Appendix A

The adiabatic approximation

The essence of the adiabatic approximation is as follows. Following [118] we suppose that we have a system of linear differential equations:

$$\frac{d}{dt} \rho(t) = \dot{Z}(t) \rho(t),$$  \hfill (A.1)

where $\rho(t)$ is the n-dimensional vector, while $\dot{Z}(t)$ is the n by n matrix. We choose initial conditions: $\rho(t)|_{t_0} = \rho_0$. If eigenvectors $a_n(t)$ and eigenfrequencies $\omega_n(t)$ of the matrix $\dot{Z}(t)$ vary slowly with $t$:

$$\frac{d}{dt} \omega_n(t) \ll (\omega_n(t))^2, \quad \frac{d}{dt} |a_n(t)| \ll \omega_n(t)|a_n(t)|,$$  \hfill (A.2)

then the adiabatic approximation of the solution of the set of linear equations Eq. [A.1] may be represented in the form:

$$\rho_n(t) = a_n(t) \exp(\int_{t_0}^{t} \omega_n(t')dt').$$  \hfill (A.3)

Indeed, let us substitute Eq. [A.3] in Eq. [A.1] and get

$$\dot{a}_n(t) \exp(\int_{t_0}^{t} \omega_n(t')dt') + \omega_n(t)a_n(t) \exp(\int_{t_0}^{t} \omega_n(t')dt') =$$

$$\dot{Z}(t)a_n \exp(\int_{t_0}^{t} \omega_n(t')dt').$$  \hfill (A.4)

Under the slowness condition Eq. [A.2], the first term on the left-hand side of Eq. [A.4] is small compared to the second one and may be omitted. Then Eq. [A.4] only keeps the terms yielding the identity that determines the eigenfrequency $\omega_n$. 