Chapter 5

Introduction

The thermostatistical properties of systems of N classical particles under long–range attractive potentials have been extensively studied since the seminal work of Antonov [ANT62], [PAD90, LYN67, LW68, HT71, THI70, SAS85]. One of their more specific and interesting properties is that they are unstable for all N [RUE63, PAD90] and therefore not thermodynamically extensive, i.e. they exhibit negative specific heat capacity regions even when the system is composed by a very large number of particles [LW68].

It is natural to ask whether the total angular momentum L, which is an integral of motion for systems of relevance in the astrophysical context, plays a non–trivial role on the equilibrium properties of these systems. Indeed, L is considered as an important parameter in order to understand the physics of systems like galaxies [LGM99, LDOP00, BT87]; globular clusters [LL96A, LL96B, HK77, KM00, LB00]; molecular clouds in multifragmentation regime [COM98, DVSC] which might eventually lead to stellar formation [BAT98, KFMT98, KB97, BB96, WASBW96, WHI96, CT90].

Previous works have already studied the effect of L in the mean field limit with a simplified potential [LAL99] or imposing a spherical symmetry [KM00]; or at $L = 0$ [HK77].

In this part an attempt to overcome some of these approximations is presented [a]. No symmetry is imposed a priori and a "realistic" gravitational potential is used.

Thermodynamical equilibrium does not exist for Newtonian self–gravitating systems, due both to evaporation of stars (the systems are not self–bounded) and short distance singularity in the interaction potential. However there exists intermediate stages where these two effects might be neglected and a quasi–equilibrium state might be reached [HK77, CP01] (dynamical issues like ergodicity, mixing or "approach to equilibrium" [SAS85, YM97, RN93] are not considered in this chapter). In order to make the existence of equilibrium configurations possible one has, first, to bound the system in an artificial box and, second, to add a short distance cutoff to the potential. The latter point can be seen as an attempt to take into account the appearance of new physics at very short distances (about the influence of this short cut see e.g. [ROM97, SLVH97, FL00]). Another way to avoid the difficulties due to the short distance singularity is to describe the function of distribution of the "stars" within a Fermi–Dirac statistic [LYN67, CHA98].

The box breaks the translational symmetry of the system, therefore, strictly speaking the total linear momentum $\mathbf{P}$ and angular momentum with respect to the system center of mass $\mathbf{L}$ are not conserved. Nevertheless, it is assumed that the equilibration time is smaller than the characteristic time after which the box plays a significant role [HK77,

[a] Most of the material presented in this part can be found in [FG01].

LAL99, CP01]. Therefore **P** and **L** are considered as (quasi–)conserved quantities (see discussion in sec. 1.1.2). The center of the box is put at the center of mass R_{CM} which is also set to be the center of the coordinates, thus $P = 0$.

As already mentioned, self–gravitating systems are non–extensive, i.e. "small" in the sense given in the general introduction of this thesis. A statistical description based on their intensive parameters should be taken with caution since the different statistical ensembles are only equivalent at the thermodynamical limit far from first order transitions as defined in the first part of this thesis. Hence, following the discussions in part I the equilibrium properties of these self–gravitating systems are worked out within the microcanonical ensemble (ME).

In order to perform the computation in a reasonable time and as a first step an only two dimensional system is considered. Thus, the total angular momentum is a pseudo–vector characterized by one number; it is noted from now on by L.

The next chapter is organized as follow. First the analytical expressions for entropy and its derivatives are recalled (sec. 6.1), then the two first moments of the distribution of the linear momentum of a given particle at a fixed position (sec. 6.2) are derived and commented, and finally a numerical method based on an importance sampling algorithm in order to estimate suitable observables is presented (sec. 6.3). Numerical results are presented in section 6.4. First thermostatistical properties are shown and discussed. Then the link between the average mass distribution and the thermostatistical properties is made in sec. 6.4.2. In sec. 6.4.3 the definition of phase introduced in chap. 2 is used to construct the phase diagram of the self–gravitating system as a function of its energy E and angular momentum L. The ensemble introduced by Klinko and Miller in [KM00] is discussed. In this paper this ensemble is used to treat another model of rotating and self–gravitating system. This ensemble is a function of the (intensive) variables conjugate of E and L^2. For the present model it is shown how the predictions using this ensemble are inaccurate and misleading (sec. 6.4.4); the more "standard" CE where the intensive variables are the conjugate of E and L, i.e. the inverse temperature β and the angular velocity ω is also discussed in sec. 6.4.4. Results are summarized and discussed in section 6.5.