Chapter 3

Inspecting the evil: Larsen's Signal-Noise Separation

In this chapter, a modern method to cope with magnetotelluric data containing a high level of correlated noise is introduced and evaluated. It has been published by Larsen et al. in 1996. The requirement to develop such a method occurred in connection with audiomagnetotelluric measurements in Italy, where railways are driven by DC current, too, and data are accordingly noisy.

I have told already in section 1.3.3, that correlated noise is so dangerous because it possesses features enabling it to get into transfer functions, what makes it nearly impossible to keep it away from the impedances and induction arrows with the single-site method only. Larsen et al. [1996] make a virtue of that necessity. They acknowledge the affinity of correlated noise to transfer functions providing it in their approach with extra impedances and induction arrows, where it can get into without biasing the magnetotelluric ones, and then solving the whole problem simultaneously. In section 3.1 I will show how this is done in detail.

This approach has two important consequences. First, it is possible to obtain information from the transfer functions of correlated noise about the location of its origin. Second, since the correlated noise goes into its own transfer functions instead of into the residual of the magnetotelluric ones, the latter are better determined than with the Remote Reference method, according to Larsen et al. [1996] and some of their followers. This claim opened a certain debate among the contemporary authorities of magnetotelluric data processing about whether that can really be true or not. Section 3.2 is dedicated to that question.

3.1 Description of the method

First, I have to emphasize that I do not the same as Larsen et al. do in their code. I calculate spectra as described in section 1.2 and solve equations as it will be shown in the following. None of the procedures referred to as “independent of conditional equations”in the introduction of this thesis is present in my code.

3.1.1 Discerning signal and noise

The nomenclature and derivation in this description follows Oettinger et al. [2001] with small modifications.
Just as in the Remote Reference technique, the property of correlated noise to have vanished at a distant remote site is used here to distinguish between relevant magnetotelloric signal and irrelevant noise, the latter may be correlated or not. Concerning the required distance, the same criteria as described in section 2.3 hold for selection of the remote site or its appropriateness, respectively. However, there hold stricter rules for statistic noise in the remote channels: It is forbidden here. Furthermore, in contrast to the Remote Reference technique, the Separation tensor (cf. eq. 2.5), which is used there only for derivation and indirectly, for error estimation, is applied here very explicitly: With its help that signal is reconstructed which would be measured on the horizontal magnetic channels of the local site if there was no noise. This is done very simply by multiplying the noise-free remote horizontal magnetic signal $\mathbf{B^R}$ (eq. 2.3) with the Separation tensor $\mathbf{T}$. Since the obtained signal is just the base for undistorted magnetotelluric (MT) transfer functions, this quantity gets the index MT:

$$\mathbf{B^{MT}} = \mathbf{B^R T^T}, \tag{3.1}$$

where

$$\mathbf{B^{MT}} = \begin{pmatrix} B_{x1}^{MT} & B_{y1}^{MT} \\ B_{x2}^{MT} & B_{y2}^{MT} \\ \vdots & \vdots \\ B_{xN}^{MT} & B_{yN}^{MT} \end{pmatrix}. \tag{3.2}$$

The background for this step is the following:
In general (i.e. if noise is absent) the horizontal magnetic field components of two stations differ only, if there are lateral conductivity anomalies close-by or even in-between them. In one-dimensional cases (i.e. if the subsurface consists only of horizontal layers),

they would be equal, **T** would be the unity matrix, and the effort of reconstructing would be unnecessary. This holds approximately, too, if the stations are very close to each other, as already mentioned in section 2.3. On the other hand, we need remote sites that are far away from the source of the given correlated noise, and the real world is hardly one-dimensional. But in lateral conductivity anomalies, there are induced currents having anomalous magnetic fields on their part and causing that the horizontal magnetic field becomes different in their proximity. Such differences are covered by the Separation tensor. So **T** will differ from unity according to the given lateral conductivity anomalies. "According to lateral conductivity anomalies" means that the Separation tensor possesses properties which are typical for induction processes and exist in an analogous manner also in the impedance tensor, or rather its imaging representatives ρ_a and ϕ, like:

- 2×2 tensor where, in general, all elements are different from zero,
- complex-valued,
- typical interdependency between modulus and phase,
- continuous, only slowly changing function of period.

In contrast to the impedance tensor, here the main-diagonal elements are of greater significance, or illustrative meaning, respectively. An exemplary Separation tensor demonstrating all the characteristics listed above is shown in fig. 3.1. It might be interesting to mention that the Separation tensor is an adequate transfer function that allows to derive information about the conductivity distribution of the subsurface and that can be modeled, as happened in Soyer [2002] and Varentsov and EMTESZ-Pomerania Working Group [2006]. However, for our requirements it is only important that with its help, the induction-produced and exclusively MT-relevant horizontal magnetic field at a disturbed site can be reconstructed. The mentioned properties of the Separation tensor can help to see whether the determination of it has succeeded or failed due to some problems. Returning to our reconstructed MT-data for the horizontal magnetic field of the local station, we can now calculate the residual to the measured field **B** (cf. eq. 1.21). It gets the index CU since it consists of *c*orrelated and of statistic, *u*ncorrelated noise:

$$\mathbf{B^{CU}} = \mathbf{B} - \mathbf{B^{MT}} = \mathbf{B} - \mathbf{B^{R}T^{T}} \tag{3.3}$$

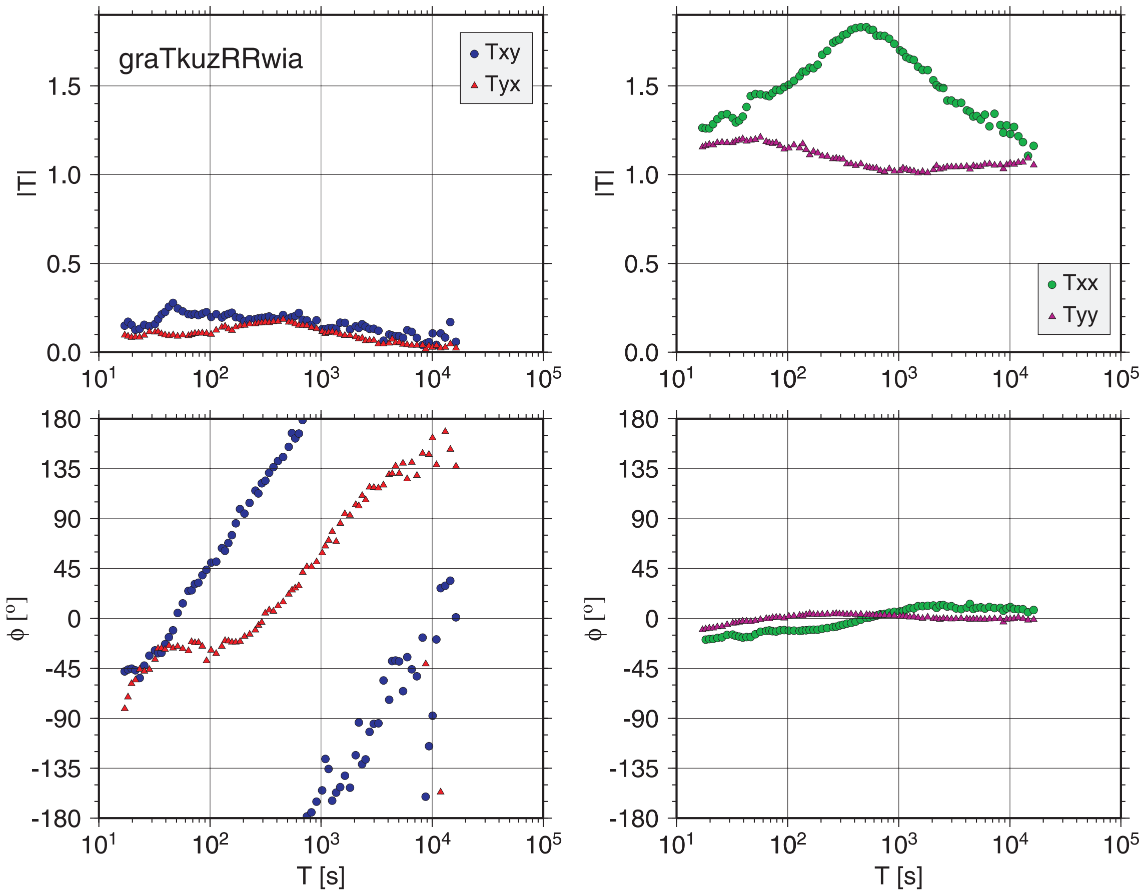

Figure 3.1: The Separation tensor which translates the horizontal magnetic field of site KUZ into that of site GRA (cf. fig. 1) stabilized by WIA in the way described in chapter 4, demonstrates that this tensor can assume also shapes far from the real unity matrix. However, it shows that the period-dependency remains induction-like with slow changes and modulus extremal values at zero transitions in phase. In this case, the local site KUZ was located on the resistive, uplifted Czaplinek Block belonging to the anticline of the inverted Polish trough, whereas the reference GRA was measuring in the corresponding syncline filled with very conductive sediments. The block-like subsurface of KUZ probably explains the odd off-diagonal phases transgressing all quadrants, while the conductivity contrast is reflected by the big extent of T_{xx}. For geological background, see Dobracka and Piotrowski [2002].

Of course, $\mathbf{B^{CU}}$ is of the shape

$$\mathbf{B^{CU}} = \begin{pmatrix} B_{x1}^{CU} & B_{y1}^{CU} \\ B_{x2}^{CU} & B_{y2}^{CU} \\ \vdots & \vdots \\ B_{xN}^{CU} & B_{yN}^{CU} \end{pmatrix} \tag{3.4}$$

as well.
Thereby the horizontal magnetic field $\mathbf{B}$ measured at the local site could be separated into a magnetotelluric and a noise part. In the next section, both $\mathbf{B^{MT}}$ and $\mathbf{B^{CU}}$ will play an important and equitable role. It may be called unusual, interesting, or even ingenious that onto a residual is laid such a meaning, too.

3.1.2 A transfer function for correlated noise

With the separation of the horizontal magnetic channels the base has been created for the introduction of a new transfer function. It is supposed to translate the noise from the input variables B_x^{CU} and B_y^{CU} into a part of the output variable E, if we stay with the impedance example first. The problem is regarded now in a way, that the measured output channel, e.g. E_x consists of a MT-relevant part E_x^{MT}, a part due to correlated noise E_x^{CN}, and a residual part δE_x not correlated with anything:

$$\vec{E_x} = \vec{E_x^{MT}} + \vec{E_x^{CN}} + \delta\vec{E_x} \tag{3.5}$$

$\vec{E_x^{MT}}$ is determined by the independent variables B_x^{MT} and B_y^{MT}, and connected to them via Z_{xx}^{MT} and Z_{xy}^{MT}. Similarly, Z_{xx}^{CN} and Z_{xy}^{CN} transfer B_x^{CU} and B_y^{CU} into $\vec{E_x^{CN}}$. Altogether, it holds

$$\vec{E_x} = Z_{xx}^{MT}\vec{B_x^{MT}} + Z_{xy}^{MT}\vec{B_y^{MT}} + Z_{xx}^{CN}\vec{B_x^{CU}} + Z_{xy}^{CN}\vec{B_y^{CU}} + \delta\vec{E_x}, \tag{3.6}$$

and analogous

$$\vec{E_y} = Z_{yx}^{MT}\vec{B_x^{MT}} + Z_{yy}^{MT}\vec{B_y^{MT}} + Z_{yx}^{CN}\vec{B_x^{CU}} + Z_{yy}^{CN}\vec{B_y^{CU}} + \delta\vec{E_y} \tag{3.7}$$

and

$$\vec{B_z} = T_x^{MT}\vec{B_x^{MT}} + T_y^{MT}\vec{B_y^{MT}} + T_x^{CN}\vec{B_x^{CU}} + T_y^{CN}\vec{B_y^{CU}} + \delta\vec{T} \tag{3.8}$$

for the tipper. Equations 3.6, 3.7, and 3.8 can be subsumed and written in matrix form:

$$\begin{pmatrix} E_{x1} & E_{y1} & B_{z1} \\ \vdots & \vdots & \vdots \\ E_{xN} & E_{yN} & B_{zN} \end{pmatrix} = \begin{pmatrix} B_{x1}^{MT} & B_{y1}^{MT} & B_{x1}^{CU} & B_{y1}^{CU} \\ \vdots & \vdots & \vdots & \vdots \\ B_{xN}^{MT} & B_{yN}^{MT} & B_{xN}^{CU} & B_{yN}^{CU} \end{pmatrix} \begin{pmatrix} Z_{xx}^{MT} & Z_{yx}^{MT} & T_{x}^{MT} \\ Z_{xy}^{MT} & Z_{yy}^{MT} & T_{y}^{MT} \\ Z_{xx}^{CN} & Z_{yx}^{CN} & T_{x}^{CN} \\ Z_{xy}^{CN} & Z_{yy}^{CN} & T_{y}^{CN} \end{pmatrix} + \begin{pmatrix} \delta E_{x1} & \delta E_{y1} & \delta B_{z1} \\ \vdots & \vdots & \vdots \\ \delta E_{xN} & \delta E_{yN} & \delta B_{zN} \end{pmatrix}, \tag{3.9}$$

or, with the symbols

$$\mathbb{E} = \begin{pmatrix} E_{x1} & E_{y1} & B_{z1} \\ \vdots & \vdots & \vdots \\ E_{xN} & E_{yN} & B_{zN} \end{pmatrix}, \tag{3.10}$$

$$\mathbb{B} = \begin{pmatrix} B_{x1}^{MT} & B_{y1}^{MT} & B_{x1}^{CU} & B_{y1}^{CU} \\ \vdots & \vdots & \vdots & \vdots \\ B_{xN}^{MT} & B_{yN}^{MT} & B_{xN}^{CU} & B_{yN}^{CU} \end{pmatrix} = \begin{pmatrix} \mathbf{B^{MT}} & \mathbf{B^{CU}} \end{pmatrix}, \tag{3.11}$$

$$\mathbb{Z} = \begin{pmatrix} Z_{xx}^{MT} & Z_{xy}^{MT} & Z_{xx}^{CN} & Z_{xy}^{CN} \\ Z_{yx}^{MT} & Z_{yy}^{MT} & Z_{yx}^{CN} & Z_{yy}^{CN} \\ T_{x}^{MT} & T_{y}^{MT} & T_{x}^{CN} & T_{y}^{CN} \end{pmatrix} = \begin{pmatrix} \mathbf{Z^{MT}} & \mathbf{Z^{CN}} \end{pmatrix}, \tag{3.12}$$

and

$$\delta\mathbb{E} = \begin{pmatrix} \delta E_{x1} & \delta E_{y1} & \delta B_{z1} \\ \vdots & \vdots & \vdots \\ \delta E_{xN} & \delta E_{yN} & \delta B_{zN} \end{pmatrix} \tag{3.13}$$

it can be written shorter

$$\mathbb{E} = \mathbb{B}\mathbb{Z}^T + \delta\mathbb{E}. \tag{3.14}$$

The solution for $\mathbb{Z}$ is, analogous to e.g. 1.22 (cf. Oettinger et al. [2001])

$$\mathbb{Z}^T = \left(\mathbb{B}^\dagger\mathbb{B}\right)^{-1}\mathbb{B}^\dagger\mathbb{E}. \tag{3.15}$$

The CN transfer functions are quasi the essence of the bias in single-site processed data that contain correlated noise. An example is shown in fig. 3.2. More about their meaning will follow in section 4.3.

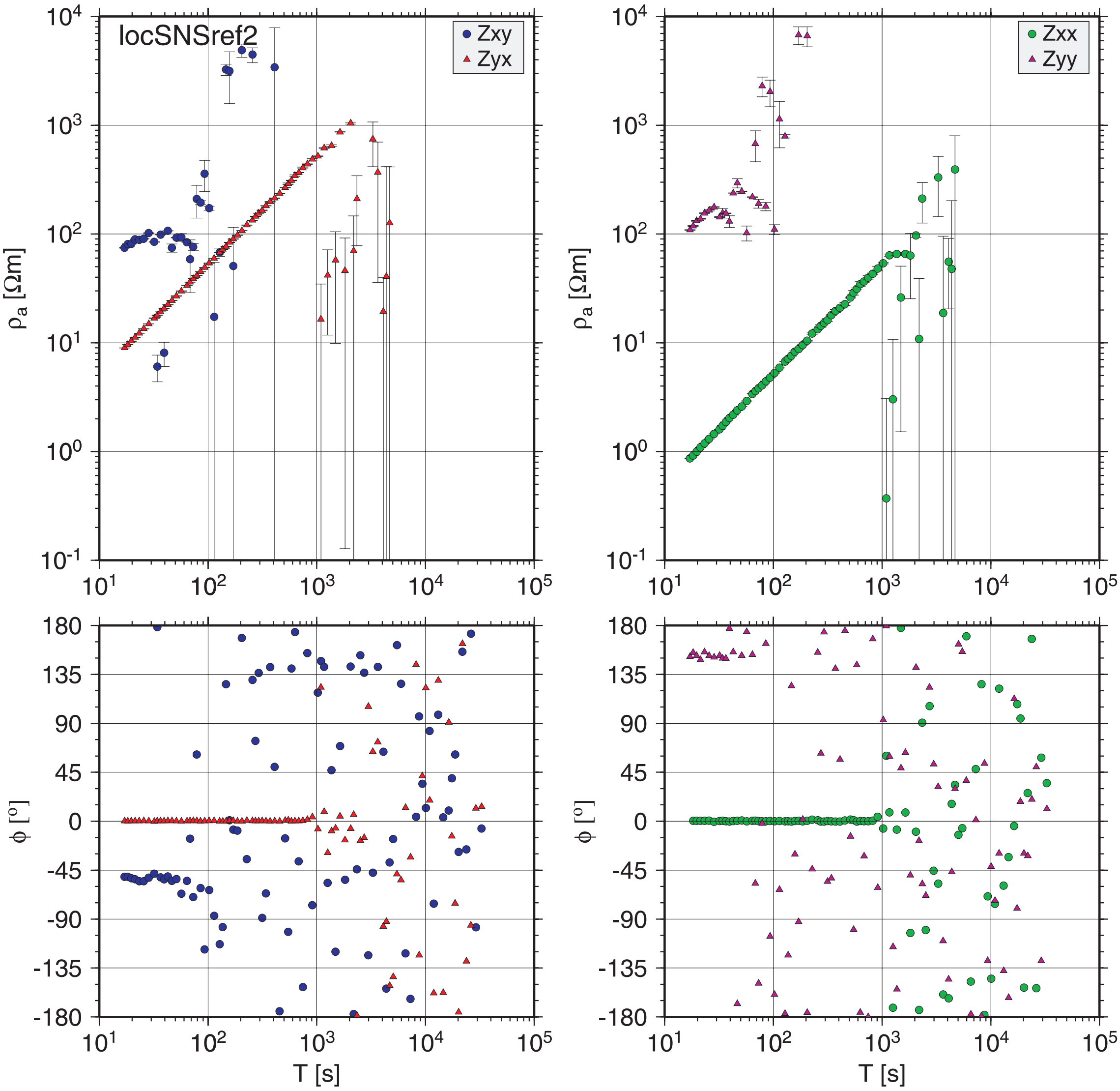

Figure 3.2: Apparent resistivities and phases for correlated noise obtained with the Signal-Noise Separation technique for synthetic data. Correlated noise exists between B_x, E_x, and E_y, hence Z_{xx} and Z_{yx} are concerned. They show the typical near-field behavior to be constant with period, manifesting itself in 0^o phases and a 45^o rise of ρ_a in an equidistant log-log plot.

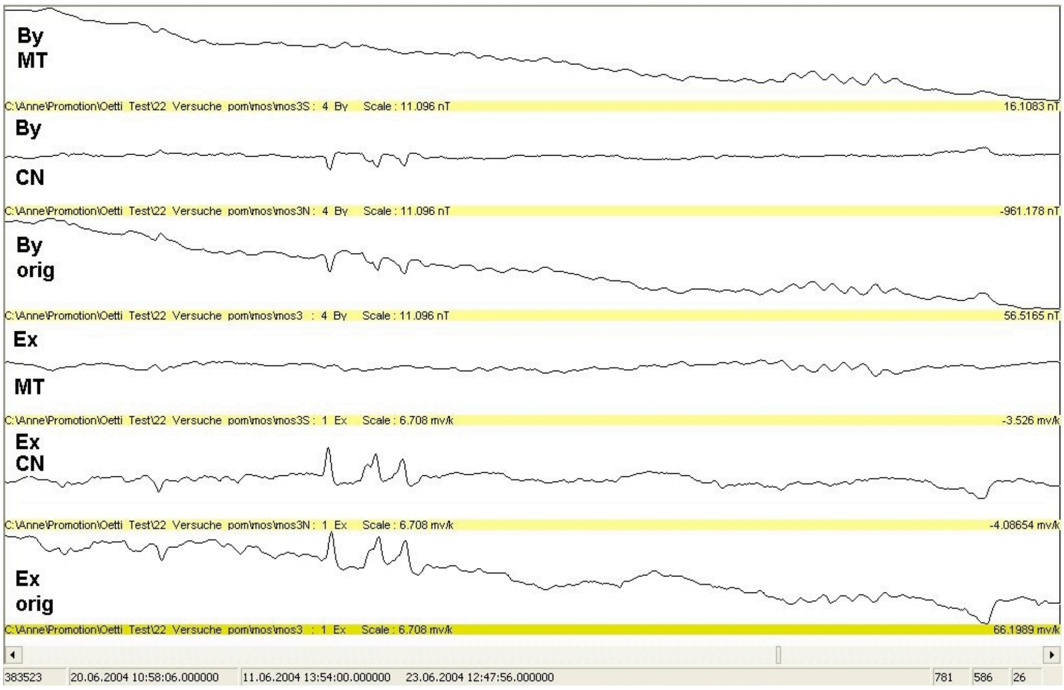

Figure 3.3: Separated time series for channels B_y and E_x of station MOS (cf. fig. 1). The scales are 11.1 nT and 6.7 mV/km, respectively, the window length, again, 34 minutes. The original records contain events with features typical for natural magnetic pulsations as well as for artificial noise. Obviously, the separation has classified them correctly.

3.1.3 Some general consequences

The distinction of magnetotelluric (MT) signal and noise in the magnetic data as well as the extra transfer functions for correlated noise (CN) offer the interesting possibility to separate time series, too. This is done by applying the Separation tensor onto the Fourier coefficients of the entire horizontal magnetic time series of the remote site (instead of, as in equation 3.1, onto those Fourier coefficients only that are supplied by the cascade decimation). This provides the Fourier coefficients of the magnetic MT series. Subtracting this from the local measured ones (i.e. again, the Fourier coefficients of the entire time series) gives the magnetic CU time series. The electric ones can be obtained by multiplying them with the corresponding MT and CN impedances. After transformation back into time domain the separated time series can be admired as in fig. 3.3.

The aesthetic look of those time series can mislead to the suggestion to use the “cleaned”MT time series as input for a further processing step, and to improve the results thereby in a quasi iterative way. Although the idea that “clean”looking time series lead to better processing results than noisy ones is very convincing, it does not work. Correlated noise exists not longer in the MT time series, they contain only what has already been recognized as correlated to the remote site. A processing method counting on presence of correlated noise would cope even worse with such an input. It is important to note that not the noise is identified and eliminated by the Signal-Noise Separation method, but rather the MT signal. This means that it's possible that there remains some unidentified part of the MT signal in the elements of the system of equations that are declared as belonging to correlated noise, whereas no "original" correlated noise can remain in the MT elements. Hence, it is imaginable that a repeated application of the SNS method onto the CN time series would improve the CN transfer functions. But nobody will do this in magnetotellurics, since the transfer functions of the correlated noise are only of marginal interest.

However, an iterative estimation of only $\mathbf{T}$ with $\mathbf{B^{MT}}$ instead of $\mathbf{B}$ is, in my opinion, not a hopeless idea. It could stabilize the sometimes scattering results, so an attempt could be worthwhile.

There is yet another feature in Larsen's processing that has to be mentioned, although is has not been investigated here primarily. This processing approach differs from other methods not only due to "robust procedures" as mentioned in the introduction and the system of equations described above, but also due to a step taking place prior to all others: The rotation of time series.

This is a rather complicated technique which in the article (Larsen et al. [1996]) is described not exactly enough to allow a reprogramming beyond doubt. My attempts ended up in trivial results, so I abandoned that implementation. On the other hand, the rotation of time series is not causeless, and the possible need for it throws a first light on facts that will require our attention later on in chapter 4: The introduction of additional transfer functions does not only potentially improve the results and offer the possibility to obtain information about the CN source unaccessible without them, it also can create problems that did not exist before. Again, it is a matter of a condition *sine qua non* for the success of the linear regression, which is not fulfilled.

This condition says: In a multivariate linear regression (i. e. there is more than one input channel determining the output) these input channels must not be linearly correlated with each other. This is probably the deeper meaning of the term "independent variables". If this condition is breached, there is no unique solution of the problem. The results get instable or even impossible. An extreme example would be a B_x completely correlated with B_y in the single-site case. Then there would hold

$$\sum_i B_{xi} B_{xi}^* \sum_i B_{yi} B_{yi}^* = \sum_i B_{xi} B_{yi}^* \sum_i B_{yi} B_{xi}^*, \tag{3.16}$$

and the denominator in, e. g., equation 1.19 would become zero.
The situation will never get so bad in the real world, since the natural variations of the Earth's magnetic field providing the magnetotelluric source signal are sufficiently independent in x and y direction. However, if there are close-by artificial sources of electromagnetic signals that are not orientated N-S or E-W, than the disturbing signal will be correlated not only between output channels and horizontal magnetic field, but also between B_x and B_y, the *per definitionem* independent variables. According to my experiments, this correlation is too weak against the independency of the natural variations to cause any harm if B^{MT} and B^{CU} are treated as a unit, as it happens in the single-site and the Remote Reference method. However, after splitting up both components in the Signal-Noise Separation, one gets with B_x^{CU} and B_y^{CU} two highly correlated input channels. This can fail, especially for the correlated-noise transfer functions which can get very instable.
The rotation of time series prior to processing provides that the correlated noise is located completely on one of the polarizations, which can, however, hardly be called x or y since they are twisted against each other in a complicated manner. This is compensated after determination of the impedances in Larsen's code. Again, I emphasize that the time series' rotation is not implemented in my code. Maybe, it is due to this, that my best results for correlated-noise transfer functions stem from the railways running between BLE and SAR and between BAK and DAB, cf. fig. 1. Their course is almost N-S or E-W, respectively, so the correlated noise is located mainly on one component even without rotation.

3.2 Comparison with Remote Reference

This section is a discussion of statements made by other MT workers, partly themselves creators of processing methods, about the question Signal-Noise Separation (SNS) versus Remote Reference (RR) method, i.e. which technique is the superior one. In this debate, the pure least-square solution of both problems plays an important role. So my code will be used to make a (hopefully clarifying) contribution to that discussion.

3.2.1 The debate and open questions

Reading the works of Larsen and Oettinger one gets the clear impression that their claim is to outclass the RR technique with the SNS method.
So there is in Larsen et al. [1996] and in Oettinger [1999] a table connecting several processing methods with the noise regime in which they are able to yield unbiased transfer functions. Therein these authors disallow the ability of RR to cope with correlated noise. According to them, this ability is limited to several versions of the SNS method, as some citations may show:
"If there is correlated noise at the local site then the single-source method *[meaning single site and RR]* yields an estimate of the MT transfer function that combines $Z^{MT}(\omega)$ and $Z^{C}(\omega)$[1] and is therefore wrong in both amplitude and phase. It is therefore important to use the two-source relationship *[SNS]* whenever the correlated noise is large."(Larsen et al. [1996], italic parts by me),
"If most or all time-series contain correlated noise *[...]* the RR method yields wrong estimates of the MT transfer function."(Oettinger et al. [2001]).
In fact, there has been found evidence that in a concrete case of very noisy data, Larsen's method yields better results than Egbert's (Egbert [1997], another method that claims to improve RR estimates) by Müller and Haak [2004].
Egbert's answer is a derivation showing that if the RR and SNS equations are solved solely by least-squares, the MT transfer functions obtained are equal.
Obviously, not all statements can be true in such a situation. I summarize below the contradictions that will have to be clarified in the following:
So if RR and SNS yield in principle (i.e. when solved by least-squares) the same result,

- how can SNS be destined to cope with correlated noise and RR fail with it?
- how can SNS results in practice be better than RR ones?
- why does Larsen's method require a noise-free reference and RR not?

[1] corresponds to our Z^{CN}. $\omega = \frac{2\pi}{T}$ is the angular frequency.

3.2.2 Answers from a consistent least-square point of view

I will outline Egbert's derivation here first, since it is not published[2], but has rather extensive implications.
Its "linchpin" is the fact that

$$\mathbf{B^{CU}}^{\dagger}\mathbf{B^{MT}} = \mathbf{0}. \tag{3.17}$$

Egbert shows this convincingly by substituting $\mathbf{B^{CU}}$ and $\mathbf{B^{MT}}$ by its definitions to the point of $\mathbf{T}$, and simplifying. However, (3.17) can also be proven by a simple contemplation of the origin of both quantities (cf. section 3.1.1): $\mathbf{B^{MT}}$ is that part of $\mathbf{B}$, which can be projected onto $\mathbf{B^{R}}$. $\mathbf{B^{CU}}$ is the residual part of $\mathbf{B}$, which can not at all be correlated with $\mathbf{B^{R}}$. The quantity dividing $\mathbf{B}$ into $\mathbf{B^{CU}}$ and $\mathbf{B^{MT}}$ is $\mathbf{T}$, determined by the method of least squares. It is the main feature of this method that the "distance"[3] between original output variables and their reconstruction via input variables and the solution obtained is minimized. Therefore it is inevitable that reconstruction and residual are orthogonal to each other. That's what is expressed by equation 3.17. By the way, this property of the separated magnetic signals holds always with least-squares, even if the separation itself is doubtful due to some noise in $\mathbf{B^{R}}$ (K. Nowożyński, pers. comm.).
Rewriting equation 3.15 into the shape

$$\begin{pmatrix} (\mathbf{Z^{MT}})^T \\ (\mathbf{Z^{CN}})^T \end{pmatrix} = \left[\begin{pmatrix} \mathbf{B^{MT\dagger}} \\ \mathbf{B^{CU\dagger}} \end{pmatrix} \begin{pmatrix} \mathbf{B^{MT}} & \mathbf{B^{CU}} \end{pmatrix} \right]^{-1} \begin{pmatrix} \mathbf{B^{MT\dagger}} \\ \mathbf{B^{CU\dagger}} \end{pmatrix} \mathbf{E} \tag{3.18}$$

and carrying out the matrix multiplications taking (3.17) into account, one gets with

$$\begin{pmatrix} (\mathbf{Z^{MT}})^T \\ (\mathbf{Z^{CN}})^T \end{pmatrix} = \begin{bmatrix} \mathbf{B^{MT\dagger}B^{MT}} & \mathbf{0} \\ \mathbf{0} & \mathbf{B^{CU\dagger}B^{CU}} \end{bmatrix}^{-1} \begin{pmatrix} \mathbf{B^{MT\dagger}E} \\ \mathbf{B^{CU\dagger}E} \end{pmatrix} \tag{3.19}$$

the proof, that $\mathbf{Z^{MT}}$ is completely independent of $\mathbf{B^{CU}}$ and $\mathbf{Z^{CN}}$ of $\mathbf{B^{MT}}$.
This has the maybe deflating consequence, that the simultaneous solution for $\mathbf{Z^{MT}}$ and $\mathbf{Z^{CN}}$ is not necessary; two subsequent bivariate procedures each with either $\mathbf{B^{MT}}$ or $\mathbf{B^{CU}}$ as input would yield the same results for $\mathbf{Z^{MT}}$ and $\mathbf{Z^{CN}}$.

[2]This derivation is mentioned in Larsen et al. [1996]. I got it in form of a written pers. comm. between A. Müller and G. Egbert (2001).

[3]This quantity is also referred to as L2-norm.

Further it follows with equation 3.1

$$
\begin{aligned}
(\mathbf{Z^{MT}})^T &= (\mathbf{B^{MT\dagger}B^{MT}})^{-1}\mathbf{B^{MT\dagger}E} \\
&= \left\{(\mathbf{B^R T^T})^\dagger \mathbf{B^R T^T}\right\}^{-1} (\mathbf{B^R T^T})^\dagger \mathbf{E} \\
&= \left\{\mathbf{T}^* \mathbf{B^{R\dagger} B^R T^T}\right\}^{-1} \mathbf{T}^* \mathbf{B^{R\dagger} E} \\
&= (\mathbf{T^T})^{-1} \left(\mathbf{B^{R\dagger} B^R}\right)^{-1} (\mathbf{T}^*)^{-1} \mathbf{T}^* \mathbf{B^{R\dagger} E} \\
&= (\mathbf{T^T})^{-1} \left(\mathbf{B^{R\dagger} B^R}\right)^{-1} \mathbf{B^{R\dagger} E} \\
&= \left(\mathbf{T^T}\right)^{-1} \mathbf{Z^{IT}}.
\end{aligned}
\tag{3.20}
$$

The last line is exactly the definition of the Remote Reference impedance we have introduced in equation 2.8. The line before last has been transformed using equation 2.2. Thereby, the equality of SNS and RR transfer function results following Egbert has been stated.
Equipped with this confirmation, I return to the questions raised up in section 3.2.1 now.
The first one can be treated of shortest: the information that the RR technique is not able to deal with correlated noise in a local site is simply not true. The only way to get a biased transfer function after a RR processing is to use a reference site within the reach of the correlated noise. I have demonstrated this in chapter 2 as well as the success of that technique with local data containing correlated noise. Larsen et al. [1996] themselves acknowledge this facility in the conclusions: "*[...]* correlated noise *[...]* requires use of *[...]* the remote-reference single-source method "contradicting thereby former statements in the same article.
Concerning the second question, better practical results of the SNS method are not verifiable in the least-square solution. The results of the SNS method shown in fig. 3.4 are the same as for the RR technique displayed in fig. 3.5 at the synthetic data example used. With real data that are used in chapter 4, one can sometimes observe even the opposite of the expected effect: Single points of the SNS transfer function lie far off the general curves giving a scattered picture, whereas the RR curves are relatively smooth (of course, with most of the values still being equal), cf. figs. 4.3 and 4.2 for station DAM with Belsk as reference. I explain such differences with difficulties at the inversion of the 4×4 matrix in equation 3.19, if the $\mathbf{B^{CU}}$ part is rather small. Then the matrix becomes almost singular and the inversion result can get numerically unstable. On the other hand, there is a point of view allowing the statement that SNS results are "better"than RR ones. The latter possess larger error bars. As explained in section 1.3.1, error bars, or variances, respectively, depend on the residual. In fact, the residual is significantly smaller with the SNS method, because the

correlated-noise part of the electric field is covered by a corresponding transfer function there and only an uncorrelated rest $\delta\mathbf{E}$ goes into the residual. However, in the RR case the residual is enlarged by $\mathbf{E^{CN}}$. Fig. 3.6 shows that difference for the synthetic data example. Larsen et al. [1996] observe something similar applying their original code. They write "the robust remote-reference single-source estimates tend to track the MT transfer function but give much larger errors than the robust least-squares two-source estimates." Obviously, the contradictions between statements concerning results of Larsen's original code and those obtained after Egbert's least-square derivation can be attenuated.

There remains the third question: If RR transfer functions are relatively insensitive to noise in the remote data and if SNS and RR results are equal, why do Larsen et al. [1996] insist on the requirement of a noise-free reference? A synthetic experiment with noisy remote data shows, that the SNS result for the MT part indeed hardly suffers from that noise (fig. 3.8). However, both Separation tensor (fig. 3.7) and CN transfer function (fig. 3.9) are significantly biased downwards, what is not unexpected according to the analysis in section 1.3.2 and the fact that the true $\mathbf{B^{CU}}$ is enlarged by a part of the MT signal due to the down-weighted Separation tensor. It is probably these obvious errors that make Larsen et al. insist on that noise-free reference. In the next chapter I will describe a way to obtain unbiased $\mathbf{T}$ and CN transfer functions in spite of noisy references.

Concluding for this chapter, one can say that Larsen's extended equations ("two-source") do not improve the results of the obtained MT transfer functions compared to the RR technique. If the output of Larsen's code is better than results of other standard processing methods, it is not a merit of the method described here, but of other features of Larsen's code.

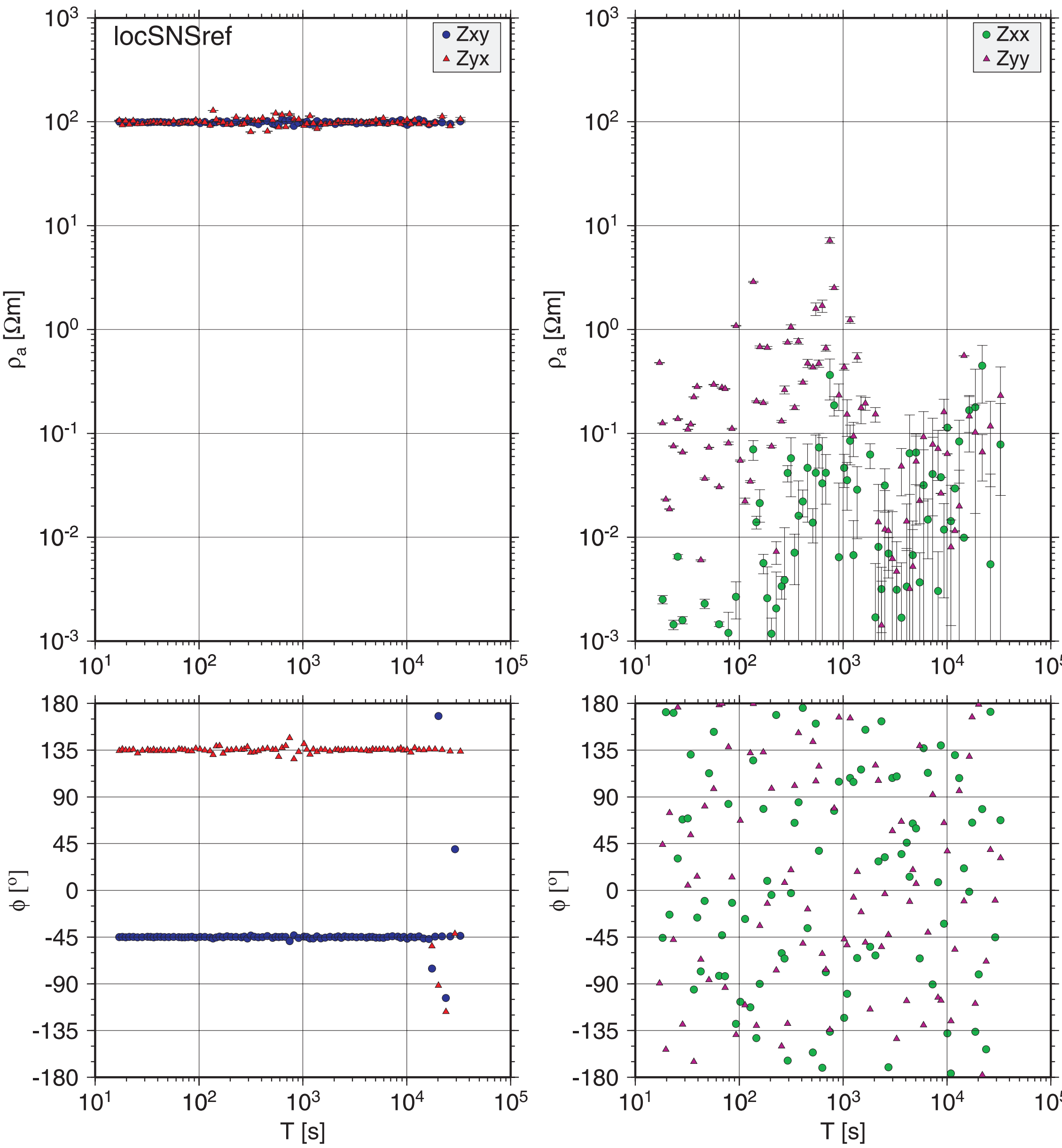

Figure 3.4: Processing results obtained with the Signal-Noise Separation for magnetotelluric transfer functions for synthetic data. The same data processed with the Remote Reference technique are displayed in fig. 3.5. The corresponding correlated-noise transfer functions have been shown in fig. 3.2.

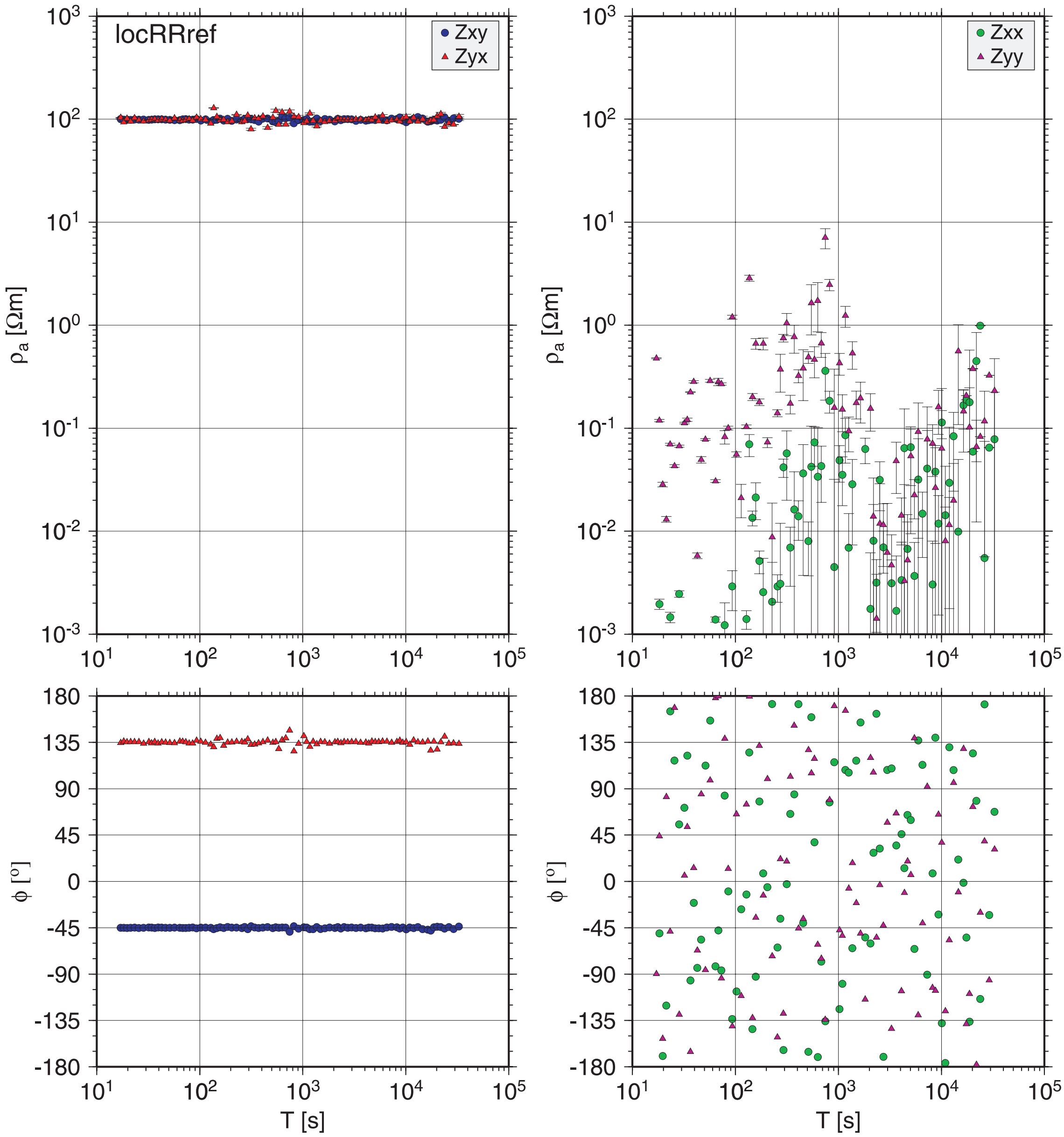

Figure 3.5: Processing results obtained with the Remote Reference technique. The data is the same as in the Signal-Noise Separation example shown in fig. 3.4. The results are equal except for the error bars.

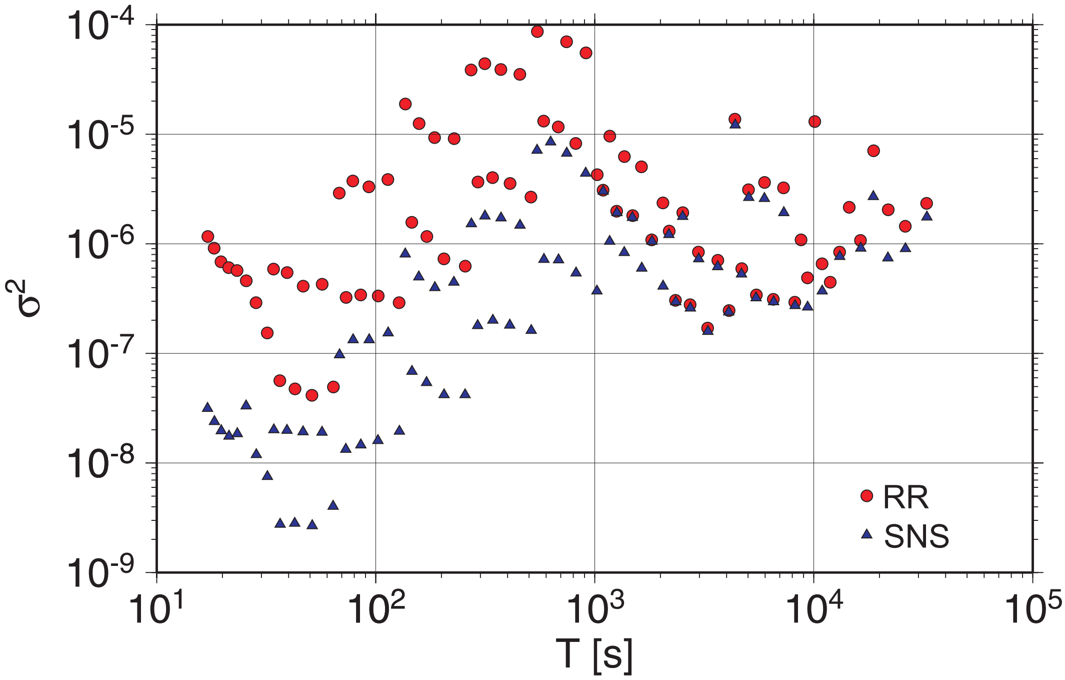

Figure 3.6: Variances of Z_{yx} of Remote Reference and Separation results for the synthetic data example shown in figs. 3.4 and 3.5. The RR transfer function has clearly higher variances due to the included E^{CN}, which is defined for periods up to 1000 s. The unit of the y axis is km^2/s^2.

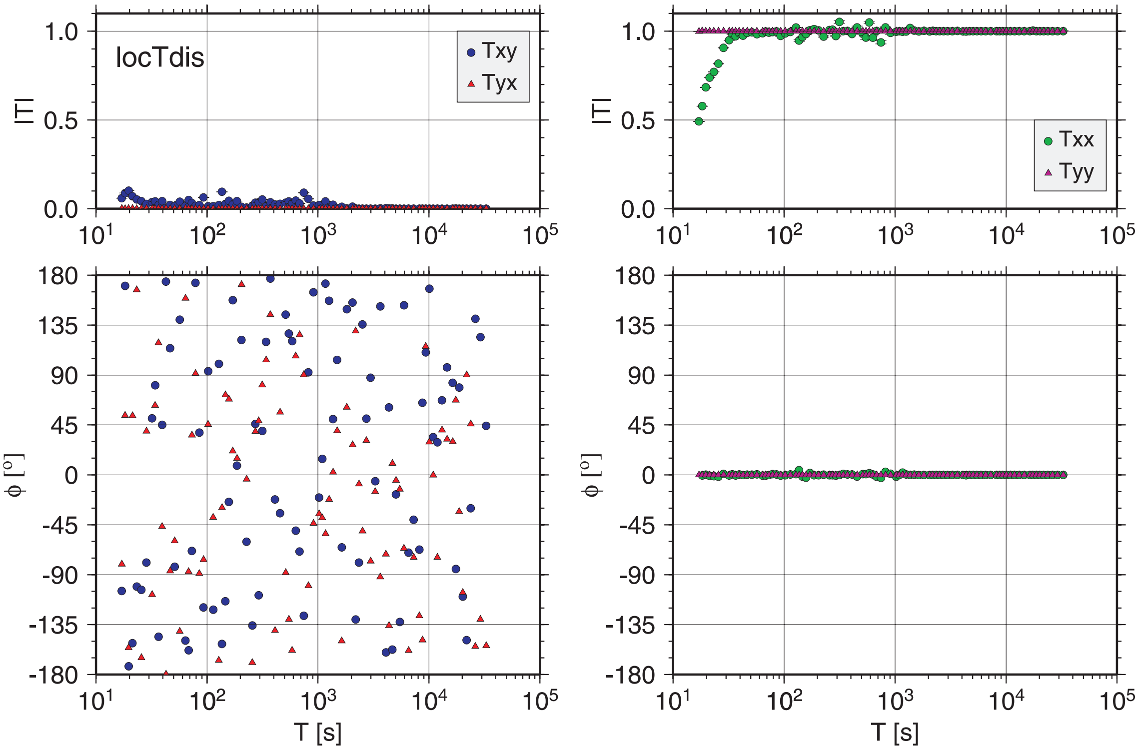

Figure 3.7: Disturbances in the remote site (here 128 synthetic peaks of $64nT$ height on channel B_x) can heavily bias the Separation tensor downwards. Due to the laws of least-squares (cf. text), this has hardly consequences for the MT Separation result (fig. 3.8), but is harmful for the CN transfer function (fig. 3.8). The unbiased tensor is the unity matrix.

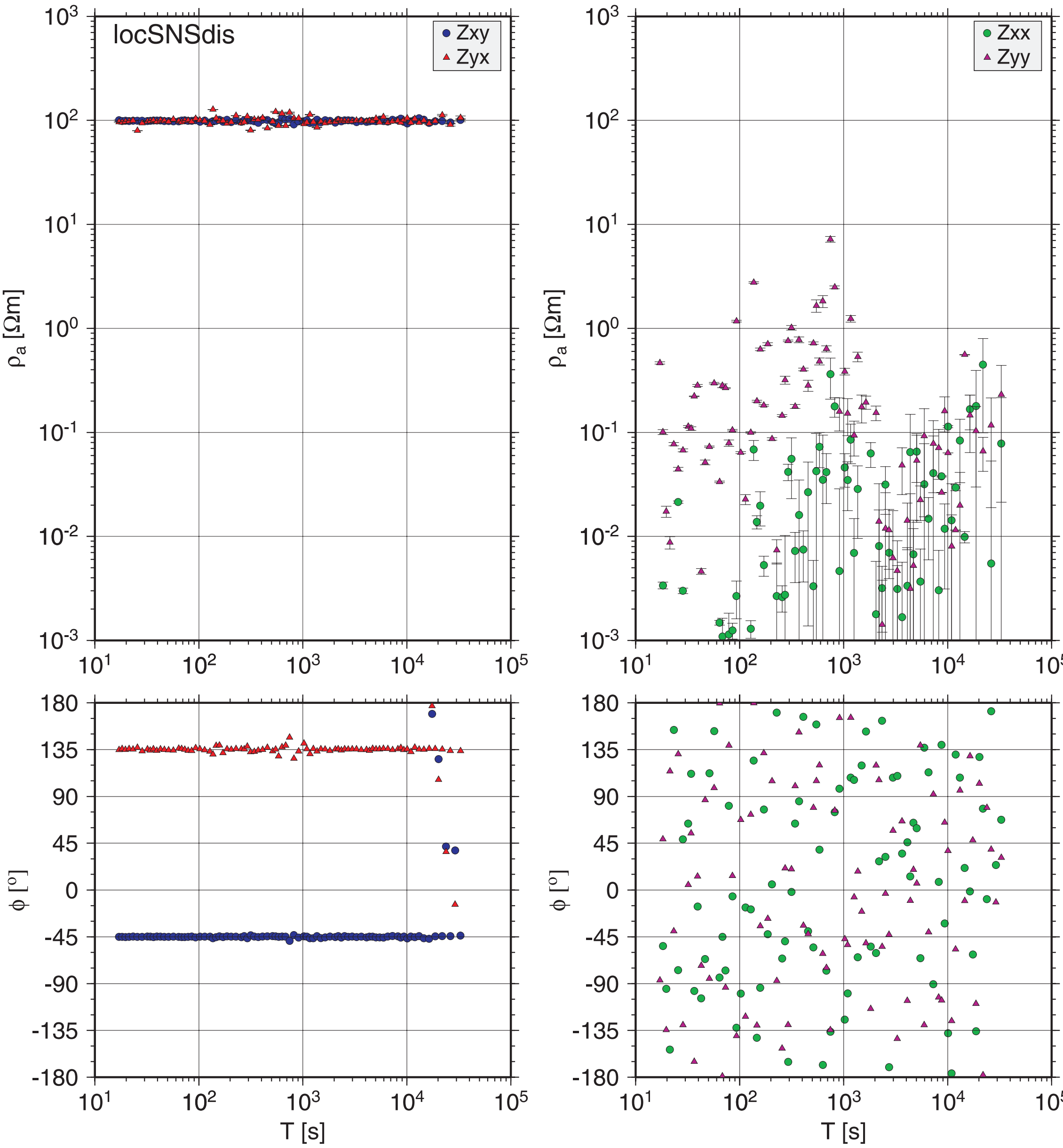

Figure 3.8: Although scattering a bit wider than in fig. 3.4 where the reference was noise-free, the magnetotelluric transfer functions obtained by the SNS method do somehow cope with the noisy remote records.

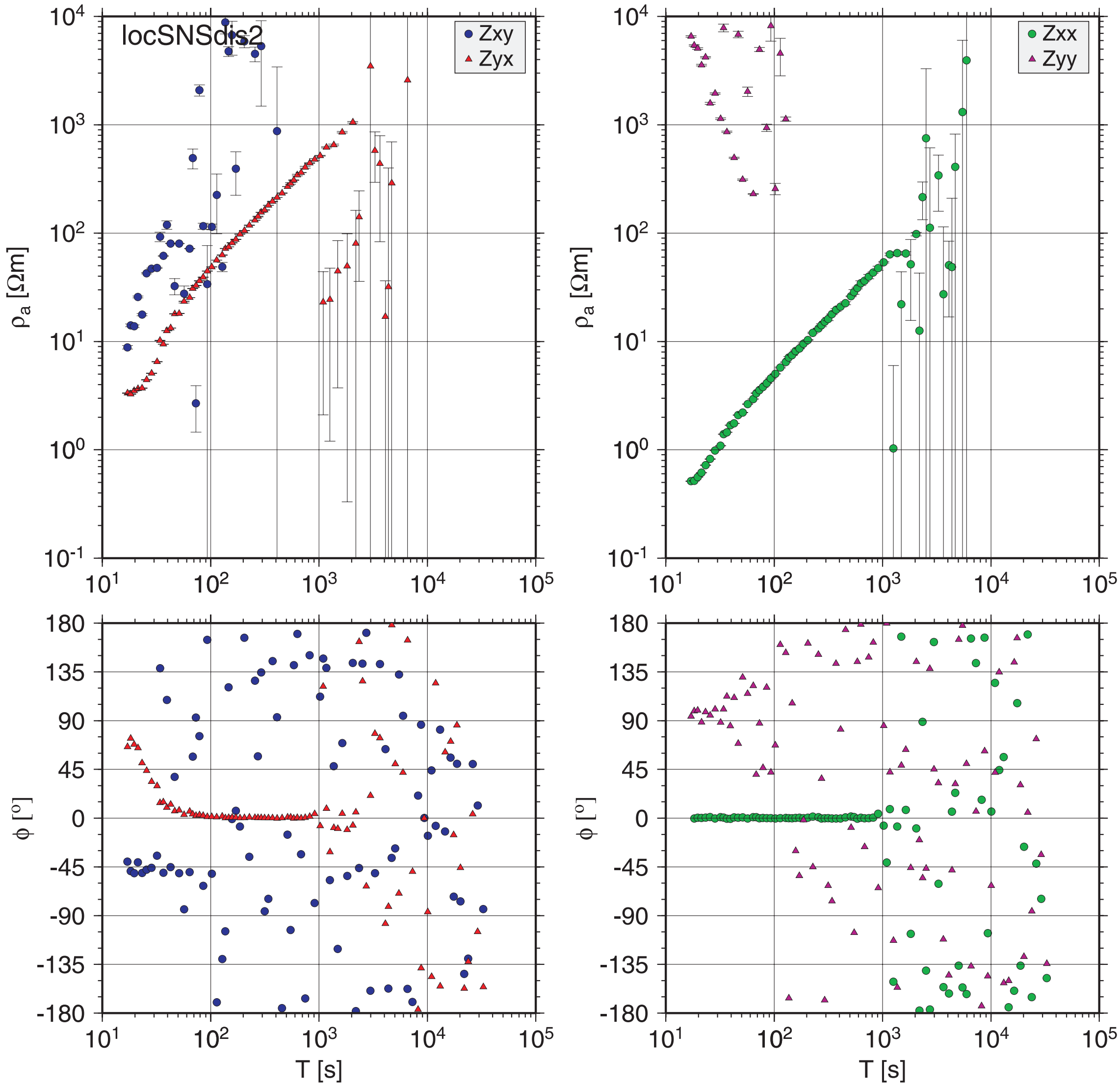

Figure 3.9: The transfer functions of the correlated noise have been biased downwards due to the noisy reference site in ρ_a. The Z_{yx} phase is distorted as well. Confer the "proper"results in fig. 3.2.