law scientists to demonstrate people’s resistant deficiency in dealing with uncertainty. By analyzing this problem’s cognitive aspects we discovered synergistic relationships among four elements from the cognitive psychologists’ toolbox. These elements are *natural frequencies*, *mental models*, *perspective change*, and the *less-is-more effect*. Our first step in this chapter is to show that these four elements can serve as building blocks of a comprehensible solution to the problem; yet certain features of the problem’s standard version block the intuitive pathway to their use. Secondly, we review ongoing debates on the problem, in particular the one on “missing information”. We argue that participants’ difficulties in solving the problem are not due to their wrong assumptions that arise from a lack of task information but rather due to a lack of appropriate information representation. Finally, we experimentally manipulate the problem’s formulation along the lines of each of the four cognitive elements. These manipulations combined indeed lead to an increase in the proportion of novice participants who respond correctly from the typical range of 5–15% to over 50% (Studies 3, 4). In a training study (Study 5) we showed that with advance tutoring people’s widely cited resistance can be broken and their performance can even be increased up to 82% correct responses.

INTRODUCTION

For 28 years, Monty Hall hosted a game show on American television called “Let’s Make a Deal”. Contestants on this show were often confronted with a dilemma in which they had to decide whether to stick with an initial choice or switch to an alternative. What contestants should do in this situation sparked a heated public debate in 1991, after a reader of *Parade* magazine asked the following question (see vos Savant, 1997), today known as the *Monty Hall problem* or the *three-door problem*:

“Suppose you’re on a game show and you’re given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what’s behind the doors, opens another door, say number 3, which has a goat. He then says to you, ‘Do you want to switch to door number 2?’ Is it to your advantage to switch your choice?”⁶

In three of her weekly columns, vos Savant⁷ attempted to convince readers that switching is to the contestant’s advantage. First, she declared: “Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance.” Then she explained her statement by asking readers to visualize one million doors: “Suppose there are a million doors, and you pick number 1. Then the host, who knows what’s behind the doors and will always avoid the one with the prize, opens them all except door number 777,777. You’d switch to that door pretty fast, wouldn’t you?”

Responses to these columns were numerous, passionate, and, in some cases, vitriolic.⁸ Many of vos Savant’s disbelievers had Ph.D.s and worked in the field of

⁶ In the following we will refer to this as the *standard version* of the problem. In the real game show Monty Hall played several variations of this setting. But it is important to note that the discussion about the problem started only after vos Savant’s columns appeared in *Parade*. Readers there were explicitly referred to the version posed by the inquisitive reader, and no mention was made of the real game show.

⁷ Marilyn vos Savant’s column in *Parade* magazine is called “Ask Marilyn”. According to the *Guinness book of world records of 1991* she was, at the time of the controversy, said to be the person with the highest IQ in the world (IQ: 228) and readers could ask her whatever they wanted. In 1997 she summarized the exploding discussion about the Monty Hall problem in her book *The power of logical thinking*.

⁸ In total, she received 10,000 letters replying to her three columns. For a collection of the most interesting (and the most amusing) ones, see vos Savant (1997).

statistics. A Ph.D. from the University of Florida wrote: “Your answer to the question is in error. But if it is any consolation, many of our academic colleagues have also been stumped by this problem.” A member of the U.S. Army Research Institute responded thus to her second attempt to convince readers of the correct solution: “You made a mistake, but look at the positive side. If all those Ph.D.s were wrong, the country would be in some very serious trouble.” Some people even offered to wager large sums of money (e.g., \$20,000) on their belief that switching has no advantage. In addressing these replies, vos Savant wrote: “Gasp! If this controversy continues, even the postman won’t be able to fit into the mailroom. I’m receiving thousands of letters, nearly all insisting that I’m wrong, including the Deputy Director of the Center for Defense Information and a Research Mathematical Statistician from the National Institutes of Health! Of the letters from the general public, 92% are against our answer, and of the letters from universities, 65% are against our answer [...]. But math answers aren’t determined by votes” (vos Savant, 1997, p. 10). From reading vos Savant’s (1997) recollection, it becomes clear that it is not only difficult to find the correct solution to the problem, but it is even more difficult to make people accept its solution.

PREVIOUS RESEARCH

This seemingly simple problem has since drawn the attention of several authors, for instance, in *American Statistician* and *Skeptical Inquirer* (e.g., Frazier, 1992; Posner, 1991). *The New York Times* also reported on the debate in a front-page story (Tierney, 1991). These discussions have verified vos Savant’s conclusion that the mathematically correct solution for the benefit of the contestant is to switch, if the rules of the game show are so: Monty Hall has in any case to reveal a goat after the contestant’s first choice and he cannot open the door chosen by the contestant.⁹

In von Randow’s book about the Monty Hall problem (1993), the German science journalist described how he shifted his interest from mathematical to psychological issues after he realized that switching is indeed better. He raised the following three questions (p. 9): Why were so many people, even those who were

⁹ For a detailed discussion on ambiguity of the wording of the Monty Hall problem see section “Are There Possible Effects of Incomplete Information?” in this chapter.

highly educated, deceived? Why are so many of them still convinced of the wrong answer? Why are they so enraged?

Similarly, Piattelli-Palmarini remarked (see vos Savant, 1997, p. 15): “No other statistical puzzle comes so close to fooling all the people all the time. [...] The phenomenon is particularly interesting precisely because of its specificity, its reproducibility, and its immunity to higher education.” He went on to say “even Nobel physicists systematically give the wrong answer, and [...] *insist* on it, and are ready to berate in print those who propose the right answer.” In his book *Inevitable illusions: How mistakes of reason rule our minds* (1994), Piattelli-Palmarini singled out the Monty Hall problem as the most expressive example of the “cognitive illusions” or “mental tunnels” in which “even the finest and best-trained minds get trapped” (p. 161).

Experimental psychologists have used the Monty Hall problem to study various psychological aspects of human probabilistic reasoning and decision making.¹⁰ In fact, before the Monty Hall problem became famous, Shimojo and Ichikawa (1989) investigated a problem that is mathematically very similar, namely, *the problem of three prisoners*.¹¹ Shimojo and Ichikawa examined the beliefs of participants experimentally, whereas Falk (1992), for instance, looked at the same issue theoretically. The main aim of both lines of work was to provide explanations for people’s reasoning errors with this kind of problem. Granberg and Brown (1995) later conducted the first comprehensive experimental study of the Monty Hall problem. They presented participants with the

¹⁰ Generally there are two kinds of possible experiments related to the Monty Hall problem. First, one can ask participants for a (justified) decision, when they are provided with a written version. Second, one can let people play the game repeatedly with feedback (e.g., against a computer) and can investigate how they change their behavior by observing the outcome. In this chapter we will only focus on experiments of the first kind. For experiments of the second kind see, for instance, Friedman (1998) or Granberg and Dorr (1998).

¹¹ The *problem of three prisoners* is the following: Tom, Dick, and Harry are awaiting execution while imprisoned in separate cells in some remote country. The monarch of that country arbitrarily decides to pardon one of the three, but the name of the lucky one is not immediately announced, and the warden is forbidden to inform any of the prisoners of his fate. Dick argues that he already knows that at least one of Tom and Harry must be executed, thus convincing the compassionate warden that by naming one of them he will not be violating his instructions. The Warden names Harry. Did this change the chances of Dick and Tom of being freed? (Paraphrasing of Falk’s 1992 problem formulation). Corresponding to the Monty Hall problem, the chances of Dick being freed remain one third, while the chances of Tom increase to two thirds.

standard version of the Monty Hall problem with slight changes in wording and found that only 13% of them correctly chose to switch doors.

Until now, all experimental studies have had similar results: The vast majority of participants thinks that switching and staying are equally good alternatives and then decide to stay. Falk (1992) calls the belief in the equiprobability of the two remaining alternatives “uniformity belief” (p. 202). But if for most participants there is no reason to favor one option over the other, why do a vast majority decide to stick to the original choice? To answer this question, Granberg and Brown (1995) gave a new group of participants hypothetical histories of choices made by previous contestants (for example “contestant switched and lost” or “contestant stayed and lost”) and asked how they would feel in the described situations. Participants reported that they would feel worse if they switched from a door with the car behind it than if they stuck to a door with a goat behind it because in the first case they would have already won the prize.

To date, efforts to encourage people to solve the Monty Hall problem *with mathematical insight* have not been very successful. Expressed in terms of the percentage of participants who switch, even the most encouraging findings (e.g., Aaron & Spivey, 1998) have not exceeded 30%.

PRESENT APPROACH AND OBJECTIVES

Most of the research on the Monty Hall problem has focused on beliefs that might lead to the mathematically incorrect choice. We are instead interested in the mental processes that lead to correct reasoning. Knowledge of these cognitive mechanisms should allow us to formulate a version of the Monty Hall problem better adapted to human cognition than the standard version.

Despite the difficulties people have with the Monty Hall problem, there are people who do find the correct solution intuitively. The natural questions are: Which reasoning processes are employed by these few successful problem solvers? And: Given we find these mechanisms, how can we develop appropriate ways to represent and to explain the brain-teaser to eliminate the typical resistance? In the brain-storming phase preceding the experiments we confronted colleagues and students with the problem and later discussed their intuitions with them. This led us to the insight that the reasoning

processes of successful problem solvers have a common denominator whose essence is expressed in Figure 2.1.

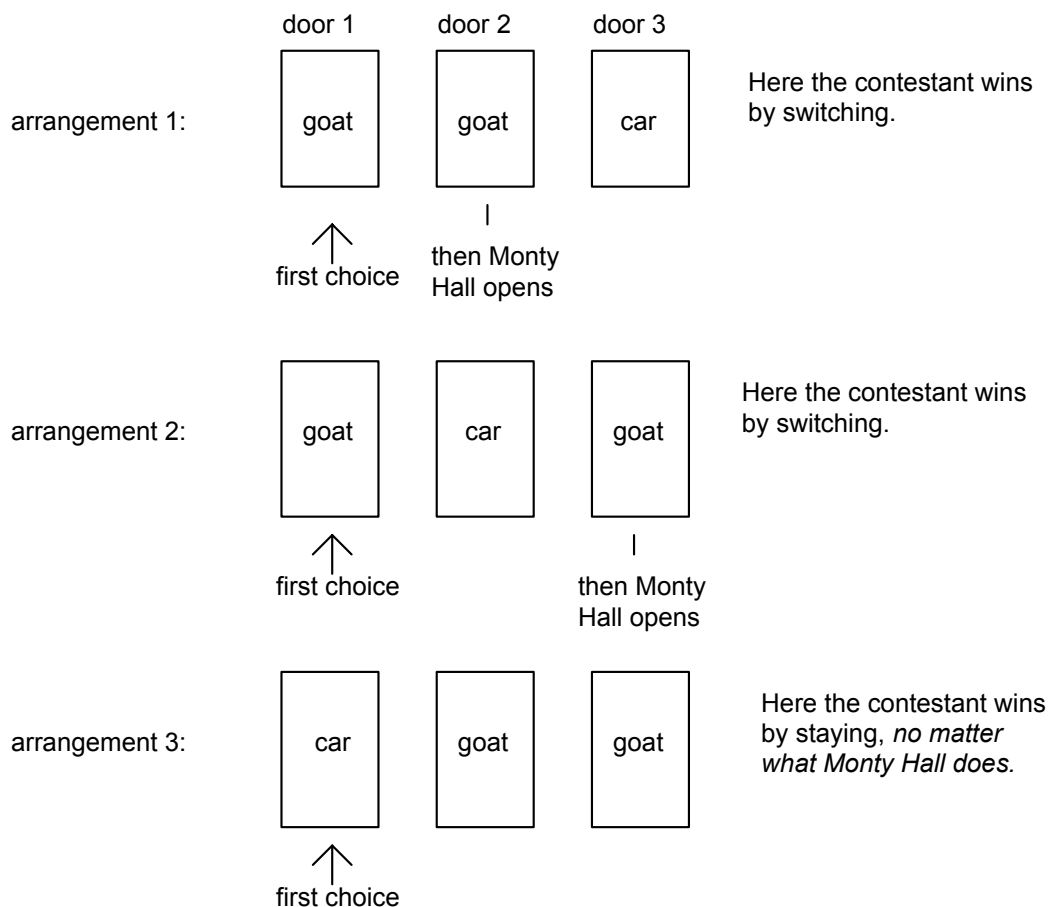


Figure 2.1: Explanation of the solution to the Monty Hall problem that has a good chance of being accepted: In two out of three possible car-goat arrangements the contestant would win by switching and therefore she should switch

Note that Monty Hall's behavior in arrangement 3 of Figure 2.1 is *not* specified (“... no matter what Monty Hall does”). In the literature one finds mostly arrays consisting of more than three single arrangements in which Monty Hall's behavior is executed in each arrangement (see, for instance, Table 2.1 below). We will later demonstrate why *ignoring* Monty Hall's behavior in arrangement 3 turns out to be a crucial building block of an intuitive solution.

By performing a “mental simulation” of the three possible arrangements (i.e., considering the whole sequence of actions specified by each arrangement in Figure 2.1), one can see that in two out of three possible arrangements the contestant would win the car by switching (namely in arrangements 1 and 2). Let us identify some

inter-related features in Figure 2.1 and express them in terms of psychological elements:

1. Rather than reasoning with probabilities one has to count and compare frequencies.
2. These frequencies correspond to possible arrangements of goats and cars behind the doors. One has to compare the number of arrangements in which the contestant would win the car by switching to the number in which she would win by staying.
3. One has to consider the possible arrangements as they would appear from *behind* the doors.
4. One has to ignore the last piece of information provided in the standard version (Monty Hall opens door 3). Taking this information for granted would eliminate arrangement 1 in Figure 2.1, because the host will not open a door concealing a car.

Item 4 demands some clarifications: Although semantically door 3 in the standard version is just labeled by example (“Monty Hall opens another door, *say* number 3”), most participants take the opening of door 3 for granted and base their reasoning on this fact.¹² In a pre-test we gave participants ($N = 40$) the standard version, asking them to illustrate their view of the situation described by drawing a sketch. After excluding 4 uninterpretable drawings, 34 out of the remaining 35 participants (97%) indeed drew an open door 3 and only 1 of them (3%) indicated that also other constellations remain possible according to the wording of the standard version. Note that the assumption of a definitely opened door 3 is further confirmed by the specific closing question: “Do you want to switch to door *number 2*?” Problem solvers seem to have a strong tendency to clutch to concrete numbers present in the problem’s wording – regardless of whether these numbers are fixed or just labeled by example. Note that whenever someone takes the information on the opening of door 3 for granted she no longer has access to the intuitive solution pathway suggested by Figure 2.1.

¹² This claim is also supported by the fact, that participants’ beliefs and justifications in the Monty Hall problem do not differ from the corresponding ones in the “problem of three prisoners” (see footnote 6), where Harry (not: “say Harry”) is named explicitly.

Each of the four features specified above has theoretical underpinnings eliciting correct responses from naive participants. In the following sections we discuss these four features in terms of four psychological elements, namely, *natural frequencies*, *mental models*, *perspective change*, and the *less-is-more effect*.

NATURAL FREQUENCIES

According to Figure 2.1 one needs to consider the three single arrangements and to reason in terms of frequencies, like “in one – or two – out of three arrangements ...”.

In Chapter 1 we saw that representing probabilistic information in natural frequencies facilitates participants’ performance in solving complex Bayesian reasoning problems. Information format may also be an important factor affecting the search for a mathematically correct solution to the Monty Hall problem. Gigerenzer and Hoffrage’s (1995) proposal for improving probabilistic reasoning by translating single-event probabilities (i.e., consider the story of a single woman) into natural frequencies (i.e., consider a whole sample of women) is readily applicable to the Monty Hall problem. Aaron and Spivey (1998) indeed presented the Monty Hall problem in both probability and frequency versions, to different groups of participants.¹³ In one of their experiments, 12% of participants given the probability version gave the correct answer, whereas 29% of participants given the frequency version did. This improvement was achieved after participants given the frequency version saw a pictorial presentation of the problem and answered a series of 11 frequency questions (e.g., “Imagine 30,000 game shows like this [...]. Of the 30,000 rounds in which the player chooses door 1, in how many of them is the car actually behind door 1?”, etc.). The disadvantage of Aaron and Spivey’s frequency version is that the wording no longer has much in common with the standard version. Their formulation of all 11 questions required adding a large number of lines and their manipulation thus looks heavy handed.

In contrast to the frequency procedure used by Aaron and Spivey (1998), the diagram shown in Figure 2.1 does not involve imagining multiple rounds such as

¹³ Although in the standard version of the Monty Hall problem the format is not obviously determined, it clearly does not ask for frequencies. The question “Is it to your advantage to switch?” refers rather to a single-event probability, that is, to the possible outcome of one specific game show.

“30,000 game shows.” Instead, it uses the concept of frequencies in the actual context of a *single-shot game*.

MENTAL MODELS

Following the diagram in Figure 2.1, one has to count and compare conditional outcomes of possible arrangements (e.g., if the car is actually behind door 2, I would win by switching) of one single game show. This sort of case-based mental simulation relates to Johnson-Laird’s (1983) work on the dynamics of logical reasoning. According to his theory, people reason about logical problems, for example, syllogisms, by constructing *mental models*. Recently, Johnson-Laird, Legrenzi, Girotto, Legrenzi, and Caverni (1999) extended this theory to probabilistic reasoning – including reasoning about the Monty Hall problem. In a section of their paper entitled *Pedagogy of Bayesian reasoning* they suggested six mental models, which are illustrated in Table 2.1, that people might use to represent the Monty Hall problem in an intuitive way. In Table 2.1 the word “open” indicates the door that Monty Hall opens after the contestant chooses door 1, and the word “car” indicates the door behind which the car actually is:

Mental Model	Door 1 (Chosen Door)	Door 2	Door 3
1	car	open	
2	car		open
3		car	open
4		car	open
5		open	car
6		open	car

Table 2.1: Mental models to represent the Monty Hall problem (by Johnson-Laird et al., 1999)

The rows correspond to the mental models, each of which represents a possible situation of the Monty Hall problem, given the contestant first chooses door 1. If the car actually is behind door 1, Monty Hall can open either door 2 (mental

model 1) or door 3 (mental model 2). Taking Monty Hall's behavior into account, Johnson-Laird et al. (1999) had to replicate our arrangement 3 in Figure 2.1. Because all arrangements in Figure 2.1 are equally probable, Johnson-Laird et al. consequently also had to replicate our arrangement 1 (which now corresponds to mental models 5 and 6) and our arrangement 2 (which now corresponds to mental models 3 and 4), in which Monty Hall does not have to make a decision, as to what door he opens. Johnson-Laird et al. (1999) did not run an empirical study on whether people actually use these mental models to solve the Monty Hall problem. Although they admitted that the artificial replicating of models (model 3 corresponds with model 4 and model 5 corresponds with model 6) might be difficult to grasp, they proposed that these six mental models can serve as a means of explaining the problem to people.

Interestingly, Marilyn vos Savant also used six "mental models" in her second attempt to explain the Monty Hall problem (vos Savant, 1997, p. 8). Her models had a 3×2 structure, in which the dimensions were the three possible locations of the car and the two possible choice strategies (i.e., stay vs. switch). Yet, as we have learned, this approach did not convince her readers.

Both vos Savant (1997) and Johnson-Laird et al. (1999) suggested six models to explain the Monty Hall problem. We argue that the three-model presentation of Figure 2.1 is a more effective way to represent the problem.

PERSPECTIVE CHANGE

The three mental models from Figure 2.1 are constructed as if one were standing behind the doors and could see each possible arrangement of goats and car. This perspective, which is the one of the game show host, makes it possible to imagine what the host would have to do contingent upon which door the car is behind. Taking the contestant's perspective, in contrast, may block or even blind participants' "view" of the three possible arrangements behind the doors. The idea of investigating the impact of changing perspective on human reasoning has been applied with different aims and in different reasoning tasks (e.g., Gigerenzer & Hug, 1992; Wang, 1996; Fiedler, Brinkmann, Betsch, & Wild, 2000).

With respect to the Monty Hall problem we suggest the following theoretical connection between a perspective change and the structure of Bayes' rule: Assuming

that the contestant first chooses door 1 and that Monty then opens door 3 (the standard version), the probability that the car is behind door 2 can be expressed in terms of Bayes' rule in the following way:

$$p(C_2 | M_3) = \frac{p(M_3 | C_2) \cdot p(C_2)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)} \quad (2.1)$$

with C_i = car behind door i ; $i = 1, 2, 3$; and M_3 = Monty reveals door 3.

The theoretical connection between the perspective change and the structure of Bayes' rule is apparent: When calculating a conditional probability of an arbitrary event A given a condition B, that is, $p(A | B)$, Bayes' rule stipulates that one has to consider the inverse conditional probabilities $p(B | A)$ and $p(B | \bar{A})$. For the Monty Hall problem this means that to judge $p(C_2 | M_3)$ one has to insert the three conditional probabilities $p(M_3 | C_3)$, $p(M_3 | C_2)$, and $p(M_3 | C_1)$ into Bayes' rule. The cognitive process for assessing these three probabilities is independent of the behavior of the contestant but relies on imagining Monty Hall's behavior in all three arrangements. Thus, a Bayesian solution of the problem — whether a formal one based on Bayes' rule or an intuitive one based on Figure 2.1 — focuses on the behavior of the host rather than on that of the contestant. Consequently, the change from the contestant's perspective to Monty Hall's perspective corresponds to a change from non-Bayesian to Bayesian thinking. In the sum-up section we will see that the idea of perspective change in Bayesian reasoning is not restricted to agents' perspectives.

LESS-IS-MORE EFFECT

A common question encountered by both a user and a provider of information is what is the best amount of information that should be used or provided? Goldstein and Gigerenzer (1999) reported empirical evidence that sometimes “knowing less is more”. A clear example provided by the authors is the use of the *recognition heuristic*, which exploits the potential of recognition to help people make inferences. When a situation requires inferring which of two objects has a higher value on some criterion (e.g., which is faster, higher, stronger), the recognition heuristic is simply stated: If one of the two objects is recognized and the other is not, then infer that the recognized object has a

higher value. One of the surprising findings of the authors was that Germans were better than Americans at judging which of two cities in the United States (e.g., San Diego and San Antonio) had the larger population. Why? The German participants, many of whom did not know of San Antonio, could use the recognition heuristic (e.g., infer that San Diego has a larger population than San Antonio because they recognized the former but not the latter). The recognition heuristic is not only frugal in its use of information: It actually *requires* a lack of knowledge to work. This research finding shows that under certain conditions, a counterintuitive *less-is-more effect* appears, in which a lack of knowledge is actually beneficial for inference.

Regarding the door opened by Monty Hall, someone solving the three-door problem can have two possible states of knowledge: First, she just knows that after her first choice Monty Hall *will* open another door revealing a goat, or, second, she already has learned *the number* of this door. Note that participants are only able to provide the intuitive solution (see Figure 2.1) if the specific number of the door which Monty Hall actually opened is *not taken into account*.

The easiest way to make sure that participants' reasoning processes are not interfered with by knowing the door Monty Hall opened, is simply not to give them this information. The corresponding formulation would be: "Monty Hall now opens *another* door and reveals a goat". Although the cognitive situation here differs from the one treated in the recognition heuristic, the underlying principle is the same: Having less information can be beneficial for inference.

The issue of "door information" is of great relevance for the cognitive processes to solve the Monty Hall problem. Before inserting the four psychological elements into the problem's wording, let us have a closer look at the different possible *scenarios* of the problem based on different "door information". Since we learned from the pre-test that the formulation "say number 3" *psychologically* is interpreted as "door 3 is open", we will call expressions such as in the standard version ("you pick a door, say number 1, and the host [...] opens another door, say number 3") *specifications* of doors.

NO-DOOR SCENARIO

If no door were specified in the formulation of the Monty Hall problem (*no-door scenario*), that is, neither that chosen by the contestant nor that chosen by Monty Hall,

then the participant has no restriction for mentally simulating the game show. The contestant's three possible choices and the three possible locations of the car would then yield a total of nine possible arrangements, as illustrated in Figure 2.2:

	A	B	C
1	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">door 1 car</div> <div style="text-align: center;">door 2 goat</div> <div style="text-align: center;">door 3 goat</div> </div> <p style="text-align: center;">↑ first choice</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">door 1 car</div> <div style="text-align: center;">door 2 goat</div> <div style="text-align: center;">door 3 goat</div> </div> <p style="text-align: center;">↑ first choice</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">door 1 car</div> <div style="text-align: center;">door 2 goat</div> <div style="text-align: center;">door 3 goat</div> </div> <p style="text-align: center;">↑ first choice</p>
2	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">goat</div> <div style="text-align: center;">car</div> <div style="text-align: center;">goat</div> </div> <p style="text-align: center;">↑ first choice</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">goat</div> <div style="text-align: center;">car</div> <div style="text-align: center;">goat</div> </div> <p style="text-align: center;">↑ first choice</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">goat</div> <div style="text-align: center;">car</div> <div style="text-align: center;">goat</div> </div> <p style="text-align: center;">↑ first choice</p>
3	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">goat</div> <div style="text-align: center;">goat</div> <div style="text-align: center;">car</div> </div> <p style="text-align: center;">↑ first choice</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">goat</div> <div style="text-align: center;">goat</div> <div style="text-align: center;">car</div> </div> <p style="text-align: center;">↑ first choice</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">goat</div> <div style="text-align: center;">goat</div> <div style="text-align: center;">car</div> </div> <p style="text-align: center;">↑ first choice</p>

Figure 2.2: The nine possible arrangements in a no-door scenario

In Figure 2.2 we label the rows that denote actual car location by numbers and the columns that denote first choice by letters. For instance, the arrangement in which the car stands behind door 3 and the contestant first chooses door 1, is named A3. Figure 2.2 illustrates that a contestant would win the prize in six of the nine possible arrangements by switching doors, but in only three of the nine arrangements (A1, B2, and C3) by sticking with the door initially chosen. Hence, switching affords a better chance of winning. Not specifying a door in the wording would thus allow one to use an intuitive representation that is likely to lead to the correct response. However, this may be suboptimal because it would be difficult to simulate all nine scenarios mentally.

ONE-DOOR SCENARIOS

If the contestant's first choice is specified (*one-door scenario*), then only three arrangements remain possible. If, for instance, the wording is such that the contestant

chooses door 1, then only arrangements A1, A2, and A3 remain (see left column in Figure 2.2). By switching, the contestant would win in two of the three arrangements (A2 and A3) and lose in only one arrangement (A1).

A second type of one-door scenario that may lead to a similar path of intuitive thinking would entail specifying the location of the car. The arrangements then can be illustrated using any row in Figure 2.2. For example, the first row would represent the three possible first choices by a contestant when the car is specified to be behind door 3. One can see that the contestant would win in two of three arrangements by switching (A3 and B3), and in only one arrangement by staying (C3). Thus, whether the door specified is the contestant's first choice or the car position, one-door scenarios allow one to use just one column or one row in Figure 2.2 to gain insight into the correct solution.

In a nutshell, both no-door and one-door scenarios allow unrestricted mental simulations, thereby making the counting and comparison of the frequency of wins possible, given that the contestant switches or stays. Another crucial advantage of all no-door and one-door scenarios is that one does *not* have to think about the behavior of Monty Hall in the cases in which he can choose which door to open. In a one-door scenario where, for instance, the contestant has chosen door 1 first, a correct and sufficient chain of reasoning might go in the following way:

If the car is actually behind door 3, then Monty Hall *must* open door 2, and I win by switching (A3).

If the car is actually behind door 2, then Monty Hall *must* open door 3, and I win by switching (A2).

If the car is actually behind door 1, then I win by sticking to my first choice, *no matter what Monty Hall does* (A1).

In a one-door scenario, Monty Hall's behavior either is determined (A3, A2) or irrelevant for the decision (A1). The problem becomes cumbersome, however, when the door opened by Monty Hall now is specified in addition to the contestant's first choice, as it is the case in *two-door scenarios*.

TWO-DOOR SCENARIOS

The additional specification of the door opened by Monty Hall in the standard version of the problem leaves only two of the three arrangements in the left column in Figure 2.2 (A1 and A2). A3 is impossible because Monty Hall cannot open the door concealing the car. As a result, one cannot simply count and compare the frequencies of winning, given that the contestant switches or stays, but rather one has to reason in probability terms to reach the Bayesian solution. That is, Monty Hall's opening door 3 has a lower probability in A1 than in A2, because in A1 he could have opened either door 2 or door 3, whereas in A2 he had to open door 3. Thus, one has to make assumptions about what Monty Hall would do in A1 and estimate the probability that Monty Hall would open door 3 rather than door 2. Some authors have argued that participants' assumptions about Monty Hall's strategy in A1 may affect their probability judgments, and that the lack of information about this strategy in the standard version may therefore help to explain participants' poor performance on the problem (c.f., von Randow, 1993).

To illustrate this "strategy" argument we use the standard version, in which the contestant chooses door 1. In the left column of Figure 2.2 let us consider one arrangement after the other. This means considering the three conditional probabilities

$p(M_3 | C_3)$, $p(M_3 | C_2)$, and $p(M_3 | C_1)$ according to Equation 2.1:

Arrangement A3: According to the wording "the host ... opens another door ... which has a goat", A3 is no longer possible, which means that the probability $p(M_3 | C_3) = 0$.

Arrangement A2: It also follows from the wording that the probability that Monty Hall opens door 3 given that the contestant first chose door 1 is unity, that is, $p(M_3 | C_2) = 1$.

Arrangement A1: Investigating arrangement A1 reveals that $p(M_3 | C_1)$ reflects *Monty Hall's strategy*.

We now consider three different strategies that he might use concerning arrangement A1:

First, if one assumes that Monty Hall's strategy is to choose randomly when he has a choice, then the probability of his opening door 3, $p(M_3 | C_1)$, equals $1/2$. The probability of the contestant's winning by switching (to door 2) can now be expressed in terms of Bayes' rule:

$$\begin{aligned} p(C_2 | M_3) &= \frac{p(M_3 | C_2) \cdot p(C_2)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)} \\ &= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} = \frac{2}{3} \end{aligned} \quad (2.2)$$

Thus, assuming Monty Hall uses this random strategy, the probability of the contestant winning by switching is indeed equal to what it is in the no-door and one-door scenarios, namely, $2/3$.

Second, if one assumes that Monty Hall's strategy is "to open door 3 whenever possible", then $p(M_3 | C_1)$ equals 1, and the probability of the contestant winning by switching changes to $1/2$:

$$\begin{aligned} p(C_2 | M_3) &= \frac{p(M_3 | C_2) \cdot p(C_2)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)} \\ &= \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} = \frac{1}{2} \end{aligned} \quad (2.3)$$

Third, if one assumes that Monty Hall's strategy is to open door 2 whenever possible, then $p(M_3 | C_1)$ is 0, and the probability of the contestant winning by switching would become 1:

$$\begin{aligned} p(C_2 | M_3) &= \frac{p(M_3 | C_2) \cdot p(C_2)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)} \\ &= \frac{1 \cdot \frac{1}{3}}{0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} = 1 \end{aligned} \quad (2.4)$$

As demonstrated above, different assumptions about Monty Hall's strategy indeed lead to different Bayesian solutions. Note that these different solutions are possible only in *two-door scenarios*, such as the standard version. Taking Monty Hall's strategy into account not only can lead to different solutions but also forces one to reason in terms of probabilities. Furthermore, there is no intuitive diagram that can reflect Monty Hall's strategy appropriately. The advantage of the no-door and one-door scenarios, in which Monty Hall's behavior is not specified, is that participants do not need to consider possible strategies that Monty Hall might use.

ARE THERE POSSIBLE EFFECTS OF INCOMPLETE INFORMATION?

After the Monty Hall problem became famous, many questions on possible effects of incomplete information in the standard version arose. Besides not mentioning *Monty Hall's strategy* (1), the standard version refers to neither the *exact rules of the game show* (2) nor to the *a priori probability distribution of car and goats* (3) (c.f., Nickerson, 1996; von Randow, 1993; Mueser & Granberg, 1999).

(1) The standard version provides no information about Monty Hall's strategy. Is the problem therefore mathematically underspecified and insoluble? The answer is no, because the standard version does not ask for a probability, but only for a decision. The general Bayes' rule for the standard version of the Monty Hall problem in the absence of information about Monty Hall's strategy is:

$$\begin{aligned}
 p(C_2 | M_3) &= \frac{p(M_3 | C_2) \cdot p(C_2)}{p(M_3 | C_1) \cdot p(C_1) + p(M_3 | C_2) \cdot p(C_2) + p(M_3 | C_3) \cdot p(C_3)} \\
 &= \frac{1 \cdot \frac{1}{3}}{p(M_3 | C_1) \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} = \frac{1}{p(M_3 | C_1) + 1} \quad (2.5)
 \end{aligned}$$

where $p(M_3 | C_1)$ is a "strategy" parameter that can vary between 0 and 1.

Since the strategy-dependent probability $p(M_3 | C_1)$ varies between 0 and 1, the conditional probability $p(C_2 | M_3)$ can vary only between 1/2 and 1 (see Equation 2.5). Therefore, whatever strategy one assumes Monty Hall uses, the conclusion is that the contestant should switch. Only if Monty Hall always opens door 3, that is, $p(M_3 | C_1) =$

1 (an assumption for which the wording of the problem does not provide the slightest hint of support) do staying and switching afford the contestant equal chances of winning. Given all other assumptions about Monty Hall's strategy (an infinite set of possible strategies), switching is better than staying. Thus, Equation 2.5 implies that switching is better even in two-door scenarios, regardless of Monty Hall's strategy. Even after clarifying this mathematical question a psychological question remains: Does the lack of information on Monty Hall's strategy hinder participants in choosing the right alternative? It can be read in Ichikawa (1989, p. 271) that letting participants know Monty Hall's strategy does not help them find the solution either.

(2) The conditional probabilities $p(M_3 | C_1)$, $p(M_3 | C_2)$, and $p(M_3 | C_3)$ describe Monty Hall's behavior in different arrangements. This behavior can be influenced either by his personal strategy or by the *official rules* of the game show (in the standard version the intended rule is "after the contestant chooses a door, Monty Hall has to open another door and reveal a goat"). If the rule were, instead, that the host has to reveal a goat if the contestant first chooses the car-door and should otherwise do nothing, then $p(M_3 | C_2) = 0$, which makes the probability of winning by switching 0 (see Equation 2.1).¹⁴ Nickerson (1996) writes: "... without information or an assumption about the host's behavior, the situation is ambiguous, and the question of whether one should switch is indeterminate." (p. 420). Most experimental psychologists consequently insert the intended rule "Monty has to open another door and reveal a goat" into the standard version to avoid criticism about ambiguity in the wording.¹⁵ But this does not seem to help participants: Although Granberg and Brown (1995) stressed this rule, they observed only 13% switch decisions.

(3) As we have seen, we cannot be sure of the values of the conditional probabilities $p(M_3 | C_1)$, $p(M_3 | C_2)$, and $p(M_3 | C_3)$ in the standard version, because we know neither the complete rules of the show nor Monty Hall's personal strategies. What about the remaining terms in Equation 2.2, namely, $p(C_1)$, $p(C_2)$, and $p(C_3)$? One may

¹⁴ Actually, Monty Hall did not use the same rule in every show (see also footnote 1). For a description of the real game show, see, for instance, Friedman (1998).

¹⁵ Interestingly with regard to this aspect the Monty Hall problem differs from the problem of three prisoners. In the problem of three prisoners the rule automatically is specified by Dick asking for the name of another prisoner who will be executed. In the Monty Hall problem the corresponding specification would be that the contestant explicitly asks Monty Hall to open a door with a goat after his first choice.

wonder whether the car was randomly placed behind one of the three doors. In other words, is the assumption of an equal a priori distribution $p(C_1) = p(C_2) = p(C_3) = 1/3$ justified? Perhaps the car is more likely to be placed behind door 1 because it is closest to the entrance of the stage.

A formulation of the Monty Hall problem providing all of this missing information and avoiding possible ambiguities of the expression “say number 3” would look like this (*mathematically explicit version*):

Suppose you’re on a game show and you’re given the choice of three doors. Behind one door is a car, behind the others, goats. The car and the goats were placed randomly behind the doors before the show. The rules of the game show are: After you have chosen a door, the door remains closed for the time being. The game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door. Imagine that you chose door 1, and the host opens door 3, which has a goat. He then asks you “Do you want to switch to door number 2?” Is it to your advantage to change your choice?

Even though the Bayesian solution (Equation 2.2) is now wholly justified, fleshing out the problem in this manner would fail to foster insight into its mathematical structure. The problem is that people still do not have access to an intuitive solution (such as Figure 2.1). We argue that most of the criticisms of the standard version regarding its unstated assumptions are *mathematically* relevant, but not *psychologically* relevant, since the intended assumptions will hold anyway.

Evidence supporting this claim comes from the observation that a vast majority of people wrongly regards the stay and switch choices as equally likely to result in winning. Let us give examples of how assumptions, different from the intended ones, would make this “uniformity belief” in the standard version impossible:

1. If participants assumed that *Monty Hall’s strategy* is “always open middle door when possible” they would know that it was not possible for Monty Hall to open

the middle door because it had the car.” Thus they would not conclude equiprobability of the remaining alternatives but rather switch to door 2.

2. If participants assumed the *game show rule* is that Monty Hall only reveals a goat when the first choice was a car, they also would not follow the uniformity belief but rather have an obvious reason to stay.

3. If participants did not assume the *a priori distribution* $p(C_1) = p(C_2) = p(C_3) = 1/3$ they also would not have any reason to come up with an equiprobable a posteriori distribution.

In sum, when solving the standard version, in which the required assumptions are not made explicit, people seem to assume the intended scenario anyway. Along the same lines, vos Savant observed (1997): “Virtually all of my critics understood the intended scenario. I personally read nearly three thousand letters (out of the many additional thousands that arrived) and found nearly every one insisting simply that because two options remained (or an equivalent error), the chances were even. Very few raised questions about ambiguity, and the letters actually published in the column were not among those few” (p. 15).

In short, people seem to struggle not with the ambiguity of the standard version’s assumptions but with the mathematical structure of the scenario.¹⁶ As we will see in the next section, what blocks correct intuitive reasoning is not lack of information, but lack of the right information representation.

INTUITIVE VERSIONS OF THE MONTY HALL PROBLEM

To formulate wordings of the Monty Hall problem that should elicit the correct solution we take into consideration the four psychological elements discussed earlier as well as the discussion on missing information. The four psychological elements were:¹⁷ (1) perspective change, (2) the less-is-more effect, (3) mental models, and (4) natural frequencies. We incorporated these elements via the following manipulations:

¹⁶ Ichikawa and Takeychi (1990) found that the same is true for the related problem of three prisoners.

¹⁷ The new order 1-4 now follows the order of appearance in the manipulated wording.

1. We manipulated perspective by asking participants to “imagine you are the host of this game show” instead of assigning them the role of the contestant.
2. We used a one-door scenario (this means *not* specifying the number of the door opened by Monty Hall). Relative to the two-door scenario in the standard version, this can be considered an incorporation of a less-is-more effect. The beneficial lack of information about the door opened by Monty Hall would allow participants to reason in terms of frequencies (e.g., Figure 2.1) instead of probabilities.
3. We explicitly mentioned the three possible arrangements of goats and car behind the doors to participants to prime the relevant mental models (i.e., A1, A2, and A3; see Figure 2.2).
4. We asked participants to tell us the frequencies with which the contestant would win by switching and by staying:¹⁸ “In how many of the three possible arrangements would the contestant win the car after the opening of a “goat-door”,
 - if she stays with her first choice (door 1)?
in ___ out of 3 cases
 - if she switches to the last remaining door?
in ___ out of 3 cases

Various versions of the Monty Hall problem can be constructed by incorporating combinations of these manipulations. Note that not all of the possible resulting versions are meaningful. For example, without the *less-is-more* manipulation none of the other manipulations can work. In a two-door scenario, only two arrangements are possible and – as we have seen – considering just two arrangements can never lead to an intuitive correct solution, regardless of whether the right perspective or an intuitive frequency question is provided. Therefore, we consider the less-is-more manipulation a “basic” manipulation that is required before implementing the others. This and other dependencies among the manipulations will be analyzed in detail in the sum-up section.

¹⁸ Thus the incorporation of frequency formats in the Monty Hall problem is done by formulating the *question* in terms of frequencies.

In two studies (Studies 3, 4) we tested seven meaningful and theoretically relevant versions. All versions had similar layouts (see Figures 2.3, 2.4). Our prediction was that the more manipulations are incorporated in the wording of the Monty Hall problem, the better the performance of participants becomes. Furthermore, we expected that when all manipulations are incorporated (we call such a version a *full intuitive version*), the mathematical structure will become accessible to humans' reasoning and participants' performance will no longer support the claim that the Monty Hall problem is an insurmountable cognitive illusion.

As the *control version* (Figure 2.3), we used a variant of the standard version, which was clear without ambiguity.

LET'S MAKE A DEAL	
Rules of the game show:	<p>There is a game show called “Let’s Make a Deal”, where a contestant is allowed to choose one of three closed doors. Behind one door is the first prize, a new car; behind each of the other doors is a goat. After the contestant has chosen a door, the door remains closed for the time being. According to the rules of the game, the game show host, Monty Hall, who knows what is behind the doors, now has to <i>open one of the two unchosen doors and reveal a goat</i>. After Monty Hall shows a goat to the contestant, he asks the contestant to decide whether s/he wants to stay with the first choice or to switch to the last remaining door.</p> <p>Task:</p>
Perspective of the contestant:	Imagine you are the contestant and you do not know which door the car is behind.
Specifying the first choice:	You now randomly choose a door, say number 1.
Diagram of three doors:	<p>HOST ×</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>door 1</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto; display: flex; align-items: center; justify-content: center;">—</div> </div> <div style="text-align: center;"> <p>door 2</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto; display: flex; align-items: center; justify-content: center;">—</div> </div> <div style="text-align: center;"> <p>door 3</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto; display: flex; align-items: center; justify-content: center;">—</div> </div> </div> <div style="text-align: center; margin-top: 10px;"> <p>↑</p> <p>×</p> <p>CONTESTANT (You)</p> </div>
Specifying the door opened by Monty Hall:	<p>Afterwards Monty Hall opens door 3 according to the rules of the game, and shows you a goat. Now he asks you whether you want to stay with your first choice (door 1) or to switch to door 2.</p> <p>After Monty Hall’s opening of a goat-door, what should you therefore do?</p>
Asking just for a decision:	<p><input type="checkbox"/> stay <input type="checkbox"/> switch</p>
Asking for justification:	<p>Important: Please tell us in writing what went on in your head when you thought of your answer. In explaining your answer, you may make use of things like sketches, etc. Please also tell us if you learned this game before ___ (Yes) ___ (No) and knew what the correct answer should be ___ (Yes) ___ (No).</p>

Figure 2.3: Control version of the Monty Hall problem (left column was not provided to participants)

Note that in addition to the standard version our control version contained the following features: We added the rule of the game show to give total clarification of the intended scenario and to guarantee comparability with other studies. Furthermore, to reduce variance in participants' assumptions, we eliminated the word "say" when specifying the door opened by Monty Hall.¹⁹ To describe the scenario more vividly we added a diagram of the three doors. Finally, because we were not only interested in participants' actual decisions but also in how many "switchers" have genuine mathematical insight into the problem, we asked participants for justifications of their decisions. The impact of these additional changes, which are not the four intended manipulations, can be assessed as a byproduct by comparing participants' performance in our control version (Figure 2.3) with that usually obtained with standard versions.

Figure 2.4 now illustrates a *full intuitive version*, which incorporates all four psychological elements into our control version.

¹⁹ Note that there is no need to delete the "say" belonging to the contestants' first choice because the following diagram delivers total clarification. Since the opening of a goat-door by Monty Hall is not displayed in the diagram, the specification of this door by deleting "say" is at least *semantically* appropriate (even if not *psychologically* required – as our pre-test suggests).

LET'S MAKE A DEAL	
The rules of the game show:	There is a game show called “Let’s Make a Deal”, where a contestant is allowed to choose one of three closed doors. Behind one door is the first prize, a new car; behind each of the other doors is a goat. After the contestant has chosen a door, the door remains closed for the time being. According to the rules of the game, the game show host, Monty Hall, who knows what is behind the doors, now has to <i>open one of the two unchosen doors and reveal a goat</i> . After Monty Hall shows a goat to the contestant, he asks the contestant to decide whether s/he wants to stay with the first choice or to switch to the last remaining door.
Perspective change to the perspective of Monty Hall:	Task: Imagine you are Monty Hall, the host of this game show, and you know behind which door the car is.
Specifying the first choice:	The contestant chooses a door, say number 1.
Diagram of three doors:	<p>× host (you)</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>door 1</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto; display: flex; align-items: center; justify-content: center;">—</div> </div> <div style="text-align: center;"> <p>door 2</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto; display: flex; align-items: center; justify-content: center;">—</div> </div> <div style="text-align: center;"> <p>door 3</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto; display: flex; align-items: center; justify-content: center;">—</div> </div> </div> <div style="text-align: center; margin-top: 20px;"> <p>↑</p> <p>× contestant</p> </div>
Not specifying the door opened by Monty Hall (less-is-more manipulation):	Afterwards you open another door according to the rules and show the contestant a goat. Now you ask her whether s/he wants to stay with the first choice (door 1) or to switch to the last remaining door.
Motivate building mental models:	There are <i>three doors</i> behind which the car can be.
Frequency question:	<p>In how many of these three possible arrangements would the contestant win the car after your opening of a “goat door”,</p> <p>- if s/he stays with the first choice (door 1)? in ___ out of 3 cases</p> <p>- if s/he switches to the last remaining door? in ___ out of 3 cases</p>
Asking for justification:	<p>What should the contestant therefore do?</p> <p style="text-align: center;">___ stay ___ switch</p> <p>Important: Please tell us in writing what went on in your head when you thought of your answers. In explaining your answers, you may make use of things like sketches, etc. Please also tell us if you learned this game before ___ (Yes) ___ (No) and knew what the correct answer should be ___ (Yes) ___ (No).</p>

Figure 2.4: “Full intuitive version” of the Monty Hall problem (left column was not provided to participants)

To examine the impact of incorporating certain combinations of the four manipulations on participants' performance, we conducted two studies, one in Germany (Study 3) and one in the United States (Study 4).

STUDY 3

Method

In this study, we had three groups of participants (Groups 1-3). We compared the control version (Group 1; see Figure 2.3 for English translation) with the full intuitive version of the Monty Hall problem (Group 3; see Figure 2.4 for English translation). We also tested a version in which only the less-is-more manipulation was incorporated²⁰ (Group 2). After excluding participants who had already heard of the Monty Hall problem we had 135 students (47 men and 88 women) whose average age was 24.7 years. Participants were students of different disciplines and were recruited from various universities in Berlin. They were tested at the Max Planck Institute for Human Development in small groups of up to five people. Each participant received only one version of the Monty Hall problem: In Group 1 (control group) we had 67 participants; in Groups 2 and 3 we had 34 participants each. After the experiment, every participant received a payment of 10 DM (approximately U.S. \$5).

²⁰ Why this manipulation is a basic manipulation, which is required before implementing the other manipulations, we will elucidate in the sum-up section. For the wording of the version of Group 2 see Appendix II.1.

Results of Study 3

The results of Study 3 are summarized in Table 2.2:

	Group 1 Control version	Group 2 Basic manipulation	Group 3 Full intuitive version, all four manipulations
Participants (<i>N</i>)	67	34	34
Less-is-more effect	No	Yes	Yes
Specified door(s)	first choice and door opened by Monty Hall specified	only first choice specified	only first choice specified
Perspective	Contestant	Contestant	Monty Hall
Frequency question for mental models ("Frequency simulation")	No	No	Yes
Switch choice total	21%	38 %	59 %
Mathematically correct justification of switch choice	3%	12 %	38 %

Table 2.2: Percentages of switch choice total and mathematically correct justification as a function of experimental manipulations in Study 3; the version of Group 2 is shown in Appendix II.1

Before we present a detailed analysis of Table 2.2, let us summarize the main results: Incorporating the combination of all four psychological elements into the Monty Hall problem evidently had a strong effect. The full intuitive version (Group 3) greatly facilitated switch choices. The percentage of switch choices (59%) in this group sets a new standard in the literature on the Monty Hall problem.²¹ The performance of participants who received the less-is-more manipulation only (Group 2) lies, as expected, between those of the other two groups. However, the focus of our study is on fostering mathematical insight. It can be seen in Table 2.2 that 38% of participants in

²¹ Mueser and Granberg (1999) obtained more than 70% switch decisions by offering participants an additional monetary incentive if they switched. Yet, this certainly does not qualify as an attempt to foster insight into the mathematical structure of the problem.

Group 3 demonstrated total understanding of the underlying problem structure. How should we rank this percentage?

Unfortunately, most studies on the Monty Hall problem just report the percentage of switch decisions, which usually is a number around 10-15%. We expect that given a standard or a similar version of the Monty Hall problem, the percentage of participants who have the correct mathematical insight would be much lower. This assumption is reinforced by participants of our control group (Group 1) of whom only 3% solved the problem by mathematically correct reasoning. The difference in terms of real insight from Group 1 (3%) to Group 3 (38%) must be considered with respect to how notorious the Monty Hall problem is for its counter-intuitiveness.

Let us look at the performance of paradigms differing from ours. Hell and Heinrichs (2000) obtained 65% switch decisions by investigating a variant of the problem with 30 doors, where 28 doors were opened after the first choice. Yet, increasing the number of doors changes the problem's structure substantially: Opening all doors except the first chosen and door number 21 clearly suggests a reason for not opening door 21. Indeed, the probability of winning by switching in this scenario is 97%.

Also in studies using simulation of multiple trials remarkable performance after several rounds of feedback was reached (Granberg & Brown, 1995; Friedman, 1998). However, in these repeated game settings, participants' insight into the mathematical structure of the Monty Hall problem is not a given: From the first rounds participants can realize through feedback that switching pays – yet, they do not necessarily know *why*. Our approach – in contrast – is to increase performance and insight in the notorious original problem – neither by changing the number of doors nor by changing the number of rounds.

In Table 2.2, as in the following tables displaying empirical results, the mental model and the frequency manipulations are combined. The reason for this is that both elements are theoretically and practically connected to each other: On the one hand, the frequency question alone automatically evokes building mental models, because these are the instances to be counted and compared. On the other hand, building mental models can be only half of the process that leads to a problem's solution. The correct answer can only be reached if the mental models are then counted and compared with

respect to their outcomes. In the following we will refer to this combined manipulation as “frequency simulation”. We choose the word “simulation” instead of “model” because in Figure 2.1 just building the three “models” is not enough. Each model first has to be “simulated” (i.e., considering the whole sequence of actions specified by the model) until the outcome becomes apparent. As we know, such a cognitive procedure within a model is not intended in Johnson-Laird’s notion of mental models.

The significant improvement when applying all elements together motivates the analysis of these underlying elements and their individual impact on participants’ performance. For example, at first glance the frequency question alone seems like a heavy-handed hint for how to solve the problem. Yet, as Study 4 will reveal, this seemingly powerful hint does not work effectively if a certain perspective is not provided at the same time.

STUDY 4

A key question concerning the findings of the German study (Study 3) is whether we need all four conceptual manipulations to foster people’s insight into the structure of the Monty Hall problem. Is any one of the manipulations more crucial than the others? Are there synergistic effects of the manipulations? In a second study conducted in the United States (Study 4), we examined four different versions that were designed to partition the effects of the four crucial manipulations.

Method

Participants of Study 4 were students recruited from the University of South Dakota. After excluding participants who had already heard of the Monty Hall problem, we had a total of 137 participants (96 women and 41 men) with an average age of 22.7 years. Participants were randomly assigned to four different groups (Group 4-7) and were tested in a classroom with 10-30 students in each session. The experimental groups 5-7 included 34 participants and Group 4 had 35. Each participant received only one version of the Monty Hall problem. As in Study 3, participants in Study 4 were asked to report in writing a justification for their choices. Participants were rewarded with extra course credit. The four groups received the following versions of the problem:

- Group 4: US-Control version (the same as Group 1 of Study 3; see Figure 2.3)
- Group 5: A one-door scenario in which the first choice is specified (“less-is-more” manipulation). The participants were asked to take the contestant’s perspective and make the stay-switch choice after answering the frequency question (“frequency simulation”).
- Group 6: A one-door scenario (“less-is-more” manipulation) in which the car position is specified. The participants were asked to take Monty Hall’s perspective (“perspective change”) and make the stay-switch choice without first having answered the frequency question.
- Group 7: A one-door scenario (“less-is-more manipulation”) in which the car position is specified. The participants were asked to take Monty Hall’s perspective (“perspective change”) and make the stay-switch choice after answering the frequency question (“frequency simulation”). Since all four manipulations are incorporated this is – besides the version of Group 3 – another full intuitive version.

The versions of groups 5 – 7 are shown in Appendix II.1. Note that in the versions for groups 6 and 7, the one-door scenario is incorporated by specifying the car position instead of the contestant’s first choice (as in the versions of groups 2, 3 and 5). In reference to Figure 2.2, specifying the car position requires reasoning “row-wise” and the frequency question now demands imagining the three possible first choices of the contestant instead of the three different car-goat arrangements. Interestingly, in conditions where the car position is specified (groups 6 and 7) no meaningful version from the perspective of the contestant can be formulated – regardless of whether the frequency simulation is implemented or not: Revealing participants in such a one-door scenario the car position (e.g., “the car is behind door 1”) would provoke the following answer: As a contestant, I would choose door 1, and then I would stay with this choice.

Results of Study 4

The results of Study 4 are presented in Table 2.3:

<i>All versions are shown in Appendix II.1</i>	Group 4 Control version	Group 5	Group 6	Group 7 Full intuitive version, all four manipulations
Participants (<i>N</i>)	35	34	34	34
Less-is-more effect	No	Yes	Yes	Yes
Specified door(s)	first choice and door opened by Monty Hall specified	only first choice specified	only car position specified	only car position specified
Perspective	Contestant	Contestant	Monty Hall	Monty Hall
Frequency question for mental models (“Frequency simulation”)	No	Yes	No	Yes
Switch choice total	23 %	26 %	29 %	50 %
Mathematically correct justification of switch choice	0 %	9 %	9 %	32 %

Table 2.3: Percentages of switch choice total and mathematically correct justification as a function of experimental manipulations in Study 4

The full intuitive version (Group 7) elicited the best performance in Study 4, comparable to that on the full intuitive version in Study 3 (Group 3): Half of participants in Group 7 chose to switch, and 32% of them provided a mathematically correct justification for their choice. The performances of participants in groups 5 and 6 lie, as expected, between that of the control group (Group 4) and that of the full intuitive version (Group 7). For a detailed analysis of the results of Study 4 let us consider the results of studies 3 and 4 together.

COMPARISON OF STUDIES 3 AND 4

The similar performance of the two identical control versions (Groups 1 and 4) suggests that it would be reasonable to look at the results collapsed between studies. Let us consider participants' performance in all seven groups simultaneously (Table 2.4).

	Group 1 Control version	Group 4 Control version	Group 5	Group 6	Group 2	Group 7 Full intuitive version	Group 3 Full intuitive version
Participants (<i>N</i>)	67	35	34	34	34	34	34
Less-is-more effect	No	No	Yes	Yes	Yes	Yes	Yes
Specified door(s)	First choice + door opened	First choice + door opened	First choice specified	Car position specified	First choice specified	Car position specified	First choice specified
Perspective	Contestant	Contestant	Contestant	Monty Hall	Contestant	Monty Hall	Monty Hall
Frequency simulation	No	No	Yes	No	No	Yes	Yes
Switch choice total	21%	23 %	26 %	29 %	38 %	50 %	59 %
Mathematically correct justification	3%	0 %	9 %	9 %	12 %	32 %	38 %

Table 2.4: Complete results of Study 3 and Study 4, ordered according to the percentages of switch choice in each group; visual display of same results is provided in Figure 2.5

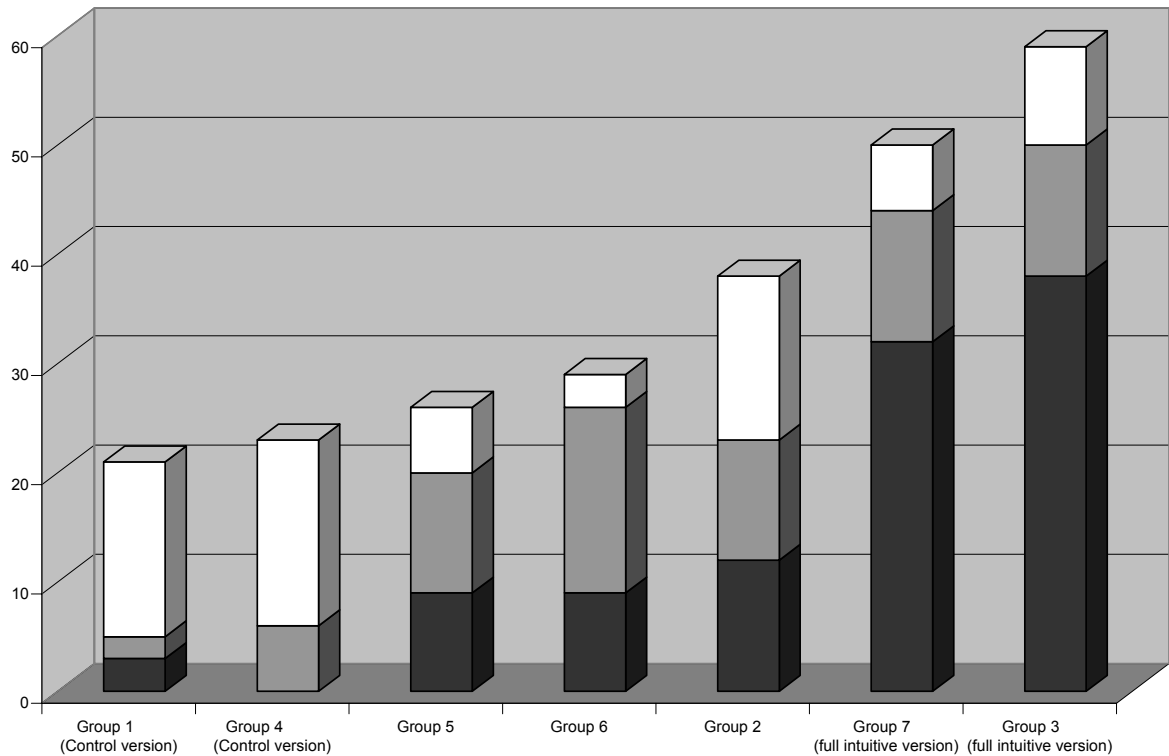
For a more detailed analysis of participants' performance we furthermore classified the "switch choices" of all seven groups into the following categories:

- Participants who gave correct solutions and exhibited full insight into the mathematical structure
- Participants who had the right intuition but could not provide a mathematically correct proof
- Participants who switched randomly, meaning that switching and staying are equally good

To summarize: The category "switched with mathematical insight" remained the same, whereas the other switchers were classified according to their "degree of

insight”. All participants’ protocols could easily be classified within this scheme. Interestingly the “stay choices” did not require a similar categorization: None of these participants had the feeling that staying might be better from a mathematical point of view.

Figure 2.5 illustrates the results for all seven versions, according to our new categorization and ordered by observed percentage of switch decisions:



- Switched randomly, meaning that switching and staying are equally good
- Had the right intuition, but could not provide a mathematically correct proof
- Gave the correct solution and exhibited full insight into the mathematical structure

Figure 2.5: Complete results of Study 3 and Study 4, ordered according to the percentages of switch choice in each group

The difference between the performance on the two control versions (on the left in Figure 2.5) and that on the two full intuitive versions (on the right in Figure 2.5) demonstrates the strong impact of the simultaneous incorporation of all four

manipulations. In groups 3 and 7 the better performance in terms of really grasping the mathematical structure of the task clearly can be attributed to the four psychological manipulations. The similar performance of groups 3 and 7 – as well as analyzing participants' protocols – suggest that the kind of one-door scenario (specifying first choice vs. specifying car position) makes no remarkable difference. The synergistic effect of the combination of the perspective change and the frequency simulation is particularly intriguing. The perspective change alone (Group 5) and the frequency simulation alone (Group 6) both failed to facilitate understanding: The percentage of mathematically correct justifications in Group 5 and Group 6 were not significantly different from that in Group 2, where just the one-door scenario was given. Thus the good performance of participants in the full intuitive versions *cannot* be attributed to the seemingly powerful frequency question alone. In fact there seems to be a synergistic effect between the frequency simulation and the perspective change since both manipulations have to be applied simultaneously. From the trichotomous categorization (Figure 2.5) it can also be seen that the counter-intuitively better performance in Group 2 compared to groups 5 and 6 is in fact due to a relatively high proportion of participants who had no insight but switched by following the uniformity belief (empty bars).

Let us finally consider the impact of our “additional changes”. Our control version was a modification of the standard version: We specified the rule of the game, we used a diagram of the three doors, we deleted the ambiguous word “say” when specifying the door Monty Hall opens, and we asked for justification of the choice. These changes alone indeed did increase the proportion of switch decisions somewhat, compared to the standard version, namely from the usual 10-15% up to about 22%. However, the percentage of participants who gained insight into the problem's structure only because of these non-experimental changes in both control versions was negligible.

STUDY 5

From Piattelli-Palmarini we learned that even Nobel physicists *insist* on the wrong answer to the Monty Hall problem and are ready to berate in print those who propose the right answer (quoted in vos Savant, 1997, p. 15). People's resistance to the correct solution has been widely cited; even Marilyn vos Savant repeatedly failed in explaining the teaser to her readers (vos Savant, 1997). Thus, the questions of Study 5 are pedagogical ones: Is it possible to get insight into the mathematical structure of the problem across to the problem solvers? What kind of complete demonstration of the entire solution might help here? In Study 5 we investigated this question by checking whether people were able to apply different explanations to other, similar problems.

Method

We divided participants of Study 3 (German study) – independent of the previous divisions into groups 1-3 – randomly into three new groups. After they had finished solving the Monty Hall problem – successfully or not – they were asked to continue the session for an extra payment. The interested participants got one of three different explanations on how to solve the Monty Hall problem (all three explanations are shown in Appendix II.2). All explanations were based on demonstrations of the correct solution explicated above: The first explanation was based on Figure 2.1, the second on Table 2.1, and the third on Equation 2.1.

1. Figure 2.1 Frequency simulation of the three arrangements, shortened to “FS”
2. Table 2.1 Six mental models by Johnson-Laird et al. (1999), shortened to “6MM”
3. Equation 2.1 Bayes' rule, shortened to “BR”

Table 2.5: Different explanations of the Monty Hall problem given to participants after Study 3

According to Table 2.5, we provided participants with written two-page explanations that we call “FS” (*Frequency Simulation Explanation*), “6MM” (*Six Mental Models Explanation*), and “BR” (*Bayes' Rule Explanation*). To investigate the benefit that participants can take out of each of these explanations we provided them with four new problems. These problems were closely related to the original Monty

Hall problem but required additional transformations or generalizations. To the best of our knowledge, empirically testing explanations of the Monty Hall problem is virgin soil in this type of research.

After completing their work on the original problem participants received the two-page explanation and had plenty of time to read it.²² Once they said that they had understood it they were provided with problem A (for the wording see Appendix II.3). Problem A was the so-called “Russian roulette dilemma” which is identical to the Monty Hall problem except that now there are *two* cars and *one* goat behind the doors.²³ After 10 weeks participants were recruited again and had then to solve problems B, C, and D (for the wordings see Appendix II.3). Problems B and C were extended Monty Hall problems, both with four doors, and problem D, finally, was the “problem of three prisoners”. In Study 5, problems A, B, C, and D were provided in their “standard version”, that is, without any manipulations.

We offered all participants of Study 3 the opportunity to participate in Study 5. Yet, to extract the pure impact of the different explanations we had to exclude all participants who had already solved the original problem with full mathematical insight, because these presumably would profit from their insight into the original problem. In Study 3 we had 145 participants with wrong – or at least not correctly justified – solutions. Of these 145 participants 95 volunteered to continue the session for an extra-payment and worked on problem A. When 10 weeks later the same participants were asked to come again for the second measurement date, 68 participants, who were *not* told that they would be tested again with tasks related to the Monty Hall problem, appeared and worked on problems B, C, and D. Luckily, the distribution of students across explanations still roughly was an equal distribution: Ten weeks after Study 3, 22 participants of the “FS” condition, 25 of the “BR” condition, and 21 of the “6MM” condition showed up again.

²² Because mathematical formulas are difficult to remember we gave Bayes’ rule an advantage: Whereas participants with explanation “BR” could make use of their written two-page explanation when solving problems A, B, C, and D, participants receiving explanations “FS” and “6MM” had to give their explanation sheet back after studying it. When solving problems A, B, C, and D they had no access to the explanation sheet but had to rely on their memory exclusively.

²³ Here instead of a goat, the host has to reveal a car after the contestant’s first choice. Because the contestant is not allowed to choose the opened door, in the Russian roulette dilemma she should stay. Since most participants in this problem would do this intuitively anyhow, the problem here is not to find the *right decision* but exclusively the *mathematical justification*.

Results

The results of Study 5 are displayed in Figure 2.6.

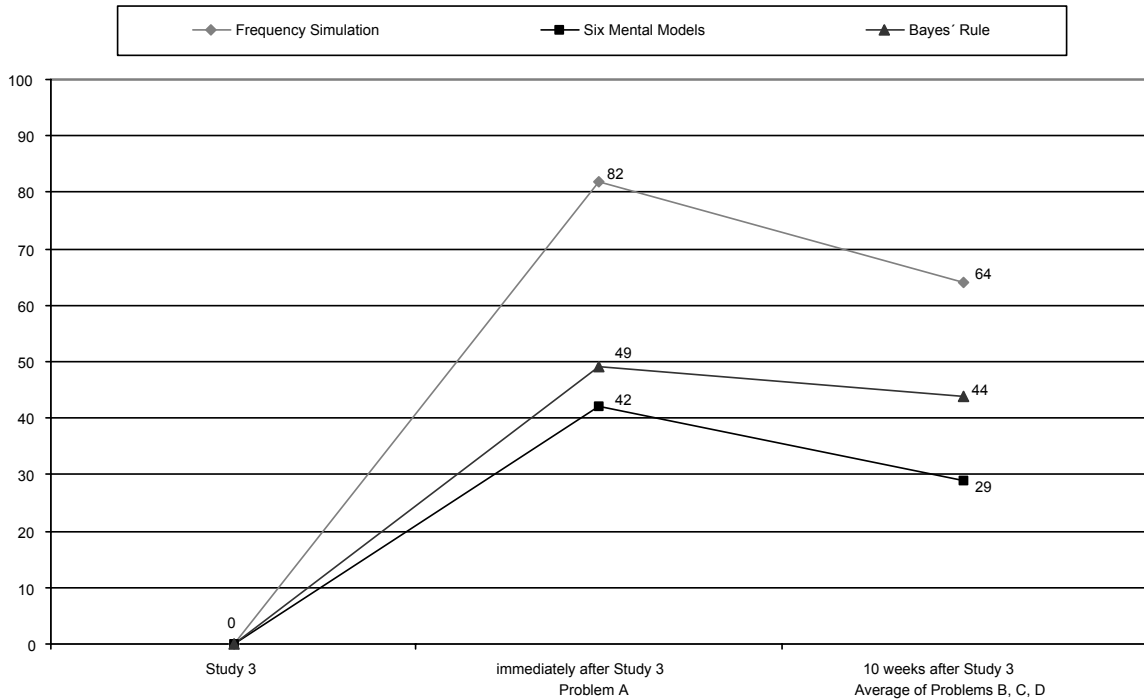


Figure 2.6: Percentage of correct solutions of problems A, B, C, and D contingent on the received explanation

First note that percentages of correct solutions in Figure 2.6 stand for full mathematical insight according to the didactical aim of Study 5. Thus, the 0% marks on the left side indicate that none of the participants solved the original problem in Study 3 with full mathematical insight. Further note that on the right side of Figure 2.6 (10 weeks later) the average performance across all three tasks for all participants is displayed: Since 10 weeks later they had to solve *three* tasks, each participant could contribute to the total performance with 0%, 33%, 67% or 100%.

Which explanation could best break people's widely cited resistance? It can be seen from Figure 2.6 that "FS" participants exhibited the best performance, both immediately after the explanation and 10 weeks later. 82% of participants in the "FS" condition could apply their gained knowledge afterwards to problem A and even 10 weeks later 64% of them solved problems B, C, and D, which indicates that they still

had the “FS” explanation in mind. Due to the mathematical similarity of problems B, C and D to the original Monty Hall problem, we hypothesize that under normal circumstances these problems will be solved *insightfully* by at most 5-10% of unaided people.

Table 2.6 displays participants’ performance on each of the problems individually.

	FS (<i>N</i> = 22)	BR (<i>N</i> = 25)	6MM (<i>N</i> = 21)	Correct solutions across explanations
Problem B	73%	60%	29%	54%
Problem C	73%	28%	43%	48%
Problem D	46%	44%	14%	35%
Correct solutions across problems	64%	44%	29%	

Table 2.6: Participants’ performance (correct solutions) on problems B, C and D – 10 weeks after Study 3

Interestingly, it seems to be more difficult for participants, when the mathematical structure is identical to the explained problem but the *content* is changed (Problem D: problem of three prisoners; 35% correct solutions across all explanations) than when the content stays identical but the mathematical structure is extended (Problems B and C: extended Monty Hall problems; 54% and 48% correct solutions across all explanations).

Remarkably, in the “BR” condition almost half of participants could make use of Bayes’ rule – even 10 weeks later (note that in problems B and C they had to extend the denominator of Equation 2.1 from three to four summands). Yet, it has to be taken into account that, in the session 10 weeks later, the “BR” explanation sheet had again been distributed (see footnote 14). We speculate that the need to construct Bayes’ rule from memory after 10 weeks would drop the performance down to close to 0%. The reason why “FS” participants – and even “BR” participants – outperformed “6MM” participants can be found by looking at protocols: Specifying Monty Hall’s behavior entails the need to artificially double the mental models (see Table 2.1). Trying to perform this doubling – already considered problematic by Johnson-Laird et al. (1999)

– seemed to be the stumbling block for participants’ reasoning in problems A, B, C, and D. Furthermore, we assume that the manner of displaying the 6 mental models – which we accurately adopted from Johnson-Laird et al. (1999) – is not the most advantageous one. Possibly, their 6 mental models would work better if presented like our 3 arrangements (using arrows etc.).

To conclude, inducing in participants a mental simulation according to Figure 2.1 – either through the problem’s *formulation* (full intuitive versions of studies 3 and 4) or through an *explanation* (“FS” explanation of Study 5) – seems to be a powerful means of fostering insight into the structure of the notorious Monty Hall problem – as well as into related problems.

SUM UP

We now recapitulate and consider the relevance of the crucial findings in this chapter.

Frequency question and mental models

In our tables presenting the results of studies 3 and 4 (Tables 2.2, 2.3, and 2.4) the two theoretical elements, *mental model* and *natural frequencies* were collapsed into the *frequency question for mental models*, or shortened, the “frequency simulation”. Johnson-Laird et al. (1999) claimed that the mental model concept provides another theory of probabilistic reasoning that is different from the natural frequency approach (see Gigerenzer & Hoffrage, 1995). It is true that both elements stress different aspects of knowledge representation: On the one hand, the term natural frequencies emphasizes external information representation. In nature we observe frequencies of outcomes instead of probabilities and our minds are adapted to this kind of information. Thus the natural frequency approach provides an ecological explanation for why humans are good at dealing with frequencies. On the other hand, the term mental models stresses internal information representation. Yet, when considering the actual reasoning process natural frequencies and mental models are deeply intertwined: The frequency question (“in how many of these three possible arrangements ...”) is answered by counting arrangements, which actually are mental models. This intertwining is not restricted to the Monty Hall problem. Both the *natural frequency* approach proposed by Gigerenzer and Hoffrage (1995) and Johnson-Laird et al.’s (1999) *numerical mental model*

approach make use of mental models as frequency assessments. Therefore – with respect to probabilistic thinking – mental models are not an alternative to the natural frequency approach, but rather a redescription of the same underlying cognitive process. However, in the Monty Hall problem just considering and counting mental models is not sufficient: Only the *mental simulation of these models* (i.e., considering the whole sequence of actions specified by the model) leads to their outcomes, which then can be counted and transferred into a frequency answer.

Perspective change and Bayes' rule

Taking the game-show host's perspective helped participants imagine which door they would open as the host after the contestant has chosen door 1. Monty Hall's perspective opens a pathway to the insight that the game show host in two out of three arrangements has no choice: Whenever the contestant first chooses a goat, Monty Hall *has to reveal the other goat* and the contestant wins by switching. Taking the game-show host's perspective is to take a Bayesian view: The question in the standard version ("Is it to your advantage to switch your choice?") corresponds to the left side of Equation 2.1, that is, $p(C_2 | M_3)$. To calculate this conditional probability with Bayes' rule one has to assess the ingredients of the right side of Equation 2.1. Although clear on the equal distribution for the car position, that is $p(C_1) = p(C_2) = p(C_3) = 1/3$, the conditional probabilities $p(M_3 | C_1)$, $p(M_3 | C_2)$, and $p(M_3 | C_3)$ remain to be assessed. In our view this is the step that requires looking at the possible arrangements through Monty Hall's eyes: What is the probability that Monty Hall will open door 3, if the car actually is behind door 1, behind door 2, or behind door 3? Arriving at the correct solution requires detecting the constraints posed on Monty Hall. Focusing on his behavior leads to a straightforward Bayesian solution, be it the formal one (Bayes' rule) or an intuitive one (according to Figure 2.1).

Interestingly, the idea of perspective change in Bayesian reasoning is not restricted to agents' perspectives. Consider, for instance, another well-known probabilistic puzzle, the "Three cards problem":

"Three cards are in a hat. One is black on both sides (the black-black card). One is white on both sides (the white-white card). One is black on one side and white on the other side (the black-white card). A card is randomly drawn such that one can only

see one side. The side is black. What is the probability that it is the black-black card?”

Bar-Hillel and Falk (1982) reported that only 7% of their participants found the correct solution. In terms of Bayes’ rule the correct solution is:

$$p(\text{BC} | \text{BS}) = \frac{p(\text{BS} | \text{BC}) \cdot p(\text{BC})}{p(\text{BS} | \text{BC}) \cdot p(\text{BC}) + p(\text{BS} | \neg\text{BC}) \cdot p(\neg\text{BC})} = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{2}{3}} = \frac{2}{3} \quad (6)$$

where BC: Card is the black-black card, BS: A black side can be seen

How can a perspective change be realized? The original version asks what the probability of a *card* is, namely $p(\text{BC} | \text{BS})$. Bayes’ rule solves the problem by switching BC’s and BS’s position and thus by concentrating on the *sides* of the cards. Thus, instead of asking “What is the probability that it is the black-black card?”, the perspective change could be implemented by asking: “What is the probability that the other side is also black?” Of course, this probability question now in addition can be turned into a frequentistic one: “How many of the possible black sides are black also on the other side?”

Implementing the perspective change and the frequency question along these lines into the “Three card problem” and testing the resulting version informally with psychology students of a seminar ($N = 30$) led to 43% correct responses.

The less-is-more manipulation

Deleting useless information can facilitate reasoning performance for various reasons. Consider, for instance, the famous *Linda problem*:

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which probability has a higher value?

- (a) Linda is a bank teller
- (b) Linda is a bank teller and is active in the feminist movement

The “conjunction fallacy” – which is assigning (b) a higher probability than (a) – disappears if we simply ask the question without the prefixed description. Whether the grasp of the conjunction rule in principle is a genuine human skill or not, this ability can be misguided by distracting information.

In the Monty Hall problem assuming – for whatever reason – that Monty Hall actually opened door 3 prevents the construction of a mental model in which the car is behind door 3. Only if this “door information” does not interfere with the problem solving process, participants can adopt the intuitive path of reasoning according to Figure 2.1.

The cognitive variability in the three examples – the recognition heuristic (inferring that San Diego is larger than San Antonio merely from never having heard of San Antonio), the Linda problem, and the Monty Hall problem – shows the widespread validity of “less-is-more” effects.

Synergistic effects of manipulations

Among the manipulations we introduced there exist various dependencies, either mathematical or psychological. Why is the less-is-more manipulation a “basic” manipulation? The frequency simulation (frequency question for mental models) requires the less-is-more manipulation, because only in a one-door scenario can the frequency counting be applied to the relevant mental models: Not implementing the less-is-more manipulation would lead to the uniformity belief, since participants would be limited to two possible arrangements. The perspective change requires the previous incorporation of the less-is-more manipulation for the following reason: Simulating Monty Hall’s behavior concerning the relevant arrangements becomes impossible when this behavior is already specified. In a two-door scenario the door opened by Monty Hall is given to the problem solver. Thus she can no longer exploit the perspective change by simulating the opening of different doors in the different arrangements. Consequently, we consider the *less-is-more* incorporation as a prerequisite manipulation for incorporating the remaining three psychological elements *natural frequencies*, *mental models*, and *perspective change* into the wording of the Monty Hall problem.

In addition to the less-is-more manipulation either the frequency simulation (see Group 5) or the perspective change (see Group 6) can be incorporated. Yet, neither of

these single implementations leads to a remarkable improvement in terms of understanding compared to the version of Group 2, where only the less-is-more manipulation was incorporated. Only when the frequency simulation and the perspective change are incorporated simultaneously will a large improvement in understanding occur (see groups 3 and 7). Indeed, analyzing participants' protocols reveals that each manipulation needs the other, now for *psychological reasons*. Not asking for a frequency simulation makes it difficult for the problem solver to grasp the relevant arrangements – even if she perceives the problem from Monty Hall's perspective. On the other hand, when only the frequency simulation is provided, the problem solver cannot “feel” the constraints introduced by the contestant's first choice: This would require perceiving the arrangements from Monty Hall's perspective. Thus, the seemingly powerful frequency simulation alone is not sufficient to gain insight into the problem's structure. Let us summarize the three important inter-relationships among the four manipulations:

- (a) The frequency question asks for mental models. Both elements were combined into one manipulation, namely the frequency simulation.
- (b) The frequency simulation and the perspective change need each other to improve participants' performance (psychological dependency).
- (c) Both the frequency simulation and the perspective change require the less-is-more manipulation as a pre-condition to express meaningful versions (mathematical dependency).

Does the lack of information on Monty Hall's strategy matter?

Although in the present studies we included the rule in every version of the Monty Hall problem (“Monty Hall has to open one of the unchosen doors, which has a goat”), we did not specify Monty Hall's strategy. As we saw, in one-door scenarios, Monty Hall's strategy does not influence the solution given by Figure 2.1, whereas in two-door scenarios (our control versions) it might make a difference (see Equations 2.2-2.4). But the protocols revealed that even in our control versions no participant in either study struggled with this problem. All participants either found the correct solution or failed without even reaching the point where Monty Hall's strategy would come into play. No participant justified her decision to stay in terms of Monty Hall's strategy, nor

did any state that she could not solve the problem because she did not know what Monty Hall would do if the car were behind door 1.

Uniformity belief

Almost all participants who decided to stay held the uniformity belief. But if they had no reason to favor one option over the other, why then were their choices not evenly distributed between switching and staying? Our analysis of the protocols yielded substantial support for Granberg and Brown's explanation ("participants feel worse if they switch away from a prize than if they stick to an initial choice and miss the prize"). But we also found some instances of another reasoning line. Some participants argued like this: "Why should I change anything if this would not increase my chances of winning?" Although the uniformity belief is mathematically unjustified, from this point of view it seems to be economically rational: Think of a foraging animal that expects the same amount of return from two alternative foraging patches and picks one for the day. It would be perfectly rational to stay in that patch for the whole day rather than to expend extra time and energy to travel to the other patch. The adaptive strategy would be to stick to an initial choice unless a better alternative is available.

We want to indicate that the laws of consistency also tell us not to reverse preferences when options (dis)appear (see, e.g., Gigerenzer, 2001). For example, if one prefers option A over option B, she should not reverse this preference just because a third option C appears (or disappears). What does this mean for the Monty Hall problem? A contestant who first chooses door 1 prefers – for some reasons – door 1 over doors 2 and 3. In the Monty Hall problem deleting one choice option, for instance by opening door 3, brings new information about the unselected door 2, and taking this into account, one should switch. Yet, if this new information is not realized – as with the uniformity belief – deleting door 3 simply means that this option has vanished and now changing the previous preference (i.e., choosing now door 2 instead of door 1) would be a violation of the laws of consistency.

Granberg and Brown's (1995) "regret explanation", our "economical rationality" approach, and even the laws of consistency can be seen as rational justifications for staying: The uniformity belief in the Monty Hall problem does not and should not lead to a random choice.

CONCLUSION

A remarkable proportion of naive people can gain full insight into the Monty Hall problem's structure when elements from the cognitive psychologists' toolbox are applied. During the last 10 years the claim has persisted that there is no way to break the resistance of a majority of naive people to grasping its mathematical structure. In fact, all previously tested explanations of the problem (e.g., vos Savant, 1997; Aaron & Spivey, 1998; Johnson-Laird et al., 1999, tested in the present chapter) displayed no great power of persuasion. It is our claim that our manipulations could have prevented Marilyn vos Savant from receiving thousands of protest letters. Note that our manipulations do not "destroy a fascinating cognitive illusion", but – as we learned from our participants – the Monty Hall problem displays its whole fascination only when one realizes that switching is indeed better.