Appendix B

Channeling model

B.1 Analogies between the Channeling analysis and the Groom & Bailey tensor decomposition

The determination of the twist ($t = tan(\beta_t)$) and shear ($e = tan(\beta_e)$) distortion parameters of the Groom & Bailey tensor decomposition (introduced in section 1.2.5) can be obtained directly in terms of the tensor element columns. In the regional coordinate system the measured (or distorted) impedance tensor $\mathbf{Z}$:

$$\mathbf{Z} == \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}$$

is decomposed (with unknowns anisotropy and gain factors) as:

$$\mathbf{Z} = TS\mathbf{Z^r} = \begin{bmatrix} (e-t)Z^r_{yx} & (1-te)Z^r_{xy} \\ (1+te)Z^r_{yx} & (e+t)Z^r_{xy} \end{bmatrix} \qquad \text{(B.1)}$$

where $\mathbf{Z^r}$ is the 2-D tensor, T and S are the matrices containing the twist and shear distortion parameters, respectively, and their product (TS) constitutes the "distortion matrix" of the telluric model.
From eq.B.1, the impedance element quotients of each tensor element column are:

$$\frac{Z_{xx}}{Z_{yx}} = \frac{e-t}{1+te} \qquad \frac{Z_{yy}}{Z_{xy}} = \frac{e+t}{1-te} \qquad \text{(B.2)}$$

Applying the inverse of the tangent function to the above equations, the following relations are obtained:

$$arctan(\frac{e-t}{1+te}) = arctan(e) - arctan(t)$$
$$arctan(\frac{e+t}{1-te}) = arctan(e) + arctan(t)$$

for $|t|$, $|e| <= 1$. The relations for the twist and shear as function of the measured tensor elements quotient (assumed in the regional coordinate system) are obtained directly from the above equations and eq.B.2:

$$arctan(\frac{Z_{xx}}{Z_{yx}}) = \beta_e - \beta_t \qquad arctan(\frac{Z_{yy}}{Z_{xy}}) = \beta_e + \beta_t \, , \tag{B.3}$$

where $\beta_e = arctan(e)$ and $\beta_t = arctan(t)$.

The solution of the G & B decomposition is valid for negligible magnetic distortion parameters. Thus the distortion matrix TS (eq.B.1) is equivalent to the telluric distortion matrix D_e proposed by Smith (1997) (section 1.2.6) for unknown anisotropy an gain factors:

$$TS = D_e = \begin{bmatrix} 1 & c \\ b & 1 \end{bmatrix}$$

Considering the above relation to determine the sign of the tensor element quotients, by replacing the twist (t) and shear (s) parameters in the telluric parameters b and c in eq.B.2 we obtain:

$$sign(\frac{Z_{xx}}{Z_{yx}}) = sign(c) \qquad sign(\frac{Z_{yy}}{Z_{xy}}) = sign(b) \, ,$$

which leads to the final expressions for twist and shear after subtracting and adding the two terms written in eq.B.3:

$$\beta_e = \frac{1}{2}[arctan(\frac{|Z_{yy}|}{|Z_{xy}|}sign(b)) + arctan(\frac{|Z_{xx}|}{|Z_{yx}|}sign(c))] \tag{B.4}$$

$$\beta_t = \frac{1}{2}[arctan(\frac{|Z_{yy}|}{|Z_{xy}|}sign(b)) - arctan(\frac{|Z_{xx}|}{|Z_{yx}|}sign(c))] \tag{B.5}$$

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B.1.1 The equivalence of the twist and shear telluric parameters

In the following, it will be shown that eqs. B.4 and B.5 are equivalent to the twist and shear relations in terms of the complex matrix C (introduced in section 6.1.1) for the telluric and magnetic model of the form:

$$\mathbf{Z} = C\mathbf{Z^r} = \begin{bmatrix} c_2 Z^r_{yx} & c_1 Z^r_{xy} \\ c_4 Z^r_{yx} & c_3 Z^r_{xy} \end{bmatrix} \tag{B.6}$$

if the channeling assumption is valid. The elements of C are defined in polar form (eq.6.2) as:
$c_2 = r\, e^{i\beta}$, $c_3 = t\, e^{i\alpha}$, $c_4 = w\, e^{i\delta}$, $c_1 = g\, e^{i\varphi}$,
where β, δ, α and φ represent the "phase deviations" due to the magnetic distortion.
Expressing eq.B.3 in terms of the elements of the distortion matrix C:

$$arctan(\frac{c_2}{c_4}) = \beta_e - \beta_t \qquad arctan(\frac{c_3}{c_1}) = \beta_e + \beta_t \, ,$$

the expressions of the twist and shear as derived in the eqs. B.4 and B.5, after replacing the parameters c_1,.., c_4 by the the polar complex form, result in:

$$\beta_e = \frac{1}{2}[arctan(\frac{t}{g}e^{i(\alpha-\varphi)}) \; + \; arctan(\frac{r}{w}e^{i(\beta-\delta)})] \tag{B.7}$$

$$\beta_t = \frac{1}{2}[arctan(\frac{t}{g}e^{i(\alpha-\varphi)}) \; - \; arctan(\frac{r}{w}e^{i(\beta-\delta)})] \tag{B.8}$$

Under the strong current channeling assumption the absolute value of the phase deviations from the magnetic distortion effect should be equal between each tensor element column (section 6.1.4)

$\alpha = \varphi$ and $\beta = \delta$ if $\theta >= 0$

$|\alpha - \varphi| = \pi$ and $|\beta - \delta| = \pi$ if $\theta < 0$

with θ defined as the angle conformed between the regional and the local axis, positive c.c.w. with respect to the local azimuth (i.e., the elongated conductor strike). Thus the sign of the phase deviations subtraction $(\alpha - \varphi)$ and $(\beta - \delta)$ are given by that of θ. Taking the sign of θ into account, eqs. B.7 and B.8 simplify to

$$\beta_e = \frac{1}{2}[arctan(\frac{t}{g}sign(\theta)) \; + \; arctan(\frac{r}{w}sign(\theta))] \tag{B.9}$$

$$\beta_t = \frac{1}{2}[arctan(\frac{t}{g}sign(\theta)) \; - \; arctan(\frac{r}{w}sign(\theta))] \, . \tag{B.10}$$

Both equations should be equivalent to eqs. B.4 and B.5. From the telluric and magnetic model of eq.B.6 and assuming that the channeling model is valid (i.e., equal phase deviations), it is clear that the magnitude of the polar form elements of the matrix C approach:

$$\frac{t}{g} = \frac{|Z_{yy}|}{|Z_{xy}|} \qquad \frac{r}{w} = \frac{|Z_{xx}|}{|Z_{yx}|} \, .$$

Introducing both terms to eqs. B.9 and B.10 results in:

$$\beta_e = \frac{1}{2}[arctan(\frac{|Z_{yy}|}{|Z_{xy}|}sign(\theta)) \; + \; arctan(\frac{|Z_{xx}|}{|Z_{yx}|}sign(\theta))]$$

$$\beta_t = \frac{1}{2}[arctan(\frac{|Z_{yy}|}{|Z_{xy}|}sign(\theta)) \; - \; arctan(\frac{|Z_{xx}|}{|Z_{yx}|}sign(\theta))] \, .$$

It follows to proof that:

$sign(\theta) = sign(b) = sign(c)$

in order to have the equivalent twist and shear expressions as that from eqs. B.4, B.5 of the telluric model.

The sign of θ can be deduced from the current channeling condition (eq.6.15):

$$tan(\theta) = \frac{r\,cos\beta}{w\,cos\delta} = \frac{g\,cos\varphi}{t\,cos\alpha} \qquad tan(\theta) = \frac{r\,sin\beta}{w\,sin\delta} = \frac{g\,sin\varphi}{t\,sin\alpha}$$

where (from eq.6.4)

$$\frac{r\,cos\beta}{w\,cos\delta} = \frac{c - \varepsilon\,ReZ_{xy}}{1 - \varepsilon\,ReZ_{yy}} \qquad \frac{g\,cos\varphi}{t\,cos\alpha} = \frac{1 - \gamma\,ReZ_{xx}}{b - \gamma\,ReZ_{yx}} \, .$$

The above relations can be rewritten in terms of the measured tensor:

$$\frac{r\,cos\beta}{w\,cos\delta} = \frac{ReZ_{xx}}{ReZ_{yx}} \qquad \frac{g\,cos\varphi}{t\,cos\alpha} = \frac{ReZ_{xy}}{ReZ_{yy}}$$

$$\frac{r\,sin\beta}{w\,sin\delta} = \frac{ImZ_{xx}}{ImZ_{yx}} \qquad \frac{g\,sin\varphi}{t\,sin\alpha} = \frac{ImZ_{xy}}{ImZ_{yy}}\,.$$

And therefrom the following equivalences are found :

$$\frac{c-\varepsilon\,ReZ_{xy}}{1-\varepsilon\,ReZ_{yy}} = \frac{ReZ_{xy}}{ReZ_{yy}} \qquad \frac{1-\gamma\,ReZ_{xx}}{b-\gamma\,ReZ_{yx}} = \frac{ReZ_{xx}}{ReZ_{yx}}\,.$$

From the first equation system the telluric parameter c is:

$$c = \frac{ReZ_{xy}}{ReZ_{yy}}\,,$$

and from the second equation system the telluric parameter b is:

$$b = \frac{ReZ_{yx}}{ReZ_{xx}}\,.$$

Thus under the current channeling condition we have that:

$$c = \frac{1}{b} = tan(\theta)\,. \tag{B.11}$$

This implies that:

$$sign(\theta) = sign(b) = sign(c)$$

and the equivalence between eqs. B.4, B.5 and eqs. B.9, B.10 for the twist and shear are true for the channeling model, where magnetic effects are not necessarily negligible.

B.1.2 The maximal shear value

The maximal value for the shear angle ($\beta_e = \pm\pi/4$) was derived in section 6.1.5 after determining the zeros of the vanishing tensor elements aligned with the elongated conductor strike (section 6.1.4). From the equation of the channeling condition (eq.6.15), the product of the measured tensor elements quotient in the regional coordinate system follows the relation

$$\frac{Z_{xx}}{Z_{yx}} \cdot \frac{Z_{yy}}{Z_{xy}} = tan(\theta)\,cot(\theta)\,. \tag{B.12}$$

Which means that :

$$\frac{Z_{xx}}{Z_{yx}} = \frac{Z_{yy}}{Z_{xy}}^{-1}$$

This expression can be rewritten in terms of the twist (t) and shear (e) parameters by considering eq.B.2 of the G & B decomposition:

$$\frac{e-t}{1+te} \cdot \frac{e+t}{1-te} = 1 \implies e^2 = 1\,.$$

and the maximal shear value ($e = \pm 1$) is obtained:

$$\beta_e = \arctan(e) = \pm\frac{\pi}{4}.$$

The sign of e can be determined as follows. Since $|te| <= 1$ and $|e| >= |t|$, then $sign(e-t) = sign(e+t) = sign(e)$.
Thus from eq.B.3

$$sign(\frac{Z_{yy}}{Z_{xy}}) = sign(\beta_e + \beta_t) = sign(\beta_e)$$

$$sign(\frac{Z_{xx}}{Z_{yx}}) = sign(\beta_e - \beta_t) = sign(\beta_e)\,.$$

By considering eq.B.12:

$$sign(\theta) = sign(\frac{Z_{xx}}{Z_{yx}}) = sign(\frac{Z_{xy}}{Z_{yy}})\,,$$

then

$$sign(e) = sign(\beta_e) = sign(\theta)$$

and eq.6.18 introduced in section 6.1.5 has been again verified:

$$\beta_e = \frac{sign(\theta)\pi}{4}. \tag{B.13}$$

B.2 Tensor rotation from the regional coordinate system to the local azimuth

The impedance tensor Z of the local azimuth coordinate system (x,y) is rotated counter-clockwise to the regional coordinate system (x',y'). Then Z as a function of the regional coordinate system Z' is expressed as:

$$Z = R(\theta) \cdot Z' \cdot R^T(\theta)\,.$$

where

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

is the rotation matrix.

And Z' is expressed as a function of the 2-D regional tensor Z^r and the distortion matrix C:

$$Z' = CZ^r = \begin{bmatrix} rZ^r_{y'x'}e^{i\beta} & gZ^r_{x'y'}e^{i\vartheta} \\ wZ^r_{y'x'}e^{i\delta} & tZ^r_{x'y'}e^{i\alpha} \end{bmatrix} \tag{B.14}$$

Then each tensor element of Z takes the form:

$$Z_{xx} = \left\{\cos^2\theta(re^{i\beta}) - \sin\theta\cos\theta(we^{i\delta})\right\} Z^r_{y'x'} + \left\{-\sin\theta\cos\theta(ge^{i\vartheta}) + \sin^2\theta(te^{i\alpha})\right\} Z^r_{x'y'}$$

$$Z_{xy} = \left\{\sin\theta\cos\theta(re^{i\beta}) - \sin^2\theta(we^{i\delta})\right\} Z^r_{y'x'} + \left\{\cos^2\theta(ge^{i\vartheta}) - \sin\theta\cos\theta(te^{i\alpha})\right\} Z^r_{x'y'}$$

$$Z_{yx} = \left\{\sin\theta\cos\theta(re^{i\beta}) + \cos^2\theta(we^{i\delta})\right\} Z^r_{y'x'} + \left\{-\sin^2\theta(ge^{i\vartheta}) - \sin\theta\cos\theta(te^{i\alpha})\right\} Z^r_{x'y'}$$

$$Z_{yy} = \left\{\sin^2\theta(re^{i\beta}) + \sin\theta\cos\theta(we^{i\delta})\right\} Z^r_{y'x'} + \left\{\sin\theta\cos\theta(ge^{i\vartheta}) + \cos^2\theta(te^{i\alpha})\right\} Z^r_{x'y'}$$

In an arbitrary measured coordinate system, the local azimuth, rotated clockwise with respect to the measured coordinates, (θ_l) is:

$$\theta_l = \theta^r - \theta$$

where θ^r is the regional strike rotated clockwise from the the measured coordinates.

B.3 Parameters estimation in the Channeling model

B.3.1 3-D induction strength and frequency independent strike angle

The frequency independent regional strike is found for the minimum standard deviation of the distortion parameters averaged in the period band. The function to minimize is

$$\partial g^2 = \frac{\sum_{i=1}^{N}\left[\left(b^i - \hat{b}\right)^2 + \left(c^i - \hat{c}\right)^2 + \left(\varepsilon^i - \hat{\varepsilon}\right)^2 + \left(\gamma^i - \hat{\gamma}\right)^2\right]}{4\,(N-1)}$$

where N is the period number, where b and c are the telluric distortion parameters, and where ε and γ are the magnetic distortion parameters. The function ∂g^2 is called the *3-D induction strength* parameter, in the sense that it is a measure of the superposition model departure. Greater values are indicative of induction. The superscripts i and $\wedge$ indicate the single frequency and the arithmetic averaged values, respectively.
The single frequency distortion parameters depend on the impedance coordinate system, since they are solved directly in terms of the rotated measured tensor elements (section 1.2.6, eqs.1.26, 1.27).

The function ∂g^2 is calculated 36 times, corresponding to a tensor which is in the coordinate system of a strike angle rotated every 5°, starting at −90° and ending at 90°. The single site regional strike is hence that where ∂g^2 approaches a minimum, which means that the distortion parameters at this coordinate system are most frequency independent (i.e., galvanic distortion).
It is of course possible to obtain constant strike angles for selected frequency bands. For example, two regional strikes can be estimated independently for a single site, corresponding to the short and the long period bands, respectively.

Another form for the ∂g^2 function can be recommended for data having very different error magnitudes as function of the period (usually the errors are much greater by longer period data). In this case, i.e., by unequal tensor element errors, the standard deviation function will not account for the biased distortion parameters in the period band. Thus, a minimum value could point to a trend rather than to a frequency independent distortion. Therefore, the *function which can eliminate such possible trends* (e.g. Johnson and Kotz [1972]) is

$$\partial g^2 = \frac{\sum_{i=1}^{N-1} \left[\left(b^{i+1} - b^i\right)^2 + \left(c^{i+1} - c^i\right)^2 + \left(\varepsilon^{i+1} - \varepsilon^i\right)^2 + \left(\gamma^{i+1} - \gamma^i\right)^2 \right]}{8\,(N-1)}$$

B.3.2 Local azimuth

The single frequency local azimuth (θ_l) is calculated by imposing that the shear angle (β_e) is maximal in the assumed regional coordinate system, as well as that
$sign(\theta) = sign(\beta_e)$, as shown in app. B.1. Thus from eq.B.13 and the twist angle of the channeling model:

$$\beta_t = sign(\theta)\frac{\pi}{4} - \theta$$

the angle between the regional and the local structure is:

$$\theta = sign(\beta_e)\,\frac{\pi}{4} - \beta_t \quad ,$$

and the local azimuth with respect to the measured coordinates is

$$\theta_l = \theta^r - \theta\,.$$

The local azimuth coincides with the the local structure orientation for the vanishing electric field component aligned with it. If the local structure strikes along the x-axis, the correct azimuth will be that where the tensor element quotients fulfill the condition

$$\frac{|Z_{xx}|}{|Z_{yx}|} < \frac{|Z_{yy}|}{|Z_{xy}|}$$

for the impedance in the local coordinate system. Since the tensor decomposition is realised in the regional coordinate system (x', y'), the tensor must be rotated c.w. by the angle θ_l in order to bring it back to the local coordinates. If the previous inequality is not achieved, then the azimuth should be shifted by $\pm 90°$:
$\theta_l \rightarrow \theta_l + \pi/2$ if $\theta_l < 0$
otherwise
$\theta_l \rightarrow \theta_l - \pi/2$ if $\theta_l > 0$,

and thus determining the correct local azimuth.

The error of θ_l is given by the departure from the expected maximal shear angle:

$$\Delta\theta_l = |\beta_e - sign(\theta)\,\pi/4|\,.$$

The single site local azimuth ($\hat{\theta}_l$) is simply the arithmetical average of the single frequency values.

The error ($\Delta\hat{\theta}_l$) of the averaged local azimuth ($\hat{\theta}_l$) is estimated by considering the single frequency local azimuth errors with the standard deviation formula:

$$(\Delta\hat{\theta}\,)^2 = \frac{\sum_{i=1}^{N}\left[\left((\theta_l^i + \Delta\theta_l^i) - \hat{\theta}\right)^2 + \left((\theta_l^i - \Delta\theta_l^i) - \hat{\theta}\right)^2\right]}{2\,(N-1)}$$

where N is the number of periods, and θ_l^i, $\Delta\theta_l^i$ are the azimuth and error for the ordered period i, respectively.

B.3.3 Frequency dependent strike angle

The frequency dependent regional strike angle can be determined for the elements of the complex distortion matrix C (eq.B.14) approaching the channeling condition (section 6.1.4)

$$\min[|(r/w) - (g/t)|^2 + |\beta - \delta|^2 + |\varphi - \alpha|^2] \tag{B.15}$$

Considering that the measured impedance tensor is a function of the distortion parameters (eq.B.14), where:

$$\frac{|Z_{xx}|}{|Z_{yx}|} = \frac{r}{w}$$

$$\frac{|Z_{xy}|}{|Z_{yy}|} = \frac{g}{t}$$

and the difference between the measured impedance phases ϕ_{xx} and ϕ_{yx}, which are distorted by the "phase deviations" β and δ, respectively (arisen from the magnetic effect):

$$\phi_{xx} = \beta + \phi_{yx}^r \quad and \quad \phi_{yx} = \delta + \phi_{yx}^r$$

results in

$$\phi_{xx} - \phi_{yx} = \beta - \delta.$$

Analogously for the difference between the impedance phases ϕ_{xy} and ϕ_{yy}, distorted by φ and α,

$$\phi_{xy} - \phi_{yy} = \varphi - \alpha\,,$$

the functional of eq.B.15 will be equivalent to find the minimum directly in terms of the measured impedance tensor

$$\min[(|Z_{xx}|/|Z_{yx}| - |Z_{xy}|/|Z_{yy}|)^2 + |\phi_{xx} - \phi_{yx}|^2 + |\phi_{xy} - \phi_{yy}|^2] \tag{B.16}$$

The functional is varied by rotating the tensor in step strike angles to every 5°, between −90° and 90°. Thus the frequency dependent regional strike corresponds to the minimal value found for this single frequency functional [1].

Since field data are measured with errors, the confidence limit of the tensor elements and hence the bias must be considered, which requires more appropriate functional. Therefore,

[1] A value of zero means that shear is maximal($\beta_e = \pm\pi/4$, or $e = \pm 1$). In addition, the phase deviations from the magnetic distortion are the same between each tensor element column

the first term of the above function will be transformed to

$$(|Z_{xx}|/|Z_{yx}| - |Z_{xy}|/|Z_{yy}|)^2 \longrightarrow$$

$$\frac{1}{6}[(\frac{|Z_{xx}+\Delta Z_{xx}|}{|Z_{yx}|} - \frac{|Z_{xy}|}{|Z_{yy}+\Delta Z_{yy}|})^2 + (\frac{|Z_{xx}|}{|Z_{yx}+\Delta Z_{yx}|} - \frac{|Z_{xy}+\Delta Z_{xy}|}{|Z_{yy}|})^2 +$$

$$+(\frac{|Z_{xx}+\Delta Z_{xx}|}{|Z_{yx}|} - \frac{|Z_{xy}+\Delta Z_{xy}|}{|Z_{yy}|})^2 + (\frac{|Z_{xx}|}{|Z_{yx}+\Delta Z_{yx}|} - \frac{|Z_{xy}|}{|Z_{yy}+\Delta Z_{yy}|})^2 +$$

$$+2(\frac{|Z_{xx}|}{|Z_{yx}|} - \frac{|Z_{xy}|}{|Z_{yy}|})^2]$$

where ΔZ_{ij} $(i, j = x, y)$ are the corresponding tensor element errors. The values 1/6 and 2 are given weight factors.
Meanwhile the term for the phases are transformed to

$$|\phi_{xx} - \phi_{yx}|^2 \longrightarrow$$

$$\frac{1}{4}\{[\phi_{xx} + \triangle\phi_{xx} - (\phi_{yx} - \triangle\phi_{yx})]^2 + [\phi_{xx} - \triangle\phi_{xx} - (\phi_{yx} + \triangle\phi_{yx})]^2 + 2[\phi_{xx} - \phi_{yx}]^2\}$$

with $\Delta\phi_{ij} = \Delta Z_{ij} / |Z_{ij}|$ defining the phase linear propagated errors.
Analogous is the transformation for the other phase differences written in the third term of eq.B.16.
By means of the transformed functional, the same procedure as explained above follows for the regional strike determination; the strike angle is that of the coordinate system where the tensor approaches the functional minimum value.

B.3.4 Channeling misfit

A magnitude for the departure of the channeling model hypothesis can be proportional to the functional of eq.B.16 of the tensor in the regional coordinate system, normalized by the linear propagated errors:

$$\frac{1}{3}[\frac{(|Z_{xx}|/|Z_{yx}| - |Z_{xy}|/|Z_{yy}|)^2}{\partial Z^2} + \frac{\sin^2(\phi_{xx} - \phi_{yx})}{\partial\phi_x^2} + \frac{\sin^2(\phi_{xy} - \phi_{yy})}{\partial\phi_y^2}]$$

where

$$(\partial Z)^2 = \frac{\Delta Z_{xx}^2}{|Z_{yx}|^2} + \frac{(\Delta Z_{yx}\,|Z_{xx}|)^2}{|Z_{yx}|^4} + \frac{(\Delta Z_{xy})^2}{|Z_{yy}|^2} + \frac{(\Delta Z_{yy}\,|Z_{xy}|)^2}{|Z_{yy}|^4},$$

$$(\partial\phi_x)^2 = \Delta\phi_{xx}^2 + \Delta\phi_{yx}^2 \quad and \quad (\partial\phi_y)^2 = \Delta\phi_{yy}^2 + \Delta\phi_{xy}^2$$

are the corresponding numerator errors estimated with the first order Taylor expansion.
The single site channeling misfit is the arithmetic average of the single frequency values, estimated in the assumed regional coordinate system.