Concluding Remarks

CHAPTER 13

Summary

The main aspect of this thesis was to extend the range of applicability for functional integrals in quantum statistics and quantum field theory. Distributed over four parts, this thesis combines the formal justification of dealing with continuous path integrals from a perturbative point of view and a general solution for Gaussian path integrals in phase space with variational perturbation theory as a powerful resummation method which is also applicable for strongly coupled systems, where perturbative methods fail. The perturbative column on the one hand and the nonperturbative one on the other hand are bridged by a recursive graphical construction method which permits a systematic generation of all topologically different Feynman diagrams contributing to any order of perturbation with their correct multiplicities. As an interesting detail, the applicability of this method in quantum field theory is demonstrated for quantum electrodynamical scattering processes.

Motivated by the partial nonexistence of analytic results we have applied variational perturbation theory for atomic systems at arbitrary temperature and thermodynamical properties of fluctuating membranes. To this end, we have extended and generalized variational perturbation theory in a manifold way. For calculating density matrices, we generalized the smearing formula which accounts for the effects of thermal and quantum fluctuations. This was essential for the treatment of nonpolynomial interactions. We applied the theory to calculate the particle density in the double-well potential, and the electron density in the Coulomb potential, the latter as an example for nonpolynomial application. In both cases, the approximations were satisfactory.

We have also calculated the effective classical potential for the hydrogen atom in a magnetic field. For this we have extended variational perturbation theory to phase space to make it applicable to physical systems with uniform external magnetic field. The effective classical potential containing the complete quantum statistical information of the system was determined in first-order variational perturbation theory. For zero-temperature, it gave the binding energy of the system. Our result consists of a single analytic expression which is quite accurate at all temperatures and magnetic field strengths. The different asymptotic behavior of the perturbation series for the binding energy for weak and strong magnetic fields has been investigated in detail. In the weak-field case, we confirmed the power series character of the expansion, while for strong magnetic field strengths a deeply structured logarithmic behavior occurs.

As an application for strong-coupling theory in membrane physics, we have calculated the universal constant α occurring in the pressure law of a membrane fluctuating between two walls. This has been done by replacing the walls by a smooth potential with a parameter m^2. This potential approaches the wall potential in the limit $m^2 \to 0$. The anharmonic part of the smooth potential was treated perturbatively. The limit $m^2 \to 0$ corresponds to a strong-coupling limit of the power series, and was calculated by variational perturbation theory. Extrapolating the lowest four approximations to infinity yields a pressure constant, which is in very good agreement with Monte Carlo values.

We have also calculated the pressure constants for a stack of different numbers of membranes between two walls in excellent agreement with results from Monte Carlo simulations. The requirement that the membranes cannot penetrate each other was accounted for by introducing a repulsive potential and going to the strong-coupling limit of hard repulsion. We have used the similarity of the membrane system to a stack of strings enclosed by line-like walls, which is exactly solvable, to determine the potential parameters in such a way that the two-loop result is exact. This minimizes the neglected terms in the variational perturbation expansion, when applying the same potential to membranes.

It was shown in this thesis that variational perturbation theory can successfully be applied to a large variety of problems in quantum statistics and membrane physics. The results obtained for fluctuating strings and membranes open the gate to a large field of applications to be harvested with the help of this strong-coupling theory.