Customers' Preferential Choice for Manifestations of Flexible Products

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Abstract

Flexible products consist of several alternative products, where the seller determines the final manifestation after a product is sold. This determination step is called the allocation of the flexible product. In revenue management, flexible products let firms deal with uncertainties during the sales process. An airline, for example, may sell a flexible product that contains flights to Hamburg, Berlin, or Munich. Before departure, the airline assigns the flexible product to a flight and informs the passenger. Existing research showed that flexible products can induce new demand segments, generate more revenue, and improve the capacity utilization.

The success of flexible products is closely linked with their design and integration into existing frameworks. Current research on revenue management for flexible products assumes customers to be indifferent between the alternatives and to impulsively decide when considering such products. Several contributions and insights from practical applications of flexible products raised concerns about this assumption. Websites such as betterbidding.com or biddingtraveler.com suggest that customers have started to approach even the concept of flexible products strategically. Especially customers' expectations about possible manifestations can diminish benefits of flexible products. Considering customers' preferential choice when selling flexible products, creates needs for new demand models. They have to describe how customers evaluate the alternatives defining a flexible product. Considering customers' preferential choice when allocating flexible products, opens new perspectives and enables new allocation methods.

This thesis examines the impacts on revenue management when customers are able to express their preferences for alternatives included in a flexible product. In this regard, this thesis presents several models of customers' preferential choice and pays particular attention to customers acting strategically. Based on the additional information about customer preferences, several decision heuristics and a multi-objective optimization program are formulated. A computational study shows the reliability of these models and compares the performance of various allocation methods. In addition, this thesis shows implications of updating allocations over the sales period, flawed input parameters, and wrong model assumptions on the performance. Among others, the results demonstrate that strategic customers can significantly reduce the benefits of flexible products in specific circumstances and imply that protecting flexibility and reactivity can firms help to counteract strategic behavior.

Zusammenfassung

Flexible Produkte bieten Verkäufern die Möglichkeit erst nach erfolgtem Verkauf die finale Ausprägung eines Produktes zu spezifizieren. Dieser Schritt wird Allokation des flexiblen Produktes genannt. In Bezug auf das Revenue Management helfen flexible Produkte Firmen bei der Bewältigung von Unsicherheiten. Eine Fluggesellschaft kann zum Beispiel ein flexibles Produkt anbieten, das Flüge nach Hamburg, Berlin und München beinhaltet. Den tatsächlichen Zielort für eine Buchung legt die Fluggesellschaft erst kurz vor dem Abflug fest und informiert den Kunden darüber. Untersuchungen zeigen, dass das Anbieten flexibler Produkte neue Nachfragesegmente ansprechen, den Ertrag steigern und die Auslastung vorhandener Kapazitäten verbessern kann.

Der Erfolg flexibler Produkte hängt stark von ihrer Ausgestaltung und Integration in das übrige Produktportfolio eines Unternehmens ab. Aktuelle Forschungen unterstellen Kunden meistens, dass sie keinerlei Präferenz bezüglich der in einem flexiblen Produkt enthaltenen Alternativen haben und sich außerdem spontan für den Kauf entscheiden. Diese Annahmen sollten überdacht werden, wie Beispiele aus der Praxis zeigen. Webseiten wie betterbidding.com oder biddingtraveller.com unterstützen diese Überlegungen und lassen vermuten, dass Kunden bei flexiblen Produkten immer mehr strategisch entscheiden, basierend auf Erwartungen über mögliche Spezifikationen. Dies beeinträchtigt jedoch die Vorteile, die durch den Verkauf flexibler Produkte entstehen. Firmen, die mögliche Präferenzen ihrer Kunden berücksichtigen wollen, benötigen neue Kundenwahlmodelle, die beschreiben wie Kunden die Alternativen eines flexiblen Produktes bewerten. Eine Berücksichtigung solcher Präferenzen bei der Zuteilung eröffnet für Unternehmen aber auch neue Möglichkeiten. Um diese nutzen zu können, werden neue Zuteilungsverfahren benötigt.

Die vorliegende Arbeit untersucht die Auswirkungen von Kundenpräferenzen für die Alternativen eines flexiblen Produktes auf das Revenue Management. Verschiedene präferenzbasierte Kundenwahlmodelle werden vorgestellt, die auch ein mögliches strategisches Entscheidungsverhalten von Kunden abbilden. Es werden neue Zuteilungsverfahren entwickelt, die zusätzliche Informationen bei der Allokation flexibler Produkte berücksichtigen. Dazu werden verschiedene Entscheidungsheuristiken und ein Optimierungsproblem mit mehreren Zielfunktionen formuliert. Simulationsexperimente zeigen die Auswirkungen und die Anwendbarkeit der Modelle und vergleichen deren Einfluss auf den Ertrag. Zusätzlich werden Möglichkeiten untersucht, die Allokation während des Verkaufszeitraumes zu aktualisieren, sowie der Einfluss ungenauer Eingabeparameter auf den Erfolg. Unter anderem zeigen die Ergebnisse, dass strategisches Kundenverhalten die Vorteile flexibler Produkte abschwächen kann. Erhalten sich Unternehmen jedoch ein Maß an Flexibilität und Reaktionsfähigkeit, kann das helfen diesem Effekt entgegenwirken.

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Part I

Motivating Preferential Choice for Manifestation Sets

1 Introduction

Firms with a value chain affected by restrictions like fixed capacities or perishable products use Revenue Management (RM) methods to optimize their profits. The core idea of RM is to sell the right product to the right customer at the right time for the right price. Quantity-based RM optimizes the quantity of offered products for a set of discrete prices (Talluri & van Ryzin, 2004b, Chapter 5). Price-based RM continuously optimizes the setup of prices instead of quantities (Bitran & Caldentey, 2003).

For the airline industry, RM is a successful story since its beginnings in 1978 when the U.S. airline market was deregulated (cf. Littlewood, 1972; Smith, Leimkuhler, & Darrow, 1992). Its methods and algorithms provide a powerful toolbox for airlines to estimate future demand and to set up a suitable strategy of price-discrimination (Talluri & van Ryzin, 2004b). RM has also been applied to various other industries such as hotels (Choi & Mattila, 2004), rental cars (Geraghty & Johnson, 1997), retail (Vinod, 2005), cruise lines (Ladany & Arbel, 1991), and advertising (Kimms & Müller Bungart, 2007).

Airline RM research started with flight-based models (Belobaba, 1987b; Littlewood, 1972) and evolved into network-oriented models (Buhr, 1982; Wang, 1983). Several contributions incorporate the ability to react to unforeseen changes in RM concepts (cf. Curry, 1990; Lardeux, Goyons, & Robelin, 2010; Wong, Koppelman, & Daskin, 1993). For example, overbooking methods try to hedge against bookings recalled in advance (cancellations) and bookings not utilized at time of execution (no-shows) (cf. Alstrup, Boas, Madsen, & Vidal, 1986; Chiu & Tsao, 2004; Rothstein, 1971; Subramanian, Stidham Jr, & Lautenbacher, 1999). In order to react to newly available information during the sales period, RM methods are updated several times. The performance of an RM system largely depends on the frequency and accuracy of such updates (McGill & van Ryzin, 1999).

More sophisticated setups including networks or customer choice models increase the complexity of RM. Together with changing exogenous conditions it is clear that RM has to follow these trends to still be successful and helpful (McGill & van Ryzin, 1999). Talón Ballestero and González Serrano (2013), for example, clarify the importance of more customer centricity in RM methods. In discussions, subject matter experts of Deutsche Lufthansa AG emphasized the importance of reactivity and flexibility during the RM process. In this context, they highlighted the applicability of new methods into state-of-the-art distribution systems and the impacts on customer satisfaction as the main challenges in practical RM.

Following Mandelbaum and Buzacott (1990), this thesis defines flexibility as ensuring a certain number of alternatives left over after an initial decision has been made. A flexible concept with highly practical relevance for RM is offering flexible products: products for which one or more attributes are not fully specified when they are sold (cf. Gallego & Phillips, 2004). The set of possible specifications for a flexible product can be termed manifestation set.

Example. *An airline offers direct flights from Frankfurt to Hamburg, Munich, and Berlin as specific products. In addition, it offers a flexible product that includes all three destinations. After a customer has booked the flexible product, the airline allocates the booking to a flight and informs the customer about the resulting destination.*

Selling flexible products causes an amount of flexibility for the airline while simultaneously introducing uncertainty for customers regarding the outcome. The gain in flexibility for the airline results from specifying the unknown attributes later. This preserves a certain amount of reactivity for the airline. Using flexible products hedges selling strategies against exogenous and endogenous uncertainties. Customers, however, accept the uncertainty when buying flexible products only in case of an adequate monetary discount.

One of the first airlines that offered flexible products was Freedom Air International. According to their concept, customers define their individual level of uncertainty for a flight ticket through a web interface. The underlying RM system adapts the offered price respectively. Mang, Post, and Spann (2012) analyzed this concept of selling flexible products and showed the potential to generate incremental demand and revenues. Several firms currently offer flexible products, e.g., Hotwire, Priceline, Germanwings, and AIDA Cruises. Germanwings offers flexible products that enable customers to exclude alternatives before allocation. This allows customers to achieve a higher level of certainty regarding the allocation later. Both practical examples, however, suggest that customers have certain preferences regarding the outcome of a flexible product.

An increasing market transparency induced by faster and more detailed ways to exchange information enhances the possibility that customers anticipate outcomes of flexible products. Several websites, such as betterbidding.com or biddingtraveler.com, provide platforms for customers to communicate the results of previous purchases of flexible products. This additional information can change the expectations regarding future decisions of other customers. As a result, the balance of uncertainty for customers and flexibility for airlines characterizing flexible products is impacted. This may lead to cannibalization of demand for specific products towards the flexible products.

First contributions to extend RM for flexible products are Gallego and Phillips (2004) and Gallego, Iyengar, Phillips, and Dubey (2004). Both formulate basic algorithms to calculate booking limits for flexible products in various setups including the choice between specific and flexible products. Petrick, Gönsch, Steinhardt, and Klein (2010) focus on dynamic capacity control mechanisms to allocate flexible products. The authors benchmark the method's flexibility and practicability and characterize the correlation between forecast quality and revenue gain. Later, Petrick, Steinhardt, Gönsch, and

Klein (2012) extend these models and demonstrate that, given uncertain demand, flexible products and late notification dates are beneficial. All these contributions, however, assume that customers are indifferent regarding the manifestations of a flexible product.

Only very few contributions consider customers' preferences for flexible product manifestations so far. Post (2010) lets customers define several characteristics of the products booked. In Lee, Khelifa, Garrow, Bierlaire, and Post (2012) the customers' likelihood to limit the manifestation set is investigated. Both references emphasize the importance of considering customers' preferences. Therefore, in addition to the practical examples of Freedom Air International and Germanwings, the research gap thus identified motivates this thesis.

This thesis focuses on customers' preferential choice between manifestations of a flexible product. It investigates how RM methods for flexible products have to be modified in order to establish a more customer-centric RM process with flexible products. Furthermore, the relevant aspects for implementing customers' preferential choice between manifestations are examined.

One focus will be on allocation methods specifying flexible bookings. This aspect is currently neglected in relevant research contributions. Here, four models will be presented differing in the applicability with previously formulated choice models. This thesis will incorporate another relevant aspect for allocations that deals with the dynamic and the possibility to update allocations throughout the sales period.

Moreover, as the Internet enables more transparent markets, a strategic behavior of customers can evolve. Based on additional knowledge from past decisions or other individuals' experiences, customers may anticipate possible outcomes when buying flexible products and try to exploit the concept. This thesis characterizes and analyzes such strategic customer behavior for flexible products. To this end, the properties of appropriate choice models will be investigated. Together with insights about allocation methods for flexible products, we will evaluate the impacts on revenue performance and characterize possible counteractions.

As RM methods and models are subject to flawed input parameters, this thesis investigates the effects of wrongly observed parameters on the proposed preferential choice models. Furthermore, the allocation methods for flexible bookings have to be tested against errors in input parameterizations. The aim of this thesis is to evaluate the impacts on revenue performance for different RM process configurations when various input parameters are flawed.

In order to examine the consequences and resulting opportunities from these new and adapted models, extensive computational studies will be done. By varying the demand setup and the applied models and methods, we will create a sensitivity analysis and evaluate different parameterizations. The results of this thesis are expected to support considerations for a trend towards more customer-centric RM research and application. Therefore, this thesis focuses on implications and considerations that affect the practicability of our methods.

As RM is highly relevant for the airline industry, all models, examples, and computational studies in this thesis are formulated in this context. However, all considerations are easily extendable to more sophisticated setups as well as other industries using RM.

Part I specifies the research gap of this thesis by evaluating current applications and research prospects. Chapter 2 reviews relevant literature about RM and flexible products. This chapter sets up a basis for later modifications and extensions. Building on that, Chapter 3 reveals the research gap regarding RM with customers' preferential choice between manifestations and allocation of flexible products. The gap will be characterized by several research questions structuring subsequent parts of this thesis.

Mathematical models for customers' preferential choice, strategic customer behavior and the allocation of flexible products are formulated in **Part II**. Chapter 4 presents models for customers' preferential choice. Dependencies between customers' decisions and the current set of offered products and possible allocations for flexible products are presented as a separate customer choice model in Chapter 5. Having this additional information in mind, airlines may postulate new allocation methods for flexible products: Chapter 6 provides the mathematical models to extend current methodology. Chapter 7 discusses the impacts of wrong assumptions and flawed parameters.

Finally, in **Part III**, the simulation environment and experimental setup is presented (Chapter 8). The performance of the mathematical models is examined by three computational studies focusing on impacts of customers' preferential choice (Chapter 9), strategic behavior of customers (Chapter 10), and the effect of flawed input parameters (Chapter 11). The numerical results provide a reliable basis to evaluate the interaction between different choice models and allocation methods. Chapter 12 closes the thesis with a discussion of the findings and an outlook toward future research directions.

2 State-Of-The-Art: Revenue Management and Flexible Products

This chapter provides an overview about current methodology and research prospects in Revenue Management (RM) with flexible products. Section 2.1 deals with RM in general and gives a short introduction to research about customer choice in this application area. In Section 2.2, we take a look at flexible products as a tool to introduce flexibility into RM. In this context, we also briefly focus on flexibility in general as well as with regard to RM.

2.1 Revenue Management

RM methods support airlines in making price and availability decisions of products in the presence of fixed capacities. One of the first concepts of RM related to the airline industry was provided by Littlewood (1972). A decision rule uses the estimated demand to come per product and resource to determine the selling strategy for a setup including two products. This concept was later extended by Belobaba (1987b) for a more general setup, called Expected Marginal Seat Revenue (EMSR).

Several extensions to EMSR, e.g., Alstrup et al. (1986); Wollmer (1992), as well as completely new approaches dealing with network setups were developed, e.g., Buhr (1982); Wang (1983). McGill and van Ryzin (1999), and more recently Chiang, Chen, and Xu (2007) document research prospects in RM and present relevant future directions. An overview of important process elements, methods, and algorithms in airline RM can be found in Talluri and van Ryzin (2004b). A review about current trends in RM with a focus on the mathematical formulation of models and methods as well as on the importance of simulations is provided by Talluri, van Ryzin, Karaesmen, and Vulcano (2008).

As Talluri and van Ryzin (2004b) state, RM can address two management decision areas: quantity and pricing of offered products. With the quantity decision, airlines try to maximize the revenue by reserving capacity for higher valuated requests while minimizing the unused capacity at the end of the sales period. Control policies determine the offered products at each point in time. As opposed to this, pricing focuses on general price structures and price-variations in order to maximize revenue. The focus is turned to this when no clear separation between products is possible and when they are separated only by price.

This thesis is restricted to the quantity-based aspects of RM and neglects all pricing aspects: exactly one price is associated with a product and is valid over the whole sales period. For a basic reflection of RM as pricing decision, we refer to Gallego and van Ryzin (1994), Gallego and van Ryzin (1997), or Feng and Gallego (1995). A comprehensive overview of dynamic pricing approaches is presented by Bitran and Caldentey (2003).

Figure 2.1 depicts a general view of the airline related RM process adapted from Talluri and van Ryzin (2004b). The airline does not know the real demand at the beginning of the sales period. Instead, historical booking data, inventory controls, and forecasts are used to estimate future demand. This is the input for the optimization, considering capacity and prices as fixed in advance. The resulting control policies are confronted with demand in the inventory. Demand is strongly influenced by market conditions. The resulting bookings are input data for the next demand forecast and so forth, which is indicated by dashed lines.

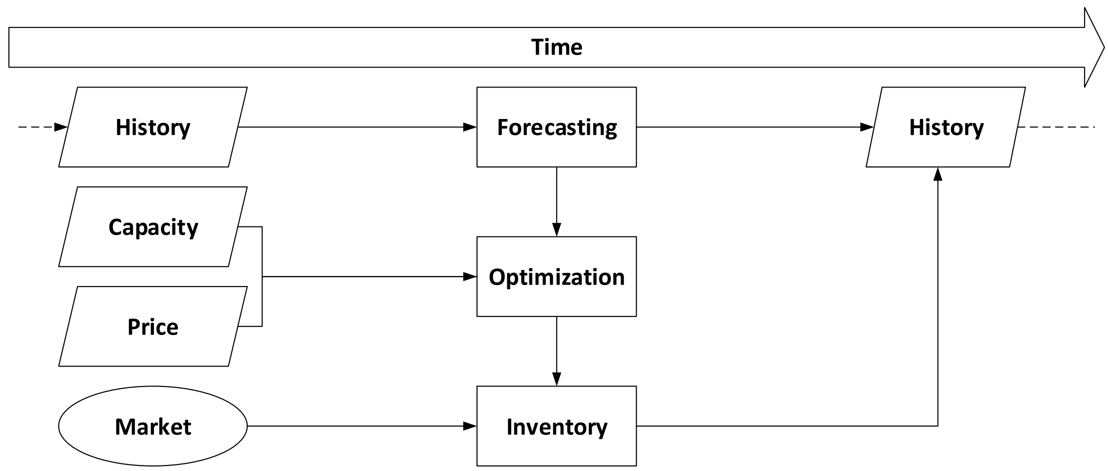


Figure 2.1: Revenue management process adapted from Talluri and van Ryzin (2004b)

This process view as well as all following considerations neglect any competition, as it plays no crucial role when investigating customers' preferential choice between manifestations. Furthermore, we assume resources with fixed capacities and the nonexistence of cancellations during the sales period. Subsequent sections review all process steps in detail, following the structure of Figure 2.1.

Notation and prerequisites. We denote the sales period by $T = \{\bar{t}, \ldots, 0\}$ where $0 \in T$ is the time of execution. An arbitrary point in time before execution is denoted by $0 < t \leq \bar{t}$. In later formulations, we will need a discretization of T where at most one request occurs in a certain time slice τ.

Let $R = \{1, \ldots, \bar{r}\}$ be the index set of resources (flights) used by the airline to offer their service. With respect to the production industry the set R represents the set of machines available for usage. The initial capacity for each resource is denoted by

$c_r \in \mathbb{Z}$. The remaining capacity for sale for each resource at time $t \in T$ is denoted by $c_r^t \leq c_r$.

Let $S = \{1, \dots, \overline{s}\}$ be the index set of specific products offered by the airline. Let $y_{sr} \in \mathbb{Z}$ be a parameter indicating the capacity used by a specific product on a resource. With regard to the airline industry, each specific product uses exact one unit of capacity:

$$y_{sr} = \begin{cases} 1, & \text{specific product } s \text{ uses capacity on resource } r, \\ 0, & \text{else.} \end{cases} \tag{2.1.1}$$

The set of products corresponding to a resource is denoted by $S_r \subseteq S,\ \forall r \in R$.

Let f_s be the price for a specific product. The set S can be ordered following a nesting order. Index 1 denotes the specific product that has the highest price and $\overline{s}$ the specific product that has the lowest price. Depending on the current availability situation, $s^t \in S,\ \forall t \in T$ denotes the cheapest product offered. Because of the nesting order, control policies describe the current availability situation with s^t, which implies that all products with $f_s > f_{s^t}$ are available for requesting, too.

2.1.1 Forecast

The first statistical models that describe booking and cancellation behavior of customers can be found in Beckmann and Bobkoski (1958). Martinez and Sanchez (1970) use empirical data from Iberia Airlines to estimate probability distributions concerning air travel demand behavior. Demand forecasting and the estimation of arrival rates using a Poisson model is addressed in Lee and Hersh (1993). Recent challenges for forecasting methods in RM regarding the use of dynamic pricing methods and a restriction free product structure are described in Cleophas, Frank, and Kliewer (2009). Cleophas (2009) uses a simulation tool to describe and evaluate measures of forecast performance under theoretical aspects.

Applying control policies allows only a constraint snap of the true demand. Historical data represents constrained data, as the set of available products is constrained by the valid control policies when a request occurs. Dependent on the assumed customer behavior, different methods can be applied to estimate the true demand using constraint observations and information about situational characteristics. These methods are termed **unconstraining methods** and are addressed, for example, in Swan (1990), McGill (1995), Pölt (1998), and Weatherford and Pölt (2002).

Most forecast and optimization techniques rely on the assumption that demand is independent, despite the fact that researchers know that this assumption may not be appropriate. The assumption states that demand for certain products is independent from the set of currently available products. In early RM research, independent demand could be seen as valid, because restrictions[1] and prices fence different products. With increasing success of low-cost airlines using simple product structures demand can no longer be seen as independent (Zeni, 2007). Several contributions extend the concepts using a hybrid forecasting model including price-sensitive customers, e.g., Bartke (2013); Belobaba and Hopperstad (2004); Cleaz Savoyen (2005); Weatherford and Ratliff (2010).

As stated in Talluri and van Ryzin (2004a) and Fiig, Isler, Hopperstad, and Belobaba (2009), RM systems provide control policies in a nested form to avoid that capacity is unavailable for expensive products and simultaneously available for cheaper products. Nested policies reserve capacity for products in an overlapping hierarchical way. More expensive products are ranked higher and can grab capacity reserved for cheaper/ lower ranked products but not vice versa.

Dependent demand leads to more customer-oriented forecasting techniques by assuming that customers first consider a set of appropriate classes and then choose the most attractive option. First contributions modeling this choice discrete are Akçay, Natarajan, and Xu (2010) and Cirillo and Hetrakul (2011). Another approach described by Winter (2010) represents a firm's product in an ordered graph. Depending on the current availability situation, demand is modeled as a flow within this graph. Fiig et al. (2009) describe a transforming approach for fares and dependent demand to an equivalent independent demand model. Assuming dependent demand creates the need to precisely characterize the availability situation when a request occurs. For a more detailed view on choice modeling we refer to Section 2.1.4.

The forecast process consists of two steps. First, the airline estimates the true demand that has occurred in the last sales period. Using this unconstrained demand, the airline computes a forecast for the upcoming demand. This estimation and combination process is conducted on a very detailed level, depending on available data and proposed requirements (Bartke, 2013).

Let $b_s^t \in \mathbb{Z}$ be the observed bookings for a particular specific product at a certain time during a sales period. We denote the corresponding unconstrained demand by $\hat{d}_s^t \in \mathbb{R}$. The demand forecast that is used to calculate the current control policy leading to b_s^t is denoted by $d_s^t \in \mathbb{R}$.

A common way for combining the estimates of upcoming demand is exponential smoothing. Let $\lambda \in [0, 1]$ be the weighting parameter used for smoothing. The demand reference $\tilde{d}_s^t \in \mathbb{R}$ calculates the estimated demand for the next sales period using

$$\tilde{d}_s^t = \lambda \cdot \hat{d}_s^t + (1 - \lambda) \cdot d_s^t. \tag{2.1.2}$$

The unconstrained demand $\hat{d}_s^t$ is calculated based on the underlying demand model. As stated above, a basic assumption in early RM theory was the independence of demand from current available alternatives. In an environment where customers decide

[1] Airlines differentiate products by restricting the usability, e.g., applying minimum-stay conditions or fees for changing the product.

independently from available alternatives, a simple pick-up technique can be used to estimate the true demand for a product (Weatherford & Pölt, 2002). As long as the product is available, historical bookings are used as estimate and in all other cases the corresponding demand reference of the previous sales period is used. This definition is independent from the current availability for other products

$$\hat{d}_s^t = \begin{cases} b_s^t, & \text{if } s \leq s^t, \\ d_s^t, & \text{else.} \end{cases} \tag{2.1.3}$$

Weatherford and Kimes (2003) state that unconstraining demand using equation (2.1.3) provides robust estimates for the true demand, given that the independence assumption is correct. One serious handicap of independent forecast techniques, however, is the emergence of a **spiral down** effect in the presence of insufficient demand segmenting restrictions. In a spiral down, buy-down from higher into lower products is not able to be averted by RM. For a more detailed description and a mathematical model formulation, we refer to Cooper, Homem-de Mello, and Kleywegt (2006).

For a detailed introduction to forecasting methods assuming dependent demand, we refer to Talluri and van Ryzin (2004b), Fiig et al. (2009), and Bartke (2013).

2.1.2 Optimization

First approaches to maximize revenue by using the marginal revenue are made by Littlewood (1972) and are then extended to EMSR by Belobaba (1987b), Belobaba (1987a) and Belobaba (1989). Similar results are provided by Wollmer (1992), who determines a critical value for the number of accepted requests per product. This critical value is defined as a decreasing function of the price. A first consideration of complete flight networks is done by Buhr (1982), who assumes only a single product. An extension to this concept for multiple products can be found in Wang (1983).

A linear programming approach for the network formulation is provided by Wollmer (1986), followed up by a nonlinear model introduced by Vinod (1991). Curry (1990) combines the concepts of marginal seat revenue with nested products and the network formulation.

A first approach towards a more risk-sensitive EMSR logic can be found in Weatherford (2004). The aim of this optimization is not only to maximize the expected revenue, but also the utility that can be achieved by earning this revenue. The first approach to use revenue risk in the context of dynamic pricing is done by Levin, McGill, and Nediak (2008), to the best of the author's knowledge.

The aim of the optimization step is to determine revenue maximizing control policies depending on the estimated demand. As outlined before, several methods exist but they differ in the assumption about the demand model and the design of the RM system. A common way in practice is to use control policies based on bid prices as threshold values to decide about accepting or rejecting a request. The term bid price denotes an estimation for the opportunity costs of a unit of capacity. It was introduced by Williamson (1992). The work of Talluri and Van Ryzin (1998) reviews different concepts of bid prices used as booking control policies in the context of network RM.

For reasons of simplicity, this thesis is restricted to optimal control policies computed via dynamic programming. A Bellman equation for every combination of resource, remaining capacity, and time evaluates recursively the marginal costs for a unit of available capacity per time slice. Let $p_s^\tau \in [0,1]$ denote the probability that a request for product $s \in S$ will occur in time slice $\tau \in T$.

The achievable revenue $R^\tau \in \mathbb{R}$ at a time slice in case a request for a product arrives can be written as

$$R^\tau = \begin{cases} f_s, & \text{if request for product } s \text{ occurs at time } \tau, \\ 0, & \text{else.} \end{cases} \tag{2.1.4}$$

Using the definition of R^τ and p_s^τ the expected revenue can be calculated. Let $z^\tau \in \{0,1\}$ be the decision variable for each time slice if the request is accepted or not

$$z^\tau = \begin{cases} 1, & \text{accept request,} \\ 0, & \text{deny request.} \end{cases} \tag{2.1.5}$$

Following Talluri and van Ryzin (2004b), a value function denoted by $V(r,\tau,c_r^\tau),\ \forall r \in R, \tau \in T$ for each time slice can be calculated as

$$\begin{aligned} V(r,\tau,c_r^\tau) &= E\left[max_{z^\tau}\left\{R^\tau \cdot z^\tau + V(r,\tau+1,c_r^\tau - z^\tau)\right\}\right], \\ &= V(r,\tau+1,c_r^\tau) + E\left[max_{z^\tau}\left\{(R^\tau - \Delta V(r,\tau+1,c_r^\tau)) \cdot z^\tau\right\}\right]. \end{aligned} \tag{2.1.6}$$

We denote the expected marginal revenue for selling one unit of capacity in the next time slice by

$$\Delta V(r,\tau+1,c_r^\tau) = V(r,\tau+1,c_r^\tau) - V(r,\tau+1,c_r^\tau - 1). \tag{2.1.7}$$

By solving (2.1.6) we have to consider the following boundary conditions

$$V\left(r,0,c_r^0\right) = 0, \quad \forall c_r^0 \in 1,\dots,c_r \quad \text{and} \tag{2.1.8}$$

$$V(r,\tau,0) = 0, \quad \forall \tau \ \in 1,\dots,T. \tag{2.1.9}$$

Caused by perishability of products as well as capacity restrictions of the underlying problem, the marginal revenue decreases with increasing time and capacity usage. Remaining units of capacity are less likely sold when time elapses. Therefore, the benefit of an additional unit of capacity at any point in time decreases.

Solving equation (2.1.7) provides a value $\Delta V(r,\tau,c_r^\tau)$ that is used as optimal control policy for each resource, time slice, and amount of remaining capacity.

2.1.3 Booking Control Policies

Booking control policies are formalized rules helping the airline to decide if a request should be accepted or not in order to maximize overall revenue. Control policies can be defined for example using the marginal costs $\Delta V\left(r, \tau, c_r^\tau\right)$ or the bid price denoted by $\pi_r^\tau\left(c_r^\tau\right) \in \mathbb{R}$ for each resource r, time slice τ, and remaining capacity.

Equation (2.1.6) implies to accept a request for a certain product only in case that the achievable marginal revenue exceeds the marginal costs caused by using one unit of capacity on the corresponding resource. For the independent demand case the achievable marginal revenue is equal to the price of the product

$$f_s \cdot y_{sr} \geq \Delta V\left(r, \tau, c_r^\tau\right) = \pi_r^\tau\left(c_r^\tau\right). \tag{2.1.10}$$

A method to formulate control policies for the dependent demand case is presented in Fiig et al. (2009). Their model relies on parameters considering the current availability situation and each possible change of the availability. The marginal revenue per booking is calculated by using the incremental demand and revenue for changes in the availability situation.

2.1.4 Customer Choice Models in Revenue Management

Especially economic and consumer theory focus on the decision process of individuals with regard to pricing and offering of alternatives. Beside the overall decision between buying or not buying, the choice between the offered alternatives plays a crucial role. RM uses choice models either to improve forecasting (Fiig et al., 2009) or optimization (Talluri & van Ryzin, 2004a).

Discrete choice models focus on situations where the set of alternatives is discrete. Valuations for alternatives result from a function including a deterministic and an uncertainty term modeling aspects that cannot be included in calculations or observed by other people (cf. Ben Akiva & Lerman, 1985; McFadden, 1973). The foundation for the concept of discrete choice models is done in McFadden (1973). A framework characterizing the relevant elements of discrete choice models is introduced in Domencich and McFadden (1975): a decision maker, the alternatives on offer and their attributes, and a decision rule that helps the decision maker to choose between alternatives. Several contributions extend and elaborate the concept towards Multinomial Logit models (cf. Fader, Lattin, & Little J., 1992; McFadden, 1987; Zhen Chen & Lynn Kuo, 2001)

Discrete choice models in transportation research. A comprehensive review about application and development of choice models is done by Hensher, Rose, and Greene (2005). This contribution discusses the main techniques for data collection, analyzes, model estimation, and interpretation with regard to all established modeling approaches.

An overview about current research trends in the area of discrete choice models with a focus on latest advances is provided by Train (2009).

A systematic introduction into the field of discrete customer choice models with a focus on transportation is done by Ben Akiva and Lerman (1985). The authors present several choice models for individuals as well as forecasting and estimation techniques. Ben Akiva and Bowman (1998) extend the view on discrete choice models for travel demand towards activity-based models. The application of various discrete choice models in short-term travel decisions is reviewed in Ben Akiva and Bierlaire (2003).

Regarding the airline industry, a sensitivity analysis about customer itinerary choice based on statistical data about preferences is done in Garrow (2009). The author uses online travel data to get insights about choice behavior by combining multiple choice models. Furthermore, Garrow analyzes the impact of time and itinerary preferences on customer choice decisions.

Beside customer preferences for offered products the choice between different itineraries or airlines is one of the most relevant dimensions in air travel. Manifold contributions exist discussing this aspect, e.g., Coldren and Koppelman (2005); Coldren, Koppelman, Kasturirangan, and Mukherjee (2003); Garrow and Koppelman (2004).

Airline revenue management using choice models. The work of Shen and Su (2007) focuses on the general application of discrete choice models in RM. An overview of relevant discrete choice models especially in the context of air travel demand is done by Garrow (2012). The importance of applying choice models in RM theory is highlighted and basic concepts are reviewed and described with a focus on applicability and understandability.

First approaches incorporating customer choice in RM models are presented in Belobaba (1989) and Belobaba and Weatherford (1996). Numerical results proving superiority can be found in Belobaba and Hopperstad (1999). Talluri and van Ryzin (2004a) introduce a general discrete choice model for a set of alternatives. The authors extend existing RM methods to improve results by using the discrete choice model. To this end they formulate maximum expected revenue functions and a structural characterization of optimal control policies. An extension to a network RM formulation is made by Gallego et al. (2004) and Liu and Van Ryzin (2008).

Customer choice among parallel flights in the same market is investigated by Zhang and Cooper (2005). The authors formulate a model neglecting buy-down or sell-up behavior between such flights. Zhang and Cooper (2009) extend this approach and formulate an adapted dynamic program to solve the choice-based RM problem.

Improvements resulting from applying discrete choice models in RM are presented by Vulcano, van Ryzin, and Chaar (2010). Beside the theoretical analysis of the proposed Likelihood approach, Vulcano et al. use numerical experiments to show that a choice-based EMSR optimization method improves revenue by $1\% - 5\%$.

An alternative modeling approach for customer choice using a Markov Chain is made by Blanchet, Gallego, and Goyal (2013). This approach shows robustness against model selection errors and provides a tractable data-driven modeling and assortment optimization approach. Gallego, Ratliff, and Shebalov (2015) extend the basic attraction model formulation (Luce, 1959) towards a generalized attraction model with improvements in parameter quality and flexibility as well as in the resulting RM performance.

A critical view on existing discrete choice models in RM is done by Ferguson, Garrow, and Newman (2012). The authors analyze different existing models and show that they have to deal properly with alternatives that are part of a competitors' portfolio to get usable results. Ferguson et al. conclude that these alternatives have to be incorporated in the own no-purchase alternative.

Models of strategic customer choice behavior. Andersson (1998) states that customers may anticipate selling and pricing strategies of firms to maximize revenue and incorporate this into their decisions. This time-dependent choice behavior based on customers' expectations is termed **strategic customer choice**. Relevant literature in the field of strategic customers is reviewed in Gönsch, Klein, Neugebauer, and Steinhardt (2013).

Li, Granados, and Netessine (2014) estimate the share of air travel customers acting strategic to be between 5.2% and 19.2% depending on market characteristics. Anderson and Wilson (2003) and Anderson, Davison, and Rasmussen (2004) describe possible implications and counteractions preserving airline's success for specific products. A simple optimization model is developed by Wilson, Anderson, and Kim (2006) and extended by Kim (2015) and Gorin, Walczak, Bartke, and Friedemann (2012).

2.2 Flexible Products as Tool to Increase Flexibility

In literature, several definitions of flexibility regarding a system, individual, or process exist. In a general context the term flexibility describes the capability to adapt to new, different, or changing requirements with the aim to be more successful (Pye, 1978). Mandelbaum and Buzacott (1990) characterize these changing requirements, for example, as changing environmental influences "or [...] a change in the decision maker's perception of reality".

Several contributions define flexibility as aspect of robustness. For example, Ionescu and Kliewer (2011) describe flexibility in the context of airline scheduling as part of robustness. Kaluza (2005) and Roy (2010) characterize robustness as combination of two separate aspects: stability and flexibility. A basic definition of flexibility as part of robustness in the context of production- and resource planning is provided by Scholl (2001). Related to RM various contributions deal with robustness, e.g., Curry (1990) or dynamic and adjustable methods, e.g., De Boer (2004).

The work on supply chain management by Christopher and Peck (2004) defines the term resilience as synonym to flexibility. Resilience is the ability to perform well even though the situation is not as expected in the planning stage (Christopher & Peck, 2004). This definition allows various interpretations how one can reach this ability: by introducing flexibility or stability in a system. However, this perspective extends the definition of flexibility from Mandelbaum and Buzacott (1990) towards the possibility of a decision maker or process to return to an initial state instead of adapting to a disturbance.

This thesis defines flexibility as the ability of a system or process to be adapted to a changing and uncertain environment during execution. Flexibility ensures a certain number of alternatives left over after an initial decision was made (adapted from Mandelbaum & Buzacott, 1990). The concept was successfully applied to improve the performance of systems or processes in several research areas, e.g., linear programming (Ben Tal & Nemirovski, 1999) and scheduling (Ionescu & Kliewer, 2011).

Flexibility is an important current research topic, e.g., in manufacturing, supply-chain, and transport management. Existing RM literature also contributes several approaches, but does not yet apply a universal definition of resilience, flexibility, or stability. The approach of Wong et al. (1993) introduces flexibility in RM by accounting explicitly for the stochastic of demand. The authors define a flexible capacity segment in a two-leg itinerary that can be used either by the local demand or by the connecting passengers demand. This approach combines the benefits of the bucket control (Smith & Penn, 1988) and the decomposition approach (Curry, 1990).

A risk sensitivity analysis regarding decision makers in an uncertain RM environment is presented by Barz and Waldmann (2007). Perakis and Roels (2010) make an attempt to calculate control policies in a robust way. Lardeux et al. (2010) give a general introduction how robustness can be integrated in optimization methods.

Several other RM related contributions avoid the term flexibility when talking about dynamic methods or adjustable strategies. Referring to the definitions of Mandelbaum and Buzacott (1990) and Christopher and Peck (2004), these adjustments can be classified as flexibility. The work of Wang and Regan (2006) deals with a dynamic use of different existing capacities for two flights. An extension to the EMSR heuristic for a dynamic capacity management is done by De Boer (2004). Referring to the introduced dynamic the author term her approach EMSRd.

Gallego and Phillips (2004) introduce the concept of flexible products in the RM process. To the author's knowledge, this is the first contribution where flexibility is created through conceptual changes of the RM process instead of modifying methods or process steps. By using flexible products, the airline has the opportunity to adapt prior decisions with regard to a changed situational framework.

2.2.1 Flexible Products in Revenue Management

A comprehensive overview of flexible products in RM with regard to the broadcast industry is given in Müller-Bungart (2007). The author introduces the general concept and formulates RM relevant models and methods.

Note that sometimes the term **opaque products** is used as synonym to describe this particular type of products. As this thesis takes place in the context of the work of Petrick et al. (2010), we refer to the term opaque product as a flexible product that is immediately allocated after it is sold. This contrasts the understanding of opaque products always being sold through an intermediary, as presented for example in Fay (2008).

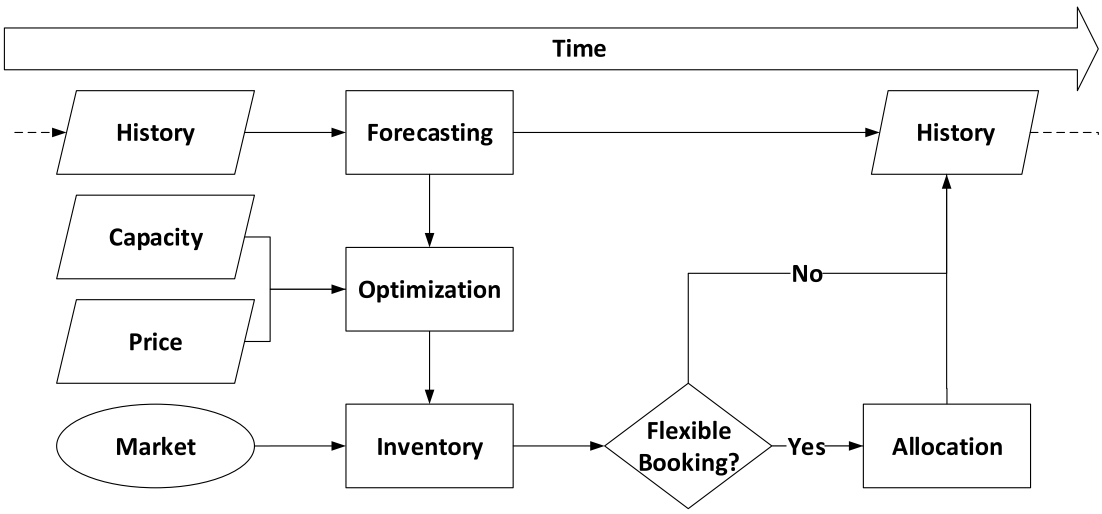


Figure 2.2: Revenue management process with flexible products

Figure 2.2 extends the RM process from Figure 2.1 to include flexible products. After accepting the request for a flexible product it has to be allocated and stored in the inventory.

Gallego and Phillips (2004) first examine the concept of flexible products in the context of airline RM. They introduce them in a two-class, single-leg context and formulate a basic optimization model. Furthermore, Gallego and Phillips develop algorithms to calculate booking limits. Numerical studies show that apart from a more efficient use of capacity and improvements in revenue, the concept has potential benefits of addressing more customers without cannibalizing existing demand. In a follow-up paper, Gallego et al. (2004) extend the concept to a network formulation and incorporate demand dependency between specific and flexible products. By formulating a stochastic RM approach for this setup, they show that it can be approximately solved as a deterministic control problem. These contributions focus on the extension of the availability optimization to handle flexible products.

The work of Petrick et al. (2010) focuses on dynamic capacity control mechanisms used to determine the allocation of flexible products over the sales period. The authors compare different methods with regard to their proposed flexibility and practicability. Furthermore, they establish a correlation between forecast quality and revenue gain for the different methods. In a second paper, Petrick et al. (2012) extend the basic model from Gallego et al. (2004) while formulating control mechanisms allowing arbitrary notification dates. The benefits from using flexible products and late notification dates in an uncertain environment are explicitly shown with numerical studies.

Fay (2008) takes a look at firms selling flexible products through an intermediary. He formulates an analytical model and discusses questions about assumptions and requirements. Furthermore, managerial implications when using an opaque selling channel and some future research directions are presented. Another approach dealing with a special form of flexible products is proposed by Post (2010). The author introduces the concept of variable opaque products, where customers are able to select the acceptable degree of uncertainty themselves. After presenting the theoretical framework, a pricing heuristic is proposed.

Chen, Gallego, Li, and Lin (2010) present a slightly different view on flexible products. The authors assume a flexible demand segment where customers are willing to take multiple flights. As a result, an optimal control policy is formulated and characteristics revealed by a numerical study. Jiang (2007) presents guidelines for effectively selling flexible products with a focus on price-discrimination and market segmentation.

An empirical analysis of customer choice behavior is done by Mang et al. (2012). They use historical data from a low-cost airline, in order to be able to individually determine the level of flexibility of the flexible product. Beside the identification of behavioral drivers regarding the amount of flexibility, they present estimates for the effect of introducing flexible products.

Talluri (2001) presents an approach where an airline serves several itineraries on the same route. He assumes that customers are indifferent between the exact routing, but have preferences regarding other characteristics. The assignment to the specific routing is made right after booking. As a result, Talluri formulates a bid price control for this setup. A similar work in the context of air cargo is done by Chen, Günther, and Johnson (2003). The authors extend a route-based bid price control to deal with a dynamically generated set of routes. Fay and Xie (2010) investigate two selling strategies involving buyer uncertainty and show that both can be used to address unobservable buyer heterogeneity.

A more theoretical view on flexible products is provided by Gönsch and Steinhardt (2013) by extending the traditional dynamic program decomposition models to cope with flexible products. Numerical results show the improvements of their model formulation as well as general effects when flexible products are introduced. Similar results are provided by Gönsch, Koch, and Steinhardt (2014) when they extend the deterministic linear program formulation to consider the benefits of flexible products.

The valuation of flexible products is adjusted to capture the monetary benefits that will be considered when calculating an optimal booking control policy.

2.2.2 Models for Availability Decision of Flexible Products

Introducing flexible products in an existing RM system changes the requirements for optimization and inventory and calls for adaption of the corresponding models. To clarify the concept of handling flexible products, the following section provides an overview of state-of-the-art models and methodological concepts.

Let $F = \left\{1, \ldots, \overline{f}\right\}$ be the index set of flexible products. Following Gallego and Phillips (2004) a flexible product is defined as a set of alternatives. We term this set manifestation set M_f. It can be defined as

$$M_f \subseteq S, \quad \forall f \in F. \tag{2.2.1}$$

We can impose some assumptions about the supply offered by the airline without restricting the applicability of the subsequently introduced models. First, the airline offers only direct flights without the possibility of connections. Flexible products are offered as an alternative to a set of specific products and the manifestation set includes one specific product for each flight offered. The last assumption is ensured by two conditions

$$\sum_{m \in M_f} y_{mr} = 1, \quad \forall r \in R, \forall f \in F \quad \text{and} \tag{2.2.2}$$

$$\sum_{r \in R} \sum_{m \in M_f} y_{mr} = \overline{r}, \quad \forall f \in F. \tag{2.2.3}$$

Based on these assumptions we can define the manifestation set for a flexible product as the set of resources: $M_f = R$.

Furthermore, let $c_m^t,\ \forall m \in M_f,\ \forall f \in F$ denote the remaining capacity at a certain point during the sales horizon regarding a manifestation. In case booking controls based on bid prices are used, we denote the bid price related to a manifestation by $\pi_m^t\left(c_m^t\right),\ \forall m \in M_f,\ \forall f \in F$.

These generalizations allow us to simplify the following formulations for optimization and control policies to clarify the overall understanding. Beside the model of Petrick et al. (2012), we discuss a second approach to handle flexible products without restricting the general applicability of the models. We will discuss relevant implications later in this section.

Handling flexible products similar to specific products. A simple approach to include flexible products in an existing RM system is to handle them similar to specific products. The methods introduced in Section 2.1 can be easily applied if the flexible bookings are immediately allocated after acceptance. The remaining capacity at each point in time represents the true remaining capacity. Therefore, optimization models and control policy formulations are still valid and lead to a feasible result. In this case, however, benefits resulting from selling flexible products are only partially exploited (Petrick et al., 2012).

To exploit the full benefits of flexible products, the methodology has to be extended to handle them more efficiently. Formulations regarding a dynamic programming approach and a bid price control for flexible products are presented by Petrick et al. (2010) and Petrick et al. (2012). A simplified version of the following optimization model and the ensuing booking control policy are used later in the mathematical models and the computational studies.

Handling flexible products using the model of Petrick et al. (2012). This model proposes a more sophisticated way to calculate the optimal booking control policy. Its application provides the airline with an increased amount of flexibility. Petrick et al. (2012) use a dynamic programming approach and define a modified Bellmann equation that is solved recursively.

Let $Y \in \mathbb{Z}^{\bar{f}}$ be a vector counting the number of requests accepted for all flexible products. These flexible bookings are not directly allocated to a resource. The point in time when flexible bookings are finally allocated is denoted by $t^* \in T$. Let $q_f^t \in [0,1]$ be the request probability for a flexible product and $p_s^t \in [0,1]$ the request probability for a specific product. Accordingly, the probability of no request occurring is $p_0^t = 1 - p_s^t - q_f^t$. The proposed model allows varying the allocation time. Let $c \in \mathbb{R}^{\bar{r}}$ be the vector denoting the remaining capacities for all resources. An allocation of flexible bookings in the model of Petrick et al. bases on variable costs. We denote the variable costs for a booking of a flexible product when allocated to a certain manifestation by $v_{fm} \in \mathbb{R}$.

Petrick et al. explicitly differentiate flexible and specific products regarding the request probability and capacity allocation in the value function $V(t, c, Y)$. Furthermore, they divide the sales horizon into three different parts depending on t^* and therefore define an adapted Bellman equation consisting of three parts. For $t \geq t^*$, the general formulation is extended to handle sales for specific and flexible products and to memorize accepted flexible requests for allocation. For specifying the allocation, an additional decision variable $y_{fm} \in \{0, 1\}$ is introduced that denotes the allocation of a flexible product to a particular manifestation.

For $t > t^*$, specific as well as flexible products are sold. Specific products are handled as described in 2.1 whereas accepted requests for a flexible product are memorized in $e(j)$, denoting the jth standard basis vector in $\mathbb{R}^{\bar{f}}$. The capacity for the resources is not subsequently reduced in this case. The adapted Bellman equation for calculating the value of expected future requests is formulated as

$$
\begin{aligned}
V(t,c,Y) = & \sum_{s\in S} p_s^t \cdot max\left\{V(t-1,c,Y), f_s + V(t-1,c-e(s),Y)\right\} \\
& + \sum_{i\in F} q_i^t \cdot max\left\{V(t-1,c,Y), f_i + V(t-1,c,Y+e(i))\right\} \\
& + p_0^t \cdot V(t-1,c,Y).
\end{aligned}
\tag{2.2.4}
$$

For $t = t^*$, the following modification of the value function is used where flexible bookings are finally allocated and the capacity of the respective resources is reduced

$$
\begin{aligned}
V(t,c,Y) = \max\nolimits_{y_{fm}} & \left\{ -\sum_{f\in F}\sum_{m\in M_f} v_{fm}\cdot y_{fm} + V\left(t-1, c-\sum_{f\in F}\sum_{m\in M_f} y_{fm}, 0\right)\right. \\
& \left. \mid \sum_{m\in M_f} y_{fm} = Y_f,\ \forall f\in F, y_{fm}\in\mathbb{N},\ \forall f\in F, m\in M_f \right\}.
\end{aligned}
\tag{2.2.5}
$$

During the third period ($t < t^*$) only specific products are offered and the formulation of the Bellman equation corresponds to the one for RM without flexible products (2.1.6).

Petrick et al. state that this theoretical model is hardly applicable in practice due to its computational complexity. In their work they propose a dynamic approximation method called Certainty Equivalent Control (see Bertsekas, 2005; Bertsimas & Popescu, 2003) to reduce the complexity. This approximation, however, can be further reduced by applying a bid price formulation. For a comprehensive formulation of the optimization model, we refer here to the work of Petrick et al. (2012).

Relying on the formulation based on bid prices, we will now establish a booking control policy for flexible products extending (2.1.10). This control policy that decides to accept a request for a flexible product or not has to account for all possible manifestations of the flexible product. The decision to accept or deny a request bases on the current valid bid prices: at time $t \in T$ a request for a flexible product $f \in F$ should only be accepted if at least one manifestation $m \in M_f$ exists fulfilling the following conditions

$$f_f \geq \pi_m^t\left(c_m^t\right) \quad \text{and} \tag{2.2.6}$$

$$c_m^t > 0. \tag{2.2.7}$$

The customers' preferential choice models and the methods to allocate flexible bookings proposed in this thesis partially rely on the concepts formulated in this section. However, the aim is to explore the effects when customers' preferential choice between manifestations is assumed.

2.2.3 Choice Models for Flexible Products

Gallego and Phillips (2004) introduce a first discrete customer choice model for RM with flexible products. Under the first-choice-only-choice assumption they model cannibalization and demand-induction between specific and flexible products. An extended customer choice model in a network setting is provided by Gallego et al. (2004). The authors show that column generation is an effective solution approach for optimization methods assuming independent demand models, the multinomial Logit model, and other attraction models.

Lee et al. (2012) provide an extensive study about the probability to exclude elements from the manifestation set after buying a flexible product. Based on real data from a European airline, several attributes of manifestations could be extracted influencing the exclusion probability. Lee et al. apply a multidimensional binary Logit model to predict these probabilities.

Anderson and Xie (2009) develop a customer choice model for flexible products that aims to improve demand segmentation. The impact of different distribution channels for flexible products on potential customers' valuation of unfairness is explored by Lee and Jang (2013). The authors state that approximately one third of potential customers for flexible products will avoid distribution channels, they evaluated as unfair.

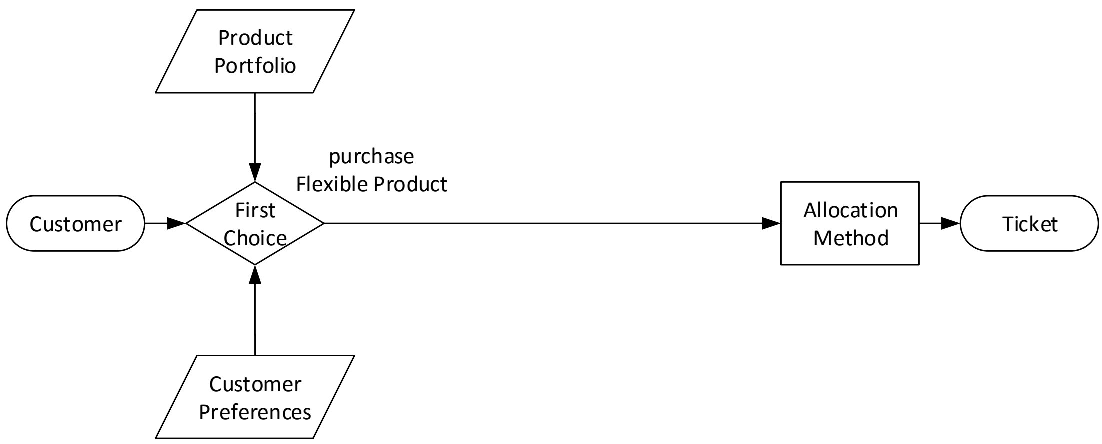


Figure 2.3: Decision process for customers of flexible products under the indifference assumption

The decision process for customers of flexible products is depicted in Figure 2.3. It is assumed that customers are indifferent between the manifestations of the flexible product. They are only faced with the decision between buying the flexible or specific product. This decision depends on their preferential choice and the offered product portfolio. In case they buy a flexible product, the airline has to allocate the booking to be able to create a final ticket for the customer.

The transparency of a flexible product and the role of information when customers make decisions is discussed in Granados, Gupta, and Kauffman (2012). The authors state that market transparency can be used to attract customers. However, airlines have to be aware of the diffusion of information as they highly influence consumption decisions of customers. Especially the concept of selling flexible products is discussed, as they represent a particular strategy to segment markets based on incomplete information. Furthermore, Granados et al. highlight the importance of synergizing information design, underlying selling mechanism, and supporting technological platform.

A similar aspect is investigated in Bai, Yan, and Liu (2015). The authors look at customers' price-elasticity in relation to the opacity of flexible products. They develop a customer choice model depending on the design of the flexible product and discuss price function design and opacity design. As a result, they propose analytical formulations to design variable flexible products.

The impact of strategic customers in the presence of flexible products is documented in Jerath, Netessine, and Veeraraghavan (2009) and Jerath, Netessine, and Veeraraghavan (2010). They state that airlines start to use an opaque intermediary as a corrective action for strategic customer behavior induced by last minute offerings. Their research shows that the use of such intermediaries leads to an improved capacity usage and an increasing demand level. The work of Fay and Lee (2015) shows potential benefits of strategic behavior. The authors focus on customer expectations in name-your-own-markets and state that it could be beneficial for the airline if customers learn the true behavior and formulate accurate expectations.

2.2.4 Practical Applications of Flexible Products

This section presents various practical applications of flexible products and several websites collecting information about flexible product sales.

Hotwire offers flexible products for hotel rooms, airline seats, or rental cars (Hotwire Inc., 2015). Their contingent results from unsold inventories of various travel companies. This implies that a flexible product can consist of specific products from different travel companies. Right after the customer has paid, Hotwire allocates the flexible product. Afterwards, there is no possibility for customers to cancel their purchase.

Priceline sells flexible products for hotel rooms, car rentals and complete vacation packages combined with a “Name Your Own Price” system (priceline.com LLC, 2015). In addition to the company serving the flexible product, Priceline hides the exact location of the hotel and details about the travel itinerary until the purchase has gone through. There is no possibility to later cancel the purchase.

Currently, **Germanwings** is the only airline selling flexible products (Germanwings GmbH, 2015). They offer various flexible products consisting of multiple destinations in Europe that have a common theme. A booking is allocated right after the purchase was successful, without the possibility to cancel the purchase later. Germanwings implemented a web-interface allowing customers to exclude manifestations on payment of a fee. Germanwings terms this concept Blind Booking.

Digital and mobile platforms allow to share and distribute information between a large number of individuals. Customers use online platforms and digital communication to share their knowledge and information on experiences and opinions about offers of flexible products.

The website **betterbidding.com**, for example, provides lists of possible descriptions for the different hotels in Hotwire offers, so that customers can guess which hotel may be hidden behind the flexible product (BetterBidding.com, 2015). Furthermore, the website collects successful deals for Hotwire and successful bids for Priceline offers. These are stored and visualized, so that new customers can review the history including the dates and bids of successful bookings for certain hotels. Based on such a history, customers can form expectations about their manifestation when booking via Hotwire or Priceline.

The website **biddingtraveller.com** helps customers to find promising bidding strategies for Pricelines' Name Your Own Price system (The Bidding Traveller, 2015). A bidding strategy consists of the lowest and highest price a customer wants to pay. Using historical data, the website suggests bids for the destination and time period the customer had chosen that will be accepted from Priceline. The historical data is continuously updated by customers submitting their successful and failed bids.

A GoogleTrends analysis starting from 2009, the time Germanwings introduced Blind Booking, to 2015 shows an increasing number of queries for the term "Blind Booking". In several forums, e.g., vielfliegertreff.de, vielfliegerforum.de, or flyertalk.com, customers discuss tricks and strategies for purchasing Blind Booking tickets. Searching for the term "Blind Booking" at vielfliegertreff.de shows 27,914 views[2] for the thread about general tips and experiences about Blind Booking. A thread discussing how to predict the outcome of Blind Booking achieves 27,374 views[2] and another thread about the predictability of Blind Booking reaches even 41,448 views[2]. The overall amount of registered users on this website is 30,250[2]. This indicates that customers try to exploit the concept of flexible products to get a desired offer for a small price while minimizing uncertainty.

[2] Accessed: 12.12.2015

3 Research Gap: Customers' Preferential Choice Between Manifestations

The previous chapter reviewed contributions in the areas of RM and flexible products. In addition to a comprehensive overview, notation and models relevant for this thesis have been introduced.

One aim of current RM research is to develop methods handling the increasing uncertainty (see Talluri & van Ryzin, 2004b, Chap. 1). The Journal of Revenue and Pricing Management, for example, highlights the importance of this aspect by publishing a special issue on robustness in 2016. Section 2.2 introduced flexibility as a possibility to prepare methods against uncertainty. One of the promising concepts are flexible products, established by Gallego and Phillips (2004) in the context of airline related RM. In a follow-up paper, Gallego et al. (2004) extend the concept to a network-based RM formulation including the choice between specific and flexible products. Figure 2.2 illustrates the modification of the RM process towards the usage of flexible products.

Starting with the work of McFadden (1973), choice models have become a powerful tool to model customer decision making. Contributions regarding customer choice models focus on presenting valid formulations concerning the deterministic part of the choice function, e.g., Ben Akiva and Bierlaire (1999) and Talluri and van Ryzin (2004a), or the estimation of parameters when a specific choice model is assumed, e.g., Lee et al. (2012).

Various contributions to flexible products assume indifference of customers between the manifestations (cf. Gallego et al., 2004; Petrick et al., 2010, 2012). This implies that customers valuate each manifestation equally. In case the manifestation sets of two flexible products have common elements, this assumption implies indifference between both alternatives. This contradicts the existence of a unique utility maximizing decision when multiple flexible products are offered.

The aim of this thesis is to model customers' preferential choice between manifestations of flexible products. These new choice models account for individual customer behavior outlined by descriptive research on flexible products (cf. Lee et al., 2012) and practical applications (see Section 2.2.4). To this end, this thesis extends the customer decision process depicted in Figure 2.3 to incorporate a second decision regarding the manifestations of a flexible product. To quantify the effects of such customer choice on RM performance, computational studies are used to evaluate a variety of experiments.

Figure 3.1 shows the extended decision process. New parts are surrounded by a gray box. The additional decision depends on the customers' preferences and the manifestation set defining the flexible product. Obviously, this decision is only relevant in case that the customer has decided to buy a flexible product at the first decision.

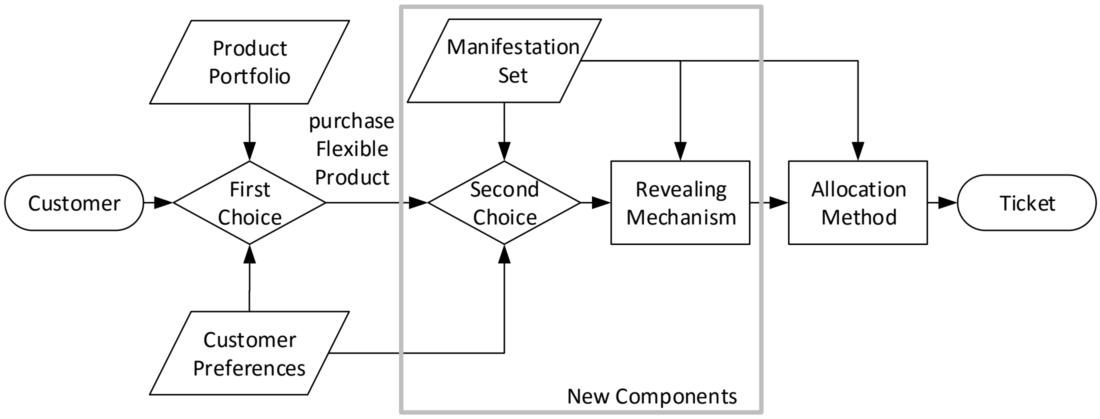


Figure 3.1: Decision process for customers of flexible products assuming preferential choice for manifestations

The agenda of this thesis has five research questions. First, the general aspects of customers' preferential choice are addressed (research question 1 and research question 2). This includes the evaluation of newly introduced revealing and allocation methods. Research question 3 extends the perspective towards strategic customer behavior between specific and flexible products and accounts for the resulting trade-off. Finally, research question 4 and research question 5 focus on the effects of parameter uncertainty and misleading model assumptions to present a proper understanding of the impacts on RM performance.

Research Question 1. *What are the implications for RM with flexible products resulting from different models of customers' preferential choice between manifestations?*

Including flexible products in RM creates demand to extend the theory about customer decisions as customers may face two sequential decisions (see Figure 3.1). In a first decision, they choose between buying a specific or a flexible product. If they decide to buy a flexible product, a second choice between the manifestations of the flexible product follows. Current research on flexible products assumes indifference between manifestations (cf. Gallego et al., 2004; Gallego & Phillips, 2004; Petrick et al., 2010, 2012).

However, customers are not indifferent between manifestations. Descriptive research about customer choice showed that various attributes of manifestations exist, which influence their popularity (Lee et al., 2012). In practical applications, several concepts exist where customers can express preferences for manifestations. Hotwire, for example, allows customers to modify the manifestation set based on attributes such as hotel category, recommendation, or facilities. Germanwings let customers decline certain manifestations in return for an additional fee. Limiting the manifestation set in any way reduces the uncertainty carried by the flexible product.

Mathematical modeling of appropriate customers' preferential choice models can help in evaluating the impacts of such concepts already applied in practice (Chapter 4). When analyzing these models with regard to RM in general implications for other process steps become clear.

Research Question 2. *How does RM performance depend on the parameterization of customers' preferential choice models, revealing mechanisms, and allocation methods?*

Relevant literature (cf. Gallego & Phillips, 2004; Petrick et al., 2012) assumes a simple allocation method focusing only on the airlines' benefit. When airlines establish more sophisticated choice models, however, various objectives are possible. Implementing these creates the need for an interface for customers to communicate their preferences to the airline. Such preference revealing mechanisms have to present possible manifestations and have to allow customers to state their preferences.

For example, the control mechanisms for flexible products described in Petrick et al. (2010) define the way flexible products are handled in the inventory. The success of control mechanisms depends on their suitability to the interaction of demand and supply. Petrick et al., however, neglect the impacts of customers' preferential choice between manifestations.

This thesis presents various allocation methods (Chapter 6) that emphasize customer preferences to different degrees. In a setup with flexible products, allocation methods represent a highly relevant cause for the airlines' success. Only by combining well-defined and matching revealing and allocation methods, airlines are enabled to apply a more sophisticated and customer-oriented process for handling flexible products. To the best of the author's knowledge, no allocation method that uses revealed customer preferences exists.

To evaluate the impact of different parameterizations and combinations of models, the mathematical concepts are implemented in a simulation tool to perform a computational study (Chapter 9). The results from various experiments help to compare the impacts of different parameterizations on the RM process and the possible benefits or disadvantages when using choice models.

Research Question 3. *What are the consequences for RM with flexible products if customer choice includes strategic choice between flexible and specific products? Does some degree of strategic behavior offset the benefits of flexible products?*

Introducing flexible products can induce bookings from new demand segments (cf. Gallego & Phillips, 2004; Mang, Spann, & Post, 2009). Selling them, however, creates the danger of cannibalizing demand for specific products. Cannibalization can be traced back for example to a dependency of customer choice on the current set of available products. A special form of dependent demand occurs when customers rely on previous experiences and expectations in their decision process, which is called strategic customer behavior. The way flexible products are currently used in practice does not fence against a possible strategic customer behavior. Research disregards such exploiting customer behavior.

However, strategic customer behavior cannot be precluded even if customers request flexible products (Jerath et al., 2009, 2010). When buying flexible products, expectations, experiences of themselves and of other customers concerning possible allocations render the decision behavior of customers more strategic. Due to an increasing transparency in the hospitality market, a trend in customer manner towards a more strategic behavior is encouraged. The existence and popularity of websites like betterbidding.com indicates this trend. Such websites provide information about possible allocations of flexible products based on previous purchases.

Shen and Su (2007) highlight the importance of modeling strategic customer behavior regarding a portfolio with only specific products. To the best of the author's knowledge, only Jerath et al. (2009) present a first step to achieve a better understanding of customer behavior in the presence of flexible products. The authors state that assuming strategic customer behavior and designing appropriate choice models can overcome shortcomings and drawbacks.

Practitioners fear about the fact that customers buying flexible products may act strategically. Discussions with subject matter experts at Deutsche Lufthansa AG clarified that from the practitioners' view, the most feared dimensions are predictability and transparency. The airline's benefits of selling flexible products are diminished if customers can outline the allocation of a specific flexible product. An induced revenue gain from selling flexible products may be completely offset due to strategic customers. This fear is strengthened and becomes more relevant as market transparency increases and communication between customers becomes even more intensive.

This thesis formalizes a customer choice model that incorporates strategic customer behavior for flexible products (Chapter 5). To evaluate the consequences of strategic decision behavior, this choice model considers price- and uncertainty-sensitive customer behavior regarding the outcome of a flexible product. A computational study implements the strategic choice model in various experiments (Chapter 10). The numerical results help to explore the correlation of demand setup and impacts of strategic customer behavior. Further, experiments evaluate the effects of variations in the transparency of the allocation process and in the reliability of information about the outcome. The numerical results will examine implications and possible counteractions for airlines, for example, by improving their control mechanisms for flexible products.

Research Question 4. *What is the effect of flawed input parameters on different combinations of customers' preferential choice model, revealing mechanism, and allocation method?*

RM methods have to deal with inaccuracies in demand estimation and with exogenous uncertainties affecting the optimality of calculated control policies. Several contributions investigate the impact of model or parameter inaccuracies on the RM performance. In general, especially the impact of forecast precision on revenue is well researched (cf. Weatherford & Belobaba, 2002; Weatherford & Pölt, 2002). Petrick et al. (2012) state that selling flexible products can improve the results of an airline if uncertainties (like imprecise forecasts) arise during a sales period. The authors show that the improvements increase with an increasing level of uncertainty.

If airlines start introducing additional aspects like considering the fulfillment of customer preferences, they have to handle new sources of uncertainty. Only by extending existing allocation methods based on different RM parameters, the potential of uncertainty within the process increases. To this end, after examining the theoretical potential of a customer-centric RM the practicability of the proposed models in the presence of parameter uncertainty has to be evaluated.

This thesis uses computational experiments to examine revenue robustness of allocation methods (Chapter 11). The computational experiments include setups where various input parameters for allocation methods are distorted. The numerical results are interpreted to show the relevance of imprecise parameter assumptions with regard to actual RM methods and extensions to model customers' preferential choice between manifestations.

Research Question 5. *What happens if the airline implements a faulty customers' preferential choice model? How does this affect revealing mechanisms and allocation methods?*

RM became more successful as models changed from flight-based to network-orientation. Also, the success of dependent customer choice models used in forecasting was a result of better fitting assumptions. These cases clarify that RM performance strongly depends on the match between assumptions and reality.

For general RM, Queenan, Ferguson, Higbie, and Kapoor (2007) compare different customer choice models and the impacts when demand behaves differently. Assuming customer choice for the manifestation set of a flexible product, uncertainties resulting from model mismatch become more relevant. They have a significant impact on the performance of flexible products in RM. To the best of the author's knowledge, no contribution dealing with this aspect exists. Current research neglects models for customers' preferential choice between manifestations as well as formulations of revealing mechanisms and allocation methods.

This thesis highlights and discusses relevant combinations of customers' preferential choice, revealing mechanism, and allocation method using mathematical models (Chapter 7 and 11). A theoretical analysis and discussion about the impacts of model mismatch and wrong assumptions helps to understand the effects of false assumption in the context of flexible products.

Part II

Mathematical Models of Customers' Preferential Choice and Allocation for Flexible Products

4 Modeling Customers' Preferential Choice for Manifestation Sets

A first step to examine the effects of customers' preferential choice with regard to the manifestation set of flexible products is the formulation of mathematical models. Such models formalize the customers' preferences for manifestations. They help to formulate methods to allocate flexible products considering different objectives later.

Section 4.1 starts with introducing the general notation, followed by the simplest choice model assuming indifference in Section 4.2. A choice model where customers can limit the manifestation set is presented in Section 4.3. Finally, Section 4.4 introduces the actual preferential choice model.

4.1 Prerequisites and Notation

Modeling customers' preferential choice between manifestations of a flexible product becomes relevant when customers have already decided to buy a flexible product. This section introduces several prerequisites and constraints valid for all upcoming modeling approaches. As air transportation is the industry where Revenue Management (RM) is mostly used, the following notation relies on the airline industry. However, the models and formulations are easily transferable to other use cases and industries.

Following the framework of Domencich and McFadden (1975), a choice model can be characterized by: a decision maker, the alternatives and their attributes, and a decision rule that helps the decision maker to choose between the alternatives. This framework is applied by Ben Akiva and Lerman (1985) and Ben Akiva and Bierlaire (1999) towards general travel decisions. Garrow (2012) addresses decision making in the context of air travel demand by using and extending this framework. This thesis applies this general framework in the following paragraphs to explicate decision making characteristics between manifestations of a flexible product.

In the context of air travel demand, a **decision maker** is an individual, a group of individuals, or a company. With regard to the application to flexible products, we focus in the following on individuals as decision makers. Therefore, regarding the first decision (see Figure 3.1), each customer that considers booking a flexible product is a decision maker. Regarding the second decision, the set of decision makers is restricted to customers that already requested a flexible product and were accepted by the airline.

When customer decide to buy a flexible product instead of a specific one, this choice is based on the current set of offered products and the **manifestations** included in the flexible product. For the second decision, the manifestations represent the alternatives a customer may choose from in a general choice model. Based on his preferences, each customer has an individual choice set of manifestations he is willing to accept as allocation.

Each customer evaluates the **attributes for each manifestation** differently. Using their individual utility function, customers calculate utility values for each manifestation. The attributes for the manifestations may be deterministic or stochastic quantities. For example, regarding the manifestation set for a flexible product in the airline industry, the geographic and time characteristics of the manifestations are deterministic attributes. However, especially the attributes quantified by stochastic values such as the allocation probability may primarily influence the utility valuation and therefore the individual choice.

A **decision rule** formalizes the process of customers comparing the manifestations. They use information about the manifestations' attributes to choose a manifestation. Referring to the definition in Ben Akiva and Lerman (1985), we assume that individuals have consistent and transitive preferences. Transitivity ensures that for preference values $p(A),\ p(B),\ p(C)$ for three alternatives A, B, C it holds that

$$[p(A) \geq p(B)] \ \wedge \ [p(B) \geq p(C)] \quad \Rightarrow \quad p(A) \geq p(C). \tag{4.1.1}$$

In case an individual is faced with an identical choice situation several times, consistency captures the fact that an individual will always choose the same alternative.

Customer decisions can be described using a discrete choice model and a utility maximizing decision rule (Garrow, 2012). However, this chapter does not explicitly formulate an actual model or estimate the coefficients. To model preferential choice for manifestations, this thesis uses the output of an existent model: the utility value. The following formulations rely on the existence of such utility values for all manifestations. Referring to Figure 3.1, we focus on the second decision, influenced by the preferential choice model and the manifestation set.

Products offered by competitors do not affect this second decision. The customers' choice is already restricted to the manifestation set of a flexible product. Therefore, we consider a setup where only one airline acts in the market and offers various specific products and one flexible product.

Let M_f denote the manifestation set for a flexible product $f \in F$. R denotes the set of resources and $y_{mr} \in \{0, 1\}$ the amount of capacity utilized by manifestation $m \in M_f$ on resource $r \in R$.

Let B denote the set of customers acting in the model. Each customer $b \in B$ associates a utility value $u_{bm} \in \mathbb{R}^+$ with each manifestation $m \in M_f$. The minimal utility that a customer accepts a manifestation as possible outcome of the flexible product is denoted by $u_{bf} \in \mathbb{R}^+,\ \forall b \in B, \forall f \in F$.

To exploit the benefits of flexible products, it suffices that M_f contains one specific product for each resource. Adding more than one product will not improve the reactivity of flexibility. Therefore, we define

$$\sum_{m \in M_f} y_{mr} = 1, \quad \forall r \in R. \tag{4.1.2}$$

For each customer exists a manifestation $m_b \in M_f$ maximizing his utility value for flexible product $f \in F$:

$$m_b = \text{argmax}_{m \in M_f} \left\{ u_{bm} \right\}, \quad \forall b \in B. \tag{4.1.3}$$

To render utility values for different customers comparable, we define preference values derived from the actual utility values by transformation. Given that a customer has chosen a flexible product, we denote a preference value for a manifestation by $q_{bm} \in \mathbb{R}^+$. We define an interval $Q \subseteq \mathbb{R}^+$ including the preference values for all customers. Let $T\left(u_{bm}\right)$ be a transitive function transforming the utility value for each customer and specific product into a preference value:

$$T : \mathbb{R} \rightarrow Q, \quad u_{bm} \mapsto q_{bm}. \tag{4.1.4}$$

The transformation of utility values into preference values has to preserve a given order:

$$u_{bk} < u_{bj} \quad \Rightarrow \quad q_{bk} < q_{bj}, \quad \forall j,k \in M_f, \forall b \in B \quad \text{and} \tag{4.1.5}$$
$$u_{bk} = u_{bj} \quad \Rightarrow \quad q_{bk} = q_{bj}, \quad \forall j,k \in M_f, \forall b \in B. \tag{4.1.6}$$

As $T\left(u_{bm}\right)$ fulfills transitivity, the decision rule (4.1.3) is also well-defined if applied to preference values. The assumptions and definitions provided so far do not restrict the application to any of the following modeling approaches. The notation is completely independent from supply characteristics to ensure general applicability.

If an airline aims to account for preferential choice regarding the manifestation set, it needs to implement a mechanism for customers to communicate their actual preference values. For each proposed customers' preferential choice model, we will characterize respective revealing mechanisms. Given existing online booking technology, implementing a wide range of revealing mechanisms does not pose a technical problem for airlines.

4.2 Indifference Between Manifestations

Existing choice models for RM with flexible products mostly assume that customers are indifferent between all manifestations.

4.2.1 Modeling Indifference Between Manifestations

Indifference implies that customers link the same utility value with all possible manifestations of a flexible product. For a certain flexible product $f \in F$ offered by the airline and given a utility value $u_{bm} \geq u_{bf},\ \forall m \in M_f$, we can formulate the indifference assumption for customers as follows

$$u_{bk} = u_{bj}, \quad \forall k, j \in M_f,\ \forall b \in B. \tag{4.2.1}$$

Airlines have to consider two different situations: whether they offer more than one flexible product and if they suppose customers to be indifferent. Assume a situation where two flexible products $f, g \in F$ are offered.

Case 1: $M_f \cap M_g = \emptyset$**.** In this situation the indifference assumption (4.2.1) just implies indifference between the manifestations for each flexible product. As a customer already decided to book f or g follows

$$u_{bk} \neq u_{bj}, \quad \forall k \in M_f,\ \forall j \in M_g. \tag{4.2.2}$$

Although he is indifferent between the elements of both manifestation sets, the first choice between the two flexible products is not affected because of condition (4.2.2). This choice obviously depends on additional attributes, e.g., the sizes of M_f and M_g.

Case 2: $M_f \cap M_g \neq \emptyset$**.** Now, because of equality (4.2.1) customers are indifferent between all manifestations in M_f and M_g. It follows $u_{bk} = u_{bj},\ \forall k \in M_f,\ \forall j \in M_g$. This implies indifference between both flexible products g and f itself. The airline should not offer f and g simultaneously or adapt the manifestation sets to enforce a unique decision. As customers are forced to book a particular flexible product, the airline is not able to use the additional flexibility induced by the indifference between both flexible products, e.g., by offering only one flexible product with a manifestation set $M_f \cup M_g$.

A generalization of these two cases for an arbitrary number of flexible products can be made without any restrictions.

4.2.2 Implications of Indifferent Customers

If customers are indifferent, the airline does not need any additional information for processing a flexible booking.

Researchers assume indifference between manifestations because of its simplicity. Practitioners assume indifference because they want to protect the flexibility gained through selling flexible products. Indifference allows airlines to transfer the complete uncertainty to customers. As the airline allocates the flexible bookings, they keep the ability to handle uncertainties during the sales period.

Practical examples and the contribution of Lee et al. (2012) show that practitioners already relax the indifference assumption. Some practical applications account for existing customer preferences (see Section 2.2), e.g., Germanwings offers the possibility for customers to book a Blind Booking ticket to limit the manifestation set based on their individual preferences. The following section will introduce a choice model regarding customers' individual acceptance decision for each manifestation.

4.3 Limited Acceptance of Manifestations

A simple model to incorporate customer preferences for manifestations is to account for binary decisions. Such decisions express the general acceptance for each manifestation by modifying the customers' choice set.

4.3.1 Modeling Limited Acceptance

This approach models limitations of the manifestation set based on customer preference values. Such limitations allow customers to reduce the uncertainty of the flexible product.

Let $C_{bf} \subseteq M_f$ denote the choice set for customer $b \in B$ and flexible product $f \in F$. Each customer has a minimal utility $u_{bf} \in \mathbb{R}^+, \ \forall f \in F, \ \forall b \in B$. The choice set restricts the manifestation set to manifestations with a utility value greater than u_{bf}

$$C_{bf} = \{m \in M_f \mid u_{bm} \geq u_{bf}\} \subseteq M_f, \quad \forall f \in F, \ \forall b \in B. \tag{4.3.1}$$

Definition (4.3.1) does not impose any requirements on the way in which the underlying utility is evaluated. Based on the definition of the threshold value, the choice set C_{bf} is independent from the choice to buy a particular flexible product. For a different flexible product $f' \in F$ with the same manifestation set, the minimal utility may be different: $u_{bf'} \neq u_{bf}, \ f, f' \in F$ and therefore it is possible that $C_{bf} \neq C_{bf'}$.

Example. *Consider the example introduced in Chapter 1. A customer departs from Frankfurt. The manifestation set of the flexible product includes three alternative destinations: Munich, Hamburg, and Berlin connected via direct flights from Frankfurt. Munich is the most unpopular destination and for the utility the customer associates with Munich holds that $u_{b,Munich} < u_{bf}$. Now the remaining choice set for this customer only includes Berlin and Hamburg.*

The easiest way for the airline to account for limitations is to allow customers to modify the manifestation set during the sales process.

4.3.2 Revealing Limited Acceptance

In general, we can differentiate between two forms of revealing mechanisms allowing customers to limit the manifestation set. On the one hand, it can be bottom-up: the airline presents the whole manifestation set and customers are able to pick manifestations to build up their individual choice set. On the other hand, the mechanism can be top-down: the airline allows customers to exclude some alternatives given the manifestation set. In both cases, the airline has to define a minimal size of the choice set C_{bf}.

The number of elements in the manifestation set defines the flexibility gained by the airline through offering the flexible product. Simultaneously, it defines the uncertainty taken over by the customers. We denote the minimum cardinality for C_{bf} by $\overline{m}_f$. Then, for each flexible product and customer it is valid that

$$\overline{m}_f \leq |C_{bf}| \leq |M_f| \, . \tag{4.3.2}$$

Allowing customers to limit the manifestation set is a possibility to account for their preferences regarding manifestations in a binary way. To ensure a certain level of flexibility or uncertainty in the model, airlines can apply monetary incentives. For example, postulating an additional fee for excluding manifestations or offering an additional discount for choosing additional manifestations in bottom-up models.

The presented formulation does not account for preferences for the remaining manifestations in the choice set. Such preferences may exist and can be handled in various ways, e.g., the airline can assume indifference for the remaining manifestations or present a second preferential choice model.

4.3.3 Implications of Limiting the Manifestation Set

Limited acceptance of manifestations allows customers to reduce the size of the manifestation set. In case customers decline some manifestations, this still enables them to book a flexible product as they can exclude these manifestations as allocation. This can increase the number of customers booking flexible products compared to a setup where the airline assumes indifference.

Example. *Let the customer be able to book only the flexible product and the customer's choice set only includes Berlin and Hamburg. If the airline allows him to exclude Munich, he will book the flexible product. Otherwise, if the airline assumes indifference, the customer will not book the flexible product as there is the possibility that he will be allocated to Munich. Because the customer's budget is not large enough to book the specific product, he will not book any product at this airline.*

Enabling customers to limit the manifestation set decreases their uncertainty regarding the outcome. At the same time, it reduces the flexibility of the airline to allocate the booking. If customer choice behavior conforms to this model, not allowing customers to reduce the manifestation set will negatively impact their decision because of deprecating particular manifestations.

A first step towards a more sophisticated preferential choice model is to account for the actual preference values instead of only binary decisions.

4.4 Preferential Choice Between Manifestations

The previous model framed customers' preferences for manifestations as a binary decision of accepting or rejecting particular manifestations for allocation. This section introduces a more sophisticated customers' preferential choice model explicitly accounting for the customers' preference values for manifestations.

4.4.1 Modeling Preferential Choice

The following choice model assumes that customers base their choice on individual utility values for each manifestation of a flexible product. Instead of modifying the manifestation set they state their individual utility values as valuation.

As stated for limitations of the manifestation set in Section 4.3, an explicit formulation of a utility function or value is not required to model customers' preferential choice. Instead, the model assumes that customers express their valuations as preference values. This accounts for individual utility functions per customer and makes utility values comparable over all customers.

We will discuss characteristics, limitations, and opportunities for three variations according to the scale of the resulting preference values: ordinal, interval, and ratio. For each variation, we formulate a transformation function following definition (4.1.4).

Defining preferences on an ordinal scale. Ordinal scales establish a ranking within a given set. An example for an ordinal scale to evaluate a manifestation set of a flexible product could be: popular, average, unpopular.

Ordinal scales restrain from the possibility to quantify the degree of difference between elements. For a set of valuations on an ordinal scale, it is not possible to calculate the arithmetic mean. The median, however, is well-defined. Using an ordinal scale to valuate customer preferences, automatically makes different customers comparable since they have to choose from the same set of preference ranks to order the manifestations. The elements on the scale correspond to the customers' preference values.

Equation (4.4.1) defines an ordinal scale as the set of possible discrete ranks:

$$\Omega = \{1, \ldots, \omega\} \text{ with } 1 \leq \ldots \leq \omega. \tag{4.4.1}$$

Let $c_m \in \Omega$ be the rank of a manifestation $m \in M_f$ within the ordinal scale Ω defining the preference value of a manifestation. The transformation function (4.4.2) calculates the rank as a measure for the position of utility value u_{bm} within Ω

$$T_\Omega(u_{bm}) = c_m = |\{u_{bk} \mid k \in M_f \wedge u_{bk} \geq u_{bm}\}| \in \Omega. \tag{4.4.2}$$

This formulation includes the possibility that two manifestations carry the same preference value because they have the same rank. Depending on the number of elements on the scale, some will not be used or some will be used multiple times. Possible implications of this fact regarding the allocation of flexible products will be addressed later.

Example. *Consider the previous example. A possible ordinal scale for customers can consist of the ranks popular, average, unpopular. Now the customer associates his most preferred manifestation Hamburg with popular and the most unpopular destination Munich with unpopular.*

Defining preferences on an interval scale. This scale type allows to numerically express the difference between two elements. A well-known example for an interval scale is the Celsius scale for temperature. A possible example for an interval scale regarding four manifestations could be the set $\{1, 2, 3, 4\}$.

Interval scales allow to calculate the degree of difference between two items. However, it is not possible to set two items in relation with each other. For an interval scale, we can calculate the median and arithmetic mean, as well as the range and standard deviation to describe the statistical dispersion. Regarding our example scale, we can state that a preference of 4 is 2 higher than 2, but not necessarily twice as much. This depends on the fact that the zero-point on interval scales can be arbitrarily defined.

Let $I \subseteq \mathbb{Z}^+$ be an interval scale. Each customer can individually set a zero-point. Regarding the statement of preferences, we assume that the zero-point refers to the manifestation with the minimal utility value. For simplicity's sake, we assume that the minimal utility value over all manifestations for each customer is set to 1, which has to be contained at least by the scale to ensure feasibility.

Given an interval scale, the transformation function (4.4.3) formulates a way to transform utility values into rankings on the interval scale

$$T_I(u_{bm}) = i_m = |\{u_{bk} \mid k \in M_f \wedge u_{bk} \leq u_{bm}\}| \in I. \tag{4.4.3}$$

Based on the zero-point definition above, $i_m \in I$ defines the value and rank of a manifestation within the scale.

Example. *Consider the example from the airline industry: then a possible individual interval scale for a customer contains three values* $\{3, 2, 1\}$. *Now the customer associates the value* 1 *with Munich and* 3 *with Hamburg. Obviously, we can calculate the mean valuation for the given set as* 2, *however, we cannot follow that the customer likes Hamburg three-times more than Munich.*

Defining preferences on a ratio scale. Using a ratio scale to express customer preferences allows to quantify the relation between preference values. In addition to median and arithmetic mean, on a ratio scale the geometric and harmonic mean can be additionally calculated. Therefore, customers have to be able to set individual, continuous preferences, specifying "how much" of a unit they associate with a manifestation.

Let $P \subseteq \mathbb{R}^+$ denotes a ratio scale. Based on the definition of preference values, this ratio scale can directly correspond to the preference interval $Q = P$.

Let $\overline{q}_b \in Q$ be for each customer $b \in B$ the maximal possible preference value. Each customer associates this value with his utility maximizing manifestation $m_b \in M_f$. A possible transformation function is

$$T_P\left(u_{bm}\right) = q_{bm} = \frac{u_{bm}}{u_{b,m_b}} \cdot \overline{q}_b. \tag{4.4.4}$$

This definition includes the case where two manifestations are associated with the same utility value.

Example. *Again, consider the example with a manifestation set of three alternatives: Munich, Hamburg, Berlin and a price of* 70 *Euro for the flexible product. An individual ratio scale for the customer representing the monetary equivalent values contains three values* $\{5\ Euro, 10\ Euro, 15\ Euro\}$. *Now the customer associates an additional monetary value of* 5 *Euro with manifestation Munich and* 15 *Euro with Hamburg to express his preferences.*

4.4.2 Revealing Preferences for Manifestations

The design of an appropriate revealing mechanism has to fulfill several requirements to ensure applicability. First of all, a revealing mechanism has to make the preference values comparable for different customers. This is the most important requirement and the most difficult, as it has to incorporate the individual utility valuations of different customers.

Based on the discussion and considerations before, we differentiate between revealing mechanisms where utility values are transformed into preference values on an ordinal, interval, or ratio scale.

When using an **ordinal scale**, the airline has to define a set of discrete elements that sets up a unique ranking. If designations are used, they have to be understandable and well-defined to avoid misunderstandings. Customers are then able to transform their utilities by using for example a function as defined by equation (4.4.2) to get preference values. The exact valuation in following process steps bases on assumptions and valuations the airline imposes for the different preference values. This ensures comparability over customers.

To define the elements of the ordinal scale the airline uses a valuation framework based on certain assumptions. If the scale includes less elements than possible manifestations, customers have to valuate several manifestations equally. Offering more elements than manifestations enables customers to skip some valuations. This imposes the illusion for customers that the difference between the valuations may be relevant leading to a misunderstanding regarding the mechanism. Implementing a more sophisticated mechanism based on an interval scale can overcome this illusion.

When the airline assumes preferential choice and formalizes it using an **interval scale**, the revealing interface has to account for this. Customers have to valuate manifestations individually within the given scale. Beside a pure ranking, an interval scale establishes the possibility to numerically compare the difference between pairs of manifestations. To ensure a minimum level of variety and to enable preferential choice, the interval scale has to consist of at least two elements. The airline can apply a function as defined in equation (4.4.3) to get interval scaled preference values from individual utility values.

A revealing mechanism based on a **ratio scale** enables customers to reveal as much information as possible about their preferences. Beside a ranking they can individually valuate the ratio between manifestations. The airline has to ensure that the revealing mechanism offers an interval that is large enough to evaluate all manifestations. Using a monetary equivalent to express the preferences seems to be an appropriate way. Customers can easily compare monetary valuations and the airline can use the information to learn about the customers' price sensitivity. A revealing mechanism defined in this way ensures comparability between customers and between different valuations of a single customer.

4.4.3 Implications of Preferential Choice

Accounting for preferential choice allows more variance in customers' valuation of the different manifestations. It extends possibilities for designing revealing mechanisms. The most important requirement for the airline is to ensure comparability over customers. Because of customers' individual perceptions, a revealing mechanism has to transform utility values into comparable preference values. Due to the design of the revealing mechanism the airline imposes assumptions about preference values as discussed before.

This section presented three scales for valuating preference values. Looking at the applicability of the different scale types based on the mathematical formulations, we can follow that using a ratio scale enables a larger degree of freedom for customers to evaluate the manifestations. Based on current technical possibilities all three scale types are easily implementable and applicable.

5 Strategic Customer Choice With Flexible Products

The term strategic choice of specific products describes a time-dependent choice behavior. Customers strategically delay their purchases when expecting cheaper products to be offered in the future. This chapter models strategic choice of flexible products: customers anticipate the allocation of the flexible product.

Section 5.1 starts with a formulation of a strategic customer choice model for flexible products, followed by a formalized view on utility values underlying this choice model in Section 5.2. Section 5.3 concludes this chapter with a summary and discussion of implications for the remaining RM process.

5.1 Modeling Strategic Customer Decisions for Flexible Products

Let us assume that a customer has preferences for the manifestations of a flexible product, but intentionally he decides to buy a specific product because it maximizes his utility. Possible information regarding the upcoming allocation for a simultaneously offered flexible product can increase the customers' utility. This can lead to a different buying decision: the customer may decide to buy the flexible product instead of the specific one. We term this behavior strategic, as customers try to take advantage of expectations about the future.

Assuming strategic behavior for flexible products incorporates the first decision between flexible or specific products referring to Figure 3.1 in our customers' preferential choice model. The consideration of choice is not restricted to customers' choice between manifestations of a flexible product. The following model connects the decision between specific and flexible products with preferences for manifestations.

Note, that here the term strategic is used in a different context compared to existing research (Jerath et al., 2010; Li et al., 2014). As the notation and the proposed model is restricted to RM with flexible products, using the same terminology can be seen as an extension to existing research.

Example. *Consider the previous example and a strategic customer having Hamburg as desired destination. Due to consistently low demand for the corresponding specific flight, a* 90% *chance to travel to Hamburg may turn out to be associated when booking the flexible product. If the customer knows this, he may decide to book the flexible product instead of the specific product for the flight to Hamburg.*

Current technical developments empowering communication between individuals support strategic customer behavior. Digital, mobile, and social media platforms allow to share and distribute information between individuals. Air travel, hospitality, or other markets that partially relied on a nontransparent pricing are now challenged with an increasing market transparency: customers share their knowledge and information on experiences and opinions about offers.

This additionally accessible information impacts customer expectations and decisions. Several contributions deal with customers using external and internal information, e.g., Jerath et al. (2009); Li et al. (2014). Such customers anticipate developments in pricing and schedule their purchases based on this. Especially in the context of flexible products, an increasing market transparency counteracts potential benefits for the airline. With additional information available, the opaqueness of flexible products decreases. Of course, this change in conditions affects the customers' decision behavior.

Modeling strategic customer decisions for flexible products assumes the anticipation of additional information about potential allocations. The following model formalizes such customer decisions based on their expectations.

Let the airline assume customers acting based on the indifferent choice model described in Section 4.2. No revealing mechanism is needed and established: customers are not able to reveal anything when requesting a flexible product.

The airline offers a set of specific products S and a set of flexible products F. When allocating the flexible bookings immediately after they are sold, the airline focuses on revenue maximization. For all customers $b \in B$ exist utility values $u_{bs} \in \mathbb{R}, \forall s \in S$ and $u_{bf} \in \mathbb{R}, \forall f \in F$. They establish a unique ranking of all product alternatives for each customer.

For each customer exists a specific product $x \in S$, maximizing his utility in case of booking. For the sake of clarity, the following model restricts to the case where only one flexible product exists and the utility maximizing specific product is part of the manifestation set of the flexible product $x \in M_f$. Based on these assumptions and the uncertainty incorporated in the flexible product, follows for the utilities

$$u_{bx} \geq u_{bf}. \tag{5.1.1}$$

Inequality (5.1.1) implies that without any additional information or restriction all customers are going to request the specific product based on their choice model and decision rule.

Let us now assume that additional information about allocations of previous flexible bookings exist and customers can estimate future allocations. We denote this forecasted allocation by $y \in M_f$ and the additional utility induced through this information by $u_y \in \mathbb{R}$. This utility has to be considered when modeling the choice of customers.

As the information about the allocation is based on historical data and information from other individuals, a particular level of uncertainty remains. Let $\varphi_y \in [0,1]$ denote the level of reliability regarding this information. The boundary cases regarding the reliability represent situations where perfect information about the allocation are available ($\varphi_y = 1$) or no additional information exist ($\varphi_y = 0$).

Because of risk awareness, customers may not trust this information at all. Let $\vartheta_b \in [0,1], \forall b \in B$ describe the minimum reliability a customer requires to rely on the information. If the customer now requests the flexible product and $y = x$, he gains the additional utility value $u_y > 0$. We can differentiate three possible cases regarding the consequences of u_y on a customer's decision.

Case 1. The additional information is too uncertain to change the customer's decision towards the flexible product. This represents the most obvious case. The customer can access the information about the allocation, however, it is not reliable enough to meet the requirements of certainty. This situation can be characterized by

$$\varphi_y < \vartheta_b. \tag{5.1.2}$$

In this situation, the customer still books the specific product. RM and the airline's revenue is not affected.

Case 2. The certainty of the additional information is superior to the customer's risk aversion. However, the additional utility does not affect the customer's choice. Let $\varphi_y \geq \vartheta_b$, the customer considers the additional information. Let us assume the case where the value or reliability of the additional information induces not enough utility to change the decision behavior of the customer. In this situation it holds that

$$u_{bx} \geq u_{bf} + \varphi_y \cdot u_y. \tag{5.1.3}$$

Therefore, the customer still buys the specific product. As we assume that the additional information is relevant for the customer ($u_y > 0$), reasons can be that the reliability φ_y or the additional utility u_y are too small to change the customer decision.

In this situation, there is no need to fear cannibalization between specific and flexible products. A booking control policy based on basic optimization methods remains applicable.

Case 3. The certainty of the additional information is superior to the customer's risk aversion and the additional utility alters the customer's choice. Let again $\varphi_y \geq \vartheta_b$. Now, let the accessible information change the customer's decision. In this case, it is valid that

$$u_{bx} < u_{bf} + \varphi_y \cdot u_y. \tag{5.1.4}$$

In order to fulfill inequality (5.1.4), φ_y or u_y have to be large enough.

In this situation, the customer buys the flexible product instead of the specific one. This leads to a loss in airline's revenue. To provide optimal booking control policies RM is expected to consider this customer behavior.

Inequality (5.1.4) can be reformulated to better characterize the case customer's decision is affected by φ_y for arbitrary $u_y > 0$:

$$\varphi_y > \frac{u_{bx} - u_{bf}}{u_y}. \tag{5.1.5}$$

Now, a customer trusts on the additional information and changes his decision towards booking the flexible product if

$$\varphi_y \in \left(\frac{u_{bx} - u_{bf}}{u_y}, 1 \right]. \tag{5.1.6}$$

The term $u_{bx} - u_{bf}$ in formula (5.1.6) describes the differential utility between specific and flexible product.

5.2 A Formalized View on Utilities

The strategic choice model extends a discrete choice model between specific and flexible products. It includes a decision step influenced by additional available information about the allocation. Customers use this information to act strategically. In the following, we formalize the differences in utilities for flexible and specific products. We explicitly consider the value of information and derive possible corrective actions for the airline to protect against strategic behavior.

Let the utility maximizing specific product and the possible allocation of the flexible booking be identical ($x = y$) and $\varphi_y \geq \vartheta_b$. The monetary utility values for specific and flexible products are denoted by $u_p(x)$ and $u_p(f)$. For simplicity's sake, we transform the utility scale in a way that $u_p(x) = 0$. Such a transformation is well-defined. Using a general price sensitivity of customers and $f_s > f_f$, $\forall s \in S$ regarding the fares it follows that $u_p(f) > 0$.

Let $u_c(x)$ and $u_c(f)$ denote the utility of certainty about the outcome of a booking. As all specific products are more certain than flexible products, we can set $u_c(f) = 0$. From this follows that $u_c(x) > 0$.

Let u_x denote the whole utility for the specific product calculated as

$$u_x = u_p(x) + u_c(x) \tag{5.2.1}$$

and u_f the complete utility for the flexible product calculated as

$$u_f = u_p(f) + u_c(f). \tag{5.2.2}$$

For the differential utility between the flexible and the specific product follows now:

$$0 < u_x - u_f = u_c(x) + u_p(x) - [u_c(f) + u_p(f)] = u_c(x) - u_p(f). \tag{5.2.3}$$

In a next step, we use equation (5.2.3) and $u_c(x) = u_y$ to reformulate the condition regarding the minimum reliability in inequality (5.1.5) to

$$\varphi_y > \frac{u_c(x) - u_p(f)}{u_c(x)} = 1 - \frac{u_p(f)}{u_c(x)}. \tag{5.2.4}$$

This conversion is well-defined as still $\varphi_y \in [0,1]$. Equation (5.2.4) implies that given a parameterization for φ_y, the airline has to lower $\frac{u_p(f)}{u_c(x)}$ to prevent strategic behavior of customers. This could be done by increasing the value of certainty for the specific product $u_c(x)$ or by decreasing the monetary benefit of the flexible product $u_p(f)$.

The value of certainty for the specific product represents a utility penalty for the flexible product resulting from its uncertainty. Therefore, increasing $u_c(x)$ can be achieved by adding more alternatives to the manifestation set or by using allocation mechanisms where the final specification cannot be estimated with appropriate certainty. To reduce the monetary utility, the airline can increase the price level or include more manifestations in the flexible product.

If customers act strategically and rely on information about previous allocations, the revenue achieved from a particular transaction shrinks. They will more likely buy flexible products instead of specific products. Thereby, the difference in revenue between their actual choice and the flexible product represents a direct loss. Additionally, the changed customers' decisions impact the inventory. This affects the optimality of the imposed control policy as the assumed conditions do not exist anymore. Purchases occur for different products as expected in the forecast, which was used to calculate the booking control policy. The resulting difference in the airlines' revenue can be characterized as indirect loss due to strategic customer behavior.

5.3 Implications of Strategic Customer Behavior

This choice model extends the focus of customers' preferential choice between manifestations towards the first decision. To this end, we assume the existence of additional available information about allocations. Customers may use this information to act strategically. We formalized the differences in utilities for flexible and specific products with respect to the value of information. Subsequently, we derived possible corrective actions to prevent such strategic customer behavior.

The analysis in Section 5.2 showed that airlines have several possibilities to react to this behavior. These adaptions aim either to influence the available information or the attractiveness of the flexible product. Using these thoughts, we can formulate expectations about possible impacts on the RM process.

If customers act strategically and use the information about the allocation, the airlines' revenue achieved from this particular transaction shrinks. The difference between the revenue of the actual choice and the price of the flexible product can be specified as direct loss. Additionally, the changed customer decision impacts the inventory and affects the optimality of the imposed booking control policy. Obviously, the affected control policy does not optimally decide and leads to inferior revenue. The difference between the revenue without strategic behavior and the actual revenue can be characterized as indirect loss due to strategic customer behavior.

Choice models allowing customers to limit the manifestation set or to communicate their preferences account for the relevant information inducing strategic behavior. However, just the knowledge of customer preferences does not help to improve the fulfillment of customer preferences in order to counteract strategic behavior. Allocation methods for flexible products that make use of this information and hedge against strategic customer behavior are necessary for airlines to act successfully.

6 Allocation Methods for Flexible Products

The preferential choice models in Chapter 4 handle customer preferences for manifestations of flexible products. In order to efficiently use the additional information about preferences, existing methodology has to be extended. Referring to the structure of the customer decision process (Figure 3.1) and the Revenue Management (RM) process with flexible products (Figure 2.2) the airline has to impose extended allocation methods for flexible bookings.

Section 6.1 introduces general assumptions and notation first. Section 6.2 presents a stochastic allocation method using different ways to calculate the allocation probabilities for a manifestation. A deterministic allocation method using bid prices is presented in Section 6.3 and a corresponding allocation method using revealed customer preferences is formulated in Section 6.4. Finally, as airlines may consider bid prices and revealed customer preferences simultaneously, Section 6.5 presents a multi-objective allocation method.

6.1 Preliminaries to Define Allocation Methods

Given indifferent customers, allocation is a matter of assigning unused capacity with regard to the current inventory situation. The airlines' aim is to maximize revenue. Given customers' preferential choice, the objective of the allocation decision becomes a trade-off between reserving capacity for specific bookings and complying with revealed customer preferences.

For the airline, allocating flexible bookings means to choose a manifestation based on a decision rule following an objective. Various combinations of decision rule, e.g., optimization program, heuristic, and objective, e.g., maximizing revenue, satisfying customer preferences, are possible. In the following, the term **allocation method** denotes a combination of a decision rule and objective.

Talluri and van Ryzin (2004b) state that it is beneficial for RM to update the control policy throughout the sales period. This fact accounts for the dynamic of the sales process, ensures feasibility, and approximates revenue optimality. The following allocation methods rely on network RM methods as introduced in Chapter 2 with regularly scheduled update points. Then, the RM system provides information about the expected demand-to-come until the end of the sales period and the currently valid bid prices.

With this in mind, it is suspected that it can be beneficial also to recalculate the allocation of flexible bookings. Several contributions state that additional benefits arise, as the airlines' flexibility increases when updating the allocation several times (cf. Petrick et al., 2010, 2012).

Considering such dynamic allocation methods, we have to bear in mind that not only the final allocation of a flexible booking affects the optimality of control policies, but also temporary allocations during the sales process restrain the optimality. Therefore, the update points of the RM system provide well-suited possibilities to recalculate the temporary allocation of flexible bookings.

We term allocation methods finalizing the allocation right after accepting the flexible booking **ad-hoc methods**. Ad-hoc methods allow the airline to immediately communicate the allocation to the customer after the booking was accepted. Methods that repeatedly update the allocation over the sales period are termed **re-allocation methods**. As long as the specification is concealed, the allocation can be recalculated.

Due to the tendencies toward a more strategic behavior (Anderson & Wilson, 2003; Gönsch et al., 2013) and following the strategic choice model for flexible products presented in Chapter 5, we highlight the predictability of allocation methods. For each method, we will discuss possible extensions or modifications impacting the predictability to improve a successful application.

The allocation method based on bid prices as well as the basic concept to re-allocate flexible bookings are replicated from Gallego and Phillips (2004) and Petrick et al. (2012). Additionally, to the notation and definitions introduced in Chapter 4 we now introduce some allocation specific prerequisites and notation.

Thereafter, the set of bookings B denotes all customers that have already decided to request a flexible product and their request was accepted by the airline. Let the sales period T denote all time points where products can be offered for sale. The execution date is defined to be at $t = 0$. The time point during the sales period when a certain booking occurs is denoted by $t_b \in T,\ \forall b \in B$.

Let $D_b \subseteq S,\ \forall b \in B$ denote the set of the available specific products regarding the time point during the sales period when booking $b \in B$ occurred.

Let $s_r^t \in \mathbb{R}$ denote the number of bookings for specific products accepted until time $t \in T$ that use capacity on resource $r \in R$. Analogue, let $f_r^t \in \mathbb{R}$ denote the number of bookings for flexible products allocated to resource $r \in R$ at time $t \in T$.

The remaining capacity $c_r^t \in \mathbb{R}$ at time $t \in T$ is calculated dependent on the initial capacity $c_r \in \mathbb{R}$ and the number of specific and flexible bookings as

$$c_r^t = c_r - s_r^t - f_r^t \geq 0, \quad \forall t \in T,\ \forall r \in R. \tag{6.1.1}$$

We impose the usual assumption that one booking uses only one unit of capacity on the respective resource. However, all formulations can be easily extended to a more general capacity utilization assumption.

Capacity-based RM systems frequently rely on the expected marginal revenue per unit of capacity, denoted as bid price (see Williamson, 1992), to optimize product availability. This parameter represents the opportunity costs associated with the usage of the next available unit of remaining capacity given a resource and given the remaining time until the execution date. We denote the bid price for a resource $r \in R$ at time $t \in T$ given a remaining capacity of c_r^t by $\pi_r^t\left(c_r^t\right) \in \mathbb{R}$. For a more detailed introduction into bid prices and opportunity costs we refer to Section 2.1.

As each manifestation represents a single resource (see definition (4.1.2)), the remaining capacities and bid prices can be defined in relation to a manifestation instead of a resource:

$$\pi_m^t\left(c_m^t\right) = \sum_{r \in R} y_{sr} \cdot \pi_s^t\left(c_r^t\right), \quad \forall m \in M_f. \tag{6.1.2}$$

The set of all revealed customer preference values Q_{bf} for a booking $b \in B$ of flexible product $f \in F$ is defined as

$$Q_{bf} = \left\{q_{bm} \mid m \in M_f\right\}. \tag{6.1.3}$$

We term the sum of all preference values realized through allocating flexible bookings over all customers the **sum of allocated preferences**.

To follow the RM control policy a possible manifestation for a flexible booking has to be available when the allocation is calculated. Therefore, the definitions in the following are restricted to the set of available manifestations for a flexible product defined as

$$M_{bf} = \left\{m \in M_f \;\mid\; m \in D_b\right\} = D_b \bigcap M_f, \quad \forall f \in F. \tag{6.1.4}$$

Let $m_b^* \in M_{bf}$ denote the manifestation that is selected as allocation for a flexible booking by an allocation method.

Relying to previous assumptions about capacity usage of flexible bookings in equation (2.1.1) and the composition of the manifestation set in equation (4.1.2), manifestations correspond to resources. Therefore, we use the term resources when formulating allocation methods. This allows major simplifications regarding some of the following formulations. However, when referring to the customers' view we refer to manifestations.

6.2 Stochastic Allocation Method

The composition of the manifestation set may support some manifestations to be more often used for allocating flexible bookings. Market characteristics or features of the control policy may also cause concentrations. Therefore, airlines may not only rely on RM related indicators to specify an allocation. Using stochastic allocation methods may overcome the issue of predictability of the outcome. Customers cannot completely rely on previous experiences and therefore are not able to postulate any expectations. Applying a stochastic allocation method, however, will not benefit the airline or the customer.

This section presents a concept to specify allocations using discrete random variables. Several ways and indicators will be considered to calculate the allocation probabilities. To ensure general applicability, we formulate a stochastic decision rule based on discrete weights. Subsequently, we focus on the derivation of these weights using various indicators. Finally, we discuss implications and characteristics of the resulting allocations of flexible bookings.

6.2.1 Stochastic Decision Rule

Let $w_{bm} \in [0,1]$ be the probability for a manifestation $m \in M_{bf}$ to be selected as allocation for booking $b \in B$. We term this parameter **allocation probability**.

As at least one manifestation has to be selected for allocation, it has to hold that

$$\sum_{m \in M_{bf}} w_{bm} = 1, \quad \forall f \in F, \forall b \in B. \tag{6.2.1}$$

To formalize a stochastic allocation of flexible bookings, we model a probability distribution based on allocation probabilities. The set of possible outcomes is the set of available manifestations for a flexible product. Consider the following discrete probability space: $(M_{bf}, \mathcal{P}(M_{bf}), \mu)$. The probability measure μ can be defined as

$$\mu : \mathcal{P}(M_{bf}) \to [0,1]. \tag{6.2.2}$$

The corresponding probability distribution is denoted by $\mathcal{R}$. For the manifestation set it has to hold $|M_{bf}| < \infty$. We define the selection of a manifestation as event. Selecting multiple manifestations defines a finite union of singular events $A = \bigcup_{m \in M \subseteq M_{bf}} m$. Accordingly, μ can be written as

$$\mu(A) = \sum_{a \in A} w_a. \tag{6.2.3}$$

Let $X \sim \mathcal{R}$ be a random variable. To specify the allocation of a flexible booking, a single realization x_b of X is drawn to select the manifestation m_b^* where the booking is going to be allocated to

$$m_b^* = x_b \in M_{bf}, \quad \forall b \in B. \tag{6.2.4}$$

There is no need to define a second order decision rule, because equation (6.2.4) always provides a unique m_b^*. As long as condition (6.2.1) is fulfilled by the allocation probabilities, selection rule (6.2.4) can be applied to specify an allocation.

This decision rule enables several possibilities to determine the allocation weights. The following sections introduce various concepts based on different parameters. First, the same probability for all manifestations is chosen. This neglects the current state of the RM process and corresponds to a totally random approach. Beside this simple case, we use several RM indicators to calculate the weights.

6.2.2 Uniform Allocation Probabilities

The most obvious and rudimentary way to set allocation weights for all manifestations is to choose uniform probabilities. The allocation probabilities for a manifestation are then calculated as

$$w_{bm} = \frac{1}{|M_{bf}|}, \quad \forall m \in M_{bf}. \tag{6.2.5}$$

We denote the allocation method using decision rule (6.2.4) and allocation probabilities calculated by equation (6.2.5) by **sUNI**.

6.2.3 Allocation Probabilities Based on the Current Situation

Defining allocation probabilities based on the current booking situation partly accounts for the airlines' objective to maximize revenue. Several indicators describe the booking situation, for example the number of specific bookings and the number of overall bookings. Both indicators are easily accessible and postulated by state-of-the-art RM systems. Flexible bookings should be allocated to resources where both indicators are minimal.

Note, that to definitely ensure revenue maximization, additional indicators have to be considered for allocation. A possible allocation should enable the most opportunities to react. Therefore, choosing a manifestation with enough remaining capacity is equivalent to the manifestation with minimal number of bookings. As both indicators behave in the same way, we can generally define the calculation of allocation probabilities.

We denote the parameter describing the current number of bookings for a manifestation in the following by $\delta_m \in \mathbb{R}$, $\forall m \in M_{bf}$. The allocation probability is calculated as

$$w_{bm} = \frac{1}{|M_{bf}| - 1} \cdot \left[1 - \frac{\delta_m}{\sum_{k \in M_{bf}} \delta_k}\right], \quad \forall m \in M_{bf}. \tag{6.2.6}$$

The allocation method consisting of the decision rule defined by (6.2.4) and the booking situation related allocation probabilities resulting from (6.2.6) is denoted by **sCUR**.

6.2.4 Allocation Probabilities Based on Demand-Estimates

To overcome the drawback of accounting only for the current situation, indicators estimating future developments can be used. In quantity-based RM, several candidates exist describing the estimated demand until the end of the sales period to various extents: the estimated amount of unused capacity at the end of the sales period or the estimated number of upcoming customer requests from now on until time of execution.

The following definition restricts to the indicator for the estimated amount of unused capacity at the end of the sales period. To support revenue maximization this parameter has to be maximized. We denote it by $\tilde{c}_m \in \mathbb{R}$, $\forall m \in M_{bf}$. The allocation probabilities can be calculated as

$$w_{bm} = \frac{\tilde{c}_m}{\sum_{k \in M_{bf}} \tilde{c}_k}, \quad \forall m \in M_{bf}. \tag{6.2.7}$$

This indicator involves the current booking situation, the estimated future demand, and the control policy. Additionally, the included demand estimation also accounts for specific market characteristics, e.g., the overall demand level.

By abstracting and simplifying the analytical correlation between revenue and capacity usage, minimizing the unused capacity at the execution date seems to be a prerequisite in case of maximizing the revenue. An equivalent condition can be formulated as maximizing the number of accepted bookings for this resource. Price-based RM methods (see Talluri & van Ryzin, 2004b) calculate both indicators as result of the forecasting methods.

We denote the allocation method defined by decision rule (6.2.4) and allocation probabilities resulting from (6.2.7) by **sEST**.

The other mentioned candidate can be easily applied, even he has to be minimized to ensure maximal revenue. A well-defined transformation exists to apply decision rule (6.2.4).

6.2.5 Allocation Probabilities Based on Bid Prices

Applying bid prices to calculate the allocation probabilities is another way to simultaneously employ the RM systems' information on future demand and information about the current booking situation. Bid prices include the current bookings on hand by evaluating the current remaining capacity per resource. Furthermore, the number of expected requests until the execution date is included. In general, they represent a valuation for the next unit of available capacity (see Talluri & van Ryzin, 2004b, Chapter 2 and 3) given the current demand forecasts.

Allocating a flexible booking to a manifestation supports revenue maximization if the bid price of the corresponding resource is minimal. To fulfill condition (6.2.1) and support revenue maximization, we have to transform the bid prices before calculating the allocation probabilities. The bid price for manifestation $m \in M_{bf}$ and current time $t \in T$ is denoted by $\pi_m^t\left(c_m^t\right) \in \mathbb{R}^+$.

The transformed values of all manifestations are denoted with $\overline{\pi}_m^t\left(c_m^t\right) \in \mathbb{R}^+$. Let $\tilde{\pi}$ be an upper bound ($\tilde{\pi} > max_{m \in M_{bf}} \pi_m^t\left(c_m^t\right)$) for the bid prices. For example, $\tilde{\pi}$ can be set to the maximum over the prices for all specific products as bid prices are always smaller by definition. Now, we can calculate $\overline{\pi}_m^t\left(c_m^t\right)$ as

$$\overline{\pi}_m^t\left(c_m^t\right) = \tilde{\pi} - \pi_m^t\left(c_m^t\right), \quad \forall m \in M_{bf}. \tag{6.2.8}$$

The transformed bid prices at time $t \in T$ define the allocation probabilities as

$$w_{bm} = \frac{\overline{\pi}_m^t\left(c_m^t\right)}{\sum_{k \in M_{bf}} \overline{\pi}_k^t\left(c_k^t\right)}, \quad \forall m \in M_{bf}. \tag{6.2.9}$$

We denote the allocation method defined by decision rule (6.2.4) and the definition of allocation probabilities resulting from (6.2.9) by **sBID**.

6.2.6 Allocation Probabilities Based on Customer Preferences

To consider customer preferences, the airline can use them to calculate the allocation probabilities. While doing so, the airline has to account for customer preferences and establish a corresponding revealing mechanism. Based on the underlying choice model and the way customer preferences are communicated, different variants are possible.

The objective is to maximize preference values realized by the allocation. Using the revealed preference values $q_{bm} \in Q_b$ for each customer, equation (6.2.10) calculates the allocation probabilities as

$$w_{bm} = \frac{q_{bm}}{\sum_{k \in M_{bf}} q_{bk}}, \quad \forall m \in M_{bf}. \tag{6.2.10}$$

Note, that these weights are defined per customer $b \in B$ and manifestation $m \in M_{bf}$, as preference values differ between customers. Therefore, the allocation probabilities are different between customers, too. The allocation method defined by combining decision rule (6.2.4) and allocation probabilities resulting from (6.2.10) is denoted by **sPRE**.

6.2.7 Implications of Using a Random-Based Allocation Method

sUNI sets the same probabilities for each manifestation. Customers are not able to predict the allocation or to formulate expectations based on previous observations. However, the airlines' objective is also completely neglected. From a theoretical point of view, the airline could benefit from using flexible products combined with this allocation method. In the worst case, the allocation may totally counteract the control policy by always selecting the manifestation with the largest bid price.

Allocating flexible bookings using **sCUR** partly neglects the objective of revenue maximization, as it only accounts for available information about the current situation. The resulting allocation can be disturbing the optimality of the underlying booking control policy. This may benefit revenue maximization if the current booking situation represents future developments more or less correctly. As the current booking situation can be estimated by customers based on the current offerings and prices, the predictability of this approach increases compared to Stochastic Allocation with Uniform Weights (sUNI).

sEST follows the airlines' objective as stated before. The estimated number of accepted bookings consists of two parts. At a particular point in time all the information about the past is certain (the number of already accepted bookings) and possible future developments can be estimated (the number of onwards accepted bookings). Therefore, this concept should show superiority in revenue over Stochastic Allocation Based on Bookings (sCUR).

For customers, it is difficult to predict the outcome of this approach. They are not able to observe and to evaluate the number of accepted bookings or the number of estimated future bookings. Hence, this approach may counteract possible strategic customer choice between flexible and specific products.

Bid prices represent a valuation of upcoming requests in terms of revenue. Therefore, **sBID** considers additional information about the monetary performance. This implies a superior behavior compared to Stochastic Allocation Based on Estimations (sEST).

Using allocation probabilities calculated from bid prices guarantees a low predictability of the allocation method. Customers can only partly evaluate the exact current valid bid prices for a set of resources. The set of currently offered products actually is an indicator for the magnitude of the bid prices. However, there are many other factors influencing the availability preventing a direct correlation. This approach may partially restrict customers to act strategically regarding the choice between flexible and specific products.

sPRE neglects the airlines' objective to maximize revenue. As there may be a concentration of customer preferences for certain manifestations, allocation probabilities for these will be very large. In this case, using equation (6.2.10) to calculate the allocation probabilities leads to a defective design and therefore to an inferior solution in revenue. This can potentially offset the benefits achieved from selling flexible products.

The predictability of this approach depends largely on the choice model implemented by the airline. However, in general, the predictability is larger if the airlines follow the revealed customer preferences for allocating flexible bookings.

Have in mind, that there exist possibilities to formulate stochastic allocation methods with more sophisticated designed weights. For example, the combination of customer preferences and current or estimated RM indicators is easily conceivable. Based on the illustrations given in this section relevant indicators may easily be populated using price-based RM systems.

The concepts relying on bid prices or customer preferences represent the best case (sBID) and the worst case (sPRE) in terms of revenue maximization. Evaluating them will suffice to investigate the behavior of more sophisticated designs for the weights. To this end, this thesis is restricted to the basic concepts and does not explicitly formulate any modified requirements for alternative approaches.

6.3 Allocation Method Based on Bid Prices

Selling flexible products allows airlines to maximize revenue while dealing with demand uncertainty by postponing the allocation decision. The allocation has to be done based on the airlines' objective to maximize these benefits.

Thereby, the decision to allocate flexible bookings has to preserve as much flexibility as possible for later steps while not affecting the current valid booking control policy. As we will see in this section, using a deterministic allocation method based on bid prices will meet these conditions. In Section 2.2 we already outlined contributions, e.g., Gallego and Phillips (2004); Petrick et al. (2010), using opportunity costs to allocate flexible bookings. The following approach replicates these models.

6.3.1 Decision Rule Based on Bid Prices

The bid price for a resource depends on time and remaining capacity. With decreasing capacity and time bid prices increase monotonously. An allocation for a flexible booking has to compare the current bid price for each manifestation against each other. The expected revenue is maximized when the manifestation with minimal bid price is chosen.

Let M_b^* denote the set of optimal manifestations given a flexible booking:

$$M_b^* = \operatorname{argmin}_{m \in M_{bf}} \left\{ \pi_m^t \left(c_m^t \right) \right\}, \quad \forall b \in B. \tag{6.3.1}$$

As the bid prices for several manifestations may be equal, we have to formulate a second order decision rule applying in such situations. For example, the method formulated in Section 6.2 is conceivable. In case the minimal bid price can be associated uniquely to a manifestation, the set of optimal manifestations contains only one element. Let m_b^* denote the manifestation used to allocate the flexible booking. We refer to the decision rule (6.3.1) as allocation method based on **bid prices or opportunity costs (OCB)**.

6.3.2 Using the Allocation Method Based on Bid Prices

The allocation method based on bid prices (6.3.1) generally aims to maximize revenue. From a customer oriented and choice modeling perspective it is assumed that customers' preferences are irrelevant or that the fulfillment rate of customer preferences does not affect revenue. The impacts of applying the stochastic allocation method as second order decision are already discussed in Section 6.2, especially regarding the objective to maximize revenue.

The predictability of an allocation calculated by this method is moderate. As bid prices are used to determine the current availability situation, customers can compare all possible manifestations and formulate assumptions. Therefore, the use of a second order decision rule based on random variables supports the uncertainty and lowers the predictability of this allocation method. However, this occurs only in case the second order decision rule is needed to calculate a unique allocation.

In case the airline implements the possibility for customers to limit the manifestation set, this allocation method can be applied as well. The above presented decision rule accounts for the set of available manifestations. The formulation can easily be adapted to account for limitations of the manifestation set. If a more sophisticated customer choice model is assumed and applied, decision rule (6.3.1) could be used as well. However, in this case the additional information about customer preferences is not used regarding the first decision. One possible modification could be a second order decision, based on customers' preferences. A stand-alone allocation method using customer preferences is presented in the following section.

6.4 Preference-Based Allocation Method

Using choice-based RM methods enables researchers and practitioners to support estimations based on time series models or probability distributions with models from choice theory (Garrow, 2012). Beside improving the RM performance, these models could be used to address customer wishes and needs. However, in current applications choice-based RM is only used to improve revenue.

In case an airline assumes the existence of preferential choice for the manifestation set, opportunities for more customer oriented allocation methods arise. A first stochastic method that uses revealed customer preference values for allocating flexible bookings was introduced in Section 6.2. The following allocation method, however, uses a deterministic decision rule to specify M_b^*. As a consequence, we can speak of an actual customer oriented allocation method.

Using truthfully revealed preference values enables the airline to maximize the sum of allocated preferences when specifying flexible bookings. However, this neglects the initial objective to maximize revenue as both objectives may be in conflict with each other.

We assume that a revealing mechanism as formulated in Section 4.4 exists. Customer preferences are expressed by preference values. The revealing mechanism enables customers to communicate these values for the manifestations based on a ratio scale. It is possible that two manifestations carry the same preference values for a certain customer. The following decision rule formalizes the selection criteria and accounts for the existence of equal valuations.

6.4.1 Preference-Based Decision Rule

The objective of an allocation method based on preference values is to maximize the sum of allocated preferences. Therefore, for each booking, the decision rule has to compare the preference values for the set of available manifestations and to select the one with maximal value. Doing so for all flexible bookings maximizes the sum of realized preference values over all bookings.

For each flexible booking, the customer reveals his preferences for the whole manifestation set to the airline. Using this information, the set of manifestations with maximal preference values can be defined as

$$M_b^* = \text{argmax}_{m \in M_{bf}} \left\{ q_{bm} \in Q_{bf} \right\}, \quad \forall b \in B. \tag{6.4.1}$$

In case this set contains more than one manifestation, we have to apply a second order decision rule to get a unique selection. There exist various criteria that may be applied. Several of them have been already presented and discussed in Section 6.2 and Section 6.3. Each of them can be applied without any modifications to find a unique specification.

We term the allocation method resulting from the definition (6.4.1) and an optional second order condition customer **Preference-Based Allocation (CPR) method**.

6.4.2 Implications of Allocating Flexible Bookings Using Preferences

By applying CPR, the allocation neglects the airlines' objective to maximize revenue. Therefore, the specified allocation may not exploit the full potential of flexible products. In case a reasonable second order decision rule is applied, e.g., sCUR or Allocation Based on Bid Prices (OCB), the airlines' interests will still be partly considered.

Allocation methods based on RM indicators will always outperform customer-oriented allocation methods in terms of revenue. There has to be a trade-of between both competing objectives. Ideally, an allocation method for flexible bookings uses customer preferences and supports the airlines' objective. Considering all relevant dimensions of input parameters is always more successful than neglecting parts and focusing on certain objectives. Additionally, it may be beneficial to simultaneously allocate multiple flexible bookings instead of processing only one booking at a time, to consider dependencies between particular allocations. The following section introduces an allocation method using linear programming techniques to meet these requirements.

6.5 Multi-Objective Allocation Method

All previously introduced allocation methods focus on a single objective. Accounting for airlines' purposes is contrary to accounting for customer preferences regarding the manifestation set. Simultaneously considering both objectives may help to find an economic equilibrium.

This section introduces the formulation of a multi-objective allocation method in terms of a linear optimization program. We discuss particular requirements on both objectives and the necessary constraints. Subsequently, two extensions to the linear program and the applicability of this allocation method will be discussed with regard to different parameterizations.

6.5.1 Formulation as Binary Linear Program

Using a multi-objective allocation method only makes sense if the preceding revealing mechanism accounts for customer preferences. Therefore, we assume that the preference values for all manifestations are truthfully revealed to the airline.

An allocation of a flexible booking is a step within the RM process. Subsequent steps can only be processed when the allocation method is finished. The remaining RM system has to be in a steady state while computing an allocation. Therefore, we can assume that RM parameters used for allocating flexible bookings are fixed during the solution process. This implicates a fixed number of accepted bookings for specific and flexible products as well as fixed bid prices.

As there may exist dependencies between the allocation of multiple flexible bookings, the linear program is designed to simultaneously allocate a set of flexible bookings. Furthermore, the formulation allows the airline to offer multiple flexible products differing in the manifestation set. Therefore, all following parameters are defined on the set of specific products S instead of the manifestation set M_{bf}.

The decision variables of the optimization program indicate the allocation of a flexible booking to a certain specific product that is part of the manifestation set. We define the decision variable $x_{bs} \in \{0, 1\}$ as

$$x_{bs} = \begin{cases} 1, & \text{if flexible booking } b \text{ is allocated to product } s, \\ 0, & \text{else.} \end{cases} \tag{6.5.1}$$

One objective of the optimization program maximizes the allocated customer preferences, whereas the second objective minimizes the opportunity costs caused by the allocation in terms of bid prices.

The objective considering the sum of allocated preferences can be written as

$$\max \sum_{b \in B} \sum_{s \in S} q_{bs} \cdot x_{bs}. \tag{6.5.2}$$

To ensure usability of this objective, we have to consider artificial preference values for all specific products not included in the individual choice set of a booking. We set

$$q_{bs} = 0, \quad \forall s \in S \setminus M_{bf},\ \forall b \in B. \tag{6.5.3}$$

The objective minimizing the sum of caused opportunity costs is defined as

$$\min \sum_{b \in B} \sum_{s \in S} \pi_s^t \left(c_s^t \right) \cdot x_{bs}. \tag{6.5.4}$$

To ensure feasibility regarding conditions derived from the RM system, the optimization program has to consider several constraints. In general, we differentiate two groups of constraints. The first group accounts for feasibility of the solution concerning the underlying RM problem. We term this group **RM constraints**. The second group accounts for characteristics of flexible bookings and is termed **product constraints**.

RM constraints. A feasible solution has to ensure that the remaining capacity for each resource is not exceeded. Inequality (6.5.5) defines the **capacity constraint**

$$\sum_{b \in B} \sum_{s \in S} y_{sr} \cdot x_{bs} + s_r^t + f_r^t \leq c_r, \quad \forall r \in R. \tag{6.5.5}$$

Additionally, an optimal solution has to ensure that a flexible booking is allocated to a specific product that was available at time of booking. Therefore, we define an availability parameter $a_{bs},\ \forall s \in S,\ \forall b \in B$ memorizing the availability situation for each booking and specific product as:

$$a_{bs} = \begin{cases} 1, & \text{if product } s \text{ was available at } t_b, \\ 0, & \text{else.} \end{cases} \tag{6.5.6}$$

The **availability constraint** (6.5.7) is then defined as

$$\sum_{s \in S} a_{bs} \cdot x_{bs} = 1, \quad \forall b \in B. \tag{6.5.7}$$

Product constraints. When accepting the request for a flexible product a common assumption is to guarantee the serving of the booking (cf. Gallego & Phillips, 2004; Petrick et al., 2010). Therefore, the following constraint ensures that each booking is allocated to one and only one specific product. We define the **allocation constraint** as

$$\sum_{s \in S} x_{bs} = 1, \quad \forall b \in B. \tag{6.5.8}$$

Despite the primary objective of maximizing revenue and allocated preference values, there may be additional restrictions on the manifestation set based on the implemented revealing mechanism. To ensure general applicability, the following constraint considers limitations of the manifestation set in case customers are enabled to do this as well. The **limitation constraint** (6.5.9) accounts for feasibility regarding an individual customer choice set

$$\sum_{s \in C_{bf}} x_{bs} = 1, \quad \forall b \in B. \tag{6.5.9}$$

The limitation constraint can be seen as a more restrictive version of the allocation constraint (6.5.8). There is no need to consider both constraints. Therefore, only for the limitation constraint (6.5.9) is used.

In order to evaluate the trade-off between both objectives later, we will use a weighted-sum formulation in the computational studies. Here, we consider the difference between both objectives weighted by a parameter $\alpha \in [0, 1]$. The allocated preferences have to be maximized by an allocation, whereas the realized bid prices have to be minimized. The following formulation accounts for this fact by subtracting both components

$$\max \sum_{b \in B} \sum_{s \in S} \left(\alpha \cdot q_{bs} - (1 - \alpha) \cdot \pi_s^t \left(c_s^t \right) \right) \cdot x_{bs}. \tag{6.5.10}$$

We term (6.5.1), (6.5.5) – (6.5.10) the **Multi-Objective Allocation (MO)** method.

6.5.2 Incentivizing Allocation Stability

MO is a binary integer linear program. Therefore, it belongs to the family of NP-hard (Non-deterministic Polynomial-time hard) problems. Following Karp (1972) it is even a NP-complete (NP and NP-hard) problem and computational-efficient approaches may not find the exact solution. Instead, only an approximation of the optimal solution will be achievable in polynomial time. Especially in re-allocation setups where MO is solved several times for multiple several flexible bookings this may affect the overall solution quality, as there may be allocations of a certain flexible booking switching between equivalent manifestations over several re-allocations.

Benchmarking tests for small prototype instances showed the existence of such a switching behavior. The objective value of MO is not negatively affected by this instability over re-allocations. Such switching allocations for a flexible booking, however, have significant negative effects on revenue regarding the whole sales period. Due to shifts of flexible bookings between manifestations, the optimality of the booking control policy is no longer given.

Equation (6.5.11) defines an abbreviation representing a part of MOs objective:

$$z_{bs}^{\alpha} = \alpha \cdot q_{bs} - (1 - \alpha) \cdot \pi_s^t \left(c_s^t \right), \quad \forall b \in B, \forall s \in S. \tag{6.5.11}$$

To incentivize a more stable solution and to prevent the switching behavior, we introduce two auxiliary parameters. The parameter $\sigma_{bs} \in [0.1, 1.9]$ weights parts of the objective function whether the new allocation is the same as the previous or not. Let $\overline{x}_{bs} \in \{0, 1\}$ indicate the memorized allocation of booking $b \in B$ to specific product $s \in S$. It is calculated using another parameter $\rho \in [0, 0.9]$ as follows:

$$\sigma_{bs} = \begin{cases} 1+\rho, & \text{if } \overline{x}_{bs} = 1 \wedge z^{\alpha}_{bs} \geq 0, \\ 1-\rho, & \text{if } \overline{x}_{bs} = 1 \wedge z^{\alpha}_{bs} < 0, \\ 1, & \text{else.} \end{cases} \tag{6.5.12}$$

The parameterization of ρ relies on the aim of the current computational experiment. Objective (6.5.13) combines σ_{bs} and the previous definition of the objective of MO to an extended formulation:

$$\max \sum_{b \in B} \sum_{s \in S} \sigma_{bs} \cdot z^{\alpha}_{bs} \cdot x_{bs}. \tag{6.5.13}$$

Using objective (6.5.13) prevents alternating allocations of flexible bookings over certain re-allocations and supports revenue maximization as one of MO's objectives.

6.5.3 Approximating Bid Prices for Objective Definition

Using MO to specify the allocation of flexible bookings, we can differentiate two applications. MO can be used to allocate a single flexible booking or to simultaneously allocate a set of flexible bookings. As we will see in this section, the distinction is relevant regarding the evaluation of the bid prices used.

Without re-allocating already accepted flexible bookings, only new accepted flexible bookings have to be allocated by MO. In this case it holds that $|B| = 1$. The current capacity utilization for each resource is exactly known and we can evaluate the actual bid prices.

If we use MO to allocate multiple flexible bookings at a scheduled update point, the capacity utilization is dynamic during the solution of MO. Therefore, bid prices are dynamic and have to be approximated. This is not trivial, because there is no specific sequence of allocation and we are not able to compute an exact capacity situation for a particular flexible booking. Therefore, we have to use a bid price approximation for all flexible bookings. The approximation has to consider the dynamic of the bid price regarding time and capacity utilization. We denote the approximated bid prices with $\overline{P^t_m} \in \mathbb{R}$, as they no longer depend on a particular flexible booking respectively the capacity utilization.

We have to consider that MO was formulated using the set of specific products S instead of the manifestation set M_{bf} as dimension for the decision variables. For all specific products not included in the manifestation set, we set $\overline{P^t_s} = 0, \ \forall s \in S \setminus M_{bf}$.

The approximation only considers the remaining capacity depending on the number of accepted bookings for specific products. This reflects the current fixed capacity usage and leads to a feasible solution. To evaluate this parameter, we modify equation (6.1.1) as follows

$$\overline{c_m^t} = c_m - s_m^t. \tag{6.5.14}$$

In order to account for the dynamic of bid prices within a single approximation value, we average bid prices for different capacity scenarios. As defined before, the bid price for c_m^t remaining capacity is denoted with $\pi_m^t\left(c_m^t\right)$. We use this variable to define a set of four different approximations. We denote a valuation by $\pi_{mi}^t,\ \forall i \in \{1, \ldots 4\}$. Test cases during modeling MO showed good results of using these four different approximations. As we will see, the four approximations are derived from aspects of the RM process. However, any arbitrary number of approximations can be used.

The most obvious approximation (6.5.15) bases on the remaining capacity, where flexible bookings are neglected in the capacity utilization

$$\pi_{m1}^t = \pi_m^t\left(\overline{c_m^t}\right), \quad \forall m \in M_{bf}. \tag{6.5.15}$$

The second evaluation (6.5.16) assumes that the accepted flexible bookings are equally distributed on all manifestations. To this end, we denote the number of manifestations with $\overline{m} = |M_f|$. This leads to a situation where revenue maximization will not be achieved. The following approximation accounts for this consideration

$$\pi_{m2}^t = \pi_m^t\left(\max\left\{\overline{c_m^t} - \frac{f_m^t}{\overline{m}}, 1\right\}\right), \quad \forall m \in M_{bf}. \tag{6.5.16}$$

Similarly, the accepted specific bookings can be considered instead of flexible bookings. There is no practical interpretation for approximation (6.5.17):

$$\pi_{m3}^t = \pi_m^t\left(\max\left\{\overline{c_m^t} - \frac{s_m^t}{\overline{m}}, 1\right\}\right), \quad \forall m \in M_{bf}. \tag{6.5.17}$$

A critical value for the remaining capacity is achieved when only one unit of capacity is left. In this case the corresponding bid price valuates the estimated revenue for the last available unit. Therefore, (6.5.18) evaluates bid prices for $c_m^t = 1$:

$$\pi_{m4}^t = \pi_m^t\left(1\right), \quad \forall m \in M_{bf}. \tag{6.5.18}$$

After defining the evaluations of the bid prices we calculate the weighted average. We define weighting parameters $\beta_1, \beta_2, \beta_3, \beta_4 \in [0, 1]$ for each valuation, fulfilling

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1. \tag{6.5.19}$$

The approximated bid prices are denoted with P_m^t and are calculated as weighted sum over the approximations.

$$P_m^t = \sum_{i=1}^{4} \beta_i \cdot \pi_{mi}^t, \quad \forall m \in M_{bf}. \tag{6.5.20}$$

As the approximation P_m^t takes on values in $[0, \infty)$, it has to be transformed to arrive at $\overline{P_m^t} \in [0,1]$. Bid prices represent an approximation of the benefit resulting from an allocation for the airline to a certain resource. The aim of bid prices is to compare and evaluate the scarcity for different resources. To transform P_m^t we define an upper bound P_+^t and a lower bound P_-^t

$$P_+^t = \max\left\{P_m^t \mid m \in M_f\right\} \quad \text{and} \tag{6.5.21}$$

$$P_-^t = \min\left\{P_m^t \mid m \in M_f\right\}. \tag{6.5.22}$$

These values are now applied to two auxiliary parameters defining the transformed bid price

$$\overline{P_m^t} = \frac{P_m^t - P_-^t}{P_+^t - P_-^t}, \quad \forall m \in M_{bf}. \tag{6.5.23}$$

Last but not least we modify the objective function (6.5.4) to account for the approximated bid prices

$$\min \sum_{b \in B} \sum_{s \in S} \overline{P_s^t} \cdot x_{bs}. \tag{6.5.24}$$

The extended optimization model is formulated in (6.5.25). It can be solved for an arbitrary number of flexible bookings. Therefore, it can be employed in ad-hoc allocation methods as well as re-allocation methods.

$$\begin{aligned}
\max \sum_{b \in B} \sum_{s \in S} q_{bs} \cdot x_{bs} & & \qquad (6.5.25)\\
\min \sum_{b \in B} \sum_{s \in S} \overline{P_s^t} \cdot x_{bs} & & \\
\sum_{b \in B} \sum_{s \in S} y_{sr} \cdot x_{bs} + s_r^t + f_r^t \le c_r, & \qquad \forall r \in R & \\
\sum_{s \in S} a_{bs} \cdot x_{bs} = 1, & \qquad \forall b \in B & \\
\sum_{s \in C_{bf}} x_{bs} = 1, & \qquad \forall b \in B & \\
x_{bs} \in \{0,1\}, & \qquad \forall b \in B,\ \forall s \in S &
\end{aligned}$$

For ad-hoc allocation, the set of flexible bookings contains only the newly accepted request. Re-allocations can be implemented according to two alternative setups. On the one hand, we can use the described model to solve an offline problem: after accepting a request for a flexible product, we solve the ad-hoc problem considering only this booking. Then, at each update point MO considers all accepted flexible bookings and calculates an optimal solution. On the other hand, we can solve the corresponding online problem: after accepting a request, perform the same steps as for scheduled re-allocations in the offline version. As the online implementation is a conceptual change, we distinct it by terming it **Online Multi-Objective Allocation (OMO)** method.

6.5.4 Discussing the Multi-Objective Approach

This section presented a multi-objective allocation method that simultaneously accounts for bid prices and revealed customer preference values. We formulated a binary linear program. In this formulation several linear constraints account for the feasibility of the underlying RM problem and special requirements resulting from the concept of flexible products.

The formulation as binary linear program allows to compute the allocation of multiple flexible bookings. We highlighted the problem to exactly evaluate the bid prices for a single resource and discussed the usability of this approximation in our program formulation. Additionally, we outlined insights from numerical tests regarding the behavior when recalculating the allocations several times during the sales horizon. We modified the objective function to incentivize more stable allocations for a particular flexible booking over time.

Regarding the formulation of the objective as weighted sum, we can propose some expectations for different α values. In case $\alpha = 0$, solving MO only minimizes the bid prices realized during allocation. We expect the allocation and the achieved revenue to be the same as when OCB is used for allocation. For $\alpha = 1$, we expect the opposite case, now only the sum of allocated preferences is minimized. Therefore, the achieved revenue should be the same as when using CPR. Regarding all other parameterizations, we expect a linear relationship between the revenue and the sum of allocated preferences.

7 Analysing the Effects of a Flawed Model

Chapter 4 presented several ways to model customers' preferential choice between manifestations. For each model, we discussed attributes of a corresponding revealing mechanism. Chapter 6 formulated various allocation methods, using information about customers' preferences to a certain extent. Both chapters revealed that the success of these methods is strongly linked to their correct application.

Section 7.1 discusses implications in case the assumptions made by the airline about customers' decision behavior do not match. False assumptions in the presence of strategic customer behavior are highlighted in Section 7.2. Section 7.3 analytically evaluates the effects of flawed input parameters to RM with flexible products. As we will see, in the concluding discussion in Section 7.4, on the one hand flexible products are beneficial to mitigate the effects of flawed parameters, on the other hand, they introduce additional dimensions of parameter defectiveness.

7.1 Effects Caused by False Assumptions

This section particularly looks at the effects of false assumptions regarding the implemented revealing mechanism and allocation method. These aspects only become relevant when the airline assumes customers' preferential choice between manifestations.

Assuming indifference. When discussing implications for limitations of the manifestation set, we concluded that assuming no customer preferences for the manifestation set can decrease the overall number of bookings (see Section 4.2). Customers may not accept some manifestations and therefore not book a flexible product at all. This decision is caused by the fact, that the possibility to be allocated to an unacceptable manifestation exists. In this case, the wrong assumption worsens the performance of the RM process as it affects the number of bookings.

In case customers have preferences for manifestations and the airline assumes indifference, this will merely decrease the attractiveness of the flexible product. If, however, customers have enough utility to buy a specific product, they will probably buy the specific product instead. As the general audience for flexible products is mostly more price-sensitive, horizontal spill will be more likely. Horizontal spill indicates customer choice changing to a similar product of the same airline or to a similar product of a competitor. In case horizontal spill leads to an outflow of bookings to a competitor, there will be a substantial loss in revenue.

Assuming limited acceptance. Assuming limited acceptance for manifestations has no impact when customers have no preferences regarding the manifestation set. They will not exclude any manifestations, so that the airline always keeps a certain flexibility. Obviously, this may result in a revenue gain compared to the situation where customers have preferences and limit the manifestation set.

In case customers have preferences for the manifestations, they limit the manifestation set. This is expected to negatively impact the revenue performance, but to ensure a minimal level of benefits compared to a setup without flexible products. The magnitude of the revenue loss depends on the imposed allocation method.

Assuming preferential choice. Last but not least, the airline may implement a preference-based revealing mechanism. Even this assumption may not meet the actual perception of customers.

In case customers are indifferent, the information about their revealed preferences does not directly affect revenue. The effect depends on the imposed allocation method. If the airline uses CPR or MO to allocate flexible bookings, uninformative and probably randomly chosen preference values may negatively impact the revenue. No clear expectations can be formulated, as the effect largely depends on the current inventory situation, bid prices, and the revealed preference values.

In general, we expect that revenue performance significantly decreases if preference values are used to allocate flexible bookings.

7.2 False Assumptions in the Presence of Strategic Customers

Let us consider the situation where customers behave following the strategic choice model (see Chapter 5) and where the airlines assume indifference. The resulting effect largely depends on the imposed allocation method. Using a more predictable allocation method, e.g., OCB, will increase the reliability and therefore encourage customers to act strategically. Therefore, it may be beneficial to impose an RM indicator-based stochastic method, e.g., sCUR or sEST, or a completely stochastic method (sUNI) to decrease the reliability and discourage customer to act strategically.

If customers act strategically, allowing them to limit the manifestation set increases the reliability. Imposing a monetary top-up for excluding manifestations or setting up a large minimum number of manifestations included in the choice set may counteract strategic behavior. In order to efficiently protect RM with flexible products against strategic behavior, the allocation method has to include a certain level of randomness.

Assuming myopic customers when implementing customers' preferential choice may lead to a significant loss in revenue. Customers reveal their preferences and in case the airline applies a preference-based allocation method, the predictability of a possible allocation and the reliability of this information increases. If customers are not myopic, this leads to a behavior which exploits the system. To counteract such strategic behavior, again the airline has to implement allocation methods with increased randomness. This lowers the predictability and prevent customers from acting strategically.

7.3 Flawed Input Parameters in Revenue Management

RM methods are highly sensitive to the correctness of input parameters, as existing literature, e.g., Petrick et al. (2012); Pölt (1998), shows and the formulation of allocation methods in Chapter 6 implies. Referring to the simplified RM process with flexible products shown in Figure 2.2, we can identify three relevant input parameters that may be flawed: demand forecasts, bid prices, and revealed customer preferences.

Demand forecasts incorporate a significant amount of uncertainty. The estimation of future demand bases on previous forecasts and historical booking data. As estimations already may be defective and historical booking data is constrained by the imposed control policies (cf. Weatherford & Pölt, 2002) subsequent forecasts are highly uncertain by definition. Additionally, unexpected occurrences like political or economic crises or changes in the competitive environment of an airline increase the level of uncertainty.

Referring to the process in Figure 2.2, forecasting is the first step when starting a new iteration of the RM process. The results are used in following steps to compute an optimal booking control policy. Variations in the estimated values from the real demand affect all subsequent process steps. Several contributions outline a negative impact on the achieved revenue for increasing errors in forecast values (see Section 2.1). Regarding RM with flexible products, the work of Petrick et al. (2012) provides extensive numerical results about the performance when forecasts are flawed. As a result, the authors contribute a weakening effect of flexible products on the revenue loss induced by forecast errors.

Bid prices are calculated using forecasts, hence they are uncertain by definition. Flawed bid prices affect the RM performance and the allocation of flexible products in particular. In contrast to uncertain forecast values, the impacts on the allocation may be precisely observable because bid prices are used as input parameters for example for OCB.

The certainty about customer preferences values depends on two aspects: the correctness of the implemented revealing mechanism and the honesty of customers when revealing their preferences. Similar to uncertain bid prices, distorted customer preference values generally only affect the allocation of flexible bookings. Impacts from distorted customer preference values can be quantified exactly by comparing the achieved revenue for different distortion setups. Immediate consequences for the RM process do not exist (see Figure 2.2).

7.4 Implications for Revenue Management Performance

The mathematical model in Chapter 5 indicates a significant impact of strategic customers on the advantage of flexible products. These negative effects seem to be independent from the assumed choice model for the manifestation set. This chapter, however, introduces another dimension impacting the benefits of using flexible products that largely depends on the assumption about customers' choice behavior made by the airline. A mismatch does not necessarily lead to a loss in revenue. Some combinations exist where wrong assumptions can have a positive revenue impact. Implementing customers' preferential choice between manifestations seems to be robust against a mismatch of concepts.

Section 7.3 shows how uncertain input parameters affect the results of the RM process with flexible products. Based on the process scheme depicted in Figure 2.2, three relevant parameters exist: demand forecasts, bid prices, and revealed customer preference values. Each of these indicators has several impacts on the performance of the RM process with flexible products.

Flawed forecasts have the largest impact, as they disturb the imposed booking control policy as well as the decision about the allocation of flexible products. However, quantifying the particular impact of flawed forecasts and bid prices on the allocation step is not possible. Both impact the calculation of the control policy and the allocation step if they are used as input parameters. In computational experiments it seems to be simpler to distort bid prices just before using them for allocation. Here, the exact relationship between wrongness and performance can be quantified. We expect that with increasing distortion of bid prices the achieved revenue decreases. Similar to uncertain forecast values, we expect allocation methods that use no bid prices as input to be robust against flawed values.

As several allocation methods depend on customer preferences, they are considered as third parameter. By definition customer preferences have no immediate impact on the RM process performance. However, they affect the allocation of flexible bookings. Then again, flawed allocations have consequences on the current valid booking control policy as the remaining capacity situation differs. For MO and OMO in re-allocation setups, a modified capacity value is used. Flawed allocations therefore do not impact the recalculation of the RM control parameters. Just between two scheduled re-allocations, flawed allocations are stored in the inventory and therefore impact RM performance. This leads to the expectation that distorting bid prices and customer preference values does not significantly impact RM performance in case of re-allocating flexible bookings throughout the sales period.

In general, we expect different effects when different parameters are flawed. Imprecise forecasts are expected to have a negative impact on RM performance compared to a setup with correct input parameters. This behavior is assumed for all allocation methods. Flawed bid prices or customer preference values are expected to only impact the outcome if allocation methods that use them as input parameters are applied.

Part III

Computational Studies

8 General Simulation Setup

This chapter introduces the Revenue Management (RM) simulation system used for the following computational studies. Due to the high relevance of RM for the airline industry and the extensive availability of data to calibrate the simulation tool, the computational studies are conducted in this context.

Section 8.1 reviews existing research on simulation models for RM and customer choice. The implementation of the applied simulation tool is described in Section 8.2. Finally, the parameterization of setups in general and for the strategic customer choice model is introduced in Section 8.3.

8.1 Simulation in Revenue Management

Simulations are powerful tools to virtually examine the impacts of changes in strategies, parameters, or environmental variables on a system. In case the system can be completely described using mathematical formulations and solving them is computationally efficient, this suffices to examine the impacts of changes. Without knowledge of dependencies or in case the system is highly complex, however, a computationally solvable analytical formulation is not possible. Computational experiments in a simulation environment have to be used to determine the effects of changes (Belobaba & Hopperstad, 2004; Louviere & Woodworth, 1983; North & Macal, 2007).

The work of Law and Kelton (2000) extended by Law (2003) provides a general introduction in simulation modeling, validation, and result analysis. Beside a comprehensive overview, both contributions review the consideration of complete manufacturing systems in simulation environments. A general introduction into modeling and simulation is presented in Carson (2004). Gilbert (2008) and Grimm and Railsback (2011) give a detailed introduction into agent-based modeling techniques and simulation.

Simulation tools are often used to evaluate alternative modeling approaches, especially for customer choice modeling. Compared to empirical data, simulations provide strictly controlled environments and conditions (Louviere & Woodworth, 1983). A comprehensive review of existing research on experimentally analyzing customer choice is made by Carson et al. (1994). The authors provide an overview about research gaps and address issues about reliability and validity of simulation tools.

One of the first simulation tools examining customer choice in the travel industry based on the concept of Hägerstraand's time-space prisms (Hägerstraand, 1970) is presented by Kitamura, Fujii S., and Otsuka Y. (1996). It is called Prism-Constrained-Activity

Travel Simulator (PCATS) and produces trips within prisms established in the time-space dimension. Relevant extensions and improvements to PCATS are contributed in Kitamura, Kikuchi, Fujii, and Yamamoto (2005). Further work includes other travel activity simulation systems, e.g., Kitamura and Fujii (1998); Kitamura, Lula, and Pas (1993); Pendyala, Kitamura, Kikuchi, Yamamoto, and Fujii (2005), and contributions that extend the concept of exploratory analysis towards agent-based modeling (Zhang & Levinson, 2004).

Ben Akiva, Bierlaire, Koutsopoulos, and Mishalani (1998) present a tool with application in traffic prediction and guidance. An environment for testing and evaluating dynamic traffic management systems is contributed in Yang, Koutsopoulos, and Ben Akiva (2000). Simulation tools concerning path choice of pedestrians are provided by Abdelghany, Abdelghany, Mahmassani, and Al Gadhi (2005) and Zhang and Han (2011).

An approach to use a micro-simulation to model air travel demand is presented by Abdelghany, Abdelghany, and Ekollu (2008). They set up a formulation that considers multiple factors like schedule attractiveness or seat inventory control. Examining changes with analytical models is hardly possible in airline RM. Therefore, simulations are often used for research and practical purposes. The work of Cleophas (2009) uses simulations to evaluate the performance of various forecasting concepts. Cleophas and Bartke (2011) present an approach to use simulations to model strategic customer behavior in airline RM. The authors contribute a mathematical formulation of strategic customers and use a stochastic simulation environment to analyze conditions and resulting performance. In addition to decision support, simulation environments can also be used to establish training possibilities for practitioners (Cleophas, 2012b).

Several simulation tools support research and practice in RM. Frank, Friedemann, and Schröder (2008) set up a framework and guidelines for developing a well-defined RM simulation tool. The first simulation systems related to airline operations were developed during the 1990's. Boeing build the Decision Window Path Preference Model to model passenger behavior on paths, focusing on scheduling and fleet planning. Later it has been extended to serve for research questions regarding the RM process. The new simulator was termed Passenger Origin-Destination Simulator (PODS) and was developed and built in cooperation with the Massachusetts Institute of Technology (MIT) (Hopperstad, 1995). This simulator still exists and is now operated by researchers from MIT and practitioners from different airlines (cf. Fiig et al., 2009; Weatherford, 2014).

Deutsche Lufthansa AG pursued a slightly different objective when developing REvenue MAnagement Training for Experts (REMATE) in cooperation with several partner universities (Frank, Cleophas, Schröder, & Gerlach, 2010). REMATE serves as a combination of research and training tool including various state-of-the-art forecasting and optimization methods. It implements a sophisticated customer choice model, to valuate different travel itineraries and products based on a customer-type specific set of preferences. Furthermore, several features like competition, code-sharing, and point-of-sale distinction are integrated (cf. Gerlach, Cleophas, & Kliewer, 2013; Zimmermann, Cleophas, & Frank, 2011). An important feature of REMATE is the existence of user influences. This functionality implements the manipulation of control policies as it is done by RM analysts in practice (Cleophas, 2012a). However, it limits the extensibility of REMATE by making the simulation tool more complex.

Almost every RM related contribution presenting numerical results uses a simulation tool. In most cases simple and use case specific simulation tools are used. The simulation tools mentioned in this section aim to be applicable for a broader range of research questions.

The contributions to simulation reviewed in this section are the framework for the simulation system applied in this thesis. It relies on experiences and guidelines postulated by research regarding existing simulation systems for RM and customer choice.

8.2 Simulation Environment

The simulation system used for the following computational studies is the Airline Revenue Management Simulation (ARMS). ARMS is a modular RM simulation environment that can be used to examine the impacts of flexible products in the airline industry. This section provides an overview of the general implementation and functional extensions relevant for this thesis.

ARMS is developed by a group of researchers from several German universities: RWTH Aachen University, Freie Universität Berlin, and Technische Universität Kaiserslautern. The aim of this project is to create a widely applicable academic simulation tool for airline RM. A high degree of modularity ensures plenty of opportunities to extend ARMS. ARMS is implemented in Java and the code is available to all involved researchers. This allows them to easily extend the existing core framework to provide a vast functionality.

Applying the principles for the design of RM simulations formulated by Law and Kelton (2000) and Frank et al. (2008) ensures a high degree of reliability for results generated with ARMS. In contrast to REMATE, ARMS is not designed as a training and decision support tool for RM practitioners. To this end, there is no need for a graphical user interface or any integrated statistic functionality. This reduces overall complexity and accounts for the favor of clarity for ARMS as research tool.

Like any simulation tool, ARMS implements a simplified view of dependencies and complexities of the real world. The general RM process structure implemented in ARMS is shown in Figure 8.1. Setups are shown as rhombuses, process steps are shown as rectangles, and internal data is illustrated as boxes. A gray box surrounds methods and data located in the actual RM process.

The remainder of this section is structured along the process in Figure 8.1. Each setup, process step, and resulting data will be discussed.

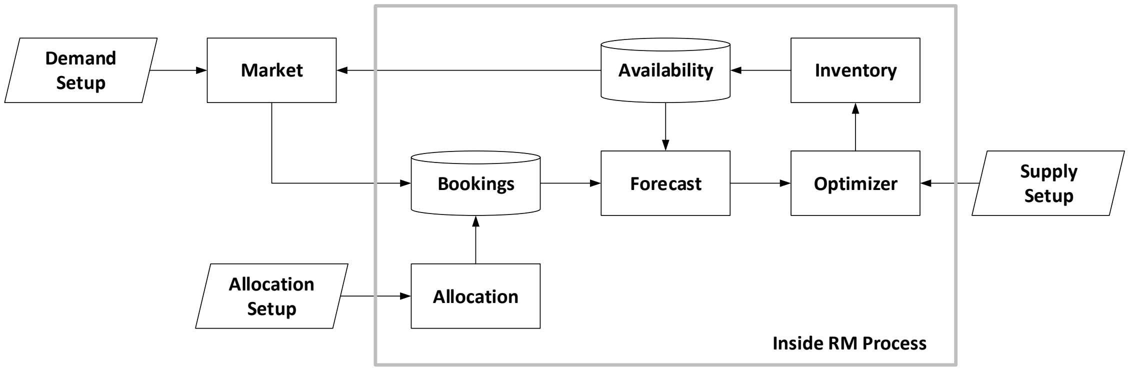


Figure 8.1: Flowchart of the RM process in ARMS

8.2.1 Defining the Experimental Setup

Each experiment in ARMS consists of multiple simulations repeating a sales period several times. Each simulation is characterized by a combination of supply, demand, and allocation setup. The supply setup defines the basic framework; the demand setup defines the antagonist of the supply setup. An allocation setup is only required for experiments where flexible products are offered.

We term the repetitions of the sales period **simulation runs**. The supply remains constant over all simulation runs. The actual demand, however, differs between the simulation runs. All demand parameters are stochastically defined and independently realized for each run. This stochastic variation over the simulation runs ensures valid and reliable results.

Supply setup. Relevant components are the schedule with the offered flights and the product portfolio with prices and restrictions for each product. A valid schedule includes the set of flights with origin, destination, and flight times.

Each product is linked to a price and subset of restrictions defining the characteristic features of this product. Flights have fixed capacities and each product consumes one seat on a single flight.

Demand setup. The decision behavior of customers follows a customer choice model. Beside the actual decision behavior, the demand setup includes additional parameters, e.g., the time a customer places the request.

Demand parameters are independently realized for each simulation run. Therefore, the demand for each simulation run slightly differs. However, the realizations are all derived using the same random variables defined in the demand setup. Each customer is a realization of these random variables. This allows us to calculate averages over all simulation runs for all numerical indicators.

Customers carry a set of attributes defining their decision behavior:

- The willingness-to-pay is the maximal amount of money a customer is willing to pay for any product. The willingness-to-pay is derived from a Gaussian distribution $N(\mu, \sigma^2)$ with an expected value of μ and a variance of σ^2.
- The time customers place their requests. This is relevant for ensuring a well-defined order between all requests. The distribution modeling the arrival of customer requests during the sales period can be described by a Poisson process $P_{\lambda,t}$. The piecewise constant intensity between two discrete time steps is denoted by λ and t is defined in $[0, \infty)$.
- The restriction costs as monetary equivalent for the utility diminished by a restriction. Active restrictions diminish the products' usability for customers and therefore the associated utility. Restriction costs diminish the customers' particular willingness-to-pay for this product. This means in general: the utility a customer connects with this product decreases.
  The restriction costs are modeled as random variables distributed following an independent Gaussian distribution $N(\mu, \sigma^2)$ with an expected value of μ and a variance of σ^2. The additional uncertainty of flexible products is modeled as an additional restriction connected with these products.
- The preference value for each specific product included in the manifestation set of any flexible product. The distribution modeling the preference values is a uniform distribution $U([0, 1])$.

The actual customer choice to buy a particular product follows a two-step process. Customers only consider products included in their individual choice set. A product is included in the choice set if the customers' willingness-to-pay exceeds the price plus the monetary equivalents of the restrictions of this product.

After determining the choice set, customers compare the sum of price and restriction costs for each included product. Each customer books the currently available product with the lowest sum.

The share of strategic customers is defined in relation to the overall number of customers. We denote it by $o \in [0, 1]$. Demand creation independently decides for each customer if he is strategic or not. To this end, a realization x_b, $\forall b \in B$ from a uniform random variable $X \sim U([0, 1])$ is drawn. A customer is strategic if

$$x_b \leq o, \quad \forall b \in B. \tag{8.2.1}$$

A strategic customer books the flexible product depending on the reliability of the additional information available (see Chapter 5). The parameter for the individual minimum reliability per customer is denoted by $\vartheta_b \in [0, 1], \forall b \in B$.

A strategic customer's decision to actually book strategically depends on multiple criteria. During the simulation process, the following conditions are checked for each customer requesting a specific product:

1. Check if this customer is a strategic customer.
2. In case this customer is, check if the manifestation proposed by the current allocation method matches the preferred manifestation m_b by the customer. The preferred manifestation is defined as the manifestation with the maximal preference value.
3. In case of a match, check if the reliability about the allocation information is high enough by comparing it to the individual minimum reliability of this customer: $\vartheta_b \leq \varphi_{bm_b}$.

After successfully checking all three conditions, this customer books the flexible product instead of a specific one. In case any of the three checks fails, this customer does not act strategically and books his originally chosen specific product.

This implementation of strategic customer behavior can be combined with all allocation methods presented in Chapter 6. However, the current implementation is restricted to experiments using ad-hoc allocation. Re-allocation is not possible as the manifestation for allocating the flexible product may change and the second condition may be unsatisfied later without knowing this when initially checking for a customer to act strategically.

Allocation setup. The allocation setup defines the allocation method used and specifies if flexible bookings are re-allocated at each update point. Furthermore, for Multi-Objective Allocation (MO) and Online Multi-Objective Allocation (OMO) the allocation setup specifies the parameterization of the weighting parameter α.

8.2.2 Revenue Management Methods

In this section we detail the RM methods implemented in ARMS, following the models introduced in Chapter 2, the preferential choice models in Chapter 4 and Chapter 5, and the allocation methods in Chapter 6.

Even though demand continuously arrives over the sales period, optimization requires a discretization of time (see Section 2.1). Therefore, we define a set of time intervals discretizing the sales period. At the beginning of each time interval a re-optimization is scheduled. The beginning of each time intervals is termed **update point**.

Forecast. The forecaster estimates the expected demand for upcoming sales periods. For each itinerary, product, and update point the number of expected customer requests is calculated. The forecast implementation in ARMS uses a customer choice model assuming independent demand (see Section 2.1).

The first simulation run requires an initial demand reference. This reference is provided by an initialization method. For subsequent simulation runs, the forecaster can use historical booking data to update the reference. This forecast is termed **demand**

learning. Following the work of Weatherford and Kimes (2003), ARMS includes a simple pick-up forecasting technique. This is sufficient to merge the estimations resulting from the unconstraining step. For unconstraining, the number of bookings for each itinerary, product, and update point is counted as long as the product was available. In case the product was not offered the past reference demand is used as estimate (cf. Pölt, 1998). These demand estimates are now used to update the existing demand reference via exponential smoothing. For each itinerary, product, and update point exists an independent estimate. For a more detailed view on this forecasting method, we refer to Section 2.1.

The initialization method in ARMS is termed **omniscient initialization** and uses knowledge of the generated demand in the particular experiment. This leads to forecasts that provide the airline with a good fit of the underlying demand model. The initialization counts customers towards the demand for the most expensive product that they can afford. Each run is independently counted and the initial values are provided by averaging the references over all runs. In order to create a simulation without forecast dependencies between runs, the initialization reference may be used for each simulation run to create a benchmark setting. This forecast is termed **omniscient forecast**.

Optimizer. The optimizer calculates a revenue maximizing control policy given the demand forecasts. ARMS does neither implement sub-modules for finding the best capacity configuration nor consider cancellations. The dynamic programming approach as described in Section 2.1 is used to calculate bid prices as valuation of remaining capacity.

The handling of flexible products in ARMS is adapted from Petrick et al. (2012) (see Section 2.2). Flexible products are offered over the complete sales period (in contrast to Gallego & Phillips, 2004; Petrick et al., 2012). The decision to make them available depends on the current valid booking control policy (cf. Petrick et al., 2012). The optimizer applies the dynamic programming formulation to determine bid prices. To ensure feasibility regarding capacity and optimality for upcoming requests, flexible products are temporarily allocated and the respective capacities are immediately adjusted. For simplicity's sake, no evaluation of flexibility is included in the products' valuation as described for example in Gönsch et al. (2014).

The optimizer in ARMS is implemented to schedule re-optimizations at the beginning of each update point (see Section 2.1). The inventory is modified, as optimization should only account for accepted specific bookings. Therefore, all flexible bookings are removed from the inventory before re-optimizing. This step implies a separation of the allocation problem and the underlying RM problem. As it keeps the implementation in ARMS simple, it allows a clear evaluation of the actual effects when introducing customers' preferential choice for flexible products.

Inventory. The inventory stores the results from previous steps and translates these results into actual instructions for the market module regarding accepting or denying customer requests. To this end, the inventory stores prices, capacities from the supply, and observed bookings. Furthermore, the bid prices resulting from the optimization are stored. ARMS implements an inventory based on bid prices. This type of inventory relies on bid price vectors representing the valuation of a free seat given various amounts of remaining capacity.

Allocation methods. Allocation methods depend on the current allocation setup and follow the mathematical formulations presented in Chapter 6.

8.2.3 Market Module

Last but not least, ARMS implements a market module. Here, the generated demand and the calculated availability situation meet. The market module communicates the current availability situation based on the data stored in the inventory to the customers to subsequently retrieve their decision about requesting a product. Afterwards, the market module generates the acceptance decision and updates all relevant parameters in the inventory to proceed to the next time slice.

8.3 Parameterizing of Experimental Setups

This section describes the experimental setups used in the computational studies. In general, we differentiate three types of setups: supply setups, demand setups, and allocation setups. An experiment consists of several combinations of these three setups. Numerical results are calculated by comparing these combinations against each other.

Each supply setup includes a single airline offering a fixed set of direct flights ($\bar{r} \in \{5, 10\}$) with a capacity of 50 seats ($c_r = 50,\ \forall r \in R$) each. The flights share a common duration, departure location, and departure time.

Hence, the performance indicators per flight can be averaged over all flights as each flight has the same characteristics. By doing so, we increase the amount of data used to calculate the numerical indicators. This improves the reliability and validity of these indicators and of the derived implications. All numerical results presented in the following chapters are calculated as average over all results per combination of flight and run.

The airline can offer three different sets of products in one compartment. First, the airline can offer three specific products ($S = \{1, \dots, 3\}$). This setup is termed Setup with Three Specific Products (3SP). The second setup is termed Setup with Four Specific Products (4SP). Here, the airline offers four specific products ($S = \{1, \dots, 4\}$). Finally, the cheapest product from 4SP, $s = 4$, can be offered as flexible product:

$S = \{1, \ldots, 3\}$ and $F = \{1\}$. This setup is termed based on the allocation method used to allocate the flexible bookings, e.g., OCB, if the allocation method based on bid prices is applied.

Prices for all products are fixed over the complete sales period. The price structure is calibrated using empirical airline data from Deutsche Lufthansa AG.

Demand is modeled as independent (see Chapter 2). Each customer requests only one product and his choice function does not consider the offered alternatives. The overall demand volume and the arrival time distribution for customer requests were calibrated using empirical airline data. We consider that customers reveal their preference values using a ratio scale and ARMS uses the omniscient initialization.

Re-optimization to update control policy parameters is performed 12 times during a single sales period. The sales period consists of 360 days. The following set of update points is denoted in days before departure: $\{360, 180, 90, 45, 20, 10, 5, 4, 3, 2, 1\}$. These update points were derived from empirical data from Deutsche Lufthansa AG. However, these update points only represent a subset of the update points used in practice in order to ensure operability of ARMS. To this end, the set of update points represents relevant times within the sales period regarding the demand arrival process and therefore relevant times to re-optimize the RM parameters as well as to re-allocate the flexible bookings.

All experiments consist of 200 simulation runs. For a particular indicator, this number of runs (stochastically different instances) ensures convergence of the averaged results over all 200 runs against the real value.

General RM performance strongly depends on the demand-to-capacity ratio established in a demand setup. To this end, we use a Gaussian distribution $N(\mu, \sigma^2)$ to model unbiased distributions of the overall demand level and the demand-per-product level. Multiple demand setups, differing in the demand level and the distribution over specific and flexible products, will be used in a sensitivity analysis later on. We denote the overall demand level by l, defined relative to the capacity included in the supply setup. The demand level indicates the mean of the Gaussian distribution: $\mu = \overline{r} \cdot c_r \cdot \frac{l}{100}$ and the standard deviation is defined as $\sigma = 0.1 \cdot \mu$. The following parameterizations are used: $l = \{80, 90, 100, 110, 120\}$.

We refer to the division of demand for flexible and specific products as demand distribution d. This parameter is defined as the percentage of customers requesting a specific product. For example, $d = 80$ indicates that 80% request specific products and $100\% - 80\% = 20\%$ percent of all customers request flexible products. The distribution of requests over the specific products follows a Gaussian distribution $N(\mu, \sigma^2)$ with $\mu = d$ and $\sigma = 0.1 \cdot d$. The share of demand for a particular specific product can be calculated as

$$\frac{100 - d}{|S|} \xRightarrow{S=\{1,\ldots,3\}} \frac{1}{3} \cdot (100 - d)\,. \tag{8.3.1}$$

The demand distribution is varied between $d = \{50, 60, 70, 80, 90\}$.

To parameterize customer preferences $q_{bm},\ \forall b \in B$ with regard to the manifestation set of the flexible product, we define a uniformly distributed random variable $X \sim U\left([0,1]\right)$. Drawing a realization of X defines the preference value for each combination of customer and specific product. When applying MO, we do not assume any restrictions of the manifestation set for all customers: $C_{bf} = M_{bf}$. This is relevant regarding the feasibility constraint (6.5.9) and the assumed revealing mechanism.

For simulating strategic behavior of customers, we assume uniformly distributed minimum reliabilities in between $[0, 1]$ over all customers. This is equivalent to customers requiring a reliability of 50% on average to trust the additional information.

To approximate the bid prices using equation (6.5.20) for MO and OMO, we set $\beta_i = 0.25,\ \forall i = 1, \ldots, 4$.

9 Impacts of Accounting for Customers' Preferential Choice

This computational study examines the implications of customers' preferential choice for Revenue Management (RM) with flexible products. The computational experiments are designed to answer the following research questions as presented in Chapter 3:

Research Question 1. *What are the implications for RM with flexible products resulting from different models of customers' preferential choice between manifestations?*

Research Question 2. *How does RM performance depend on the parameterization of customers' preferential choice models, revealing mechanisms, and allocation methods?*

We start with evaluating the benefits of flexible products in Section 9.1. In Section 9.2 the airline lets customers limit the manifestation set. A setup where customers reveal their actual preference values is used in Section 9.3. The particular effects on revenue in case the Multi-Objective Allocation (MO) method is modified as detailed in Section 6.5 are presented in Section 9.4. A biased parameterization of customer preferences is evaluated in Section 9.5. Finally, Section 9.6 concludes this chapter by discussing the results.

9.1 General Effects of Flexible Products

This section presents numerical results to quantify the benefits for the airline from selling flexible products. Allocating flexible bookings to one of several alternatives allows the airline to gain a certain degree of freedom by specifying the allocation. However, the degree of freedom for allocation is constrained by the overall demand level and the demand composition (cf. Gallego & Phillips, 2004; Petrick et al., 2012).

This experiment uses 25 demand setups. These setups differ in the demand level $l = \{80, 90, 100, 110, 120\}$ and distribution $d = \{50, 60, 70, 80, 90\}$. Six supply setups are considered differing in the size of the manifestation set ($|M_f| \in \{5, 10\}$) and the set of offered products (3SP, 4SP, OCB). We use the Setup with Three Specific Products (3SP) as base supply setup to segregate the benefits of inducing additional demand from the benefits of introducing flexibility. When flexible products are sold we apply two different allocation setups: Allocation Based on Bid Prices (OCB) with ad-hoc allocation and re-allocation. This leads to an overall number of 200 combinations of supply, demand, and allocation setup included in this experiment.

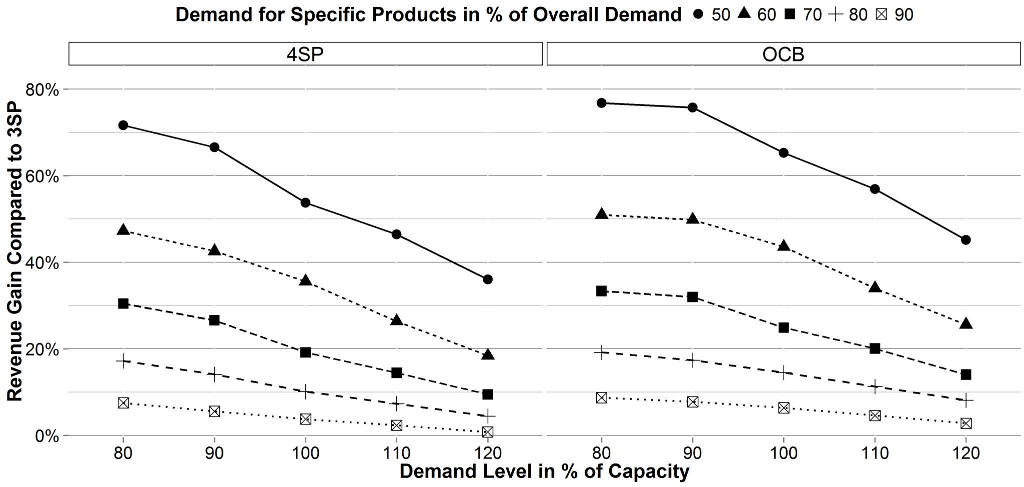


Figure 9.1: Revenue gain given different supply and demand setups and $|M_f| = 5$

Figure 9.1 and Figure 9.2 display the revenue gain for the Setup with Four Specific Products (4SP) and OCB compared to Setup with Three Specific Products (3SP) on the y-axis. Each line graph connects the results of a particular demand distribution. The x-axis depicts the different demand levels.

Figure 9.1 shows the results of the smaller manifestation set ($|M_f| = 5$). The revenue decreases when the demand level increases. Further, the benefits of offering the cheapest product increase with a decreasing share of customers buying the three other products. Comparing the results of 4SP and OCB shows an additional revenue gain by offering the cheapest product as flexible product.

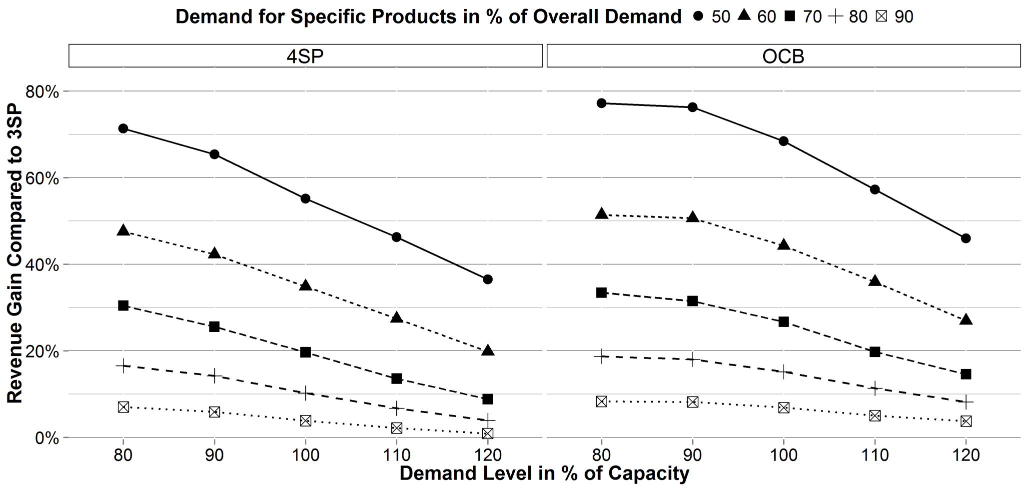


Figure 9.2: Revenue gain given different supply and demand setups and $|M_f| = 10$

The results of a manifestation set with ten elements ($|M_f| = 10$) are shown in Figure 9.2. Generally, the same characteristic behavior for variation in demand level and distribution is shown as in Figure 9.1. Comparing particular demand setups identifies additional benefits from the increased size of the manifestation set.

Figure 9.3 and Figure 9.4 emphasize the effect of offering a flexible product. This revenue gain is achieved by selling the cheapest product as a flexible product instead of a specific product. Both figures compare 4SP to two allocation setups: OCB with ad-hoc allocation and re-allocation. Again, we evaluate the impacts of different manifestation sets.

The numerical results indicate a significant revenue gain induced by offering the flexible product. For the smaller manifestation set ($|M_f| = 5$), Figure 9.3 shows at last 7.5% revenue gain. The effects of demand levels 80%, 90%, and 100% are contrary to those in Figure 9.1 and 9.2. For demand levels less than 100% of capacity, the revenue gain increases with an increasing demand level. For all other demand levels, the revenue gain decreases or at least stagnates. Between 100% and 110% exists a particular demand level that maximizes the revenue gain for most demand distributions. A possible explanation is, that for demand levels much larger than 100% the flexible product will not be often available. There is enough demand for more expensive products such that offering the flexible product will waste capacity for less-paying customers. Regarding demand levels much smaller than 100%, the airline is not able to profit from selling the flexible product because there is enough capacity available to serve all customers.

Figure 9.4 illustrates similar results of the larger manifestation set ($|M_f| = 10$). In comparison to Figure 9.3, the benefits from introducing flexible products are significantly positively related to the number of manifestations. This revenue gain due to the larger manifestation set is caused by an increased reactivity for the airline. The revenue gain is at last more than 8.5%: 1% more than for less manifestations. Also, the revenue difference between the different demand distributions is larger. This implies that each additional customer for the flexible product is more valuable when the manifestation set is larger. Additionally, all demand distributions have their revenue maximum at a demand level of 100%. This supports our explanation proposed for the smaller manifestation set. The increased airlines' reactivity has to be efficiently handled and in case the airline is not able to use it ($l > 100\%$) or in case it does not improve the situation ($l < 100\%$) the results are inferior to the maximal benefits.

Based on the numerical results of this first experiment, we restrict for following experiments in this computational study to a single demand setup where demand is equal to capacity ($l = 100\%$) and 80% of demand requests the specific products ($d = 80\%$).

9.2 Effects of Limiting the Manifestation Set

This experiment examines the implications if customers are allowed to limit the manifestation set. The corresponding customers' preferential choice model is introduced in Section 4.3.

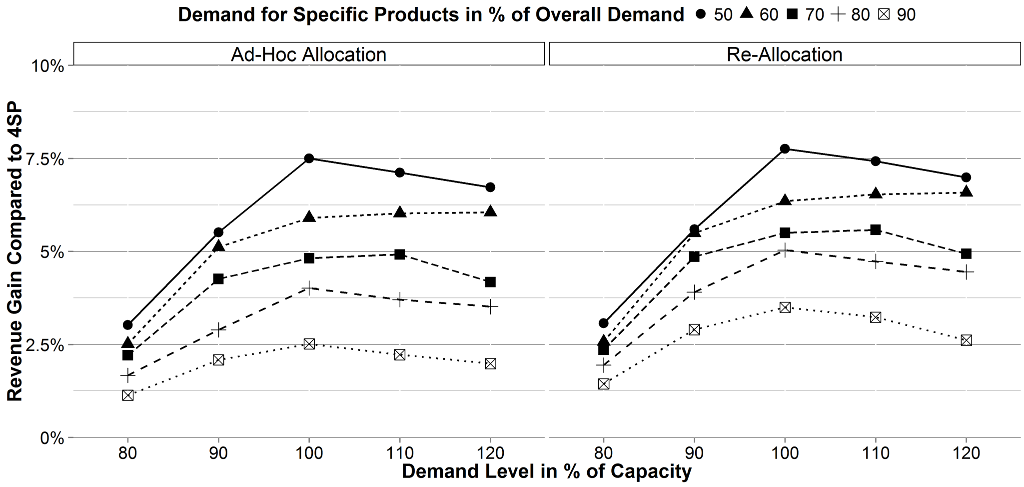


Figure 9.3: Revenue gain for OCB given different demand setups and $|M_f| = 5$

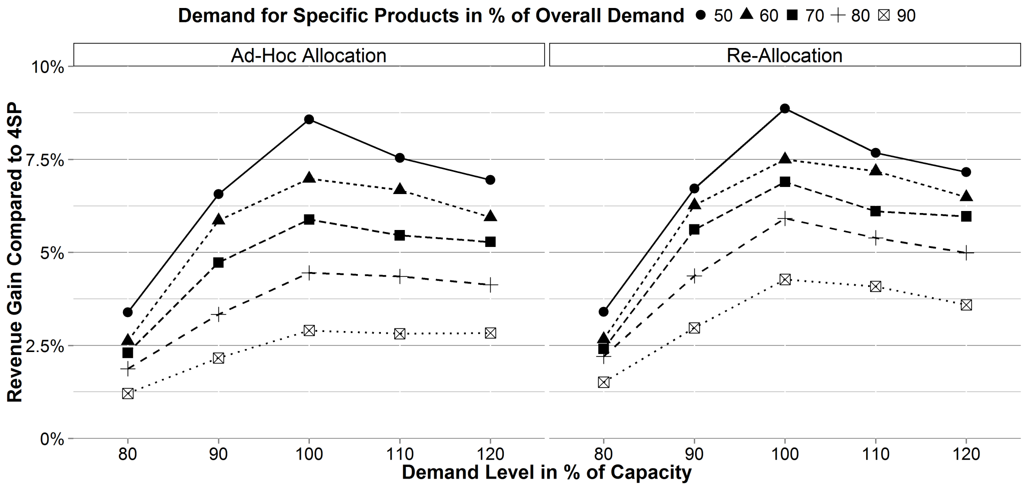


Figure 9.4: Revenue gain for OCB given different demand setups and $|M_f| = 10$

We evaluate two supply setups differing in the size of the manifestation set; for the setup with $|M_f| = 5$ we apply five demand setups and ten demand setups for the supply setup with $|M_f| = 10$.

The demand setups differ in the number of manifestations customers are allowed to exclude. We assume that all customers exploit the full potential by excluding the maximum number of manifestations allowed. Further, customers always exclude those manifestations having the smallest preference values. No fee is claimed for excluding manifestations. The resulting scenario represents the worst-case: the loss in flexibility is not monetarily compensated and the airlines' flexibility decreases as much as possible. In general, excluding manifestations is expected to diminish revenue as the possibilities to allocate flexible bookings for the airline decrease.

The allocation setups consist of OCB, Stochastic Allocation Based on Preferences (sPRE), Stochastic Allocation Based on Estimations (sEST), Stochastic Allocation Based on Bookings (sCUR), and Stochastic Allocation with Uniform Weights (sUNI). They are applied as ad-hoc allocation and re-allocation method. Overall, this experiment evaluates 150 different setups.

Figure 9.5 visualizes the revenue loss in case a manifestation set with five elements is subject to limitations. The y-axis displays the revenue loss compared to OCB without limitations. The x-axis differentiates the demand setups denoted by the number of manifestations excluded. The left part shows the results of ad-hoc allocation setups and the right part depicts the results of re-allocation setups. For comparison only, the revenue for 4SP is shown as horizontal line.

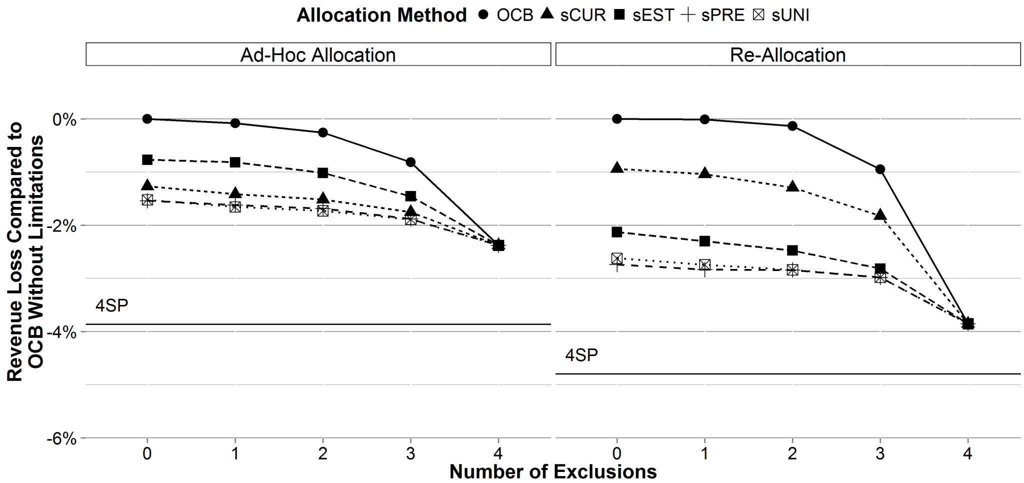


Figure 9.5: Revenue loss when the manifestation set ($|M_f| = 5$) is limited

Given ad-hoc allocation, small but significant losses in revenue occur when customers exclude one or two manifestations. For OCB, the mean revenue decreases only by about 0.1% for excluding one and less than 0.3% for excluding two manifestations. Comparing these results to OCB with three exclusions, however, shows a nonlinear relationship between the number of excluded manifestations and revenue. Excluding three manifestations reduces revenue by more than 0.8%. The setup with four exclusions leads to loss in revenue of about 2.5%.

The stochastic allocation methods show a similar characteristic behavior. The incremental loss between one or two exclusions is small. For each additional exclusion, the incremental loss increases up to 1% for sEST and 0.6% for sCUR, sPRE, and sUNI. The overall behavior of sEST is similar to the one of OCB. All other stochastic allocation methods show less revenue sensitivity to exclusions.

For re-allocation, we use OCB with re-allocation as base case. The general behavior is similar to the one for the smaller manifestation set shown in Figure 9.5. Revenue is highly robust in case a few manifestations are excluded. The impact of more than three exclusions, however, significantly increases.

The nonlinearly increasing loss can be explained by the fact that selling flexible products when three or four manifestations are excluded becomes similar to selling a specific product instead. Re-allocation increases this loss as the current implementation separates the re-allocation problem from the RM problem. When updating the RM parameters, only specific bookings remain in the inventory. All flexible bookings are removed, as the booking control policy should be updated independently. In case only one manifestation remains for allocation, flexible bookings are more or less specific and the separation leads to a defective booking control policy.

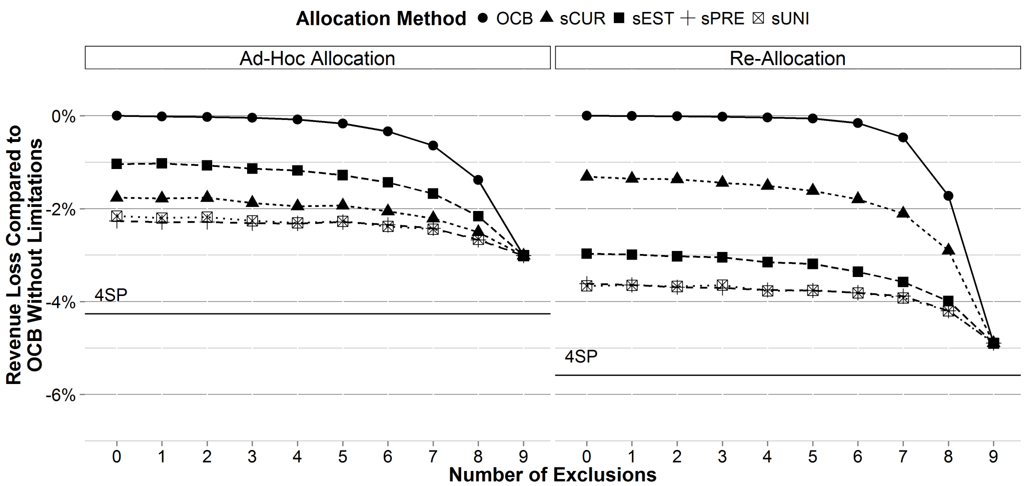


Figure 9.6: Revenue loss when the manifestation set ($|M_f| = 10$) is limited

Figure 9.6 depicts the revenue loss for setups with a larger manifestation set. The overall behavior looks similar to Figure 9.5 and seems to be independent from the size of the manifestation set. Again, the number of excluded manifestations and the revenue loss show a nonlinear relationship for all allocation methods.

The possibility to recalculate the allocation and to dynamically react makes the revenue loss nonsignificant for excluding up to four or five manifestations. For more than seven manifestations setups with re-allocation show a steep increase in revenue loss. Again, this behavior can be explained by the loss in flexibility to allocate the flexible bookings for the airline and by separation of allocating flexible bookings and updating the RM parameters.

Figure 9.5 and Figure 9.6 indicate that the magnitude of revenue loss in case of limitations does not depend on the absolute number of manifestations. We can rather conclude that the performance depends on the relation between excluded manifestations and the initial size of the manifestation set.

All Figures show a revenue gap between setups with four excluded manifestations and 4SP. This additional revenue loss for 4SP is caused by the variation in demand, leading to a biased distribution of customers over flights. However, the preference values are derived from a uniformly distributed random variable. Therefore, the distribution of the last remaining manifestation over the set of resources is more regular.

9.3 Comparing Different Allocation Methods

This section includes two experiments comparing the performance of different allocation methods with regard to revenue and the fulfillment of customer preferences.

The experiments include two supply setups with five manifestations and a single demand setup ($l = 100\%$, $d = 80\%$). 4SP is used as base case to compare with the supply setup with flexible products. We apply 46 allocation setups for eight allocation methods: OCB, Preference-Based Allocation (CPR), Stochastic Allocation Based on Bid Prices (sBID), sPRE, sEST, sCUR, sUNI, MO, and Online Multi-Objective Allocation (OMO). 30 setups differing in the objective weighting and allocation dynamic are evaluated alone for MO and OMO.

In general, we expect that re-allocating flexible bookings will increase revenue even when considering customer preferences. This expectation derives from similar improvements given exclusions from the manifestation set. For the different allocation methods, we expect sPRE and sUNI to perform similar to CPR as they rely on similar objectives. sBID and sEST are expected to be superior to sCUR, because they use more information about the current situation and possible future developments to calculate the allocation probabilities. OCB should be the most beneficial method in terms of revenue and CPR the worst. Therefore, in Figure 9.7, CPR is shown as a horizontal line.

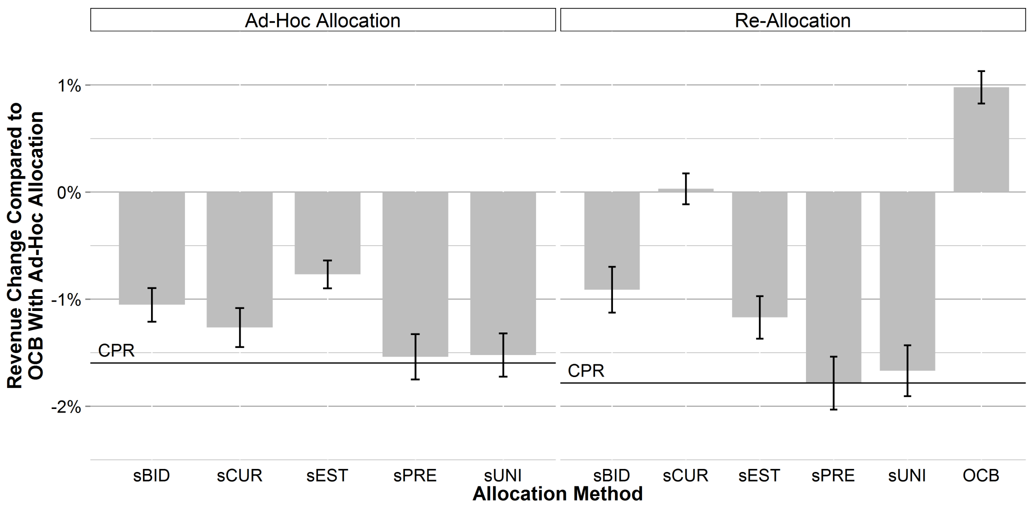


Figure 9.7: Comparing revenue change for ad-hoc and re-allocation methods

Figure 9.7 shows the revenue change compared to OCB with ad-hoc allocation and CPR. The left part depicts results for the setups with ad-hoc allocation and the right part the results for re-allocation setups. The vertical black lines indicate the confidence intervals calculated to a confidence level of 95%.

For ad-hoc allocation and re-allocation, no stochastic allocation method is able to achieve the same revenue as for the corresponding variant of OCB. For ad-hoc allocation, sEST performs best, followed by sBID and sCUR. sPRE and sUNI perform similar to CPR. Overall, CPR leads to a loss in revenue of more than -1.5% compared to OCB. For re-allocation, OCB shows a significant improvement with an increase in revenue of 1%. sCUR significantly outperforms the other stochastic allocation methods, followed by sBID performing slightly better than sEST. sPRE and sUNI show similar characteristics as their corresponding ad-hoc allocation variants.

The numerical results largely confirm our expectations regarding the different allocation methods. Comparing the results of sCUR and sEST with re-allocation to their behavior with ad-hoc allocation in Figure 9.7, shows an unexpected behavior. Table 9.1 analyzes the distribution of re-allocations over the manifestation set calculated as average over all simulation runs. These indicators outline the willingness of an allocation method to re-allocate flexible bookings to a different manifestation. It helps to examine if there is a concentration of allocations to a subset of manifestations.

The average number of re-allocations per manifestation indicates the number of flexible bookings allocated to a certain manifestation during re-allocation that were previously allocated to another manifestation. Large standard deviation values depict a concentration on several manifestations, whereas a small standard deviation indicates a well distributed allocation behavior over the manifestations.

Re-allocating flexible bookings is only useful if the allocation method actually takes advantage of the opportunity to revise a previous allocation. We can therefore expect that a larger mean number of re-allocations per manifestation should benefit the RM performance. The aim of re-allocating flexible bookings is to create capacity on manifestations with a large number of estimated customers. As mentioned in Section 6.5, an alternating behavior between two manifestations, e.g., only exchanging bookings between manifestations, may worsen the performance. As a consequence, we can expect that a larger standard deviation over all manifestations indicates an improved re-allocation behavior. We term an allocation behavior exchanging bookings between two manifestations **alternating allocation** behavior.

Table 9.1 shows the mean and standard deviation characterizing the behavior of sCUR, sEST, and OCB for benchmarking purposes. No result is statistical significant to a confidence level of 95%. Therefore, the following explanation approach is only a suggestion.

For the mean number of re-allocations per manifestation, no significant difference exists between both stochastic allocation methods. OCB, however, has a slightly larger number of re-allocations. In contrast to this, the standard deviation differs between the allocation methods. OCB shows the largest standard deviation with a value of 42.66. This validates our expectation about improvements when mainly concentrating on a subset of manifestations. The standard deviation for sEST is smaller than for

Table 9.1: Mean and standard deviation for the number of re-allocations per manifestation for sCUR, sEST, and OCB

	Allocation Method		
	sCUR	sEST	OCB
Mean	68.12	68.20	68.60
Std.-Dev.	31.34	29.06	42.66

sCUR, although the mean is larger. These results imply that instead of using the slightly superior number of re-allocations, sEST seems to allocate flexible bookings more equally over all manifestations.

Figure 9.8 shows the change in the fulfillment of customer preferences for different allocation methods compared to OCB. For the sake of readability, CPR is not included as it completely outperforms all other methods with a gain of 62.5%. Comparing ad-hoc and re-allocation setups, re-allocation seems to diminish the fulfillment of preferences significantly. Overall, for both cases sPRE increases the level of satisfaction by more than 25% compared to OCB. The remaining stochastic methods do not largely change the fulfillment of preferences. Again, confidence intervals for a significance level of 95% are shown for each bar.

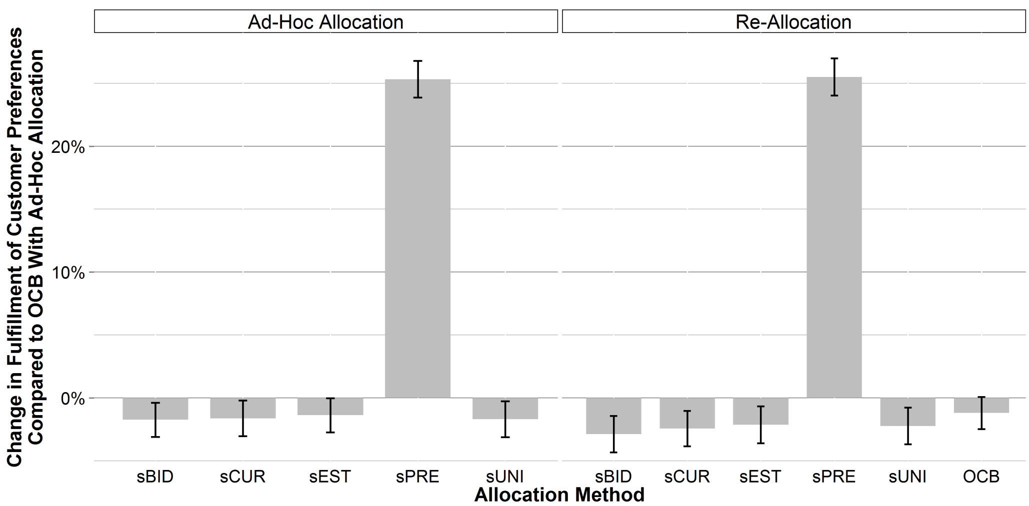


Figure 9.8: Change in fulfillment of customer preferences for different allocation setups

Figure 9.9 shows results for MO and OMO differing in the parameterization of the objective. The graph depicts the trade-off between revenue and fulfillment of customer preferences. These results are achieved by varying the weighting parameter α in the objective in steps of 0.1 within the interval $[0, 1]$. The revenue gain compared to 4SP is shown on the y-axis and the gain in the fulfillment of customer preferences compared to 4SP on the x-axis. Table A.1 in the appendix shows the actual numerical results for each parameterization of α.

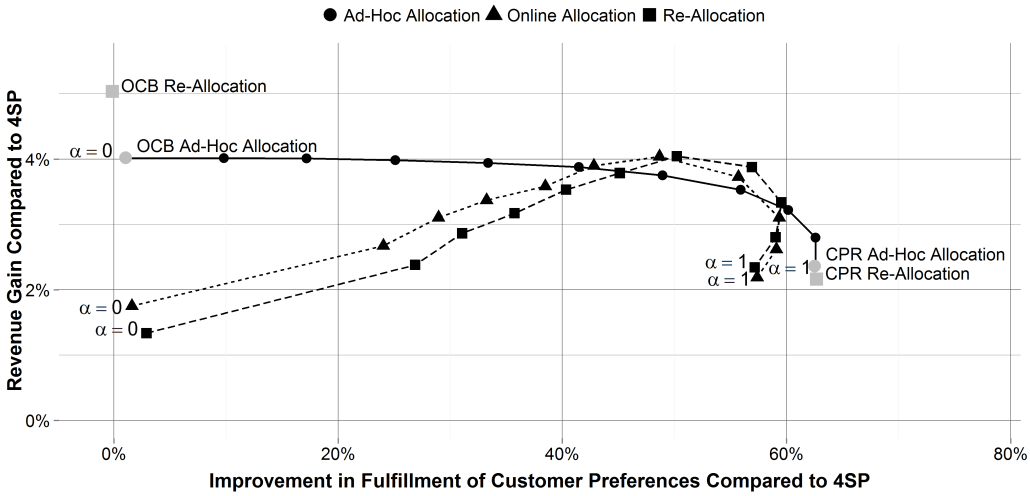


Figure 9.9: Trade-off between revenue and fulfillment of customer preferences for multi-objective allocation given varied α-values

For benchmarking purposes, Figure 9.9 shows results of OCB and CPR. The respective points are marked by gray shapes and labeled with the methods' name. The results for MO with ad-hoc allocation are shown as solid line with dots indicating the particular results for a certain parameterization. MO with re-allocation is depicted as dashed line with triangles and the online allocation setup as dashed line with squares. Further, the particular results for $\alpha \in \{0, 1\}$ are annotated with their α-values.

For some parameterizations of α, we can formulate expectations based on the definition of the objective as weighted-sum (see definition (6.5.13)). For $\alpha = 0$ and ad-hoc allocation, we expect equivalent results to those observed for OCB. In this parameterization the objective function of MO only accounts for bid prices. For $\alpha = 1$ and ad-hoc allocation, we expect the same behavior as for CPR. When using the multi-objective allocation with re-allocation or online allocation we expect slightly inferior revenue gains for small α-values due to the approximation of bid prices.

First of all, Figure 9.9 shows that a gain in revenue and fulfillment of customer preferences is independent from the actual α-value. The results for MO compared with the benchmarking cases largely confirm our expectations. For ad-hoc allocation the revenue gains and improvements in fulfillment of customer preferences is similar to OCB for $\alpha = 0$ and similar to CPR for $\alpha = 1$. However, comparing the results for re-allocation and online allocation shows significantly less revenue and fulfillment of customer preferences as expected.

For MO with ad-hoc allocation, the results show a clear trade-off between the achieved revenue gain and improvement in fulfillment of preferences. An improvement of 40% in fulfillment of customer preferences is achievable by a decrease in revenue from 4% to 3.8%. The relationship, however, is nonlinear: a decrease in revenue from 3.8% to 3.6% only leads to less than 10% improvements in fulfillment of preferences. For all three allocation methods, the upper bound for the fulfillment of preferences is achievable with various parameterizations. However, as the revenue gain is different this indicates some inefficient parameterizations. The pareto-efficient result is obviously achievable with a revenue gain of more than 3.2% and an improvement of more than 60% in fulfillment of preferences compared to 4SP. In this particular case α was set to 0.8.

For MO with re-allocation and OMO, the curves for varied parameterizations of α look similar. The relationship between fulfillment of preferences and revenue gain is positive up to 50% improvement in fulfillment of customer preferences. Furthermore, the online allocation setups always outperform the re-allocation setups in terms of revenue. For parameterizations with more than 50% improvement in fulfillment of customer preferences, we observe a negative relationship: revenue of both curves is monotonously decreasing. Now, the re-allocation setups perform better. Similar to the ad-hoc allocation setup, there are parameterizations ($\alpha \in \{0.9, 1\}$) where the achieved results are inefficient. Surprisingly, for both setups the pareto-efficient results are achieved with a parameterization of $\alpha = 0.7$, revenue gain maximal results with $\alpha = 0.6$, and fulfillment of customer preferences maximal results with $\alpha = 0.8$.

Figure 9.9 shows a significant gap between $\alpha \in \{0, 0.1\}$ for re-allocation and online allocation. The fulfillment of customer preferences increases from less than 5% up to more than 25%. The revenue gain simultaneously increases only by about 1%. Furthermore, a clear range exists where MO with re-allocation and OMO perform better than MO with ad-hoc allocation. This range can be characterized in terms of fulfillment of preferences as the interval $[45\%, 55\%]$ and in terms of α-values as the interval $[0.6, 0.8]$. Revenue increases by about 4% compared to 4SP. For all other parameterizations, ad-hoc allocation outperforms the dynamic allocation methods.

Figure 9.10 depicts the change in the number of bookings when MO and OMO are compared to 4SP on the y-axis. The x-axis shows the α value. Black horizontal lines depict the results of CPR and of OCB with ad-hoc allocation. For both allocation methods, dashed horizontal lines indicate re-allocation.

The results look similar to the ones' shown in Figure 9.9. For ad-hoc allocation, Figure 9.10 shows a monotone decrease in bookings. For re-allocation and online allocation, a maximum exists for $\alpha = 0.6$. The change in the number of bookings is similar to OCB with ad-hoc allocation.

The results of this experiment follow our expectations for preference-based allocation methods: maximizing fulfillment of preferences contradicts revenue maximization. Overall, the numerical results show significant improvements for both revenue and fulfillment of customer preferences while slightly considering them in the objective. This indicates a high sensitivity of the allocation problem to the objective formulation.

Figure 9.9 and Figure 9.10 underlined the usefulness of MO to balance both objectives. However, the numerical results illustrate an unexpected nonlinear relationship between revenue, fulfillment of customer preferences, and values of α. Slightly considering preferences reduces revenue, but shows significant gains in fulfillment of customer

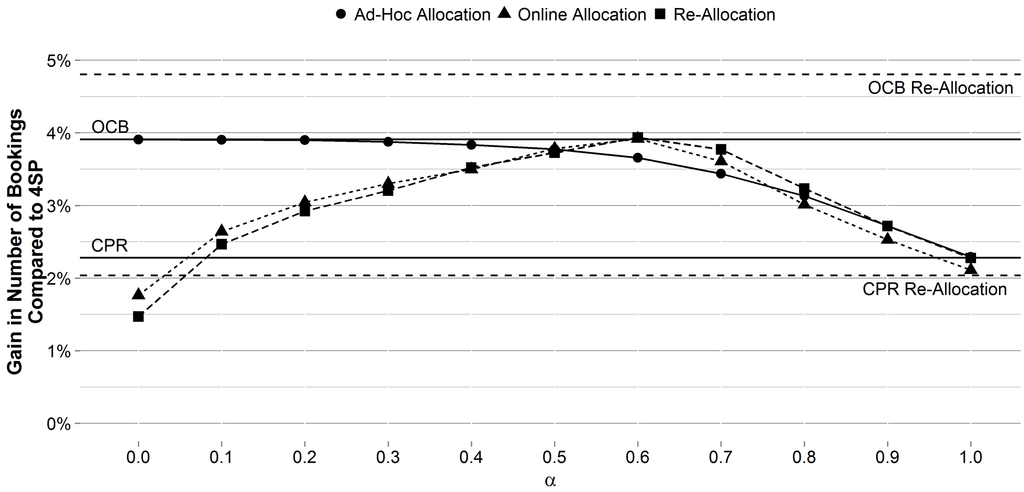


Figure 9.10: Change in number of bookings for MO and OMO given varied α-values

preferences. Starting from a case were only preferences are considered, the results indicate large improvements in revenue when α is slightly decreased.

The results in Figure 9.7 for the stochastic allocation methods support the sensitivity of the allocation problem to the objective formulation. These allocation methods are mainly stochastic, even they partially rely on RM indicators. A clear relationship between the extent of supporting revenue maximization and performance is not observable. However, when re-allocating flexible bookings throughout the sales period the results depict a clear improvement in revenue.

9.4 Revenue Sensitivity to Different Modeling Approaches

Section 6.5 highlighted two modeling approaches that extend the formulation of MO: the modification (6.5.13) of the objective to support more stable allocations over the sales period and the consideration of the initial availability in equation (6.5.7). This section quantifies the effects of both modifications. The demand and supply setup are the same as used in Section 9.3. Only two allocation setups are relevant for this experiment: MO with re-allocation and OMO.

Both modifications of MO aim to improve the allocation behavior of MO. We expect that MO with re-allocation and OMO show an increase in revenue when including them. The availability constraint includes additional information about the underlying booking control policy into the allocation step. This is expected to support revenue maximization of RM in general and the allocation of flexible bookings in particular. The modification of the objective improves the stability of allocations over the sales period and therefore is expected to support revenue maximization.

Note that the modified formulation of MO and OMO has been already used in the previous experiment and will be used for following experiments. The aim of this section is to particularly examine the effects of both modifications to show their suitability and necessity with regard to the performance of MO and OMO.

Revenue sensitivity to solution stability. First benchmarking experiments when developing MO showed unexpected results of re-allocation setups. Comparing the detailed results on a single-booking level for OCB, MO, and OMO for the same instances indicated a possible cause: MO and OMO show alternating allocations of a particular flexible booking over multiple re-allocations within a single sales period.

A possible counteraction has to increase the allocation stability of a particular flexible booking over the sales period. To this end, we modified the objective of MO by introducing the parameter $\rho \in [0, 0.9]$. This parameter supports the stability by increasing the valuation of the manifestation, the booking is currently allocated to, in the objective. For more details, we refer to Section 6.5.2.

To quantify the effects of ρ on solution stability and revenue we evaluate the two boundary parameterizations ($\rho \in \{0; 0.9\}$) and a mid-interval parameterization ($\rho = 0.5$). We simultaneously vary α between $\{0, 0.6, 0.9\}$.

Table 9.2 shows the relative revenue change for MO and OMO for the resulting six parameterizations. The revenue change is calculated relative to the setup with the same α and $\rho = 0$. Statistical significant results to a confidence level of 95% are marked with an asterisk.

Table 9.2: Relative change in revenue compared to $\rho = 0$ given an increased allocation stability

	MO			**OMO**		
ρ	$\alpha = 0$	$\alpha = 0.6$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.6$	$\alpha = 1$
0.5	0.25	0.39	*0.02	*0.05	*0.07	*0.01
0.9	0.49	*-0.02	*-0.06	0.16	*-0.02	*-0.06

For $\rho = 0.5$, a revenue gain can be achieved for re-allocation and online allocation for all values of α. For re-allocation and $\alpha = 0.0$, the modification of the objective increases the achieved revenue by 0.25% for $\rho = 0.5$ and 0.49% for $\rho = 0.9$. For online allocation, only the improvement of 0.16% for $\rho = 0.9$ and $\alpha = 0$ is not significant. In case customer preferences are weighted more in the objective ($\alpha \in \{0.6, 1\}$), prohibiting a switching behavior leads to significant revenue losses. Obviously, in this situation the modification changes the valuation of manifestations too much.

Figure 9.11 shows the mean change in the number of re-allocations per update point for different parameterizations of ρ and α. The update points are denoted in days before departure on the x-axis. In almost all cases the number of re-allocations decreases or remains the same. Only for $\alpha = 0$, the results indicate not significantly more re-allocations at the end of the sales period, especially for OMO.

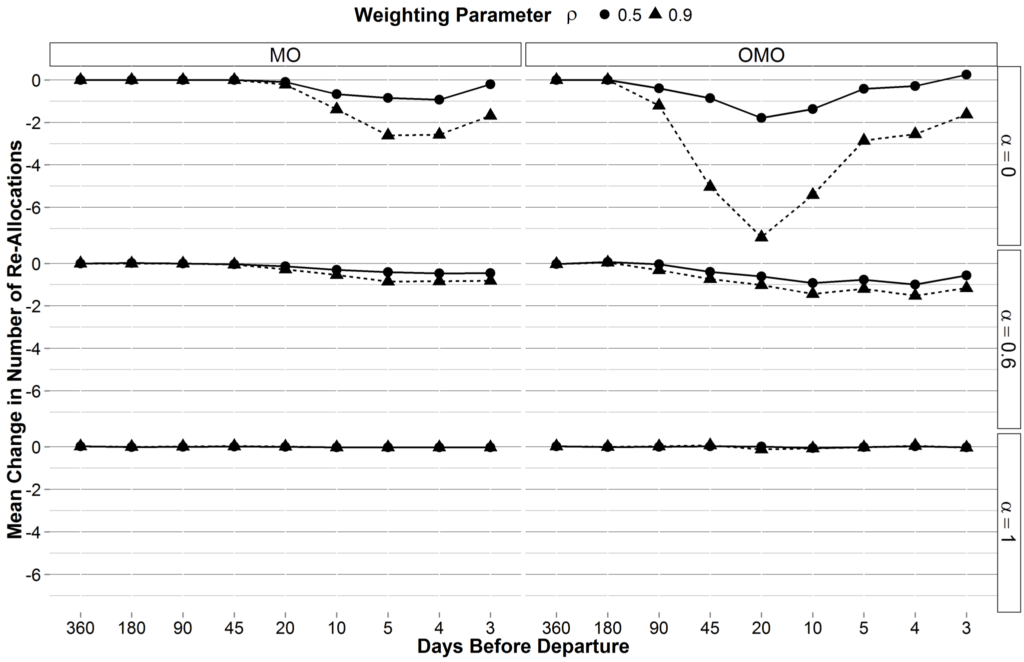


Figure 9.11: Mean difference of number of re-allocations when ρ is varied

These results indicate that preventing alternating allocations for MO and OMO supports revenue maximization by inducing more stability over the sales period. The results in Section 9.3 implicated that an increased number of re-allocation enhances RM performance. However, the results in Table 9.2 and Figure 9.11 show a different relationship for MO. Even a minor negative difference in the mean number of re-allocations corresponds to significant improvements in revenue.

A more detailed analysis of the mean number of re-allocations per manifestation and the standard deviation between manifestations is presented in Table 9.3. Again different parameterizations for α and ρ are compared for MO and OMO.

Table 9.3 indicates that an increasing α-value leads to a smaller number of re-allocations. This behavior results from a dominating effect of customer preferences in the objective function. As the customer preferences remain constant over the sales period, the only difference between two re-allocations results from changes in the inventory. For small α-values, however, the bid prices are prioritized in the objective formulation. Their valuation changes throughout the sales period based on the booking control policy, because the remaining capacity decreases or just because time goes on. This makes bid prices highly variable and significantly influences the optimality of a previous allocation. Therefore, the possibility that a previous allocation will be revised during re-allocation increases.

Table 9.3: Mean and standard deviation for the number of re-allocations per manifestation given different parameterizations of ρ and α

		MO			OMO		
ρ		$\alpha = 0$	$\alpha = 0.6$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.6$	$\alpha = 1$
0	Mean	90.85	67.73	60.22	177.76	122.56	103.49
	Std.-Dev.	27.01	15.84	8.86	59.84	40.39	26.62
0.5	Mean	88.09	65.94	60.20	172.89	118.30	103.50
	Std.-Dev.	26.97	15.25	9.14	59.67	41.03	26.90
0.9	Mean	82.38	64.35	60.25	151.66	115.18	103.43
	Std.-Dev.	26.65	14.92	9.16	58.30	40.50	26.92

No clear relationship exists between the parameterization of ρ and the indicators for re-allocations of flexible bookings. For MO and $\alpha = 0$, the mean number of re-allocations per manifestation and the corresponding standard deviation only slightly decreases. For the same parameterization of OMO, the mean is larger. Table 9.2, however, indicates no significance for the corresponding changes in revenue. Therefore, the results in Table 9.2 indicate no reliable impact on the revenue when using the modified objective. The results for the standard deviation in Table 9.3 state that modifying the objective does not impact the general re-allocation dynamic between the manifestations. The results only imply an improvement regarding the alternating allocation behavior.

For MO and OMO, the mean number of re-allocations shown in Table 9.3 differs only slightly between the setups with $\alpha = 0.6$ and $\alpha = 1$. Nevertheless, the corresponding standard deviation is significantly smaller when $\alpha = 1$. This confirms to our expectations about an improved performance of parameterizations with a larger standard deviation value.

Revenue sensitivity to using the availability constraint. MO and OMO can be used to simultaneously allocate multiple flexible bookings. Measuring and modeling the current availability situation correct at each time is difficult and may affect the revenue performance. To this end, we formulated an additional constraint modeling the initial availability when the booking originally happened in Section 6.5.2. Using the availability constraint (6.5.7) instead of the allocation constraint (6.5.8) restricts the set of feasible solutions of MO. The following experiment evaluates the effect of using the availability constraint for different values of α and ρ.

Table 9.4 shows the revenue gain induced by this constraint compared to a setup using the weaker allocation constraint (6.5.8). Statistically significant results to a confidence level of 95% are marked with an asterisk.

In case of online allocation and $\alpha = 0$ revenue increases by at least 3.14%. The results indicate no correlation between the choice of ρ and the achieved revenue gain. The

Table 9.4: Revenue gain for MO including the availability constraint compared to a setup only including the weaker allocation constraint

	MO			OMO		
ρ	$\alpha = 0$	$\alpha = 0.6$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.6$	$\alpha = 1$
0	*1.66	*0.25	*1.63	*3.14	*0.39	*1.52
0.5	*1.59	*0.25	*1.66	*3.28	*0.93	*1.57
0.9	*1.62	*0.25	*1.76	*3.22	*1.00	*1.70

smallest effects occur when $\alpha = 0.6$ for both online allocation and re-allocation. Here, the achievable improvements are in an interval of at least 0.25% and at most 1.00%.

Considering the initial availability situation as constraint in MO and OMO ensures feasibility of the solution and significantly increases the revenue. Especially setups with boundary parameterizations of $\alpha \in \{0, 1\}$ show substantial improvements.

9.5 Effects of Biased Customer Preferences

Lee et al. (2012) analyze customer behavior in case an airline offers the possibility to limit the manifestation set. Using empirical data, Lee et al. estimate the impact of various factors on the exclusion probability. For example, the distance between origin and destination, differences in the spoken language, length of stay, or cost of living are found to be significant variables.

The results from Lee et al. (2012) imply that modeling customer preferences using a uniform distribution may not be realistic. Therefore, the following experiment evaluates the impact if few manifestations are more popular than others. We use the published characteristics of Blind Booking tickets offered by Germanwings and the results from Lee et al. of limitation probabilities to set up a data-driven parameterization of customer preferences. However, the detailed analysis as shown in the appendix (Table A.2 and Table A.3) reveals that the manifestation set is more or less homogenous. No clear implications about biased preferences arise. Therefore, we define an analytical setup where customer preferences are biased for a subset of manifestations.

To parameterize customer preference values for a scenario with ten manifestations for the flexible product ($|M_f| = 10$), the following experiment relies on a slightly modified setup than presented in Chapter 8. In a first step, the preference values for each customer are drawn using a uniformly distributed random variable $X \sim U([0, 1])$. To model biased preferences, the preference values q_{bm}, $\forall m \in M_f, \forall b \in B$ are increased for a subset of manifestations $M_f^I \subseteq M_f$ and decreased for all remaining manifestations $m \in M_f^D = M_f \setminus M_f^I$ by a parameter $q^* \in [0, 1]$:

$$q'_{bm} = \begin{cases} \min\{1, q_{bm} + q^*\}, & m \in M_f^I, \\ \max\{0, q_{bm} - q^*\}, & m \in M_f^D, \end{cases} \quad \forall b \in B. \tag{9.5.1}$$

The modified preference values q'_{bm}, $\forall m \in M_f$ are now used for allocating flexible bookings. In all following experiments, we set $q^* = 0.3$ as this leads to mean preference values of 0.8 for M_f^I and 0.2 for M_f^D ensuring a significant concentration. Furthermore, we set $\left|M_f^I\right| = 2$ to leave enough possibilities for allocating the flexible bookings. We expect that the revenue will be negatively impacted by the preference concentration for all preference-based allocation methods. The performance of OCB should not be impacted. For limitations of the manifestation set, significant effects are expected only for setups where plenty manifestations are excluded.

Table 9.5 shows the revenue change in case customers can limit the manifestation set and OCB is used as allocation method. Statistical significant results to a confidence level of 95% are marked with an asterisk. The relative revenue change is calculated with respect to the corresponding setups with uniformly distributed preference values.

In general, the results in Table 9.5 indicate only minor changes in revenue. The revenue loss exceeds 1% only if customers exclude more than eight manifestations for ad-hoc allocation and more than seven manifestations for re-allocation.

Table 9.5: Revenue impact for limitations of M_f with biased customer preferences

	Number of Exclusions								
	1	2	3	4	5	6	7	8	9
Ad-Hoc All.	0.00	0.01	-0.01	*-0.04	*-0.07	*-0.18	*-0.64	*-2.01	*-1.59
Re-All.	*0.02	*0.02	*0.03	*0.02	*-0.07	*-0.34	*-1.57	*-4.20	*-3.22

Again, the same nonlinear relationship exists between number of exclusions and revenue as for uniformly distributed preferences. Up to two excluded manifestations for ad-hoc allocation or four manifestations for re-allocation, a concentration of customer preferences seems to be beneficial. However, the results of ad-hoc allocation are statistically not significant.

Surprising results are shown for both allocation setups when increasing the number of exclusions from eight to nine. Here, revenue increases in contrast to the overall relationship between exclusions and revenue. A possible explanation is that the bid prices significantly increase caused by the bias in preferences and the flexible product is no longer offered for the more popular manifestations. As each flexible booking has to be served, customers with a choice set including only one of the more popular manifestations are declined.

Figure 9.12 shows the revenue loss for preference-based allocation methods in case the customer preferences are biased. The revenue loss is calculated relative to the respective setup with uniformly distributed preference values. The vertical black lines indicate the confidence intervals to a confidence level of 95%. As the stochastic allocation methods perform within the revenue interval generated by OCB and CPR, we will not conduct particular simulations for them.

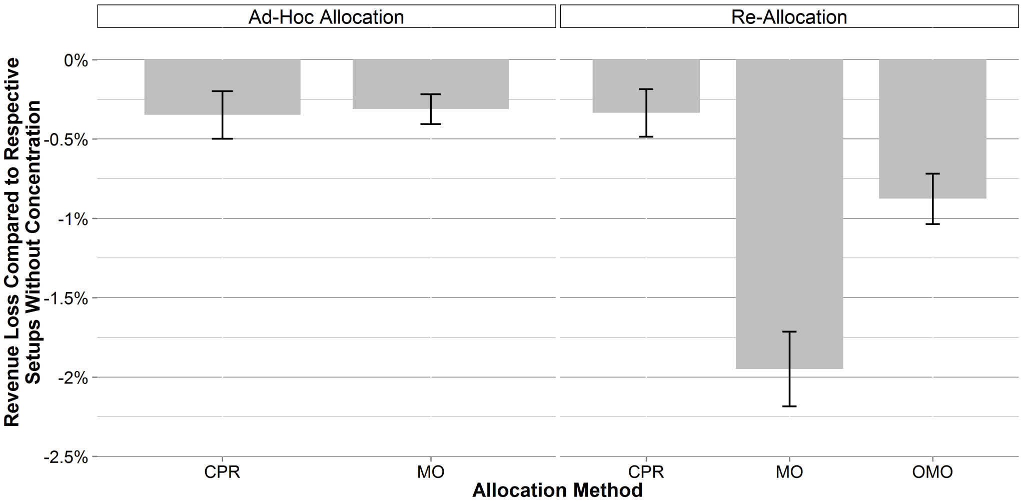


Figure 9.12: Relative revenue change when customer preferences are biased

For ad-hoc allocation on the left side, we observe a minor change in revenue for CPR and MO of about -0.3%. A revenue loss in the same magnitude is observable for CPR with re-allocation. For MO and OMO, however, re-allocating flexible bookings lead to an increasing loss in revenue. With slightly less than -2% for MO the revenue loss is more than twice compared to OMO. Here the revenue decreases only by less than -0.8%.

Even, all revenue changes are minor the confidence intervals indicate statistical significance. In general, the results meet our expectations by showing a significant revenue loss if customer preferences are biased for a subset of manifestations. However, in case of limitations, the experiment indicates minor positive effects for only a few exclusions and significant losses in revenue for more than eight exclusions.

9.6 Discussing Impacts of Customers' Preferential Choice Between Manifestations

The results in Section 9.1 illustrate benefits from selling flexible products. We expected fewer improvements of flexible products in setups with excess demand. RM methods will be more restrictive and lead to fewer flexible bookings. Regarding different demand distributions, we expected the benefits of flexible bookings to decrease if demand for specific products increase. The numerical results confirmed these expectations: revenue benefits from selling flexible products.

Flexible products provide most benefits when the demand-to-capacity ratio is about 100%. The results indicate further improvements if the size of the manifestation set increases or more customers request flexible products. Re-allocating flexible bookings throughout the sales period positively impacts the revenue.

The experiments in Section 9.2 show that minor limitations have no significant effects, revenue is only minimally reduced. Comparing the respective ad-hoc and re-allocation setups shows improvements when re-allocating flexible bookings.

Excluding a larger number of manifestations significantly reduces revenue, even for re-allocation setups. This behavior can be explained by the separation of optimizing the RM parameters and allocating flexible bookings. In case the maximum number of manifestations is excluded, just a single possibility for allocation is left over. This actually implies that no flexibility for the airline remains and flexible bookings are the same as specific bookings. Based on the re-allocation algorithm all flexible bookings are removed from the inventory before updating RM methods. This leads to a sub-optimal booking control policy regarding the current inventory situation. All subsequent decisions about accepting or rejecting booking requests are not revenue maximizing. The inferiority of re-allocation in this situation supports our expectations about the benefits of selling flexible products.

The numerical results of preference-based allocation methods in Section 9.3 demonstrate that improving fulfillment of customer preferences contradicts revenue maximization. MO was formulated in order to balance both objectives while allocating flexible bookings. The numerical results depict an unexpected nonlinear relationship between revenue and the fulfillment of customer preferences. Considering preferences only slightly reduces revenue compared to the base case while significantly increases the fulfillment of customer preferences.

The sensitivity analysis in Section 9.3 indicates great improvements in revenue when the weighting slightly changes to consider bid prices. When the customers' wish fulfillment has reached a certain level, the incremental revenue gain decreases. The numerical results indicate no further improvements in fulfillment of customer preferences, whereas revenue simultaneously changes. This implies inefficiency for some parameterizations with large α values.

The results of re-allocation setups support our expectations: re-allocation increases the airline's flexibility when using OCB. At each re-allocation additional information regarding the real demand situation are accessible and can be used to adjust the allocation; this supports revenue maximization. Also, for MO and OMO, we expected re-allocations to improve the performance. Instead, the numerical results of setups with $\alpha \leq 0.5$ show inferior revenue performance compared to ad-hoc allocation. For $\alpha \in \{0.6, 0.7, 0.8\}$ the results show only minor improvements due to re-allocations. There exists no linear relationship between revenue and fulfillment of preferences when α increases. For $\alpha > 0.8$, the results show a worse behavior in setups with re-allocation and inefficiency in terms of trade-off between revenue and fulfillment of preferences.

We modified MO in two different ways. The first modification accounts for the allocation stability of a flexible booking over time. Table 9.2 showed only minor improvements for various parameterizations of ρ. Additionally, in most setups the changes are not statistically significant. This may be caused by the stochastic included in the simulation tool. Note, that for MO and OMO and $\alpha = 0$ the results show nonsignificant improvements for $\rho = 0.9$. As all other cases perform worse than OCB, we use a parameterization of $\rho = 0.5$ in following experimental setups.

The second modification introduces an additional constraint considering the availability when the booking happened. The numerical results in Section 9.4 depict significance and superiority for most parameterizations. Especially the results of the boundary parameterizations of α are improved. This indicates that MO and OMO perform inferior when considering only one indicator.

Biased customer preferences for some manifestations diminish the revenue. The results meet our expectations and show that a worse composition of the manifestation set impacts the performance significantly. However, the experiments with uniformly distributed preference values are still valid. Subsequent experiments further rely on uniformly distributed preferences. While interpreting the results, we just have to remember that in case biased customer preferences exist revenue will be further reduced.

Regarding research question 1, we can conclude that implementing different customers' preferential choice models diminishes flexible products' benefits. However, independent from the actual choice model substantial benefits are preserved. Comparing different allocation methods showed a clear dominance of deterministic methods over stochastic methods and the applicability of multi-objective methods.

In order to answer research question 2, several experiments outlined the effects of different parameterizations and mostly showed nonlinear relationships between parameter variations and RM performance of the allocation methods.

For example, varying the size of the manifestation set only slightly impacts the revenue performance. As long as enough possibilities for allocation exist, flexible products remain beneficial. However, re-allocating flexible products during the sales period increases the benefits for the airline even if customers exclude manifestations. For the weighting parameter α, the results show significant impacts on the performance of MO and OMO. The relationship between the weighting of the bid price concerned objective and the achieved revenue, however, is not linear. Especially for setups with re-allocation, the coherence between weighting and revenue is nonlinear and not monotonously increasing.

Further experiments in Section 9.4 showed that approximations of the actual bid prices or inclusion of additional constraints can improve the solution quality. Especially for setups with re-allocation and online allocation the numerical results show a significant worse performance of MO compared to OCB and CPR.

10 Impacts of Strategic Customer Behavior

The current chapter addresses effects when flexible products are sold and customers act strategically. Section 10.1 introduces relevant extensions to the basic setups of computational experiments with strategic customers. Section 10.2 presents numerical results for varied demand setups. Experiments on stochastic allocation methods and deriving the reliability from the allocation probabilities are presented in Section 10.3. The effects of using the demand learning forecast are examined in Section 10.4. Section 10.5 concludes this chapter by discussing the results and answering the third research question:

Research Question 3. *What are the consequences for Revenue Management (RM) with flexible products if customer choice includes strategic choice between flexible and specific products? Does some degree of strategic behavior offset the benefits of flexible products?*

10.1 Parameterizing Strategic Customers

The general experimental setup is the same as used in the previous computational study. Referring to the framework presented in Section 8.2 and the choice model introduced in Chapter 5, two additional parameters for modeling strategic customers exist: the reliability of information and the share of strategic customers.

The strategic demand behavior in this computational study includes a dependency of customer choice regarding the current situation. Therefore, this study does not generally follow the assumption of independence of demand.

Reliability characterizes the trustworthiness of information about the allocation of a flexible booking. The parameter expresses the probability for this information for each customer and manifestation to be correct. The reliability is denoted by $\varphi_{bm} \in [0,1]\,,\ \forall m \in M_f, \forall b \in B$. We create a sensitivity analysis by varying this parameter in steps of 0.1. The share of strategic customers is denoted by $\sigma \in [10\%, 90\%]$ and defined relative to the overall demand level. This parameter will be varied in steps of 20%.

By testing each combination of these two parameters, we get 50 strategic setups used in three experiments:

Revenue sensitivity to demand level. The reliability φ_{bm} is the same for all customers and manifestations and fixed over the sales period. Only the Allocation Based on Bid Prices (OCB) method is used to clearly evaluate the impact of changes in reliability and variation of the demand setup.

Revenue sensitivity to dynamic reliability parameterizations. The second experiment is designed to evaluate effects if the reliability varies throughout the simulation. In this experiment, the stochastic allocation methods are used. These allocation methods rely on allocation probabilities that can be used for each booking and manifestation to parameterize the reliability: $\varphi_{bm} = w_{bm},\ \forall m \in M_f, \forall b \in B$. Furthermore, the share of strategic customers σ is varied between $[0\%, 100\%]$ in steps of 10%.

Revenue sensitivity to demand learning and concentration of demand. The third experiment examines the effect if historical data is used to forecast future demand. To this end, we use the demand learning forecast as described mathematically in Section 2.1. The implementation details are presented in Section 8.3. The learning rate of the exponential smoothing function is set to 0.2. The parameterization of φ_{bm} depends on the used allocation methods.

10.2 Revenue Sensitivity to Demand Level

This experiment evaluates 50 strategic setups differing in reliability parameterization and share of strategic customers. To evaluate the impact of different demand setups, three demand levels $l \in \{80\%, 100\%, 120\%\}$ are used, each with a demand distribution of $d = 80\%$. This leads to an overall number of 150 combinations of demand setups and strategic setups.

We expect that with increasing reliability and number of strategic customers the revenue decreases as more customers book the flexible product. The share of customers that actually book strategically is expected to rise with an increasing demand level. However, the additional demand will lead to a more restrictive control policy avoiding strategic customers by not offering the flexible product or diminishing the reliability if the stochastic allocation methods are used.

Figures 10.1–10.3 depict the revenue change of OCB for the different demand levels and the different experimental setups. OCB without strategic customers represents the reference case for calculating the relative revenue change. Each figure contains two horizontal lines indicating benchmarks using the Preference-Based Allocation (CPR) method without strategic customers and the Setup with Four Specific Products (4SP).

CPR without strategic customers is only a weak benchmark for the other setups. Using a preference-based allocation method obviously increases the reliability of allocations.

If strategic customers exist, CPR as allocation method is the worst case. Customers are aware of the allocation method and they know that they will get their desired manifestation when booking the flexible product. Therefore, all strategic customers will book the flexible product instead of a specific one. However, CPR is used in this study to represent the worst case for selling flexible products with myopic customers.

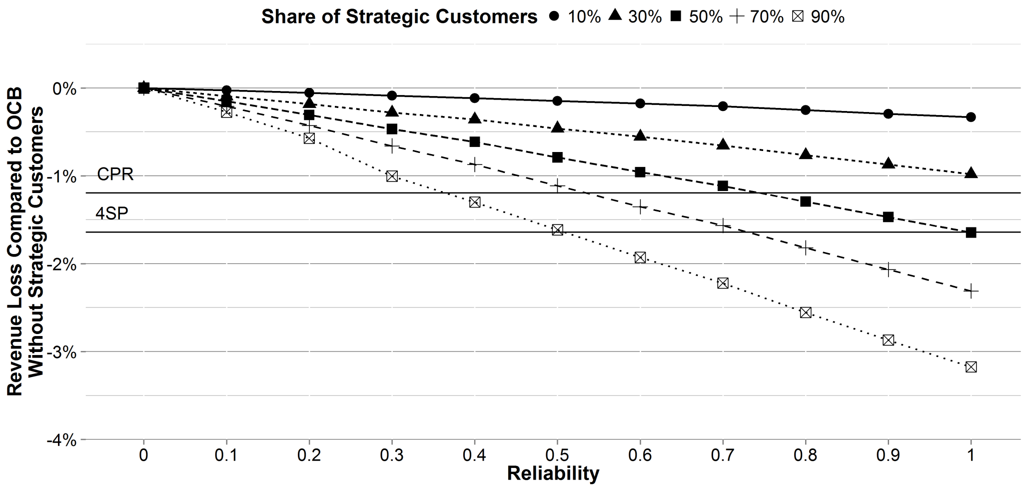


Figure 10.1: Revenue loss of OCB due to strategic customers given a demand level of $l = 80\%$

Figure 10.1 depicts the revenue change for the setup with less demand than capacity ($l = 80\%$) on the y-axis and the reliability on the x-axis. As expected, revenue decreases when the share of strategic customers increases. Comparing the results for varied reliabilities and a fixed share of strategic customers shows a negative linear impact on the revenue.

In case of 90% strategic customers, a reliability of 0.4 suffices to offset the advantages of OCB compared to CPR. Obviously, enough customers have a minimum reliability $\vartheta_b < \varphi_{bm} = 0.4,\ \forall m \in M_f$ that they actually act strategically. A reliability of 0.5 offsets the complete monetary benefits of selling flexible products. For a smaller share of strategic customers, the reliability of these breakpoints increases. In case of only 50% strategic demand, the reliability has to be larger than 0.8 to decrease revenue below the level of CPR. No reliability parameterization exists to decrease revenue below the level of 4SP. Setups with less than 50% strategic demand always protect the benefits of flexible products.

Figure 10.2 shows the numerical results for the setup where the demand level is equal to capacity ($l = 100\%$). The qualitative revenue change is similar compared to the setup with $l = 80\%$. However, the loss in revenue for each setup increases. Given 90% strategic customers and full reliability, the revenue loss increases to more than 3.5%. For $l = 80\%$, it is only slightly more than 3%. Further, the limits, where the advantages of selling flexible products are offset, differ. For 50% strategic customers, the reliability has to be greater than 0.8 to decrease revenue below the level of CPR.

For 70% and 90% strategic customers, the reliability breakpoints with CPR decrease to 0.7 and 0.5 respectively.

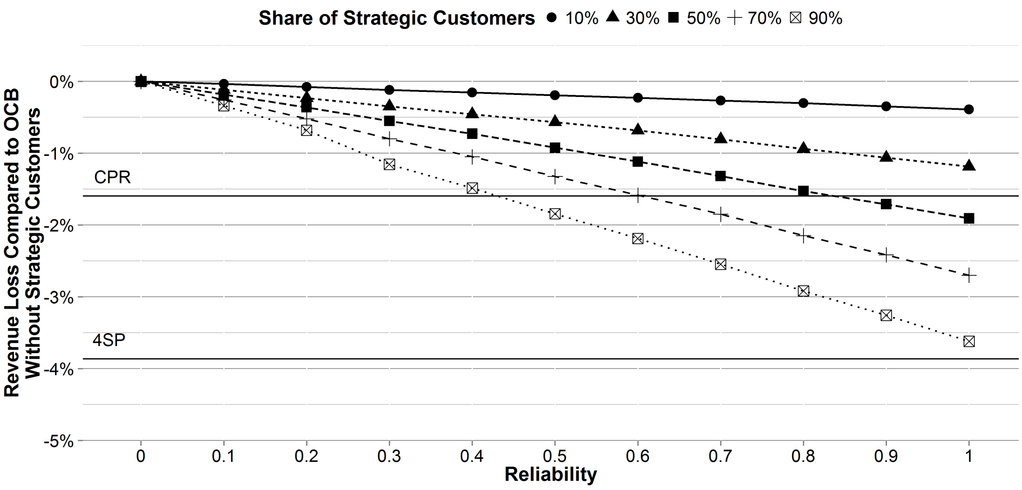


Figure 10.2: Revenue loss of OCB due to strategic customers given a demand level of $l = 100\%$

For $l = 100\%$, the revenue gap between CPR and 4SP is five times larger than for $l = 80\%$. No demand setup exists where the revenue decreases below the level of 4SP. If demand is equal to capacity, there is enough demand to always preserve benefits from selling flexible products even if strategic customers exist.

The revenue loss in case demand exceeds capacity ($l = 120\%$) is illustrated in Figure 10.3. Again, revenue loss increases compared to setups with a demand-to-capacity ratio of 1 and 0.8. For 90% strategic customers and a reliability of 1, the revenue loss amounts to more than 8.5%. However, the relationship between reliability and revenue as well as between share of strategic customers and revenue remains linear.

Figure 10.3 depicts a different situation regarding the reference cases CPR and 4SP. The revenue gap when using CPR instead of OCB is less than 0.5%. However, the revenue gap between CPR and 4SP increases to more than 2.0%. This significantly decreases the breakpoints' reliability values. In case of 10% strategic demand, a reliability of less than 0.4 is enough to decrease revenue below the level of CPR. The benefits of selling flexible products for these parameterizations still exist, yet the reliability increases up to 1. If at least 50% customers act strategically, breakpoints exist for both, CPR and 4SP. A share of 90% strategic customers shifts the reliability breakpoint to 0.4.

10.3 Revenue Sensitivity to Dynamic Reliability Parameterizations

Instead of statically parameterizing the reliability to perform sensitivity analyses, the allocation probabilities of the stochastic allocation methods can be used: $\varphi_{bm} =$

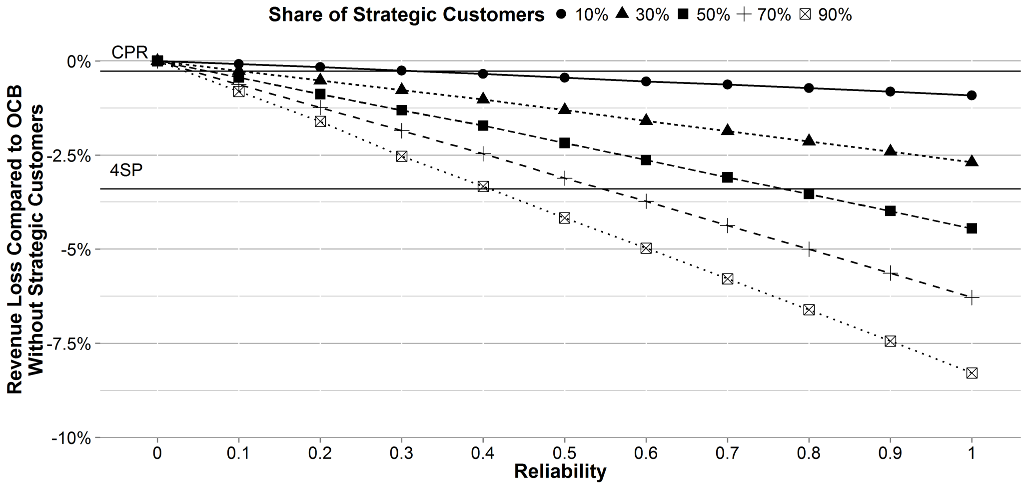


Figure 10.3: Revenue loss of OCB due to strategic customers given a demand level of $l = 120\%$

w_{bm}, $\forall m \in M_f, \forall b \in B$. This leads to different reliabilities for different manifestations and customers. Furthermore, the reliability varies over the sales period and between different allocation methods. This parameterization increases the dependencies between RM system and strategic customers. Now, the customers' decision to book strategically completely depends on the current inventory situation. This is expected to induce indirect revenue loss caused by misfitting control policies due to strategic bookings.

The following experiment compares five stochastic allocation setups to a setup without strategic customers and OCB as allocation method. We include all stochastic allocation methods in this experiment. Previous experiments outlined similar results for different demand levels. Therefore, we restrict to the demand setup with a demand-to-capacity ratio of 1 ($l = 100\%$). Ten different shares of strategic customers are evaluated to perform a sensitivity analysis. Altogether, this experiment evaluates 50 different setups.

Figure 10.4 depicts the revenue loss compared to OCB without strategic customers on the y-axis. The x-axis shows the share of strategic customers. In contrast to the results with fixed reliability, the results depict a nonlinear relationship between revenue and share of strategic customers. In general, the curve reminds of a sigmoid curve for an increasing share of strategic customers. For small shares of strategic customers, the curve seems to be almost linear.

The curves run parallel to each other for all allocation methods and shares of strategic customers. The best performing allocation method is the Stochastic Allocation Based on Estimations (sEST). The second best method is the Stochastic Allocation Based on Bid Prices (sBID), followed by the Stochastic Allocation Based on Bookings (sCUR). The performance of the Stochastic Allocation Based on Preferences (sPRE) is very similar to the one of one with uniform weights (sUNI); both depict the worst case.

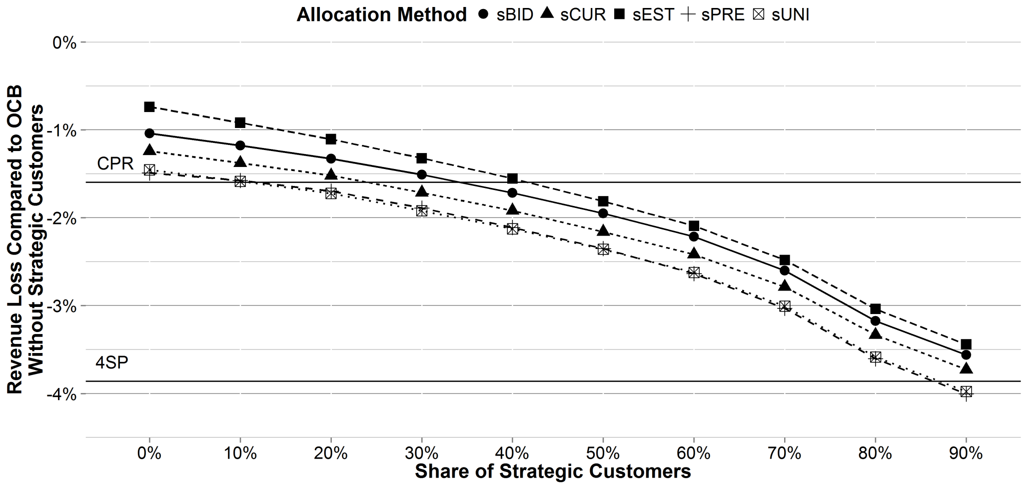


Figure 10.4: Revenue loss due to strategic customers for dynamic reliability parameterizations

Revenue for these two allocation methods decreases even below the level of 4SP for a share of 90% strategic customers.

These results fully correspond to our expectations regarding the behavior of the stochastic allocation methods. The nonlinearity implies the existence of some amount of indirect revenue loss caused by misfitting control policies due to strategic customers.

10.4 Revenue Sensitivity to Demand Learning and Concentration of Demand

Section 8.3 introduced two forecasting methods: the omniscient forecast and demand learning. So far, computational experiments rely on the omniscient forecast for all runs. The following experiments include another setup using demand learning as forecasting method: at the end of each run the realized bookings are used to update the forecast. As previous experiments outlined similar impacts for different demand levels, this experiment restricts to the demand setup with a demand level of 100% and 50% strategic customers. Different allocation methods will be used dependent on the respective experiment.

We expect that using the demand learning forecast induces a spiral down that creates more strategic bookings and a significant revenue loss. The omniscient forecast excludes strategic customer behavior as it is not identifiable when initializing. The realized bookings, used by the demand learning forecast to update the estimations, include strategic bookings. As strategic customers substitute specific for flexible products, the airline will observe more bookings for the flexible product whenever both products are available. Therefore, the forecast for the flexible product increases and consequentially fewer seats will be reserved for specific products. Thus, in the next simulation run, both products are more often available at the same time and the number of strategic customers will further increase. Additionally, the effect is strengthened as the increased availability raises the chance for customers to be allocated to their preferred manifestation. The resulting situation encourages additional customers to book strategically.

Varying the capacity for one flight. In order to analyze if strategic bookings induce a spiral down, three different supply setups are compared differing in the capacity for one flight. We vary the capacity between $\{25, 50, 75\}$ seats for flight 1 while the other flights' capacities remain at 50 seats.

We expect that a capacity reduction to 25 seats for one flight will lead to a more restrictive control policy for this flight. This implies that OCB allocates fewer flexible bookings to this flight. The number of strategic bookings for this flight should decrease, too. For the remaining flights, we expect the number of flexible and strategic bookings to be slightly increased. A capacity of 75 seats for one flight leads to decreasing bid prices as capacity exceeds demand. This implies more strategic bookings on this flight encouraging a spiral down over simulation runs. Overall, the number of strategic bookings and flexible bookings are expected to increase.

For setups where capacity is larger, we expect all allocation methods to profit from this modification. More flexible bookings are allocated to this flight as there is enough capacity. This reserves capacity for more valuable bookings on the remaining flights. The stochastic allocation methods relying on RM related indicators will benefit more from the increased capacity for one manifestation. The allocation probabilities for the four flights without changes in capacity, calculated by these methods, will drop significantly. This counteracts strategic customer behavior for flexible products.

Table 10.1 shows the relative change in revenue, flexible bookings, and strategic bookings for OCB and each of the capacity setups. To analyze the dynamic between runs the indicators are calculated for five intervals segmenting the simulation runs. The reference case is run interval $[1, 10]$. The capacity setups are denoted with the number of seats on the influenced flight. Statistical significant results to a confidence level of 95% are marked with an asterisk.

The numerical results show no significant revenue loss due to a spiral down for all setups. Over all runs, the revenue is more or less constant. The observable minor changes result from the differences in demand between runs. For the capacity setups with 50 and 75 seats, the revenue slightly increases but not significantly over the runs.

For both capacity setups with 25 and 50 seats, the flexible bookings slightly decrease until run 80 and increase for the following runs with a slight stagnation at the end. The strategic bookings show the largest change; they significantly increase by at last more than 50%. For the capacity setup with increased capacity for one flight, the

Table 10.1: Relative change in revenue and bookings (BKD) for OCB given demand learning and different capacity setups

		Runs				
Cap. Setup	Indicator	11-30	61-80	111-130	161-180	191-200
	Revenue	-0.15	-2.08	-1.23	-0.14	-0.03
25	Flexible BKD	-1.95	-6.54	2.29	3.31	0.51
	Strategic BKD	*40.26	*29.22	*37.01	*40.26	*45.45
	Revenue	1.04	-1.04	0.14	0.57	0.62
50	Flexible BKD	-0.85	-2.82	2.39	6.66	1.37
	Strategic BKD	*43.75	*42.19	*28.91	*51.56	*42.19
	Revenue	2.97	-0.15	1.88	2.90	1.71
75	Flexible BKD	-3.52	-5.03	2.43	5.53	-2.01
	Strategic BKD	9.47	13.33	*22.67	12.00	1.33

strategic bookings only increase by more than 20%. These results imply a spiral down effect for all capacity setups. However, for the setup with increased capacity, the effect only arises to a minor degree.

As the flights are different now, it could be useful to separately analyze the indicators for each flight. Table 10.2 shows the number of flexible and strategic bookings per flight for the two extreme capacity setups and two run intervals. Further, the average relative capacity utilization on each flight is shown. The results for flight 1, whose capacity is changed, are formatted bold.

For the setup with decreased capacity, comparing both run intervals shows an increasing number of strategic bookings for all flights. The number of flexible bookings increases for flight 1 (the one with modified capacity) and for flight 5. The increase for flight 5 is caused by the variation in demand between runs. For the capacity utilization, the results indicate a slightly improved distribution of bookings over the flights. However, this effect is not significant. These results indicate a spiral down especially leading to an increased number of strategic bookings. The reactivity due to selling flexible products allows the airline to absorb the monetary drawbacks as Table 10.1 shows.

For the setup with increased capacity, the overall number of flexible bookings is larger than for the other capacity setup. More than 50% of all flexible bookings are allocated to the flight with increased capacity. Contrary to our expectations, the increase in flexible bookings over runs happens on almost all flights. The overall number of strategic bookings in the last 50 runs is smaller compared to the setup with decreased capacity. These results indicate weakened effects due to the spiral down when capacity is larger for one flight. The reactivity of the RM system caused by the flexible products seems to counteract the spiral down effect.

An increased capacity for one flight, however, not only induces strategic bookings, but

Table 10.2: Booking (BKD) indicators per flight given demand learning and different capacity setups

Indicator	Cap. Flight 1	Runs	Flight 1	2	3	4	5
Flexible BKD	25	1-10	**1.50**	13.60	16.40	15.20	13.40
		151-200	**2.26**	13.39	15.34	13.94	16.14
	75	1-10	**33.10**	6.14	9.00	7.67	7.11
		151-200	**33.46**	8.68	9.07	7.41	8.34
Strategic BKD	25	1-10	**1.50**	2.75	2.00	2.50	2.67
		151-200	**1.94**	3.22	3.05	2.66	3.69
	75	1-10	**3.70**	1.67	2.29	1.00	3.00
		151-200	**4.37**	1.92	2.04	1.88	2.30
Cap. Util. in %	25	1-10	**99.20**	97.80	96.40	97.60	99.60
		151-200	**98.00**	96.52	97.32	96.28	95.88
	75	1-10	**91.47**	86.60	89.00	87.60	92.20
		151-200	**90.53**	88.64	89.68	89.32	88.16

also leads to additional allocations of flexible bookings to this flight. This effect has to be considered as it is very strong from the first run on. The number of flexible bookings for flight 1 increases only from 33.10 bookings for the first runs to 33.46 bookings for the last runs. For most of the other flights, the number of flexible bookings increases more. In general, we can conclude that the horizontal shift of flexible bookings between the flights helps to mitigate the spiral down as it leads to rapidly increasing bid prices for the resource with more remaining capacity. This partially counteracts the buy-down from strategic customers.

Revenue sensitivity to changes in demand variation. Another effect that influences the results, is the underlying demand calibration. As most of the requests for specific products occur late in the sales period, this implies that the flexible product will be less available when strategic customers arrive. Now, strategic customers will not book strategically as the airline only offers them specific products. This fact explains the substantial increase in flexible bookings and the minor increase in strategic bookings. Furthermore, we have to consider that the variation in demand between the simulation runs could diminish the visibility of spiral down.

In order to clearly examine the effects of demand variation, another experiment evaluates setups with less variation: instead of a standard deviation $\sigma = 0.1 \cdot \mu$ the modified setups use $\sigma = 0.06 \cdot \mu$ and $\sigma = 0$. We expect that diminishing the stochastic between runs improves the visibility of spiral down. Less variation in demand for the different flights restricts the reactivity of the airline due to flexible products. Based on the conclusion that a horizontal shift of flexible bookings allows counteracting the spiral down, the results of this modified experiment are expected to clearly show a loss in revenue and an increase in strategic bookings.

Table 10.3 shows the relative change in revenue and the number of flexible and strategic bookings for simulation runs 5–200 compared to runs 1–4. In addition to OCB, this experiment uses CPR as it will encourage all customers for specific products to act strategically.

The results of OCB and CPR show that the number of strategic bookings significantly increases for both allocation methods when $\sigma = 0.06 \cdot \mu$. Also, revenue significantly decreases for the setups with less stochastic. The number of flexible bookings does not change significantly for both allocation methods. These results clearly show the existence of a spiral down effect. Furthermore, they support our explanation that an increased variability in demand helps the airline to counteract spiral down.

A further reduction in stochastic leads to deterministic demand: $\sigma = 0$. Now, the same set of customers is used for each simulation run. Deterministic demand is expected to further improve the visibility of spiral down over runs. As expected, for the number of flexible and strategic bookings, the numerical results in Table 10.3 for both allocation methods substantially increase. However, the revenue loss induced by the spiral down is reduced to -0.55% for OCB and -1.46% for CPR. This is caused by the improved quality of the forecast reference. As each simulation run consists of the same customers, the reference resulting from omniscient initialization provides a perfect forecast of decision behavior for this customer set. The only aspect neglected by the forecast reference is the strategic customer behavior that causes the observed loss in revenue. Therefore, this magnitude may be a rough guess for the share of loss attributed to spiral down. For the sake of completeness, analog results to Table 10.1 for both setups with less stochastic can be found in the appendix (Table A.4 and Table A.5).

Experiments including varied setups with changes in demand level, share of strategic customers, and smoothing rate for the demand learning forecast showed comparable results. These variants were chosen in order to strengthen the spiral down to achieve more significant results. However, these results indicate that the robustness of RM with flexible products is not restricted to those particular experimental setups illustrated in Table 10.1, Table 10.2, and Table 10.3.

Table 10.3: Relative change of revenue and bookings (BKD) for OCB and CPR given demand learning and different magnitudes of variation in demand

	$\sigma = 0.1 \cdot \mu$		$\sigma = 0.06 \cdot \mu$		$\sigma = 0$	
Indicator	OCB	CPR	OCB	CPR	OCB	CPR
Flexible BKD	-1.74	12.35	-3.36	15.93	*12.43	*19.41
Strategic BKD	*12.58	*43.72	*28.14	*70.54	*85.37	*43.81
Revenue	*-2.34	*-4.12	*-4.88	*-5.42	*-0.55	*-1.46

Evaluating the stochastic allocation methods. The last experiment evaluates the effect of demand learning on the stochastic allocation methods. Compared to OCB, these methods show at least a decrease in revenue by about -1%. A possible explanation is that the allocation probabilities always reflect the relative situation between manifestations. In case of less capacity for one flight, the bid prices will not significantly differ and therefore lead to similar allocation probabilities. As expected, the gap gets smaller for an increasing capacity. In case capacity is reduced to 25 seats for one flight, the revenue gap for sBID and sEST is more than 2%. For setups with increased capacity, the gap decreases to less than -1%. A significantly smaller bid price for the flight with more capacity explains this behavior: the corresponding allocation probabilities increase and induce additional allocations.

The results of the stochastic allocation methods that neglect revenue maximization in their allocation probabilities support this finding. They are not able to benefit from more than 20% additional capacity and the revenue gap increases up to more than -1.5% compared to OCB. The complete comparison between OCB and the stochastic allocation methods can be found in the appendix (Figure A.2).

10.5 Implications of Strategic Customers for Flexible Products

There exist quantities of strategic customers offsetting the advantages of flexible products. However, these breakpoints largely depend on the reliability and the demand setup. When the reliability is fixed over the sales period and for each customer, the relationship between the quantity of strategic customers, the reliability, and the revenue loss is linear. This linearity can be explained by the revenue difference from cannibalizing specific demand by offering the flexible product. Apparently, the results indicate no effects caused by sub-optimal RM control policies. Such effects were expected to be caused by the indirect consequences of strategic customers booking flexible products instead of specific products.

Comparing the results of different demand levels illustrates a decreasing impact of strategic customers when demand increases. At first glance, this supports the existence of only direct loss from strategic customers: a not-optimal control policy has more impact when demand exceeds capacity. However, it also indicates that RM methods are not able to efficiently counteract the strategic behavior in situations with sparse demand.

For the next experiment, the reliability is dynamically parameterized while using the stochastic allocation methods. A curve similar to a sigmoid curve for different shares of strategic customers not only indicates direct loss through cannibalization (compare Figure 10.4), but also effects caused by an affected control policy. However, these indirect impacts seem to be nonsignificant.

By varying the capacity in the last experiment, we encourage (increased capacity) or discourage (decreased capacity) customers to act strategically. This is supposed to strengthen the spiral down occurring when demand learning is used as forecast. However, at a first glance the revenue shows no impact caused by spiral down.

The results in Table 10.1 and 10.2 show a spiral down effect for specific bookings. However, the possibility to allocate flexible bookings enables the airline to slightly counteract this drawback. Furthermore, in the setup where capacity for one flight is larger many flexible bookings are allocated to this flight. This changes the allocation behavior for the remaining sales period and obviously discourages customers to book strategically. As expected, the spiral down becomes more visible in terms of revenue and number of strategic and flexible bookings when the variation in demand between runs is reduced. However, for the setup with deterministic demand, revenue loss is reduced caused by an improved forecast quality.

The performance of the stochastic allocation methods largely depends on the direction of the capacity change. In case of decreased capacity, these allocation methods perform significantly worse than OCB. First improvements have become apparent for setups with increased capacity. These improvements are caused by an increased uncertainty for strategic customers created by the stochastic allocation methods.

On the one hand, the results without demand learning only show direct revenue losses caused by strategic customers. No indirect loss results from RM policies flawed by strategic customers. On the other hand, as the share of strategic customers and the reliability of the additional information increases, RM methods are not able to efficiently counteract this behavior. Only the stochastic allocation methods show some improvements compared to OCB. The experiment with demand learning as forecast shows that a high variability in demand for the specific products defining the manifestations helps to reduce the drawbacks from spiral down. This leads to the conclusion, that airlines can work against strategic customers by ensuring a certain amount of uncertainty in their allocation and a good design of flexible products.

With regard to research question 3, we can conclude that strategic customers have a significant impact on RM performance. Although the results of the computational study indicate only minor indirect effects that are caused by the changed booking decision of customers, airlines should be aware of this behavior. The experiments with demand learning show one efficient way to counteract the impacts of strategic customers: flexibility. As long as the airline has enough flexibility for allocation, a revenue loss caused by strategic customers can be prevented. Furthermore, using stochastic allocation methods helps to decrease the reliability of the additional information in situations where demand exceeds capacity.

11 Impacts of Flawed Input Parameters

The models presented in Chapter 2 and the discussion in Chapter 7 outlined a strong dependency of Revenue Management (RM) methods on uncertain input parameters. Several contributions underlined this fact, e.g., Pölt (1998); Weatherford and Belobaba (2002); Weatherford and Pölt (2002). This computational study investigates the relationship between the correctness of three input parameters and the performance of RM methods for flexible products to answer the following research questions:

Research Question 4. *What is the effect of flawed input parameters on different combinations of customers' preferential choice model, revealing mechanism, and allocation method?*

Research Question 5. *What happens if the airline implements a faulty customers' preferential choice model? How does this affect revealing mechanisms and allocation methods?*

Section 11.1 characterizes relevant input parameters for this computational study and introduces the implementation to distort them. Afterwards, Section 11.2 presents numerical results for flawed forecast values. Impacts of flawed bid prices are examined in Section 11.3 and numerical results for flawed customer preferences are presented in Section 11.4. Finally, results and implications are discussed in Section 11.5.

11.1 Modeling Flawed Input Parameters

RM with flexible products has three performance relevant input parameters: demand forecasts, bid prices, and revealed customer preferences. This section introduces the implementation to distort these parameters in the Airline Revenue Management Simulation (ARMS).

Let $P = 1, \dots, \overline{p}$ denote the index set of all parameters for one type. To model a flawed input parameter, ARMS distorts the correct parameters using a stochastic error term $i_p \in \mathbb{R},\ \forall p \in P$.

The basic idea is to skew each parameter independently to create uncorrelated distortions. Independence between the error terms i_p is achieved by drawing separate realizations of a random variable for each parameter. The minimal levels of this independence are simulation runs to ensure comparability and reliability of the distortion process. Additionally, particular dimensions for each parameter have to be considered, e.g., forecast values are defined per itinerary, product, and update point.

To skew a parameter, we multiply it with the respective error term i_p. A concrete parameterization of i_p, $\forall p \in P$ is calculated by drawing a realization of a uniformly distributed random variable. Before specifying the detailed implementation of i_p, we highlight parameter type related details first.

Demand forecasts. Flawed forecast values impact the RM performance significantly as they are input values into the optimization (cf. Pölt, 1998; Weatherford & Pölt, 2002). However, demand forecasts naturally include a significant level of uncertainty as they predict future developments by using only historical data that is additionally influenced by RM analysts. Flawed demand forecasts impact the accuracy of RM methods throughout the whole process. Calculations made in following process steps can strengthen or weaken the effects of forecast inaccuracy.

Flexible products are affected by flawed forecast values in two ways. First, control policies may be skewed impacting the availability of the flexible product. This affects upcoming requests leading to a sub-optimal revenue result. Second, some allocation methods use RM parameters to valuate manifestations. Using skewed values for allocation affects the optimality of the determination step.

Demand forecasts exist on a very detailed level. For each itinerary, product, and update point, the expected number of customer requests is estimated. Therefore, distorting them has to be done independently for each combination of these three dimensions and each simulation run. Let the index set P include indices for all relevant combinations. Let d_p, $\forall p \in P$ denote the true demand and $\tilde{d}_p$, $\forall p \in P$ the distorted demand forecast calculated as

$$\tilde{d}_p = d_p \cdot i_p, \quad \forall p \in P. \tag{11.1.1}$$

Independent demand implies that underestimating demand affects revenue less than overestimating. As capacity is restricted, overestimating creates a situation where capacity is smaller than the estimated demand. RM optimization assumes an increased rivalry for seats and the control policy reserves more capacity for expensive products. In case the real demand is less than estimated, too many customers will be rejected and the revenue is affected significantly. On the opposite, underestimating demand leads to an increased availability for cheaper products. Caused by the independence of demand, buy-down to cheaper products is neglected. However, the booking control policy will accept too much customers for cheap products and this will diminish the revenue. For the unbiased case, we expect that revenue will be diminished. Caused by the stochastic of the error terms, we are not able to formulate more detailed expectations.

Bid prices. Flawed bid prices immediately affect the availability decision in the inventory as they are one of two main input parameters. Therefore, the magnitude of inaccuracy directly impacts RM performance. However, this computational study is restricted to measuring the impact of flawed bid prices on the allocation of flexible products.

Bid prices exist for each manifestation and amount of remaining capacity and are used as part of the booking control policy and in allocation methods. This computational study restricts to distort bid prices used in allocation methods. Following the definition in Chapter 6, the bid price per resource is denoted by $\pi(r, \tau, c_r^\tau)$. Analog to the specification of flawed forecast values, we define the index set P to include indices for all relevant combinations of resources $r \in R$, time slices $\tau \in T$, and remaining capacities c_r^τ. We denote the distorted bid price by $\tilde{\pi}_p$ and calculate it as

$$\tilde{\pi}_p = \pi_p \cdot i_p, \quad \forall p \in P. \tag{11.1.2}$$

For bid prices, we can formulate clear expectations about the impacts of biased inaccuracies. Overestimated bid prices lead to a control policy rejecting more customer requests. Analogously, underestimated bid prices lead to a larger number of accepted customer requests, especially for cheaper products.

Customer preferences. If the established revealing mechanism follows the customers' preferential choice model (see Chapter 4), preference values exist for each manifestation and customer. Different from the previously discussed parameters, flawed customer preferences do not directly impact the calculations of RM methods as they are only used for allocating flexible bookings. However, there will be an indirect effect caused by a difference in allocations changing the capacity situation and therefore indirectly impacting the RM performance.

Customer preferences q_{bm} exist for each booking and manifestation of the booked flexible product. Therefore, the index set P has to include indices for all possible combinations. Let the distorted value for customer preferences be $\tilde{q}_p$ that is calculated as

$$\tilde{q}_p = q_p \cdot i_p, \quad \forall p \in P. \tag{11.1.3}$$

We expect that only allocation methods using customer preferences are affected. For Preference-Based Allocation (CPR), we can state that increased or decreased preference values for all customers will not affect the general allocation behavior. Modeling the objectives of the Multi-Objective Allocation (MO) method as a weighted sum leads to the assumption that flawed preference values distort the balance between both objectives. Larger preference values lead to a more customer-oriented allocation, whereas smaller values will weigh the bid price in the objective more.

Parameterizing the error term. The random variable determining the error term $i_p, \forall p \in P$ can be independently parameterized from the parameter type. As i_p is multiplied with a parameter and all considered parameter types take only positive values, we state $i_p \geq 0, \forall p \in P$. Based on subject matter experts, it is unlikely that parameters will be distorted by more than 100%. Following this, we set $i_p \leq 2, \forall p \in P$.

Let $\mathcal{I} \subseteq [0, 2]$ denote the error interval and $i_p \in \mathcal{I}, \forall p \in P$. The actual definition of $\mathcal{I}$ depends on the error type and the modeled variation of the error term. To this end, we introduce two parameters $\varepsilon^-, \varepsilon^+ \in [0,1]$ denoting the magnitude and symmetry of $\mathcal{I}$. Both are specified per experiment and are used for all realizations in this experiment. The error interval can be written as

$$\mathcal{I} = \left[1 - \varepsilon^-, 1 + \varepsilon^+\right] \subseteq [0,2], \quad \varepsilon^-, \varepsilon^+ \in [0,1]. \tag{11.1.4}$$

Note that ε^- and ε^+ are independent. For deriving implications later, the trend and characteristic to distort a parameter is highly relevant. To this end, we differentiate three types of error terms, however, formula (11.1.4) can be used to define intervals for all types.

- **Unbiased** distorted parameters have an expected value over all error terms of 0. A practical interpretation is that on average all parameters are not systematic larger or smaller. For an unbiased error interval, obviously holds that $\varepsilon^- = \varepsilon^+$ that defines a symmetric interval $\mathcal{I} = [1 - \varepsilon^-, 1 + \varepsilon^+]$. To create different setups $\varepsilon^-, \varepsilon^+$ are varied in the interval $[0,1]$.
- **Positively biased** parameters are on average larger than the original parameter. Here the expected value over all error terms is larger than 0. To examine overestimation ε^- is set to 0 that defines the interval $\mathcal{I} = [1, 1 + \varepsilon^+]$ while simultaneously varying $\varepsilon^+ \in [0,1]$.
- **Negatively biased** parameters are only distorted into smaller values. The expected value over all error terms is smaller than 0. To model underestimation of parameters, we set $\varepsilon^+ = 0$ that defines the interval $\mathcal{I} = [1 - \varepsilon^-, 1]$. Simultaneously, ε^- is varied in $[0,1]$.

A particular error value i_p, $\forall p \in P$ is now drawn as realization of the uniformly distributed random variable $X \sim U(\mathcal{I})$. For each error term, we define various experiments by varying ε^- and/ or ε^+ in steps of 0.1 depending on the error type. Considering all possible cases leads to 33 different setups for each input parameter.

To clearly denote a setup, we calculate the mean relative error $\bar{\varepsilon} \in \mathbb{R}$. The calculation differs for the different error types. For a biased experiment, $\bar{\varepsilon}$ is calculated as

$$\bar{\varepsilon} = \frac{\varepsilon^+ - \varepsilon^-}{2} \cdot 100. \tag{11.1.5}$$

For the unbiased cases, we calculate the mean relative error term as

$$\bar{\varepsilon} = \frac{\varepsilon^-}{2} \cdot 100 = \frac{\varepsilon^+}{2} \cdot 100. \tag{11.1.6}$$

Equality (11.1.6) holds because of $0 \leq \varepsilon^- = \varepsilon^+$ for experiments with unbiased error terms. We use the mean relative error $\bar{\varepsilon}$ as x-axis label in all following figures.

11.2 Revenue Sensitivity to Flawed Demand Forecasts

The aim of this chapter is to quantify the impacts of flawed demand forecasts on the performance of RM with flexible products. Parameterizing error terms for demand forecasts follows the implementation described in the previous section.

We conduct three experiments differing in the error type: unbiased, positively biased, and negatively biased. For each experiment, 13 different allocation setups are compared for 11 different error parameterizations. Overall, 429 setups are evaluated to investigate the impacts of flawed demand forecasts.

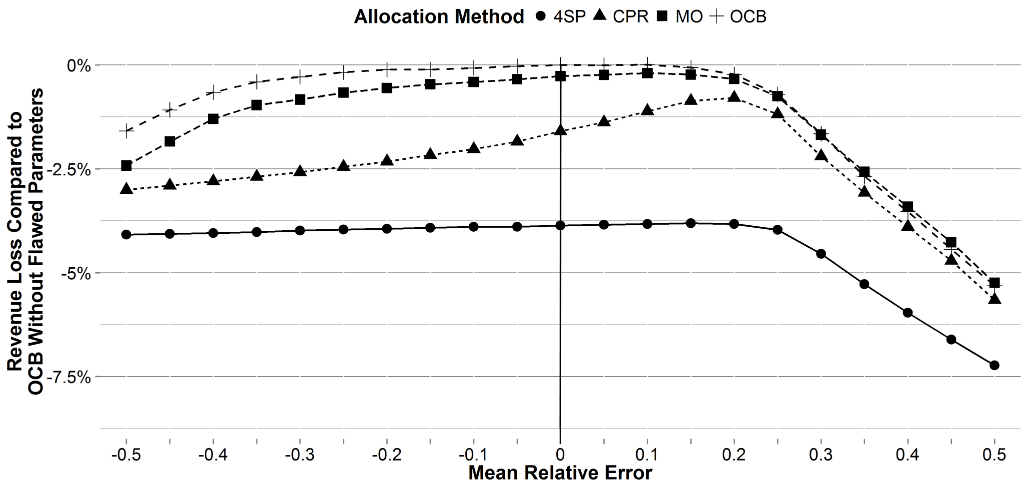
Figures 11.1–11.3 show the revenue change compared to the setup using Allocation Based on Bid Prices (OCB) as allocation method with correct forecast values. First, we evaluate the effects for ad-hoc allocation (six allocation setups), turning later to re-allocation and online allocation (seven allocation setups). Note that we only evaluate the setup with $\alpha = 0.6$ for MO and the Online Multi-Objective Allocation (OMO). The Setup with Four Specific Products (4SP) relying on the same flawed forecasts is included in each figure for benchmarking purposes.

Evaluating the effects of flawed demand forecasts for ad-hoc allocation. Figure 11.1 depicts the revenue change for ad-hoc allocation setups when demand forecasts are distorted by biased error terms. Figure 11.1a shows the effects for the deterministic allocation methods, whereas Figure 11.1b shows the effects for three stochastic allocation methods. The revenue change is calculated relative to OCB without flawed forecasts.

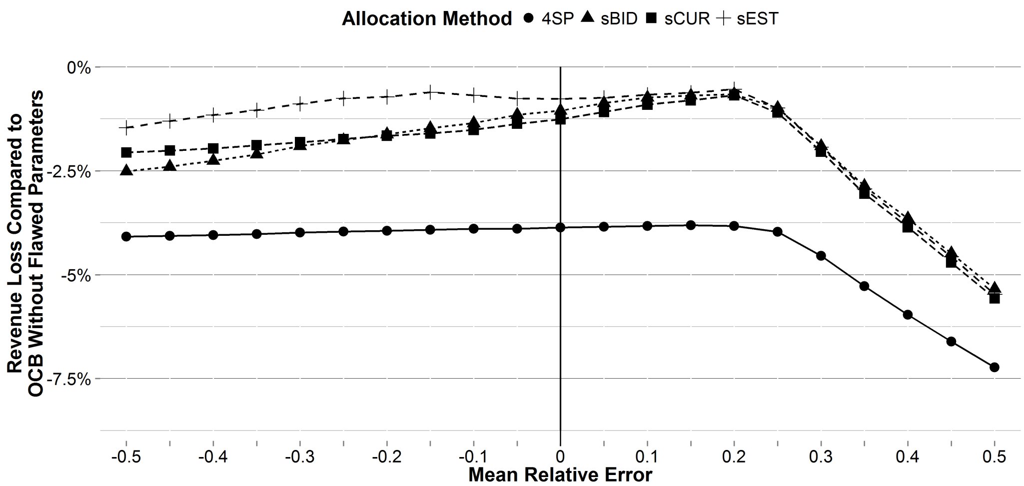
Figure 11.1a shows only minor changes for small positive error terms for OCB and MO. However, for CPR revenue increases by more than 1%. For a mean relative error larger than 0.2, revenue of CPR drops significantly. Although MO considers customer preferences, the revenue loss is less than for CPR: it is more similar to OCB. With increasing error terms, the revenue gap between CPR, MO, and OCB is getting smaller. The maximal revenue loss is more than 6% for a mean relative error of +0.5.

For increasing negatively biased error terms, revenue monotonously decreases for CPR and MO. For large mean errors, the coherence between error term and revenue loss differs for all allocation methods. MO shows a significant drop in revenue, whereas CPR decreases continuously. The overall behavior of MO is similar to that of OCB, however, OCB dominates the other allocation methods. The maximal loss in revenue for -0.5 mean relative error is less than 3%.

For positive mean error terms, the stochastic allocation methods in Figure 11.1b behave similar to the deterministic allocation methods. For small distortions, they show a robust behavior; instead of a decreased revenue they show slight improvements up to +0.2 mean relative error. The Stochastic Allocation Based on Bid Prices (sBID) and Stochastic Allocation Based on Estimations (sEST) even show superior results compared to the Stochastic Allocation Based on Bookings (sCUR). For large mean



(a) Deterministic allocation methods



(b) Stochastic allocation methods

Figure 11.1: Revenue change in ad-hoc allocation setups for biased errors terms applied to forecasts

relative errors, however, they perform similar and show more than −5% revenue change for a mean relative error of 0.5.

For negative mean error terms, the stochastic allocation methods perform better than deterministic methods. The overall loss in revenue is less than 2.5%. sCUR and sBID perform worse than sEST. Yet, sEST shows slight improvements for small distortions and overall a more robust behavior. The maximal loss is less than 2% revenue compared to the setup with correct forecasts.

The results for unbiased error terms can be found in the appendix (Figure A.3). The overall behavior is similar to the behavior for positively biased error terms. Particular setups, however, are slightly more robust against distortions in demand forecasts. This is expressed in smaller changes in revenue.

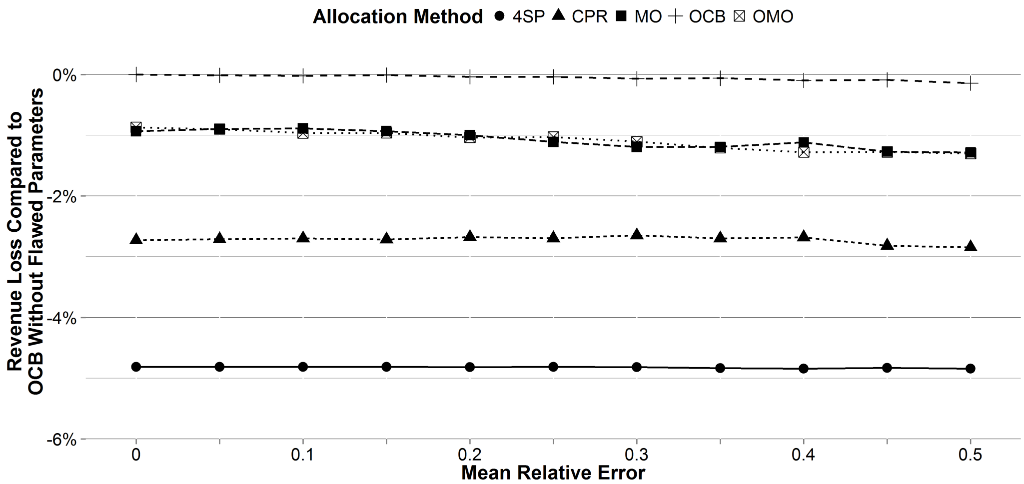
Evaluating the effect of flawed forecasts for re-allocation. The second part of this section evaluates the effects of flawed demand forecasts for re-allocation. The share of uncertainty in forecasts decreases over the sales period. More and more observations exist and replace the respective uncertain estimations. Figure 11.2 illustrates the results for the same setups as for ad-hoc allocation. Again, the revenue is compared to OCB without flawed forecasts.

The results for deterministic allocation methods in Figure 11.2a show a stable behavior. CPR represents the worst case with more than 2.5% revenue loss compared to OCB. MO and OMO show an overall robust behavior with a slight worsening for mean relative errors larger than 0.25. OCB is not significantly affected.

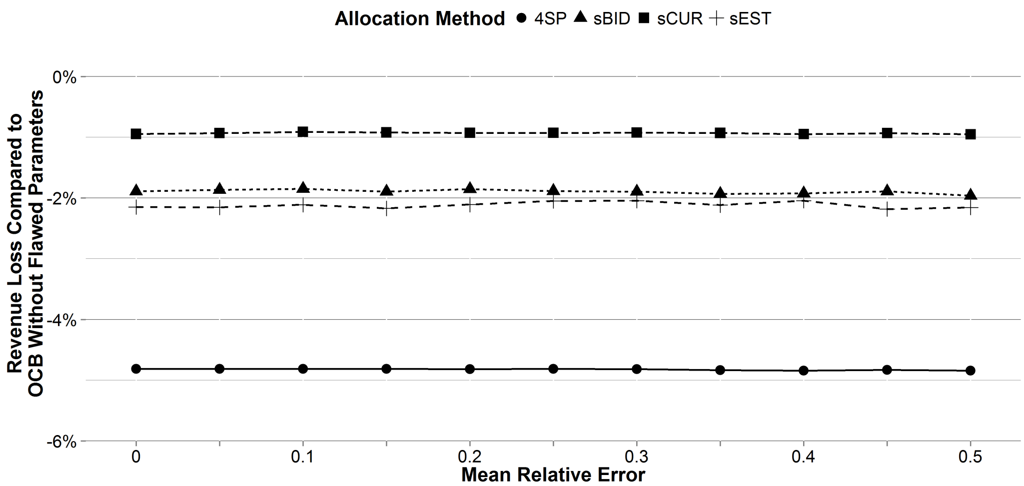
Figure 11.2b depicts similar results for the stochastic allocation methods, indicating the same robustness against flawed forecast values. The loss magnitude is less than 1% for sCUR and more than 2% for sEST. Re-allocating flexible bookings completely compensates the effects of flawed forecasts when symmetric error terms are used. For all stochastic allocation methods, Figure 11.2b does not show a significant loss in revenue due to flawed forecasts.

The results show that RM performance in a setup with re-allocation is not vulnerable to unbiased flawed forecasts. No matter which allocation method is used, a substantial loss in revenue does not occur.

Figure 11.3a illustrates the effects when biased error terms are applied to demand forecasts. Only for OCB the results show a robust characteristic, shown by losing less than 0.5% revenue for −0.5 mean relative error. MO and OMO perform similar to OCB, with slight more revenue loss for OMO. Both show a decreasing revenue when the mean relative forecast error decreases. For positive error terms, all allocation methods show a similar behavior. A mean relative error up to a point of 0.2 does not significantly affect revenue. For larger distortions, the revenue decreases linearly, showing more than −6% change in revenue for OCB, MO, and OMO. For CPR and all stochastic allocation methods, revenue decreases by more than 7% compared to OCB without flawed forecasts.

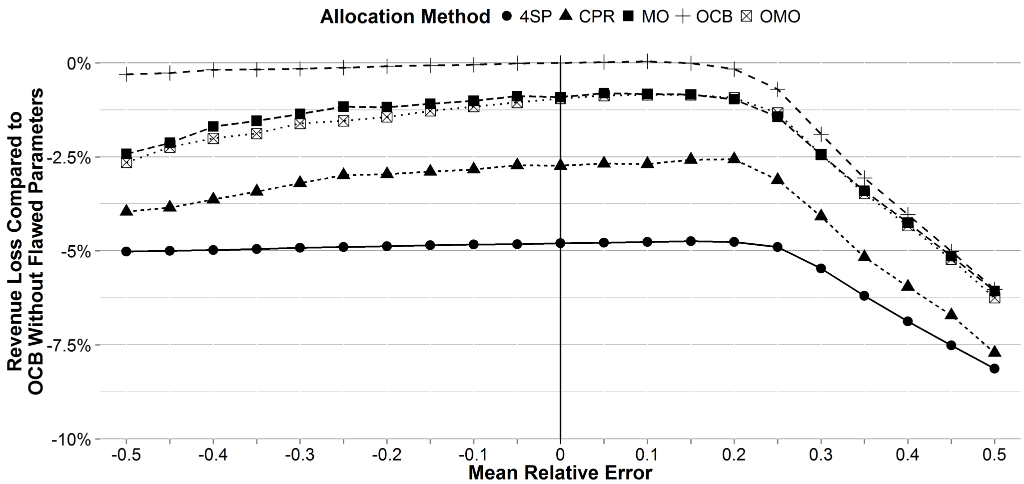


(a) Deterministic allocation methods

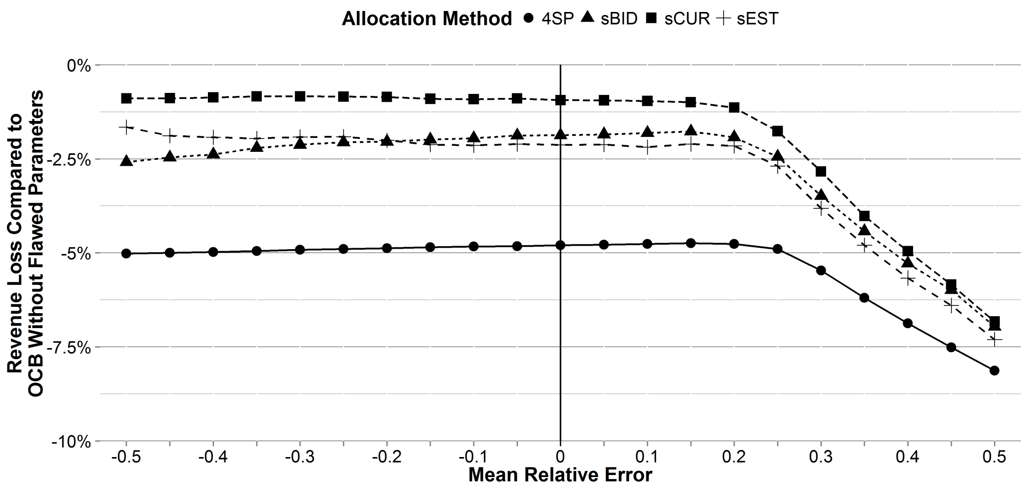


(b) Stochastic allocation methods

Figure 11.2: Revenue change in re-allocation setups for unbiased errors terms applied to forecasts



(a) Deterministic allocation methods



(b) Stochastic allocation methods

Figure 11.3: Revenue change in re-allocation setups for biased errors terms applied to forecasts

The results of biased flawed forecasts on the stochastic allocation methods are shown in Figure 11.3b. For negative mean errors, all of them exhibit a robust performance and do not show additional loss in revenue compared to the setup without error terms. The maximal revenue change for each allocation method is about ±0.5% compared to the respective setup with correct demand forecasts. sEST and sBID show a contrary behavior. sBID loses revenue with increasing error values, whereas sEST can slightly improve the achieved revenue compared to the setup with true demand forecasts. For a mean relative error of −0.2, they perform similar with a diverging tendency for larger error values.

11.3 Revenue Sensitivity to Flawed Bid Prices

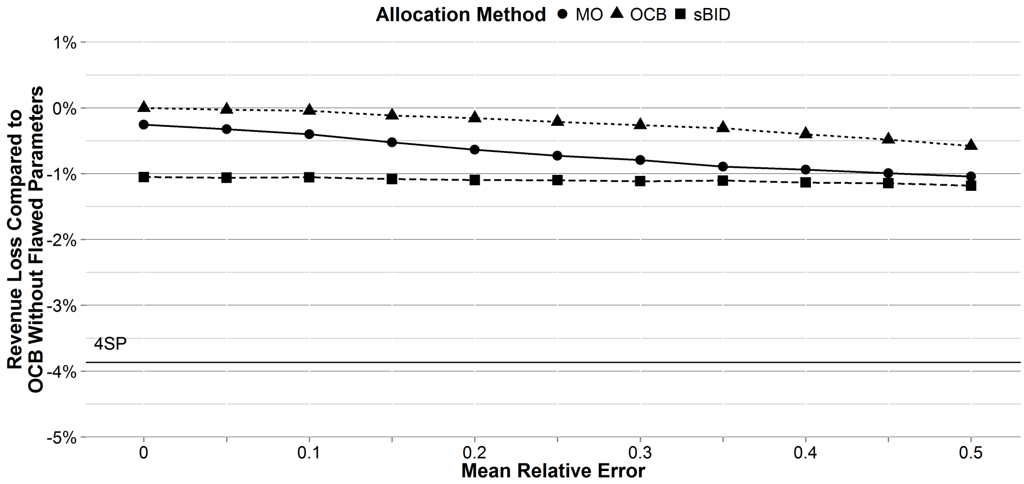
In contrast to uncertainty in demand forecasts, flawed bid prices as modeled in this thesis do not directly affect the optimality of the booking control policies. We distort bid prices only for allocation purposes, the acceptance decision of customer requests still depends on correct bid prices. However, an indirect impact on revenue performance exists. Allocating flexible bookings leads to a decrease in free capacity and therefore, by definition, affects the valuation of the next available seat: the bid price used for the acceptance decision.

This section evaluates seven allocation setups and 33 different error setups. Three ad-hoc allocation setups (OCB, CPR, MO) and four re-allocation setups (OCB, CPR, MO, OMO). This leads to 231 combinations of allocation and error setup. The revenue change is calculated relative to OCB without flawed bid prices. Again, the 4SP without flawed bid prices is included as a benchmark case.

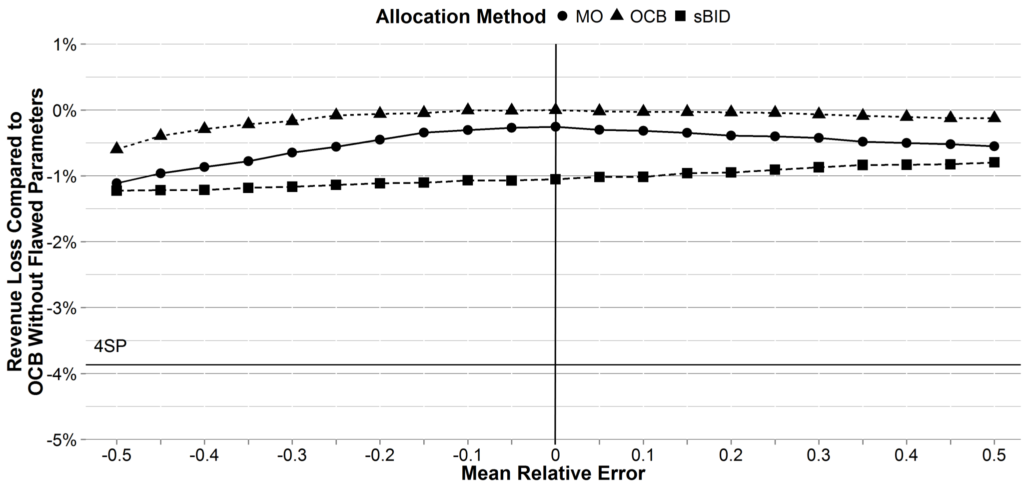
Evaluating the effects of flawed bid prices for ad-hoc allocation. Figure 11.4 illustrates the results when flexible bookings are immediately allocated for different types of error terms used to distort the bid prices.

Figure 11.4a shows the results for unbiased error terms. OCB shows a slightly negative relationship between revenue and magnitude of the error terms. For errors larger than 0.15, the revenue significantly decreases, leading to an overall loss of more than 0.5% for 0.5 mean relative error. sBID examines a completely different behavior, it does not show any impact on the revenue when bid prices are flawed. For large error values, sBID seems to represent a lower bound for MO. Compared to the setup without flawed parameters, the revenue loss for MO is larger than for OCB and sBID. For 0.5 symmetric mean error, the revenue decreases by more than 0.7%. Errors larger than 0.35 induce a slight convergence against sBID.

Figure 11.4b depicts the revenue change when applying biased error terms. OCB examines a robustness for positive mean errors. Only for large error terms a slight nonsignificant loss in revenue occurs compared to the undistorted case. Up to 0.25 mean relative error, the graph illustrates no significant change in revenue. For MO,



(a) Unbiased error terms



(b) Biased error terms

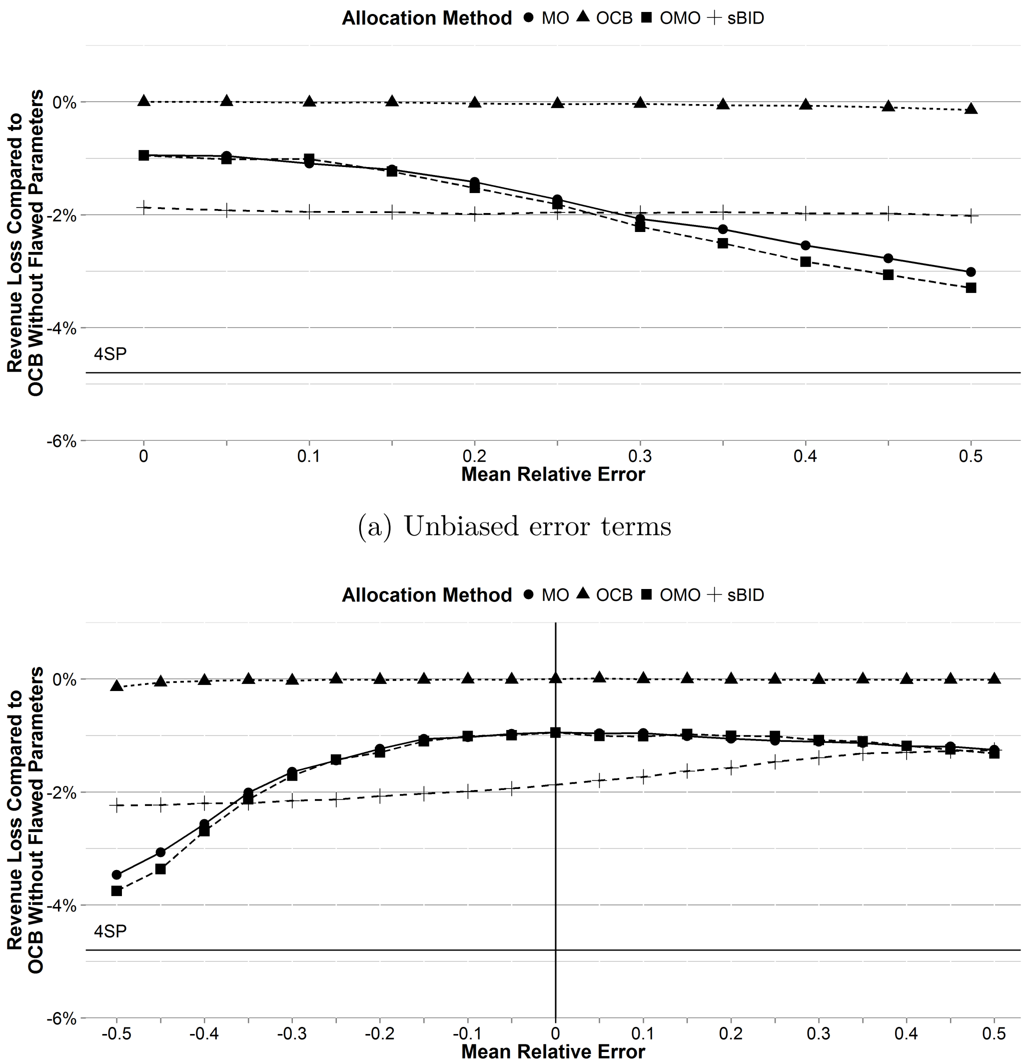
Figure 11.4: Revenue change in ad-hoc allocation setups given flawed bid prices

the stability increases compared to the setup with symmetric error terms in Figure 11.4a. Overall, a minor loss in revenue of less than 0.25% is observable. Also, the results of sBID are superior compared to symmetric error terms. Instead of only showing robustness against flawed bid prices, the results indicate a positive trend. The regression in Figure 11.4b exhibits less than 0.25% revenue gain when applying a mean relative error of 0.5.

By applying negatively biased error terms on bid prices, the effects are similar to the effects for positive error terms. However, for all allocation methods the absolute loss compared to the respective setup without flawed parameter is twice as much. For sBID and MO, the maximal loss is more than 1%.

Evaluating the effect of flawed bid prices for re-allocation. Figure 11.5 shows the results when flexible bookings are re-allocated given flawed bid prices for biased and unbiased error terms.

The effects for symmetric error terms for OCB, sBID, and MO are illustrated in Figure 11.5a. The revenue for OCB and sBID is completely stable over all applied error terms. Only for maximal distortion, a slight revenue gap compared to the undistorted case exists. For MO and OMO, the results show a similar inferior behavior as depicted in Figure 11.4. Revenue decreases by more than 3% for a mean relative error of 0.5. The relationship is reminiscent of a sigmoid curve having a point-of-inflection at 0.25 mean relative error. For larger error terms MO performs slightly superior to OMO by losing 2% revenue compared to the undistorted case.



(a) Unbiased error terms

(b) Biased error terms

Figure 11.5: Revenue change in re-allocation setups given flawed bid prices

Figure 11.5b shows the results for distorting bid prices with biased error terms. The results are similar to the results of ad-hoc allocation (Figure 11.4a), but the effects are much better visible. OCB depicts a completely stable behavior without any significant loss in revenue when error terms increase. Only for -0.5 mean relative error a small loss of -0.2% revenue occurs compared to the case without distorted bid prices.

sBID shows the same positive trend in revenue as for the ad-hoc allocation setup. The overall gain in revenue compared to the undistorted case, however, increases up to 0.8% for a mean relative error term of 0.5. Again, MO and OMO seem to converge against sBID for large positive error terms. Both allocation methods show a high sensitivity to negative error terms. The revenue loss is more than 2% compared to the base case without distortions. For a positive mean error of 0.5, the revenue loss is less than 0.25% compared to the setup with correct bid prices.

We can conclude that MO and OMO are sensitive to flawed bid prices, especially when flexible bookings are re-allocated. Here, the same flawed bid prices are used again and again to allocate flexible bookings and lead to worse allocation decisions. OCB does not show a significant revenue dependence on symmetric and asymmetric error terms for re-allocation. Re-allocating flexible bookings even improves the robustness of OCB against flawed bid prices. Obviously, using only flawed values to allocate flexible bookings is not as bad as using preferences and bid prices.

11.4 Revenue Sensitivity to Misconceived Customer Preferences

Some allocation methods presented in Chapter 6 use the revealed customer preferences, indicating customers' choice between manifestations. Customer preferences can be misconceived caused by manifold reasons. Customers may for example manipulate their preferences during the revealing step to affect the outcome. Also, a misunderstanding of the provided revealing mechanism can distort the parameters. The experiments in this section evaluate the impacts of misconceived preferences for 231 different setups resulting from the combination of 7 different allocation methods and 33 error setups.

The following figures show the relative revenue change compared to OCB on the y-axis. As OCB allocates flexible bookings independent from customer preferences it is unaffected and proposes an adequate base case. The x-axis displays the mean relative error. For another benchmark, all figures contain two horizontal black lines depicting the revenue for 4SP and CPR without flawed preferences.

For all experiments in this section, we expect an overall robust revenue performance when distorting customer preference values. As the allocation methods neglect completely the airlines objective to maximize revenue, it does not matter if the allocation method uses misconceived customer preferences. The actual indicator that is affected is the sum of allocated preferences. This is the objective pursued by using CPR or Stochastic Allocation Based on Preferences (sPRE). However, the effect of misconceived preferences on this indicator is directly observable: the mean relative error. The aim of the following experiments is to evaluate if an impact on revenue performance exists, when the customer preferences are misconceived. Overall, we expect that distorting customer preferences does not significantly affect revenue performance.

Regarding MO, we can formulate more sophisticated expectations. Maximizing customer preference values represents only one half of MO's objective. Let us consider the objective to be modeled as weighted sum. In case the flawed preference values are negatively biased, the performance of MO in terms of revenue may benefit. Decreased preference values considered in the objective of MO imply an increased weighting of bid prices: the actual value of α will be smaller than initially defined. On the opposite, for a positive bias we expect a slightly inferior performance when using MO. This expectation is valid for ad-hoc allocation as well as for re-allocation. We do not expect any benefits or drawbacks when flexible products re re-allocated over the sales period. For the setups with unbiased error terms, we expect a significant impact on the revenue performance. Here, particular bookings are weighted more or less within the objective giving them an advantage. This violates the assumption underlying MO to treat all bookings equally.

Evaluating the effects of faulty customer preferences for ad-hoc allocation. Figure 11.6 shows the impacts of symmetric error terms applied on customer preferences when using ad-hoc allocation methods. The results of CPR and sPRE indicate a strong robustness against uncertain valuations. Both methods rely only on revealed preference values to allocate flexible bookings. However, distorted input parameters do not significantly affected revenue. CPR shows a slight improvement compared to the undistorted setup, whereas sPRE depicts a nonsignificant loss. For MO, however, the wrong valuations impact the revenue significantly. Compared to the undistorted setup, this allocation method loses more than 0.5% revenue.

For biased error terms, the results show a similar characteristic behavior. For simplicity's sake, we only discuss the findings and the corresponding figures can be found in the appendix (Figure A.4). In case of overestimating customer preferences, CPR and sPRE show the same robustness without any dependence on the magnitude of the applied error terms. The results show only minor differences for large error terms. Instead of inferior behavior as seen for symmetric error terms, sPRE performs similar to or slightly above the benchmark of the undistorted CPR case. In case of underestimating preferences, CPR and sPRE show the same characteristic behavior over varied preference errors.

For MO, however, the impact of preference distortions significantly differs between negatively biased, positively biased, and unbiased error terms. Figure 11.7 compares the effects on revenue relative to the undistorted setup using MO as allocation method. The results show that applying positively biased errors or symmetric errors leads to the same inferior behavior with a negative correlation between error magnitude and revenue. Underestimating preference values, however, leads to a reverse progression. Despite the fact that the improvement is rather small and statistically not significant,

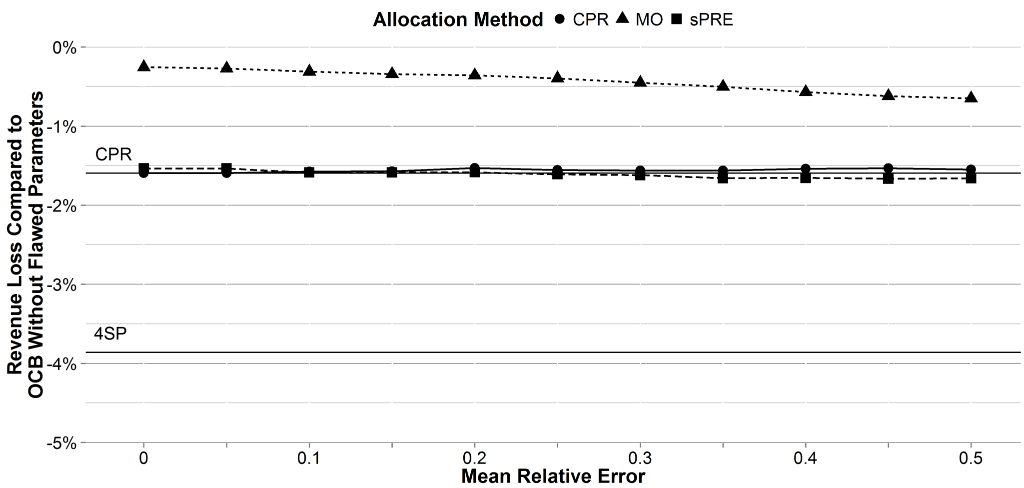


Figure 11.6: Revenue change in ad-hoc allocation setups given unbiased distorted revealed customer preferences

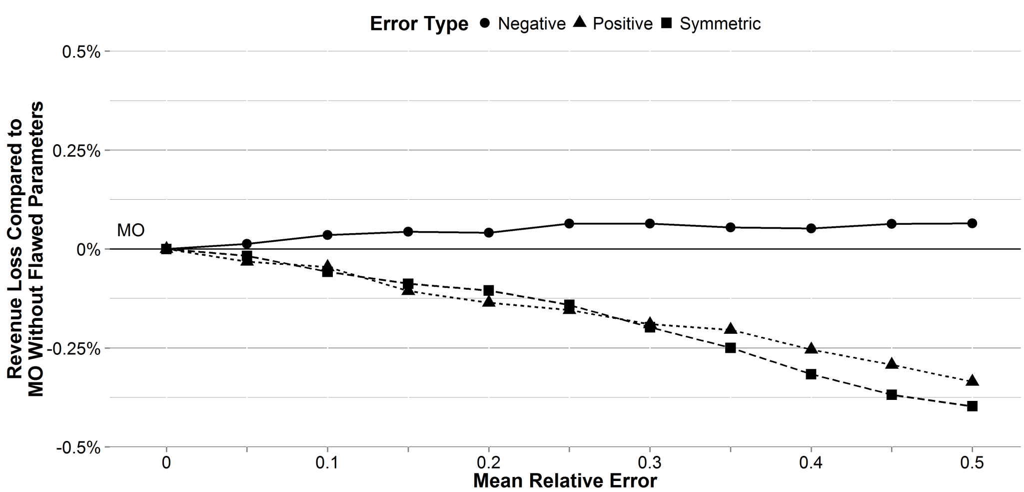


Figure 11.7: Revenue change for MO with ad-hoc allocation given distorted revealed customer preferences

we can conclude a clear superiority of MO with positively biased uncertain preference values.

Evaluating the effect of faulty customer preferences for re-allocation. Experiments with re-allocation of flexible bookings show similar results as for ad-hoc allocation. Flawed customer preferences impact revenue only slightly. Similar to previous observations, only MO shows minor differences when using biased error terms to distort customer preferences. To this end, we restrict to similar analyses for MO as shown in Figure 11.7 for ad-hoc allocation.

Figure 11.8 illustrates the results for the different error types for re-allocation setups. Compared to Figure 11.7, the revenue of MO decreases in all cases. Re-allocating flexible products obviously diminishes the improvements that MO could achieve for negative error terms and ad-hoc allocation.

For all other allocation methods, the numerical results are similar to the results for ad-hoc allocation. Compared to the unbiased case, the overall performance is slightly superior. The setups with biased error terms exhibit a stable behavior for an increasing error magnitude. Only for a mean relative error of 0.5, revenue decreases significantly. The behavior of OMO for increasing errors shows minor improvements. This indicates a higher robustness against flawed preferences when increasing the number of re-allocations. However, these improvements are not significant. For 0.5 mean relative error, the loss in revenue is still larger than 0.4%.

11.5 Implications of Flawed Parameters on Revenue Management

Flawed input parameters marginally affect the quality of RM systems. Optimizing product availability using wrong valuations leads to sub-optimal decisions and therefore diminishes RM performance. Selling flexible products can help to make RM more robust against such uncertainties. The experiments in this chapter explored this behavior and outlined several results.

For flawed demand forecasts, the largest revenue loss occurs in case demand forecasts are overestimated. This result is independent from the applied allocation method. For 0.5 mean relative error and ad-hoc allocation, the loss exceeds 5% compared to OCB with correct valuations. However, for setups with small error terms a few allocation methods could even improve their revenue caused by a more restrictive availability situation due to larger demand forecasts. The flexible product is sold less often and this leads to additional capacity available for bookings of more expensive products. This potential for improvements results from the natural uncertainty and inaccuracy incorporated in demand forecasts used to calculate the booking control policy.

For re-allocation, results vary for different allocation methods and error terms. In setups where demand is underestimated revenue decreases. The magnitude, however, is smaller than in case of positive and symmetric error terms. Overall, the results show

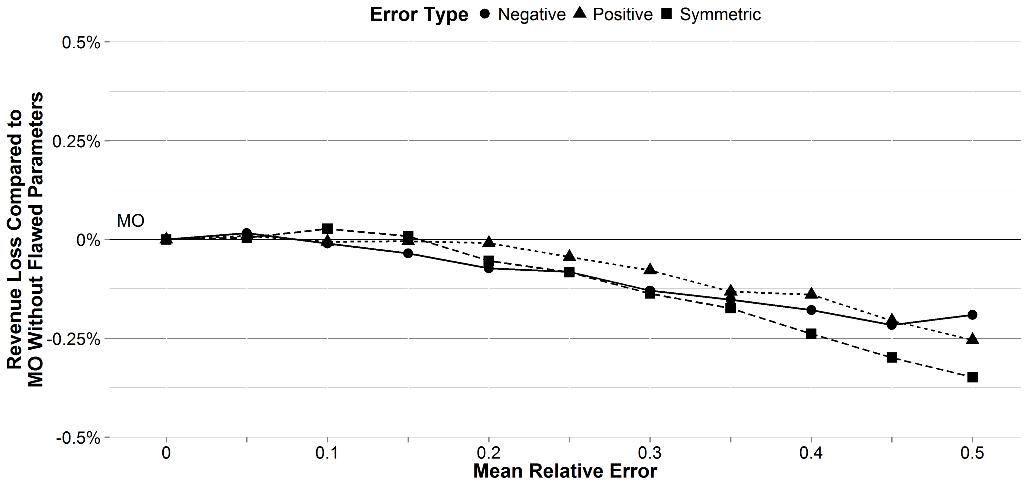


Figure 11.8: Revenue change for MO with re-allocation given distorted revealed customer preferences

a great dependency of RM performance on uncertain demand forecasts. One cause is that demand forecasts are used as input parameter for subsequent process steps. Almost all other RM parameters depend on calculations using demand forecasts. In case of negatively biased demand forecasts, a loss by about 5% for MO and OMO is observable similar to the case with ad-hoc allocation. For CPR, the similarity between ad-hoc and re-allocation increases. In contrary to previous results, re-allocation does not support revenue maximization.

Distorting bid prices only for allocation isolates the effects of false allocation on RM. Impacts on the booking control policy and general RM exist, however, they result only from flawed allocations. The performance of sBID shows a large robustness against increasing error terms applied on bid prices. For a re-allocation setup with positive error terms (overestimating the bid prices), sBID even shows a slight improvement in revenue compared to the case without error terms applied. For MO and OMO, we can conclude that, although bid prices are only partly considered in the objective formulation, these methods are sensitive to flawed bid prices.

In case of unbiased distortions, the sensitivity is even larger compared to the corresponding setups with biased error terms. This outlines that MO is extremely sensitive to the correctness of bid prices. As discussed for biased customer preference values, a bias just shifts the weighting between both objectives. Note that this is only valid in case a weighted sum is used to combine both objectives. Also, unbiased flawed input parameters significantly affect the performance of MO. Particular bookings are weighted more or less, depending on the associated bid price and preference value and whether they are increased or decreased compared to the actual values.

For the impact on RM performance when preference values are distorted, we derived expectations for our experiments using the mathematical models for customers' preferential choice (see Chapter 4) and the formulation of different allocation methods for flexible bookings (see Chapter 6). We expected only minor impacts when using distorted preference values, because using preference values for allocation handicap the RM performance by definition.

The numerical results of the third experiment meet our expectations. The mainly preference-based allocation approaches (CPR and sPRE) show a robust behavior. The results of MO are slightly different in case of positively biased errors. In this case we expected an inferior behavior as this impacts the weighting of dimensions within the objective towards more customer-centricity. The results support these expectations even if using re-allocation helps to counteract this trend.

With regard to research questions 4 and 5, we can conclude that flawed input parameters affects performance of RM with flexible products. Even if flexible products are a tool to counteract uncertainties, an increasing magnitude of uncertainty in input parameters diminishes revenue. This computational study outline these effects for all three parameter types, though the dimension of decline differs between the three parameters. Further, we can conclude that the preference-based allocation methods not significantly affect revenue in case of distorted input parameters. Derived from the fact that they considering fulfillment of customer preferences instead of revenue maximization, they obviously represent a general lower bound for revenue.

12 Conclusion

Section 12.1 summarizes this thesis' motivation, the proposed targets, the applied solution approaches, and the results. Managerial implications are discussed in Section 12.2. Finally, Section 12.3 addresses limitations of the models and experiments and discusses opportunities for extending this thesis.

12.1 Summary of Findings

Chapter 1 outlined the challenge for airlines applying Revenue Management (RM) while acting in an uncertain environment. We discussed the power of flexible products to deal with this challenge. Increasing flexibility for airlines, however, usually implies rising uncertainty for customers. This correlation imposes new challenges to simultaneously handle fulfillment of customer preferences and environmental uncertainties.

Chapter 2 reviewed relevant literature on RM, flexible products, and customer behavior. We revealed that only a few contributions integrate the aspect of customer choice for the manifestation set in the application of flexible products. Consequently, we formulated five research questions in Chapter 3. They frame the research prospect of this thesis: evaluating the impacts of customers' preferential choice between manifestations of flexible products.

Research Question 1. *What are the implications for RM with flexible products resulting from different models of customers' preferential choice between manifestations?*

We developed three customer choice models for the manifestation set of flexible products in Chapter 4. Various allocation methods were formulated in Chapter 6.

Analyzing the choice model where customers have no preference for one of the manifestations (Section 4.2) revealed no direct implications for RM when airlines assume this choice model. However, based on the considerations in Chapter 3 customers may carry preferences for the manifestation set. Considering these preferences can improve the customers' satisfaction. If airlines assume preferential choice (Sections 4.3 and 4.4), various challenges for RM arise. On the one hand, supposing a wrong choice model can lead to fewer booking requests as customers may decide to not buy at all (compare Chapter 7). On the other hand, considering customer preferences creates the need for appropriate mechanisms to handle flexible products: revealing mechanisms and allocation methods.

The knowledge about customers' preferential choice between manifestations enables airlines to consider additional aspects for allocation. This extends the initial ambition behind flexible products as a tool to protect airlines against uncertainties. Existing benefits achieved by selling flexible products can be protected, although customers' preferences are considered. The possibility to easily implement and integrate all presented choice models, revealing mechanisms, and allocation methods into the RM process shows the practicability of this extension.

> Implementing customers' preferential choice for flexible products still improves airlines' performance while enabling them to consider customers' wishes for allocation.

Research Question 2. *How does RM performance depend on the parameterization of customers' preferential choice models, revealing mechanisms, and allocation methods?*

Based on the mathematical reflections in Chapter 4 and 6, we differentiate our findings into conceptual parameters and model parameters. Conceptual parameters are the size of the manifestation set and the dynamic of the allocation. For model parameters, we consider the objective weighting parameter α in Multi-Objective Allocation (MO) and the quality of input parameters for MO.

Varying the size of the manifestation set only slightly impacts the revenue performance. When customers are allowed to limit the manifestation set, results show robustness of revenue for a few exclusions. For an increasing number of exclusions, however, the revenue reduction is nonlinear. As long as enough possibilities for allocation exist, a significant monetary benefit can be obtained. This implies that flexible products remain beneficial, even if limiting the manifestation set prunes the airlines' reactivity.

Varying the dynamic of allocations, e.g., re-allocating flexible products during the sales period, increases the benefits for the airline (see Chapter 9 and 11). With increasing uncertainty, the positive effects increase leading to a certain robustness against flawed input parameters used for allocating flexible products. Increasing the allocation dynamic by doing online allocation shows no significant improvements. Regarding limitations of the manifestation set, re-allocating flexible bookings substantially improves revenue. However, this only happens if the number of excluded manifestations is low (see Section 9.2).

Model parameters become especially relevant when the airline implements the actual preferential choice model (Section 4.4) and MO is used to allocate flexible bookings. The weighting parameter α, used for combining both objectives as a sum, significantly impacts the performance. The relationship between the weighting of the bid price concerned objective and the achieved revenue is nonlinear. Also for setups with re-allocation, the coherence between weighting and revenue is nonlinear and especially not monotone.

The quality of input parameters for MO, e.g., the used bid prices is the second relevant model parameter. Several experiments in Section 9.4 showed that extending the modeling of MO through approximations of the actual bid prices or inclusion of additional constraints improves the solution quality. However, MO with re-allocation and Online Multi-Objective Allocation (OMO) perform worse than the Allocation Based on Bid Prices (OCB) and Preference-Based Allocation (CPR) methods. Nevertheless, modeling, implementing, and using MO substantially increases reactivity and revenue for the airline if α is appropriately chosen.

A successful integration of flexible products and customers' preferential choice in RM largely depends on conceptual and model parameters. The appropriate choice of an allocation objective is one of the most critical steps.

Research Question 3. *What are the consequences for RM with flexible products if customer choice includes strategic choice between flexible and specific products? Does some degree of strategic behavior offset the benefits of flexible products?*

From a theoretical perspective, we identified two possible consequences of strategic customer behavior: direct loss through dependent customer choice and indirect loss through the thus affected control policy. Chapter 5 discusses the different consequences, stating that indirect loss seems to be more dangerous as it also affects the revenue achieved from bookings for other products.

The computational study in Chapter 10 shows that direct loss is the main consequence. Various experiments outline a linear relationship between revenue loss and reliability of information. A dynamic reliability parameterization leads to slight indirect effects. However, for an increasing share of strategic customers the indirect effects again disappear. These effects largely depend on the demand configuration and the accessibility and reliability of information about future allocations. Further, setups with demand learning forecast clearly show indirect effects: spiral down caused by strategic customers occurs. Using the concept of flexible products as a selling strategy, however, is an autonomous countermeasure against spiral down. Ensuring a certain amount of variance in demand between manifestations preserves reactivity for allocation and improves self-recovery of flexible products against spiral down. Therefore, in general, the danger from indirect loss is smaller than from direct loss as countermeasures exist.

The concept of flexible products relies on opaqueness and the resulting uncertainty for customers. Offering flexible products induces demand with a lower willingness-to-pay. Changing the information balance between airlines and customers without simultaneously adapting pricing of flexible products, leads to a disequilibrium in opaqueness.

Experiments show no explicit degree of strategic behavior that offsets the benefits of flexible products. The revenue loss induced by strategic customers largely depends on the share of strategic customers and the reliability of expectations and information about allocations. For most experiments, a parameterization exists where the benefits of flexible products are wiped out. However, the vast majority of experimental setups still retains a certain amount of benefits of selling flexible products. The most influential parameter is the share of strategic customers that actually books strategically caused by the current situation. An effective countermeasure is to decrease the information reliability, although it cannot completely offset the induced drawbacks.

RM with flexible products suffers from strategic choice between specific and flexible products. However, an appropriate use of flexible products as a selling strategy and a balanced design of the manifestation set preserves substantial benefits for the airline.

Research Question 4. *What is the effect of flawed input parameters on different combinations of customers' preferential choice model, revealing mechanism, and allocation method?*

Various experiments in Chapter 11 show that selling flexible products helps to counteract the drawbacks of uncertain parameterizations. However, the results outline a sensitivity of allocation methods to uncertain input parameters. This sensitivity is observable for flawed demand forecasts, flawed bid prices, and misconceived revealed customer preferences.

We examined the reaction to unbiased and biased distortions for all three parameters and different allocation methods. The results show a substantial difference between positively and negatively biased parameters. Overestimating demand forecasts leads to three times more revenue reduction than underestimating, independent from the used allocation method. For bid prices, the numerical results show an opposite behavior: revenue is more robust against positively biased parameters. Misconceived customer preferences generally impact revenue to a lesser extent. Nevertheless, a minor loss in revenue arises if too large preference values are used for allocation.

Flexible products efficiently counteract existing parameter uncertainties within the RM process. No setup with flawed parameters exists that completely offsets the initial benefits of flexible products. The increased reactivity allows airlines to always handle the situation more efficiently and enables them to assume the existence of a lower bound for the achievable revenue.

Research Question 5. *What happens if the airline implements a faulty customers' preferential choice model? How does this affect revealing mechanisms and allocation methods?*

Chapter 7 discusses different combinations of assumptions and choice models. We summarize that RM performance strongly depends on the implemented choice model and the actual demand behavior.

If the airline assumes indifference between manifestations and implements corresponding methods, this leads to a no-buy decision in the worst case. Customers may have strong aversions against some manifestations and therefore choose not to buy any product at all. However, this situation is robust regarding the expectations of customers. Without the possibility to postulate preferences they accept the whole uncertainty and are not displeased by any allocation.

Further, no situation exists where faulty assumptions are beneficial for RM performance. However, assuming preferential choice and implementing an appropriate revealing and allocation method does not worsen the results in case of any other real demand behavior.

Regarding strategic customers, the effects of a model mismatch largely depend on the imposed allocation method. Partly predictable allocation methods, e.g., OCB, increase the reliability and therefore encourage customers to act strategically. However, RM with flexible products counteracts this strategic behavior itself. The only real danger is to assume myopic customers while implementing preferential choice methods. This increases the predictability of allocations and customers will no longer be myopic: they tend to act strategically and start to exploit the concept of flexible products.

> Faulty assumptions about customer choice significantly affect RM performance. Especially disregarding strategic choice negatively affects the current success and future effectiveness of flexible products.

12.2 Managerial Implications

Current applications of flexible products indicate that they are primarily used to sell unsold capacities. They represent a good alternative to cheap last-minute offerings as they increase the airlines' reactivity and flexibility. However, the mathematical and numerical results of this thesis show that the scope of flexible products can be extended to incorporate customer preferences. Considering customers' preferential choice between manifestations can improve the fulfillment of customer preferences when they book flexible products. Simultaneously, airlines can retain the conceptual benefits: increased reactivity and flexibility throughout the sales period, additional revenue gains, and higher load factors as flexible products help to induce price-sensitive customers.

Changing the objective of allocating flexible products towards fulfillment of customer preferences extends the scope of flexible products. From short-term selling of unsold products towards a customer acquisition tool for price-sensitive customers with less willingness-to-pay. As these customers' willingness-to-pay may increase over time, e.g., students start to work, airlines can use flexible products to initialize a long-term relationship with these customers. To this end, considering the fulfillment of customer preferences as objective becomes more important.

The customers' preferential choice models, revealing mechanisms, and allocation models presented in this thesis show that it is possible to sell flexible products not only to fill up remaining capacities, but also to establish a customer relationship. Certainly, the numerical experiments show that considering customer preferences diminishes the short-term benefits of flexible products. Relying on discussions with subject matter experts from Deutsche Lufthansa AG, however, clarified that selling flexible products does not really provide a significant profit margin at all. Therefore, every conceptual change preserving the initial benefits and focusing on the long-term view could result in more substantial improvements.

As flexible products primarily address price-sensitive demand, sell-up to specific products is more or less improbable. Especially, if RM models only consider the short-term customer behavior. Therefore, sell-up is excluded in the computational studies. When looking at the long-term relationship between airlines and their customers, a sell-up can be quite possible. However, this implies that airlines have to partly focus on long-term customer relationships, even for customers booking flexible products.

This thesis formulates two mechanisms for customers to reveal their preferences for manifestations. The numerical results outline that practitioners can apply both concepts without losing much reactivity. However, the approach where customers limit the manifestation set seems to have a few benefits compared to the actual preferential choice model.

Considering current applications of flexible products, this thesis recommends to airlines that they should introduce the possibility to re-allocate flexible bookings. The computational studies outline significant improvements for re-allocating flexible bookings compared to ad-hoc allocation setups.

Practitioners should consider strategic behavior for flexible products. Numerical results show a significant impact on revenue, although the magnitude largely depends on the amount of strategic customers and the reliability of their expectations. If airlines retain enough reactivity for allocation, the concept of flexible products can autonomously counteract strategic behavior. However, the greatest risk is to neglect strategic customers and to impose a preferential choice between manifestations. This situation unavoidably leads to serious losses in revenue and an exploiting customer behavior.

Flexible products are already applied in different industries, although the main application is still the hospitality sector. Currently, various other industries start to apply flexible products, e.g., communication provider or the broadcasting industry.

For them and other possible application areas, the models and concepts discussed in this thesis are also relevant as customers may have preferences for manifestations. All formulations and implementations can be easily transferred to different applications. Especially, the existence of strategic customers is relevant and independent from the actual industry and must be considered when offering flexible products.

12.3 Limitations and Future Research Opportunities

This thesis focuses on the impacts of customers' preferential choice between manifestations. As only a few contributions deal with this aspect, the research agenda concentrates on basic considerations on a conceptual level. To this end, the models rely on the existence of a discrete customer choice model providing utility values for the manifestations. Following that, we formulated customers' preferential choice models (Chapter 4) and proposed a model of strategic customer behavior for flexible products (Chapter 5). However, the preferences derived from the underlying discrete choice model largely impact the performance of allocation methods.

Appropriate concepts to model customer choice for traditional RM already exist (cf. Garrow, 2012; Talluri & van Ryzin, 2004a). Therefore, researchers should formulate own choice models for the manifestation set of flexible products to extend the preferential choice models presented in this thesis. This will allow them to examine the dependencies between the design of flexible products, customer choice, and revenue performance in much more detail. For example, the choice model from Bai et al. (2015) includes an integrated view of price-sensitivity, the uncertainty overtaken by the customers, and the transparency of the allocation of flexible products. A possible extension should include these aspects in combination with different revealing mechanisms and allocation methods. Including additional fees for limitations or to stagger the price levels for different flexible products may also increase the applicability.

Chapter 5 introduced a model for strategic customer behavior for flexible products. Assumptions about the set of manifestations and number of offered flexible products were made a priori to simplify the model formulation. Furthermore, we modeled the reliability and minimal confidence of customers in a static way. For the corresponding simulation study (Chapter 10), we restricted to a fixed price for all products and simplified assumptions about customers accessing the information about allocation. Since our results have shown that the revenue loss through strategic behavior largely depends on the share of strategic customers and the reliability, further research should include empirical data to calibrate simulation systems. However, neither this data really exists, nor is it accessible. Extending research should further include more agent-based components modeling the individual behavior and decisions of customers and airlines. This will allow for example to implement more sophisticated models regarding the reliability for the strategic customer choice model. Again, the contribution of Bai et al. (2015) outlines a framework that can be extended to an integrated approach for reliability and pricing information improving the relevance and amount of achieved insights.

The RM model used relies on the assumption that demand is independent. However, strategic customer behavior is a sort of dependent demand. Evaluating the impact of strategic customer behavior in a different simulation setup may yield additional results and allow further implications. For example, modeling the customer choice in more sophisticated ways, e.g., including dependent choice behavior or communication between customers, or using different RM methods accounting for such behavior are three possible extensions.

Most of the computational studies modeled the customer preference values as a uniformly distributed random variable. This completely neglects the composition of the manifestation set, as there may be popular and unpopular manifestations included. Such popularity aspects exist as shown in Lee et al. (2012) and implied by our research gap in Chapter 3. To this end, we introduced a small excursion in Chapter 9 where we modeled the distribution of preference values adapting empirical results from Lee et al. (2012). For simplicity's sake, this experiment restricted to results of only one particular preference setup. Results derived from computational studies hardly rely on the underlying setup calibration. A more extensive study including more setups and using additional empirical data for calibration would ensure general applicability and reliability of the implications.

To enable the consideration of firms' benefits and customer satisfaction simultaneously, this thesis formulated the MO allocation method. For ad-hoc allocation, re-allocation and online allocation (OMO), however, we observed unexpected results for some parameterizations. This impacts the practical applicability of this allocation method. Through extensive analysis of the behavior and characteristics of the different allocation methods, we could detect the approximation of bid prices as one cause for this bad performance. More extensive research in this field, for example, examining different modeling approaches for the multi-criteria objective formulation or other ways to approximate the bid prices could be beneficial.

The contributions of Petrick et al. (2010) and Petrick et al. (2012) include several ways to incorporate flexible products in a more sophisticated way into the inventory. Extending these approaches by using the probability of an allocation as weighting factor may be beneficial for the resulting booking control policy. In this context, a mathematical analysis of the achieved level of reactivity when flexible products are introduced may be helpful to evaluate the success of different approaches.

The research aim of this thesis was to contribute concepts for customers' preferential choice between manifestations. To this end, the research agenda and especially the computational experiments are designed to cover a wide scope of scenarios. This enables manifold possibilities to adapt and extend the design to fit different application areas. Further research can build on the proposed setups and models as they can be easily modified to focus on particular aspects of customers' preferential choice between manifestations of flexible products.

Appendix

Numerical Results for MO and OMO With Different Objective Parameterizations

Table A.1 shows numerical results for different parameterizations of α in MO and OMO that are illustrated in Figure 9.9 and Figure 9.10. All results are calculated relative to the Setup with Four Specific Products (4SP). Statistical significant results to a confidence level of 95% are marked with an asterisk.

Table A.1: Relative change in revenue (REV), number of bookings (BKD), and fulfillment of customer preferences (PREF) for MO with ad-hoc and re-allocation and OMO

	MO						**OMO**		
	Ad-Hoc Allocation			**Re-Allocation**			**Online Allocation**		
α	REV	PREF	BKD	REV	PREF	BKD	REV	PREF	BKD
0	*4.01	*0.91	*3.91	*1.34	*2.91	*1.47	*1.75	*1.61	*1.76
0.1	*4.01	*9.80	*3.91	*2.38	*26.87	*2.47	*2.67	*24.03	*2.64
0.2	*4.01	*17.18	*3.90	*2.86	*31.07	*2.92	*3.10	*28.95	*3.04
0.3	*3.98	*25.09	*3.87	*3.17	*35.75	*3.20	*3.37	*33.24	*3.30
0.4	*3.94	*33.38	*3.84	*3.53	*40.34	*3.52	*3.58	*38.49	*3.50
0.5	*3.88	*41.48	*3.78	*3.79	*45.13	*3.73	*3.90	*42.82	*3.78
0.6	*3.75	*48.95	*3.66	*4.04	*50.23	*3.94	*4.04	*48.67	*3.92
0.7	*3.53	*55.92	*3.44	*3.88	*56.92	*3.77	*3.73	*55.70	*3.61
0.8	*3.22	*60.16	*3.13	*3.34	*59.55	*3.23	*3.10	*59.35	*3.01
0.9	*2.80	*62.60	*2.72	*2.80	*59.03	*2.72	*2.62	*59.10	*2.53
1	*2.37	*62.59	*2.29	*2.34	*57.18	*2.28	*2.19	*57.38	*2.11

Analysis of Published Characteristics for Blind Booking Products Offered by Germanwings

This section details the analysis of published characteristics for currently offered Blind Booking tickets as mentioned in Section 9.5 with respect to the exclusion probabilities provided by Lee et al. (2012). Two different Blind Booking products are analyzed: the culture package and the party package, both having Cologne/Bonn (IATA: CGN) as departure location (Germanwings GmbH, 2015). Four variables are evaluated: the language spoken at the destination, the flight distance, the cost of living at the destination, and the attractiveness.

The language variable and the flight distance were parameterized in the same way as done by Lee et al. (2012).

For the cost of living variable, four intervals regarding the "Bic Max Index" published by The Economist (2015) in January 2015 were chosen. These intervals are based on the study design of Lee et al. (2012) to rank the destination city ($1 = \text{low} = [1.20; 2.79]$, $2 = \text{medium} = [2.80; 4.37]$, $3 = \text{high} = [4.38; 5.96]$, $4 = \text{very high} = [5.97; 7.54]$).

For evaluating the attractiveness, the *Travelers' Choice Destinations 2015* of TripAdvisor LLC (2015) are used. If a destination was located among the top ten, the attractiveness was set to 4. Destinations with a rank between 11 and 20 are classified as *somewhat attractive* (attractiveness = 3). Destinations with ranks below 20 are classified as *somewhat unattractive* (attractiveness = 2) and all other destinations, which are not listed, are classified as *unattractive* (attractiveness = 1).

For the party package, various websites are used for classifying destinations' attractiveness. The rank column contains the results for the following rankings: *Best Party Cities in the World* of Clubplanet Inc. (2015), *The top 10 party cities* of The Daily Telegraph (2015), and *9 of the best cities to party in Europe* of Hostelworld.com Ltd. (2015). The attractiveness variable was set the same way as defined for the culture package.

Table A.2 shows the classification for the destinations included in the culture package and Table A.3 the classification for destinations included in the party package.

Impacts of Strategic Customer Behavior

This section contains additional tables and figures to support the findings of Section 10.4. Table A.4 contains the relative change in revenue, flexible bookings, and strategic bookings over runs for the setup with a smaller demand variation between runs. The allocation method used is OCB. These results are analog to Table 10.1 for a larger demand variation. Statistical significant results to a confidence level of 95% are marked with an asterisk.

Table A.5 contains the relative change in revenue, flexible bookings, and strategic bookings over runs for the setup with deterministic demand ($\sigma = 0$). These results are only included for the sake of completeness. They rely on a single demand realization, hence all observable characteristics are caused by this realization and are not generally transferable. Statistical significant results to a confidence level of 95% are marked with an asterisk.

Table A.2: Classifying the current culture package offered by Germanwings

	Language	**Distance**		**Cost of Living**		**Attractiveness**	
Destination	Variable	km	Variable	Index	Variable	Rank	Variable
Barcelona	0	1,124	3	4.23	2	7	4
London	0	500	2	4.37	2	3	4
Salzburg	1	560	2	3.93	2	-	1
Budapest	0	956	2	3.17	2	11	3
Mailand	0	631	2	4.46	3	-	1
Venedig	0	730	2	4.46	3	14	3
Dresden	1	474	1	4.25	2	-	1
Prag	0	536	2	2.92	2	2	4
Wien	1	743	2	3.93	2	22	2
Leipzig	1	380	1	4.25	2	-	1
Rom	0	1,090	3	4.46	3	4	4

Table A.3: Classifying the current party package offered by Germanwings

	Language	**Distance**		**Cost of Living**		**Attractiveness**	
Destination	Variable	km	Variable	Index	Variable	Rank	Variable
Barcelona	0	1,124	3	4.23	2	-/2/1	4
Berlin	1	478	1	4.25	2	7/3/3	4
Budapest	1	956	2	3.17	2	-/6/8	4
Dublin	0	940	2	4.04	2	-/-/-	1
Edinburgh	0	872	2	4.37	2	-/-/-	1
Hamburg	1	357	1	4.25	2	-/-/-	1
Leipzig	1	380	1	4.25	2	-/-/-	1
London	0	500	2	4.37	2	8/10/-	3
Mailand	0	631	2	4.46	3	-/-/-	1
Manchester	0	687	2	4.37	2	-/-/-	1
Prag	0	536	2	2.92	2	-/-/6	2
Wien	1	743	2	3.93	2	-/-/-	1

Table A.4: Relative change in revenue and bookings (BKD) for OCB given demand learning, different capacity setups, and $\sigma = 0.06 \cdot \mu$

		Runs				
Cap. Setup	Indicator	11-30	61-80	111-130	161-180	191-200
25	Revenue	*-2.04	-0.66	*-2.49	-0.64	-1.27
	Flexible BKD	4.49	2.70	*6.64	*5.61	*7.16
	Strategic BKD	*38.55	*37.21	*44.66	*60.11	*51.91
50	Revenue	*-3.25	*-1.89	*-4.32	-1.74	-1.77
	Flexible BKD	2.15	0.36	4.76	2.54	5.73
	Strategic BKD	*27.54	*28.12	*40.87	*44.64	*44.06
75	Revenue	*-4.10	*-3.52	*-6.26	*-2.83	-2.81
	Flexible BKD	-0.21	-2.88	1.58	-1.75	1.17
	Strategic BKD	*21.66	*20.32	*34.49	*31.68	*31.82

Table A.5: Relative change in revenue and bookings (BKD) for OCB given demand learning, different capacity setups, and deterministic demand ($\sigma = 0$)

		Runs				
Cap. Setup	Indicator	11-30	61-80	111-130	161-180	191-200
25	Revenue	*-0.30	*-0.16	*-0.21	*-0.28	-0.01
	Flexible BKD	*4.88	*4.92	*3.20	*8.21	-0.64
	Strategic BKD	*16.53	*16.67	*10.84	*27.78	-2.17
50	Revenue	*-0.27	*-0.15	*-0.19	*-0.26	-0.01
	Flexible BKD	*5.00	*5.20	*3.64	*8.33	-0.56
	Strategic BKD	*16.94	*17.62	*12.33	*28.18	-1.90
75	Revenue	*-0.42	*-0.28	*-0.25	*-0.36	-0.08
	Flexible BKD	*8.42	*8.38	*8.49	*11.35	3.47
	Strategic BKD	*22.28	*22.19	*22.47	*30.06	9.18

Figure A.1 shows the relative revenue for a setup with demand learning forecast and a setup with omniscient forecast given a setup with strategic customers. The revenue change is calculated to OCB without strategic customers and omniscient forecast.

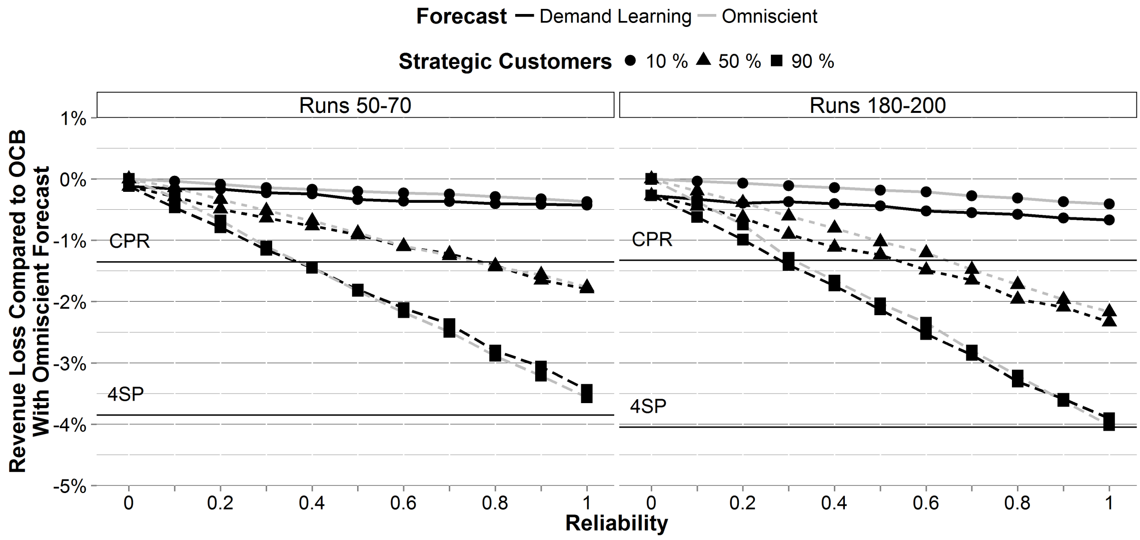


Figure A.1: Comparing the relative revenue change for omniscient forecast and demand learning given two run intervals and variation in reliability and share of strategic customers

Figure A.2 illustrates results for the stochastic allocation methods given varied capacity for one flight. The revenue change is calculated compared to OCB with strategic customers, where also capacity is varied. The horizontal line shows the revenue gain for a setup using 4SP without capacity change and strategic customers.

Impacts of Flawed Demand Forecasts

Figure A.3 shows the relative revenue change for stochastic and deterministic allocation methods with ad-hoc allocation given symmetrically distorted demand forecasts. The revenue change is calculated in relation to OCB without distorted forecasts.

Figure A.4 shows the relative revenue change for stochastic and deterministic allocation with ad-hoc allocation methods given biased error terms applied on customer preferences. The revenue change is calculated in relation to OCB without distorted forecasts.

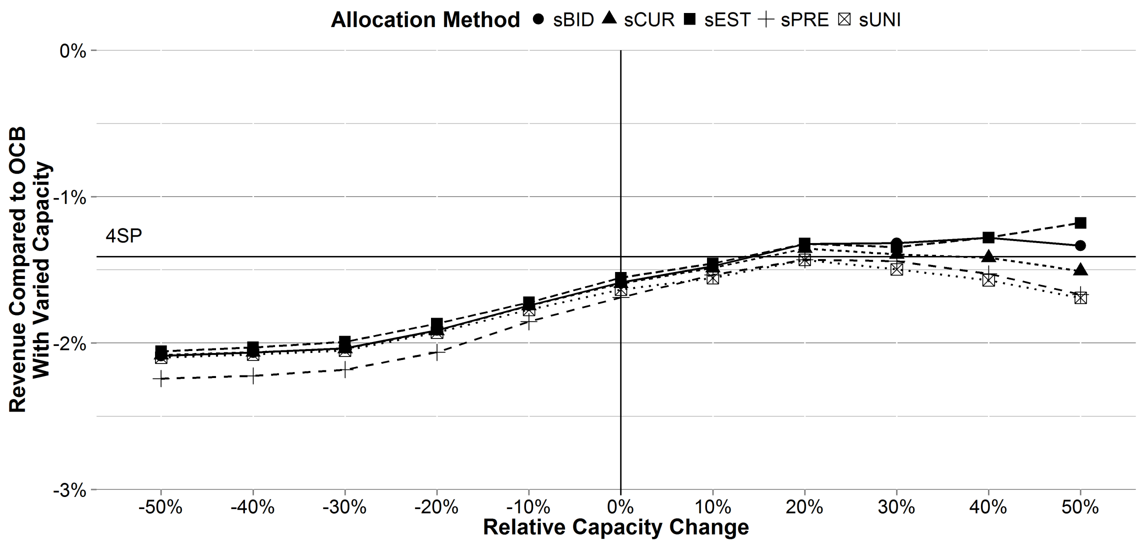
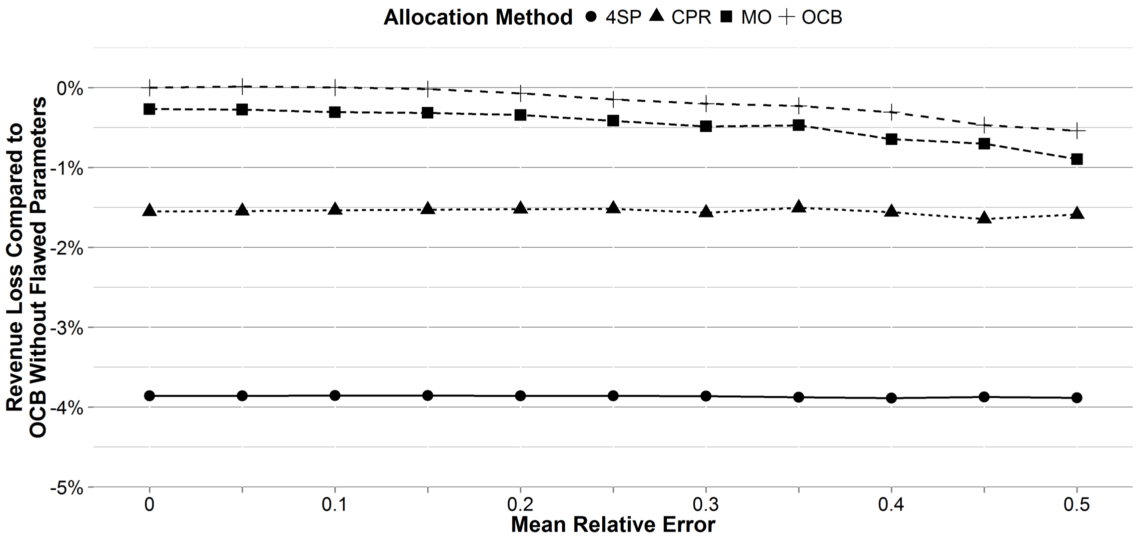
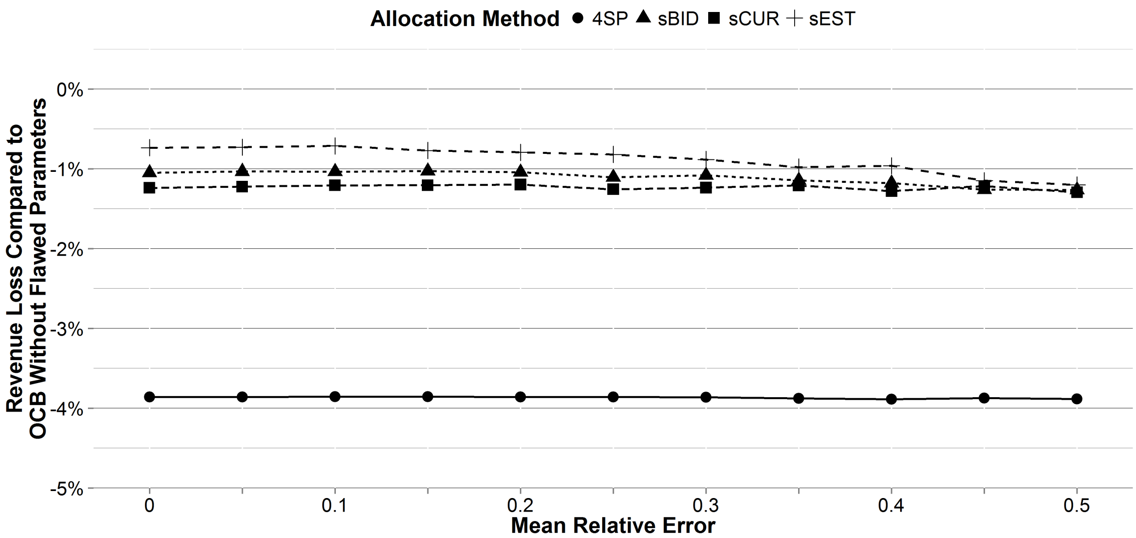


Figure A.2: Relative revenue change for the stochastic allocation methods given demand learning and different capacity setups



(a) Deterministic allocation methods



(b) Stochastic allocation methods

Figure A.3: Revenue change in ad-hoc allocation setups for unbiased errors terms applied to forecasts

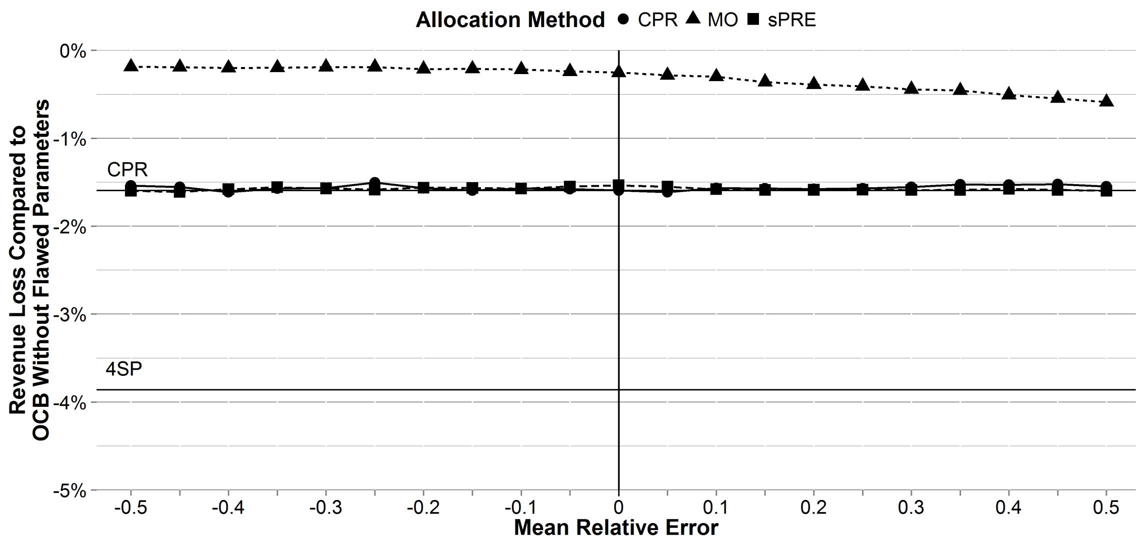


Figure A.4: Revenue change in ad-hoc allocation setups for biased errors terms applied to revealed customer preferences

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Acronyms

sPRE	Stochastic Allocation Based on Preferences. 105, 107–109, 125, 145, 146, 150
sUNI	Stochastic Allocation with Uniform Weights. 74, 86, 105, 107, 108, 125

Symbols

Note, that this list includes only symbols that are used across more than one subsection. The symbols are ordered by their first occurrence in the thesis.

Table B.1: Variables

Symbol	Definition
$z^{\tau} \in \{0,1\}$	Variable indicating if the customer request occurring at time slice $\tau \in T$ is accepted or not
$x_{bs} \in \{0,1\}$	Variable indicating if booking $b \in B$ is allocated to specific product $s \in S$

Table B.2: Sets

Symbol	Definition
$T \subseteq \mathbb{Z}_0^+$	Set of discrete time points defining the sales period, so that $0 \in T$ is the time of execution
B	Set of customers booking flexible products
R	Set of resources available to the airline
S	Set of specific products offered
S_r	Set of specific products using capacity on resource $r \in R$
F	Set of flexible products offered
$M_f \subseteq S$	Manifestation set of flexible product $f \in F$
$C_{bf} \subseteq M_f$	Choice set of customer $b \in B$ with regard to flexible product $f \in F$
Q_{bf}	Set of preferences for customer $b \in B$ and flexible product $f \in F$
$M_{bf} \subseteq R$	Set of available manifestations when booking $b \in B$ for flexible product $f \in F$ occurs
$M_b^* \subseteq R$	Set of optimal allocations for customer $b \in B$ calculated by OCB or CPR

Table B.3: Parameters

Symbol	Definition
$y_{sr} \in \{0,1\}$	Indicator for capacity usage of product $s \in S$ with regard to resource $r \in R$
$f_s, f_f \in \mathbb{R}^+$	Price for specific product $s \in S$ respectively flexible product $f \in F$
$s^t \in S$	Cheapest specific product available at time $t \in T$
$t_b \in T$	Discrete time point when booking $b \in B$ happened
$\tau \in T$	Discrete time slice where at most one request occurs
$\overline{m}_f \in \mathbb{R}$	Number of elements in the manifestation set of flexible product $f \in F$
$m_b \in M_f$	Manifestation maximizing utility for customer $b \in B$ that books flexible product $f \in F$
$u_{bm} \in \mathbb{R}^+$	Utility value of manifestation $m \in M_f$ for customer $b \in B$
$u_{bf} \in \mathbb{R}$	Threshold value for minimal acceptable utility for customer $b \in B$ and flexible product $f \in F$
$q_{bm} \in Q_{bf}$	Preference value of manifestation $m \in M_f$ for customer $b \in B$
$c_r \in \mathbb{R}$	Capacity of resource $r \in R$
$s_r^t \in \mathbb{R}$	Number of specific bookings for resource $r \in R$ at time $t \in T$
$f_r^t \in \mathbb{R}$	Number of flexible bookings for resource $r \in R$ at time $t \in T$
$c_r^t \in \mathbb{R}$	Residual capacity for resource $r \in R$ at time $t \in T$.
$\pi_r^t\left(c_r^t\right) \in \mathbb{R}$	Bid price for resource $r \in R$ at time $t \in T$ and residual capacity c_r^t
$\vartheta_b \in \mathbb{R}$	Minimal acceptable reliability of customer $b \in B$
$\varphi_{bm} \in \mathbb{R}$	Reliability/ estimated probability that flexible booking $b \in B$ will be allocated to manifestation $m \in M_f$
$w_{bm} \in [0,1]$	Allocation probability for manifestation $m \in M_f$ regarding flexible booking $b \in B$
$\alpha \in [0,1]$	Weighting parameter in objective of (O)MO
$\beta_i \in [0,1]$	Weighting parameter for approximation $i \in I$ of bid prices
$m_b^* \in M_f$	Manifestation that is selected as allocation for flexible booking $b \in B$ by an allocation method
$\rho \in [0,1]$	Auxiliary variable for objective modification in (O)MO