Thermal Infrared photometry: NOTES

The flux incident on the detector follows a Poisson distribution, and the probability of receiving *n* photons within the time interval *t* is given by

$$p(n,t) = \left(\frac{\bar{n}^n}{n!} \right) e^{-\bar{n}} \tag{C-2}$$

where $\bar{n}$ is the mean number of photons in the time t. The variance of the arrival number of photons is equal to its mean value. The uncertainty associated to $\bar{n}$ is therefore equal to $\sqrt{\bar{n}}$ .However, it is not only the number of incident photons which fluctuates, but the measuring process of the charge carriers (electrons) is statistical and Poissonian. The mean number of photo-generated electrons is $\bar{n}' = Q_E \bar{n}$ and its error $\sigma_{n'} = \sqrt{\bar{n}'} = \sqrt{Q_E \bar{n}}$. (see for example Robberto 1988, PhD thesis). The final S/N equation for a thermal IR observation must be written in terms of electrons.

A final medium infrared image results from the coadding of the 4 chop-nod channels: A-B – (A'-B') For each channel A, B, A' and B' several elementary frames are summed up together. For example, at Keck/LWS a 120s-observation consists of 12000 elementary 0.01s frames. In the S'/N' equation (I used the primed letters since the quantities are in unit of numbers of electrons) the noise term results from the quadratic sum of the various (independent) noise contributions in each elementary exposure:

$$N'^2 = \sum_{i=1}^{6000} \left[S'_i + A_{aperture} \times \left(B'_i + D'_i + R'^2 \right) \right] + \sum_{i=1}^{6000} \left[A_{aperture} \times \left(B'_i + D'_i + R'^2 \right) \right] \tag{C-3}$$

where *S'* is the total number of electrons from the source (within the aperture), $A_{aperture}$ is the area in pixel of the photometric aperture, *B'* is the number of electrons per pixel from the background, *D'* is the dark current of the detector per pixel and *R'* is the readout noise in electrons per pixel. There are two terms since for half of the time the source is off and it does not contribute. It is clear we can write:

$$N'^2 = N_{frames} \times \left(\frac{1}{2} S'_i + A_{aperture} \times \left(B'_i + D'_i + R'^2 \right) \right) \tag{C-4}$$

where N_{frames}=12000 in this example. The final S/N ratio is therefore given by:

$$\frac{S'}{N'} = \frac{\frac{1}{2} N_{frames} S'_i}{\sqrt{N_{frames} \times \left(\frac{1}{2} S'_i + A_{aperture} \times \left(B'_i + D'_i + R'^2 \right) \right)}} = \frac{S'}{\sqrt{S' + A_{aperture} \left(B' + N_{frames} D'_i + N_{frames} R'^2 \right)}} \quad \text{(C-5)}$$

which closely resemble the well-known "CCD equation" (see Howell 1989). However, in the background-limited case, detector read noise, dark current noise, etc., are negligible compared to fluctuations in the incident background photon flux rate. In fact the environmental 10 μm thermal background flux in the Cassegrain focal plane of a large, ambient-temperature telescope is of the order of 10^9 photons s^{-1} m^{-2} μm^{-1} $arcsec^{-2}$. (Gezari et al. 1992). For 0.01s integration time, 0.7 quantum efficiency, 0.08 arcsec/pixel scale there are about 4.5×10^6 electrons/pixel; about half of the pixel full well capacity (1.1×10^7 electrons). The dark current is about 4×10^5 electrons/pixel for such integration time while the readout noise is about 1000 electrons. We have thus $\left(B'_i + D'_i + R'^2 \right) = \left(4.5 \times 10^6 + 4 \times 10^5 + 10^6 \right)$ in the denominator term of the S/N equation. The dark current contribution is completely negligible, and the background dominates the readout noise contribution. Actually, the measured background value in several LWS exposures taken on the night of February 21, 2002 is about to 3500 counts per pixel in a 0.01 second frame-time. This means 7.8×10^6 electrons (given the inverse gain of 2200 electrons/counts) making the detector readout noise even more negligible in the S/N equation.

The flux from the star α Lyr at 10 μm is about 5.8×10^7 photons s^{-1} m^{-2} μm^{-1} above the atmosphere, which gives about 4.0×10^7 electrons integrated within the photometric aperture for an integration time of 0.01s, 0.7 of detector quantum efficiency and 1μm filter bandwidth. Using a photometric aperture of 13 pixels of radius (corresponding to about 1 arcsec) $A_{aperture}$ is equal to 530 pixels.

$$\frac{S'}{N'} = \frac{S'}{\sqrt{S' + A_{aperture} \left(B' + N_{frames} D'_i + N_{frames} R'^2 \right)}} = \frac{2 \times 10^7}{\sqrt{2 \times 10^7 + 530 \times \left(4.5 \times 10^6 \right)}} \approx \frac{2 \times 10^7}{\sqrt{530 \times \left(4.5 \times 10^6 \right)}} \quad \text{(C-6)}$$

The source "shot noise" contribution to the denominator term in the signal-to-noise ratio equation is clearly negligible compared with the background one ($2 \times 10^7 << 2.3 \times 10^9$)