Optische Tomographie basierend auf der Gleichung für Strahlungstransport

im Fachbereich Physik
der Freien Universität Berlin
eingereichte Dissertation

von

Alexander Klose

aus Berlin

Berlin, Oktober 2001

Betreuer: Prof. A.H. Hielscher, PhD

Erster Gutachter: Prof. Dr. J. Beuthan

Zweiter Gutachter: Prof. Dr. K.D. Kramer

Tag der Disputation: 14. November 2001

Optical Tomography Based on the Equation of Radiative Transfer

A thesis presented
to the Department of Physics
Freie Universität Berlin
Germany

by

Alexander Klose

Berlin, October 2001

Optical Tomography Based on the

Equation of Radiative Transfer

Abstract

Optical tomography is a non-invasive medical imaging modality that utilizes measurements of transmitted near-infrared light to reconstruct the distribution of optical properties inside the human body. Clinical studies are currently being conducted that use this imaging technique for the determination of blood oxygenation, functional imaging of brain activities, and early diagnosis of rheumatoid arthritis in finger joints. These studies incorporate the fact that optical properties are closely related to physiological and pathological differences between healthy and diseased human tissue types. The instrumentation for highly precise measurements of light intensities is nowadays widely available. However, the development of algorithms that efficiently transform these measurements into accurate cross-sectional images of optical parameters remains a major challenge.

The majority of currently applied image reconstruction schemes rely on the validity of the diffusion equation for the description of light propagation in tissue. Unfortunately, the diffusion equation does not accurately describe light propagation in media that contain low-scattering regions, such as the cerebrospinal fluid that cushions the brain or the synovial fluid that lubricates joints. Therefore, the usefulness of diffusion-theory-based image reconstruction algorithms is questionable.

This work addresses these shortcomings by developing a novel model-based iterative image reconstruction scheme for optical tomography. It consists of two major parts: (1) a *forward model* for light propagation and (2) an *inverse model*. The forward model predicts the detector readings on the tissue boundary given a source and distribution of optical parameters inside the medium. The equation of radiative transfer, unlike the diffusion equation, describes correctly as a forward model the photon propagation in turbid media containing low-scattering areas. In contrast, the inverse model determines the optical parameters inside tissue, given a set of detector readings on the boundary of tissue. The inverse model is viewed as a nonlinear optimization problem. The measured fluence on the boundary is compared to the predicted detector readings by defining an objective function. The objective function is iteratively minimized by a nonlinear conjugate gradient technique, or by quasi-Newton methods. These techniques use the first derivative of the objective function with respect to the optical parameters for calculating search directions towards the minimum. Forward and inverse model are iteratively employed until self-consistency is reached.

A major obstacle is the computationally efficient calculation of the first derivative of the objective function within the inverse model. We calculate the derivative by utilizing an adjoint differentiation technique that is a particular numerical implementation of an adjoint model. We apply the adjoint differentiation technique for the first time to the equation of radiative transfer.

Two-dimensional images of the scattering and absorption coefficients are reconstructed by using experimental data. Never before executed, cross-sectional images of scattering phantoms with water-containing areas are reconstructed. Furthermore, we show reconstructed sagittal images of optical parameters of a human finger joint. We emphasize the potential application for the early diagnosis of rheumatoid arthritis in a numerical study.

Contents

Titelseite (*German*) i
Title Page iii
Abstract v
Table of Contents vi
List of Figures x
List of Tables xv
Glossary of Notation xvi
Acknowledgments xix

1 Introduction and Motivation **1**
1.1 Tomographic Imaging in Medicine 2
1.2 Basics of Optical Tomography 5
1.2.1 Forward Model - Photon Propagation in Tissue 9
1.2.2 Inverse Model - Reconstruction of Optical Properties 11
1.3 Rheumatoid Arthritis of Finger Joints 14
1.4 Objectives 14
1.5 Outline of the Content 16

I Forward Model **19**

2 Photon Transport in Turbid Media **23**
2.1 Equation of Radiative Transfer 25
2.2 Numerical Methods to the Solution of the Equation of Radiative Transfer . 32
2.3 Finite-Difference Discrete-Ordinates Method 33
2.3.1 Spatial and Angular Discretization 34
2.3.2 Successive Overrelaxation Method 36
2.3.3 Boundary Conditions 38
2.4 Discussion 40

3 Error Estimation **41**
3.1 Grid Size and Number of Ordinates 41
3.2 Discussion 45

4 Experimental Validation **49**
4.1 Experimental Set-up 49
4.1.1 Scattering Phantoms 50
4.1.2 Light Source 51
4.1.3 Light Detection 52
4.2 Results 53
4.2.1 Homogeneous Phantom 54
4.2.2 Phantom with Void Ring 60
4.3 Discussion 60

II Inverse Model **65**

5 Image Reconstruction as Nonlinear Optimization Problem **69**
5.1 Objective Function 69
5.2 Optimization Methods 70
5.2.1 Line Search 72
5.2.2 Nonlinear Conjugate Gradient Method 74
5.2.3 Newton's Method for Nonlinear Equations 77
5.2.4 Quasi-Newton Method 78
5.2.5 Positive Definite Inverse Hessian 81
5.3 Discussion 82

6 Derivative Calculation **83**
6.1 Adjoint Model 84
6.2 Numerical Implementation of the Adjoint Model 88
6.3 Differentiation of Algorithms 92
6.4 Adjoint Differentiation of the Objective Function 96
6.4.1 Decomposition of the Forward Model 96
6.4.2 Adjoint Differentiation 98
6.5 Scaling Factor 101
6.6 Example of a Derivative Calculation Based on Experimental Data 102

7 Numerical Reconstruction Results **105**
7.1 Comparison of BFGS, lm-BFGS, and CG Methods 105
7.1.1 Problem Set-up and Method 106
7.1.2 Definition of Image Accuracy 107
7.1.3 Impact of Noise 108
7.1.4 Impact of Different Initial Guesses 115
7.1.5 Impact of Noise and Initial Guess 119
7.1.6 Discussion 123
7.2 Source and Detector Configuration 124
7.2.1 Problem Set-up and Method 124
7.2.2 Impact of Different Numbers of Source-Detector Pairs 125

7.2.3 Discussion . . . 128
7.3 Void Areas . . . 129

8 Experimental Reconstruction Results 131
8.1 Phantom with Single Scattering Heterogeneity . . . 131
8.2 Phantoms with Void Regions . . . 133
8.3 Discussion . . . 138

9 Imaging of Rheumatoid Arthritis 140
9.1 Rheumatoid Arthritis . . . 141
9.2 Optical Monitoring of Rheumatoid Arthritis . . . 143
9.2.1 Finger Joint Model . . . 143
9.2.2 Numerical Results . . . 145
9.3 Reconstruction of a Human Finger Joint . . . 152
9.4 Discussion . . . 157

III Summary and Outlook 159

10 Summary 163

11 Outlook 170
11.1 Time-Dependent Forward Model . . . 171
11.1.1 Numerical Example . . . 173
11.1.2 Conclusion . . . 174
11.2 Stochastic Optimization Methods . . . 175
11.2.1 Evolution Strategy . . . 176
11.2.2 Numerical Example . . . 179
11.2.3 Conclusion . . . 180

Bibliography 183

A Coefficients of the Phase Function 205

B Example: Differentiation of a Composite Function 207
B.1 Total Derivative . . . 209
B.2 Forward Mode of Differentiation . . . 210
B.3 Reverse Mode of Differentiation . . . 212

C Total Derivative of Objective Function 214

List of Figures

1.1 Components of the MOBIIR scheme in OT for determining cross-sectional images of the optical parameters. The forward model for light propagation in tissue is iteratively employed to calculate detector predictions. The image is obtained by updating the optical parameters within the inverse model. . 8

2.1 The laboratory coordinate system describes the global geometry of the scattering medium that contains all points $\boldsymbol{r}$. The local coordinate system describes the local scattering process into directions $\boldsymbol{\omega}$ at the point $\boldsymbol{r}$. 26

2.2 Hemispheres S^1 and S^2 around the point $\boldsymbol{r}$. S^1 is on top and S^2 is underneath of the $x-y$ plane at z_0. All directions $\hat{\boldsymbol{\omega}}$ point towards $\boldsymbol{r}$. 30

2.3 Finite-difference grid. 35

2.4 Boundary conditions at a grid point (i,j). The incident radiance ψ_i is partly reflected due to the refractive index mismatch at the air-tissue interface. . . 38

3.1 Schematic of the test medium with dimensions of 3 cm × 3 cm and source position A. The relative fluence profiles were taken on the boundary along the x-axis and y-axis. 43

3.2 Relative fluence ϕ for different numbers of ordinates. The medium was isotropically scattering (g=0). The source was located at position A. 44

3.3 Relative fluence ϕ for different numbers of ordinates. The medium was anisotropically scattering (g=0.8). The source was located at position A. . . 44

3.4 Relative fluence ϕ for different numbers of grid points. The medium was isotropically scattering (g=0) and had optical parameters μ_s = 11.6 cm^{-1} and μ_a = 0.35 cm^{-1}. The source was located at position A. 46

3.5 Relative fluence ϕ for different numbers of grid points. The medium was anisotropically scattering (g = 0.8) and had optical parameters μ_s = 58 cm^{-1} and μ_a = 0.35 cm^{-1}. The source was located at position A. 46

4.1 Experimental set-up. 50

4.2 Schematic of phantoms in x-y plane. 52

4.3 Source and detector positions of the homogeneous phantom. Sources: (A) 0.3 cm, (B) 0.9 cm, and (C) 1.5 cm from the edge of the phantom. 55

4.4 Relative fluence ϕ for different source positions A, B, and C of the homogeneous phantom. 56
4.5 Relative fluence $\phi(x)$ along the x-axis for source position C. Different optical parameters were varied. 57
4.6 Relative fluence $\phi(y)$ along the y-axis for source position C. Different optical parameters were varied. 58
4.7 Source and detector positions of the phantom with a void-like ring. Sources: (A) 0.4 cm, (B) 1.2 cm, and (C) 2 cm from the edge of the phantom. 60
4.8 Relative fluence ϕ for different source positions A, B, and C of the phantom with void-like ring. 61

6.1 Three different ways of implementing the adjoint model to calculate the gradient of the objective function. The forward model can either be based on the diffusion equation or the ERT. Method III is represented by the adjoint differentiation technique. 91
6.2 Computational graph of the function evaluation by stepping through all subfunctions from left to right. 93
6.3 Computational graph of the forward mode of differentiation of algorithms. The derivative $\frac{\partial G}{\partial \boldsymbol{x}}$ is evaluated by stepping from left to right. 94
6.4 Computational graph of the reverse or adjoint mode of differentiation of algorithms. The derivative $\frac{\partial G}{\partial \boldsymbol{x}}^T$ is evaluated by stepping backwards from right to left. 95
6.5 Computational graph of the transport forward model. The objective function Φ is calculated by stepping through all sub-functions from left to right. The sub-functions are given by the SOR method for solving the discretized ERT. 97
6.6 Computational graph of the adjoint differentiation technique applied to the transport forward model. The derivative $\frac{d\Phi}{d\boldsymbol{\mu}}^T$ is calculated by stepping backwards through the computational graph of the forward model (see Figure 6.5). 99
6.7 Schematic and source-detector configuration of the phantom that contained a single scattering heterogeneity. The phantom was illuminated from all four sides. The measurements were taken on the sides opposite the sources. . . . 103
6.8 Comparison of predictions and measurements for source A. The predictions were calculated by assuming a homogeneous medium, whereas measurements were performed on the phantom containing a scattering heterogeneity (see Figure 6.7). 104
6.9 Scaled gradient $\nabla_{\boldsymbol{\mu}_s}\Phi$ of the objective function with respect to the scattering coefficient. It depicts alterations of the assumed homogeneous distribution of the scattering coefficient μ_{s_0} to the scattering coefficients μ_s of the original medium. 104

7.1 Scattering coefficients μ_s of original medium with dimensions of 3 cm $\times$ 3 cm containing three heterogeneities ($\mu_{\rm s}$ = 2.9 cm^{-1}, $\mu_{\rm s}$ = 8.7 cm^{-1}, and $\mu_{\rm s}$ = 11.6 cm^{-1}). The bulk medium had a scattering coefficient of $\mu_{\rm s}$ = 5.8 cm^{-1}. 107

7.2 Objective functions for different signal-to-noise ratios of the synthetic measurement data. 111
7.3 Final image reconstructions of μ_s. No noise was present in the synthetic measurement data. Distance between adjacent isolines is 1 cm^{-1}. 112
7.4 Image reconstructions of μ_s after 92 basic operations. No noise was present in the synthetic measurement data. Distance between adjacent isolines is 1 cm^{-1}. 112
7.5 Final image reconstructions of μ_s. The SNR of the synthetic measurement data was 20 dB. Distance between adjacent isolines is 1 cm^{-1}. 114
7.6 Image reconstructions of μ_s after 23 basic operations. The SNR of the synthetic measurement data was 20 dB. Distance between adjacent isolines is 1 cm^{-1}. 114
7.7 Objective functions for different initial guess μ_{s_0}. 116
7.8 Final image reconstructions of μ_s. Initial guess μ_{s_0} was 30% higher than the background scattering of the original medium. Distance between adjacent isolines is 1 cm^{-1}. 117
7.9 Image reconstructions of μ_s after 84 basic operations. The initial guess μ_{s_0} was 30% higher than the background scattering of the original medium. Distance between adjacent isolines is 1 cm^{-1}. 117
7.10 Objective functions using the BFGS method with/without a positive definite inverse Hessian. An initial guess 50% higher than the background scattering of the original medium led to a Hessian that was not positive definite. . . . 118
7.11 Objective functions starting from an initial guess μ_{s_0} that was 30% higher than the background scattering of the original medium. Additionally, the synthetic measurement data were corrupted by noise with a SNR of 20 dB. 120
7.12 Final image reconstructions of μ_s. The initial guess μ_{s_0} was 30% higher than the background scattering of the original medium. The SNR of the synthetic measurement data was 20 dB. Distance between adjacent isolines is 1 cm^{-1}. 121
7.13 Image reconstructions of μ_s after 31 basic operations. The initial guess μ_{s_0} was 30% higher than the background scattering of the original medium. The SNR of the synthetic measurement data was 20 dB. Distance between adjacent isolines is 1 cm^{-1}. 121
7.14 Scattering coefficients μ_s of original medium with dimensions 3 cm × 3 cm containing three heterogeneities (μ_s = 5.8 cm^{-1}, μ_s = 17.4 cm^{-1}, and μ_s = 23.2 cm^{-1}). The bulk medium had a scattering coefficient μ_s of 11.6 cm^{-1}. 125
7.15 Image reconstructions of μ_s with 12 detectors. Distance between adjacent isolines is 2 cm^{-1}. 126
7.16 Image reconstructions of μ_s with 28 detectors. Distance between adjacent isolines is 2 cm^{-1}. 126
7.17 Image reconstructions of μ_s with 56 detectors. Distance between adjacent isolines is 2 cm^{-1}. 127
7.18 Image reconstructions of μ_s with 126 detectors. Distance between adjacent isolines is 2 cm^{-1}. 127

7.19 Original medium and reconstructed image of scattering coefficients μ_s. The original medium contained a void-like ring with $\mu_\mathrm{s} = 1$ cm^{-1}. The bulk medium had a scattering coefficient μ_s of 58 cm^{-1}. Distance between adjacent isolines is 10 cm^{-1}. 130

8.1 Reconstructed scattering coefficient μ_s after 5, 8, and 13 iterations. The region with an elevated scattering coefficient is clearly seen in the lower left corner of the images. The initial guess of the reconstruction was $\mu_{\mathrm{s}_0} =$ 50 cm^{-1}. Adjacent isolines are separated by $\mu_\mathrm{s} = 2$ cm^{-1}. 132
8.2 Schematic and source-detector configuration of phantom with three void-like heterogeneities. The phantom was illuminated by one source at each side. . 134
8.3 Schematic and source-detector configuration of phantom with a void-like ring. The phantom was illuminated by three sources at each side. 135
8.4 Reconstruction results of the phantom with three water-filled holes after 34 basic operations by using the BFGS method. Distance between adjacent isolines is $\mu_\mathrm{s} = 4$ cm^{-1} and $\mu_\mathrm{a} = 0.04$ cm^{-1}. 136
8.5 Reconstruction results of the phantom with a water-filled ring after 150 basic operations by using the CG method. Distance between adjacent isolines is $\mu_\mathrm{s} = 4$ cm^{-1} and $\mu_\mathrm{a} = 0.02$ cm^{-1}. 137

9.1 Sagittal MRIs of human PIP joints with the interior surface on the left. Images were taken from a 60 years old male patient with RA. The image of the healthy condition of the PIP joint was taken without a contrast agent. The image of the rheumatoid condition of the PIP joint was taken on the other hand with the contrast agent Gadolinium. The inflamed joint capsule appears very bright in the image. 142
9.2 Numerical PIP joint model with sagittal cross-section. Sources were placed on the interior side of the finger. 145
9.3 Reconstructed scattering coefficient μ_s of the numerical PIP joint model of the healthy and early rheumatoid condition. The optical parameters of the rheumatoid condition were altered according to Table 9.1. The homogeneous initial guess of the reconstruction process was $\mu_{\mathrm{s}_0} = 100$ cm^{-1} and $\mu_{\mathrm{a}_0} =$ 0.7 cm^{-1}. The distance between adjacent isolines is $\mu_\mathrm{s} = 10$ cm^{-1}. 147
9.4 Reconstructed absorption coefficient μ_a of the numerical PIP joint model of the healthy and rheumatoid condition. The optical parameters of the rheumatoid condition were altered according to Table 9.1. The homogeneous initial guess of the reconstruction process was $\mu_{\mathrm{s}_0} = 100$ cm^{-1} and $\mu_{\mathrm{a}_0} =$ 0.7 cm^{-1}. The distance between adjacent isolines is $\mu_\mathrm{a} = 0.04$ cm^{-1}. 148
9.5 Relative change in μ_s of the reconstructed rheumatoid PIP joint with respect to the reconstructed healthy PIP joint. 150
9.6 Relative change in μ_a of the reconstructed rheumatoid PIP joint with respect to the reconstructed healthy PIP joint. 151
9.7 MRI of the human PIP joint with the interior side on the top. The synovial cavity between the two bones that is filled with synovial fluid is clearly visible in the center of the image. 152

9.8 Schematic of human finger (PIP joint) with source-detector configuration. Sources were placed on the interior side and detectors were placed on the posterior side of the finger. 153
9.9 Sagittal image of the reconstructed scattering coefficient μ_s. The image has a length of 4 cm and a height of 2.1 cm. The interior side of the finger is on the top. The finger tip is towards right. Small μ_s of the synovial fluid are visible in the center of the image. 155
9.10 Sagittal image of the reconstructed absorption coefficient μ_a. The image has a length of 4 cm and a height of 2.1 cm. The interior side of the finger is on the top. The finger tip is towards right. Small μ_a of the synovial fluid are visible in the center of the image. 156

11.1 Schematic and source-detector configuration of the test medium. 173
11.2 Calculated fluence profiles based on the time-dependent ERT. The medium had dimensions of 3 cm × 3 cm and optical parameters $\mu_s = 11.6$ cm^{-1}, $\mu_a = 0.001$ cm^{-1}, $g = 0$, and $n = 1.5$. 174
11.3 Objective function $\log_{10}(\Phi)$ for 100 generations of the (ν, λ)-ES. Each generation t is represented by its smallest value $\tilde{\varphi}$ of the objective function. The minimum was found after 48 generations (stopping criterion $\epsilon = 10^{-5}$). . . . 180
11.4 Objective function $\log_{10}(\Phi)$ of the optical parameters $\mu_s = 0.5..20$ cm^{-1}, $\mu_a = 0.04..1.6$ cm^{-1}, and $g = 0$. The minimum is at $\mu_s = 10$ cm^{-1} and $\mu_a = 0.6$ cm^{-1}. The search space was sampled by the (ν, λ)-ES, which is displayed by 200 dots. The minimum $\log_{10}(\tilde{\varphi}) = -4.43$ was found at $\mu_s = 9.74$ cm^{-1} and $\mu_a = 0.61$ cm^{-1}. 181

B.1 Computational graph of the composite function f. 208
B.2 Computational graph of the forward mode of differentiation. 211
B.3 Computational graph of the reverse mode of differentiation. 213

List of Tables

4.1 Average error R [%] of the predicted fluence profiles with respect to the measured fluence profiles. 59

7.1 Image accuracy of reconstructed images for all three optimization techniques (CG, lm-BFGS, and BFGS). Images with highest accuracy are represented by a large ρ_a and a small ρ_b (bold-printed). 122
7.2 Image accuracy of reconstructed images by using different source-detector configurations. The three reconstructed images with the highest image accuracy are represented by a large ρ_a and a small ρ_b (bold-printed). 128

9.1 Optical parameters of the numerical PIP joint model. The anisotropy factor was assumed to be constant with $g = 0.9$. 144

A.1 Coefficients $a_{\mathrm{k}'}\tilde{p}_{\mathrm{k}=1\ \mathrm{k}'}$ for K = 16 ordinates and different anisotropy factors g. 205
A.2 Coefficients $a_{\mathrm{k}'}\tilde{p}_{\mathrm{k}=1\ \mathrm{k}'}$ for K = 32 ordinates and different anisotropy factors g. 206

Glossary of Notation

Mathematical and physical notations:

$\mathbb{R}$	set of real numbers
x	scalar
$\lvert x \rvert$	absolute value of scalar
$\boldsymbol{x}$	vector
x_i	element of vector
$\boldsymbol{e}$	unit vector
$\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \sum_\mathrm{i} x_\mathrm{i} y_\mathrm{i}$	inner product
$\parallel \boldsymbol{x} \parallel = \sqrt{\langle \boldsymbol{x}, \boldsymbol{x} \rangle}$	Euclidean norm
f, g	function
$(f \circ g)(x) = f(g(x))$	composition
$\boldsymbol{A}$	matrix
$\boldsymbol{A}^*$	adjoint matrix
$\boldsymbol{A}^T$	transposed matrix
$\boldsymbol{I}$	identity matrix
$\boldsymbol{\mathcal{J}}$, $\boldsymbol{\mathcal{G}}$	Jacobian matrix
$\boldsymbol{\mathcal{B}}$	approximate Hessian matrix
$\boldsymbol{\mathcal{H}}$	approximate inverse Hessian matrix
δx	variation
Δx	finite difference
$\frac{d}{dx}$	total derivative
$\frac{\partial}{\partial x}$	partial derivative
ρ	overrelaxation parameter
$\boldsymbol{u}$	search direction
$\boldsymbol{u}_\mathrm{k}$	search direction of the k-th iteration step
α	step length
φ	azimuthal angle
ϑ	polar angle
x, y, z	Cartesian coordinates
$\boldsymbol{r}(x, y, z)$	spatial coordinate
$\boldsymbol{\omega}(\varphi, \vartheta)$	angular direction
$\tilde{\boldsymbol{\omega}}(\varphi)$	angular direction in $x - y$ plane
$\tilde{\boldsymbol{\omega}}_\mathrm{k}$	discrete ordinate in $x - y$ plane
θ	enclosed angle between two directions $\boldsymbol{\omega}$ and $\boldsymbol{\omega}'$
$\psi(\boldsymbol{r}, \boldsymbol{\omega})$	radiance
ψ_kij	radiance at grid point (i,j) with ordinate k
$\psi^\mathrm{z}_\mathrm{kij}$	radiance of the z-th iteration step at grid point (i,j) with ordinate k
$\boldsymbol{\psi}$	vector of radiance
$\boldsymbol{\psi}^\mathrm{z}$	vector of radiance of z-th iteration step

Mathematical and physical notations:

$\phi(\boldsymbol{r})$	fluence
ϕ_{ij}	fluence at grid point (i,j)
μ	optical parameter
μ_s	scattering coefficient
μ_s'	reduced scattering coefficient
μ_a	absorption coefficient
$\boldsymbol{\mu}$	vector of optical parameters
$\boldsymbol{\mu}_0$	vector of initial guess of optical parameters
$\boldsymbol{\mu}_{\mathrm{k}}$	vector of optical parameters at the k-th iteration step
$\boldsymbol{\mu}_s$	vector of scattering coefficients
$\boldsymbol{\mu}_a$	vector of absorption coefficients
g	anisotropy factor
p	scattering phase function
$\tilde{p}$	normalized scattering phase function in $x-y$ plane
n	refractive index
T	transmissivity
R	reflectivity
c	speed of light
$\boldsymbol{v}^{\mathrm{i}}$	i-th individual of population
$\boldsymbol{\sigma}$	strategy parameter
ν	number of individuals in parent population
λ	number of individuals in offspring population
$\gamma(\mathrm{l})$	rate of progress of the last l generations
$\boldsymbol{m}$	vector of measurement data
$\boldsymbol{p}$	vector of predicted data
Φ	objective function (as a function of $\boldsymbol{\mu}$)
$\tilde{\Phi}$	objective function (as a function of $\boldsymbol{p}$)
$\tilde{\varphi}$	value of the objective function
F	forward model
ρ_a	correlation coefficient
ρ_b	deviation factor
χ	scaling factor
D	number of source-detector pairs
I	number of grid points along the x-axis
J	number of grid points along the y-axis
N	number of grid points for μ_s and μ_a
K	number of ordinates
Z	number of iteration steps of the SOR method

Acronyms:

ART	algebraic reconstruction technique
BFGS	Broyden-Fletcher-Goldfarb-Shanno
CAT	computer aided tomography
CG	conjugate gradient
CS	coordinate system
CSF	cerebrospinal fluid
DOT	diffuse optical tomography
EP	evolutionary programming
ERT	equation of radiative transfer
ES	evolution strategy
FDM	finite-difference method
FEM	finite-element method
GA	genetic algorithm
lm-BFGS	limited-memory BFGS
MC	Monte-Carlo
MOBIIR	model-based iterative image reconstruction
MRF	Markov random field
MRI	magnetic resonance imaging
NEP	noise equivalent power
NIR	near-infrared
NMR	nuclear magnetic resonance
OT	optical tomography
QN	quasi-Newton
P_N-method	spherical harmonics method
PET	positron emission tomography
PIP	proximal interphalangeal
PMT	photon migration tomography
RA	rheumatoid arthritis
RF	radio frequency
RWT	random walk theory
S_N-method	discrete-ordinates method
SA	simulated annealing
SNR	signal-to-noise ratio
SOR	successive overrelaxation

Acknowledgments

Oft bedrückt mich der Gedanke, in welchem Maße mein Leben auf der Arbeit meiner Mitmenschen aufgebaut ist, und ich weiß, wieviel ich ihnen schulde.

Albert Einstein, Mein Glaubensbekenntnis, Caputh, 1932.

I would like to express my deepest and sincere appreciation to my thesis advisor, Prof. Andreas Hielscher (Columbia University New York), for his continuous support, encouragement, guidance, and patience throughout the course of this work. Without his inspirations, this work would have been impossible.

Especially I am obliged to Prof. Jürgen Beuthan and Prof. Klaus Kramer for their helpful suggestions, their effort and significant time devoted to advising me as the thesis evolved. I am greatly indebted to Dr. Ken Hanson (Los Alamos National Laboratory) for his support and many fruitful discussions concerning adjoint differentiation techniques and quasi-Newton methods. I am very grateful to my colleagues Avraham Bluestone and Dr. Gassan Abdoulaev (Columbia University New York) for sharing their excellent knowledge on mathematics and their helpful feedback.

Among the many others who contributed to the experimental part of this work, I would like to thank Uwe Netz and Hans-Joachim Cappius for providing me with experimental data on tissue phantoms and finger joints in Chapter 8 and 9, Jürgen Massuthe, Thomas Fricke, and Dr. Hans-Georg Eberle (Institut für Lasermedizin, FU Berlin) for their expertise and help concerning the experimental set-up and its electronic data acquisition system. Last but not least, I thank Dr. Uwe Sukowski (PTB Berlin) for the tissue phantom preparation.

Many thanks to Prof. Randall Barbour, Dr. Harry Graber, and Dr. Yaling Pei (SUNY Downstate Medical Center Brooklyn) for their helpful discussions on image reconstruction techniques, and Kai Schnitker and Allison Chaitkin for their contributions to software development and editing.