Chapter 2

Evading noise: The Remote Reference technique

The Remote Reference technique is a very helpful remedy for statistic noise on input channels (described in section 1.3.2) and, if the remote station fulfills certain conditions that will be treated of in section 2.3, also for correlated noise (see section 1.3.3). It is without influence on statistic noise on output channels as described in 1.3.1.
The formulas used in this technique are barely more complicated than the single-site ones. The difficulty having the most important impact within this method is rather a matter of measurement instrumentation and logistics, since there must be an additional station recording horizontal magnetic channels simultaneously with the station of interest and preferable with the same sampling rate. Because of its simplicity and undoubted efficiency the Remote Reference technique is a favored tool in practical magnetotellurics. Its history goes back more than a quarter century having been described first by Gamble et al. [1979]

2.1 New transfer functions due to a second site

The following derivation is based on Schmucker [1984].
In middle latitudes, natural magnetic variations are correlated over several hundred kilometers, see fig. 2.1. Hence it is possible to estimate transfer functions between simultaneously recorded channels of different stations. Two kinds of them are of importance here. The first one is the "remote-reference" analogon to the local impedances

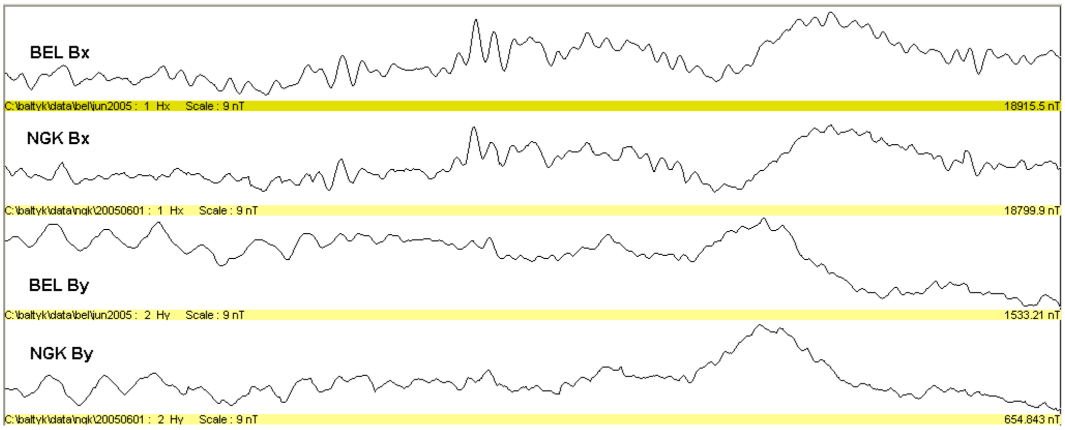

Figure 2.1: Horizontal magnetic time series of the geomagnetic observatories Belsk (BEL) and Niemegk (NGK). The scale is $9\,nT$, the length of the window 25 minutes. Obviously, the components of both locations are highly correlated in spite of the spacial distance of 560 km between them (cf. fig. 1).

(e. g. $\vec{Z}_x$ given by equation 1.10 and solved by 1.22). This means that the local electric field is referred to the horizontal magnetic field $\mathbf{B^R}$ of the so-called remote site:

$$\begin{pmatrix} E_{x1} \\ E_{x2} \\ \vdots \\ E_{xN} \end{pmatrix} = Z^I_{xx} \begin{pmatrix} B^R_{x1} \\ B^R_{x2} \\ \vdots \\ B^R_{xN} \end{pmatrix} + Z^I_{xy} \begin{pmatrix} B^R_{y1} \\ B^R_{y2} \\ \vdots \\ B^R_{yN} \end{pmatrix} + \begin{pmatrix} \delta E_{x1} \\ \delta E_{x2} \\ \vdots \\ \delta E_{xN} \end{pmatrix}. \tag{2.1}$$

The solution for $\vec{Z}^I_x$ is

$$\left(\vec{Z}^I_x\right)^T = \left(\mathbf{B^R}^\dagger \mathbf{B^R}\right)^{-1} \left(\vec{E_x}^\dagger \mathbf{B^R}\right) \tag{2.2}$$

where $\mathbf{B^R}$ is constructed like

$$\mathbf{B^R} = \begin{pmatrix} B^R_{x1} & B^R_{y1} \\ B^R_{x2} & B^R_{y2} \\ \vdots & \vdots \\ B^R_{xN} & B^R_{yN} \end{pmatrix}. \tag{2.3}$$

The second relevant type of transfer function refers the horizontal magnetic field of the remote station to that of the local one:

$$\mathbf{B} = \mathbf{B^R}\,\mathbf{T^T} + \delta\mathbf{B}. \tag{2.4}$$

Its solution for minimum error in $\mathbf{B}$ $\delta\mathbf{B}$ is the transfer function

$$\mathbf{T^T} = \left(\mathbf{B^R}^\dagger \mathbf{B^R}\right)^{-1} \left(\mathbf{B^R}^\dagger \mathbf{B}\right) \tag{2.5}$$

of the shape

$$\mathbf{T} = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix}, \tag{2.6}$$

which has miscellaneous names among magnetotelluric workers, e. g. inter-station transfer function (Egbert [1997], Soyer and Brasse [2001]), Separation tensor (Oettinger et al. [2001]), horizontal magnetic tensor (Varentsov and EMTESZ-Pomerania Working Group [2006]), and perturbation tensor (Schmucker [1970], there defined in a slightly different way, but meaning almost the same).
Since $\mathbf{T^T}$ transfers $\mathbf{B^R}$ into $\mathbf{B}$ and $\mathbf{Z^T}$ $\mathbf{B}$ into $\mathbf{E}$, the product of both operators must be equal to $\mathbf{Z^I}^T$ which transfers $\mathbf{B^R}$ immediately into $\mathbf{E}$:

$$\mathbf{T^T}\,\mathbf{Z^T} = \mathbf{Z^I}^T \tag{2.7}$$

In reverse, there must hold

$$\mathbf{Z^T} = \left(\mathbf{T^T}\right)^{-1}\mathbf{Z^I}^T. \tag{2.8}$$

This can be written in detail and transformed in the following way:

$$\begin{aligned}
\mathbf{Z^T} &= \left\{\left(\mathbf{B^R}^\dagger\mathbf{B^R}\right)^{-1}\left(\mathbf{B}^\dagger\mathbf{B^R}\right)\right\}^{-1}\left(\mathbf{B^R}^\dagger\mathbf{B^R}\right)^{-1}\left(\mathbf{E}^\dagger\mathbf{B^R}\right) \\
&= \left(\mathbf{B}^\dagger\mathbf{B^R}\right)^{-1}\left(\mathbf{B^R}^\dagger\mathbf{B^R}\right)\left(\mathbf{B^R}^\dagger\mathbf{B^R}\right)^{-1}\left(\mathbf{E}^\dagger\mathbf{B^R}\right) \\
&= \left(\mathbf{B}^\dagger\mathbf{B^R}\right)^{-1}\left(\mathbf{E}^\dagger\mathbf{B^R}\right)
\end{aligned} \tag{2.9}$$

This new expression 2.9 for $\mathbf{Z}$ is free of bias. The pernicious influence of noise in the input channels B^R_{xi}, B^R_{yi} has been canceled down together with the auto spectra of $\mathbf{B^R}$ due to the combination of $\mathbf{T}$ and $\mathbf{Z^I}$. On the other hand, both operators bring their statistical errors to $\mathbf{Z}$, so its variance σ^2_Z is the sum of the variances of $\mathbf{T}$ and $\mathbf{Z^I}$ and hence, in general, it will be bigger than in case of a single transfer function (Schmucker [1984]).

Thus, the Remote Reference method transmutes the imminence of a "dropping" bias as described in section 1.3.2 into a matter of statistic errors comparable to that of section 1.3.1. The following consideration may show that in principle, the problem of correlated noise (cf. section 1.3.3) is solved as well by this technique.
Correlated noise is dangerous because it is (like the magnetotelluric signal) coherent between input and output channels of the given station and hence enters the transfer function. However, if the Remote Reference technique is applied, the horizontal magnetic channels of the remote site instead of the local ones play the role of input channels during the evaluation of $\mathbf{Z^I}$ and $\mathbf{T}$. So, if the remote site is located beyond

the reach of the source of the correlated noise, the latter will not get into those transfer functions since it is impossible for it to find a counterpart in $\mathbf{B^R}$. Rather the correlated noise remains in the minimized residual where it cannot cause any harm exceeding increased error bars or scattering of transfer functions, respectively. Thus, the issue of correlated noise is reduced to that of section 1.3.1, too.

2.2 Application to data in need

For a brief demonstration of the effectivity of the Remote Reference method, there will be shown transfer functions obtained with it in this section. It is, of course, advisable to take just the examples which got distorted with the single-site equations.
Additional I show comparisons between cross and both possible auto spectra in these examples (figs. 2.2 and 2.4). The first one may illustrate the meaning of sentences like "Noise increases auto spectra" and "Cross spectra reduce this effect", that are often used to introduce the Remote Reference method. The second one shows how difficult it is from a spectral point of view to find a distinct solution within very noisy data.
The Z_{yx} transfer function of the case study affected by a dropping bias in section 1.3.2 is clearly cured when processed with the undisturbed horizontal magnetic channels as reference (fig.2.3). Figs. 2.5 and 2.6 show Remote Reference results (remote site: WIA, cf. fig. 1) obtained with my non-robust code and the robust one by Egbert and Booker [1986], respectively. Both are free of correlated-noise features (cf. fig. 1.13), but scattering in the lower-valued components worryingly, especially in fig. 2.5.
There is a method that promises to cope with correlated noise better than the Remote Reference solution. I will investigate whether this is true in chapter 3. However, first there will be addressed another important question in the next section.

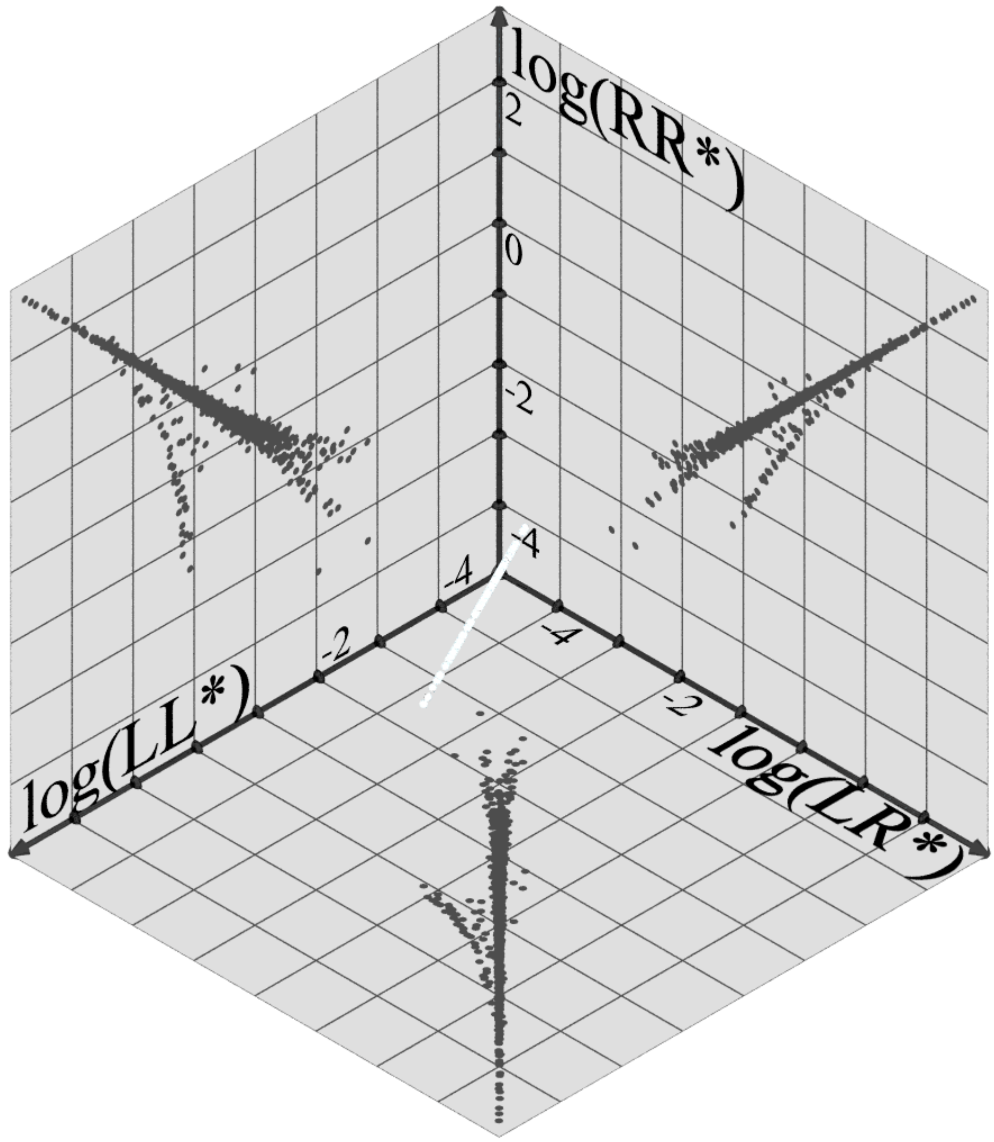

Figure 2.2: Comparison of spectra occurring in the synthetic data example (see text and fig. 2.3) for the B_x component at $32\,s$.

The conception behind this figure is to associate three different kinds of spectra, i. e. auto spectra of the local site(LL*), auto spectra of the remote site (RR*), and cross spectra of both (LR*), with the axes of a three-dimensional cartesian coordinate system. A comparison between two kinds of spectra is enabled by a projection onto the surface spanned by the corresponding axes. If the data was completely noise-free, all their spectra would be equal and on all three surfaces there would appear a straight line having an angle of 45^o with the axes. The view is chosen in a way that the perspective distortions for all surfaces are equal and the mentioned line is visible only as a point close to the axes' intersection in the ideal case.

Obviously, some local auto spectra are highly increased with respect to the remote ones due to the artificial noise in this case. The corresponding cross spectra are much smaller, although still larger than the remote auto spectra which have arisen from "clean" time series.

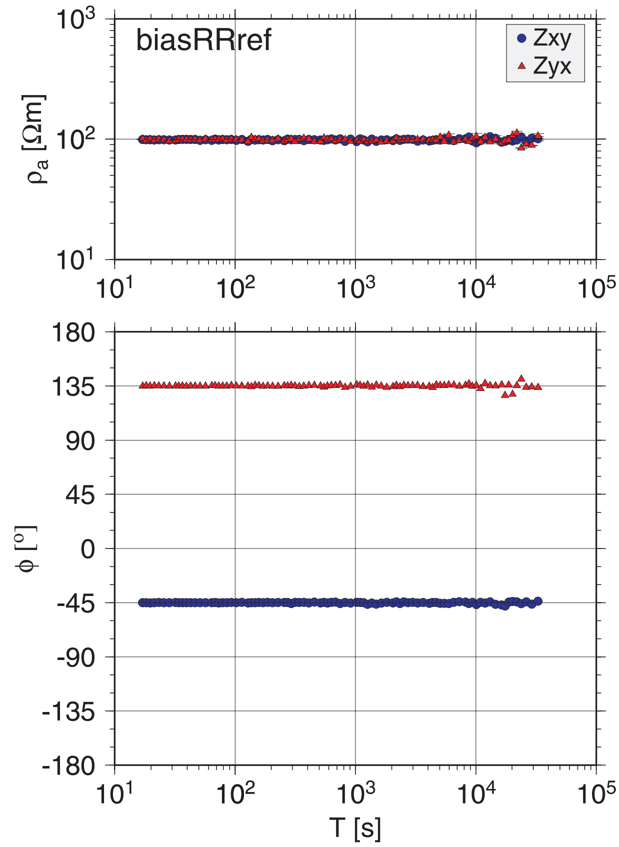

Figure 2.3: Transfer functions of the synthetic data example described in section 1.3.2 obtained with the Remote Reference technique. The undisturbed B_x and B_y time series have been taken as reference. Thereby the dropping bias of fig. 1.11 has vanished.

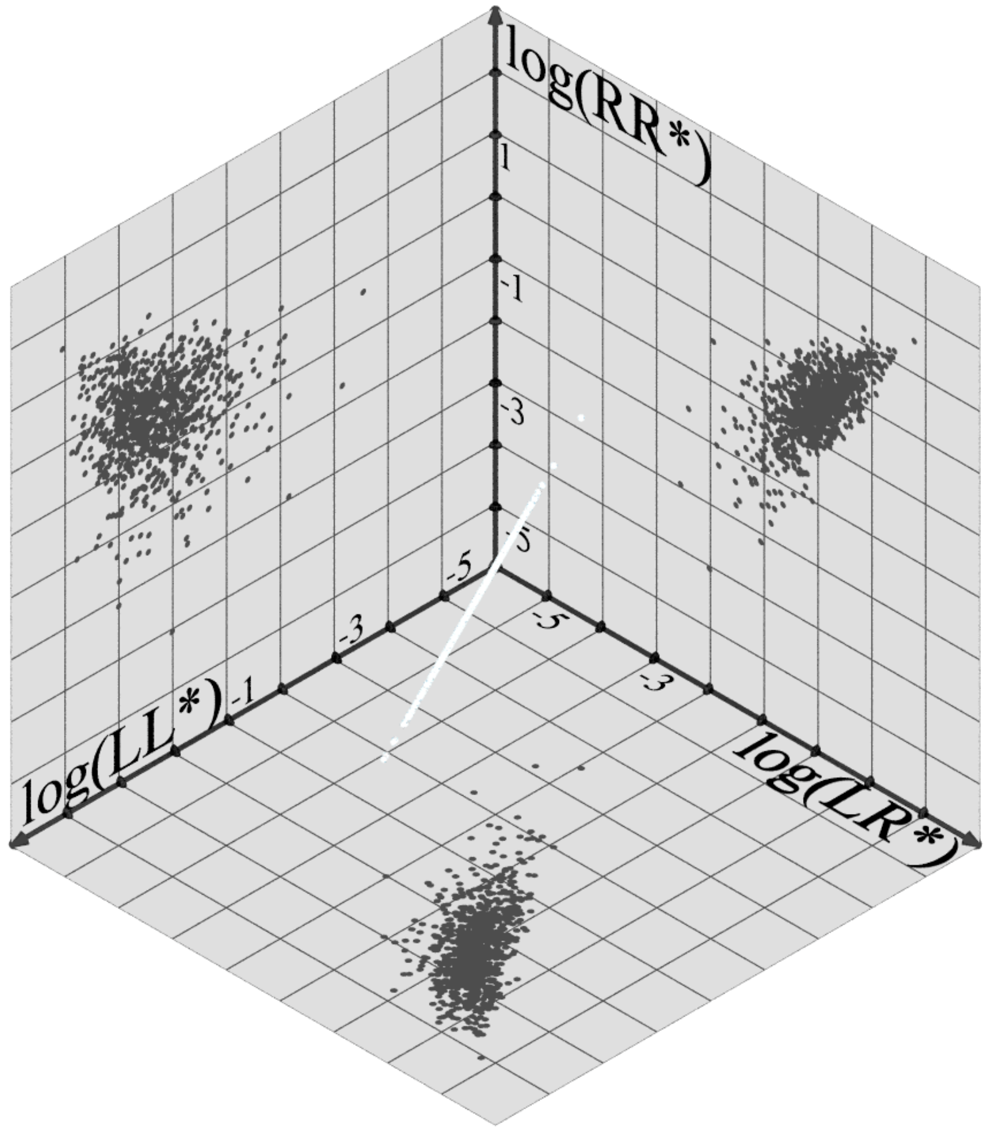

Figure 2.4: Comparison of spectra occurring in the data example sarRRwib (see text and fig. 2.5) for the B_x component at $32\,s$. See fig. 2.2 for explanation of the conception behind this picture. In this case, the distribution of spectra is far from the ideal line. This results not only from railway noise at SAR. In this case the scatter plots would possess a convex shape only at the side directed to the log(LL*) axis. Since the axis of reference auto spectra also faces a bellied shape, WIA must be a bit disturbed as well. Here the cross spectra are really beneficial in terms of mutual noise control. The hardly defined shape of the scatter plot also explains the poor results for Z_{yx} in fig. 2.5.

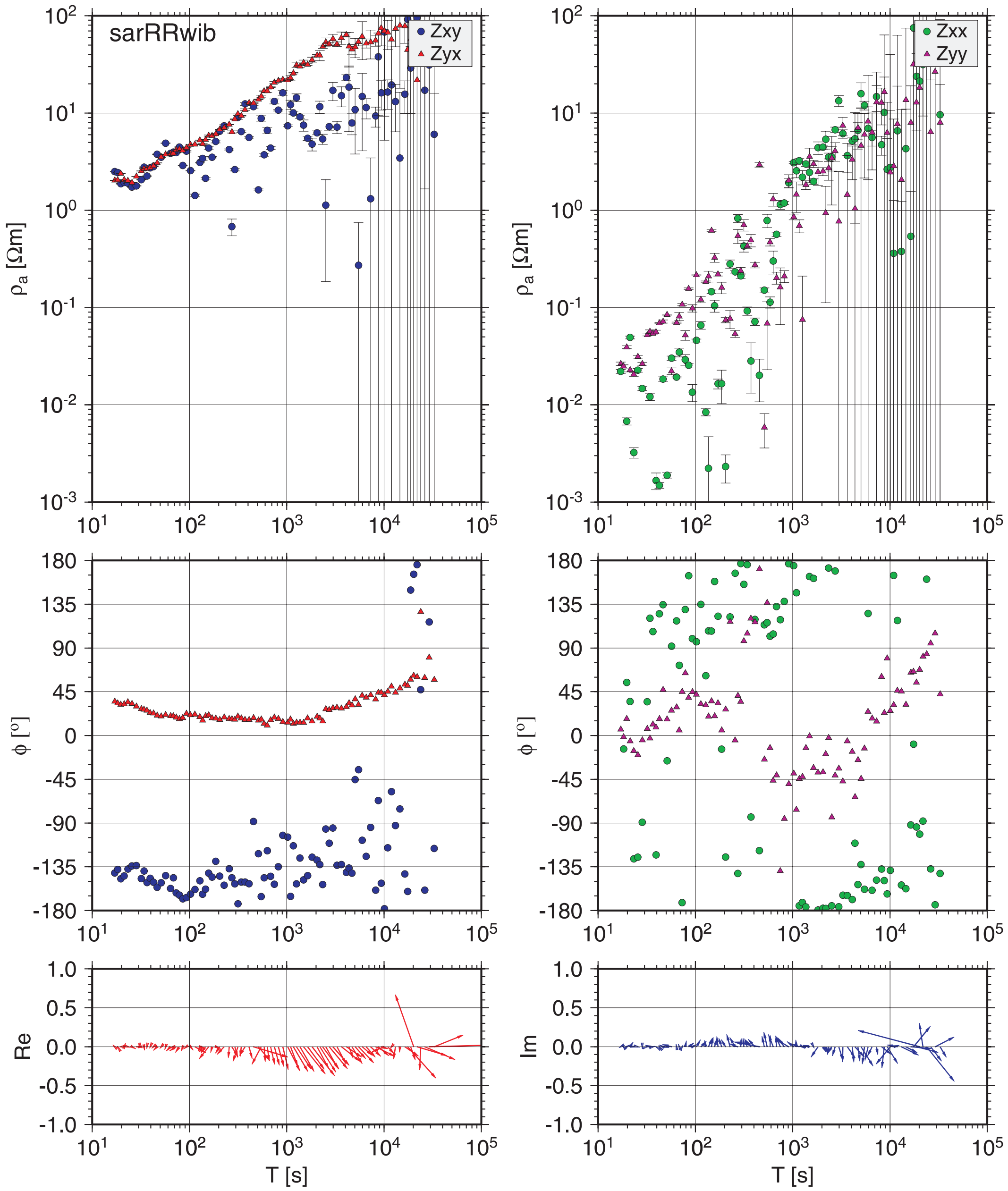

Figure 2.5: Transfer functions of station SAR obtained with the Remote Reference method with WIA as remote site. The features induced by correlated noise have vanished as a comparison with fig. 1.13 shows. The scattering especially of component Z_{yx} is too large for modeling purposes, but maybe preferable to bias. It shows that the strength of the natural magnetotelluric signals is significantly lower than that of the railway emissions.

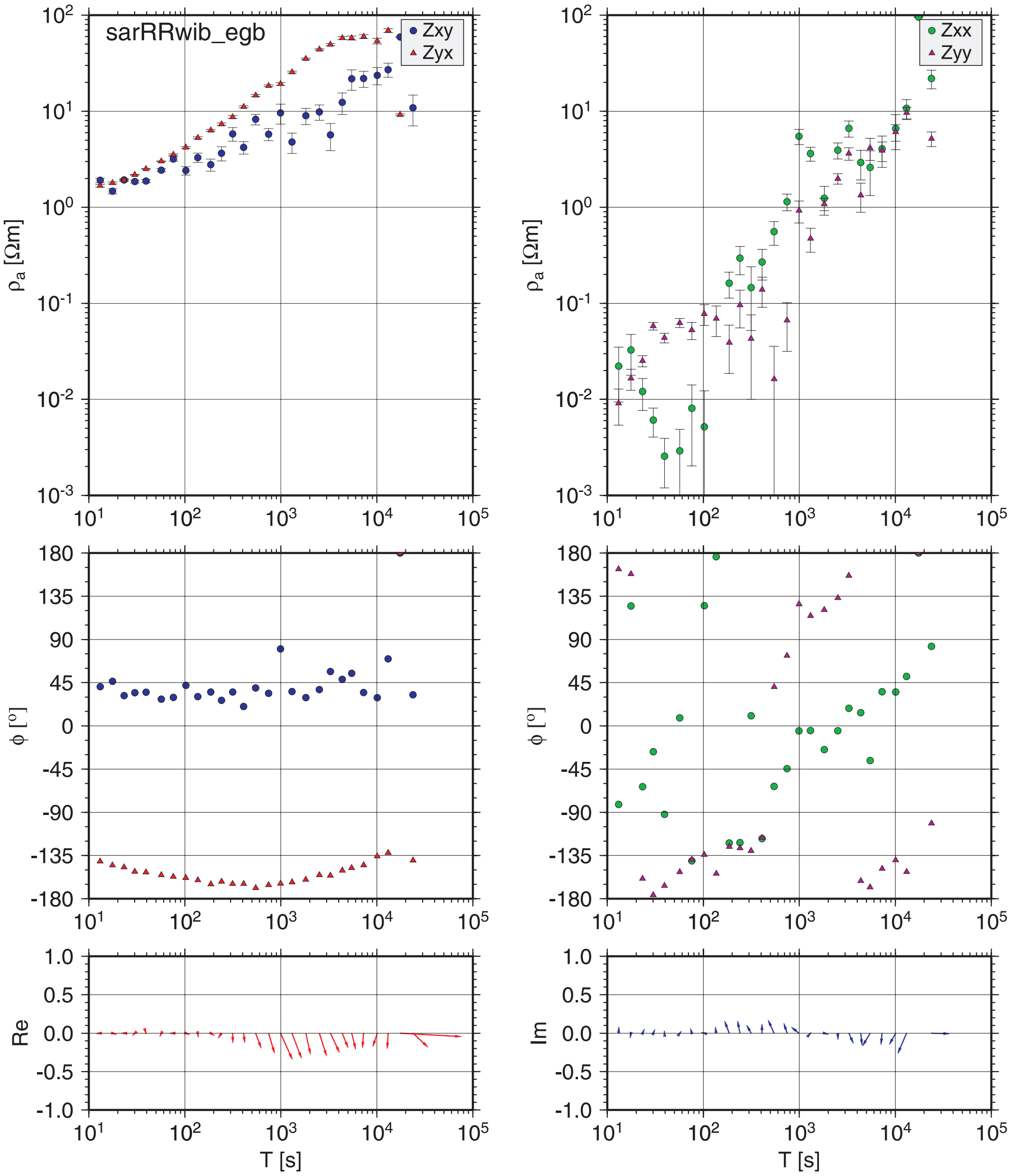

Figure 2.6: Transfer functions for site SAR obtained with the Remote Reference processing by Egbert and Booker [1986]. WIA is used as reference station. Although the results are much less scattering than those determined with the non-robust code (fig. 2.5), it is obvious that especially in the Z_{yx} component there are stability problems, too.

2.3 Requirements to a remote site

This section is supposed to address practical aspects of the application of remote stations. It will become clear that despite all theoretical elegance of the Remote Reference technique, there can occur problems if certain conditions are not fulfilled. That's why it is important to know these conditions when choosing or providing an appropriate reference station.

First, there has to be provided that some preconditions of fundamental and technical character are met.
The remote station has to record simultaneously with the measurement of the local station one is interested in. The time bases of both data loggers must be equal, this makes a synchronizing technique like via the GPS time signal preferable.
It is also necessary that the time interval the local and remote records are sampled with is the same or at least not very different. If it is different, the more densely sampled record has to be resampled onto the longer interval after an appropriate low-pass filtering. If the remote records are the originally less densely sampled ones, this means a loss of information for the transfer functions of the local station: They begin only at longer periods.
The quantities which have to be measured at the remote site are B_x and B_y, i. e. the horizontal magnetic field. Of course, the instruments' response functions for these channels are to be taken into account and therefore, they must be known.
I remind of things like sampling intervals and instruments' responses that, maybe, appear not worth mentioning if local and remote measurements are under control of the same worker. However, it will turn out to be desirable to use magnetic records of far-off working groups eventually obtained with completely different instruments as reference. Then it is very important to be able to deal with such realities.

Second, it may be worthwhile to address the issue of the required data quality of the reference station. It has been shown in section 2.1 that in principle, the harmful consequence (a dropping bias) of statistic noise in input channels is avoided with the Remote Reference technique. "Statistic noise" refers here to noise in the horizontal magnetic channels of the remote site that is not correlated with noise in the electric and magnetic channels of the local station. This means paradoxically, that noise correlated just between electric (if existent) and magnetic channels of the remote site fulfills the definition of statistic (i. e. uncorrelated) noise in this context. On the

other hand, it has been mentioned ibid. and shown in fig. 2.5 that such noise is quasi transmuted into an increased scattering of the transfer functions. From this follows that for stable results, there is more data necessary than in the single-site case. How much more data it is about, depends very much from the absence of noise in the remote records. Hence, it is highly desirable to install one's remote site far from all man-made infrastructure that could potentially harm the data quality to avoid a need for unrealistic amounts of data.
One technical feature more belongs to this issue. "Many data" means particularly long, uninterrupted records. Interruptions occur in long-period magnetotellurics almost necessarily due to attendance of the station. Since overlapping records of two stations are required here, this method is twice endangered by interruptions. Thus, one should try to decrease the number of needed maintenance visits at least of the remote site via providing it with appropriate technical parameters concerning memory space, power supply, and general reliance.

Third, if a remote station has to be used as a remedy for a site affected by correlated noise, it has to be located in a certain distance from that local site, or rather from the source of the correlated noise. The remote station must not be concerned by this noise for a successful application. To fulfill this condition is the harder the higher the resistivity of the subsurface and the given period are. This shall be explained a bit more detailed under reference to Oettinger [1999]:
In many cases, the source of correlated noise can be approximated as a horizontal grounded electromagnetic dipole. The propagation of the electromagnetic field emitted by it changes its character significantly with the distance. This leads to a discrimination into three zones: In the *near field* the only determining quantity for the electric and the magnetic field is the distance to the dipole, and the transfer function between them is independent of period and subsurface. In the *far field* it depends mainly on period and the electric resistivity of the subsurface ρ, whereas the distance to the source acts an underpart. Between both areas, the *transition zone* is situated, where the transfer function between electric and magnetic field depends on all three quantities. Being in the far field is a condition *sine qua non* for magnetotellurics, its default leads to all the problems referred to as correlated noise here. After Zonge and Hughes [1987] the far field is reached in a distance of the fivefold penetration depth from the dipole. The penetration or skin depth δ is a characteristic measure for induction processes near the surface of electric conductors signifying the depth in the conductor at which an electromagnetic field has decayed to one e^{th} part of its surface value. It depends on the period T of the incident electromagnetic field and the electric

resistivity ρ of the conductor. In a homogeneous half-space it is about

$$\delta \approx \frac{1}{2}\sqrt{\rho T}, \tag{2.10}$$

if ρ is taken in Ωm, T in s, and δ in km[1] So, in a distance of the fivefold of this expression from e. g. a DC railway, a magnetotelluric station will not suffer from the correlated noise, but even benefit from the artificial emitted signal. However, as long as this signal is still included in the time series, it will be correlated with the harmful one at stations closer to the source if Remote Reference is applied. Hence, a station just in the far field of a dipole source is not necessarily an appropriate reference for sites closer to this source, even if itself having reasonable transfer functions. Oettinger [1999] assesses, that multiplying the far field distance with an additional factor of 2.4 yields a sufficient attenuation of the artificial signal to make up a secure distance for a remote site.

This leads to alarming results. For a 100 Ωm homogeneous half-space and a period of 100 s (both values are ordinary) a distance of about 500 km is needed to get out of the reach of e. g. a DC railway. The results of Egbert et al. [2000], who found that signals of a Californian DC railway in a period range of $10 - 30$ s could be correlated over a distance of 300 km, are a good match for it. But, if a distance of several 100 km is not enough for a good remote site, it is an enormous effort to operate it simultaneously. It is virtually impossible for the longest periods ($\sim 10\,000\,s$). The scope of this problem is easily underestimated: When planning the measurements displayed in fig. 1, we had in mind WIA as a continuously running reference. However, it turned out that declaring just a station of the array as reference yielded poor results in some cases, in spite of the notable length of 400 km of the profile. WIA is too close-by to be a good remote site for stations in its surrounding, and had partly technical and noise problems. To take other sites of the profile as reference is also problematic. The run-time plot (fig. 2.7) shows the cause: For logistic reasons, simultaneously running stations are, in general, located close to each other rather than at different ends of the profile.

However, we were more lucky than Oettinger and Egbert with their just cited works in two different aspects: First, our measurements were carried out in a well-conducting sedimentary basin where disturbing signals decay within shorter distances (cf. equation 2.10). Second, in contrary to those audiomagnetotelluric experiments, here we sampled with a 2 s interval. This made it possible to use the 1 s records of the

[1] Note that formula 2.10 is a rough expression appropriate for, e.g. an "in-situ"estimation, not a result of an exact derivation. Namely, the units do not correspond. The exact expression for the penetration depth would be $\delta = \sqrt{\frac{2\rho}{\mu_0 \omega}}$.

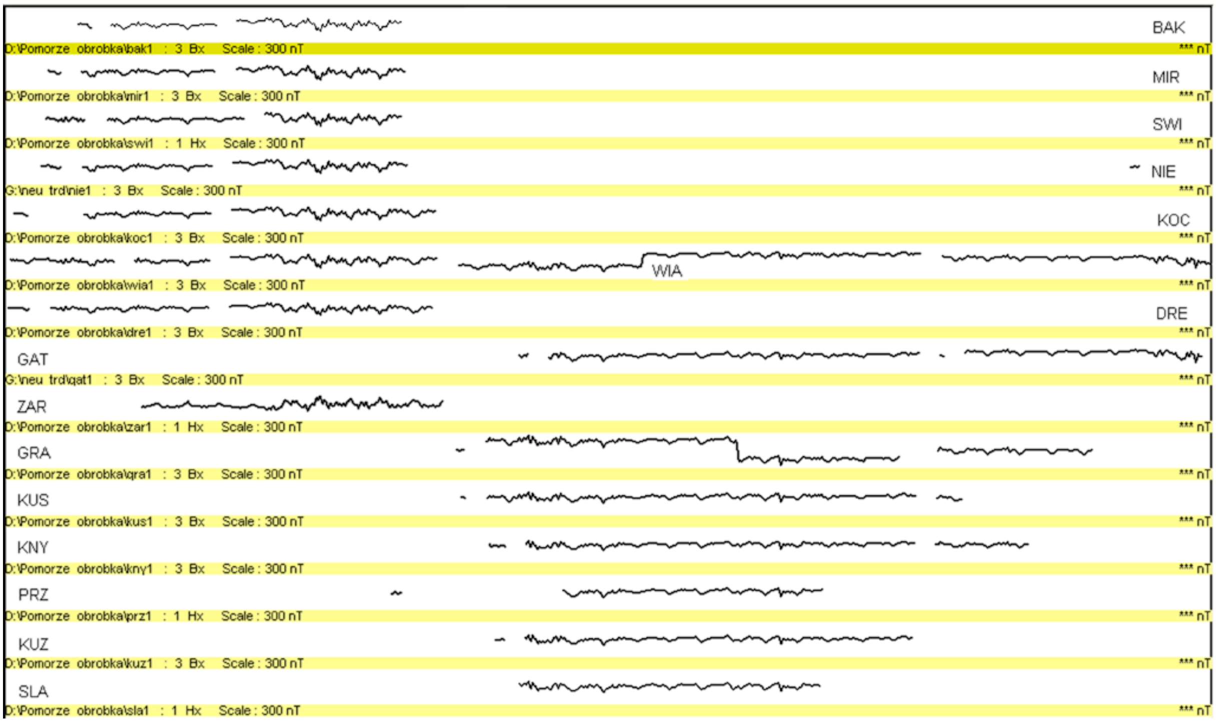

Figure 2.7: B_x records for all stations running during the 2003 campaign (window length: 37.6 days) arranged in spacial order according to the profile at fig. 1. They show which stations were operated simultaneously. The breaks between single records appear longer as they were because the records were shortened due to filtering. The plot shows the problem that mostly, simultaneously running stations are contiguous.

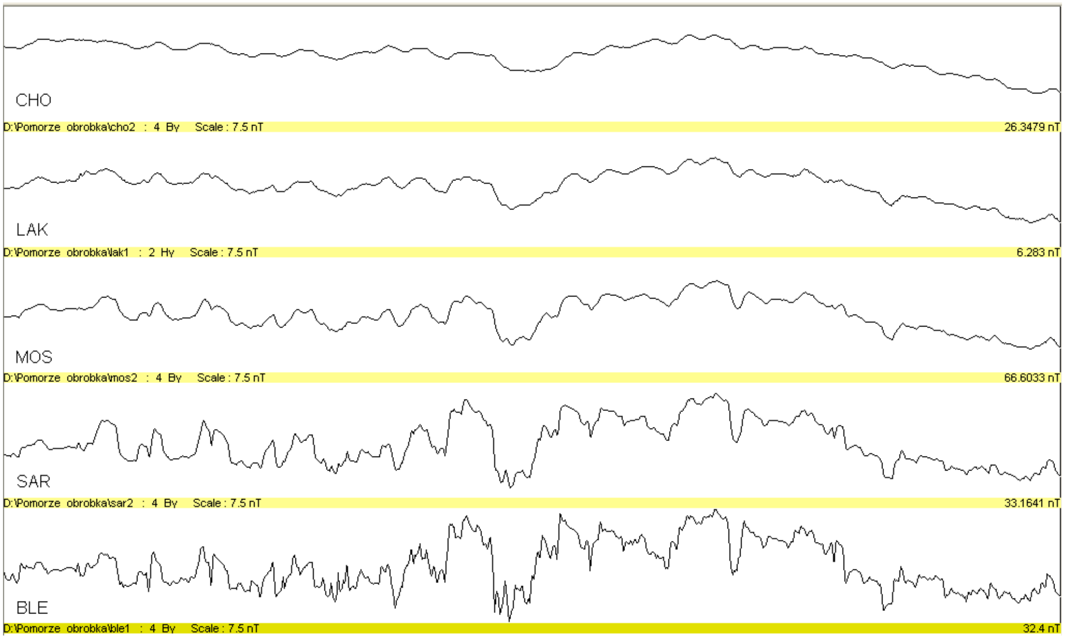

Figure 2.8: B_y time series (window length 34 min, scale $7.5\,nT$) for five synchronously measuring neighboring stations each with an average distance of $10\,km$. There runs a DC railway between sites BLE and SAR (cf. fig. 1). Its disturbing signal can be traced easily over more than $30\,km$, although it is attenuated in a frequency-dependent way that softens the signal with distance.

geomagnetic observatories Niemegk and Belsk (see fig. 1) as remote sites. This guaranteed not only continuous data, but also a distance of 350 - 450 km from all stations of the profile and was invaluable for some of the considerations following in the subsequent chapters.

Concluding, it can be said that the sufficient condition for a remote site (to be beyond the reach of any signal emitted by the given source) is hardly accomplishable for the entire long-period range. However, in practice it's often enough to see that some necessary conditions are not breached. I will show in the following some examples of how such breaches can look like. To be alarmed if such features appear in the transfer functions prevents the worker from blunders.

First, if a time series of a station in a certain distance to a DC railway still show its emissions, it is by no means an appropriate remote site for stations even closer to that railway. Fig. 2.8 demonstrates this situation: B_y of site CHO possesses features clearly correlated with the start-up peaks on the railway line between SAR and BLE. If one was looking at the time series of CHO alone, one could easily overlook that it is a matter of man-made noise, since the shape of the record looks almost natural due to

the softening effect of frequency dependent damping. That's why it's very important to display time series of different stations together to get an impression of the signal's origin.
Ignoring this reality and taking CHO as a reference for SAR leads to results displayed in fig. 2.10, that are though different, hardly better than the single-site ones (cf. fig. 1.13). Thus, if applying the Remote Reference technique does not clearly reduce the correlated-noise-features of the single-site results, one has to think about a more remote reference site as it has taken place for this example in fig. 2.5. To complete this case study, fig. 2.9 demonstrates that the transfer functions of CHO itself are affected by correlated noise in middle period ranges, too, and that they need Remote Reference to become reasonable. In other words, CHO is not even in the far field of the railway between BLE and SAR.

There is a further, maybe less obvious indication of correlated noise in transfer functions worth to know about. It is a matter of a smooth phase shift upwards or downwards at short periods in the main diagonal elements of the horizontal magnetic transfer functions that cannot be explained by induction processes. Fig. 2.11 shows this effect between KOC and WIA, the latter being reference. These stations are neighbors on the profile (see fig. 1), and the expected shape of the horizontal magnetic tensor in the case of near-by sites is something very close to the identity matrix, i.e. zero phases for the diagonal elements. The suspicion that the deviation from this has something to do with the bias due to uncorrelated noise in WIA (this effect is visible in the modulus) can be excluded by the reverse experiment: if WIA is referred to KOC, the phase shift looks alike but upwards. Other obvious suspicions like a lack of synchronization or a mistake in applying instruments' responses, could be excluded as well. What me makes to associate this astonishing effect with correlated noise is the result of an alternative way to estimate that transfer functions shown in fig. 4.6. Here they look completely as expected after applying a processing technique which removes the effect of correlated noise between both sites. This technique will be described in chapter 4. To explain the phase shift in detail is beyond the scope of this work. But, as mentioned above, in the near field and the transition zone of a dipole, the transfer function between electric and magnetic field depends on distance. This makes it rather plausible that the phase of the magnetic field changes with distance, too, and that the shift between both stations is an expression of their different distances to the railway. This would mean that either both sites or at least the closer one is not situated in the far field yet.

In practice, finding the best reference station will be a question of trial and

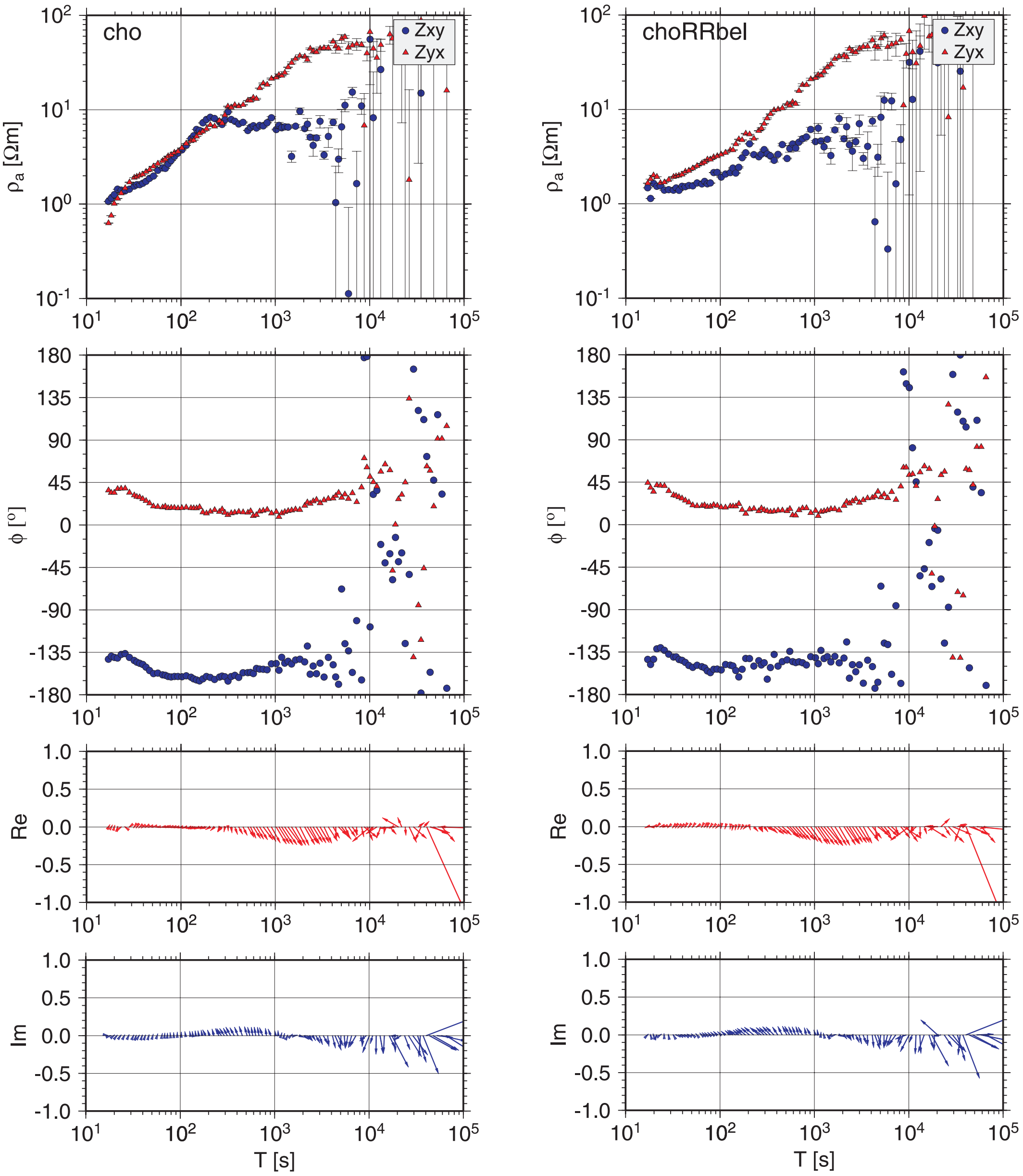

Figure 2.9: Sounding curves for off-diagonal elements of the impedance tensor of site CHO, left hand side obtained via single-site, right hand via Remote Reference processing with Belsk. As clearly visible due to the different results, the transfer functions of CHO are affected by correlated noise. Hence features like the sharp bench in ρ_a of Z_{xy} at $200\,s$ in the single-site case, must caution the worker about the presence of correlated noise, even if it's not as clear as with SAR in fig. 1.13. Particularly, such a station is not to be used as a reference for sites that could be affected by correlated noise emitted by the same source. Note also the dropping bias at the shortest periods of Z_{yx} in the single-site case.

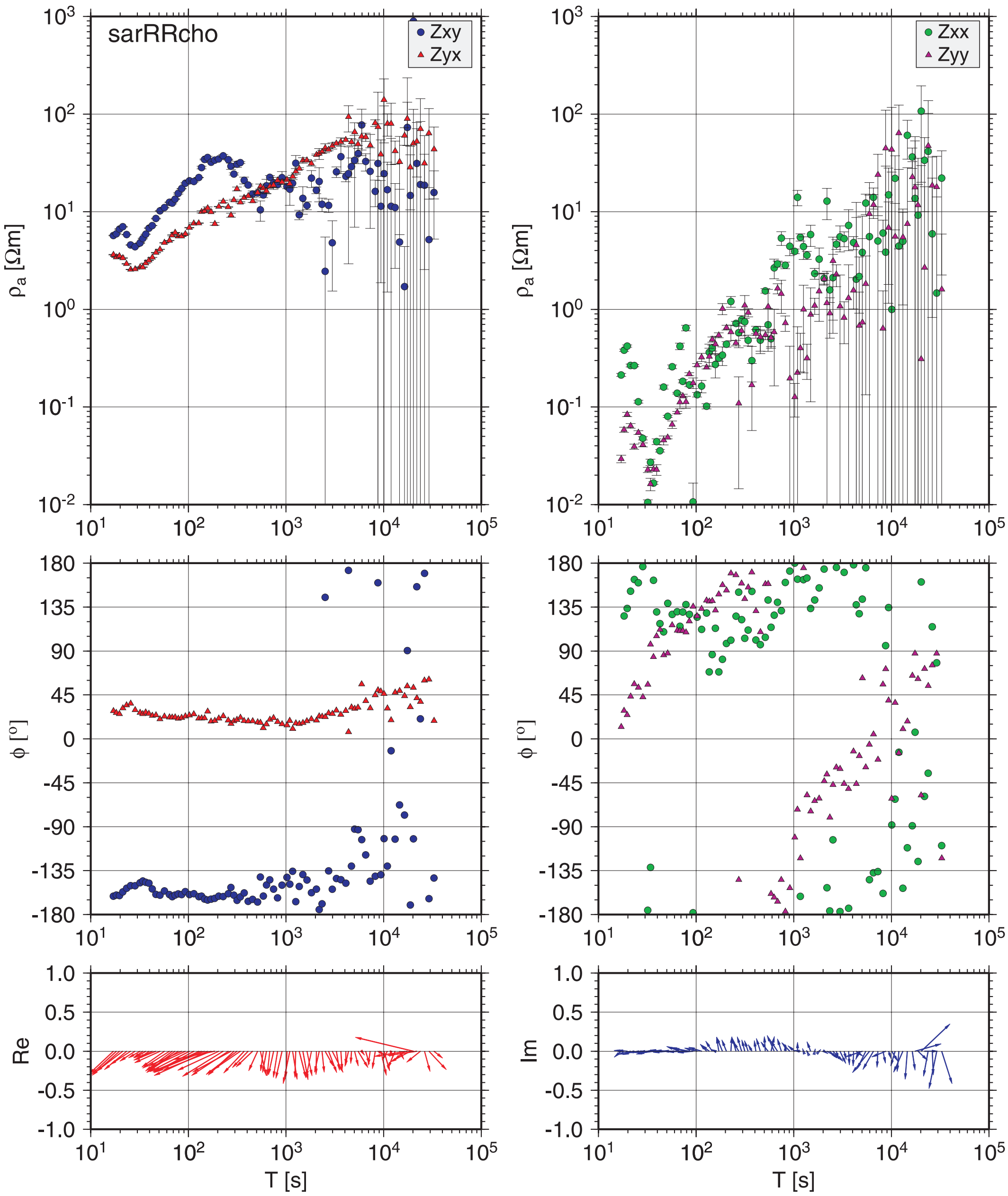

Figure 2.10: An example for a failed application of the Remote Reference technique. The transfer functions of site SAR still possess, though reduced, features of correlated noise as in the single site case (cf. fig. 1.13) due to the too close-by reference CHO (see fig. 1). The reasonable induction-like shapes of curves and arrows are achieved only with a far-off site like WIA in figs. 2.5 and 2.6.

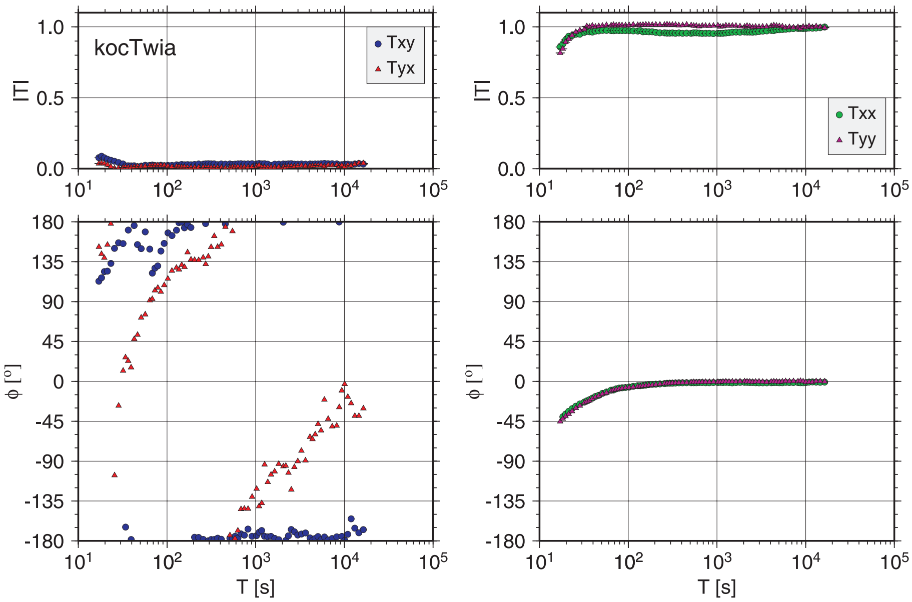

Figure 2.11: The horizontal magnetic tensor between sites KOC and WIA (displayed are above modulus and below phase for all elements) is affected not only by a dropping bias due to statistic noise in WIA, but also by a smooth phase shift at short periods in the components T_{xx} and T_{yy}. This shape is very untypical for induction processes. However, comparison with fig. 4.6 shows that these distortions are removable in a way suggesting that correlated noise between both locations is the reason for the phase shift, which can, in reverse, be used as an indication for at least one station not being in the far field yet.

error. Often it will be necessary to choose the lesser evil between potential remote stations having either too few or poor quality data or being to close for evaluating longer periods reasonably. However, it is important to understand why certain combinations cannot succeed with the Remote Reference technique. Equipped with the warnings given here one should be able to avoid the most unpleasant surprises in the business with remote sites.