8 Delayed Feedback Control

Chaotic systems are characterized by an extreme sensitivity to perturbations. Nevertheless, it has been shown [1] that it is possible to select a desired behavior naturally present in a chaotic system and then stabilize this behavior by applying only tiny changes to an accessible parameter of the system.

The stabilization of a chemical system with unstable periodic orbits (UPO) in the phase space can be achieved by introducing additional degrees of freedom. For system characterized by UPO embedded within a strange attractor a discrete time-dependent perturbation approach was first introduced by Ott and coworkers [2]. Pyragas [3] proposed an experimentally more convenient continuous control. The stabilization of unstable periodic orbits of a chaotic system was achieved either by combined feedback, with the use of an external oscillator, or by delayed self-controlling feedback. Mere external forcing (like a periodic perturbation) has the same effect, but requires a comparatively large perturbation, since the obtained periodic orbits differ from the ones of the unperturbed system.

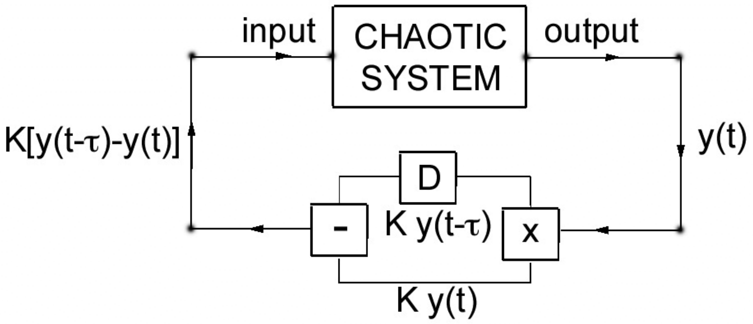

Fig. 8.1 Block diagram of the delayed feedback control. D is a delay line.

The delayed feedback control principle (see the block diagram in Fig. 8.1) consists in supplying the system with a perturbation of the form:

$$F(t) = K [y (t - \tau) - y (t)]. \qquad (1)$$

Here τ is a delay time: if this delay coincides with the period of the unstable periodic orbits of the system, and the strength of the feedback K is appropriate, then stabilization can be achieved.

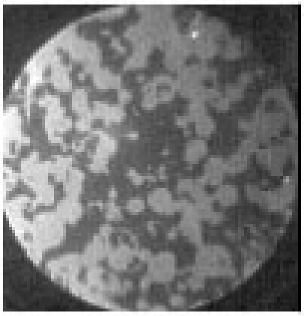

Fig. 8.2 A PEEM image of the CO oxidation on Pt in a chaotic oscillatory regime.

Examples of the application range from chemical systems such as the BZ reaction [4], [5] to the enzymatic proxidase-oxidase reaction [6], and the control of flow rate in the chlorine dioxide-iodide reaction [7].

Parmananda and coworkers [8] report numerical and experimental results indicating successful stabilization of unstable steady states and periodic orbits in an electrochemical system. Applying a continuous delayed-feedback technique periodic and chaotic oscillations are suppressed via stabilization of steady state. Chaotic dynamics can be as well converted to periodic behavior. In all cases the feedback perturbation vanishes as a target state is attained.

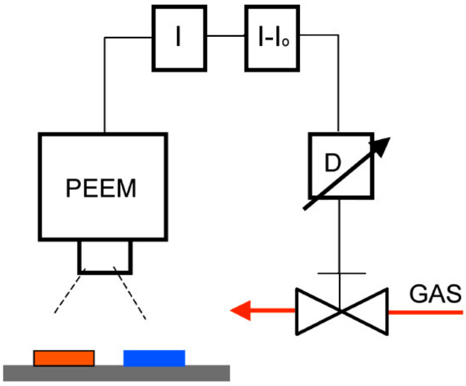

Fig. 8.3 Schematic of the delayed feedback setup used in the experiment.

In a regime of chaotic oscillations (see the PEEM image in Fig. 8.2) the rate in the CO oxidation on platinum exploits an irregular behavior have been influenced by a delayed feedback control of the CO gas inlet. The slightly modified method we used (illustrated schematically in Fig. 8.3) consists in subtracting from the integrated PEEM signal a the central value of the oscillations (estimated from the data shown on the left-hand side in Fig. 8.3), multiplying by a factor K and feeding back to the pressure control with an adjustable time delay.

As shown in Fig. 8.4, as soon the delayed feedback is switched on the oscillations become periodic. Moreover the amplitude of the oscillations can also be modified in dependence from the choice of the delay time: for long delays the amplitude remains comparable to the chaotic oscillations, whereas for short delays the system can be practically locked at the centre of the oscillations.

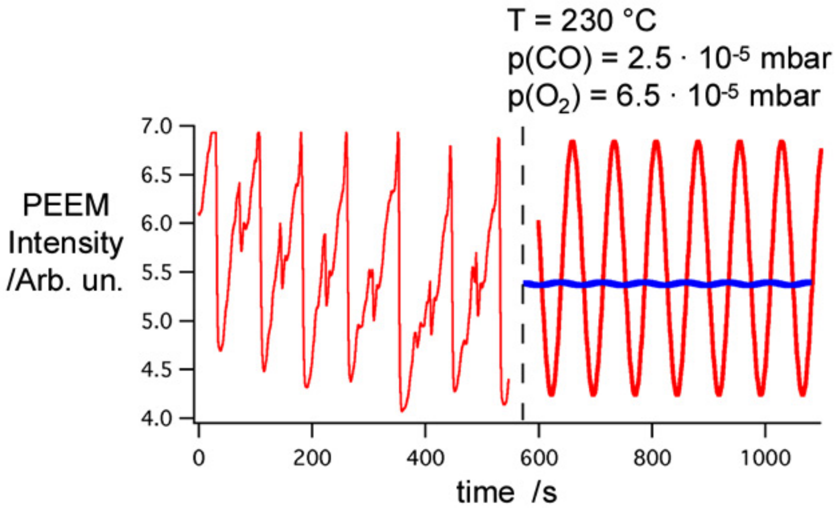

Fig. 8.4 Chaotic oscillations of the reaction rate as detected by the integrated PEEM intensity. At 580 s the feed back is switched on. The two curves represent the obtained oscillation a for long delay (large amplitude) and short delay, respectively.

An interesting application of this method is likely to be an improvement of the performance of a reactor. For instance, recent laboratory studies have shown that a modulation in the feed composition can lead to large increases in conversion in the so-called *periodic operation* of a continuous reactor [9]. Delayed feedback-driven periodic operation can therefore become an appealing engineering tool for the control of conversion or selectivity in a chemical reactor.

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