Chapter 1

Theoretical Background

1.1 Principles of Magnetotellurics and Geomagnetic Deep Soundings

The source of the Magnetotelluric (MT) and Geomagnetic Deep Sounding (GDS) measurements are the electromagnetic (EM) fields originated in space (mostly due the solar wind) which perturb the terrestrial magnetic field. Such perturbations lead to the deformation of the magnetosphere, constituted partly of ionized plasma (e. g., Villlante [1993]). Another source of electromagnetic disturbances is the ionosphere (ionized due to UV radiation, thus producing different current densities), which leads to a strong diurnal EM variation. The EM-fields propagate in the atmosphere (of conductivity around zero) and reach the earth surface as quasi-homogeneous waves, valid for the frequencies considered by MT and GDS ($> 10^{-5}$ $[1/s]$). A great part of the incident fields reflects at the surface and a small part penetrates as quasi-stationary plane waves (EM harmonic plane waves) into the conductive earth. The latter is what MT and GDS measures at the earth surface.

Maxwell's equations

The EM-fields in isotropic and homogeneous media (of constant electric conductivity σ $[S/m]$), of uniform electric permittivity ε $[\frac{As}{Vm}]$ and magnetic permeability μ $[\frac{Vs}{Am}]$ are described by the Maxwell's equations. Considering the fields with harmonic temporal variation (e^{iwt}), these equations are:

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$$

$$\nabla \times \mathbf{B} = i\omega\mu\varepsilon\mathbf{E} + \mu\sigma\mathbf{E} \approx \mu\sigma\mathbf{E} \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = q/\varepsilon \cong 0 \tag{1.2}$$

The electric current density $\mathbf{j}$ $[\frac{A}{m^2}]$ is proportional to the electric field according to Ohm's law:

$$\mathbf{j} = \sigma \mathbf{E}$$

where q $[\frac{As}{m^3}]$ is the volume density of charge, $\mathbf{B}$ the magnetic field $[\frac{Vs}{m^2}]$, $\mathbf{E}$ $[\frac{V}{m}]$ the electric field and $\omega = \frac{2\pi}{s}$ the angular frequency. The permittivity and permeability in the earth are assumed both to be approximately constant values ($\mu_r \approx 1$, $\varepsilon_r \approx 20$). The parameters $\mu = \mu_r \mu_o$ and $\varepsilon = \varepsilon_r \varepsilon_o$ approach in consequence the values of the vacuum (air; μ_o, ε_o). Due to the stationary approximation of the EM-field, the displacement current is negligible and the field propagate only by diffusion ($\omega\varepsilon \ll \sigma$; eq. 1.1).
By the diffusive process, the current density across (and perpendicular to the) conductivity interface is continuous. For a homogeneous media (i.e., $\nabla \sigma = 0$) the current density achieves thus $\nabla \cdot \mathbf{j} = 0$; otherwise surface charges q/ε would arise at the interface.
The incident field measured by MT corresponds to the TE-polarisation mode (tangential electric field at the surface; Schmucker and Weidelt [1975]). Therefore, since the propagation is a diffusive process (eq. 1.1), where the density current across interfaces is continuous, no surface charges (q/ε) at the air-earth interface occur (eq. 1.2) because the electric field component is tangential.

The penetration of the quasi-stationary fields $\mathbf{F} = \mathbf{E}, \mathbf{B}$ in a homogeneous earth (i.e, σ is constant) are described by applying the Laplacian vector operator (∇^2) to the second order differentials of the Maxwell's equations:

$$\nabla^2 \mathbf{F} = i\omega\mu\sigma\mathbf{F} = k^2 F \tag{1.3}$$

The term $k^2 = i\omega\mu\sigma$ $[1/m^2]$ is the *diffusion factor*, which describes the complex penetration depth $1/k$ ($[m]$) of the EM-field (e. g., Schmucker and Weidelt [1975]).
The penetration in depth of the EM field for a stratified earth (see below) is called the "response function" $C(\omega) = \frac{E_x}{i\omega B_y}$ by some authors (e.g., Weaver [1994]). In the case of a homogeneous earth this is $C(w) = 1/k$, as was outlined above. Also, the real part of the response function ($Re(C(w))$) represents physically the depth of the gravity center of the induced current density (Weidelt [1975]).

Skin depth
In terms of the diffusion factor describing the penetration in depth of the fields (eq. 1.3), the so-called "skin depth" ($\delta(\omega)$ $[m]$) in a homogeneous earth is defined as:

$$\delta(\omega) = \sqrt{\frac{2}{|k^2|}} = \sqrt{\frac{2}{\omega\mu\sigma}}, \tag{1.4}$$

which represents the exponential decay of the EM-field amplitude with depth. At depth $\delta(\omega)$, the EM-field amplitude has dropped by $1/e$ with respect to its value at the surface. The skin depth of the EM-fields increases with the period (T $[s]$), namely proportional to the square root of T ($T = \frac{2\pi}{\omega}$; eq. 1.4).

For a general **1-D stratified Earth** of N layers the penetration in depth of the EM-fields measured at the surface ($C_1(\omega)$) is solved iteratively, with a recursive formula (Wait [1970]) described by the EM-response function $C_i(\omega)$. The index i refers to the EM-response measured at the top of the layer i (Weaver [1994]):

$$C_i(\omega) = \frac{1 - r_i \exp(-2k_i d_i)}{k_i \left[1 + r_i \exp(-2k_i d_i)\right]} \tag{1.5}$$

where $i = N - 1, \, , N - 2, \, , .., 1$ and

$$r_i = \frac{1 - k_i C_{i+1}(\omega)}{1 + k_i C_{i+1}(\omega)} \, .$$

d_i is the thickness of the layer i and $k_i = \sqrt{i\omega\mu\sigma_i}$ the diffusion factor in the layer (of conductivity σ_i; eq. 1.3).
The bottom layer N (with depth $\rightarrow \infty$) has the response function $C_i(\omega) = \frac{1}{k_N}$.
Thereby, an analogous skin depth (eq. 1.4) for a stratified earth is:

$$\delta(\omega) = \sqrt{2|C_1(\omega)|} \tag{1.6}$$

1.1.1 Magnetotellurics

The Magnetotelluric method uses the horizontal components of the electric and magnetic fields to determine an electrical impedance Z (see below) as a function of frequency ($Z = i\omega C(\omega)$).
In a **2-D earth** with a strike along the horizontal x-axis (i.e., $\frac{\partial}{\partial x} = 0$) and conductivity $\sigma(y, z)$ (z positive downwards), the Maxwell's equations are decoupled into two polarisation modes. The decoupling is valid since the EM-fields are treated as plane waves, which means that the interaction between electric and magnetic fields are always orthogonal with each other and therefore the horizontal component of the magnetic field tangential to the conductivity strike does not depend on the magnetic field component perpendicular to it.
In this context, the so-called *TE-polarisation* mode refers to the *tangential electric* field and the *TM-polarisation* mode to the *tangential magnetic* field; both components are tangential with respect to the strike (x-axis) of the conductivity structure:

$$\begin{array}{ll}
\textbf{TE}-\textbf{polarisation}: E_x,\, B_y & \textbf{TM}-\textbf{polarisation}: B_x,\, E_y \\
\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu\sigma E_x & \frac{-\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} = i\omega B_x \\
\frac{-\partial E_x}{\partial z} = i\omega B_y & \frac{\partial B_x}{\partial z} = i\mu\sigma E_y \\
\frac{-\partial E_x}{\partial y} = i\omega B_z & \frac{\partial B_x}{\partial y} = i\mu\sigma E_z
\end{array} \tag{1.7}$$

Impedance tensor

The electrical impedance Z [mV/T] is the ratio between the electric and magnetic field components, which comes from the matrix form relation: $\mathbf{E} = \underline{\mathbf{Z}}\mathbf{B}$.
In a **homogeneous media**, the ratio of the orthogonal components is (from eq. 1.3):

$$Z = i\omega/k \tag{1.8}$$

In a general **3-D earth**, the impedance is expressed in matrix form in cartesian coordinates (x, y horizontal and z positive downwards):

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \underbrace{\begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}}_{\underline{\mathbf{Z}}} \begin{pmatrix} B_x \\ B_y \end{pmatrix} \tag{1.9}$$

Thus each tensor element is $Z_{ij} = E_i/B_j$ $(i, j = x, y)$.
In a **2-D earth** the diagonal elements of $\underline{\mathbf{Z}}$ vanish (in the 2-D strike coordinate system): $Z_{xx} = Z_{yy} = 0$.
In a **1-D layered earth**, besides having vanishing diagonal elements, the off-diagonal elements are related in the form: $Z_{xy} = -Z_{yx}$.

The tensor $\underline{\mathbf{Z}}$ can be rotated to any other coordinate system by an angle θ with the rotation matrix R:

$$\underline{\mathbf{Z}}_{\mathbf{m}} = R\underline{\mathbf{Z}}R^T \quad \text{where} \quad \mathrm{R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

with positive θ describing a c.w. rotation from the coordinate system of $\underline{\mathbf{Z}}_{\mathbf{m}}$.

Impedance phase

The phase of the impedance element describes the phase shift between the electric and magnetic field components:

$$Z = \left|\frac{E_i}{B_j}\right| e^{i\varphi} \rightarrow \quad \varphi = \psi_{E_i} - \psi_{B_j} = \arctan\left(\frac{\mathrm{Im}(Z_{ij})}{\mathrm{Re}(Z_{ij})}\right)$$

where $i, j = x$, y and φ_{E_i,B_j} is the phase of the electric and magnetic field, respectively.
In a **homogeneous earth** the impedance phase (eq. 1.8) is:

$$Z = \frac{i\omega}{k} = \sqrt{\frac{\omega}{\mu\sigma}}\sqrt{i} \ \rightarrow \ \varphi = \pi/4$$

which means that the electric field precedes the magnetic field by 45°, given by the diffusive process of the EM plane waves propagation.
In a **1-D layered earth** the phase increases over 45° when the EM-response ($C_1(\omega)$; eq. 1.5) penetrates into a higher conductivity media. By analogy, the phase decays below 45°

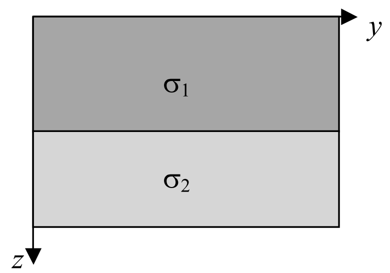

Figure 1.1:
Scheme of a 1-D layered earth of conductivity σ_1 and σ_2. The impedance phase (φ) behaviour by the diffusive process is:
$\sigma_1 < \sigma_2 \;\Rightarrow\; \varphi > \pi/4$
$\sigma_1 > \sigma_2 \;\Rightarrow\; \varphi < \pi/4$

for the EM-response penetrating into a less conductive media. This means that by the diffusive process the phase shift between the orthogonal electric and magnetic field components attenuates when the fields penetrates into a less conductive media:

In the 1-D/2-D case **the phases lie in the I or III quadrant** ($[0, pi/2]$ or $[\pi, 3\pi/2]$), which means that the real and imaginary parts of Z_{xy} (or Z_{yx}) have equal sign. This is due to the principle of causality of the interaction between electric and magnetic fields induced in the earth; i.e., any secondary field induced due to a conductivity contrast should necessarily postdate the primary incident field (the initial source).
By convention, the element Z_{xy} is defined as positive and therefore Z_{yx} is negative, implying an impedance phase in the I and III quadrant, respectively [1].
The principle of causality should be generally satisfied in a **3-D earth**. There can be particular conductivity structures, however, which can violate this principle, as was discussed for the first time by Egbert [1990]. The present thesis also gives insights into this particular case, by applying a current channeling analysis (Chapter 6) to field data and synthetic data from 3-D models (Chapters 7, 8).

Apparent resistivity

The electrical resistivity in depth (the inverse of σ) of an inhomogeneous earth can be determined indirectly by measurements realized at the surface. In this sense, an "apparent" resistivity of the true value can be inferred by the EM fields of the corresponding penetration depths.
The apparent resistivity $\rho_{a_{ij}}$ [Ωm] ($i, j = x, y$) is defined in terms of the impedance tensor element (eq. 1.9) by the form:

$$\rho_{a_{ij}} = \mu |Z_{ij}|^2 / \omega$$

In case of a homogeneous earth (of conductivity σ), the apparent resistivity reflects the true value of the earth's conductivity:
$\rho_a = 1/\sigma = \mu |Z|^2 / \omega$.

[1]The impedance phases in this thesis will be presented all in the I and II quadrants. Thus a phase originally laying in the III, IV quadrant is moved to the I, II by adding π

1.1.2 Geomagnetic Deep Sounding

The Geomagnetic Deep Sounding method (GDS) measures the horizontal and vertical components of the magnetic fields. The ratio between the horizontal and vertical components are most sensitive to the lateral variations in conductivity. The horizontal component perpendicular to a conductivity interface sharply increases at the side of the contact of higher conductivity, while the vertical component increases at the side of lower conductivity (Ritter [1996]). In a 2-D earth, the magnetic field components arisen from the TE-polarisation mode (eq. 1.7) identify the spatial lateral variations in conductivity.
For EM plane waves, the vertical and horizontal components of the sum of external and internal magnetic fields are linearly related in the form:

$$B_z = T_x B_x + T_y B_y$$

T_x and T_y are the **magnetic transfer functions** (or tipper). They contain the ratio between the vertical and the horizontal component, thus give evidence regarding the lateral conductivity distribution. They can be represented as vectors, assigned with the name **induction arrows** (Parkinson [1959], Schmucker [1970]):

$$\mathbf{T} = T_x \hat{x} + T_y \hat{y}$$

which is separated into its real **P** and imaginary parts **Q**:
$\mathbf{P}, \mathbf{Q}(\omega) = Re, Im[T_x(\omega)]\hat{x} + Re, Im[T_y(\omega)]\hat{y}$.
In the so called Wiese convention, the real parts of **T** point away from higher conductivity values (fig. 1.2).

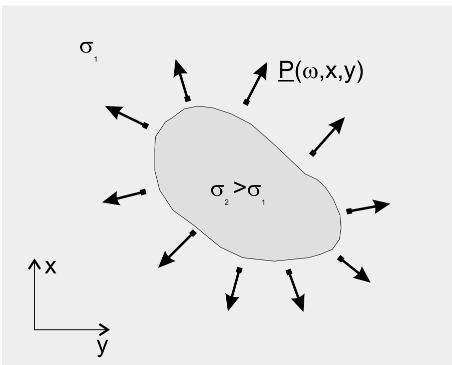

Figure 1.2:
Representation of real induction arrows ($P(\omega)$) in a lateral spatial variation of conductivity ($\sigma_{1,2}$). The arrows point away from the higher conductive media.

The length $L_{P,Q}$ and direction $\phi_{P,Q}$ of the arrow (with respect to the x-axis) is:

$$L_{P,Q} = \sqrt{Re, Im(T_x)^2 + Re, Im(T_y)^2} \qquad \phi_{P,Q} = \arctan \frac{Re, Im(T_y)}{Re, Im(T_x)}$$

In a 2-D earth the direction of the induction arrows is perpendicular to the orientation of the regional strike, provided that they are not influenced by local shallow 3-D conductivity structures (magnetic distortion; next section).

1.2 Dimensionality and distortion of the transfer functions

The transfer functions have specific characteristics according to the dimensionality of the conductivity structure. The ideal case of a 2-D model of the underground would fulfill the expectations of geophysicists, since the measurements are usually carried out along a transect. 2-D modeling is much simpler to elaborate than a 3-D one. The non-uniqueness problem becomes more remarkable in the 3-D modeling because of the larger numbers of model parameters to be solved versus an insufficient number of observations. Besides, 3-D model inversion schemes are either still under study or are seldomly available because of their complicated application. For these reasons, one deals in general with a 2-D model assumption, which is only an approximation of the real earth since 3-D conductivity structures are usually present (in terms of the penetration depths of the EM-fields).

In this chapter diverse physical aspects are presented in which the electromagnetic fields are affected by departures from an ideal 2-D model. The idea is to recover the regional 2-D information (as the regional strike direction) if and only if the electromagnetic fields are distorted by electrostatic currents (DC) produced by local 3-D structures. I will first introduce the different concepts of distortion cases (section 1.2.1) so as to describe in the further subsections the mathematical aspects of them. First is shown a method for recovering the 2-D impedance tensor where no distortion analysis is considered (section 1.2.2). The following sections refer to the distortion (section 1.2.3) and dimensionality analysis (section 1.2.4), where the sections of tensor decompositions introduce the most common methods used for recovering 2-D information (sections 1.2.5, 1.2.6).

1.2.1 Introduction

The MT transfer functions can be affected by DC-currents, in MT referred to as *galvanic distortion*[2], produced by local conductive bodies, small 3-D structures which can impede the regional exploration of the underground. In this sense, they are a cause of distortion for the regional fields[3] and therefore the goal is to remove this effect and recover the regional information. Such anomalous structures (or local anomalies) should be at shorter depths than the skin depth of the induced electromagnetic (EM) fields under consideration in order to neglect induction effects and thus have an electrostatic DC (galvanic) effect prevailing (e.g., Vozoff [1987]). Also, the local anomaly should be placed electrically far (in terms of a skin depth) from the regional conductivity contrast in order to neglect the inductive coupling between both structures. Thereby the regional fields inside the small body can be assumed uniform, implying that the anomalous electrostatic field is in-phase with the regional field (e.

[2]The term galvanic refers here to an electrostatic effect, although the word "galvanic" comes originally from electro-chemistry. Two cells (cathode, anode) conformed by silver and copper are set in a solution of nitrate. These galvanic cells are basically batteries to produce electricity. This has nothing to do with the galvanic distortion referred to in the MT literature, which is a distortion produced by DC-currents.

[3]The regional fields refer the electromagnetic response of the media without the local structures.

g., Groom and Bailey [1991]). The intensity and direction of the DC currents depend on the conductivity contrast between the regional space and the anomaly, as well as its geometry. In this sense, a 2-D regional model superposed by shallow local anomalies (i.e., the distorter) is identified in magnetotellurics as the "superposition model", where "galvanic" distortion (i.e., DC currents produced by the anomalies) affects the regional fields.

Under the above mentioned assumptions, the distortion of the regional electric fields due to local conductive heterogeneities, referred to as *telluric galvanic distortion* in MT, is frequency independent. Then, telluric distortions can effect considerably the measured electric field amplitudes in the frequency band under consideration, being especially stronger at lower frequencies where the amplitudes of the regional fields decrease. The so-called *static shift* effect refers to this type of distortion, where the apparent resistivity (ρ_a) curves are affected by a parallel offset.

The DC-current induces also anomalous magnetic fields[4], which are proportional to and in-phase with the regional electric fields (e.g., Groom and Bailey [1991]). They decrease in magnitude proportional to the square root of the frequency (Groom and Bailey [1991]). Thus this magnetic effect, in MT identified as *magnetic galvanic distortion*, is much smaller than the telluric distortion effect at lower frequencies, at which the EM-field skin depths considerably increase in comparison with the dimensions of the local structure. Therefore the magnetic effect can be considered to vanish at lower frequencies. Of course, under non-negligible magnetic galvanic distortions, the magnetic transfer functions are also effected by the local structures.

A local 2-D conductivity macro-scale anisotropy striking differently from the regional 2-D structure can also be a galvanic distorter, provided that its scale is smaller than the penetration depths of the fields. But MT measurements are usually unable to distinguish between an intrinsic anisotropy and a macro scale (or pseudo-) anisotropy because of the limited penetration depth range of the measured fields. Thus a conductivity structure consisting of a bundle of lamellae is not truly anisotropic but can have the same gross physical properties for MT.

A pseudo-anisotropy of finite extensions, conceived as highly conductive vertical thin dikes −due for example to salinary fluids in shear zones− can strongly channel all the currents into one direction. The horizontal electric field components will be thus strongly polarised into one single direction. In this case, a tensor decomposition scheme for recovering the regional fields fails, being unable to determine the regional strike. This special case of distortion was treated by assuming an anomalous DC-current density in a preferred direction to infer the presence and orientation of such strong scatterers, and the theory is presented in Chapter 6.

[4]The attribute "anomalous" for the magnetic fields is in the sense that these are produced by the local anomaly.

1.2.2 The impedance Eigenvalues for a 2-D model

In an ideal 2-D model case, the regional impedance tensor in the coordinate system of the 2-D structure has the form

$$\mathbf{Z^r} = \begin{bmatrix} 0 & Z^r_{xy} \\ Z^r_{yx} & 0 \end{bmatrix} \tag{1.10}$$

Due to the orthogonality between the electric (E-) and the magnetic (B-) field components involved in each polarization mode (XY or YX), the product between E- and B-vectors is

$$\mathbf{E} \cdot \mathbf{B} = 0 \tag{1.11}$$

This condition is a consequence of the transverse electromagnetic (EM) waves property of the fields (Eggers [1982]). Now, the regional impedance tensor ($\mathbf{Z_r}$) can be rotated by any angle θ to an arbitrary coordinate system with respect to the regional one to bring a tensor $\mathbf{Z_m}$ of the form:
$\mathbf{Z_m} = R(\theta) \cdot \mathbf{Z^r} \cdot R^T(\theta)$,

where $R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

is the rotation matrix.

By considering the relation $\mathbf{E} = \mathbf{Z_m} \cdot \mathbf{B}$, which is equivalent to
$E_x = Z_{xx}B_x + Z_{xy}B_y$
$E_y = Z_{yx}B_x + Z_{yy}B_y$

we can expand the relation given in eq.(1.11) in the form

$$\begin{aligned} \mathbf{E} \cdot \mathbf{B} &= E_x B_x + E_y B_y \\ &= Z_{xx}B_x^2 + (Z_{xy} + Z_{yx})B_x B_y + Z_{yy}B_y^2 \end{aligned} \tag{1.12}$$

Imposing eq. 1.12 to vanish (i.e., recalling the orthogonality of the transverse EM waves), implies that the tensor $\mathbf{Z_m}$ can be transformed to a tensor Λ containing the eigenvalues :
$\Lambda = \begin{bmatrix} 0 & \lambda_i \\ -\lambda_i & 0 \end{bmatrix}$

where i=1, 2 denotes two possible solutions, i.e., two different eigenvalues (λ_i) can validate the orthogonal equation $\mathbf{E} \cdot \mathbf{B} = 0$, result which will be demonstrated in the following.
The fields $\mathbf{E}$ and $\mathbf{B}$ are transformed to the vectors E^i and B^i, respectively, constituting the eigen-vectors of the equation system. We observe that Λ corresponds to the impedance tensor

of a 1-D medium. The problem reduces to find λ_i, E^i and B^i for the given impedance $\mathbf{Z_m}$ fulfilling:
$E^i = \mathbf{Z_m} B^i = \lambda B^i$ then $(Z - \Lambda)B^i = 0$,
and the solution is obtained from the characteristic function $\det(Z - \Lambda) = 0$ resulting in a quadratic equation in the eigenvalue λ_i:
$0 = \det Z - (Z_{xy} - Z_{yx})\lambda_i + \lambda_i^2$

which implies two eigenvalues thereof, each conforming the elements of a 1-D tensor Λ. Since we are dealing with a 2-D case, the eigenvalues $\lambda_{1,2}$ correspond to the TE(=XY) and TM(=YX) polarisation mode impedance elements of the regional coordinate system, with solutions obtained from the above equation:

$$Z^r_{xy}, Z^r_{yx} = -\frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 - 4\det|\mathbf{Z_m}|} \tag{1.13}$$

where $\alpha = (Z_{xy} - Z_{yx})$.

It can be easily demonstrated, that the values α and $\det|\mathbf{Z_m}|$ are independent of the rotation angle −which means that in any coordinate system the results for Z^r_{xy}, Z^r_{yx} will be the same−. The eigenvalues are therefore rotational invariant and recover the regional tensor regardless of the knowledge about the 2-D strike direction. This expression was formulated by Eggers [1982]. This solution is an approximation of the 2-D regional tensor under 3-D galvanic distortion. In this case, the eigenvalues are local site-dependent relative to the position of the 3-D anomaly and are a mixture of the 2-D regional impedances (Groom and Bailey [1991]). However, in case of weak distortion, i.e., the electric fields are slightly deviated from the regional electric fields, the eigenvalues are nearly recovering the 2-D impedances. Furthermore with the addition of noise, the eigenvalues have been shown to be generally more stable than the elements of an impedance tensor rotated to the regional coordinate system (Groom and Bailey [1991]).

1.2.3 Telluric and magnetic galvanic distortions

In the general case of telluric galvanic distortion effecting the 2-D regional response, the elements of the measured impedance tensor ($\mathbf{Z}$) in the regional coordinate system will be proportional to and in-phase with the elements of the regional impedance $\mathbf{Z_r}$ in the form (Bahr [1988]):

$$\mathbf{Z} = D_e \mathbf{Z^r} = D_e \begin{bmatrix} 0 & Z^r_{xy} \\ Z^r_{yx} & 0 \end{bmatrix} \tag{1.14}$$

where D_e is the telluric distortion matrix of frequency independent real numbers, a consequence of the distorted current which is in-phase with the regional electric field. The above relation comes from the effect observed on the total field E under the presence of galvanic currents. D_e is responsible for the frequency independent distortion of the regional electric field (E_r). Then, the total observed field E is $E = D_e E^r$.

Until now we have just considered the telluric distortion assuming that magnetic effects are negligible. In the presence of magnetic galvanic distortion, an anomalous magnetic field B^a is originated from the galvanically distorted current, which is proportional to and in-phase with the regional field E_r (e.g., Groom and Bailey [1991]): $B^a = D_m E^r$.
D_m is the magnetic distortion matrix of frequency independent real numbers. Then, the observed field B will be the sum of the anomalous field B^a and regional magnetic field B^r :
$B = B^r + B^a = B^r + D_m E^r$.
From the above relations given for the total observed E and B fields, it can be derived that the elements of the observed impedance tensor will be proportional to the elements of the regional tensor Z^r, expressed in terms of the magnetic and distortion matrix in the form (Chave and Smith [1994]):

$$\mathbf{Z} = D_e \mathbf{Z^r} (I + D_m \mathbf{Z^r})^{-1} \tag{1.15}$$

The product elements $D_m \, \mathbf{Z^r}$ are non-dimensional and correspond to the magnetic effect. Thereby the impedance is no more frequency independent distorted, since the elements of $\mathbf{Z^r}$ are contained in the magnetic effect.

1.2.4 Skew parameters of dimensionality

In the telluric galvanic model (eq.1.14), the impedance phases of each column element pair are equal, since the matrix D_e contains real numbers. This is due to the in-phase condition accomplished between the regional and the anomalous electric fields. If the anomalous magnetic fields are not negligible, then the 2-D superposition model of eq.(1.15) governs the measured impedance. Therefore a phase difference between the elements of each column is expected if magnetic effects are present. In this sense Bahr (1988) found a non-dimensional rotational invariant parameter which is a measure for these phase differences, identified as the *phase sensitive skew* (η):

$$\eta = \frac{\sqrt{2\,|\mathrm{Re}\, Z_{xx}\, \mathrm{Im}\, Z_{yx} - \mathrm{Re}\, Z_{yy}\, \mathrm{Im}\, Z_{xy} + \mathrm{Re}\, Z_{xy}\, \mathrm{Im}\, Z_{yy} - \mathrm{Re}\, Z_{yx}\, \mathrm{Im}\, Z_{xx}|}}{|\alpha_2|} \tag{1.16}$$

where
$\alpha_2 = Z_{xy} - Z_{yx}$,
which is also rotational invariant.
A skew value of zero supports the validity of the telluric distortion hypothesis, i.e., a perfect regional 2-D model can be identified. Greater values indicate the departure of this assumption. Bahr gave a limit of 0.3 to test the validity of the galvanic model, where small phase differences due to the galvanic magnetic effects could take place. A surpass of this limit could reflect the non-validity of the galvanic magnetic distortion, which means that 3-D inductive structures are present.

Another non-dimensional rotational invariant parameter which measures the departure from an ideal 2-D model is the *skew* given by Swift [1967]:

$$s = |\alpha_0| / |\alpha_2| \tag{1.17}$$

defined also in terms of the rotational invariant α_2 shown previously, and $\alpha_0 = Z_{xx} + Z_{yy}$

which is also rotational invariant.
A value of zero supports an ideal 1-D or 2-D model assumption. Values greater than 0.1 can be identified whether with a 3-D model or a galvanic 2-D model.

1.2.5 Telluric tensor decomposition

Under telluric galvanic distortion assumptions Groom and Bailey [1989b] proposed a tensor decomposition method where the distortion matrix D_e (eq.1.14) is separated into three matrices and a real number. In the coordinate system of the regional 2-D structure the measured impedance Z is expressed as:

$$\mathbf{Z} = D_e \mathbf{Z^r} = (gTSA)\mathbf{Z^r}$$

where g is a real number denominated the "gain" factor. The matrices T and S are called the *twist* and *shear* tensor of the telluric deformation, respectively:

$$T = \frac{1}{\sqrt{1+\mathrm{t}^2}} \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} \quad S = \frac{1}{\sqrt{1+e^2}} \begin{bmatrix} 1 & e \\ e & 1 \end{bmatrix} \tag{1.18}$$

The twist parameter ($t = \tan(\beta_t)$) represents the electric field rotation through a clockwise angle β_t due to additional anomalous DC-currents. T is normalised by this twist parameter. The tensor S is also normalized by the shear parameter $e = \tan(\beta_e)$ with the physical meaning of deflecting the electric field by an angle β_e, clockwise with respect to the x axis and counter-clockwise for the other horizontal axis. The name shear is motivated from the analogy made with the mechanical strain of bodies, with shear representing the deformation of its principal axes. Both twist and shear angles under maximal telluric deformation should not exceed the $|45°|$.
S represents the anisotropy or splitting tensor, which has the geometrical effect of stretching the two electric field components by different factors:

$$A = \frac{1}{\sqrt{1+s^2}} \begin{bmatrix} 1+s & 0 \\ 0 & 1-s \end{bmatrix}. \tag{1.19}$$

This distortion does not refer to the electrical anisotropy mentioned in the introduction, but is an effect produced by the small scale 2-D and/or 3-D scatters.
Groom and Bailey [1989b] have shown that the anisotropy A and the gain factor g cannot be determined due to the equivalence:

$$\mathbf{Z} = (gTSA)\mathbf{Z_r} = TS\tilde{\mathbf{Z}}_\mathbf{r}$$

where the impedance $\tilde{\mathbf{Z}}_\mathbf{r}$ still represents a 2-D tensor. The final expression for this reduced tensor decomposition form by using eqs. 1.18 is:

$$\mathbf{Z} = \begin{bmatrix} 1-te & e-t \\ e+t & 1+te \end{bmatrix} \begin{bmatrix} 0 & Z^r_{xy} \\ Z^r_{yx} & 0 \end{bmatrix} \tag{1.20}$$

The product gA represents the static shift effect mentioned at the beginning of this section, which can not be determined by the tensor decomposition. For an arbitrary coordinate system, for example the measured coordinate system, the impedance tensor will then be:

$$\mathbf{Z_m} = R(TS)\tilde{\mathbf{Z}}_\mathbf{r}R^T \tag{1.21}$$

where R is the rotation matrix.

The goal is to determine the strike direction of the regional 2-D structure, once the twist and shear have been estimated. The system consists of 7 parameters to be solved for:

$\mathrm{Re}, \mathrm{Im}(Z^r_{xy})$, $\mathrm{Re}, \mathrm{Im}(Z^r_{yx})$, t, e, θ

where θ represents the strike angle, t and e the distortion parameters twist and shear, respectively (eq. 1.18), and Z^r_{xy}, Z^r_{yx} are the regional impedances. The system is overestimated, since there are 8 known variables corresponding to the 4 complex impedance elements of $\mathbf{Z_m}$.

The impedance $\mathbf{Z_m}$ can be expressed in terms of the Pauli spin matrices:

$$\Sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \Sigma_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \Sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and

$$\mathbf{Z_m} = \frac{1}{2}\left(\alpha_0 I + \alpha_1 \sum_1 + \alpha_2 \sum_2 + \alpha_3 \sum_3\right)$$

where

$\alpha_0 = Z_{xx} + Z_{yy}, \quad \alpha_1 = Z_{xy} + Z_{yx}$

$\alpha_2 = Z_{xy} + Z_{yx}, \quad \alpha_3 = Z_{xx} - Z_{yy}.$

Thereof the following system of non-linear equations in terms of the tensor decomposition of eq. 1.21 can be obtained:

$$\begin{aligned} \alpha_0 &= t(Z^r_{xy} + Z^r_{yx}) + e(Z^r_{xy} - Z^r_{yx}) \\ \alpha_1 &= [(1-et)Z^r_{xy} - (1+et)Z^r_{yx})]\cos 2\theta - [(e+t)Z^r_{xy} + (e-t)Z^r_{yx}]\sin 2\theta \\ \alpha_2 &= -(1-et)Z^r_{xy} - (1+et)Z^r_{yx} \\ \alpha_3 &= -[(1-et)Z^r_{xy} - (1+et)Z^r_{yx}]\cos 2\theta - [(e+t)Z^r_{xy} + (e-t)Z^r_{yx}]\sin 2\theta \end{aligned} \tag{1.22}$$

From these equations, the distortion parameters t and e, the strike angle θ and the regional impedances can be estimated with a least square method. The test of misfit between the tensor decomposition model and the measured data is:

$$\chi^2 = \frac{\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{2} |\hat{Z}_{ij} - Z_{ij}|^2}{\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{2} |Z_{ij}|^2} \tag{1.23}$$

where $\hat{Z}_{ij}$ is the model impedance. If the tensor elements are assumed normally distributed, eq. 1.23 represents a χ^2 test of one degree of freedom, leading to an expected value of 1. Thus, an acceptable tensor decomposition fit with the telluric galvanic model is indicated by a $\chi^2 \approx 1$.

1.2.6 Telluric and magnetic tensor decomposition

Chave & Smith (1994) extended the telluric decomposition method to include also magnetic galvanic distortions. The tensor decomposition model to be solved is that from eq.(1.15), where the telluric matrix D_e is the product of the twist and shear matrices (eq. 1.18) as in the telluric decomposition scheme. The magnetic galvanic matrix D_m is defined:

$$D_m = \begin{bmatrix} \gamma & D_{xy} \\ D_{yx} & \varepsilon \end{bmatrix}$$

It can be demonstrated that the off-diagonal elements of the magnetic distortion matrix are not distinguishable, which means physically that the anomalous magnetic fields aligned with the induced regional magnetic field are being absorbed in it (Chave and Smith [1994]). Only the diagonal elements can be determined from the decomposition, representing the magnetic effect at right angles to the regional magnetic field. The non-linear systems of equation (1.22) contains now two more unknown variables; the magnetic distortion parameters γ and ε. Then, there are 9 parameters to be determined, which can be solved by considering at least two frequencies simultaneously.
The system was also extended to solve the magnetic transfer functions. This leads to 12 real equations and it can be solved for 12 parameters: $\mathrm{Re}, \mathrm{Im}(Z^r_{xy})$, $\mathrm{Re}, \mathrm{Im}(Z^r_{yx})$, the regional magnetic transfer function (2more),e,t,θ,γ, ε and an additional magnetic parameter associated to the vertical magnetic field component distortion. Chave & Smith (1994) used a modified Levenberg-Marquard algorithm to solve the system. They recommend also the χ^2 misfit test of eq.(1.23) but modified in the denominator. The impedance amplitudes are replaced by the data errors. This functional has now 4 degrees of freedom for normally distributed tensor elements; therefore an acceptable model decomposition fit should not exceed this value. For simultaneous frequencies the number of degrees of freedoms increases.

Smith [1997] simplified the telluric and magnetic tensor decomposition method, where, instead of inverting matrices as in eq.(1.15), he found an equivalent expression of the form:

$$\mathbf{Z} = D_e \mathbf{Z^r} - \mathbf{Z}\, D_m \mathbf{Z^r}. \tag{1.24}$$

The telluric and magnetic matrices D_e, D_m each contain only two parameters to be solved, reducing to

$$D_e = \begin{bmatrix} 1 & c \\ b & 1 \end{bmatrix} \quad and \quad D_m = \begin{bmatrix} \gamma & 0 \\ 0 & \varepsilon \end{bmatrix}, \tag{1.25}$$

respectively.
The diagonal elements of the telluric matrix are set to one because they are absorbed in the regional impedance, corresponding to the known unsolved problem of gain + anisotropy effects explained in section 1.2.5 (Groom and Bailey [1989b]).
As mentioned above, the reduction of the off-diagonal elements to zero in the magnetic matrix D_m is also due to the indetermination in solving them. This means that the anomalous magnetic fields aligned with the induced regional magnetic field are being absorbed in it. Given the indeterminacy of the parallel distorted field, the off-diagonal elements are assumed

to vanish (Smith [1997]).

From this parameterizations Smith (1997) found exact solutions for each distortion parameter as a function of the measured impedance elements assumed to be in the 2-D regional coordinate system:
magnetic

$$\gamma = \frac{\operatorname{Re} Z_{xy} \operatorname{Im} Z_{yy} - \operatorname{Im} Z_{xy} \operatorname{Re} Z_{yy}}{\operatorname{Re} Z_{xy} \operatorname{Im}(\det Z) - \operatorname{Im} Z_{xy} \operatorname{Re}(\det Z)}, \quad \varepsilon = \frac{\operatorname{Re} Z_{yx} \operatorname{Im} Z_{xx} - \operatorname{Im} Z_{yx} \operatorname{Re} Z_{xx}}{\operatorname{Re} Z_{yx} \operatorname{Im}(\det Z) - \operatorname{Im} Z_{yx} \operatorname{Re}(\det Z)} \tag{1.26}$$

telluric

$$b = \frac{\operatorname{Re} Z_{yy} \operatorname{Im}(\det Z) - \operatorname{Im} Z_{yy} \operatorname{Re}(\det Z)}{\operatorname{Re} Z_{xy} \operatorname{Im}(\det Z) - \operatorname{Im} Z_{xy} \operatorname{Re}(\det Z)}, \quad c = \frac{\operatorname{Re} Z_{xx} \operatorname{Im}(\det Z) - \operatorname{Im} Z_{xx} \operatorname{Re}(\det Z)}{\operatorname{Re} Z_{yx} \operatorname{Im}(\det Z) - \operatorname{Im} Z_{yx} \operatorname{Re}(\det Z)} \tag{1.27}$$

Thereby the solution for the regional impedance (eq. 1.24) can be directly solved for single frequencies, provided that the regional strike angle is known. Thus, in the regional coordinate system the regional response Z^r as function of the measured impedance Z is:

$$\begin{aligned} Z^r_{yx} &= Z_{yx}/(1 - \varepsilon Z_{yy}) \\ Z^r_{xy} &= Z_{xy}/(1 - \gamma Z_{xx}) \end{aligned} \tag{1.28}$$