On Knowledge and Ignorance

The strategic role of information in conflicts

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Chapter 1

Introduction

1.1 Motivation

Conflictual social interactions create incentives for participants to seek a strategically favorable position prior to the conflict. This can influence the outcome in a decisive way, both if the conflict takes the form of a situation where the participants have diverging interests (e.g. military conflict or political competition) or if the participants, in principle, share a common interest (as, for example, in the case of environmental protection). Often, ways of improving the own strategic position aim at achieving a credible commitment to a certain behavior in the conflict. For example, actions could be chosen that induce a credible threat of a certain (aggressive) behavior. Schelling (1980) gives the example of the burning of bridges behind oneself while facing an enemy. Similarly, for an incumbent firm, a deterrence effect will depend on whether the firm can make credible that it will engage in a price war should other firms decide to compete in their market segment. One instrument of achieving credibility is delegation, which allows for a commitment to future actions. Moreover, the timing of decisions can be used strategically: contestants can often benefit from having the possibility of a first move or precommitment. More generally, influencing one's own strategy space or the value or cost of choosing a certain action may lead to an advantage in an upcoming conflict. This can hold even if it, at first glance, worsens the own position, as in the example of burning bridges.

A crucial factor influencing conflict behavior and outcomes is the information that competing agents possess about their competitor, but also about their own potential gain or cost of choosing a certain action. Informational assumptions are important for explaining individual behavior, and information asymmetries can be driven by actions of individuals or firms (Hirshleifer and Riley 1992; Stiglitz 2000). Often, agents will be willing to spend a considerable amount of time or money in order to find out about the circumstances they are going to compete in, as typically ex ante they won't be fully informed of all variables determining their chances and the value of influencing a certain outcome in their favor. When architecture firms compete for a contract to build a new museum or opera house, they do not only incur costs for preparing a design. They will also expend resources to find out about what the value of winning the contest would be, and they will do so by clarifying details about the utilization of the building, negotiating with construction firms and ordering geological surveys. Automobile manufacturers will acquire information about the market potential before investing in environmental technology, and, more generally, firms often expend substantial amounts for the services of consulting companies in order to improve their investment strategies.

When contestants acquire information prior to the conflict, they are able to choose whether to invest in information that is only related to themselves or in information that relates to common circumstances. In addition, they can decide whether to conceal this information or to reveal it to their competitors, and whether to hide or make public their acquisition of this additional information. Decisions to acquire information, or to share information, may follow the balancing of direct cost and benefit of the information, but they can also be driven by strategic considerations, as the own decision can influence the behavior of the opponents. For instance, when firms publish independent market surveys, this affects the competitors' estimation of their potential profits. Moreover, observing that a competing firm has precise information about market conditions and consumer preferences can influence one's own investment decision, but this also holds if one is convinced that the competitor does not possess detailed information. In the case of environmental protection, for example, where agents' efforts serve as a means to achieve a common goal, being unaware of the cost and benefit of taking action could be used as a justification not to contribute. The informational context in which contestants act can be employed as a credible commitment to future behavior, and the strategic use of information can ensure a more favorable outcome of the upcoming conflict. An analysis of information in conflicts therefore incorporates both the fact that information can be acquired in order to resolve uncertainty and that there may be strategic reasons for decisions on information acquisition.

The issue of information acquisition plays an important role in different strands of the literature. When drilling rights for oil and gas are sold via an auction, the competitors may try to gather private information about the value of the rights by conducting seismic surveys (Milgrom and Weber 1982; Hendricks and Kovenock 1989). Firms that compete in oligopolistic markets often draw on the services of consulting companies in order to resolve uncertainty about future profits (Hurkens and Vulkan 2001). In financial markets, investors can purchase information about the return to a security (Grossman and Stiglitz 1980). Before accepting a contract, firms conduct cost analyses to obtain superior information about the necessary technology or input costs (Crémer and Khalil 1992; Kessler 1998). In many of the numerous studies in the literature, strategic considerations are involved in the decision on information acquisition. Often, these considerations are due to the fact that others will adapt their behavior when observing the information acquisition of a competitor.

In the analysis of conflicts and contests, questions of information acquisition and the strategic role of information are mostly unexamined. The theory of contests and conflicts analyzes situations where competing agents invest some effort or resources in order to achieve an outcome in their favor or appropriate or defend economic rents. The invested amount has an impact on the probability of winning the contest, but cannot be recovered even if the contest is lost. Examples for such structures are firms investing in advertising in order to increase their market share (Schmalensee 1976; Monahan 1987), decisions on how much to expend for a lawyer when being sued (Farmer and Pecorino 1999; Wärneryd 2000; Baye et al. 2005), or political competition where higher campaign costs may increase the chances of winning the upcoming elections (Che and Gale 1998; Klumpp and Polborn 2006). Many more examples can be found, for instance, in the context of labor market tournaments or rent-seeking and lobbying (see Konrad 2009). Moreover, conflicts can emerge even if agents pursue a common goal, and the interaction resembles the game of private provision of a public good. Such interactions arise on the international level when, for instance, the decision has to be taken whether to intervene in a civil war and how the contributions should be allocated, or inside a country when different institutions or layers of government share a certain responsibility.

Obviously, in the context of military actions, information not only about the forces of the enemy, but also about the (economic) gain of winning a war, about geographic conditions or about the cost of sending own forces will be acquired before the decisions are actually taken. Similarly, firms typically order market surveys before entering a new market and competing with the incumbent firms. But firms may also be able to credibly publish information about their own technology or their predicted gain of attracting a certain share of consumers, and they may, for instance, do so in order to signal a strong willingness to compete and in this way discourage their competitors. Similar motives can be present in military conflict where, for example, information about the possession of nuclear weapons may be used as a means of deterrence and hence affect the opponents' behavior.

More information, however, is not always beneficial. There can be situations where not knowing certain circumstances of an upcoming conflict can make an agent better off. While this may seem counterintuitive when the agent's decision is considered in isolation, such an incentive can be driven by strategic considerations as soon as the impact of information on the decisions of the competitors is anticipated. To sum up, whenever one's own decision on information acquisition affects the behavior of the opponents in the actual conflict, one's information obtains a strategic value, and the same can be true for not being informed about some key factors of the conflict. Thus, information acquisition should not only be considered as a comparison of direct cost and benefit, but also strategic incentives have to be taken into account.

This thesis gives consideration to the strategic relevance of decisions involving the change of the information available to contestants. Taking into account the possibility of acquiring or releasing information is important for the prediction of actual conflict behavior. The thesis adds to the literature on strategic activities ahead of conflicts on the one hand and the literature on information and uncertainty on the other hand. In the following five chapters, different conflictual situations will be analyzed, and it will be demonstrated how incentives to acquire or to share information can influence the outcome of the conflict. Information acquisition and information sharing, respectively, will be modeled as a decision that takes place before the participants choose their effort in the conflict. Moreover, the analysis will abstract from the fact that decisions on information may be continuous in the sense that the contestants may be able to decide not only whether to invest in information, but also how much (money or time) to spend on information acquisition. Instead, the focus will be on the strategic importance of the decision itself. Consequently, the most part of the thesis will assume that the contestants' decisions can be observed by the competitors, as this emphasizes the commitment character of decisions on information.

The results on the willingness to incur costs in order to obtain information and on the possibility of a purely strategic value of remaining uninformed in certain situations will show that information acquisition has several effects, as it changes both one's own and the opponents' behavior. Agents' incentives and welfare consequences depend on the characteristics of the actual conflict. The question of what the competitors can observe if somebody acquires information will prove to be important for the strategic motives. Moreover, as in the literature on strategic moves prior to conflicts, the strategic value of decisions on information will rely on the assumption that the decisions are credible: if, for instance, a contestant decides not to acquire information, there is no possibility of secretly revising this decision.

In the following, an overview of the research questions and the results of the five main chapters will be given. The first of the five chapters picks up on two widely studied information structures, and for several standard auctions, these information structures are compared with respect to agents' payoffs and revenue. The following two chapters consider incentives for information acquisition and for information sharing in contests. The analysis of these two chapters focuses on contests without noise, where, in contrast to probabilistic contest success functions, the contestant who invests the most effort wins with probability one. Such perfectly discriminating contests or all-pay auctions reflect situations where exogenous shocks do not play a decisive role for the contest outcome, and many of the properties of the all-pay auction persist in contests with small exogenous noise (see Che and Gale 2000 and Alcalde and Dahm 2009). The fourth of the five chapters considers strategic information acquisition in the context of global warming where agents share a common goal and the interaction can be described by a game of private provision of a public good. In the last chapter, incentives for information acquisition are analyzed in a war of attrition where everyone prefers that someone else concedes first. Part of this research has been carried out with co-authors. Chapters 2 and 3 are joint work with Johannes Münster from the Free University of Berlin, and Chapter 4 is joint work with Dan Kovenock from the University of Iowa and Johannes Münster.

1.2 Private versus complete information in auctions

The thesis starts with a basic analysis of the impact of the information structure on agents' payoffs in situations that can be described by standard auctions. In a standard auction, participants simultaneously submit their bid and the highest bid wins (see Krishna 2002). Auctions are very similar to certain types of conflicts, as agents may try to outbid or outperform their competitors in order to win a prize or influence an outcome in a way favorable to them. The analysis in this chapter comprises winner-pay auctions, where only the winner has to pay an amount that depends on the rules of the auction, as well as all-pay auctions, which are a standard contest model in the literature. In the all-pay auction, the contestants choose their outlay or bid, and the contestant that chooses the highest bid wins with probability one. All contestants, however, have to pay their bid.

A large part of the literature on auctions builds on one of the following two frameworks: auctions where the agents have private and independent values of winning (Vickrey 1961, 1962; Riley and Samuelson 1981; Myerson 1981), and auctions with complete information about all agents' valuations, which exhibit a structure similar to Bertrand competition. Whereas the famous revenue equivalence theorem compares the agents' bids among different auction formats for the case of private independent values, the contribution of this chapter is a comparison of the two basic informational frameworks, private independent values and complete information, for a given auction format. This can shed light on the question of how a certain information structure may form endogenously, and part of the results are used in Chapter 4 when the incentives to share information with competitors are examined.

It is shown that, for many standard auctions, the agents' payoffs under complete information are identical to their payoffs under private information. These auction formats include the first-price and the second-price winner-pay auction, combinations of these two, and the all-pay auction. In the first-price and second-price auction, expected revenue is the same under complete and under private information. In the all-pay auction, however, bids are lower under complete information than under private information. This is due to the fact that, contrary to the case of private information, the allocation is inefficient under complete information and agents do not win with probability one even if they have the highest possible valuation. Thus, their efforts in the contest are lower under complete than under private information. The result of payoff equivalence does not, however, carry over to all standard auctions. As an example, in a convex combination of the first-price all-pay auction and the second-price all-pay auction, the agents prefer complete information about all valuations to keeping all valuations private information.

1.3 Information acquisition in conflicts

After this more general comparison of two (possibly endogenously arising) information structures, the next chapter studies incentives for information acquisition prior to an all-pay auction as a standard model of conflict. Contrary to the assumption in the previous chapter, the information that is available to the contestants need not be symmetric, but there may be superior information about one of the contestants. This can, but need not, be a property caused by the nature of the problem. If, for instance following deregulation, an incumbent firm has to defend its rents against firms entering the market, there is typically more information available about the incumbent than about the market newcomers. Otherwise, such asymmetric information structures can be caused by information acquisition when only a subset of contestants acquires private information prior to the contest.

The first part of this chapter analyzes an all-pay auction between two contestants where one contestant's value of winning is publicly known and the other contestant's value of winning is his private information. Similar problems have been considered by Hurley and Shogren (1998a,b) and Wärneryd (2003) for imperfectly discriminating contests, and by Konrad (2009) for an all-pay auction where one contestant's type is drawn from a two-point distribution. This work complements the analysis in Konrad (2009) by considering general (continuous) distribution functions. It does not only characterize the behavior in many naturally arising situations, but it is also a useful building block for the analysis of information acquisition. In the second part of this chapter, the contestants can invest in information about their value of winning before competing in an all-pay auction. In this setting, the value that a contestant derives from getting information is higher if the opponent does not acquire information than if the opponent purchases information as well. Thus, for intermediate cost of information, only one contestant will invest in information and the best response of the other contestant is to remain uninformed. In this sense, asymmetric situations can arise where one contestant possesses private information, but there is common knowledge about the other contestant.

For the contestants' incentives to acquire information, it is of importance whether the opponent can observe the decision to invest in information. In addition, there could be situations where the information itself cannot be kept hidden or where the contestants can influence what the opponent can observe. The analysis in this chapter focuses on three different cases: (1) the contestants' decisions on information acquisition are publicly observable, but the information itself is only privately observable, (2) both the contestants' decisions and the information are publicly observable, and (3) neither the decisions nor the information is observable by the opponent. Whereas in cases (2) and (3) information acquisition is efficient from the point of view of a social planner, there is excessive information acquisition in case (1). This implies that the observability of the decisions on information, while the information itself is kept hidden, constitutes a strategic advantage in the upcoming contest.

1.4 Information sharing in contests

Competitors may not only have the option of acquiring information before the actual competition takes place. They may also be in a position to decide whether to conceal relevant information or to share it with their competitors. This question has attracted considerable attention in the literature on oligopolistic competition, starting with work by Ponssard (1979), Novshek and Sonnenschein (1982), Vives (1984), and Gal-Or (1985). This literature usually focuses on Cournot and Bertrand competition. In markets with intense advertising activities or research competition, however, the interaction will rather exhibit characteristics of a contest, as resources invested typically cannot be recovered. Particularly in the case of R&D competition, it is also important whether a firm's technological information affects its competitors' investment cost. The literature has studied various aspects of information disclosure in research competition, and among the contributions are Bhattacharya and Ritter (1983), Bhattacharya et al. (1990, 1992), d'Aspremont et al. (2000), and, more recently, Gill (2008) and Jansen (2008). The present work adds to this literature by analyzing a structure described by an all-pay auction, and the focus is on purely informational spillovers.

Chapter 4 studies incentives for information sharing prior to market competition that takes the form of a contest. Two different situations are analyzed: first, a situation where the information that the competitors possess is about a private variable, such as the own production cost (*private values*), and second, a situation where the information is about some common circumstances, such as demand conditions (*common values*). In addition, it is distinguished how the process of information sharing takes place. If the decisions on information sharing are taken independently, to share information is strictly dominated. This holds both for private values and for common values. If information sharing takes place as an industry-wide agreement, such an agreement to share information can arise in equilibrium if the information concerns private values, but does not arise if the information is about a common value. Possible welfare consequences depend on the social value of the competitors' contest efforts. With private values, a ban of industry-wide information sharing can result in a Pareto improvement, whereas with common values, strong positive spillover effects of the firms' efforts can make a legal requirement to share information desirable.

Interestingly, the strict preference for keeping one's own information secret holds for any possible signal a contestant can possess. Thus, there is no incentive for strategically revealing a low value of winning in order to make the opponent less aggressive, or for revealing a high value of winning in order to discourage the opponent. Rather, the value of keeping the own information private can even be higher for high values of winning. Thus, in the setting analyzed here, the strategic value of information disclosure is limited.

1.5 Strategic information acquisition and the mitigation of global warming

The strategic importance of information may increase in situations where large uncertainties about economic circumstances prevail. An obvious example is the issue of global warming. Here, controversial discussions still persist on the assessment of the cost of global warming and on the estimation of a country-specific economic value of measures for climate protection. Additional information in the form of scientific reports can lead to more precise estimates of the cost of climate change, but can often not be kept secret.

The mitigation of global warming is effectively a conflict where the countries share a common interest, but prefer to free-ride on other countries' efforts, and it is best described by a model of private provision of a (global) public good. In such settings, substantial strategic motives are present, and the literature has identified several instruments that can be used strategically in order to increase the possibility of free-riding on the opponents' contributions. For example, there can be an incentive to reduce the disposable income (Konrad 1994) or to increase the own contribution cost before the contributions are made (Buchholz and Konrad 1994). The role of information, however, has not yet been analyzed, and it seems to be particularly relevant in the context of global warming.

Although being important for removing uncertainties regarding the cost of climate change, decisions on information acquisition should not be considered in isolation, since countries will take into account the impact of additional information on the contributions of other countries. Revealing strong preferences for climate protection will make the free-riding behavior of other countries worse and shift the burden to the country itself. There is a trade-off between the possibility of improving the own contribution due to better information and the strategic effect caused by the reaction of other countries. As the analysis in Chapter 5 shows, this trade-off can result in an incentive to ignore information about the country-specific valuation of the public good even if the information is available without direct cost. Purely strategic motives can lead to an equilibrium outcome where the country with potentially the highest economic value of mitigating global warming remains uninformed of its true value, and conditions are identified under which such strategic behavior decreases the efficiency of the public good provision. Then, the provision of information by a third-party can be welfare enhancing. Moreover, if additional information is only available in the future, investments in information may be used strategically in order to credibly delay the own contribution even if future contributions are more costly.

1.6 Volunteering and the value of ignorance

While, in the context of global warming, the countries' total contributions are important for climate protection, there are many examples of public goods where only one fixed contribution is needed. In teams, there are many tasks that have a common value for all team members, but fulfilling the task involves a cost and one person is needed to incur this cost. Somebody has to clean the coffee machine, the Christmas party has to be organized, or a conference has to be hosted. Many of such tasks are allocated by asking for a volunteer to provide the public good, and as a result, a war of attrition takes place: individuals prefer to wait until someone else volunteers, but waiting may be costly. The cost of waiting is determined by the time until the first individual volunteers.

The strategic character of such situations creates incentives to take measures that increase the likelihood that somebody else volunteers first. In particular, it influences how to deal with information about the task that has to be fulfilled. There can be situations where the individuals do not know with certainty their cost of providing the public good, as they do not know exactly how much time they will need for organizing the conference. But if they decide to find out about this cost, this can reduce their opponents' willingness to volunteer.

Such strategic moves in the context of wars of attritions are so far unexplored. The literature either assumes complete information (Bilodeau and Slivinski 1996; LaCasse et al. 2002) or private information (Bliss and Nalebuff 1984; Sahuguet 2006) about the key variables. Chapter 6 endogenizes the decision on information and shows that, if there is a finite horizon of the volunteering game after which the task is allocated randomly, there can indeed be a positive strategic value of not finding out about the own contribution cost. Ignorance can constitute a commitment not to concede and in this way induce an opponent to volunteer early. Here, as in the example of global warming, using information in order to credibly commit to a certain action creates a strategic advantage in the conflict, and the analysis of information as a strategic variable in conflicts reflects the importance that strategic moves can have for the equilibrium outcome of the conflict.

Chapter 2

Private versus complete information in auctions

This chapter is joint work with Johannes Münster.

2.1 Introduction

This chapter compares two popular information structures in single-unit standard auctions (where the highest bid wins): private independent values and complete information.[1] Bidders' values are independent draws from a commonly known distribution. With private values, every bidder learns her own value, but not those of her rivals. With complete information, all values become common knowledge.

For a given auction format, we compare expected revenue and bidders' payoffs under private and complete information. We show that in several auction formats, bidders' payoffs are the same in the two information structures. These auctions include - besides the first price and the second price auction - convex combinations of first and second price auction, and the

[1] This chapter is based on the article *Private versus complete information in auctions*, published in *Economics Letters*, 2008, Volume 101 (3), p. 214-216.

all-pay auction.[2] But our payoff equivalence result does not hold for every standard auction: in a convex combination of the all-pay auction and the war of attrition, bidders prefer complete information.[3] Moreover, we consider revenue rankings. In the first (and second) price auction, revenue is the same under private information as under complete information. In the all-pay auction and in combinations of the all-pay auction with the war of attrition, expected revenue is higher under private information.

Our results could be useful for auction design. In recent work on the release of information, Kaplan and Zamir (2000, 2002) consider auctions where the seller is partially informed about the valuations of the bidders and compare revenue of first price and second price auctions. Landsberger et al. (2001) show that, if the ranking of the valuations is common knowledge among the bidders, payoffs are no longer independent of the auction format: expected revenue is generally higher in the first price auction. Assuming instead that bidders observe a noisy signal about the opponents' valuations, Fang and Morris (2006) show for a two-bidder auction that revenue equivalence of first and second price auctions does not hold, and that the revenue ranking is ambiguous. In Kim and Che (2004), bidders know the value of some others. In their setup, bidders prefer the first price auction to the second price auction, but the second price auction yields a higher expected revenue. These papers constitute intermediate cases compared with the information structures in our model. We depart from this work by extending the analysis to additional auction formats and focussing on the role of the

[2] The all-pay auction is a model of several important economic situations, including R&D races (Dasgupta 1986), election campaigns (Che and Gale 1998), rent-seeking and lobbying (Hillman and Riley 1989; Ellingsen 1991; Baye et al. 1993), and promotional competition (Konrad 2000).

[3] The war of attrition has been used in Biology to model fights between animals (e.g., Maynard Smith 1974; Riley 1980) and in Economics to model industrial competition and market exit (Fudenberg and Tirole 1986; Ghemawat and Nalebuff 1985, 1990) or the private provision of public goods (Bliss and Nalebuff 1984; Bilodeau and Slivinski 1996). See Krishna and Morgan (1997) for a comparison of the all-pay auction and the war of attrition.

informational assumptions.

Our results also shed new light on the role of information in conflicts. Wärneryd (2003) studies an imperfectly discriminating all-pay auction between two players in a common values environment. He considers three information structures: no player is informed about the value, both players are informed, or one player is privately informed. He finds that revenue is the same in the two symmetric structures; revenue is lower in the asymmetric structure where only one player is privately informed. In our private value setting, expected revenue in an all-pay auction is higher under asymmetric information (where each player knows her own value) than under complete information. Moreover, additional information may have a nonmonotone effect on revenue: when no bidder learns any of the values and there are many bidders, expected revenue is lower than under private information, but under private information it is higher than under complete information.

2.2 The model

There are N bidders $i = 1, ..., N$ competing in an auction where one object is to be sold. Bidder i values winning the object by v_i. The timing of events is as follows.

1. Nature draws the valuations v_i independently from a distribution F. The distribution F can either be continuous or discrete. Each bidder learns her own value.

2. Under private information (case P), no further information is revealed. Under complete information (case C), the vector $(v_1, .., v_N)$ becomes common knowledge.

3. Bidders simultaneously choose their bids $b_i \in [0, \infty)$. The highest bid wins. Ties are broken in favor of the bidder with the higher valuation. If

there is a tie between several bidders with the same (highest) valuation, each of these bidders wins with the same probability.[4]

We use the following terminology. Fix an equilibrium of the auction. *Interim expected payoffs* are the equilibrium payoffs calculated between stage 1 and 2, that is, they are conditional on a bidder's own value, but not on the other bidders' values. *Ex ante expected payoffs* are the unconditional expected payoffs calculated before the valuations are drawn. Note that if interim expected payoffs are the same in case P as in case C, this implies equivalence of ex ante expected payoffs as well.

2.3 Comparison of the information structures

Second price auction. Consider the second price auction (SPA). Truthful bidding is a weakly dominant strategy both under private and under complete information.[5] Interim expected payoffs are identical in cases C and P; the same holds for expected revenue (and of course for ex ante expected payoffs).

First price auction. The first price auction (FPA) with complete information has an equilibrium where the bidder with the highest value bids at the second highest value and wins with probability 1. In the following, we focus on this equilibrium. (There are other equilibria where the bids are between the highest value and the second highest value. These equilibria, however, have the unattractive feature that some bidder plays a weakly dominated strategy.)

Proposition 2.1 *In the first price auction, expected revenue and interim expected payoffs of the bidders are the same in cases C and P.*

[4]We also discuss how the analysis changes if *all* ties are broken randomly.

[5]We focus on this equilibrium. See Blume and Heidhues (2004) for a full characterization of the set of equilibria of the second price auction.

Proof. The proposition follows from three observations: (1) By the revenue equivalence theorem, in case P the FPA is revenue equivalent to the SPA (Myerson 1981). A related result (called *Myerson's Lemma* by Milgrom 2004) shows that, in case P, bidders' interim expected payoffs are the same in the FPA as in the SPA. (2) As argued above, in the SPA, expected revenue and bidders' interim expected payoffs are the same in case C as in case P. (3) In the FPA with complete information, the bidder with the highest valuation wins with probability 1 and has a payoff of her value minus the second highest value. All other bidders have a payoff of zero. Thus in case C, bidders' payoffs and expected revenue are the same in the FPA as in the SPA.[6] ■

Proposition 2.1 is related to Kim and Che (2004), who note that in the extreme cases of their model - which correspond to our cases P and C - the first price auction is revenue equivalent to the second price auction.

All-pay auction. In the next step, we analyze expected payoffs and revenue in a standard all-pay auction (APA) where the highest bid wins and every bidder pays her bid. By direct computation of the equilibrium, Konrad (2009) establishes payoff equivalence for the case of two bidders and two types. He raises the question whether there is a general pattern in this equivalence.

Proposition 2.2 *In the all-pay auction, interim expected payoffs are equivalent in cases C and P and identical to the payoffs in the first and second price auction. However, expected revenue is lower in case C.*

Proof. The proof draws again on payoff comparisons with the SPA. In case P, there is a symmetric equilibrium in increasing strategies where the

[6] The specification of the tie breaking rule is not essential for Proposition 2.1 to hold. If ties are broken randomly, there are mixed strategy equilibria of the FPA yielding the same expected payoffs (see Deneckere and Kovenock 1996 and Albano and Matros 2005). (Again, there are additional equilibria with bids between the two highest valuations where at least one bidder plays a dominated strategy.)

allocation is efficient and the payoff of the lowest type is zero.[7] Hence, in case P payoff equivalence and revenue equivalence hold between the APA and the SPA. In case C, any equilibrium of the APA is in mixed strategies. As in the SPA, the expected payoff of the bidder with the highest valuation is equal to the difference between her valuation and the second highest valuation, and the payoff of all other bidders is zero.[8] Therefore, in the all-pay auction, bidders' interim expected payoffs are the same in case P and in case C. Since the allocation is inefficient in case C, expected revenue is lower. ■

This result is particularly interesting since the nature of the equilibrium strategies of the bidders differs in cases P and C. In the APA with complete information, the equilibrium is in mixed strategies and the probability that the bidder with the highest value wins is strictly below one. Nevertheless, her expected payoff is the same as in the first price or second price auction. As a direct consequence, her expected bid must be lower (to be precise, it is one-half times the second highest value). This explains the result on expected revenue. A seller would prefer the case of private information.[9]

One implication of Proposition 2.2 is that additional information may have a nonmonotone effect on expected revenue. Consider another information structure where *no* bidder learns any of the values (case N). In a sense, there is more information as we go from case N to case P to case C. In case N, expected revenue is equal to the expected value. In case P, expected revenue is equal to the second order statistic. If there are many

[7] See e.g. Krishna (2002) for continuous distributions of types. This equilibrium is known to be unique in the case of two bidders (Amann and Leininger 1996). For the equilibrium with two bidders and two types see Konrad (2004). His analysis can be extended to the case of N bidders with K types (details available on request).

[8] See Baye, Kovenock, and deVries (1996). The equilibrium is not necessarily unique if several bidders have the same (highest or second highest) value, which happens with positive probability if F is discrete. Moreover, the equilibria do not necessarily generate the same revenue. Our proof, however, holds for all equilibria, because bidders' payoffs are the same in all equilibria.

[9] Since in both cases P and C bid distributions are continuous, the payoff equivalence in the all-pay auction does not depend on the tie breaking rule specified above. Proposition 2.2 still applies assuming that all ties are broken randomly.

bidders, expected revenue is therefore higher in case P than in case N. But by Proposition 2.2, expected revenue is lower in case C than in case P.

Combination of first and second price auction. Suppose the winner pays a convex combination of her bid and the second highest bid, whereas the losers pay nothing. Denoting the payment of bidder i by x_i, we have

$$x_i = \begin{cases} (1-\lambda)\, b_i + \lambda \max_{j \neq i} b_j, & \text{if } i \text{ wins} \\ 0, & \text{else} \end{cases} \tag{2.1}$$

for some $\lambda \in (0,1)$. In case P, revenue equivalence holds to the SPA.[10] In case C, renumber the bidders such that $v_1 \geq v_2 \geq ... \geq v_N$. There is an equilibrium where bidders 1 and 2 bid at v_2 and the other bidders bid weakly lower. Bidder 1 wins with probability one. We focus on this equilibrium. (As in the FPA there exist additional equilibria involving weakly dominated strategies where $b_1 = b_2 = b \in (v_2, v_1]$.) Comparing cases P and C shows that interim expected payoffs as well as expected revenue are identical.[11]

Combination of all-pay auction and war of attrition. Until now, it seems that the payoff equivalence result does indeed hold for many standard auctions. It does not, however, hold for all auctions: for a combination of a first and second price *all-pay* auction, we show in the following that bidders'

[10] This is shown by Riley (1989) for a discrete typespace and by Plum (1992) for a continuous typespace with two bidders.

[11] The result changes slightly if ties are broken randomly. As in the first price auction, there is an equilibrium where bidder 1 bids at v_2 and bidder 2 randomizes uniformly on $[v_2 - \varepsilon, v_2]$, ε sufficiently small. However, in this equilibrium, the expected payoff of bidder 1 is by $\lambda\varepsilon/2$ higher than in case P, and expected revenue is lower by the same amount. Hence, payoff and revenue equivalence between cases P and C do only hold approximately.

There is also an equilibrium where the expected payment of bidder 1 exactly equals v_2 : bidder 1 bids $v_2 + \frac{\lambda\varepsilon}{2-\lambda}$; bidder 2 randomizes uniformly over an intervall $\left[v_2 - \varepsilon, v_2 + \frac{\lambda\varepsilon}{2-\lambda}\right]$, ε sufficiently small; all others bid are weakly lower. In this equilibrium, payoffs and revenue are the same as in case C. However, there seems to be no reason to focus on this particular equilibrium.

payoffs are higher under complete information.

Suppose F is continuous and there are two bidders 1 and 2. Let the payment x_i of a bidder i be defined by

$$x_i = \begin{cases} (1-\lambda)\, b_i + \lambda b_j, & \text{if } i \text{ wins} \\ b_i, & \text{else} \end{cases} \tag{2.2}$$

where $\lambda \in [0,1)$ and $j \neq i$. The loser has to pay her bid while the winner has to pay a convex combination of her bid and the other bidder's bid. This structure of payments subsumes a class of all-pay auctions. For $\lambda = 0$, we obtain the standard all-pay auction whereas if $\lambda \to 1$, the game becomes a war of attrition.

Proposition 2.3 *Consider an auction with two bidders and payments defined in (2.2) and suppose that F is continuous. For all $\lambda \in (0,1)$, interim expected payoffs of the bidders are strictly higher in case C than in case P. Furthermore, expected revenue is lower in case C.*

The proof is relegated to the appendix. There, we show that payoff equivalence holds only for $\lambda = 0$ (the all-pay auction). In case P, expected payoffs do not depend on λ. In case C, expected payoffs are strictly increasing in λ. Taking the limit for $\lambda \to 1$ selects an equilibrium of the war of attrition where the bidder with the lower valuation bids zero with probability 1 and the expected revenue of the seller is zero. Of course, in case C, the war of attrition has a continuum of equilibria, including one equilibrium that is payoff equivalent to case P. However, there seems to be no reason to focus on this equilibrium.

2.4 Conclusion

We compared private-value auctions with auctions under complete information. Our main result is that in many standard auction formats, including the first price auction and the all-pay auction, the bidders' payoff is the same in both cases. We also ranked the expected revenue. For the first price auction, the second price auction, and convex combinations between the two, we showed that expected revenue is the same in the two information structures. In the all-pay auction, expected revenue is lower under complete information. In a combination of the all-pay auction and the war of attrition, revenue is lower and bidders' payoffs are higher under complete information.

2.A Appendix

2.A.1 Proof of Proposition 2.3

Concerning case P, it follows from Amann and Leininger (1996) that there is a unique Bayesian equilibrium which is symmetric and bid strategies are strictly increasing, and the payoff to the lowest type is zero. Hence, expected payoffs and revenue are equivalent to the second price auction.

Now turn to case C and suppose that $v_1 > v_2$. Riley (1999) shows that there is a unique equilibrium for all $\lambda \in [0,1)$ where the bidders randomize according to CDFs G_1 and G_2 with

$$\begin{aligned} G_1(b_1) &= \frac{1}{\lambda}\left[1-\exp\left\{-\frac{\lambda b_1}{v_2}\right\}\right], \qquad b_1 \in [0,z] \\ G_2(b_2) &= \frac{1}{\lambda}\left[1-(1-\lambda)^{1-\frac{v_2}{v_1}}\exp\left\{-\frac{\lambda b_2}{v_1}\right\}\right], \qquad b_2 \in [0,z] \end{aligned}$$

and $z = -\frac{v_2}{\lambda}\ln(1-\lambda)$.[12] Hence, $G_1(0) = 0$ and $G_2(0) = \frac{1}{\lambda}\left[1-(1-\lambda)^{1-\frac{v_2}{v_1}}\right]$.

Since bidder 2 bids zero with strictly positive probability, her expected

[12] Regarding the upper bound of the support, note that with l'Hôpital's rule, $\lim_{\lambda\to 0}\left[-\frac{v_2}{\lambda}\ln(1-\lambda)\right] = v_2$, i.e. the upper bound for the standard all-pay auction. Moreover, $z \to \infty$ for $\lambda \to 1$.

payoff must be equal to zero. However, the expected payoff of bidder 1,

$$E\left[U_1\left(\lambda\right)\right]=G_2\left(0\right)v_1=\frac{v_1}{\lambda}\left[1-\left(1-\lambda\right)^{1-\frac{v_2}{v_1}}\right]$$

is positive. In the following, we show that $E\left[U_1\left(\lambda\right)\right]>v_1-v_2$ for all $\lambda\in(0,1)$.

Consider the limit of $E\left[U_1\left(\lambda\right)\right]$ as λ approaches 0 and our game becomes a standard APA: By L'Hôpital's rule,

$$\lim_{\lambda\to 0}\frac{v_1}{\lambda}\left[1-\left(1-\lambda\right)^{\frac{v_1-v_2}{v_1}}\right]=v_1-v_2.$$

(As shown in Proposition 2.2, the payoff of the APA is equal to the payoff in the SPA.)

Differentiating $E\left[U_1\left(\lambda\right)\right]$ with respect to λ yields

$$\frac{\partial E\left[U_1\left(\lambda\right)\right]}{\partial\lambda}=-\frac{v_1}{\lambda^2}+\frac{v_1-\lambda v_2}{\lambda^2\left(1-\lambda\right)}\left(1-\lambda\right)^{\frac{v_1-v_2}{v_1}}$$

which is strictly larger than zero iff $\left(1-\lambda\right)^{\frac{v_1-v_2}{v_1}}>v_1\left(1-\lambda\right)/\left(v_1-\lambda v_2\right)$ or, by taking the logarithm, iff

$$h\left(\lambda\right):=\frac{v_1-v_2}{v_1}\ln\left(1-\lambda\right)-\ln\left(\frac{v_1\left(1-\lambda\right)}{v_1-\lambda v_2}\right)>0.$$

This is true for all $\lambda\in(0,1)$ since $h\left(0\right)=0$ and for all $\lambda\in(0,1)$

$$\frac{\partial h\left(\lambda\right)}{\partial\lambda}=\frac{\lambda v_2\left(v_1-v_2\right)}{v_1\left(1-\lambda\right)\left(v_1-\lambda v_2\right)}>0.$$

Hence, bidder 1's expected payoff is strictly increasing in λ and therefore larger than v_1-v_2 for all $\lambda\in(0,1)$.

This implies that, ex ante, both bidders strictly prefer case C over case P for all $\lambda\in(0,1)$. Since the allocation is inefficient in case C and expected payoffs are higher, it follows that expected revenue must be lower in case C

than in case P.

As $\lambda \to 1$, $\lim_{\lambda \to 1} G_2(0) = 1$ and therefore $E[U_1(\lambda)] \to v_1$ since bidder 2 bids zero with probability 1 (which corresponds to an immediate concession in the war of attrition).

Chapter 3

Information acquisition in conflicts

This chapter is joint work with Johannes Münster.

3.1 Introduction

Contest theory studies the interaction between agents who spend resources in order to increase their chances of winning a prize. A large number of economic environments can fruitfully be analyzed as contests - e.g. advertising of firms, patent races, rent-seeking and lobbying, political campaigning, or litigation. In many of these environments, the competitors do not know exactly what value they would derive from winning, or how costly it is to expend effort. They may, however, be willing to invest a significant amount of time or money in order to find out about the prize that is at stake, or about the cost of competing. Such investments in information have important implications on the interaction in the contest, both on the amount of resources spent and on allocative efficiency. Moreover, as a consequence of information acquisition, contestants may differ in the quality of the information they have about their own or their competitors' valuation.

Asymmetries with regard to the information the contestants possess are a feature of many contests. These asymmetries can arise from decisions on information acquisition prior to the conflict. In other cases, they are features of the environment the contestants compete in. As an example, consider the case of the Brent Spar oil rig that the owners, Royal Dutch Shell and Exxon, wanted to sink in the Atlantic Ocean. Following a worldwide campaign organized by the environmental group Greenpeace, they abandoned this plan and decided to re-use a large part of the rig. This contest was characterized by a one-sided asymmetry of information about the valuations of the contestants. There were publicly accessible estimations of the cost of the on-shore dismantling of Brent Spar, including estimations published by the owners themselves. There was, however, very little public information about the value Greenpeace placed on the prevention of the deep sea disposal so that the owners of Brent Spar had to rely on their guesses about how far Greenpeace would go. Such one-sided asymmetric information is also prominent in many situations where an incumbent competes with a newcomer, for example in regulated markets or in election races.

Private information of the contestants, however, often results from information acquisition. A firm entering a market will try to find out about the market conditions and the potential gains before competing with an incumbent. As another example for investments in information consider again the contest over Brent Spar. One of the first actions of Greenpeace was to enter the Brent Spar and to take some samples of how much toxic material was on it. This clearly allowed Greenpeace to learn more about the value that was at stake.

In this chapter, we first study one-sided asymmetric information in a perfectly discriminating contest or all-pay auction between two risk neutral players. The all-pay auction has been used to model a number of contests such as rent-seeking contests and lobbying (Hillman and Riley 1989; Ellingsen 1991; Baye et al. 1993; Polborn 2006), election campaigns (Che and Gale 1998), and also R&D races (Dasgupta 1986); see Konrad (2009) for a recent survey. We characterize the (unique) equilibrium of the all-pay auction between two contestants, where the valuation of one contestant is common knowledge, whereas the valuation of the other contestant is drawn from a continuous distribution and is his private information. In equilibrium, the player whose valuation is commonly known randomizes continuously, whereas the player with private information plays a pure strategy.

We then analyze information acquisition ahead of conflicts and the players' incentives for such investments. Suppose each player initially only knows the distribution of his type, but that he can learn his true valuation by investing some amount. We distinguish between three different cases depending on how much the opponent can observe if a player invests: (i) the opponent can observe whether a player has invested in information, but not the realized valuation of the opponent in case the player invests, (ii) the opponent can observe the outcome of information acquisition (open information acquisition), (iii) the opponent cannot observe at all whether a player has acquired information (covert information acquisition).

In case (i), if no player invests in information, the resulting contest is similar to an all-pay auction with complete information where, by risk neutrality, the benefit of winning is the expected valuation. If both players acquire information, the all-pay auction turns into the well-known framework with private information. If exactly one player invests in information, then the ensuing contest has one-sided asymmetric information: there are common beliefs about the type of one player, while the type of the other player is his private information. In this setting, information acquisition has a strategic effect on the behavior of the opponent in the contest. We show that players are willing to spend a considerable amount on information. Moreover, for intermediate costs of information acquisition, only one player will invest. To be more precise, there are two asymmetric equilibria where exactly one player invests, and there is also a symmetric equilibrium where both players randomize their investment decision. Thus, the case of one-sided asymmetric information can arise endogenously in an equilibrium of the game with information acquisition. Rent dissipation is incomplete, although players are symmetric ex ante. Compared with the first best, information acquisition is excessive in case (i).

In cases (ii) and (iii), the players' equilibrium investments are again guided by cut-off values concerning the cost of information acquisition. We show that, however, these cut-off values are exactly equal to the cut-off values for first best investment in cases (ii) and (iii). Since in case (i), the players' willingness to pay for information is higher, this suggests that there is a strategic value of information acquisition if the players' decisions are observable, but not the information itself.

The chapter is related to several studies of the all-pay auction under different assumptions on the information available to the contestants. Hillman and Riley (1989) study the all-pay auction for the two benchmark cases: complete information and private information about the individual valuations. Baye et al. (1996) characterize the set of equilibria of the all-pay auction with N players and complete information. Amann and Leininger (1996) show uniqueness of the equilibrium with two-sided asymmetric information and two ex ante asymmetric players. Morath and Münster (2008) compare private versus complete information in auctions, and find that for the all-pay auction, revenue is smaller under complete information, while bidders' payoffs are the same in the two information structures. Krishna and Morgan (1997) consider the case where the players' signals are affiliated. Konrad (2009) characterizes the equilibrium under one-sided asymmetric information where one player's value follows a two-point distribution. The all-pay auction with multiple prizes is studied by Moldovanu and Sela (2001) in a framework with private information, and by Clark and Riis (1998) and Barut and Kovenock (1998) with complete information.

Closely related to our work are three papers that study one-sided asymmetric information in contests. For a logit contest success function, Hurley and Shogren (1998a) analyze contests with one-sided asymmetric information, and Hurley and Shogren (1998b) compare the three information structures that also arise in our model with regard to rent dissipation and efficiency. For a more general contest success function, Wärneryd (2003) considers an imperfectly discriminating contest with two agents who have the same value of winning, but where there is uncertainty about this value. He compares a symmetric information structure to the case where one agent privately knows the value of the prize and shows that rent dissipation may be lower under asymmetric information. We add to this literature by studying the all-pay auction framework, and we focus on private values.[1]

Our work is also linked to the literature on strategic behavior ahead of contests. Konrad (2009) surveys this literature. Our contribution to this literature is to study the incentives for information acquisition in contests.

In Section 3.2, we describe the strategies and payoffs of the players in the all-pay auction for a given information structure. In Section 3.3, we analyze the all-pay auction with one-sided asymmetric information. In Section 3.4, we consider the all-pay auction in a context of information acquisition. Section 3.5 discusses how our result is affected if the assumptions on the observability of information acquisition change. Section 3.6 is the conclusion. All proofs are in the appendix.

3.2 The all-pay auction

There are two players 1 and 2 competing in an all-pay auction. Player i values winning by v_i. The *valuations*, or *types*, v_1 and v_2 are drawn independently from a cumulative distribution function F that is common knowledge. F has

[1] One-sided asymmetric information in common value first-price auctions has been studied by Engelbrecht-Wiggans et al. (1983), among others. In addition, a growing literature considers information acquisition in winner-pay auctions; recent work includes Persico (2000) or Hernando-Veciana (2009).

support $[0,1]$ and is continuously differentiable with $F'(v) > 0$ for $v \in (0,1)$.

In Section 3.3, we assume that the realized value of v_1 is common knowledge, whereas the realized value of v_2 is private information of player 2. In Section 3.4, we assume that initially no player is informed about any valuation, but players can acquire information: at a cost c, a player can learn his own value.[2] Player j can observe whether or not i has acquired information, but not the realized value v_i.

Finally, players compete in an all-pay auction. They simultaneously choose their bids $x_i \in [0,\infty)$. The player with the higher bid wins, ties are broken randomly. Both players have to pay their bids. Thus, i's payoff from the all-pay auction (gross of the direct cost of investing in information) is

$$u_i = \begin{cases} v_i - x_i, & x_i > x_j, \\ \frac{v_i}{2} - x_i, & x_i = x_j, \\ -x_i, & x_i < x_j. \end{cases}$$

3.3 One-sided asymmetric information

Suppose that player 1's valuation v_1 is common knowledge.[3] Player 2's valuation v_2 is privately known only to himself. Thus, a pure strategy of player 1 is a bid $x_1 \in [0,\infty)$, whereas a pure strategy of player 2 is a function $\beta_2 : [0,1] \to [0,\infty)$ that maps the typespace into the set of possible bids. The solution concept is Bayesian Nash equilibrium (henceforth, "equilibrium").

Denote the bid distributions of players 1 and 2 by B_1 and B_2, i.e. $B_i(x)$ denotes the probability that i's bid is weakly below x. If 1 plays a pure strategy to bid x with probability one, then B_1 is degenerate: $B_1(z) = 0$

[2] Note that the investment does not change the distribution of one's value, nor one's ability to compete in the contest. Investments in one's value or ability have been studied by Münster (2007).

[3] The analysis goes through for all $v_1 > 0$. For $v_1 = 0$, there is no equilibrium because player 1 will bid zero and player 2 has no best response since any strictly positive bid, however small, guarantees victory.

for $z < x$ and $B_1(z) = 1$ otherwise. If B_1 is not degenerate, 1 plays a non-degenerate mixed strategy. In contrast, the bid distribution B_2 captures the uncertainty concerning v_2 as well as the possible randomization of player 2.

Lemma 3.1 *In any equilibrium, the bid distributions B_1 and B_2 have the following properties:*

(i) *(Continuity) B_1 and B_2 are continuous on $(0, \infty)$.*

(ii) *(Support) The supports of B_1 and B_2 both have the same minimum $\underline{b} = 0$, and the same maximum $\bar{b} \leq v_1$.*

(iii) *(At most one mass point at zero) $\min\{B_1(0), B_2(0)\} = 0$.*

(iv) *(Monotonicity) B_1 and B_2 are strictly monotone increasing on $\left[0, \bar{b}\right]$.*

Similar properties are standard in auction theory. Continuity implies that there are no interior mass points. Monotonicity rules out any gaps in the support. Thus (ii) and (iv) imply that B_1 and B_2 have the same support.

It follows directly from Lemma 3.1 that, in any equilibrium, player 1 randomizes according to a CDF that is continuous and strictly increasing on $\left[0, \bar{b}\right]$. To get some intuition, suppose to the contrary that player 1 chooses a pure strategy, i.e. bids some amount x with probability one. Then player 2 would either like to marginally overbid player 1, or bids zero. But then bidding x is not optimal for 1, contradicting equilibrium. Thus player 1 has to randomize. In contrast, 2 plays a pure strategy.

Lemma 3.2 *In any equilibrium, player 2 plays a pure strategy $\beta_2 : [0, 1] \to \left[0, \bar{b}\right]$. There is a critical value $\underline{v} \in [0, 1)$ such that $\beta_2(v_2) = 0$ for $v_2 \leq \underline{v}$ and $\beta_2(v_2) > 0$ for $v_2 > \underline{v}$. Moreover, β_2 is continuous on $[0, 1]$ and strictly increasing on $[\underline{v}, 1]$.*

Lemma 3.2 shows that player 2, whose valuation is private information, bids according to a strategy that is increasing in his value, and low types might bid zero. The highest type of player 2 (who has $v_2 = 1$) bids exactly $\bar{b}$. The intuition behind the proof is simple. Higher types of player 2 will bid higher. Thus, if some type of player 2 randomizes over some interval, no other type of player 2 will bid in this interval. But then B_2 is constant in that interval, contradicting Lemma 3.1.

Note that β_2 has image $\left[0, \bar{b}\right]$. Since β_2 is continuous and strictly increasing on $(\underline{v}, 1]$, it is invertible on $(\underline{v}, 1]$ with $\beta_2^{-1} : \left(0, \bar{b}\right] \rightarrow (\underline{v}, 1]$. Furthermore, β_2^{-1} is continuous and strictly increasing on $\left(0, \bar{b}\right]$.

Lemma 3.3 *In equilibrium, B_1 and B_2 are differentiable on $\left(0, \bar{b}\right)$; moreover β_2 is differentiable on $(\underline{v}, 1)$.*

Given differentiability of the bid distributions, we can use first-order conditions to determine the equilibrium and show its uniqueness. The expected payoff of player 1 from a bid $x_1 \in \left(0, \bar{b}\right]$ is equal to

$$E\left[u_1\left(x_1\right)\right] = F\left(\beta_2^{-1}\left(x_1\right)\right) v_1 - x_1$$

since β_2^{-1} exists on $\left(0, \bar{b}\right]$. Because player 1 randomizes continuously on $\left(0, \bar{b}\right]$, $E\left[u_1\left(x_1\right)\right]$ must be constant in this interval. Therefore,

$$F'\left(\beta_2^{-1}\left(x_1\right)\right) \frac{v_1}{\beta_2'\left(\beta_2^{-1}\left(x_1\right)\right)} - 1 = 0. \tag{3.1}$$

Any solution to the differential equation (3.1) has to fulfill

$$\beta_2\left(v_2\right) = F\left(v_2\right) v_1 + k$$

for all v_2 such that $\beta_2\left(v_2\right) > 0$, where the constant k remains to be determined. Note that $F\left(v_2\right) v_1 + k > 0$ if and only if $v_2 > F^{-1}\left(-k/v_1\right)$. By Lemma 3.2, types $v_2 \leq F^{-1}\left(-k/v_1\right)$ bid zero, hence $B_2(0) = -k/v_1$, and thus $k \in \left(-v_1, 0\right]$. For notational convenience, let $\alpha_2 = -k/v_1$ (we use the subscript ‘2’ since $\alpha_2 = B_2(0)$). Putting things together,

$$\beta_2(v_2) = \begin{cases} 0, & v_2 \in [0, F^{-1}(\alpha_2)) \\ F(v_2)v_1 - \alpha_2 v_1, & v_2 \in [F^{-1}(\alpha_2), 1] \end{cases} \tag{3.2}$$

where $\alpha_2 \in [0, 1)$ remains to be determined.

Now consider player 2. The first-order condition for a type v_2 who bids a strictly positive amount is given by

$$B_1'(x_2)v_2 - 1 = 0. \tag{3.3}$$

Using (3.2),

$$B_1'(x_2) = \frac{1}{\beta_2^{-1}(x_2)} = \frac{1}{F^{-1}\left(\frac{x_2+\alpha_2 v_1}{v_1}\right)} \tag{3.4}$$

has to hold for all $x_2 > 0$. This is solved by

$$\begin{aligned} B_1(x_2) &= \int_0^{x_2} \frac{1}{F^{-1}\left(\frac{z+\alpha_2 v_1}{v_1}\right)} dz + \alpha_1 \\ &= \int_{F^{-1}(\alpha_2)}^{\beta_2^{-1}(x_2)} \frac{v_1}{v} dF(v) + \alpha_1 \end{aligned} \tag{3.5}$$

where α_1 remains to be determined. Note that $\alpha_1 = B_1(0) \in [0, 1)$.

To determine α_1 and α_2, we use the fact that, at most, one of the bid distributions has a mass point at zero (Lemma 3.1(iii)):

$$\min\{B_1(0), B_2(0)\} = \min\{\alpha_1, \alpha_2\} = 0. \tag{3.6}$$

Moreover, player 1 will never bid higher than the highest type of player 2, thus $B_1(\beta_2(1)) = 1$. By (3.5), we get

$$\int_{F^{-1}(\alpha_2)}^{1} \frac{v_1}{v} dF(v) + \alpha_1 = 1. \tag{3.7}$$

Equations (3.6) and (3.7) uniquely determine the mass points α_1 and α_2.

Lemma 3.4 *(i) If*

$$\int_0^1 \frac{v_1}{v} dF(v) > 1, \tag{3.8}$$

then $\alpha_1 = 0$ *and* α_2 *is the unique solution to*

$$\int_{F^{-1}(\alpha_2)}^1 \frac{v_1}{v} dF(v) = 1. \tag{3.9}$$

(ii) If (3.8) does not hold, then $\alpha_2 = 0$ *and* α_1 *is the unique solution to*

$$\int_0^1 \frac{v_1}{v} dF(v) + \alpha_1 = 1. \tag{3.10}$$

Using Lemmas 3.1-3.4, we can now state the main result of this section.

Proposition 3.1 *Suppose that player* 1*'s valuation is common knowledge and player* 2*'s valuation is his private information. The all-pay auction has a unique equilibrium. Player* 1 *randomizes according to*

$$B_1(x_1) = \begin{cases} \int_0^{x_1} \frac{1}{F^{-1}\left(\frac{z+\alpha_2 v_1}{v_1}\right)} dz + \alpha_1 & \text{for} \quad x_1 \in [0, (1-\alpha_2) v_1) \\ 1 & \text{for} \quad x_1 \geq (1-\alpha_2) v_1 \end{cases} \tag{3.11}$$

where α_1 *and* α_2 *are defined in Lemma 3.4. Player* 2 *plays the following pure strategy:*

$$\beta_2(v_2) = \begin{cases} 0 & \text{for} \quad v_2 \in [0, F^{-1}(\alpha_2)) \\ F(v_2) v_1 - \alpha_2 v_1 & \text{for} \quad v_2 \in [F^{-1}(\alpha_2), 1] \end{cases} \tag{3.12}$$

In equilibrium, player 1 randomizes according to a (concave) distribution function. The probability that he bids zero is equal to α_1. Thus, whenever $\alpha_1 > 0$, player 1's expected payoff is zero, since he is indifferent between bidding zero and any positive bid in $(0, (1-\alpha_2) v_1]$. Player 2 bids zero for all types that are smaller than $\underline{v}= F^{-1}(\alpha_2)$, i.e. with probability α_2. For all other types, player 2 bids a positive amount $\beta_2(v_2)$ and gets a positive expected payoff which is increasing in his type. From an ex ante point of view, player 2's equilibrium payoff is strictly positive. His bid distribution is given by

$$B_2(x_2) = F\left(\beta_2^{-1}(x_2)\right) = \alpha_2 + \frac{x_2}{v_1}$$

where $x_2 \in [0, (1-\alpha_2)v_1]$. Hence, player 2's bids are uniformly distributed on $(0, (1-\alpha_2)v_1)$ with (possibly) a mass point at zero. This is similar to the all-pay auction under complete information: in order to make player 1 indifferent, player 2's bids have to follow a uniform distribution with slope $1/v_1$.

Note that, if v_1 is weakly larger than player 2's expected valuation $E(V_2)$, (3.8) is always fulfilled. This follows from

$$\int_0^1 \frac{v_1}{v} dF(v) \geq \int_0^1 \frac{E(V_2)}{v} dF(v) > 1 \tag{3.13}$$

which is true by Jensen's inequality $(E(1/V_2) > 1/E(V_2))$. Thus, if v_1 is sufficiently large, $B_2(0) > B_1(0) = 0$: player 1's willingness to bid more aggressively induces player 2 to bid zero if he has a low value.

3.4 An application to information acquisition

In the following, we use our results of the previous section to analyze a game of information acquisition in conflicts, focussing on the case where the decision to acquire information can be observed by the opponent, but not the acquired information itself. (We discuss the cases of open and covert information acquisition in Section 3.5.) As before, the players' types are independent draws from a CDF F that is common knowledge. Prior to the all-pay auction, the players simultaneously decide whether to purchase a perfectly informative signal about their own valuation at a cost c. The realization of the signal is private information, but whether or not a player has acquired information is common knowledge in the all-pay auction.

Case 1: No information acquisition. Suppose that no player acquired information. Maximizing his expected payoff in the all-pay auction, a player i's optimal strategy is to choose his effort as if his valuation were equal to his expected valuation $E(V_i) = \int_{v=0}^{v=1} v dF(v)$. Hence, the all-pay auction is reduced to a game where the (expected) valuations $E(V_1)$ and $E(V_2)$ are common knowledge. The equilibrium of the all-pay auction under complete information is in mixed strategies and is derived in Baye et al. (1996): both players randomize uniformly with support $[0, E(V_1)]$.

Fact 3.1 *(Baye et al. 1996) Suppose that no player acquired information. In the unique equilibrium of the all-pay auction, expected payoffs are* $E[u_1] = E[u_2] = 0$.

If no player invests in information, and both players have the same expected valuation, there is full rent dissipation in the all-pay auction.

Case 2: Two-sided asymmetric information. Suppose that both players have acquired information and know their own type, but only know the distribution of the opponent's type. In this case, the equilibrium of the all-pay auction is well-known.[4] Each player's bid is strictly increasing in his valuation.

Fact 3.2 *(Weber 1985, Hillman and Riley 1989) Suppose both players acquired information. In the unique equilibrium of the all-pay auction, expected payoffs are*

$$E[u_1] = E[u_2] = \int_0^1 \int_0^{v_i} (v_i - v_j) \, dF(v_j) \, dF(v_i) - c. \tag{3.14}$$

[4] See, for example, Krishna (2002), pp. 33-34. Uniqueness of the equilibrium follows from Amann and Leininger (1996).

The support of the bid distributions is $[0, E(V_1)]$, as in the case without information acquisition. Without information acquisition, however, the distribution of the bids first-order stochastic dominates the bid distribution in the case of private information. Therefore, expected expenditures in the contest are lower with private information, and the players get a positive expected payoff. Moreover, the allocation of the prize is efficient in the case of private information since the player with the higher valuation wins with probability 1. Obviously, whenever c is sufficiently small, both players are better off than they are without information acquisition.

Case 3: One-sided asymmetric information. Suppose that only player 2 acquired information. Then player 2's valuation is private information, and player 1's optimal strategy is to bid as if his true valuation were $E(V_1)$. Thus, we can build on the results of Section 3.3 by just replacing v_1 with $E(V_1)$. Since $E(V_1) = E(V_2)$, it follows with (3.13) that $B_2(0) = \alpha_2 > 0$, and α_2 is defined by (3.9). Since player 2 bids zero for types smaller than

$$\underline{v} = F^{-1}(\alpha_2) > 0, \tag{3.15}$$

the uninformed player 1 has a positive expected payoff,

$$E[u_1] = F(\underline{v}) E(V_1) > 0. \tag{3.16}$$

A type $v_2 > \underline{v}$ of player 2 that bids a strictly positive amount gets a payoff of

$$B_1(\beta_2(v_2)) v_2 - \beta_2(v_2) = \int_{\underline{v}}^{v_2} \left(\frac{E(V_1) v_2}{v} - E(V_1) \right) dF(v).$$

Player 2's ex ante expected payoff is therefore equal to

$$E[u_2] = \int_{\underline{v}}^{1} \int_{\underline{v}}^{v_2} \left(\frac{E(V_1) v_2}{v} - E(V_1) \right) dF(v) \, dF(v_2). \tag{3.17}$$

We now turn to the implications for the incentives to invest in information.[5]

Proposition 3.2 *There are two critical values* $\underline{c}$ *and* $\bar{c}$ *with* $0 < \underline{c} < \bar{c}$ *such that:*

(i) *If the cost of information* c *is strictly smaller than* $\underline{c}$*, both players acquire information.*

(ii) *If* $\underline{c} < c < \bar{c}$*, there are two equilibria where exactly one player acquires information. Additionally, there is a symmetric equilibrium where player* i *acquires information with probability* $p = (\bar{c} - c) / (\bar{c} - \underline{c})$*.*

(iii) *If* $c > \bar{c}$*, no player acquires information.*

The critical value $\underline{c}$ $(\bar{c})$ is the maximum amount a player is willing to spend on information given that the opponent does (does not) acquire information. It is crucial to show that $0 < \underline{c} < \bar{c}$. Since the willingness to pay for information is smaller if the opponent acquires information ($\underline{c} < \bar{c}$), an interval $(\underline{c}, \bar{c})$ exists where only one player invests in information (or both players randomize).

Figure 3.1 illustrates this result by showing the players' expected payoffs dependent on the information cost and the information decisions for uniformly distributed types $(F(v) = v)$. Obviously, for sufficiently high cost of information, no player will buy it. On the other hand, for sufficiently low cost of information, at least one player has an incentive to acquire information due to the complete rent dissipation in the case of no private information. For any continuous distribution function F, however, there is an intermediate range of information costs where it only pays for one player to acquire information.

[5] Note that players have no private information when they decide whether to acquire information. Any reasonable belief about the opponent's type is simply the prior distribution F. Moreover, any continuation game has a unique Bayesian equilibrium. Therefore, we study the 2-by-2 game defined by the payoffs described in Facts 3.1-3.2 and equations (3.16)-(3.17). This amounts to studying the perfect Bayesian equilibria of the game defined in Section 3.2.

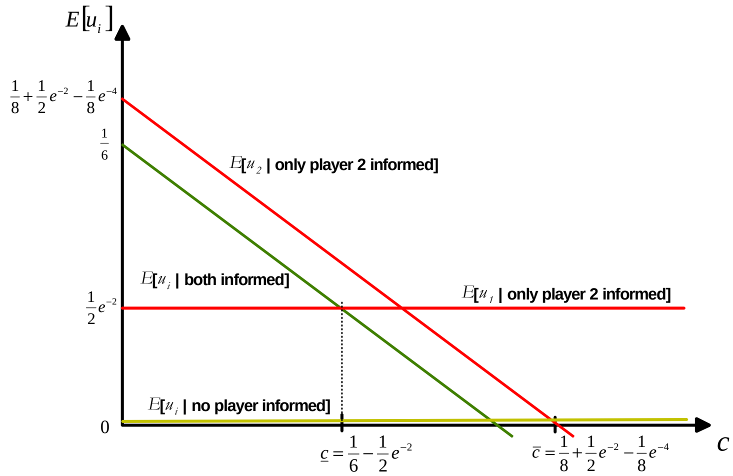

Figure 3.1: Payoffs dependent on c (for $F(v) = v$).

We conclude this section by studying the efficiency of equilibrium information acquisition. We compare equilibrium information acquisition with first best investments by a social planner who is ex ante uninformed about the valuations, but can observe the outcome of any information acquisition. For concreteness, assume that the social planner derives no value from the bids in the contest and allocates the prize to the player with the higher expected valuation.

Proposition 3.3 *In an equilibrium without randomization concerning information acquisition, the number of players acquiring information is higher than in the first best.*

In the appendix, we show that first best investments are characterized by two critical values c' and c'' (with $c' < c''$) for the cost of information: if $c < c'$, both players should invest, if $c \in (c', c'')$, one player should invest, and if $c > c''$, none should invest. The critical values that determine the social planner's investments are lower than the corresponding equilibrium thresholds of Proposition 3.2: $c' < \underline{c}$ and $c'' < \bar{c}$. Depending on the functional form

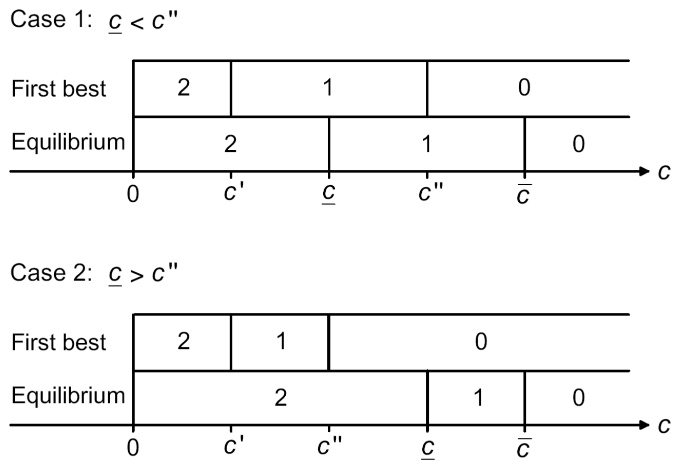

Figure 3.2: The number of contestants who invest in information, in the first best, and in an equilibrium without randomization of information acquisition, as a function of the cost of information acquisition c.

of F, c'' can be higher or smaller than $\underline{c}$.[6] Figure 3.2 compares equilibrium investments with the first best. In equilibrium, there is more information acquisition: if $c \in (c', \underline{c})$, both players acquire information although only one player at most should, and similarly, whenever $c \in (c'', \bar{c})$, at least one player acquires information, though neither of the players should.[7]

[6] For example, for $F(v) = v$, $\underline{c} < c'' < \bar{c}$, and for $F(v) = v^3$, we have $c'' < \underline{c} < \bar{c}$.

[7] In the symmetric equilibrium with randomization of information acquisition, under some parameter constellations, it may happen that ex post no player invests although in the first best one player should invest. To be more precise, if $c'' \leq \underline{c}$, then the number of players acquiring information is always weakly higher than in the first best. On the other hand, if $c'' > \underline{c}$, then for any $c \in (\underline{c}, c'')$, in the first best exactly one player acquires information, whereas in the mixed equilibrium the number of players acquiring information is zero, one, or two, depending on the realizations of players' randomization.

3.5 Observability of information acquisition

The analysis in the previous section builds on a crucial assumption on the observability of information acquisition: we assumed that the players' decisions whether to acquire information are observable, but the information itself is only privately known to a player. In the following, we discuss this assumption by modifying it in two different directions. On the one hand, we consider the case where both the players' decisions and the information is publicly observable (open information acquisition), and on the other hand, we discuss the case where neither the information nor the players' decisions are observable by the opponent (covert information acquisition).

With open information acquisition, there are three different situations that can arise in the all-pay auction. If no player acquired information, the equilibrium is as described in Fact 3.1. If only player i acquired information, i's valuation is common knowledge, and j bids as if his value was $E(V_j)$. If both players acquired information, both v_i and v_j are common knowledge. In all three cases, the equilibrium is similar to the equilibrium under complete information characterized by Baye et al. (1996). Comparing the expected payoffs in the three cases determines the amount that the players are willing to spend on information.

Proposition 3.4 *With open information acquisition, in any equilibrium without randomization concerning information acquisition, players invest as in the first best.*

If the information that players acquire is observable, cut-off values exist for the cost of information such that both, only one, or none of the players wants to invest in information. These thresholds, however, are exactly the same as the thresholds a social planner would set (c' and c''). Thus, if the information is publicly observable, players invest less in information, and information acquisition is efficient.

Now turn to the case of covert information acquisition where a player cannot observe whether or not the other player has acquired information. Intuitively, for a very low cost of information, both players invest, and for very high cost, no player invests in information.

Proposition 3.5 *With covert information acquisition, (i) there is an equilibrium where both players invest in information if and only if $c < c'$, and (ii) there is an equilibrium where no player invests in information if and only if $c > c''$.*

For the sake of brevity, we do not characterize the equilibria for the entire range of cost parameters c,[8] but, interestingly, the cut-off values for c such that both players, or none of the players, acquire information are as in the first best. Thus, a player is willing to spend more on information if the decisions are observable than if the decisions are not observable by the other player.

3.6 Conclusion

We considered the all-pay auction between two players with one-sided asymmetric information. The asymmetry accounts for the fact that there may be superior information about one of the contestants, for example an incumbent, compared to the other contestants. We showed that if one contestant's value of winning is publicly known and the value of the opponent is private information, the all-pay auction has a unique equilibrium, and we characterized the equilibrium strategies.

Building on this result, we studied the contestants' incentives to invest in information before they compete in the all-pay auction. We distinguished

[8] For $c \in (c', c'')$, one has to include situations where players randomize their information choice, in which case i bids against an informed player j with some probability. The equilibrium of the all-pay auction is then as if types are private information and drawn from a continuous distribution function exhibiting one interior discontinuity.

between three different scenarios: (i) the opponent can observe only *that* a player has acquired information, but not *what* information he received, (ii) the opponent can observe the information itself (open information acquisition), and (iii) the opponent does not observe the decision to acquire information (covert information acquisition). In all scenarios, if the cost of information is sufficiently low, it is outweighed by the value that the information has in the contest. For intermediate cost of information, however, only one player may invest in information. Therefore, in scenario (i), in the all-pay auction one contestant may have private information whereas there are common beliefs about the other contestant's value of winning. Moreover, in equilibrium, more information is acquired than in the first best. In contrast, with open or covert information acquisition, the cut-off values for the cost of information acquisition are as in the first best. In all three scenarios, although players are symmetric ex ante, rent dissipation is incomplete unless the costs of information acquisition are prohibitive.

An interesting extension of our work could be the case of N contestants and asymmetric information. For example, if a monopolist tries to defend the monopoly rents against multiple entrants, there might be asymmetric information in the sense that one contestant's type is common knowledge and the other $(N-1)$ contestants' types are private information. The structure of the equilibrium should then be similar to the two-players case.

3.A Appendix

3.A.1 Proof of Lemma 3.1

(i) *(Continuity)* Suppose that B_j exhibits a discontinuity at some $\tilde{x} > 0$. This implies that a bid of $x_j = \tilde{x}$ has strictly positive probability. Thus, there exist $\varepsilon, \varepsilon' > 0$ such that player i strictly prefers $x_i = \tilde{x} + \varepsilon$ over all $x_i \in (\tilde{x} - \varepsilon', \tilde{x})$: shifting probability mass from $(\tilde{x} - \varepsilon', \tilde{x})$ to $\tilde{x} + \varepsilon$ only involves an infinitesimally larger cost of effort, but strictly increases the probability of winning.[9] Since player i will not bid in $(\tilde{x}-\varepsilon', \tilde{x})$, player j can strictly increase his payoff by bidding $\tilde{x}-\frac{\varepsilon'}{2}$ instead of $\tilde{x}$.

(ii) *(Support)* Let $\bar{b}_i$ ($\underline{b}_i$) denote the maximum (minimum) of the support of B_i. Suppose that $\bar{b}_i > \bar{b}_j$. Then $B_j(x) = 1$ for all $x \geq \bar{b}_j$. Thus, player i prefers to bid $x_i = \left(x_i' + \bar{b}_j\right)/2$ to any bid $x_i' > \bar{b}_j$, contradicting $\bar{b}_i > \bar{b}_j$. Hence, $\bar{b}_1 = \bar{b}_2 = \bar{b}$. Since player 1 can ensure a payoff of zero by bidding zero, we must have $\bar{b} \leq v_1$.

Suppose that $\underline{b}_i > \underline{b}_j > 0$. Then any bid $x_j < \underline{b}_i$ loses with probability one; player j could increase his payoff by bidding zero instead, which is a contradiction.

Now suppose $\underline{b}_i > \underline{b}_j = 0$. Then player j strictly prefers a bid of zero over all bids in $(0, \underline{b}_i)$, thus B_j has no probability mass in $(0, \underline{b}_i)$. Since B_j has no mass points (except possibly at zero) it follows that B_j is constant on $(0, \underline{b}_i]$. But then there exists $\varepsilon > 0$ such that player i strictly prefers a bid of ε over any bid in $[\underline{b}_i, \underline{b}_i + \varepsilon)$: a bid of ε has strictly lower costs but only a marginally lower probability of winning. This is a contradiction to the definition of $\underline{b}_i$.

Finally, suppose $\underline{b}_1 = \underline{b}_2 = \underline{b} > 0$. By (i), $B_j(\underline{b}) = 0$, and there exists an $\varepsilon > 0$ such that $x_i = 0$ is preferred to any bid $x_i \in [\underline{b}, \underline{b} + \varepsilon)$, which contradicts $\underline{b}_i > 0$. Combining these arguments shows that $\underline{b}_1 = \underline{b}_2 = 0$.

(iii) *(Mass points at zero)* If $B_j(0) > 0$, there exists an $\varepsilon > 0$ such that player i prefers $x_i = \varepsilon$ to $x_i = 0$. Hence, $B_i(0) = 0$. This shows that the bid distribution of at most one player can have a mass point at zero.

(iv) *(Monotonicity)* Suppose that B_j is constant in an interval (x', x'') where $0 \leq x' < x'' \leq \bar{b}$, further suppose that $x'' = \max\{x \,|B_j(x) = B_j(x')\}$. Then $B_j(x') = B_j(x'') < 1$ since $x' < \bar{b}$. There exists an $\varepsilon > 0$ such that player i prefers $x_i = x'$ to all $x_i \in (x', x'' + \varepsilon)$: by bidding x' player i reduces his probability of winning only by (at most) an infinitesimally small amount,

[9] If $i = 2$, this argument assumes $v_2 > 0$. But this is inconsequential since type $v_2 = 0$ has zero probability.

but strictly decreases his expected cost of effort. Thus i does not bid in $(x', x'' + \varepsilon)$. Since B_i has no mass points, we have $B_i(x') = B_i(x'' + \varepsilon)$. But then j prefers bidding x' over any bid in $[x'', x'' + \varepsilon]$ and thus we must have $B_j(x'' + \varepsilon) = B_j(x')$, contradicting $x'' = \max\{x \,| B_j(x) = B_j(x')\}$.

3.A.2 Proof of Lemma 3.2

First we show that no type of player 2 randomizes. Suppose to the contrary that some type v_2' of player 2 does randomize. Let c_l (c_h) be the infimum (supremum) of the support of the distribution of bids made by type v_2'. For any $c > c_l$,

$$B_1(c_l)\, v_2' - c_l \geq B_1(c)\, v_2' - c \tag{3.18}$$

for otherwise v_2' could gain from shifting probability mass to c.[10] From (3.18),

$$c - c_l \geq (B_1(c) - B_1(c_l))\, v_2'.$$

Since B_1 is strictly increasing, for any $v_2'' < v_2'$ we have

$$c - c_l > (B_1(c) - B_1(c_l))\, v_2''$$

or

$$B_1(c_l)\, v_2'' - c_l > B_1(c)\, v_2'' - c$$

i.e. type v_2'' strictly prefers to bid c_l over bidding c. Therefore, for all $v_2'' < v_2'$, the supremum of the support of the distribution of bids made by type v_2'' must be weakly smaller than c_l. Similarly, for all $v_2''' > v_2'$, the infimum of the support of the distribution of bids made by type v_2''' must be weakly higher than c_h. Therefore only type v_2' bids in (c_l, c_h). Since the distribution of

[10]If c_l has strictly positive probability, type v_2' gains from shifting this probability mass to c. If c_l has zero probability, then, for any $\varepsilon > 0$, the interval $(c_l, c_l + \varepsilon)$ has positive probability. By continuity of B_1, if (3.18) does not hold, then for small enough $\varepsilon > 0$, $B_1(c_l + \varepsilon)\, v_2' - (c_l + \varepsilon) < B_1(c)\, v_2' - c$, and shifting probability mass from the interval $(c_l, c_l + \varepsilon)$ to c is beneficial.

types, F, is continuous, it follows that B_2 is constant on (c_l, c_h), contradicting Lemma 3.1.

It follows that player 2 plays a pure strategy $\beta_2 : [0,1] \to [0,\infty)$. Moreover, β_2 is weakly increasing. Now suppose that $v_2' < v_2''$ and $\beta_2(v_2') = \beta_2(v_2'')$. Since β_2 is weakly increasing, it follows that $\beta_2(v_2) = \beta_2(v_2')$ for all $v_2 \in [v_2', v_2'']$. Therefore B_2 has an atom at $\beta_2(v_2')$ (the size of the atom is at least $F(v_2'') - F(v_2')$). Since B_2 is continuous except possibly at zero, this atom can only be at $\beta_2(v_2') = 0$.

This shows that there is a $\underline{\mathrm{v}} \in [0,1)$ such that, first, for all $v_2 \leq \underline{\mathrm{v}}$, $\beta_2(v_2) = 0$, and second, β_2 is strictly increasing on $[\underline{\mathrm{v}}, 1]$. Since B_2 is strictly increasing, β_2 has to be continuous as well.

3.A.3 Proof of Lemma 3.3

We first show that B_1 is differentiable at any $x_2 \in (0, \bar{b})$. Let $v_2 = \beta_2^{-1}(x_2)$ and consider a strictly increasing sequence v_2^n with $v_2^n \in (\underline{v}, 1)$ and $\lim_{n\to\infty} v_2^n = v_2$. For notational brevity let $x_2^n = \beta_2(v_2^n)$. Then x_2^n is strictly increasing and $\lim_{n\to\infty} x_2^n = x_2$.

Bidding x_2^n is at least as good as bidding x_2 for type v_2^n, thus

$$B_1(x_2^n)\, v_2^n - x_2^n \geq B_1(x_2)\, v_2^n - x_2$$

or

$$1 \geq v_2^n \frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n}.$$

Taking lim sup, we get

$$\limsup \left(\frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n} \right) \leq \frac{1}{v_2}. \tag{3.19}$$

Similarly, for type v_2, bidding x_2 is at least as good as bidding x_2^n. Thus

$$B_1(x_2)\, v_2 - x_2 \geq B_1(x_2^n)\, v_2 - x_2^n.$$

Rearranging and taking lim inf, we get

$$\liminf \left(\frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n} \right) \geq \frac{1}{v_2}. \tag{3.20}$$

From (3.20) and (3.19), it follows that

$$\lim_{x_2^n \uparrow x_2} \left(\frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n} \right) = \frac{1}{v_2}.$$

A parallel argument, that considers a strictly decreasing sequence v_2^n with limit v_2, shows that

$$\lim_{x_2^n \downarrow x_2} \left(\frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n} \right) = \frac{1}{v_2}.$$

Thus B_1 is differentiable at v_2, with

$$\left. \frac{dB_1(x)}{dx} \right|_{x=x_2} = \lim_{x_2^n \to x_2} \left(\frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n} \right) = \frac{1}{v_2}.$$

We next show that the bid distribution B_2 is differentiable. Since B_1 is strictly increasing on $\left(0, \bar{b}\right)$, player 1 must be indifferent between all bids $x \in \left(0, \bar{b}\right)$. Fix one $x_1 \in \left(0, \bar{b}\right)$. Consider a sequence x_1^n with limit x_1 and with $x_1^n \in \left(0, \bar{b}\right)$ for all n. For all n, player 1 is indifferent between bidding x_1^n and bidding x_1 :

$$B_2(x_1^n) v_1 - x_1^n = B_2(x_1) v_1 - x_1$$

Rearranging,

$$\frac{B_2(x_1) - B_2(x_1^n)}{x_1 - x_1^n} = \frac{1}{v_1}$$

Thus

$$\lim_{n \to \infty} \left(\frac{B_2(x_1) - B_2(x_1^n)}{x_1 - x_1^n} \right) = \frac{1}{v_1}$$

and therefore B_2 is differentiable.

Since F is differentiable by assumption, it follows that β_2 must be differentiable as well.

3.A.4 Proof of Lemma 3.4

(i) Suppose to the contrary that $\alpha_1 > 0$. Then $\alpha_2 = 0$ by (3.6) and thus

$$B_1 (\beta_2 (1)) = \int_0^1 \frac{v_1}{v} dF (v) + \alpha_1 > 1,$$

contradiction. Thus $\alpha_1 = 0$. Inserting $\alpha_1 = 0$ in (3.7), we get (3.9). The left-hand side of (3.9) is strictly greater than one for $\alpha_2 = 0$, it strictly decreases in α_2, and is equal to zero for $\alpha_2 = 1$. By continuity, there is a unique $\alpha_2 \in (0, 1)$ such that (3.9) holds. Part (ii) can be proven similarly. From (i) and (ii), it follows that α_1 and α_2 are uniquely determined.

3.A.5 Proof of Proposition 3.1

Uniqueness follows from the discussion in the main text. It remains to establish that the strategies are an equilibrium. Consider player 1 and suppose player 2 follows (3.12). The expected payoff of player 1 for a bid $x_1 \in (0, (1 - \alpha_2) v_1]$ is equal to

$$E [u_1 (x_1)] = F \left(\beta_2^{-1} (x_1)\right) v_1 - x_1$$

since β_2^{-1} exists on $(0, (1 - \alpha_2) v_1]$. Inserting (3.12), we get $E [u_1 (x_1)] = \alpha_2 v_1$ for all $x_1 \in (0, (1 - \alpha_2) v_1]$. Moreover, if (3.8) does not hold, then $\alpha_2 = 0$ and player 1 has a payoff of zero; thus in this case he is indifferent between all $x_1 \in [0, (1 - \alpha_2) v_1]$. Bidding more than $(1 - \alpha_2) v_1$ is always suboptimal. Thus (3.11) is a best response.

Now consider player 2 and suppose he has a valuation v_2. Given B_1, his payoff $B_1(x)v_2 - x$ is strictly concave in his bid x since

$$B_1''(x) = \frac{\partial^2}{\partial x^2}\left(\int_0^x \frac{1}{F^{-1}\left(\frac{z+\alpha_2 v_1}{v_1}\right)}dz\right) = \frac{\partial}{\partial x}\frac{1}{F^{-1}\left(\frac{x+\alpha_2 v_1}{v_1}\right)} < 0.$$

If $v_2 > F^{-1}(\alpha_2)$, then the first-order condition (3.3) describes the unique maximum. If $v_2 \leq F^{-1}(\alpha_2)$, then for all $x_2 > 0$,

$$B_1'(x_2)v_2 - 1 = \frac{1}{F^{-1}\left(\frac{x_2+\alpha_2 v_1}{v_1}\right)}v_2 - 1 < 0.$$

Therefore, (3.12) is a best response.

3.A.6 Proof of Proposition 3.2

Suppose player j does not acquire information. If i does not acquire information either, he gets an expected payoff of zero by Fact 3.1; if i acquires information, his payoff is described by (3.17). Hence, i's best response is to acquire information if and only if c is smaller than

$$\bar{c} := \int_{\underline{v}}^{1}\int_{\underline{v}}^{v_i}\left(\frac{E(V)}{v_j}v_i - E(V)\right)dF(v_j)\,dF(v_i) \tag{3.21}$$

where, from (3.15), $\underline{v} = F^{-1}(\alpha_i) > 0$, and $\underline{v}$ is defined by

$$\int_{\underline{v}}^{1}\frac{E(V)}{v}dF(v) = 1. \tag{3.22}$$

Note that from (3.22), it follows that $\underline{v} < E(V)$.

Now suppose that j acquires information. If i remains uninformed, he gets $F(\underline{v})E(V)$, as in (3.16). If i acquires information, his payoff is described by (3.14). Thus, i's best response is to acquire information if and only if c is smaller than

$$\underline{c} := \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{\underline{v}} E(V)\, dF(v) \tag{3.23}$$

where again $\underline{v}$ is defined by (3.22).

Let

$$c' := E_{v_i,v_j}\left[\max\{v_i, v_j\}\right] - E_{v_j}\left[\max\{E(V), v_j\}\right].$$

(In Appendix 3.A.7, we will show that in the first best, both players acquire information if and only if $c < c'$.) The following lemmas will be used repeatedly below.

Lemma 3.5

$$c' = \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{E(V)} (E(V) - v_j)\, dF(v_j) > 0.$$

Proof. For the equality,

$$\begin{aligned} c' &= E_{v_i,v_j}\left[\max\{v_i, v_j\}\right] - E_{v_j}\left[\max\{E(V), v_j\}\right] \\ &= \int_0^1 \int_0^{v_i} v_i dF(v_j)\, dF(v_i) + \int_0^1 \int_{v_i}^1 v_j dF(v_j)\, dF(v_i) \\ &\quad - \int_0^{E(V)} E(V)\, dF(v_j) - \int_{E(V)}^1 v_j dF(v_j) \\ &= \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{E(V)} (E(V) - v_j)\, dF(v_j). \end{aligned}$$

The inequality $c' > 0$ follows from Jensen's inequality. To see this, define

$$g(v_i) := \int_0^1 \max\{v_i, v_j\}\, dF(v_j).$$

Since g is strictly convex in v_i,

$$E_{v_i}\left[g\left(v_i\right)\right] > g\left(E_{v_i}\left(v_i\right)\right)$$

or equivalently

$$E_{v_i,v_j}\left[\max\left\{v_i, v_j\right\}\right] > E_{v_j}\left[\max\left\{E\left(V\right), v_j\right\}\right].$$

■

Lemma 3.6 *(i)* $\underline{c} > c'$ *and (ii)* $\bar{c} > \underline{c}$.

Proof. (i) Using Lemma 3.5,

$$\begin{aligned}
\underline{c} - c' &= \int_0^{E(V)} \left(E\left(V\right) - v_j\right) dF\left(v_j\right) - \int_0^{\underline{v}} E\left(V\right) dF\left(v_j\right) \\
&= \int_{\underline{v}}^{E(V)} \left(E\left(V\right) - v_j\right) dF\left(v_j\right) - \int_0^{\underline{v}} v_j dF\left(v_j\right).
\end{aligned}$$

Adding and subtracting both $\int_0^{E(V)} \underline{v} dF\left(v_j\right)$ and $\int_{\underline{v}}^{E(V)} \underline{v}\frac{E(V)}{v_j} dF\left(v_j\right)$ yields

$$\begin{aligned}
\underline{c} - c' &= \int_0^{\underline{v}} \left(\underline{v} - v_j\right) dF\left(v_j\right) + \int_{\underline{v}}^{E(V)} \left(E\left(V\right) - v_j + \underline{v} - \underline{v}\frac{E\left(V\right)}{v_j}\right) dF\left(v_j\right) \\
&\quad - \int_0^{E(V)} \underline{v} dF\left(v_j\right) + \int_{\underline{v}}^{E(V)} \underline{v}\frac{E\left(V\right)}{v_j} dF\left(v_j\right).
\end{aligned}$$

First observe that

$$\begin{aligned}
\int_{\underline{v}}^{E(V)} \underline{v}\frac{E\left(V\right)}{v_j} dF\left(v_j\right) &= \underline{v}\left[\int_{\underline{v}}^{1} \frac{E\left(V\right)}{v_j} dF\left(v_j\right) - \int_{E(V)}^{1} \frac{E\left(V\right)}{v_j} dF\left(v_j\right)\right] \\
&= \underline{v}\left[1 - \int_{E(V)}^{1} \frac{E\left(V\right)}{v_j} dF\left(v_j\right)\right]
\end{aligned}$$

where the second equality uses (3.22). Therefore,

$$
\begin{aligned}
\underline{c} - c' &= \int_0^{\underline{v}} (\underline{v} - v_j)\, dF(v_j) + \int_{\underline{v}}^{E(V)} \frac{(E(V) - v_j)(v_j - \underline{v})}{v_j} dF(v_j) \\
&\quad + \underline{v} \left[1 - \int_0^{E(V)} dF(v_j) - \int_{E(V)}^1 \frac{E(V)}{v_j} dF(v_j) \right]
\end{aligned}
$$

which is strictly positive.

(ii) With (3.21) and (3.23), $\bar{c} - \underline{c}$ is equal to

$$
\begin{aligned}
&\int_{\underline{v}}^1 \int_{\underline{v}}^{v_i} \left(\frac{E(V)\, v_i}{v_j} - E(V) \right) dF(v_j)\, dF(v_i) \\
&\quad + \int_0^1 \int_0^{\underline{v}} E(V)\, dF(v_j)\, dF(v_i) - \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) \\
&= \int_0^1 h(v_i)\, dF(v_i)
\end{aligned}
$$

where

$$
h(v_i) = \int_0^{\underline{v}} E(V)\, dF(v_j) - \int_0^{v_i} (v_i - v_j)\, dF(v_j)
$$

if $v_i \leq \underline{v}$, and

$$
\begin{aligned}
h(v_i) &= \int_{\underline{v}}^{v_i} \left(\frac{E(V)\, v_i}{v_j} - E(V) \right) dF(v_j) \\
&\quad + \int_0^{\underline{v}} E(V)\, dF(v_j) - \int_0^{v_i} (v_i - v_j)\, dF(v_j)
\end{aligned}
$$

if $v_i > \underline{v}$. Then, it is sufficient to show that $h(v_i) > 0$ for all $v_i \in [0, 1]$.

Case 1: $v_i \leq \underline{v}$. From (3.22), it follows that $\underline{v} < E(V)$, and thus

$$
\int_0^{\underline{v}} E(V)\, dF(v_j) > \int_0^{v_i} v_i dF(v_j) > \int_0^{v_i} (v_i - v_j)\, dF(v_j).
$$

Case 2: $v_i \in (\underline{\mathrm{v}}, E(V)]$. Here, $h(v_i)$ is equal to

$$\int_{\underline{\mathrm{v}}}^{v_i} \frac{(v_i - v_j)(E(V) - v_j)}{v_j} dF(v_j) + \int_0^{\underline{\mathrm{v}}} (E(V) - v_i + v_j) \, dF(v_j).$$

The first term is strictly positive because $v_j \leq v_i \leq E(V)$ and $v_i > \underline{\mathrm{v}}$. The second term is strictly positive as $v_i \leq E(V)$ and, by (3.15), $\underline{\mathrm{v}} > 0$.
Case 3: $v_i \in (E(V), 1]$. Since $\underline{\mathrm{v}}$ is independent of v_i, we get

$$\begin{aligned} h'(v_i) &= \int_{\underline{\mathrm{v}}}^{v_i} \frac{E(V)}{v_j} dF(v_j) - \int_0^{v_i} dF(v_j), \\ h''(v_i) &= \frac{E(V)}{v_i} F'(v_i) - F'(v_i), \end{aligned}$$

hence, h is strictly concave for $v_i > E(V)$. Moreover, as $v_i \to 1$, h' converges to

$$\int_{\underline{\mathrm{v}}}^{1} \frac{E(V)}{v_j} dF(v_j) - \int_0^1 dF(s_j) = 1 - 1 = 0.$$

(The first integral is one by (3.22).) Thus, h' must be positive for all $v_i \in (E(V), 1)$ and thus $h(v_i) > h(E(V)) > 0$ where the last inequality follows from case 2. ■

We are now in a position to prove Proposition 3.2. From Lemmas 3.5 and 3.6, it follows directly that $\bar{c} > \underline{c} > 0$. Thus, (i) if $c < \underline{c}$, information acquisition is strictly dominant. (ii) If $\underline{c} < c < \bar{c}$, a player invests in information only if the opponent remains uninformed, and there exist two asymmetric equilibria where exactly one player invests. Moreover, there is a symmetric equilibrium where both players invest in information with probability $p = (\bar{c} - c) / (\bar{c} - \underline{c})$: if player i acquires information, he gets

$$(1 - p)(\bar{c} - c) + p(F(\underline{\mathrm{v}}) E(V) + \underline{c} - c) = pF(\underline{\mathrm{v}}) E(V)$$

which is equal to his payoff if he remains uninformed. Thus, i is indifferent between investing and not investing in information. Moreover, for all p that are strictly smaller (greater) than this critical value, i strictly prefers (not) to acquire information. Finally, (iii) if $c > \bar{c}$, not investing is strictly dominant.

3.A.7 Proof of Proposition 3.3

Consider the decision of the social planner. If she does not acquire any information, she gives the prize to any player and realizes a welfare of $E(V)$. If she acquires information about the valuation of one player, it is optimal to give the prize to this player if and only if his valuation is higher than $E(V)$. In this case, welfare is equal to $E_{v_j}[\max\{E(V), v_j\}] - c$. If the social planner acquires information about both players, welfare equals $E_{v_i,v_j}[\max\{v_i, v_j\}] - 2c$. As above, let

$$c' = E_{v_i,v_j}[\max\{v_i, v_j\}] - E_{v_j}[\max\{E(V), v_j\}]. \tag{3.24}$$

Moreover, let

$$\begin{aligned} c'' &= E_{v_i}[\max\{E(V), v_i\}] - E(V) \\ &= \int_{E(V)}^{1} (v_i - E(V))\, dF(v_i). \end{aligned} \tag{3.25}$$

If the cost of information acquisition equals c', welfare is the same if two players acquire information as if one acquires information. At c'', welfare is the same if one player acquires information as if no one does.

Lemma 3.7 *(i)* $0 < c' < c''$ *and (ii)* $c' < \underline{c}$ *and* $c'' < \bar{c}$.

Proof. **(i)** In Lemma 3.5, we have already shown that $c' > 0$. Moreover, using Lemma 3.5,

$$\begin{aligned} c' &= \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{E(V)} (E(V) - v_j)\, dF(v_j) \\ &= \int_0^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) + \int_0^1 \int_0^{v_i} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \\ &\quad - \int_0^1 \int_0^{E(V)} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \\ &= \int_0^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) + \int_{E(V)}^1 \int_{E(V)}^{v_i} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \\ &\quad - \int_0^{E(V)} \int_{v_i}^{E(V)} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \end{aligned}$$

which is strictly smaller than

$$\begin{aligned} \int_0^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) &< \int_{E(V)}^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) \\ &< \int_{E(V)}^1 \int_0^1 (v_i - E(V))\, dF(v_j)\, dF(v_i) = c''. \end{aligned}$$

(ii) The first inequality is Lemma 3.6, part (i). Moreover, by (3.21) and (3.25), $\bar{c} > c''$ is equivalent to

$$\int_{\underline{v}}^1 \int_{\underline{v}}^{v_i} \left(\frac{E(V)\, v_i}{v_j} - E(V) \right) dF(v_j)\, dF(v_i) > \int_{E(V)}^1 (v_i - E(V))\, dF(v_i).$$

By (3.17), the left-hand side is i's ex ante expected payoff if i acquired information and j remained uninformed. Since, in this case, j never bids higher than his expected value, the LHS must be weakly higher than the RHS, because the latter is the payoff i could ensure by bidding $E(V)$ for all types $v_i \geq E(V)$ and bidding zero otherwise. It remains to show that for some realizations of v_i, i can do strictly better. Note first that $F^{-1}(\alpha_i) = \underline{v} > 0$, i.e. j's maximum bid is $\bar{b} = (1 - \alpha_i) E(V) < E(V)$. Hence, for all realizations $v_i \in ((1 - \alpha_i) E(V), E(V))$, i can ensure a strictly positive payoff by bidding $(1 - \alpha_i) E(V)$, and hence the LHS must be strictly larger than the RHS. ■

The inequalities in (i) allow us to characterize first best information acquisition: if $c < c'$, both should acquire information; if $c \in (c', c'')$, exactly one player should acquire information; finally, if $c > c''$, no one should. With (ii), we can compare equilibrium investments and first best investments (see Figure 2 in the main text). If $c < c'$, both players invest as in the first best. If $c \in (c', \min\{\underline{c}, c''\})$, both players acquire information although exactly one player should. If $c \in (\min\{\underline{c}, c''\}, c'')$, in the asymmetric equilibria exactly one player acquires information, as in the first best. If $c \in (c'', \bar{c})$, at least one player acquires information, but neither of the players should. Finally, if $c > \bar{c}$, no player invests, as in the first best. Therefore, the number of players investing in information is higher than the first best.

3.A.8 Proof of Proposition 3.4

If no player invests in information, both get an expected payoff of zero. If only player i invests, i's expected payoff is $E_{v_i}[\max\{v_i - E(V), 0\}] - c$, while j gets $E_{v_i}[\max\{E(V) - v_i, 0\}]$. If both players acquire information, each of them gets $E_{v_i,v_j}[\max\{v_i - v_j, 0\}] - c$.

Now suppose that j remains uninformed. Player i's best response is to acquire information whenever c is smaller than $E_{v_i}[\max\{v_i - E(V), 0\}]$ which, with (3.25), is equal to c''. If j acquires information, i invests whenever c is smaller than

$$E_{v_i,v_j}[\max\{v_i - v_j, 0\}] - E_{v_i}[\max\{E(V) - v_i, 0\}]$$

which, by Lemma 3.5, is equal to c'. Since $0 < c' < c''$, both players (no player) acquire information if $c < c'$ ($c > c''$). If $c \in (c', c'')$, there are two equilibria where exactly one player acquires information, and a mixed strategy equilibrium where players acquire information with probability $(c'' - c) / (c'' - c')$.

3.A.9 Proof of Proposition 3.5

(i) We first analyze whether there can be an equilibrium where both players acquire information with probability 1. If this is the case, then they bid as in Fact 3.2 and both get a payoff of

$$\int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - c.$$

Now suppose that i deviates and remains uninformed. Then, his optimal bid is as if he had a value of $E(V)$ which leads to a deviation payoff of

$$\int_0^{E(V)} (E(V) - v_j)\, dF(v_j).$$

Hence, it pays off to save the cost of information whenever c is larger than

$$\int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{E(V)} (E(V) - v_j)\, dF(v_j)$$

which, by Lemma 3.5, is equal to c'. Thus, if and only if $c < c'$, an equilibrium exists where both players acquire information.

(ii) Now suppose that both players do not invest in information with probability 1. Then, both get zero payoff. If i deviates and acquires information, his optimal bid is zero if $v_i \leq E(V)$ and $E(V)$ if $v_i > E(V)$. (The type $v_i = E(V)$ is exactly indifferent. Thus, lower types prefer a bid of zero, and higher types prefer a bid at the upper bound of the support of j's bids.)

The deviation payoff is

$$\int_{E(V)}^{1} (v_i - E(V))\, dF(v_i) - c.$$

Therefore, if and only if c is larger than c'' (from (3.25)), there is an equilibrium where no player acquires information.

Chapter 4

Information sharing in contests

This chapter is joint work with Dan Kovenock and Johannes Münster.

4.1 Introduction

Competing firms can often commit to share relevant information with their competitors. Exchange of information not only takes place in joint ventures or cartels; one benefit of joining an industry association is better access to industry data. The incentives to share information have been extensively studied in the literature on imperfect competition. Competition in some oligopolistic markets is, however, best described as a contest or an all-pay auction, and the incentives to share information ahead of a contest appear to have not yet been explored. To date, the main focus of the literature has been on the implications of whether firms' decision variables are strategic substitutes or strategic complements. In the all-pay auction, however, these notions do not fit because the best replies may be nonmonotonic, involving either marginal overbidding, or spending zero effort.[1] The aim of this chapter

[1] In our analysis with continuous strategy spaces, formal best responses may not exist due to the open-endedness problem arising with discontinuous payoff functions. When referring to best replies in this context we are thinking of either $\varepsilon-$best replies or best replies in finite approximations of the continuous strategy space.

is to analyze this case.

The strategic interaction between firms in many markets has the characteristics of a contest. This is particularly true of markets with intense advertising or promotional competition (Schmalensee 1976, 1992), and in R&D races. Lichtenberg (1988) stresses the importance of 'design and technical competitions' for public procurement and points out that these competitions are best understood as contests. Dasgupta (1986) uses the all-pay auction with complete information as a model of R&D races and research tournaments. In a similar structure, Kaplan et al. (2003) analyze firms' innovation activities when potential gains are endogenous. See Konrad (2009) for a survey. This article examines a popular type of contest, often used as a benchmark in the contest literature: an all-pay auction.[2] The all-pay auction has been studied under a wide range of assumptions concerning the information possessed by competitors (Weber 1985; Hillman and Riley 1989; Baye et al. 1993, 1996; Amann and Leininger 1996; Krishna and Morgan 1997). Here we build on these results to investigate information sharing.

The literature on information sharing in oligopoly is extensive and we do not attempt to survey it here. Early contributions include Ponssard (1979), Novshek and Sonnenschein (1982), Vives (1984), and Gal-Or (1985). Raith (1996) presents a fairly general model that encompasses many of the known results; Vives (1999, Chapter 8) contains an overview. Most closely related to this chapter are studies of information disclosure in R&D competition, going back to Bhattacharya and Ritter (1983). Gill (2008) and Jansen (2008) are recent contributions, and also include overviews of the literature. Our contribution to this literature is to focus on the two-player all-pay auction structure and on the incentives to reveal one's value of winning the contest

[2] The all-pay auction captures the notion that, conditional on expenditures, exogenous shocks do not play a significant role in determining a contest's outcome. Contests with exogenous noise, such as the Tullock (1980) or Lazear & Rosen (1981) models, generally require sufficient noise to ensure pure-strategy equilibria in the complete information game. Alcalde and Dahm (2009) and Che and Gale (2000) have recently shown that contests with "small" amounts of exogenous noise share many of the same properties as all-pay auctions.

to one's competitor. We also study the social efficiency of the decision to share information and find conditions under which a legal prohibition of information sharing or, alternatively, a requirement to share information, is welfare improving.

The chapter is organized as follows. Section 4.2 sets out the model. Firms receive private information about the value they derive from winning the contest. If private information pertains to some firm-specific characteristic such as the cost structure, a model with private values may be appropriate; Section 4.3 considers this case.[3] On the other hand, if the information is about some circumstances which are common to the firms such as demand conditions, we have common values; we study this case in Section 4.4. Sections 4.2 to 4.4 assume, following most of the literature, that the decisions on information sharing are binding commitments taken ex ante, before firms receive private information. To assess the robustness of our results, Section 4.5 discusses interim information sharing, where decisions on information sharing are taken after firms have received their private information. We summarize our findings and discuss extensions in Section 4.6.

4.2 The model

There are two firms $i = 1, 2$. At stage 1, each firm decides whether or not to share information. In the literature, there are two approaches concerning how to model these decisions. We will describe each in detail below. Between stage 1 and stage 2, each firm receives a private signal s_i about its value v_i of winning the contest. We assume that the signals s_1 and s_2 are independent draws from a cumulative distribution function F with support $[s_l, s_h]$, $0 \leq s_l < s_h$. We assume that F is continuously differentiable. In the case of private values analyzed in Section 4.3 below, each firm's value of winning is

[3] At the end of Section 4.3, we also consider the case where firms receive private information about their marginal cost of effort in the contest rather than about the value of winning (see also Moldovanu and Sela 2001, 2006).

equal to its signal, $v_i(s_1, s_2) = s_i$. In Section 4.4, we investigate a common values environment in which each firm's value of winning equals a nonnegative continuously differentiable, strictly increasing, and symmetric function of the two signals, $v(s_1, s_2)$.[4]

In stage 2, firms choose their outlays or efforts $x_i \in \mathbb{R}_+$. The higher effort wins; ties are broken randomly. Thus the probability that firm i wins is given by

$$p_i = \begin{cases} 0, & \text{if } x_i < x_j, \\ \frac{1}{2}, & \text{if } x_i = x_j, \\ 1, & \text{if } x_i > x_j. \end{cases}$$

Conditional on the signals (s_1, s_2) and the efforts (x_1, x_2), firm i's expected profit is $p_i v_i(s_1, s_2) - x_i$.[5]

As noted above, there are two main approaches to information sharing in the literature: the decision whether or not to share information can be either unilateral, or a bilateral agreement. In the first approach, decisions to share information are taken simultaneously and independently. These decisions are binding commitments. Hence, if firm i has decided to share information, firm $j \neq i$ also learns the signal s_i before the efforts are chosen; otherwise, s_i is private information to firm i. In an alternative approach, the first stage decisions on information sharing are treated as an industry-wide

[4] Our assumption of independent signals is not without loss of generality. However, as demonstrated by Krishna and Morgan (1997), even in the standard symmetric environment of Milgrom and Weber (1982), affiliation is not sufficient to insure the existence of an increasing (symmetric) equilibrium bidding function in the all-pay auction. Krishna and Morgan (1997) provide a sufficient condition on the product of the value function and the conditional density for such a symmetric equilibrium to exist. Roughly speaking, for a well-behaved value function this condition requires that for all values of s_i the density of s_i conditional on s_{-i} does not change too abruptly in s_{-i} . Radhi (1994) has provided examples demonstrating that highly correlated signals may lead to perverse nonmonotonic bidding functions. Although we believe our results are robust to some degree of affiliation, it is clear that more general results for arbitrary affiliated signals would be difficult to obtain.

[5] Our analysis applies directly to the case where each firm has an identical increasing cost of effort function $c(x_i)$, $i = 1, 2$. In this case the bid can be redefined as $z_i = c(x_i)$ and all relevant calculations can be carried out with the transformed bid z_i.

agreement, where a firm shares its information before stage 2 if and only if the other firm does so as well. Here, in stage 1 both firms simultaneously indicate whether they would like an industry-wide agreement on information sharing. If both indicate that they want it, then all information is shared. If at least one firm indicates that it does not want to share, then no firm's information is shared. Note that in both approaches, 'sharing information' can be thought of as 'providing hard evidence that fully reveals the realization of one's signal'.[6]

Finally, we assume that social welfare depends on the firms' expected profits. Moreover, the firms' efforts may be socially desirable in themselves. For example, if x_i is innovative effort, it may provide positive spillovers to the rest of the economy. Thus, we assume that conditional on the signals (s_1, s_2) and the efforts (x_1, x_2) total welfare is

$$W(s_1, s_2; x_1, x_2) = \sum_{i=1,2} (p_i v_i(s_1, s_2) - x_i) + \kappa \left(\sum_{i=1,2} x_i \right).$$

Here, κ is a parameter that expresses the social value of the efforts not directly captured by the firms in the industry. If $\kappa > 0$ the efforts provide a socially valuable externality not captured by the industry and if $\kappa < 0$ this externality is negative.

Throughout, we analyze whether equilibrium information sharing is socially efficient. In particular, we study whether prohibiting information sharing, or forcing the firms to share information, increases welfare.

[6] It will become clear that, in both approaches, our findings are robust to a sequential timing of the decisions on information sharing where firm 1 decides first, and firm 2 decides after having observed the decision of firm 1.

4.3 Private values

In this section, we assume that each firm's value of winning the contest coincides with its signal, $v_i(s_1, s_2) = s_i$ for $i = 1, 2$. That is, each firm is privately informed about the value it derives from winning, and this value is independent of the other firm's value.

4.3.1 Industry-wide agreements

We begin the analysis with the simpler case of industry-wide agreements. Here we only have to consider the symmetric situations in which either both firms share their information, or both keep their information secret. The corresponding continuation equilibria are well known.

Both firms share information If both firms share their information, the resulting subgames have complete information, and the all-pay auction has a unique equilibrium in mixed strategies (Hillman and Riley 1989; Baye, Kovenock, de Vries 1996).[7] Without loss of generality, let $s_1 \leq s_2$, which corresponds to $v_1 \leq v_2$. Firms play the following mixed strategies:

$$\begin{aligned} B_1(x) &= \frac{s_2 - s_1}{s_2} + \frac{x}{s_2} \text{ for } x \in [0, s_1], \\ B_2(x) &= \frac{x}{s_1} \text{ for } x \in [0, s_1]. \end{aligned}$$

To see this is an equilibrium, note that the expected profit of firm 1 from an effort $x_1 \in [0, s_1]$ equals

$$\frac{x_1}{s_1} s_1 - x_1 = 0.$$

[7] There is a trivial technical complication if $s_l = 0$. In the event that $s_i = 0 < s_j$, firm i has a strictly dominant strategy to choose zero effort, hence firm j would like to choose the smallest strictly positive effort, which does not exist in a continuous strategy space. To fix this, we assume that in case $s_i = 0 < s_j$ and $x_i = x_j = 0$, firm j wins with probability one. A similar comment applies to Lemma 4.1 below.

A higher effort leads to a higher probability of winning, which is just outweighed by the increased costs; thus firm 1 is indifferent between all these effort levels. Moreover, choosing an effort $x_1 > s_1$ is suboptimal, because it leads to negative expected profits. Similarly, firm 2 is indifferent between all $x_2 \in (0, s_1]$, because each such x_2 gives the same expected profit:

$$\left(\frac{s_2 - s_1}{s_2} + \frac{x_2}{s_2} \right) s_2 - x_2 = s_2 - s_1.$$

To summarize, the expected profit of a firm i equals $\max \{s_i - s_j, 0\}$. As firms decide on information sharing before they know their own value, this decision is based on the *ex ante expected profit,* i.e. the expectation at the beginning of stage 1. Firm i's ex ante expected profit from an agreement to share information is equal to

$$\int_{s_l}^{s_h} \int_{s_l}^{s_i} (s_i - s_j) \, dF(s_j) \, dF(s_i) \, . \tag{4.1}$$

No firm shares information If no firm shares information, then stage 2 is characterized by two-sided incomplete information. The equilibrium is in increasing strategies: a firm that receives a signal s chooses effort

$$\xi(s) = \int_{s_l}^{s} t dF(t) \, . \tag{4.2}$$

To see that this is an equilibrium, consider the expected profit of firm i, given that firm j follows this strategy.[8] Suppose firm i chooses an effort of $x_i \in \left[0, \int_{s_l}^{s_h} t dF(t)\right]$. Equivalently, firm i bids according to ξ but as if it had

[8]The equilibrium was first derived in Weber (1985). Uniqueness follows from Amann and Leininger (1996).

received a signal z such that $x_i = \xi(z)$. The expected profit of firm i equals

$$\begin{aligned} \Pr(\xi(s_j) < \xi(z)) s_i - \xi(z) &= F(z) s_i - \int_{s_l}^{z} t dF(t) \\ &= \int_{s_l}^{z} (s_i - t) dF(t). \end{aligned}$$

As the integrand is positive if and only if s_i is greater than t, the optimal choice is $z = s_i$, and hence $x_i = \xi(s_i)$ as in (4.2).

A firm's interim expected profit, conditional on s_i, equals

$$F(s_i) s_i - \int_{s_l}^{s_i} t dF(t) = \int_{s_l}^{s_i} (s_i - s_j) dF(s_j)$$

and ex ante expected profit is

$$\int_{s_l}^{s_h} \int_{s_l}^{s_i} (s_i - s_j) dF(s_j) dF(s_i). \tag{4.3}$$

Comparing (4.1) and (4.3) shows that expected profit when both firms share information is equal to expected profit when no firm shares information.[9]

Proposition 4.1 *Consider information sharing as an industry-wide agreement, where a firm shares information if and only if the rival shares information. With private values, both information sharing and no information sharing can arise in equilibrium. Firms' profits are identical in the two cases. Expected efforts are higher without information sharing.*

Proof. The equivalence of firms' profits in the two cases has been shown above. Therefore, if firm i proposes to share information, firm j is indifferent whether or not to agree. Thus both cases can arise in equilibrium. It remains

[9] This result on payoff equivalence has been shown in Chapter 2 (Proposition 2.2), using a different method of proof than the one given here. It is also more general than presented here in that it does not rely on the assumption of only two players. Moreover, the result also holds for discrete probability distributions.

to consider the implications for expected efforts. Suppose no firm shares information and denote expected effort of firm i by $E\left(x_i^{NN}\right)$. Then

$$E\left(x_i^{NN}\right) = \int_{s_l}^{s_h} \xi\left(s_i\right) dF\left(s_i\right) = \int_{s_l}^{s_h} \int_{s_l}^{s_i} t dF\left(t\right) dF\left(s_i\right).$$

Now suppose both firms share information. Conditional on s_1 and s_2, expected effort of firm i is equal to $s_j/2$ if $s_i > s_j$, and equal to $s_i^2/\left(2s_j\right)$ if $s_i < s_j$. Therefore, the ex ante expected effort of firm i equals

$$E\left(x_i^{SS}\right) = \int_{s_l}^{s_h} \left(\int_{s_l}^{s_i} \frac{s_j}{2} dF\left(s_j\right) + \int_{s_i}^{s_h} \frac{s_i^2}{2s_j} dF\left(s_j\right)\right) dF\left(s_i\right).$$

The difference is

$$\begin{aligned} E\left(x_i^{NN}\right) - E\left(x_i^{SS}\right) &= \int_{s_l}^{s_h} \int_{s_l}^{s_i} \frac{s_j}{2} dF\left(s_j\right) dF\left(s_i\right) - \int_{s_l}^{s_h} \int_{s_i}^{s_h} \frac{s_i^2}{2s_j} dF\left(s_j\right) dF\left(s_i\right) \\ &= \int_{s_l}^{s_h} \int_{s_j}^{s_h} \frac{s_j}{2} dF\left(s_i\right) dF\left(s_j\right) - \int_{s_l}^{s_h} \int_{s_i}^{s_h} \frac{s_i^2}{2s_j} dF\left(s_j\right) dF\left(s_i\right), \end{aligned}$$

where the second equality uses Fubini's theorem. Renaming the variables of integration in the first term by exchanging i and j, we get

$$E\left(x_i^{NN}\right) - E\left(x_i^{SS}\right) = \int_{s_l}^{s_h} \int_{s_i}^{s_h} \frac{s_i}{2} \left(1 - \frac{s_i}{s_j}\right) dF\left(s_j\right) dF\left(s_i\right) > 0.$$

■

Proposition 4.1 indicates that an industry-wide agreement to share information may occur in equilibrium, but depresses effort. In the context of a procurement contest or an R&D race, for example, where it might be expected that effort has positive spillover effects, banning industry-wide information sharing would result in a Pareto improvement. As profits of the firms are unchanged, but the efforts are higher, when $\kappa > 0$, welfare is higher. The amount by which the efforts increase is exactly equal to the gain in allocative efficiency: without information sharing, the firm with the higher value wins the contest with probability one, whereas with information sharing the equilibrium is in mixed strategies and thus the firm with the lower value sometimes wins.

4.3.2 Independent commitments to share information

In this section, we turn to the two stage model, where firms independently decide whether or not to commit to share information. Here, if exactly one firm shares its information, an asymmetric situation arises at the contest stage: the signal, and hence the value, of one firm is common knowledge, whereas the value of the other firm is its private information (compare Chapter 3, Section 3.3).

Suppose, without loss of generality, that firm 1 has committed to share information, whereas 2 has committed not to share information. The equilibrium will then exhibit a mixture of the properties of the equilibria in the two symmetric cases discussed above. Firm 1, whose value is common knowledge, will randomize continuously according to a cumulative distribution function which we denote by B_1. Firm 2, on the other hand, will choose effort as an increasing function of its privately known signal. Firm 1 may choose zero effort with a positive probability, which we denote by $B_1(0) \in [0, 1)$. Similarly, there may be a signal s' such that firm 2 chooses zero effort for all signals $s_2 \leq s'$. Hence $F(s') \in [0, 1)$ is the ex ante probability that firm 2 chooses an effort of zero.[10]

Lemma 4.1 *Suppose that only firm* 1 *shares its private information. In the*

[10] There is a similar trivial technical complication here as in footnote 7 above. If firm 1 has the signal $s_1 = 0$ and shares this information, then firm 2 would like to choose the smallest strictly positive effort, which does not exist in a continuous strategy space. To fix this, we assume that firm 2 wins whenever $x_1 = x_2 = 0$. This tie-breaking rule also ensures that, in case $B_1(0) > 0$, it is optimal for the smallest type of firm 2 to choose $\xi_2(s_l) = 0$, even if $s_l > 0$.

unique equilibrium of stage 2, firm 2 plays the following pure strategy:

$$\xi_2(s_2) = \begin{cases} 0 & \textit{for} \quad s_2 \in [s_l, s') \\ (F(s_2) - F(s'))\, s_1 & \textit{for} \quad s_2 \in [s', s_h] \end{cases} . \tag{4.4}$$

Firm 1 randomizes according to

$$B_1(x_1) = \int_0^{x_1} \frac{1}{\xi_2^{-1}(z)} dz + B_1(0) \quad \textit{for} \quad x_1 \in [0, (1 - F(s'))\, s_1] . \tag{4.5}$$

$B_1(0)$ *and* s' *are uniquely defined by* $\min\{B_1(0), F(s')\} = 0$ *and* $B_1(\xi_2(s_h)) = 1$.

Proof. Here we only show that this is an equilibrium; uniqueness has been shown in Chapter 3 (using Lemmas 3.1-3.4). Consider firm 1 and suppose that firm 2 follows the strategy in (4.4). Firm 1's profit from an effort $x_1 \in (0, (1 - F(s'))\, s_1]$ equals

$$\Pr(\xi_2(s_2) < x_1)\, s_1 - x_1 = F\left(F^{-1}\left(\frac{x_1 + F(s')\, s_1}{s_1}\right)\right) s_1 - x_1 = F(s')\, s_1.$$

Thus firm 1 is indifferent between all these efforts. Higher efforts are clearly suboptimal, because they might be lowered without decreasing the chances to win. Whenever $s' > s_l$, an effort of zero is also suboptimal, because it involves the risk of losing the contest even in case that firm 2 chooses zero effort. When $s' = s_l$, any $x_1 \in [0, s_1]$ gives a profit of zero and thus firm 1 is indifferent between these efforts.

Now consider firm 2 and suppose that firm 1 follows (4.5). The profit of firm 2 from an effort $x_2 \in [0, (1 - F(s'))\, s_1]$ equals

$$\left(\int_0^{x_2} \frac{1}{\xi_2^{-1}(z)} dz + B_1(0)\right) s_2 - x_2.$$

Because ξ_2^{-1} is strictly increasing, the profit of firm 2 is strictly concave in

x_2. The maximum is thus unique and described by the first order condition

$$\frac{1}{F^{-1}\left(\frac{x_2+F(s')s_1}{s_1}\right)}s_2 - 1 \leq 0, \ x_2 \geq 0,$$

together with the complementary slackness condition. If $s' < s_2$, we have an interior solution with

$$x_2 = F(s_2)\, s_1 - F(s')\, s_1.$$

Otherwise, an effort of zero is optimal.

It remains to show that $B_1(0)$ and s' are uniquely determined. Note first that firm 1 won't choose an effort that is higher than the highest possible effort of firm 2, and thus $B_1(\xi_2(s_h)) = 1$. With the substitution $\xi_2^{-1}(z) = s$, the boundary condition $B_1(\xi_2(s_h)) = 1$ can be written as

$$\int_{s'}^{s_h} \frac{s_1}{s} dF(s) + B_1(0) = 1. \tag{4.6}$$

The first term is continuous and strictly decreasing in s'; moreover, it would vanish if s' were equal to s_h. It follows that $B_1(\xi_2(s_h)) = 1$ has a unique solution that fulfills $\min\{B_1(0), F(s')\} = 0$. ■

As in the case where both firms share information, the distribution of efforts of firm 2, considered from the point of view of firm 1, is a uniform distribution, with possibly a mass point at zero. Moreover, the slope is just $1/s_1$. For firm 1, a higher effort leads to a greater chance of winning, which is just outweighed by the higher cost. Thus firm 1 is indifferent between the efforts it randomizes over.

It is straightforward to see that, in equilibrium, at least one of the mass points $B_1(0)$ and $F(s')$ must be zero. Suppose to the contrary that $B_1(0) > 0$ and $F(s') > 0$. Then firm 1 chooses an effort of zero with strictly positive probability. But choosing a sufficiently small but strictly positive effort ε gives a higher profit: the probability of winning increases discretely at an arbitrarily small cost, contradicting equilibrium. Thus, at least one of the mass points is zero. Whether firm 1 or firm 2 has a mass point at zero depends, in general, both on the distribution function F and on the realization of the signal s_1.

For future reference, note that (4.6) together with $\min\{B_1(0), F(s')\} = 0$ implies $s' < s_1$ for all $s_1 > s_l$. To understand the economics behind this, suppose to the contrary that $s' \geq s_1$. Then firm 2 has zero profit for any signal $s_2 \leq s_1$, and firm 1 has a profit of $F(s')\, s_1$, which is strictly positive since $s' \geq s_1 > s_l \geq 0$. Therefore, the effort of firm 1 can be no greater than $(1 - F(s'))\, s_1$. But this implies that, whenever $s_2 > (1 - F(s'))\, s_1$, firm 2 can guarantee itself a strictly positive profit by bidding slightly more than $(1 - F(s'))\, s_1$, contradiction. Hence $s' < s_1$.

In contrast to the case of industry wide agreements, information sharing cannot arise in equilibrium when decisions on information sharing are taken independently.

Proposition 4.2 *Consider independent decisions on information sharing. With private values, sharing information is strictly dominated.*

Proof. We show that, for any $s_i > s_l$, sharing information is strictly worse than not sharing if the rival firm shares information (step 1), and similarly if the rival firm does not share information (step 2). (If $s_i = s_l$, the profit of firm i is zero if it shares information.) Therefore, the ex ante profit of firm i is strictly higher if i does not share information.

Step 1. Suppose that firm j shares its information. We first argue that for all realizations of s_i and s_j, the profit of firm i is *weakly* lower if it shares information than if it does not. If firm i shares its information, given s_i and s_j its profit equals $\max\{0, s_i - s_j\}$. Suppose that firm i does not share information. Any effort $x_j > s_j$ is strictly dominated for firm j. Moreover, firm j chooses $x_j = s_j$ with probability zero. Therefore, by choosing $x_i = 0$

if $s_i \leq s_j$, and $x_i = s_j$ if $s_i > s_j$, firm i can guarantee itself a profit of $\max\{0, s_i - s_j\}$, and its equilibrium profit cannot be lower.

It remains to show that firm i's interim expected profit is *strictly* higher if it does not share information. Suppose i does not share information. As argued above, for any $s_j > s_l$ the corresponding critical signal s' is strictly smaller than s_j.[11] Thus, for any $s_i > s_l$, if s_j happens to be equal to s_i, the corresponding s' is strictly smaller than s_i; hence firm i chooses strictly positive effort and has a strictly positive profit. By continuity, this is still true if $s_j \in (s_i, s_i + \delta)$ for some $\delta > 0$. On the other hand, if firm i shares information, it gets zero profit whenever $s_j \geq s_i$. It follows that whenever $s_j \in [s_i, s_i + \delta)$, firm i 's profit is strictly higher if firm i does not share information. Together with the last paragraph, this implies that firm i's interim expected profit is strictly higher if it does not share information.

Step 2. Now suppose that firm j does not share information. We focus on an interim perspective and show that given any signal $s_i > s_l$, the profit of firm i is strictly higher if it does not share information.

If firm i with signal s_i does not share information, its profit is

$$\int_{s_l}^{s_i} (s_i - s_j)\, dF(s_j)\,. \tag{4.7}$$

If firm i shares information, by Lemma 4.1 (replacing subscript 1 by i and subscript 2 by j) firm i gets a profit of

$$\int_{s_l}^{s'} s_i dF(s_j) \tag{4.8}$$

which is equal to the probability that j has a signal lower than s' and thus chooses zero effort, multiplied by i's value s_i. If $s' = s_l$, we are done because

[11] Here s' is defined in Lemma 4.1, replacing subscript 1 by j and subscript 2 by i. Remember that here firm j shares information, whereas in Lemma 4.1 firm 1 shares information. Similarly, the firm that does not share is firm i here and firm 2 in Lemma 4.1.

the profit of i equals zero if it shares its information, whereas the profit of i is strictly positive if i does not share its information. Therefore, suppose in the following that $s' > s_l$. Then the critical signal s' is determined such that

$$\int_{s'}^{s_h} \frac{s_i}{s} dF(s) = 1. \tag{4.9}$$

As argued above, $s' < s_i$. For notational convenience, let Δ denote the difference between the profits (4.7) and (4.8):

$$\Delta := \int_{s_l}^{s_i} (s_i - s_j)\, dF(s_j) - \int_{s_l}^{s'} s_i dF(s_j). \tag{4.10}$$

Straightforward manipulations show that

$$\begin{aligned} \Delta &= \int_{s_l}^{s'} (s' - s_j)\, dF(s_j) + \int_{s'}^{s_i} \left(s_i - s_j + s' - s' \frac{s_i}{s_j} \right) dF(s_j) \\ &\quad - \int_{s_l}^{s_i} s' dF(s_j) + \int_{s'}^{s_i} s' \frac{s_i}{s_j} dF(s_j). \end{aligned}$$

Rewriting the last term and using (4.9) gives

$$\begin{aligned} \int_{s'}^{s_i} s' \frac{s_i}{s_j} dF(s_j) &= s' \left(\int_{s'}^{s_h} \frac{s_i}{s_j} dF(s_j) - \int_{s_i}^{s_h} \frac{s_i}{s_j} dF(s_j) \right) \\ &= s' \left(1 - \int_{s_i}^{s_h} \frac{s_i}{s_j} dF(s_j) \right). \end{aligned}$$

Thus

$$\begin{aligned} \Delta &= \int_{s_l}^{s'} (s' - s_j)\, dF(s_j) + \int_{s'}^{s_i} (s_j - s') \frac{s_i - s_j}{s_j} dF(s_j) \\ &\quad + s' \left(1 - \int_{s_l}^{s_i} dF(s_j) - \int_{s_i}^{s_h} \frac{s_i}{s_j} dF(s_j) \right). \end{aligned}$$

The first and the second term are both strictly positive because $s_l < s' < s_i$, and the third term is nonnegative. Thus $\Delta > 0$. ■

If an industry-wide agreement on information sharing is not possible, there is a unique equilibrium where firms do not share their information. Independently of the rival's decision, they prefer to keep their own information secret.[12]

The astute reader will note that in the proofs of Propositions 4.1 and 4.2 we demonstrate that the result from Proposition 4.1 on the equality of profits with and without industry-wide information sharing and the result from Proposition 4.2 that information sharing is strictly dominated under independent decisions, carry over to an environment in which information sharing is considered at the interim stage, after firms receive their signals. Here, in the asymmetric situation where only firm i shares its signal, the interim profit of firm i is increasing in s_i, and one might conjecture that a firm with a high signal may have an incentive to share its information. However, the interim profit in the case where no firm shares its information is also increasing in one's own type. In fact, if F is the uniform distribution on the unit interval, Δ (the difference in interim profits given in equation (4.10)) is monotonically increasing in s_i: the higher one's signal, the higher is the benefit from keeping it hidden. In general, however, Δ is not monotone.[13] We examine the issue of interim information sharing further in Section 4.5.

Because the results from Propositions 4.1 and 4.2 carry over to the interim stage, our results on information sharing also apply to a model where both firms have the same constant value of winning $v > 0$, and firm $i's$ cost

[12] Proposition 4.2 goes through if the signals are drawn from a discrete distribution function and the number of possible signals is strictly larger than 2. The case of a binary distribution is an exteme case in the sense that, given that firm j does not share information, i's profit are the same whether or not i shares information, and information sharing is only weakly dominated (for the equilibrium of the all-pay auction see Konrad 2009). However, if j does share information, i is strictly better off if it keeps its information secret. Thus, there are three equilibria: one equilibrium where both firms do not share information, and two equilibria where exactly one firm shares information and the other does not share.

[13] To give an example where Δ is not monotone, suppose that signals are distributed according to $F(s) = s^3$ on the unit interval.

of effort equals $\gamma(s_i)\, x_i$, $i = 1, 2$, where $\gamma(s_i)$ is a positive and decreasing function of the signal s_i. This arises because, at the interim stage, dividing profits by $\gamma(s_i)$ is a positive affine transformation of profits that leaves behavior under uncertainty invariant. This generates an all-pay auction with prize value $v/\gamma(s_i)$ and cost of effort x_i, $i = 1, 2$, for which the results that profits are equal with and without industry-wide information sharing and that information sharing is strictly dominated under independent decisions continue to hold. Consequently, these properties hold at the interim stage for the original game and therefore at the ex ante stage as well, so that our results on ex ante information sharing policies continue to hold.

4.4 Common values

In the previous section, we assumed that the firms' values v_i are private. In many environments, however, it is reasonable to assume that the values of winning depend on the other firm's signal as well. This section studies common values where

$$v_1(s_1, s_2) = v_2(s_1, s_2) = v(s_1, s_2).$$

We assume that v is nonnegative, continuously differentiable, strictly increasing in s_1 and s_2, and symmetric, i.e. $v(s_1, s_2) = v(s_2, s_1)$ for all (s_1, s_2).

4.4.1 Industry-wide agreements

Both firms share information Here, at the contest stage, the value of winning v is commonly known. As before, under complete information, there is a unique equilibrium in mixed strategies. With $v_1 = v_2 = v$, we have complete rent dissipation, and expected profits are zero.

Conditional on (s_1, s_2), the sum of the expected efforts of both firms is equal to $v(s_1, s_2)$. The sum of ex ante expected efforts is equal to the expected value of winning

$$\int_{s_l}^{s_h} \int_{s_l}^{s_h} v(s_1, s_2)\, dF(s_2)\, dF(s_1).$$

No firm shares information Here, firm i knows s_i but not s_j. Krishna and Morgan (1997) have shown that there is a symmetric equilibrium in strictly increasing strategies $x_i = \xi(s_i)$ where

$$\xi(s_i) = \int_{s_l}^{s_i} v(t,t)\, dF(t).$$

To see that this is an equilibrium, suppose firm j follows the strategy ξ. If i chooses its effort according to ξ but as if its signal was z, it gets a profit of

$$\int_{s_l}^{z} (v(s_i, t) - v(t,t))\, dF(t).$$

As v is strictly increasing in its arguments, the integrand is strictly positive for all $t < s_i$ and strictly negative for all $t > s_i$, and it is optimal for i to choose $z = s_i$.[14]

Ex ante expected profit of firm i is equal to

$$\int_{s_l}^{s_h} \int_{s_l}^{s_i} (v(s_i, t) - v(t,t))\, dF(t)\, dF(s_i) > 0, \tag{4.11}$$

and hence higher than if both firms share their information. We summarize this argument in the following proposition:

Proposition 4.3 *Consider industry-wide agreements to share information about a common value where firm i shares its information if and only if firm j does. Then there will be no information sharing in equilibrium.*

[14] Due to our assumption that the signals are independent, the condition for existence of the equilibrium given in Krishna and Morgan (1997) is automatically fulfilled.

Contrary to the case of private values of winning, the firms' profits are higher if they do not share their information with their rival, and thus an agreement on industry-wide information sharing can not arise in equilibrium.

4.4.2 Independent commitments to share information

We now consider information sharing with independent decisions. As above, this necessitates considering the case where only one firm shares its information. Again, the equilibrium exhibits properties of each of the two symmetric cases, where either both firms share information or no firm does.

Lemma 4.2 *Consider the case of a common value $v(s_1, s_2)$. Suppose the signal of firm 1 is commonly known, whereas s_2 is private information of firm 2. In equilibrium, firm 2 plays a pure strategy*

$$\xi_2(s_2) = \int_{s_l}^{s_2} v(s_1, t)\, dF(t). \tag{4.12}$$

Firm 1 randomizes according to

$$B_1(x_1) = F\left(\xi_2^{-1}(x_1)\right). \tag{4.13}$$

Proof. Consider firm 1 and suppose it chooses an effort $x_1 \in \left[0, \int_{s_l}^{s_h} v(s_1, t)\, dF(t)\right]$. Higher efforts are obviously suboptimal because they can be lowered without changing the probability of winning. Let $z = \xi_2^{-1}(x_1)$. The profit of firm 1 equals

$$\int_{s_l}^{\xi_2^{-1}(x_1)} v(s_1, s_2)\, dF(s_2) - x_1 = \int_{s_l}^{z} v(s_1, s_2)\, dF(s_2) - \xi_2(z) = 0.$$

Therefore, firm 1 is indifferent between all these efforts.

Now consider firm 2. Its profit is

$$B_1(x_2)\, v(s_1, s_2) - x_2 = F\left(\xi_2^{-1}(x_2)\right) v(s_1, s_2) - x_2.$$

Suppose firm 2 chooses effort as if its signal were z. Then it gets

$$F(z)\,v(s_1,s_2)-\xi_2(z)=\int_{s_l}^{z}\left(v(s_1,s_2)-v(s_1,t)\right)dF(t)$$

As the integrand is strictly positive whenever $t<s_2$, and strictly negative whenever $t>s_2$, the optimal choice is $z=s_2$. ■

If exactly one firm shares its information, ex ante expected efforts are the same for both firms. In fact, the distribution of the effort of the firm that shares information,

$$\Pr(x_1\leq z)=F\left(\xi_2^{-1}(z)\right),$$

is the same as the distribution of the efforts of the firm that does not share information:

$$\Pr(x_2\leq z)=\Pr\left(\xi_2(s_2)\leq z\right)=F\left(\xi_2^{-1}(z)\right).$$

Using this characterization of equilibrium in the contest when only one firm shares information, we can derive the incentives for information sharing with independent decisions.

Proposition 4.4 *Consider the case of common values and independent decisions about information sharing. Then, sharing information is strictly dominated.*

Proof. Suppose firm i shares information. If firm j also shares information, it earns zero expected profits, as has been argued in Section 4.4.1. If j does not share information, i randomizes its contest effort as in (4.13) and j chooses an effort as in (4.12). Hence, given s_i, firm j's expected profit

$$\int_{s_l}^{s_j}\left(v(s_i,s_j)-v(s_i,t)\right)dF(t)$$

is strictly positive for any $s_j > s_l$. A fortiori, ex ante expected profit is strictly positive, and the best reply is not to share information.

Suppose that firm i does not share information. If firm j shares information, j randomizes its contest effort as in (4.13). Its expected profit is zero. If it does not share information, it gets

$$\int_{s_l}^{s_j} \left(v\left(s_j, t\right) - v\left(t, t\right)\right) dF\left(t\right)$$

which is strictly positive for all $s_j > s_l$. Thus j strictly prefers not to share information. ■

Note that the proof of Proposition 4.4 also establishes that sharing information is dominated from an interim perspective. Hence, if the decisions on information sharing were taken only after having received the signal, there is still no incentive to share information, just as in the case of private values (see the discussion following Proposition 4.2).

We now compare expected profits and efforts across the different information structures. Due to the common value, both firms value the prize identically, so there cannot be an allocative inefficiency. Ex post, the sum of profits is

$$\sum_{i=1}^{2} \left(p_i v\left(s_1, s_2\right) - x_i\right) = v\left(s_1, s_2\right) - \sum_{i=1}^{2} x_i,$$

and the sum of profits and efforts is always $v\left(s_1, s_2\right)$. Consequently, the sum of expected profits and expected efforts has to be the same in all information structures. Therefore, the ranking of expected efforts is just the opposite of the ranking of expected profits.

If both firms share information, expected profits are zero; otherwise the sum of expected profits is strictly positive. Therefore, expected efforts are highest if both firms share information. The comparison between the remaining cases, however, depends on the function v.[15]

Remark 4.1 *Suppose that firm j does not share information. Whenever v is supermodular, j's profit is higher if i shares information than if i does not share. If v is modular, j's profit is the same in both cases. If v is submodular, j's profit is lower if i shares information than if i does not share.*

Recall that i's expected profit is zero whenever i shares information. Thus, whenever v is modular or submodular, (1) the sum of expected profits is lower, and (2) the sum of expected efforts is higher if exactly one firm shares information than if no firm shares information. If v is supermodular, expected efforts may be higher if no firm shares information.

In the common values environment, firms prefer to keep their information secret, whether or not an industry wide agreement on information sharing is possible. The ranking of the expected efforts shows that they are highest if both firms share their information with their rival. Therefore, contrary to the case of private values, agreements on information sharing about a common value can be desirable from a welfare point of view if the investments in the contest are socially valuable. In fact, if the value of the efforts to society is higher than their cost to the firms (i.e. $\kappa > 1$), then a legal requirement to share information is welfare improving.

4.5 Interim information sharing

The result that, with independent decisions, there will be no information sharing does not only hold if this decision has to be taken before the firms receive their signals. Because the proofs of Propositions 4.2 and 4.4 consider an interim perspective, firm i prefers not to share information for each possible signal $s_i > s_l$. Thus firms do not have an incentive to reconsider their

[15] The proof is in the appendix. Kim (2008) obtains a similar result for the first price auction with common values.

decision and inform the rival in case they have, for instance, a high value of winning the contest.

Moreover, if the decisions on information sharing were taken only after having received the signal, results corresponding to Propositions 4.2 and 4.4 can be obtained. To be more precise, consider the following game of interim information sharing:[16]

1. Firms privately receive their signals.

2. Firms decide independently whether or not to share their signals. As above, sharing information means providing hard evidence that fully reveals the realization of one's signal.

3. The contest takes place.

We argue that this game has a perfect Bayesian equilibrium where no firm ever shares its information. In this equilibrium, the beliefs of a firm about the signal of its rival are as follows. If firm i does not reveal its signal, consistency of beliefs with strategies requires that firm j believes that s_i is distributed according to the ex ante distribution F. If firm i deviates and reveals its signal, the belief of j is pinned down by the hard evidence, that is, firm j knows s_i.

Now suppose that firm j never reveals its signal. Consider whether firm i wants to reveal its signal on stage 2. This is exactly the comparison we used in the proofs of Propositions 4.2 (step 2) and 4.4: for any $s_i > s_l$, firm i is strictly better off if it does not reveal its signal. This shows that the game of interim information sharing has a perfect Bayesian equilibrium where no firm ever shares its information.

[16] For a variety of winner-pay auctions, Benoît and Dubra (2006) study related problems and show that results may depend on the fine details of the information structure. They also present a general full disclosure result (Theorem 1); however, the assumptions of this result are not fulfilled in our all-pay auction setting.

The game has, however, also a perfect Bayesian equilibrium with full disclosure. This can be supported by off-equilibrium beliefs of the rival that make him bid very aggressively. With common values, suppose that, if firm i does not share its information, firm j believes that s_i is equal to the highest possible signal s_h. Then winning seems very important to firm j and thus it bids aggressively. Consequently firm i cannot make a positive profit, no matter what its true signal may be. Recall that, upon revealing its signal, firm i also makes zero profit. Thus all types of firm i are indifferent between revealing and not revealing. With private values, a similar construction can be given by assuming that the off-equilibrium belief of firm j is that firm i has drawn exactly the same signal as firm j itself. Again, this makes j bid aggressively, and firm i is exactly indifferent between revealing and not revealing its information. Thus, whereas this is an equilibrium with full disclosure, it is not a strict equilibrium; in fact any type of any firm is exactly indifferent.

4.6 Conclusion

This chapter considered incentives to share information ahead of competition in markets that are described by an all-pay auction. We first considered private values. We found that, with industry-wide agreements, firms are indifferent between sharing and not sharing information. Thus, an industry-wide agreement on information sharing may emerge in equilibrium. Aggregate efforts, however, are higher without information sharing, and a ban on industry-wide agreements on information sharing is a Pareto improvement whenever effort generates positive spillovers outside of the contest as, for example, may be the case in a procurement contest or a R&D race. However, with independent decisions whether or not to share information sharing information is strictly dominated.

Second, we considered a common values framework, where the true value of winning is a continuously differentiable, strictly increasing, and symmetric function of the firms' private signals. Here, efforts are highest if both firms share information. Information sharing will not arise in equilibrium - firms are strictly better of if they do not share information, no matter whether they decide individually, where information sharing is a strictly dominated strategy, or consider an industry-wide agreement. When the effort generates positive spillovers outside of the contest, information sharing may be inefficiently low. Thus, whereas there may be too much information sharing with private values, there may be too little information sharing with common values.

We conclude by discussing extensions and possible avenues for future research. In the private values case, the exact equality of profits when both firms share information and when no firm shares (Proposition 4.1) is robust to an extension to more than two firms, but it hinges on the assumptions of symmetrically distributed signals and risk neutrality of the firms. One of the main messages of our analysis, however, namely that with private values an industry-wide agreement on information sharing may arise, is reinforced if we modify these assumptions. In the private values case, for constant or decreasing absolute risk aversion, it can be shown that each firm is strictly better off if both firms share their information than if no firm does.[17] Moreover, it is possible to give examples with asymmetrically distributed signals where both firms strictly prefer an industry-wide agreement.[18] Nevertheless, when decisions on information sharing are taken independently, firms do not have a strict incentive to share information if the rival shares. Here, our result that, by keeping its information hidden, a firm can guarantee itself at least

[17] The proof extends the analysis of the all-pay auction under complete information and risk aversion from Hillman and Samet (1987) to the case where the contestants have unequal values, and uses results from Matthews (1987) and Fibich et al. (2006). Details are available upon request.

[18] Let, for example, firm i's signal be uniformly distributed on $[0, h_i]$, $i = 1, 2$, where $h_1 = 1$ and $h_2 > 1$. Then, firm 2 prefers an industry-wide agreement, and firm 1 will agree if and only if $h_2 > 2$. Details are available upon request.

the profit it gets under complete information, generalizes to risk aversion, to ex ante asymmetries, and to more than two firms. Moreover, in the case of common values, profits are zero if all firms share information. This is also true with risk aversion, ex ante asymmetries, or more than two firms, and thus we expect no industry-wide agreement to share information. Similarly, with common values and independent decisions, there cannot be a strict incentive to share information if all other firms share their information. A full analysis of independent decisions, however, seems to be more difficult when we change the model in one or the other direction, and we leave it for future research.

4.A Appendix

4.A.1 Proof of Remark 4.1

We compare the interim profit of firm 2 in the asymmetric setting where only firm 1 shares information, which we denote by $\pi_2^{SN}(s_2)$ (the first superscript indicates firm 1 does share information, the second says that firm 2 does not shares information), with the interim profit of firm 2 if no firm shares, denoted by $\pi_2^{NN}(s_2)$. We have

$$\begin{aligned}
\pi_2^{SN}(s_2) &= \int_{s_l}^{s_h} \int_{s_l}^{s_2} (v(s_1, s_2) - v(s_1, t))\, dF(t)\, dF(s_1) \\
\pi_2^{NN}(s_2) &= \int_{s_l}^{s_2} (v(s_2, t) - v(t, t))\, dF(t) \\
&= \int_{s_l}^{s_2} (v(t, s_2) - v(t, t))\, dF(t)
\end{aligned}$$

where the last line uses the symmetry of v. If $s_2 = s_l$, firm 2 chooses an effort of zero in both cases, and

$$\pi_2^{SN}(s_l) = \pi_2^{NN}(s_l) = 0.$$

Moreover,

$$\begin{aligned}
\frac{\partial}{\partial s_2}\pi_2^{SN}(s_2) &= \int_{s_l}^{s_h}\int_{s_l}^{s_2}\frac{\partial v(s_1,s_2)}{\partial s_2}dF(t)\,dF(s_1) \\
&= F(s_2)\int_{s_l}^{s_h}\frac{\partial v(s_1,s_2)}{\partial s_2}dF(s_1) \\
&= F(s_2)\,E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2}\right),
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial}{\partial s_2}\pi_2^{NN}(s_2) &= \int_{s_l}^{s_2}\frac{\partial v(t,s_2)}{\partial s_2}dF(t) \\
&= F(s_2)\,E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2}|s_1 \leq s_2\right).
\end{aligned}$$

Hence

$$\begin{aligned}
&\frac{\partial}{\partial s_2}\pi_2^{SN}(s_2) - \frac{\partial}{\partial s_2}\pi_2^{NN}(s_2) \\
= \; & F(s_2)\left(E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2}\right) - E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2}|s_1 \leq s_2\right)\right)
\end{aligned}$$

which is strictly positive if $\frac{\partial v(s_1,s_2)}{\partial s_2}$ increases in s_1. It follows that $\pi_2^{SN}(s_2) > \pi_2^{NN}(s_2)$ for all $s_2 > s_l$ whenever $v(\cdot)$ is supermodular. Similarly, $\pi_2^{SN}(s_2) < \pi_2^{NN}(s_2)$ for all $s_2 > s_l$ if $v(\cdot)$ is submodular, and $\pi_2^{SN}(s_2) = \pi_2^{NN}(s_2)$ for all $s_2 > s_l$ if $v(\cdot)$ is modular.

Chapter 5

Strategic information acquisition and the mitigation of global warming

5.1 Introduction

Global warming and the reduction of emissions of carbon dioxide have been among the most intensively debated issues in international politics in the last decade.[1] Recently, the Stern Review on the Economics of Climate Change has added to the numerous attempts to assess the costs and benefits of climate policy. Both there and in many other discussions of this topic, the relevance of uncertainty for taking action for climate protection is emphasized.[2] But reducing uncertainty does not need to be the only aim of research on the impact of climate change. Countries may use investment in information as an instrument in international climate policy. One kind of uncer-

[1] This chapter is based on the article *Strategic information acquisition and the mitigation of global warming*, forthcoming in *Journal of Environmental Economics and Management.*

[2] Cf. Stern (2006), chapters 2, 13, 14, 21. Furthermore, see e.g. McKibbin and Wilcoxen (2002) or Sandler (2004) who points out that, compared to the case of ozone-shield depletion, unresolved uncertainties inhibit the reduction of CO_2 emissions.

tainty still relates to the quantitative relationship between the accumulation of greenhouse gases and global temperatures. In addition to this more general question, however, most of the countries involved in climate policy are financing specific research programs to examine the impact of climate change at the national level. In this context, the focus of information acquisition is on country-specific costs and benefits of a global rise in temperatures as well as on the costs relative to other countries. Investments in information seem to be a decisive factor for the achievement of an effective mitigation of global warming. Some countries, however, might prefer not to acquire information.

Contributions to the mitigation of global warming are chosen in the framework of private provision of a global public good: increasing the own investment in the mitigation of greenhouse gases can reduce the effort of other countries. Likewise, decisions on information have to be considered not only from an efficiency point of view, but in the context of the contribution game. The possibility of free-riding gives countries an incentive to influence their strategic position at the international level. Persistent uncertainties about costs and benefits of climate change make investments in information a particularly valuable instrument.

In this chapter, we focus on the strategic role of information, building on the standard model for private provision of a public good. In order to keep the analysis tractable, we analyze the incentives for information acquisition in a framework with two heterogeneous countries. Ex ante, the countries are uncertain about the economic value they attach to a mitigation of global warming, but they can decide whether or not to invest in information about this country-specific value before they choose their contribution to climate protection. An important characteristic of our model consists in the observability of the information acquisition: additional information acquired by a country is publicly observable before the countries decide on their reduction of CO_2 emissions. On the one hand, this specification highlights most explicitly the strategic character of investments in information. On the other hand, observability of information acquisition reflects the fact that reports estimating the economic value of global warming at a national level are typically published by the research institutes conducting the studies. Moreover, it is likely that information acquired by a government cannot easily be kept secret, and thus, observability of information is a reasonable assumption. We rule out the possibility of acquiring information about the benefits to other countries. The strategic impact of this type of information acquisition is similar to the effect in our model.[3]

We will identify two effects of information acquisition. Better information improves the own contribution to a mitigation of global warming. But finding out that the own benefit of a mitigation is large could reduce the contributions of other countries and shift the burden of provision of the public good to the country itself. Hence, information acquisition does not only eliminate uncertainty, it also affects the behavior of other countries. We show that additional information can have a negative value even if the cost of information is zero. Thus, anticipating the impact of the information on other countries' behavior, countries may have an incentive to remain uninformed. If, in equilibrium, information is not acquired due to strategic considerations, global welfare may be lower than under complete information. We determine conditions under which the strategic information choice negatively affects the efficiency of the resulting mitigation of global warming. This may be a rationale for information provision by a supranational institution. But in addition we demonstrate that there can be too much information acquisition from a welfare point of view even if the information is available without cost. In the latter case, uncertainty helps to overcome the underprovision problem.

Since the impact of global warming differs substantially across countries, the countries' costs and benefits of emission reductions are assumed to be uncorrelated, with the implication that country-specific information does not

[3]One could imagine that some countries try to manipulate information and produce information in their favor. This type of activity, however, is excluded from our model.

inform another country of its cost of global warming. For other environmental public goods such as the protection of the ozone layer, research on ozone-depleting chemicals and the impacts of UV-B radiation provided information about all countries' damages. This restricts the possibility of benefiting from being uninformed, and, as we will show, in the extreme case of perfectly correlated valuations, all countries' valuations are uncovered in equilibrium.

An important aspect of decision-making under uncertainty is the potential irreversibility of investments which results in an *expected value* or *option value of information* revealed in the future.[4] Emissions of CO_2 are irreversible in the sense that the stock of greenhouse gases in the atmosphere depreciates very slowly. In the future, however, more information about the damages of climate change will be available. This has implications on the countries' current contributions to climate protection (Epstein, 1980; Kolstad, 1996; Ulph and Ulph, 1997; Gollier et al., 2000; Fisher, 2001). Even if countries *decide* on investments in information, the additional information may be only obtained in the future. We discuss this aspect in an extension of our model, assuming that future contributions can be based on acquired information about the country-specific benefit, but that they are more costly due to the irreversible damages that the delay has caused. Then, a further strategic effect of information acquisition emerges: investments in information can be a credible commitment to delay the own contribution to climate protection until the better information is available. This, in turn, may affect the other countries' contributions today.

Our main analysis builds on the standard model of private provision of a public good.[5] Sandler et al. (1987) indicate the role of uncertainty by showing that increased risk with regard to the contributions of the other players may make free-riding behavior worse. An important issue for the mitiga-

[4]Early papers developing this concept are Arrow and Fisher (1974), Henry (1974), and Conrad (1980).

[5]See Bergstrom et al. (1986) or Cornes and Sandler (1985). Standard textbooks such as Sandler (1992, 2004) discuss global climate in a public goods framework.

tion of global warming is the implication of uncertainty and learning on international environmental agreements (see e.g. Helm, 1998, Kolstad, 2007, and the references herein). We add to this literature by studying strategic learning decisions when contributions to climate protection are chosen non-cooperatively. Our results relate to this work since the non-cooperative equilibrium can influence a bargaining outcome.[6] A further related paper is Caplan et al. (1999) who analyze a model where there are winners and losers from global warming. Empirical studies of voluntary contributions to environmental public goods are Murdoch and Sandler (1997), Murdoch et al. (1997), and Kotchen and Moore (2007).

Within the context of voluntary contributions to a public good, the chapter is closely related to analyses of strategic behavior prior to public goods games. Konrad (1994) considers wealth in the actual contribution stage as the strategic variable. Robledo (1999) analyzes whether players may strategically abstain from purchasing insurance. The underlying effect in these two papers is similar: the players influence their expected marginal utility of income. The strategic role of transfers is considered by Buchholz and Konrad (1995). In the context of environmental public goods, Buchholz and Konrad (1994) show that investments in technology that lower the contribution cost may be reduced for strategic reasons. In Buchholz et al. (2005), citizens may strategically vote for a government with low preferences for the public good in order to improve the government's bargaining position.

Our work departs from these papers by focusing on the choice of information as a strategic variable. The observability of the information constitutes a strategic disadvantage if high preferences for climate protection are revealed. In this sense, the strategic effect is similar to the strategic voting in Buchholz et al. (2005). In our context, however, countries cannot influence their benefit of mitigating global warming (or their contribution cost, as in Buchholz

[6] Compare also Hoel (1991) who considers the impact of unilateral reductions of emissions on the outcome of international negociations.

and Konrad, 1994), but they can only decide whether or not to learn this value. Strategic behavior does not necessarily result in an advantage in the contribution game since the outcome of the information acquisition is stochastic. This allows us to study the trade-off between strategic and efficiency aspects as well as the interaction of the countries' information choices. In this way, we try to explain strategic considerations in the countries' behavior with respect to climate research.

The next section describes the model, and Section 5.3 characterizes the equilibrium of the private provision game. Section 5.4 analyzes the incentives for information acquisition and implications for welfare in a setting with two countries. Section 5.5 illustrates how irreversibility of emissions of CO_2 interacts with the decisions on information. Moreover, we discuss the two-country assumption. Section 5.6 concludes. All proofs are in the appendix.

5.2 The formal framework

Consider two countries 1 and 2. Each of them allocates a given wealth w_i between private consumption x_i and a contribution $g_i \geq 0$ to the mitigation of global warming where g_i corresponds to a country's effort invested in the abatement of CO_2 emissions (in monetary terms). Total contributions sum up to $g_1 + g_2 = G$, reflecting the substitutability of the countries' emissions reductions. The countries' preferences are described by payoff functions

$$U_i\left(x_i, G\right) = x_i + \alpha_i \varphi\left(G\right), \qquad i = 1, 2 \tag{5.1}$$

where the payoff depends on own private consumption and on the benefit of the reduction of CO_2 emissions.[7] Contribution costs are normalized to one.

[7] Quasilinearity is assumed because it bears out the strategic implications of information acquisition most strongly, and it simplifies the analysis since the optimal mitigation of global warming does not depend on the wealth distribution, but only on the cost-benefit trade-off. Qualitatively, our results do not depend on this specification.

The function φ is assumed to be strictly increasing and concave ($\varphi' > 0$, $\varphi'' < 0$), and it translates worldwide efforts into an economic value attached to the resulting mitigation of global warming. The country-specific part of this economic value is expressed by the multiplier α_i which is the key variable of our model and describes the only difference between the countries with respect to their preferences.[8] We will refer to α_i as country i's *valuation*.

Ex ante, the countries are uncertain about the individual benefit they derive from a mitigation of global warming. Both countries know the probability distribution of their own valuation and of the other country's valuation. We assume that α_1 and α_2 are asymmetrically distributed in order to account for heterogeneity among the countries with respect to potential damages from global warming. Moreover, the countries' valuations are assumed to be independent: climate change probably causes high social cost for some (developing) countries while other countries may even benefit from global warming.

Before the countries contribute to a mitigation of global warming, each country has to decide whether to invest in information about its valuation. The information is publicly observable: if i acquires information, then both countries will update their beliefs about i's valuation α_i in the same way. Hence, there is no private information about a specific country's cost of global warming. In order to focus on *strategic* incentive to invest in information, we assume that information acquisition does not involve a direct cost.

Without losing any valuable insight, we concentrate on the case where the valuations α_1 and α_2 are drawn independently from binary probability distributions F_1 and F_2 with

$$\begin{aligned} &\alpha_i \in \{l_i, h_i\}\,, \quad 0 \leq l_i < h_i, \\ &\Pr(\alpha_i = h_i) = p_i,\ \Pr(\alpha_i = l_i) = 1 - p_i, \qquad i = 1, 2\ . \end{aligned} \tag{5.2}$$

[8] Thus, the relationship between the accumulation of CO_2 in the atmosphere and global temperatures is - via φ - assumed to be known and common to all countries. The focus is on the country-specific benefit of a mitigation of global warming.

Information acquisition is assumed to yield a perfectly informative signal on the own value.[9] Note that if country i decides not to acquire information, both countries will have to choose their contributions based on the common prior about α_i.

The timing of the game is as follows. In stage 1, each country decides whether to acquire information about its valuation. The decisions are made simultaneously. At the beginning of stage 2, the decisions of the two countries and the outcomes of the stage 1 decisions become publicly known, and both countries simultaneously choose their contributions to the global public good. A strategy of a country i therefore consists of the probability of acquiring information in stage 1, denoted by $\pi_i \in [0, 1]$, and a contribution g_i in stage 2, conditioned on the information revealed at the beginning of stage 2. To solve the two-stage game, we use the concept of subgame perfect Nash equilibrium.

5.3 The private provision subgame

We first characterize the private provision equilibrium for given valuations resulting from the decisions in stage 1. Each country i maximizes

$$w_i - g_i + A_i \varphi (g_i + G_{-i})$$

subject to the budget constraint $x_i + g_i \leq w_i$ and $g_i \geq 0$. (G_{-i} are the aggregate contributions of the countries other than i.) Country i's valuation of the public good - here denoted by A_i - depends on whether or not i acquired information. If country i acquired information, A_i is equal to its true valuation. Otherwise, maximization of the expected payoff reduces to an analogous problem with A_i being the ex ante expected value of α_i.

[9] The restriction to a two-point distribution function facilitates the exposition substantially without influencing the results qualitatively, and it strongly emphasizes the different effects of information acquisition which also emerge for more general probability distributions. With quasilinear preferences, the assumption of a perfectly informative signal is not crucial as countries base their contributions on the (conditional) expected value of α_i.

Taking the quasilinear payoff functions into consideration, the solution to this problem is straightforward. Define the *stand-alone quantity* $\Gamma(A_i)$ of the public good for a valuation A_i as the solution to the first order condition

$$A_i \varphi' (\Gamma(A_i)) = 1 \quad \text{for } i = 1, 2 \ . \tag{5.3}$$

Note that this is i's desired mitigation level if $G_{-i} = 0$, given its valuation A_i and provided that its wealth is sufficiently large. It follows from monotonicity and strict concavity of φ that Γ is well-defined and strictly increasing in its argument with $\Gamma(0) = 0$ and $\Gamma(A) = (\varphi')^{-1}(1/A)$ for $A > 0$.

We will generally assume that w_i is never a binding constraint, i.e., we assume that $w_i \geq \Gamma(h_i)$.[10] Then, the equilibrium contributions are well-known to be

$$\begin{array}{llll} g_1^* = \Gamma(A_1) & \text{and} \quad g_2^* = 0 & \text{if} & A_1 > A_2 \ , \\ g_1^* = 0 & \text{and} \quad g_2^* = \Gamma(A_2) & \text{if} & A_1 < A_2 \ . \end{array} \tag{5.4}$$

If $A_1 = A_2$, then any vector $(g_1, g_2) \in [0, \Gamma(A_1)]^2$ with $g_1 + g_2 = \Gamma(A_1)$ is an equilibrium. (5.4) characterizes the solution to the private provision game for general values of A_1 and A_2: the country with the higher benefit bears the entire burden of CO_2 abatement, and the country with the lower benefit free-rides. For the equilibrium, it does not matter whether A_i is an expected value or the true value of country i's mitigation benefit. Thus, (5.4) describes the equilibrium outcome for the four possible information situations in which none of the countries have acquired information, only country 1 or country 2 has acquired information, or both countries have acquired information.

[10] Wealth constraints change the problem in a way that is interesting and related to the problem we study, and we refer back to this case in the next section.

5.4 The incentives for information acquisition

Suppose in the following that $E(\alpha_1) < E(\alpha_2)$. Let $m_i := E(\alpha_i)$, $i = 1, 2$. Additionally, we will use the short form notation $\Gamma_{A_i} := \Gamma(A_i)$. As regards the distribution of valuations, the strategic considerations are strongest if

$$\max(l_1, l_2) < m_i < \min(h_1, h_2), \quad i = 1, 2 \tag{5.5}$$

i.e. the expected value of a country i lies between the two potential valuations of the other country.[11] We will proceed in two steps. First, we determine the best response of a country to a given information decision of the other country. (Recall that $\pi_i = 1$ implies that i uncovers its true valuation with probability 1.) Second, we characterize the set of equilibria for the two-stage game.

One-sided information acquisition. Consider the best response of a country if the other country decides not to learn its valuation of climate protection. We define the *value of information* of a country i as

$$\Delta_i^{\pi_j} := EU_i(\pi_i = 1|\pi_j) - EU_i(\pi_i = 0|\pi_j)$$

where $EU_i(\pi_i|\pi_j)$ is the ex ante expected payoff (prior to the observation of the signal) as function of π_i and conditional on j's information decision π_j.

Country 1. Suppose $\pi_2 = 0$. Due to $E(\alpha_1) < E(\alpha_2)$, it follows from (5.4) that only country 2 contributes if both countries are uninformed. If country 1 uncovers a high value, equilibrium contributions are $g_1^* = \Gamma(h_1)$ and $g_2^* = 0$. Otherwise, if country 1 is learns a low valuation, $g_1^* = 0$ and

[11] If for instance $h_1 < l_2$, information acquisition of one country does not cause an externality on the contribution of the other country: independent of stage 1, only country 2 will contribute in equilibrium. See also the discussion in Section 5.5.

$g_2^* = \Gamma(m_2)$. For $\pi_2 = 0$, country 1's value of information is

$$\Delta_1^{\pi_2=0} = p_1 \left[h_1 \varphi(\Gamma_{h_1}) - \Gamma_{h_1} - h_1 \varphi(\Gamma_{m_2}) \right]. \tag{5.6}$$

With probability $1 - p_1$, country 1 uncovers a low value and its payoff is not affected by the information decision as neither the supply of the public good (Γ_{m_2}) nor country 1's contribution change. With probability p_1, however, the equilibrium contributions change and country 1 bears the burden of CO_2 reductions. However, it can adjust the contribution to climate protection to its individually optimal quantity (Γ_{h_1}). The following properties are straightforward to verify.

Observation 5.1 *The value of information of country* 1 *given* $\pi_2 = 0$ *is*

(i) *negative if* h_1 *is sufficiently close to* m_2 *(*$\lim_{h_1 \downarrow m_2} \Delta_1^{\pi_2=0} = -\Gamma_{h_1} < 0$*);*

(ii) *increasing and convex in* h_1*.*

From Observation 5.1, it follows that $\Delta_1^{\pi_2=0} > 0$ if and only if h_1 is sufficiently large, i.e. if country 1 potentially has very high cost of global warming.

Country 2. Similar to the case of country 1, we can determine country 2's value of information given that country 1 remained uninformed. If it uncovers a low value, equilibrium contributions are $g_1^* = \Gamma(m_1)$ and $g_2^* = 0$, and in case of a high value, we have $g_1^* = 0$ and $g_2^* = \Gamma(h_2)$. This yields

$$\begin{aligned} \Delta_2^{\pi_1=0} &= (1 - p_2) \left[l_2 \varphi(\Gamma_{m_1}) - (l_2 \varphi(\Gamma_{m_2}) - \Gamma_{m_2}) \right] \\ &\quad + p_2 \left[(h_2 \varphi(\Gamma_{h_2}) - \Gamma_{h_2}) - (h_2 \varphi(\Gamma_{m_2}) - \Gamma_{m_2}) \right] \end{aligned} \tag{5.7}$$

which, as we will prove, is always positive.

Lemma 5.1 *Suppose that $E(\alpha_1) < E(\alpha_2)$ and (5.5) holds. Then, (i) if country 1 does not acquire information, country 2 always prefers to uncover its valuation, and (ii) if country 2 does not acquire information, country 1 uncovers its valuation if and only if h_1 is sufficiently large.*

This result follows from Observation 5.1 and the fact that $\Delta_2^{\pi_1=0} > 0$: in the one-sided decision problem, country 2 always prefers to learn its mitigation benefit. Obviously, it cannot be worse off, since, without additional information, the other country doesn't contribute. Uncovering a high valuation, however, allows for an improvement of the own contribution. This *adjustment effect* increases its payoff. Learning a low valuation would even shift the full burden of climate protection to the other country. We refer to this effect as a *strategic effect*. Both effects increase the payoff of country 2. Contrarily, if country 1 reveals a high valuation, country 2 reduces its mitigation effort to zero. This negative strategic effect leads to an incentive for country 1 to remain uninformed. Only if h_1 is considerably higher than m_2, can the improved quantity choice of G outweigh the fact that the provision is now fully paid for by itself. In the latter case, the efficiency aspect of information acquisition dominates the strategic aspect.

It follows directly that, without direct cost of information, there is no equilibrium where both countries do not acquire information. Lemma 5.1 already determines the equilibrium strategies in a situation where only one country would have the possibility of investing in research on its benefit of a mitigation of global warming. (Alternatively, this situation could arise if one country's cost of information were prohibitively high.) Furthermore, Lemma 5.1 applies when one country's cost of global warming is publicly known.

Best response to information acquisition. Here, it is crucial to distinguish which of the countries may potentially benefit most from a mitigation of global warming, i.e. whether h_1 is larger or smaller than h_2.[12]

Case A: $l_i < l_j,\ h_i < h_j$

Let us first analyze the decision of the country $i \in \{1, 2\}$ whose potential valuations are lower than the other country's valuations. Hence, if both countries uncover a high value (or both uncover a low value), country j bears the contribution cost. If $\pi_j = 1$, country i's value of information is

$$\begin{aligned}\Delta^{\pi_j=1}_{i;h_i<h_j} &= (1-p_i)(1-p_j)\left[l_i\varphi\left(\Gamma_{l_j}\right) - \left(l_i\varphi\left(\Gamma_{m_i}\right) - \Gamma_{m_i}\right)\right] \\ &\quad + p_i(1-p_j)\left[\left(h_i\varphi\left(\Gamma_{h_i}\right) - \Gamma_{h_i}\right) - \left(h_i\varphi\left(\Gamma_{m_i}\right) - \Gamma_{m_i}\right)\right] \quad (5.8)\end{aligned}$$

which is strictly larger than zero.[13] With probability p_j, the opponent has a high valuation, and, due to $h_i < h_j$, the equilibrium contributions ($g_i^* = 0$, $g_j^* = \Gamma_{h_j}$) do not depend on i's information decision. With probability $1-p_j$, j's value is low, and country i pays for the provision if it chooses not to learn its valuation. Therefore, it is better off by uncovering its true value, since, with probability p_i, it is able to adjust its mitigation effort to Γ_{h_i}, and with probability $1-p_i$, it can free-ride on j's contribution.[14] Hence, the country i with $h_i < h_j$ prefers to learn its benefit of climate protection.

Case B: $l_i > l_j,\ h_i > h_j$

Now turn to the country $i \in \{1, 2\}$ with possible valuations l_i and h_i that are higher than l_j and h_j, respectively: i contributes if the countries learn either both a high or a low valuation. The value of information for $\pi_j = 1$ is

[12] Recall that (5.5) is still assumed to hold. The distinction of whether l_1 is larger than l_2 does not change the analysis qualitatively. We will address this issue again later.

[13] The formal proof is equivalent to the proof of Lemma 5.1 and is thus omitted.

[14] This follows from the assumption of $l_i < l_j$. With $l_i > l_j$, country i would also gain in the latter case (both have a low value) due to the adjustment of its contribution from Γ_{m_i} to Γ_{l_i}.

given by

$$\begin{aligned}\Delta_{i;h_i>h_j}^{\pi_j=1} &= (1-p_i)(1-p_j)\left[(l_i\varphi(\Gamma_{l_i})-\Gamma_{l_i})-(l_i\varphi(\Gamma_{m_i})-\Gamma_{m_i})\right] \\ &\quad +p_i(1-p_j)\left[(h_i\varphi(\Gamma_{h_i})-\Gamma_{h_i})-(h_i\varphi(\Gamma_{m_i})-\Gamma_{m_i})\right] \\ &\quad +p_ip_j\left[h_i\varphi(\Gamma_{h_i})-\Gamma_{h_i}-h_i\varphi\left(\Gamma_{h_j}\right)\right]. \qquad (5.9)\end{aligned}$$

The first two terms in (5.9) are positive: if the other country has a low value (with probability $1-p_j$), additional information always improves the individual contribution.[15] However, the third term can be negative: if country j has a high benefit of climate protection, i may want to avoid to uncover a high valuation itself. This incentive is strongest if h_i is close to h_j, and consequently a contribution of j would be close to i's standalone quantity.

Observation 5.2 *If $h_i > h_j$ and $\pi_j = 1$, country i's value of information*

(i) *decreases in p_j and h_j;*

(ii) *increases in h_i and decreases in l_i*

($\lim_{h_i\downarrow m_i, l_i\uparrow m_i, h_i>h_j} \Delta_{i;h_i>h_j}^{\pi_j=1} = -p_ip_j\Gamma_{m_i} < 0$).

If it is more likely that the other country has a high value (i.e. p_j is large), the incentive of uncovering the own value is reduced. Moreover, if the difference between h_i and l_i is sufficiently small, potential gains from an adjustment of the individual mitigation effort are limited, and country i's value of information is negative: the negative strategic effect outweighs the increase in the payoff in case the other country learns a low value.

We omit the cases where the value of information is exactly zero and a country is just indifferent between investing and not investing in information. The following proposition then describes equilibrium play in stage 1, denoting

[15] If $l_i < l_j$, $\Delta_{i;h_i>h_j}^{\pi_j=1}$ increases by $(1-p_i)(1-p_j)\left[\Gamma_{l_i}+l_i\varphi\left(\Gamma_{l_j}\right)-l_i\varphi(\Gamma_{l_i})\right] > 0$. The following arguments are still valid.

by country i the country with the higher potential valuation $h_i > h_j$. (Equilibrium contributions conditional on the history of the game up to stage 2 are characterized in (5.4).)

Proposition 5.1 *The equilibrium of the two-stage game is unique. Both countries acquire information if and only if* $\Delta^{\pi_j=1}_{i;h_i>h_j} > 0$. *Otherwise, if (a)* $\Delta^{\pi_i=0}_{j;h_i>h_j} > 0$, *country i remains uninformed and country j acquires information, and if (b)* $\Delta^{\pi_i=0}_{j;h_i>h_j} < 0$, *the equilibrium involves mixed strategies.*

Proposition 5.1 shows that, even if information about benefits of climate protection can be obtained without cost, there exists an strategic incentive to remain uninformed that can outweigh potential adjustment gains that come along with more precise information. As the above analysis shows, the equilibrium crucially depends on the underlying probability distributions F_1 and F_2, i.e. on the potential benefit and on the probabilities of the different outcomes. In particular if the distributions of valuations are similar, the risk of worsening the own strategic position in the international interaction may dominate a potential improvement of the own contribution due to better information. Then, either one country remains uninformed with probability 1, or both countries randomize their information choice. Interestingly, if a country chooses not to learn its benefit of a mitigation of global warming, it is the country that potentially attaches the largest economic value to reductions of CO_2 emissions (the country i with $h_i > h_j$). As a result, the country with the highest benefit of climate protection might not contribute to a reduction of CO_2 emissions, but free-ride on the effort of the country with the lower benefit. The following table summarizes the cases where in equilibrium (at least) one country remains uninformed with positive probability.[16]

[16] In the previous analysis, we assumed throughout that the budget constraints of the countries are never binding. Suppose that both countries uncover a high valuation and $h_i > h_j$. If $\Gamma(h_i) > w_i$, equilibrium contributions are $g_i^* = w_i$, $g_j^* = \min(w_j, \Gamma(h_j) - w_i)$. The country with the strategic disadvantage in the contribution game is still the country with the higher h. The impact of information acquisition, however, is reduced since there

$h_1 < h_2$ and $\Delta^{\pi_1=1}_{2;h_1<h_2} < 0$:		$h_1 > h_2$ and $\Delta^{\pi_2=1}_{1;h_1>h_2} < 0$:
$\Delta^{\pi_2=0}_{1;h_1<h_2} > 0$	$(\pi_1^* = 1\ ,\ \pi_2^* = 0)$	$(\pi_1^* = 0\ ,\ \pi_2^* = 1)$
$\Delta^{\pi_2=0}_{1;h_1<h_2} < 0$	*mixed*	/

Table 1: Equilibrium decisions on information.

Welfare considerations. Taking as given that the countries choose their abatement effort non-cooperatively, strategic decisions to remain uninformed may lead to additional inefficiencies: global welfare may be lower than if both countries had chosen to invest in information.[17] Due to the quasilinearity of the payoff functions, the analysis can concentrate on the aggregate surplus,

$$S(\alpha_1, \alpha_2, G) = \sum_{i=1,2} \alpha_i \varphi(G) - G\ . \tag{5.10}$$

Given the countries' true valuations, the Pareto efficient outcome is equal to $G^0(\alpha_1, \alpha_2) = \Gamma(\alpha_1 + \alpha_2)$. Note that, by assumption, costs of information are zero and do not affect welfare.

We distinguish two notions of efficiency: *ex post efficiency*, i.e. welfare depending on the true valuations of the countries, and *ex ante efficiency*, i.e. before the countries learn their valuations and depending on underlying probability distributions. Analyzing ex ante efficiency is particularly relevant for the problem of global warming since it concerns the question of whether a social planner, or supranational institution, that does not know the countries' true valuations prefers that better information is provided.

A priori, it is not clear whether information acquisition is welfare enhancing. To illustrate the impact of information on welfare, let us first consider *ex post efficiency*. As a benchmark, we compare the two cases where either no country acquires information or both countries acquire information.

is no complete free-riding and potential adjustment gains from an increased contribution are restricted. If $\Gamma(l_i) \geq w_i$, i's information decision becomes irrelevant.

[17] If a social planner could prescribe the contributions of the countries, he would always prefer to uncover the true valuations.

Observation 5.3 *With information acquisition, aggregate surplus ex post*

(i) *always increases if at least one country uncovers a high value;*

(ii) *decreases if and only if both countries uncover a low value and*

$$(l_1 + l_2)\,\varphi\left(\Gamma_{\max\{l_1,l_2\}}\right) - \Gamma_{\max\{l_1,l_2\}} < (l_1 + l_2)\,\varphi\left(\Gamma_{m_2}\right) - \Gamma_{m_2}. \quad (5.11)$$

Observation 5.3 identifies two potential welfare effects of information acquisition. On the one hand, additional information improves the efficiency of the countries' contributions. Uncovering a high value is always welfare enhancing since $G^0\left(h_i, \alpha_j\right) \geq \Gamma_{h_i} > \Gamma_{m_2}$, i.e. the equilibrium abatement is closer to the ex post efficient abatement of CO_2 emissions independent of the true valuation of the other country (Observation 5.3(i)). On the other hand, an uninformed country's abatement effort could be too high from its individual point of view. This overcontribution effect can improve ex post efficiency if both countries' true valuations are low (Observation 5.3(ii)). A sufficient condition for (5.11) to be fulfilled is $l_1 + l_2 > m_2$.

Thus, from an *ex ante* point of view, if p_1 and p_2 are sufficiently small, preventing countries from becoming informed could be welfare improving: the chance that the uncertainty about the valuations alleviates the underprovision overcompensates the welfare gain which results from uncovering a high benefit of the mitigation of global warming. However, we can formulate (sufficient) conditions such that a strategic decision to remain uninformed always has a negative impact on ex ante efficiency, and therefore, a social planner would like to induce information acquisition. Under these conditions, the positive effect of information always dominates a potential decrease in welfare as in Observation 5.3(ii), and the resulting overall effect is independent of the probabilities attached to the potential outcomes.

Proposition 5.2 *A strategic choice to remain uninformed negatively affects ex ante welfare if one of the following conditions holds:*

(C1) $\Gamma(A)$ *is convex in A;*

(C2) $\min\{l_1, l_2\} = 0$.

If the function determining a country's stand-alone quantity is convex in the valuation, the gain from the adjustment of the individual contributions is strong enough to outweigh a potential welfare gain from an overcontribution at the individual level, regardless of the probability of the two events. Therefore, a choice to remain uninformed decreases the efficiency of the mitigation outcome.[18] If $(C2)$ holds, the mitigation outcome is Pareto efficient in case both countries uncover a low value: an overcontribution reduces welfare. Thus, under $(C1)$ or $(C2)$, ex ante welfare would be highest if both countries uncovered their benefit of climate protection. The provision of information about the valuations by a third party would be welfare enhancing.

Although $(C1)$ or $(C2)$ may be reasonable assumptions in many cases, there can be situations where both conditions are violated. Consider for example $\varphi(G) = 1 - \exp(-G)$. We get $\Gamma(A_i) = -\ln(1/A_i)$ if $A_i \geq 1$, and hence, $(C1)$ is not fulfilled. Dependent on the fundamentals of the model, both countries prefer to acquire information, but preventing one country from becoming informed would be welfare enhancing.[19] In the appendix, we provide an example with concrete parameter values that lead to excessive information acquisition. Summing up, we get:

Claim 5.1 *If both (C1) and (C2) are violated, there can be too much information acquisition in equilibrium.*

If $\Gamma(A_i)$ is strictly concave and, in addition, the probabilities for low values are sufficiently large, the uncertainty leads with high probability to

[18]Note that convexity of $\Gamma(A_i)$ implies convexity of the supply G (because of $G = \max\{\Gamma(A_1), \Gamma(A_2)\}$), and therefore, the expected value of G over the possible realizations of the valuations is larger than the supply of the public good based on the expected valuations. $(C1)$ is fulfilled e.g. for $\varphi(G) = G^\gamma$, $0 < \gamma < 1$.

[19]Information acquisition of country 1 always leads to a welfare gain compared to no information acquisition because uncovering a low value does not affect the supply G. Hence, preventing *both* countries from information acquisition can never be optimal.

an overcontribution at the individual level. Thus, it might be the case that a social planner would prefer not to provide the information although it is available at no cost. From an ex ante point of view, the uncertainty will then influence in a positive way the countries' contributions to the mitigation of global warming.

The effect of correlation. The strategic advantage of remaining uninformed crucially relies on the assumption of independence of the countries' cost of global warming. Correlation of the countries' valuations can be a natural assumption for other environmental public goods such as the depletion of the ozone layer.

Assuming correlated values, an uninformed country i will update its beliefs about its own benefit in case country j acquires information. If the correlation is only weak and j uncovers e.g. a high value, i's expectation about its own benefit $(E(\alpha_i|\alpha_j = h_j))$ increases, but it may still be lower than j's valuation. In this case, the equilibrium contributions don't change, but i's value of information is affected since it becomes more likely that i would uncover a high benefit, too. If instead the valuations are highly correlated, uncovering the own value provides sufficient information about the other country's valuation.

Proposition 5.3 *If the countries' benefits are perfectly correlated, both countries' valuations are uncovered in equilibrium.*

Even if there can be an incentive for the country with the lower expected value not to invest in information, the country with the higher expected value prefers to invest in information and uncovers both countries' valuations. In this sense, inefficiencies resulting from a strategic use of information are not present in other applications such as ozone layer protection.

5.5 Extensions

Irreversibility and investments in information. Emissions of CO_2 accumulate in the atmosphere and may cause irreversible damages. If additional information is only available in the future, the irreversibility of CO_2 emissions affects the optimal timing of the abatement effort. In the following, we illustrate the interaction of decisions on information with the question of timing caused by the irreversibility.

Assume (as before) that in stage 1 of the game, the countries simultaneously decide whether to acquire information about their α_i. In stage 2, the countries' *decisions* are observed, and the countries choose an abatement effort (*period* 1 *contributions*). The *information*, however, becomes publicly observable to both countries only at the beginning of stage 3 in which the countries can again contribute to the public good (*period* 2 *contributions*). If g_i^t denotes country i's contribution in period t, the countries' payoffs are

$$U_i\left(\left(g_i^t\right)_{i,t=1,2}\right) = w_i - g_i^1 - qg_i^2 + \alpha_i\varphi\left(G\right)$$

where $G = \sum_{t=1,2}\sum_{i=1,2} g_i^t$ and $g_i^t \geq 0$. Moreover, $g_i^1 + qg_i^2 \leq w_i$. The resulting mitigation of global warming depends on the sum of the efforts in both periods. The marginal cost of contributing in period 2 is denoted by q, and we assume that $q \geq 1$, i.e. it is more costly to contribute in period 2.[20] This reflects the irreversibility effect of delaying the reduction of CO_2 emissions: if it turns out that a country has high cost of global warming, it will be more costly to mitigate the damages than it would have been if the country had invested in climate protection measures already today.

Using the fact that $\Gamma\left(A_i/q\right)$ is the *period* 2 *standalone quantity* of a country i with (expected) valuation A_i, given that there were no contributions in period 1, the equilibrium contributions in period 2 are as follows: only

[20] One could imagine that there may be uncertainty not only about α_1 and α_2, but also about q. We take q as deterministic and identical for both countries in order to keep the analysis as simple as possible.

the country i with $A_i > A_j$ contributes, and it increases potential period 1 contributions up to its desired quantity $\Gamma(A_i/q)$.[21] A country may contribute already in period 1 (based on its expectations) in order to save the higher future contribution cost if a high valuation is uncovered. This benefit caused by the irreversibility of emissions has to be weighed against the reduction of the other country's future contribution.

If countries can decide on information acquisition, the timing effect of contributing interacts with the strategic incentive to remain uninformed. If q is sufficiently close to 1 and at least one country invested in information, both countries will always postpone their contributions until the additional information is available since a contribution in period 1 may crowd out potential future contributions of the other country. The countries' equilibrium decisions on information are the same as in the previous section, and the expected value of information can be negative due to the externality the information has on the other country's contribution. On the other hand, as $q \to \infty$, the countries won't contribute in period 2. In this case, the decisions on information acquisition become irrelevant. For intermediate values of q, however, the balancing of the effects of information can result in asymmetric information decisions and different timing of the contributions.

Proposition 5.4 *For intermediate values of q, an equilibrium can exist where at least one country remains uninformed with positive probability and the countries contribute in different periods.*

If a country has an incentive to remain uninformed, it is the country i with $h_i > h_j$. Moreover, if j invests in information and $p_j q < 1$, j has a dominant strategy not to contribute in period 1: this causes expected marginal cost of $p_j q$ in period 2, but it saves marginal contribution cost of 1. If i remains uninformed and its expected marginal cost of contributing in period

[21] Note that $\Gamma(A_i/q)$ is decreasing in q: the more costly it is to contribute in the future, the lower are the countries' period 2 quantities.

2, $(1 - p_j)\, q$, is sufficiently high, it chooses a positive period 1 contribution. Nevertheless, the negative strategic effect of uncovering a high value can cause an incentive not to invest in information given that j invests, and, in equilibrium, either one country remains uninformed with probability 1, or both countries randomize their information choice (as before, this depends on whether $\Delta_j^{\pi_i=0} > 0$); an informed country contributes only in period 2, an uninformed country contributes only in period 1.[22] The strategic effect that is attached to information acquisition in this two-period model may be reflected in arguments in favor of delaying effective climate protection until better information will be available.

The case of many countries. The analysis of the two-country case already identified the two effects of information acquisition that persist in the case of more countries: investing in information allows for an adjustment of the individual contribution, but uncovering a high valuation may lead to a reduction of the other countries' contributions. Clearly, the abatement of CO_2 emissions involves a larger number of countries whose contributions are important for a mitigation of global warming. The analysis of the previous sections, however, shows that only the countries with the largest (potential) valuations may be contributors whereas countries with a low mitigation benefit free-ride. Moreover, for poor countries whose contributions are subject to budget constraints, information acquisition does not play a role since potential contributions are restricted. Hence, the strategic character of investments in information is only present for countries that may bear the burden of provision, and only a subset of countries is involved in the strategic interaction (cf. Footnote 11).

With an increasing number of important contributors, the positive effect of additional information through the improved contribution is still existent, but the strategic effect of information tends to be weakened. This can be

[22] As argued above, if q is high, there are no period 2 contributions, and it exists an equilibrium where no country invests in information.

illustrated if we replicate our economy n times and obtain two regions R_1 and R_2, each with n identical countries. Note that this implies that, if a country i_r of region R_r invests in information, all the other countries of this region can infer their cost of climate change. We assume that identical countries contribute an identical amount to the public good.

As before, if no country of region 1 invests in information, countries of region R_2 always have an incentive to learn their valuation. If no country of region R_2 invests and a country r_1 of region R_1 uncovers a high value, the contribution cost is shared among the countries in the same region, and the negative strategic effect of the information is weakened. Similar as in (5.6), the value of information of a country r_1 is

$$p_1 \left[h_1 \varphi \left(\Gamma_{h_1} \right) - \tfrac{\Gamma_{h_1}}{n} - h_1 \varphi \left(\Gamma_{m_2} \right) \right]$$

which is increasing in n and positive if n is sufficiently large. The same holds for the value of information given that a country of the other region acquires information (where in (5.8) and (5.9) all potential contributions $\Gamma(\alpha_i)$ have to replaced by $\Gamma(\alpha_{r_i})/n$). Here, not only the negative strategic effect is reduced, but also a positive strategic effect from shifting the burden of contribution to the other country.

5.6 Conclusion

Uncertainty and information are important determinants for the country-specific efforts to reduce the emissions of carbon dioxide. In this chapter, we have concentrated on the acquisition of information about the country-specific benefit of a mitigation of global warming. Based on a standard model for private provision of a public good, we showed that the choice of information prior to the interaction has a substantial impact on the equilibrium abatement efforts. We identified conditions under which countries prefer to remain uninformed of their benefit even if they do not have to pay for the information. A crucial assumption underlying this strategic incentive is the observability of the outcome of the information acquisition. It maps the nature of investments in information in the case of global warming where additional information is obtained via scientific reports estimating the country-specific cost of climate change.

In order to facilitate the exposition, we restricted our analysis to two-point probability distributions of the country-specific benefits. The two effects of information acquisition identified in this case carry over to a general distribution functions: Additional information leads to an increase in the individual payoff because the own contribution to climate protection can be adjusted. However, it bears a strategic risk since it affects the contributions of the other countries. The latter effect can be negative and, from an ex ante point of view, it can outweigh a potential adjustment gain.

We determined two sufficient conditions under which the resulting strategic information choice has a negative impact on global welfare when, in equilibrium, a country decides not to acquire information. Therefore, the provision of information on a supranational level can increase the efficiency of the mitigation outcome. This result may justify the efforts made by supranational institutions with regard to climate research. But given that these two conditions are violated, welfare could be higher if one country remained uninformed. Too high contributions from the individual point of view that are caused by uncertainty can alleviate the underprovision problem.

An argument in discussions on climate change is that large investments in the reduction of CO_2 emissions should be delayed until better information is available. The optimal timing of these investments, however, is determined by the fact that emissions of CO_2 are irreversible. We incorporated the idea of irreversible investments and learning in our model by assuming that the outcome of the information acquisition is only observable in the future, but that future contributions in case of a high benefit of climate protection are more costly due to irreversible damages that CO_2 emissions might have caused.

The strategic interaction of the timing of the contributions and the decisions on information reveals a further strategic effect of information: investments in information may be a rationale for delaying the own contributions and may in turn induce other countries to contribute already today.

5.A Appendix

5.A.1 Proof of Lemma 5.1

Part (*ii*) follows directly from Observation 5.1. Part (*i*) is true since (5.7) is positive for all (m_1, p_2, l_2, h_2) satisfying (5.5). This follows from monotonicity of φ and an optimality argument: if A_2 denotes the (expected) valuation of country 2, by definition of Γ,

$$A_2\varphi\left(\Gamma\left(A_2\right)\right) - \Gamma\left(A_2\right) > A_2\varphi\left(\Gamma\left(k\right)\right) - \Gamma\left(k\right) \quad \text{for all } k \neq A_2. \tag{5.12}$$

Hence, the second term in (5.7) is positive. The first term is larger than $[(l_2\varphi\left(\Gamma_{m_1}\right) - \Gamma_{m_1}) - (l_2\varphi\left(\Gamma_{m_2}\right) - \Gamma_{m_2})]$ which is positive since $l_2 < m_1 < m_2$ and $l_2\varphi\left(G\right) - G$ is strictly decreasing in G for all $G > \Gamma_{l_2}$.

5.A.2 Proof of Proposition 5.1

Suppose that $\Delta^{\pi_j=1}_{i;h_i>h_j} > 0$. Since $\Delta^{\pi_i=1}_{j;h_i>h_j} > 0$, there is an equilibrium where both countries acquire information. With Lemma 5.1, information acquisition is a strictly dominant strategy for country 2. Thus, the equilibrium is unique.

Now suppose instead that $\Delta^{\pi_j=1}_{i;h_i>h_j} < 0$. There exists an equilibrium where i remains uninformed and j acquires information iff $\Delta^{\pi_i=0}_{j;h_i>h_j}$ is positive. (If $j = 2$, $\Delta^{\pi_i=0}_{j;h_i>h_j} > 0$ is always fulfilled.) Due to $\Delta^{\pi_i=1}_{j;h_i>h_j} > 0$, acquiring information is strictly dominant for country j which shows uniqueness.

In the remaining case of $\Delta^{\pi_j=1}_{i;h_i>h_j} < 0$ and $\Delta^{\pi_i=0}_{j;h_i>h_j} < 0$, the latter condition implies $j = 1$. There is no equilibrium in pure strategies: country 1 always prefers to choose the same action as country 2, whereas country 2 uncovers its value if and only if 1 does not learn. Thus, consider equilibria in mixed strategies. Country i randomizes if and only if $\left(1-\pi_j^*\right)\Delta_i^{\pi_j=0}+\pi_j^*\Delta_i^{\pi_j=1}=0$. This yields

$$\pi_1^* = \frac{\Delta_2^{\pi_1=0}}{\Delta_2^{\pi_1=0}-\Delta_{2;h_1<h_2}^{\pi_1=1}} \in (0,1)\,,\quad \pi_2^* = \frac{-\Delta_1^{\pi_2=0}}{-\Delta_1^{\pi_2=0}+\Delta_{1;h_1<h_2}^{\pi_2=1}} \in (0,1)$$

in the unique equilibrium.

5.A.3 Proof of Proposition 5.2

Whenever $\Delta_{i;h_i>h_j}^{\pi_j=1} < 0$, either i remains uninformed or the equilibrium is in mixed strategies. Consider the pure strategy equilibrium and examine

$$E\left[S\left(\alpha_i,\alpha_j,G\right)|\pi_i=1,\pi_j=1\right]-E\left[S\left(\alpha_i,\alpha_j,G\right)|\pi_i=0,\pi_j=1\right]$$

which is equal to

$$\begin{aligned}&\left(1-p_j\right)\left[\left(1-p_i\right)S\left(l_i,l_j,\Gamma_{\max\{l_i,l_j\}}\right)+p_iS\left(h_i,l_j,\Gamma_{h_i}\right)-S\left(m_i,l_j,\Gamma_{m_i}\right)\right]\\&+p_ip_j\left[S\left(h_i,h_j,\Gamma_{\max\{h_i,h_j\}}\right)-S\left(h_i,h_j,\Gamma_{h_j}\right)\right]\end{aligned} \tag{5.13}$$

The second term in (5.13) is non-negative. (It is positive in the pure strategy equilibrium due to $h_i > h_j$.) The first term is positive for all p_i iff $S\left(A_i,l_j,\Gamma_{\max\{A_i,l_j\}}\right)$ is convex in A_i. For $A_i > l_j$, we have

$$\begin{aligned}\frac{\partial S(A_i,l_j,\Gamma(\max\{A_i,l_j\}))}{\partial A_i} &= \varphi\left(\Gamma\left(A_i\right)\right)+\frac{l_j}{A_i}\Gamma'\left(A_i\right)\ ,\\ \frac{\partial^2 S(A_i,l_j,\Gamma(\max\{A_i,l_j\}))}{\partial A_i^2} &= \frac{\Gamma'(A_i)}{A_i}\left(1-\frac{l_j}{A_i}\right)+\frac{l_j}{A_i}\Gamma''\left(A_i\right)\ .\end{aligned}$$

For $A_i < l_j$, $\partial S\left(A_i,l_j,\Gamma_{\max\{A_i,l_j\}}\right)/\partial A_i$ is constant and smaller than the slope of S for $A_i > l_j$. Thus, $(C1)$ is a sufficient condition for convexity of $S\left(A_i,l_j,\Gamma_{\max\{A_i,l_j\}}\right)$ and hence for $\Delta E\left[S\right]$ being strictly positive if in equilibrium only i remained uninformed.

In the mixed strategy equilibrium, the number of informed countries depends on the outcome of the randomization. Welfare without information acquisition is lower than if only country 1 uncovered its value: learning a low value has no effect on total contributions, but uncovering a high value is welfare enhancing. Moreover, as above, if exactly one country remained uninformed, welfare is lower than under complete information if $(C1)$ holds. Thus ex ante welfare is lower in the mixed strategy equilibrium than under complete information.

$(C2)$ follows directly from Observation 5.3(i) and the fact that (5.11) is violated if $\min\{l_1, l_2\} = 0$.

5.A.4 Proof of Claim 5.1

Consider as example $\varphi(G) = 1 - \exp(-G)$. Solving the FOC yields

$$\Gamma(A_i) = \begin{cases} 0 & \text{if} \quad A_i < 1 \\ -\ln\left(\frac{1}{A_i}\right) & \text{if} \quad A_i \geq 1 \end{cases},$$

and thus $(C1)$ is violated since $\Gamma''(A_i) = -1/A_i^2 < 0$. If for instance

$$l_1 = 3,\ l_2 = 2.8,\ h_1 = 8,\ h_2 = 10,\ p_1 = p_2 = 0.2,$$

we get $\Delta_{2;h_2>h_1}^{\pi_1=1} \approx 0.75$, and both countries acquire information in stage 1. Computation of the expected surplus dependent on π_1 and π_2 yields

$$\begin{aligned} E[S|\pi_1 = \pi_2 = 1] &\approx 5.01, \quad E[S|\pi_1 = 1, \pi_2 = 0] \approx 5.00, \\ E[S|\pi_1 = 0, \pi_2 = 1] &\approx 5.03, \quad E[S|\pi_1 = \pi_2 = 0] \approx 4.85. \end{aligned}$$

A social planner would choose $\pi_1 = 0$ and $\pi_2 = 1$.

5.A.5 Proof of Proposition 5.3

If a country j invests in information, it uncovers both countries' valuations and i's information decision becomes irrelevant. Suppose that $l_1 < l_2$ and $h_1 < h_2$. In this case, country 2's value of information is smallest since with information, it always contributes. Because of

$$\begin{aligned}\Delta_2^{\pi_1=0} &= (1-p_2)\left[\left(l_2\varphi\left(\Gamma_{l_2}\right)-\Gamma_{l_2}\right)-\left(l_2\varphi\left(\Gamma_{m_2}\right)-\Gamma_{m_2}\right)\right] \\ &\quad +p_2\left[\left(h_2\varphi\left(\Gamma_{h_2}\right)-\Gamma_{h_2}\right)-\left(h_2\varphi\left(\Gamma_{m_2}\right)-\Gamma_{m_2}\right)\right]>0,\end{aligned}$$

country 2 always invests in information if country 1 does not invest. If $\Delta_1^{\pi_2=0} < 0$ (which may be the case if $l_1 > l_2$ and $h_1 > h_2$), country 2 acquires the information; otherwise either of the countries invests.

5.A.6 Proof of Proposition 5.4

We show that $\Delta_{i;h_i>h_j}^{\pi_j=1}$ can be negative, which is a sufficient condition for non-existence of an equilibrium where both countries acquire information with probability 1. Note first that, if A_i denotes a country i's (expected) valuation in period 2 (dependent on whether or not i acquired information), period 2 contributions for $A_i > A_j$ are $g_i^2 = \max\left(\Gamma\left(A_i/q\right) - g_i^1 - g_j^1, 0\right)$ and $g_j^2 = 0$.

Suppose for simplicity that $l_1 = l_2 = 0$. The most interesting case is

$$q < \frac{1}{p_1} \text{ and } q < \frac{1}{p_2}. \tag{5.14}$$

(5.14) implies that a country that invested in information has a dominant strategy not to contribute in period 1. Suppose e.g. that i remains uninformed and j acquires information. Then,

$$\frac{\partial U_j\left(g_j^1\right)}{\partial g_j^1} = \begin{cases} -1 + p_j q & \text{if } 0 < g_j^1 \leq \max\left\{\Gamma_{h_j/q} - g_i^1, 0\right\} \\ -1 + p_j h_j \varphi'\left(g_i^1 + g_j^1\right) & \text{if } g_j^1 > \max\left\{\Gamma_{h_j/q} - g_i^1, 0\right\} \end{cases}$$

which, with (5.14), is negative for all $g_i^1 > 0$. (For $g_j^1 > \Gamma(h_j/q) - g_i^1$, this follows from $\partial U_j\left(g_j^1\right)/\partial g_j^1 < -1 + p_j h_j\left(q/h_j\right) < 0$.) An equivalent argument shows that contributing in period 1 is dominated for j in case both countries acquire information. Assuming $h_i > h_j$, i's expected payoff in the latter case is $-p_i q \Gamma(h_i/q) + p_i h_i \varphi(\Gamma(h_i/q))$.

Now suppose that country i remains uninformed and j acquires information. With $g_j^1 = 0$ we get

$$\frac{\partial U_i\left(g_i^1\right)}{\partial g_i^1} = \begin{cases} -1 + (1-p_j)\,q & \text{if } 0 < g_i^1 \leq \Gamma(m_i/q) \\ -1 + (1-p_j)\,m_i \varphi'\left(g_i^1\right) & \text{if } \Gamma(m_i/q) < g_i^1 < \Gamma(h_j/q) \\ -1 + m_i \varphi'\left(g_i^1\right) & \text{if } g_i^1 > \Gamma(h_j/q) \end{cases}.$$

Whenever $q > 1/(1-p_j)$, i contributes in period 1, and dependent on the parameter values, its optimal period 1 contribution is either $\Gamma((1-p_j)\,m_i)$ or $\Gamma(m_i)$. If instead $q < 1/(1-p_j)$, equilibrium candidates are $g_i^1 = 0$ and $g_i^1 = \Gamma(m_i)$. In both cases, (5.14) implies that $\Gamma(h_j/q) > g_i^1$, and j contributes in period 2 iff $\alpha_j = h_j$.

If $q > 1/(1-p_j)$, i's value of information

$$\begin{aligned} \Delta_{i;h_i>h_j}^{\pi_j=1} \leq\; & -p_i q \Gamma_{h_i/q} + p_i h_i \varphi\left(\Gamma_{h_i/q}\right) \\ & - \left[-\Gamma_{(1-p_j)m_i} + (1-p_j)\,m_i \varphi\left(\Gamma_{(1-p_j)m_i}\right) + p_j m_i \varphi\left(\Gamma_{h_j/q}\right)\right]. \end{aligned}$$

The RHS is equal to

$$\begin{aligned} & -\left[(1-p_j)\,m_i \varphi\left(\Gamma_{(1-p_j)m_i}\right) - \Gamma_{(1-p_j)m_i}\right] + \left[(1-p_j)\,m_i \varphi\left(\Gamma_{h_i/q}\right) - \Gamma_{h_i/q}\right] \\ & + (1-p_i q)\,\Gamma_{h_i/q} + p_j m_i \left(\varphi\left(\Gamma_{h_i/q}\right) - \varphi\left(\Gamma_{h_j/q}\right)\right). \end{aligned} \tag{5.15}$$

Since (by the same argument as in (5.12)) the first line in (5.15) is negative, $\Delta_{i;h_i>h_j}^{\pi_j=1}$ is negative if h_j is sufficiently close to h_i and q is sufficiently close to $1/p_i$, i.e. if the negative effect from uncovering a high value is strong. If $q < 1/(1-p_j)$, similar transformations show that, again if h_j is sufficiently close to h_i and q is sufficiently close to $1/p_i$, $\Delta_{i;h_i>h_j}^{\pi_j=1} < 0$.

Chapter 6

Volunteering and the value of ignorance

6.1 Introduction

Dragon-slaying and ballroom dancing are two famous examples[1] for the provision of a public good that induces a positive value for a certain group of individuals. One of the individuals, however, has to pay some cost in order to provide the public good. Such situations are often best described by a waiting game or war of attrition: one volunteer is needed for a certain task, and everyone prefers that someone else volunteers first and bears the cost of provision. Typically, there is a disutility or waiting cost attached to the time until a volunteer is found. In this chapter, we study the individuals' incentives to obtain information about their own cost of provision of the public good prior to a volunteering game.

Wars of attrition are used to model a large number of applications from different fields. Besides dragon-slaying, many unpleasant situations like intervening in a fight, calling the police in case of a fire or crime, household chores, fights between animals, or market exit exhibit properties of wars of

[1] Cf. Bliss and Nalebuff (1984).

attrition.[2] Organizations typically rely on the voluntary performance of a large number of tasks. These tasks may have to be performed repeatedly, and the cost of performing the task may then be well-known. But often the individuals don't know exactly how costly volunteering will turn out to be. They may, for instance, only have a guess about the time involved in chairing a university department or organizing a conference, but have the possibility of finding out about their cost of performing this task by raising questions or collecting information. If such information acquisition is observable by the other individuals, there is a strategic value attached to the information.

We analyze the individuals' incentives to acquire information about their cost of provision of the public good in a two-stage game with two individuals. In the first stage, the individuals can obtain information about their cost of provision. In order to focus on the strategic considerations, we assume that the information is available at zero cost. Whether or not an individual decided to find out about his cost can be observed by the rival before the volunteering game starts. The information, however, that an individual obtained is only privately known to this individual. In the second stage, a volunteering game or war of attrition takes place: the individuals simultaneously choose a maximum waiting time after which they provide the public good, given that nobody else volunteered before. As individuals may not be able to wait for an infinite amount of time, we impose a finite time horizon after which one of the individuals is randomly chosen to pay for the provision. At some point in time, the dragon may decide to attack itself, or, in the context of a firm, one employee will be selected by the team leader to perform the task.

As we will show, the equilibrium of the volunteering game and the incentives to learn the own cost of provision crucially depend on the length of the time horizon. For a long time horizon, both individuals prefer to find out

[2] Many more examples are given, e.g., by Bilodeau and Slivinski (1996), LaCasse et al. (2002), or Otsubo and Rapoport (2008).

about their cost of provision. If the time horizon of the volunteering game is sufficiently short, individuals without information about their provision cost prefer a random selection to an early concession. As a consequence, an individual that found out about low cost may prefer to concede immediately. Therefore, not knowing the own cost of provision can be advantageous in the volunteering game. For a sufficiently short time horizon, there are two asymmetric equilibria where one individual finds out about his cost and the other does not, and one symmetric equilibrium where both individuals randomize their decision whether to learn their cost. The choice of the time horizon is an important instrument in influencing the efficiency of the volunteering game.

The literature on wars of attrition has its origin in applications in biology, modeling fights between animals (e.g., Maynard Smith 1974, Riley 1980). Further important applications are industrial competition and market exit (Fudenberg and Tirole 1986, Ghemawat and Nalebuff 1985, 1990). The seminal paper that studies the private provision of a public good as a war of attrition is Bliss and Nalebuff (1984). In their setup, the players are privately informed of their cost of provision, and the equilibrium is efficient in the sense that the player with the lowest cost provides the public good. The provision of multiple public goods in the framework of a war of attrition is analyzed by LaCasse et al. (2002) for the case of complete information, and by Sahuguet (2006) in an environment with private information.[3] Bishop and Cannings (1978), Bilodeau and Slivinski (1996), and Myatt (2005) study models that exhibit a finite time horizon. We add to this literature by studying the effects of information on the individuals' conces-

[3] Further papers considering wars of attrition with privately informed players are Bulow and Klemperer (1999) who analyze the case of multiple prizes and Krishna and Morgan (1997) who study the case of affiliated signals. Amann and Leininger (1996) consider a general class of all-pay auctions with private information; the same class of all-pay auctions is analyzed in Riley (1999) for the case of complete information. Che and Gale (1998) study first-price all-pay auctions with caps on bidding which are similar to the finite time horizon of the volunteering game assumed here.

sion times in the private provision game, and the resulting incentives (not) to become informed. The strategic considerations involved in the decision on information are similar to the strategic aspects identified in Chapter 5 or in different settings such as principal-agent relationships (e.g. Crémer 1995, Kessler 1998): by remaining uninformed, individuals precommit to a certain behavior in the subsequent interaction.

The next section describes the setup of the model. We analyze in Section 6.3 the three different situations that may arise in stage 2 of the game: no individual has private information about his provision cost, only one individual is informed, or both individuals are informed about their cost of provision. In Section 6.4, we consider the incentives for information acquisition and discuss some implications from a designer's perspective. Finally, Section 6.5 concludes. All proofs are in the appendix.

6.2 Setup

Consider the following game with two individuals 1 and 2. One of the two individuals has to provide a public good of fixed quantity. (We assume that the contribution that is needed for the provision is indivisible.) The individuals differ with respect to their cost of provision, denoted by c_1 and c_2. These cost parameters c_1 and c_2 are independent draws from a probability distribution that is common knowledge and assumed to be a discrete function with

$$c_i \in \{c_L, c_H\}, \quad 0 < c_L < c_H,$$

and probabilities

$$\Pr(c_i = c_L) = p_L, \quad \Pr(c_i = c_H) = p_H = 1 - p_L, \quad i = 1, 2.$$

Moreover,

$$\bar{c} := p_L c_L + p_H c_H$$

is an individual's expected cost of provision. At the beginning of the game, the individuals know neither their cost of provision nor their rival's cost, but only that this cost can be high or low, and the corresponding probabilities.[4]

In stage 1 of the game, the individuals can find out about their own provision cost: if an individual decides to become informed, he privately observes his provision cost. Information acquisition does not involve any direct cost, and the decisions whether or not to obtain information are made simultaneously and become commonly known at the end of stage 1.

In stage 2, the individuals $i = 1, 2$ simultaneously choose a *time of concession* t_i, i.e., individual i plans to provide the public good in t_i given that individual $j \neq i$ has not volunteered before t_i. As soon as one individual volunteers, the game ends. However, there is a maximum waiting time T which is exogenously given and is common knowledge. Thus, the strategy space is restricted to $t_i \in [0, T]$. If both individuals volunteer exactly at the same time, the provision of the public good is allocated with equal probability to the individuals. Waiting involves a direct cost to both individuals, which is assumed to be linear in the waiting time. Stage 2 is strategically equivalent to the war of attrition or second-price all-pay auction with a cap on bidding.

Denoting by v an individual's utility from the provision of the public good, the payoff functions are given by

$$\pi_i(t_i, t_j) = \begin{cases} v - t_j, & t_i > t_j \\ v - \frac{c_i}{2} - t_i, & t_i = t_j \\ v - c_i - t_i, & t_i < t_j \end{cases}, \ i = 1, 2. \tag{6.1}$$

For all possible t_1 and t_2, the public good is provided, and its value v to the individuals is assumed to be same for both individuals and independent of

[4]The assumption of a discrete distribution crucially determines the structure of the equilibrium strategies in the war of attrition if at least one individual learned his cost. The result on incentives to become informed qualitatively carries over to the case where the individuals' cost is drawn from a continuous distribution. See the discussion in Section 6.5.

the provision time. The idiosyncrasies are captured by the provision cost. The individual that chooses the lower waiting time has to bear the provision cost, and both individuals have to pay the cost of waiting, determined by the minimum of t_1 and t_2. If both individuals decide not to concede before T, that is $t_1 = t_2 = T$, one of them is randomly selected to provide the public good, and their expected payoff in this case is equal to $v - c_i/2 - T$.

6.3 The volunteering game

In the continuation games starting in stage 2, the individuals choose their time of concession t_i, knowing the decisions on information. The time horizon T affects the properties of the equilibrium of the volunteering game for all possible stage 1 decisions. Compared to a provision in $t_i < T$, individuals can reduce their expected cost of provision by waiting until T and then possibly being subject to a random selection. This trade-off between lower expected provision cost and higher cost of waiting generates a time interval before T in which, in equilibrium, there is zero probability that an individual volunteers.

Lemma 6.1 *In any equilibrium, there is zero probability that individual i with cost c_i provides the public good in $(-c_i/2 + T, T)$.*

For large T, it will always be an equilibrium of the volunteering game that individual j volunteers immediately. In this case, the equilibrium strategy of i is not uniquely determined, and he may well choose to concede in $t_i \in (-c_i/2 + T, T)$, given that in equilibrium he will not provide the public good. Any $t_i \in (-c_i/2 + T, T)$, however, is weakly dominated, and whenever there is positive probability that j waits until T, individual i (with cost c_i) strictly prefers $t_i = T$ to any $t_i \in (-c_i/2 + T, T)$. If $T < c_i/2$, we have $-c_i/2+T < 0$, and i prefers the random selection in T to a contribution in any $t_i \in [0, T)$.

In what follows, we make the assumption of

$$(A) \quad \frac{c_L}{2} < T < \frac{c_H}{2}.$$

As will become clear in the reminder of this section, assumption (A) implies that an individual with high cost will find it optimal to wait until T, accepting the consequence that he might be randomly chosen to fulfill the task. An individual with low cost will prefer an early concession if the rival waits sufficiently long.[5]

Building on this assumption, we first determine the equilibria of the volunteering game conditional on the decisions in stage 1, and we then analyze the incentives to become informed by comparing the *ex ante expected payoffs* in these scenarios, i.e. the individuals' payoffs dependent on the decisions on information, but before they find out about their provision cost.

No individual knows his cost of provision. If neither of the individuals knows their true provision cost, they choose their waiting time based on their expected cost $\bar{c}$, and stage 2 is strategically equivalent to the war of attrition with complete information.[6]

Consider individual i and suppose that j waits until T with probability one. If i concedes in $t_i < T$, his expected payoff is $v - \bar{c} - t_i$. For $t_i = T$, he gets a payoff of $v - \bar{c}/2 - T$. Thus, if $T < \bar{c}/2$, $t_i = T$ is strictly preferred to any $t_i < T$, and there is an equilibrium where $t_1^* = t_2^* = T$. If, however, $T > \bar{c}/2$, i's best response is to concede immediately, and there are two equilibria each with one individual choosing $t = 0$. In the latter case, there are also equilibria in mixed strategies.[7] We focus on the symmetric equilibrium.

[5] This assumption ensures the strategic role of the information acquisition because the equilibrium of the volunteering game will crucially depend on the individuals' decisions whether or not to find out about their cost of provision. If $T > c_H/2$, there is always an equilibrium of the continuation game where one individual concedes immediately, independently of the decisions in stage 1 and the individuals' true provision cost. If $T < c_L/2$, in the unique equilibrium of the volunteering game, both individuals wait until T independently of the stage 1 decisions and their true cost.

[6] This holds because individuals are assumed to be risk-neutral and the payoffs are linear in the provision cost. Thus maximizing expected payoffs is equivalent to the maximization based on the expected cost.

[7] For a detailed analysis see Hendricks et al. (1988).

Proposition 6.1 *(i) If $T < \bar{c}/2$, in the unique equilibrium, $t_1^* = t_2^* = T$, and*

$$E(\pi_1) = E(\pi_2) = v - \bar{c}/2 - T.$$

(ii) If $T > \bar{c}/2$, in the unique symmetric equilibrium, individual $i \in \{1,2\}$ randomizes his concession time according to the distribution function

$$F_i(t) = \begin{cases} 1 - \exp\left(-\frac{t}{\bar{c}}\right), & 0 \leq t < -\frac{\bar{c}}{2} + T \\ 1 - \exp\left(\frac{1}{2} - \frac{T}{\bar{c}}\right), & -\frac{\bar{c}}{2} + T \leq t < T \\ 1, & t \geq T \end{cases}, \tag{6.2}$$

and expected payoffs are

$$E(\pi_1) = E(\pi_2) = v - \bar{c}. \tag{6.3}$$

For any $t_j \in (0, -\bar{c}/2 + T)$, the marginal cost of waiting are one, multiplied by the probability $(1 - F_i(t_j))$ that this waiting cost has to be paid. The marginal gain of waiting slightly longer is equal to the $\bar{c}F_i'(t_j)$, i.e. the expected provision cost multiplied by the additional probability that this cost can be saved. Individual j is indifferent to all $t_j \in (0, -\bar{c}/2 + T)$ if cost and benefit of increasing t_j are equal. This leads to (6.2). The only difference to the standard war of attrition with complete information is that, due to the time limit, no individual concedes in $(-\bar{c}/2 + T, T)$, but instead both choose a concession in T with strictly positive probability.

There are other asymmetric mixed strategy equilibria where one of the individuals places a mass point at $t = 0$, i.e. concedes immediately with strictly positive probability. Obviously, there can't be an equilibrium where both individuals have a mass point at zero because then waiting an infinitesimally small amount of time would, at a negligibly higher expected waiting cost, strictly increase the probability that the rival provides the public good. In all mixed strategy equilibria, as in the equilibrium where one individual concedes immediately with probability one, it holds that an individual that chooses $t = 0$ with positive probability has an expected payoff of $v - \bar{c}$. Thus, in the case of $T > \bar{c}/2$, in all equilibria we have

$$\min\{E(\pi_1), E(\pi_2)\} = v - \bar{c}.$$

Only one individual knows his cost of provision. Suppose that only individual j has become informed about his provision cost whereas i remained uninformed. Denote by j_L (j_H) an individual j who has found out about low (high) cost of provision. j's strategy is now contingent on his type (c_L or c_H), and i's optimal strategy is to choose his concession time as if his cost was $\bar{c}$. Recall that we still assume that (A) holds.

Proposition 6.2 *Suppose that individual i remained uninformed and individual j has become informed. If $T < \bar{c}/2$, in the unique equilibrium,*

$$t_i^* = T \text{ and } t_j^*(c_j) = \begin{cases} 0, & c_j = c_L \\ T, & c_j = c_H \end{cases}.$$

Ex ante expected payoffs are

$$E(\pi_i) = v - p_H\left(\frac{\bar{c}}{2} + T\right), \tag{6.4}$$

$$E(\pi_j) = v - p_L c_L - p_H\left(\frac{c_H}{2} + T\right). \tag{6.5}$$

A proof is omitted since this result follows from Lemma 6.1: both i and j_H prefer a random selection at T to any concession before T, and this makes it optimal for j_L to concede immediately. The expected payoff of the uninformed individual i is therefore increasing in the probability that j has a low contribution cost. Note that $E(\pi_i) > E(\pi_j)$, i.e. the individual that does not know his cost of provision has a higher expected payoff.

If $T > \bar{c}/2$, Lemma 6.1 no longer applies for all $t_i \in [0, T)$, and there is a set of equilibria where i concedes immediately and j chooses a (sufficiently) high waiting time for each of the two possible provision costs he could have been informed of (sufficiently high to make it optimal for i to concede immediately). In such a *pure strategy equilibrium*, expected payoffs are

$$E(\pi_i) = v - \bar{c} \text{ and } E(\pi_j) = v. \tag{6.6}$$

Given that (A) holds, by Lemma 6.1, j_H will never provide the public good with strictly positive probability before T. Thus, there is no further pure strategy equilibrium. To see why, suppose that i concedes in $t' > 0$ with probability one. j_L's best response is either $t_{j_L} = 0$, or $t_{j_L} > t'$, and i strictly prefers a concession in $t'/2$ over a concession in t' since in both cases this doesn't change his probability of contribution, but strictly reduces the waiting cost.

There can, however, be an additional equilibrium which is in mixed strategies. In this equilibrium, the individual without private information randomizes its concession time. Denote by F_i individual i's equilibrium distribution of concession times, and by F_{j_L} and F_{j_H} individual j's distribution of concession times dependent on his contribution cost. Note first that, by (A) and Lemma 6.1, j_H will never provide the public good before T. Thus, in any equilibrium in mixed strategies, only i and j_L contribute before T with strictly positive probability, and the equilibrium strategies exhibit similar properties as in the case of complete information.

Proposition 6.3 *Suppose that individual i remained uninformed and individual j has become informed. If $\bar{c}/2 < T < \bar{c}/2 - \bar{c}\ln p_H$. There is a mixed strategy equilibrium where*

$$F_i(t) = \begin{cases} 1 - \exp\left(-\frac{t}{c_L}\right), & 0 \leq t < -\frac{\bar{c}}{2} + T \\ 1 - \exp\left(\frac{\bar{c}}{2c_L} - \frac{T}{c_L}\right), & -\frac{\bar{c}}{2} + T \leq t < T \\ 1, & t \geq T \end{cases},$$

$$F_{j_L}(t)=\begin{cases}\frac{1}{p_L}\left[1-(1-p_L)\exp\left(-\frac{1}{2}+\frac{T}{\bar{c}}-\frac{t}{\bar{c}}\right)\right], & 0\le t<-\frac{\bar{c}}{2}+T\\ 1, & t\ge-\frac{\bar{c}}{2}+T\end{cases},$$

and j waits until T with probability one if $c_j=c_H$. Ex ante expected payoffs are

$$E(\pi_i) = v-p_H\exp\left(-\frac{1}{2}+\frac{T}{\bar{c}}\right)\bar{c}, \tag{6.7}$$

$$E(\pi_j) = v-c_L-\frac{p_H}{2}\exp\left(\frac{\bar{c}-2T}{2c_L}\right)(c_H+\bar{c}-2c_L). \tag{6.8}$$

Contrary to the case where no individual knows his cost, the mixed strategy equilibrium is uniquely determined by the condition that there is zero probability that any individual concedes in $(-\bar{c}/2+T,T)$ and that therefore j_L concedes before $-\bar{c}/2+T$ with probability one (see Appendix). This requires that F_{j_L} has a mass point at zero, and thus i's payoff in the mixed strategy equilibrium is strictly higher than $v-\bar{c}$.

The equilibrium characterized in Proposition 6.3 has several interesting properties. Whenever p_H and/or T are large, this equilibrium does not exist: as it is likely that j has high cost and the waiting time until T is costly, waiting becomes too costly; thus individual i prefers to volunteer immediately. When $T\to\bar{c}/2-\bar{c}\ln p_H$ (from below), the probability that individual j_L concedes immediately converges to zero, and i's expected payoff converges to $v-\bar{c}$ which is equal to his payoff in the pure strategy equilibrium. On the other hand, when $T\to\bar{c}/2$ (from above), the probability that j_L concedes immediately converges to one, and the probability that i concedes before T converges to zero. The equilibrium strategies in the mixed strategy equilibrium, and the individuals' expected payoffs, converge to the equilibrium for $T<\bar{c}/2$ characterized in Proposition 6.2. Hence, if the mixed strategy equilibrium is selected in case of $T>\bar{c}/2$, the individuals' expected payoffs are continuous in T. The analysis of the individuals' incentives to become informed will have to distinguish which equilibrium is selected in case exactly one individual learned his cost of provision and $T > \bar{c}/2$.

Both individuals know their cost of provision. Suppose that both individuals have decided to acquire information about their provision cost. Let i_H be an individual that has found out he has a high cost of provision. By Lemma 6.1 together with (A), there can't be an equilibrium where i_H provides the public good in $t_{i_H} < T$ with strictly positive probability. If i_H chooses a time of concession $t_{i_H} < T$ with strictly positive probability, then j_H must concede before t_{i_H} with probability one, contradicting Lemma 6.1. Therefore, in any equilibrium, i and j wait until T with probability one if they have high a contribution cost.

It remains to characterize the individuals' equilibrium strategies for low provision cost. As before, denote by i_L an individual i that has a low cost. There can't be an equilibrium where i_L choose a pure strategy. In particular, there can't be an equilibrium where an individual with low cost volunteers immediately. To see why, suppose that i_L chooses $t = 0$ with probability one. j_L's best response is to concede in $t' = \varepsilon$, ε infinitesimally small, knowing that for contribution cost c_H, i will wait until T. But then, i_L is strictly better off by choosing $t'' = 2\varepsilon$.

Hence, individuals randomize their waiting time if they have a low provision cost. By Lemma 6.1, there must be probability zero that an individual volunteers in the interval $(-c_L/2 + T, T)$, and at most one individual can have a mass point at zero. As it is a typical feature of the war of attrition, there may be a continuum of equilibria which differ in the size of the mass point at zero. Since the individuals are symmetric ex ante, we focus on the symmetric equilibrium.

Proposition 6.4 *In the symmetric equilibrium, individual $i \in \{1, 2\}$ waits until T with probability one in case of $c_i = c_H$. For $c_i = c_L$, i randomizes* *according to the distribution function*

$$F_{i_L}(t) = \begin{cases} \frac{1}{p_L}\left[1 - \exp\left(-\frac{t}{c_L}\right)\right], & 0 \leq t < \bar{t} \\ \frac{1}{p_L}\left[1 - \exp\left(-\frac{\bar{t}}{c_L}\right)\right], & \bar{t} \leq t < T \\ 1, & t \geq T \end{cases}$$

where $\bar{t} = \min\left\{-\frac{c_L}{2} + T, -c_L \ln p_H\right\}$. *Ex ante expected payoffs are*

$$E(\pi_i) = \begin{cases} v - c_L - \frac{p_H}{2}(c_H - c_L)\, e^{\frac{1}{2} - \frac{T}{c_L}}, & T < \frac{c_L}{2} - c_L \ln p_H \\ v - c_L - p_H^2\left(T + \frac{c_H}{2} - c_L(1 - \ln p_H)\right), & T \geq \frac{c_L}{2} - c_L \ln p_H \end{cases}. \tag{6.9}$$

If the probability p_H that the other individual has high cost is large, it is more attractive for an individual with low cost to volunteer early. For sufficiently high p_H, he concedes before T with probability one. This holds if $-\frac{c_L}{2} + T \geq -c_L \ln p_H$ or

$$T \geq \frac{c_L}{2} - c_L \ln p_H.$$

Otherwise, i_L puts strictly positive probability on $t_{i_L} = T$. As long as $T < \frac{c_L}{2} - c_L \ln p_H$, the ex ante expected payoffs are increasing in T since the probability that there is a concession before T increases. If $T > \frac{c_L}{2} - c_L \ln p_H$ (and (A) still holds), this result is reversed: an increase in T makes waiting more costly without changing the probability that there is a concession before T because individuals with a low provision cost concede before T with probability one.

6.4 The value of becoming informed

For the analysis of the optimal decision in stage 1, we have to distinguish whether or not $T > \bar{c}/2$. This distinction does not influence the stage 2 equilibrium in case both individuals know their provision cost, but it is crucial for the equilibrium if at least one individual does not know his cost of provision.

Let $\sigma_i \in \{N, I\}$ be an individual i's decision in stage 1 where I refers to information acquisition and N to a decision not to learn his own provision cost. Moreover, denote by $E\left(\pi_i^{(\sigma_i,\sigma_j)}\right)$ individual i's expected payoff in stage 2 conditional on the decisions (σ_i, σ_j). In case (I, I), for instance, both individuals have learned their cost of provision, whereas case (N, I) refers to a situation where exactly one individual has decided to learn his cost. Given σ_j, i's *value of information* can be defined as

$$V_i^{\sigma_j} = E\left(\pi_i^{(I,\sigma_j)}\right) - E\left(\pi_i^{(N,\sigma_j)}\right).$$

Lemma 6.2 *Suppose that* (A) *holds.*
(i) $V_i^{\sigma_j=N}$ *is strictly positive for all* T*.*
(ii) $V_i^{\sigma_j=I}$ *is strictly negative if* T *is sufficiently small and strictly increasing in* T *for* $T \in (c_L/2, \bar{c}/2)$*.*
(iii) Suppose in case (N, I) *the pure strategy equilibrium is selected. Then* $V_i^{\sigma_j=I}$ *is strictly positive for all* $T > \bar{c}/2$*.*
(iv) Suppose in case (N, I) *the mixed strategy equilibrium is selected. Then* $V_i^{\sigma_j=I}$ *is continuous and strictly increasing in* T *for* $T \in (c_L/2, \bar{c}/2 - \bar{c}\ln p_H)$*.*

Given the rival does not learn his cost of provision, learning his own cost always increases one's expected payoff (Lemma 6.2 part (i)). If the rival decides to learn his cost and T is small, this result is reversed. However, as long as $T < \bar{c}/2$, an increasing time limit makes waiting more costly in case the rival has a high provision cost, which increases the value of information (part (ii)). If $T > \bar{c}/2$, the value of information depends on which equilibrium is selected in case (N, I). For the pure strategy equilibrium, i's value of information given that j learns his cost of provision, V_i^I, exhibits a discontinuity at $T = \bar{c}/2$ and is strictly positive for all $T > \bar{c}/2$ (part (iii)). For the mixed strategy equilibrium, however, V_i^I is continuous at $T = \bar{c}/2$. This continuity in T makes the analysis for the selected equilibrium more appealing. Yet the following proposition holds independently of which equilibrium is selected in case only one individual decides to learn his provision cost.

Proposition 6.5 *(Incentives to become informed) Suppose that* (A) *holds. There exists a threshold* $\tilde{T} > c_L/2$ *such that*
(i) if $T < \tilde{T}$*, there are two asymmetric equilibria where exactly one individual acquires information and one symmetric equilibrium where both individuals randomize their information decision;*
(ii) if $T > \tilde{T}$*, it is strictly dominant to acquire information.*

If both individuals remained uninformed, this would cause a high inefficiency in the volunteering game. Therefore, it is beneficial for at least one individual to find out about his provision cost even if this leads to a higher ex ante probability of being the one who concedes first. As a consequence, there is never an equilibrium where both individuals decide not to learn their cost of provision. If, however, T is sufficiently small and only individual j acquires information, then j concedes immediately with high probability, and i's payoff is higher if he does not know his cost of provision. Being uninformed constitutes a strategic advantage in the volunteering game, being a commitment not to volunteer too early. This, in turn, induces the rival to concede immediately, which outweighs i's waiting cost in case j has high provision cost. For higher T, this waiting cost increases, and, in case of the mixed strategy equilibrium in (N, I), the probability that j concedes immediately decreases. There exists a threshold $\tilde{T}$ such that, for $T > \tilde{T}$, i is better off if he finds out about his provision cost as well. If the value of information V_i^I is negative for all $(c_L/2, \bar{c}/2)$, the location of the threshold $\tilde{T}$ is determined by which equilibrium is selected in case (N, I).

Corollary 6.1 *(i) If in case* (N, I) *the pure strategy equilibrium is selected,* $\tilde{T} \leq \bar{c}/2$*. (ii) If the mixed strategy equilibrium is selected and* p_H *is small,* $\tilde{T}$ *is strictly larger than* $\bar{c}/2$*. In this case, there may be no equilibrium where* *both individuals acquire information with probability one for all T fulfilling (A).*

If $T > \bar{c}/2$ and, in case (N, I), the pure strategy equilibrium is selected, learning the own provision cost is strictly dominant, and thus the threshold $\tilde{T} \leq \bar{c}/2$. However, if we focus on the mixed strategy equilibrium, $\tilde{T} > \bar{c}/2$ for small p_H, and the value of information V_i^I can even be negative for all $T \in (c_L/2, c_H/2)$. Thus, the strategic value of remaining uninformed is not only present in the case where an uninformed individual i has a dominant strategy not to concede before T (as in Proposition 6.2), but also if the individuals randomize their concession time (as in Proposition 6.3). The sufficiently high probability that the rival has low cost and volunteers immediately with positive probability makes it optimal for i to disregard information that is available without cost. This strategic value disappears only if the probability of having high contribution cost, p_H, is large, because, from the point of view of the rival, an early concession of the individual that knows his provision cost is less likely.

Example Consider the following example where $c_L = 2$, and $c_H = 10$.[8] Assumption (A) requires that $1 < T < 5$.
(a) Suppose that $p_H = 0.75$. If $T \to \bar{c}/2 = 4$ from below, the value of information V_i^I is positive. Hence, the critical threshold $\tilde{T} < \bar{c}/2$. Setting $V_i^I(T) = 0$ yields $\tilde{T} = 1.94$. Thus, for all $T < 1.94$, only one individual learns his cost of provision, and for all $T > 1.94$, both individuals learn their cost of provision.
(b) Now suppose that $p_H = 0.5$. V_i^I is negative if T approaches $\bar{c}/2 = 3$ (from below). Hence, if in case (N, I) the pure strategy equilibrium is selected, $\tilde{T} = \bar{c}/2 = 3$, and if the mixed strategy equilibrium is selected, $\tilde{T} > \bar{c}/2$. In the latter case, $\tilde{T} = 3.56$.

[8] Details on this example are in Appendix B.

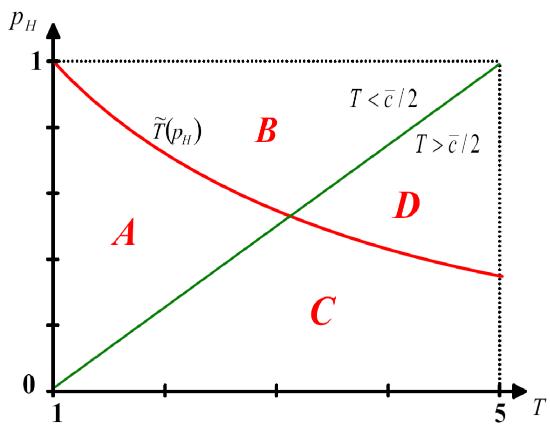

Figure 6.1: Equilibrium information acquisition (for $c_L = 2$, $c_H = 10$).

(c) If $p_H = 0.25$, again V_i^I is negative if T approaches $\bar{c}/2 = 2$, and $\tilde{T} = \bar{c}/2$ if in case (N, I) the pure strategy equilibrium is selected. If the mixed strategy equilibrium is selected, V_i^I is negative for all T satisfying (A), and thus there is no equilibrium where both individuals find out about their cost of provision with probability one.

Figure 6.1 shows the equilibrium outcome for different combinations of T and p_H. The 45-degree line describes the condition $T \gtrless \bar{c}/2$. In the areas B and D, finding out about the own cost of provision is a strictly dominant strategy; in area A, the individuals prefer to remain uninformed if the rival acquires information, and in equilibrium only one individual learns his cost. In area C, the outcome depends on which equilibrium is selected in case (N, I). Here, $T > \bar{c}/2$, and for the pure strategy equilibrium, information acquisition is strictly dominant. For the mixed strategy equilibrium, however, only one individual acquires information, or both individuals randomize their information choice.

A designer's perspective. There are several dimensions along which efficiency can be defined. On the one hand, it could be of interest that the individual with the lowest cost (highest ability) provides the public good. On the other hand, a designer might want to minimize the expected waiting time. In a framework of a contest, a designer may want to induce a large time of fighting, i.e. high waiting times. To capture these different dimensions, consider the following objective function

$$W = \lambda_1 \left[E\left(\pi_1\right) + E\left(\pi_2\right) \right] - \lambda_2 E\left(\min\left\{t_1, t_2\right\}\right) - \lambda_3 E\left(k\left(t_1, t_2\right)\right)$$

where

$$k\left(t_1, t_2\right) = \begin{cases} c_1 & \text{if} \quad t_1 < t_2 \\ \left(c_1 + c_2\right)/2 & \text{if} \quad t_1 = t_2 \\ c_2 & \text{if} \quad t_1 > t_2 \end{cases}$$

is the expected cost of providing the public good and λ_1, λ_2, and λ_3 are the weights given to the sum of expected payoffs, the expected waiting time, and the expected provision cost. To illustrate the different objectives and the consequences for information acquisition, we only consider the cases where two of the weights are zero. Moreover, we assume that the designer does not know the individuals' cost of provision and cannot change the structure of the game.

Suppose first that $\lambda_3 > 0 = \lambda_1 = \lambda_2$, that is, maximizing W is equivalent to minimizing the expected cost of provision. Then, W is highest if both individuals acquire information and know their cost of provision (case (I, I)), and an individual with low cost volunteers with probability one before the time limit is reached. With Propositions 6.4 and 6.5, this implies that $T > c_L/2 - c_L \ln p_H$ (from Proposition 6.4) and $T > \tilde{T}$ (from Proposition 6.5). In this case, information acquisition is efficient.

Another objective could be to focus on the expected waiting time. Let $\lambda_1 = \lambda_3 = 0$. Obviously, if $\lambda_2 > 0$, the time horizon should be as short as possible. Then, both individuals always wait until T, and the decisions on information become irrelevant.[9] If $\lambda_2 < 0$, setting the time limit very high

[9] If $T > \bar{c}/2$ and in case (N, I) the pure strategy is selected, W would also be maximized

and prohibiting information acquisition would result in the highest waiting times: as $E\left(k\left(t_1,t_2\right)\right)=\bar{c}$ and

$$E\left(\pi_i\right)=v-E\left(\min\left\{t_1,t_2\right\}\right)-E\left(k\left(t_1,t_2\right)\right)/2, \tag{6.10}$$

it follows from (6.3) that, in case (N,N), $E\left(\min\left\{t_1,t_2\right\}\right)=\bar{c}/2$. In case (I,I), however, the expected waiting time is lower for any T. If $T<c_L/2-c_L\ln p_H$, (6.9) implies that

$$E\left(\pi_i\right)>v-c_L-\frac{p_H}{2}\left(c_H-c_L\right)=v-\frac{\bar{c}}{2}-\frac{c_L}{2}.$$

By (6.10), if $E\left[\min\left(t_1,t_2\right)\right]$ was larger than $\bar{c}/2$, then $E\left[k\left(t_1,t_2\right)\right]<c_L$, which is a contradiction. For $T>c_L/2-c_L\ln p_H$, note first that $E\left[k\left(t_1,t_2\right)\right]=p_H^2c_H+\left(1-p_H^2\right)c_L$. Moreover, $E\left[\min\left(t_1,t_2\right)\right]$ is increasing in T. Using (6.9) and (6.10), $E\left[\min\left(t_1,t_2\right)\right]$ is therefore strictly smaller than

$$\begin{aligned}
& c_L+p_H^2\left(\frac{c_H}{2}+\frac{c_H}{2}-c_L+c_L\ln p_H\right)-\frac{1}{2}\left(p_H^2c_H+\left(1-p_H^2\right)c_L\right)\\
= & \frac{1}{2}c_L+\frac{1}{2}p_H^2c_H-\frac{1}{2}p_H^2c_L+p_H^2c_L\ln p_H\\
< & \frac{1}{2}c_L+\frac{1}{2}p_Hc_H-\frac{1}{2}p_Hc_L=\frac{\bar{c}}{2}.
\end{aligned}$$

Thus it can be in the interest of the designer to prohibit information acquisition.

A benevolent designer may want to maximize the individuals' payoffs, that is $\lambda_1>0=\lambda_2=\lambda_3$. Note that this is similar to minimizing a weighted average of waiting times and efficiency of the provision captured by $k\left(t_1,t_2\right)$.[10] If no individual knows his provision cost, the sum of payoffs is lowest: both

if exactly one individual acquires information. This, however, does not occur in equilibrium if the individuals decide on information acquisition, but only if information acquisition is forbidden for one individual. In this sense, there can be too much information acquisition in equilibrium if $\lambda_2>0=\lambda_1=\lambda_3$.

[10] More precisely, $\lambda_1=1$, $\lambda_2=0$, $\lambda_3=0$ is equivalent to $\lambda_1=0$, $\lambda_2=2$, $\lambda_3=1$.

individuals get a payoff of $v - \bar{c}$ that they can ensure in each of the other cases by volunteering immediately independently of their cost. Thus, a benevolent designer would never want to forbid both individuals to find out about their cost of provision. The pure strategy equilibrium in case (N, I) can lead to the highest sum of expected payoffs since the cost of waiting is zero, but if c_H is high, the sum of payoffs is lower than in case (I, I) : for high c_H, the inefficiency of the provision in case (N, I) (where the uninformed individual volunteers immediately) outweighs the higher waiting cost in case (I, I). In the latter case, the sum of payoffs is highest if $T = c_L/2 - c_L \ln p_H$ such that individuals with low cost concede before T with probability one. In general, the optimal choice of T depends on the balancing of expected waiting time and cost of provision.

6.5 Conclusion

The private provision of a discrete public good is likely to end up in a war of attrition: individuals may prefer to wait until someone else volunteers and provides the public good. However, they may not be able to wait for an infinite amount of time. This can be due to time constraints or to a finite time horizon imposed by a third party. In many applications, such as allocating tasks in firms or communities, time limits are a typical feature of the volunteering game.

In this chapter, we analyzed incentives for obtaining information ahead of the war of attrition. The information that is available to the individuals has an important impact on the equilibrium outcome of the volunteering game. This suggests that individuals have an incentive to use information acquisition strategically when they anticipate the private provision game. We assumed that initially the individuals do not know exactly their own cost of provision of the public good, but that they can find out about this cost prior to the volunteering game. Indeed, there can be an incentive for one individual not to become informed of his cost of provision even if the information is available without cost. For a sufficiently short time horizon, being uninformed induces an informed individual to volunteer immediately in case he has low cost of provision, whereas not knowing the own cost of provision constitutes a commitment to delay the own concession. For a sufficiently long time horizon, however, finding out about the own cost is a strictly dominant strategy. Since the time horizon has a crucial impact on information acquisition as well as on the equilibrium outcome, it may be used as an instrument to influence the efficiency of the public good provision.

Our model assumed that the individuals' costs of provision follow a two-point probability distribution. For continuous distribution functions, similar results can be obtained. The equilibrium properties change in the sense that an individual with private information about his cost of provision chooses his concession time as an increasing function of his provision cost. In the case where exactly one individual i has learned his cost we get a similar result for a small time limit T: i volunteers immediately if he has low cost of provision, which creates an incentive for the rival to remain uninformed of his own cost. For intermediate values of T, a mixed strategy equilibrium exists that exhibits similar properties to the one characterized in Proposition 6.3. The value of information is then determined by the shape of the probability distribution of the provision cost. The resulting effects are qualitatively the same, but can be most clearly demonstrated by using a two-point distribution and varying the probabilities that the cost of provision is high and low, respectively. The key assumption remains that with positive probability individuals face a rival who prefers to wait until the time limit is reached. In this sense, our approach is similar to Fudenberg and Tirole (1986) who assume that there is positive probability that the rival never concedes. For the incentives to find out about the cost of provision, the time limit is of additional strategic importance.

6.A Appendix

6.A.1 Proof of Lemma 6.1

Denote by Ψ_j the distribution of j's waiting times, from the point of view of i, that is, $\Psi_j(t) = a$ means that, from the point of view of i, j concedes before t with probability a.[11] Consider a concession of i in $t_i \in (-c_i/2 + T, T)$ and suppose that there is strictly positive probability that i provides the good in t_i, i.e. $\Psi_j(t_i) < 1$. If Ψ_j exhibits a discontinuity at t_i, then there is an $\varepsilon > 0$ such that i is strictly better off by conceding in $t_i + \varepsilon$ instead of in t_i because this would strictly decrease the expected contribution cost at only an infinitesimally higher expected waiting cost. Otherwise, i's expected payoff from a concession in t_i is

$$\begin{aligned}
& \int_0^{t_i} (v-t)\, d\Psi_j(t) + (1-\Psi_j(t_i))(v - t_i - c_i) \\
= & \int_0^{T} (v-t)\, d\Psi_j(t) - \int_{t_i}^{T} (v-t)\, d\Psi_j(t) \\
& + (1-\Psi_j(T))(v - t_i - c_i) + (\Psi_j(T) - \Psi_j(t_i))(v - t_i - c_i) \\
= & \int_0^{T} (v-t)\, d\Psi_j(t) + (1-\Psi_j(T))(v - t_i - c_i) \\
& - \int_{t_i}^{T} (t_i + c_i - t)\, d\Psi_j(t). \qquad (6.11)
\end{aligned}$$

$\Psi_j(t_i) < 1$ implies that $\Psi_j(T) - \Psi_j(t_i) > 0$ or/and $1 - \Psi_j(T) > 0$. Therefore, for all $t_i \in (-c_i/2 + T, T)$, (6.11) is strictly smaller than

$$\int_0^{T} (v-t)\, d\Psi_j(t) + (1-\Psi_j(T))(v - c_i/2 - T)$$

which is i's expected payoff for $t_i = T$.

[11] Note that Ψ_j captures both uncertainty of i over j's contribution cost and possible randomization of j.

6.A.2 Proof of Proposition 6.1

(i) As argued in the main text, best response to $t_j = T$ is $t_i = T$, and $t_1 = t_2 = T$ is an equilibrium. Moreover, since $-c_i/2+T < 0$, Lemma 6.1 rules out any further equilibrium because any individual that contributes with strictly positive probability in $t' \in [0, T)$ would strictly prefer a concession in T to a concession in t'.

(ii) The structure of the equilibrium strategies follows from Hendricks et al. (1988) and the analysis in the main text. We only show that the strategies constitute an equilibrium. Suppose that j follows (6.2). Then, by Lemma 6.1, i strictly prefers $t_i = T$ to any $t_i \in (-\bar{c}/2 + T, T)$. For $t_i \in [0, -\bar{c}/2 + T]$, i's payoff is

$$\int_0^{t_i} (v - x) \frac{1}{\bar{c}} \exp\left(-\frac{x}{\bar{c}}\right) dx + \exp\left(-\frac{t_i}{\bar{c}}\right) (v - t_i - \bar{c}) = v - \bar{c},$$

and if $t_i = T$, i gets

$$\int_0^{-\frac{\bar{c}}{2}+T} (v - x) \frac{1}{\bar{c}} \exp\left(-\frac{x}{\bar{c}}\right) dx + \exp\left(\frac{1}{2} - \frac{T}{\bar{c}}\right) \left(v - T - \frac{\bar{c}}{2}\right) = v - \bar{c}.$$

Thus i is indifferent to all $t_i \in (0, -\bar{c}/2 + T] \cup \{T\}$, and F_i and F_j are mutually best responses.

6.A.3 Proof of Proposition 6.3

If there is an equilibrium in mixed strategies, the equilibrium strategies must exhibit similar properties as in the case of complete information. In particular, for waiting times t_i and t_{j_L} in the support of the mixed strategies, it has to hold that

$$F_i(t_i) = 1 - (1 - b_i) \exp\left(-\frac{t_i}{c_L}\right), \tag{6.12}$$

$$F_{j_L}(t_{j_L}) = \frac{1}{p_L}\left[1 - (1 - p_L b_{j_L}) \exp\left(-\frac{t_{j_L}}{\bar{c}}\right)\right], \tag{6.13}$$

where the constants b_i and b_{j_L} correspond to the mass points at zero, $F_i(0)$ and $F_{j_L}(0)$, and remain to be determined. The factor $1/p_L$ in F_{j_L} takes into account the probability p_L that i faces a rival with cost c_L. It has to hold that $0 \leq b_i, b_{j_L} < 1$, and $\min(b_i, b_{j_L}) = 0$: if i (j_L) concedes immediately with strictly positive probability, j_L (i) strictly prefers a concession in $\varepsilon > 0$, ε infinitesimally small, to a concession in 0.

Assumption (A) implies that no $t_{j_H} < T$ with $F_i(t_{j_H}) < 1$ can be part of j_H's equilibrium strategy: j_H won't choose any $t_{j_H} < T$ in the support of F_i. In turn, for any $t_i < T$, we must have $F_{j_H}(t_i) = 0$, and thus i strictly prefers $t_i = T$ to all $t_i \in (-\bar{c}/2 + T, T)$. Moreover, F_i must be continuous on $(0, T)$. To see why, suppose that i concedes in $t_i \in (0, -\bar{c}/2 + T]$ with strictly positive probability. Then, there are $\delta > 0, \varepsilon > 0$ such that j_L strictly prefers $t_j = t_i + \varepsilon$ to any $t_j \in (t_i - \delta, t_i)$, hence i is strictly better off by choosing $t_i - \delta/2$ instead of t_i. Therefore, possible mass points of F_i are restricted to $t_i = 0$ and $t_i = T$.

We proceed in two steps: (1) we show that the mass points at zero are uniquely determined, and (2) we prove that (6.12) and (6.13) constitute an equilibrium.

Step 1: From (6.12), it follows that $b_i < 1$ implies $F_i(t_i) < 1$ for all $t_i < T$: whenever i chooses a mixed strategy, there is strictly positive probability that $t_i = T$. In particular, we have $F_i(-\bar{c}/2 + T) < 1$ which implies that $F_{j_L}(-\bar{c}/2 + T) = 1$. This is due to the fact that there is strictly positive probability that i waits until T, and, as in the case of F_i above, F_{j_L} must be continuous on $(0, T)$. However, as F_i is constant in $(-\bar{c}/2 + T, T)$ and $c_L < \bar{c}$, j_L strictly prefers $t_{j_L} = -\bar{c}/2 + T$ to all $t_{j_L} > -\bar{c}/2 + T$, and therefore $F_{j_L}(-\bar{c}/2 + T) < 1$ contradicts the nonexistence of interior mass points.

With (6.13), $\min(b_i, b_{j_L}) = 0$, and $F_{j_L}(-\bar{c}/2 + T) = 1$, we get

$$b_i = 0 \text{ and } b_{j_L} = \frac{1}{p_L}\left[1 - (1 - p_L)\exp\left(-\frac{1}{2} + \frac{T}{\bar{c}}\right)\right]. \tag{6.14}$$

b_{j_L} is strictly decreasing in T with $\lim_{T \downarrow \bar{c}/2} b_{j_L} = 1$ and $\lim_{T \uparrow \bar{c}/2 - \bar{c} \ln p_H} b_{j_L} = 0$. Hence, $\bar{c}/2 < T < \bar{c}/2 - \bar{c} \ln p_H$ is a necessary condition for the existence of a mixed strategy equilibrium.[12]

Step 2: It remains to show that (6.12), (6.13) and (6.14) indeed constitute an equilibrium. Consider first individual i and suppose that j follows F_{j_L} and F_{j_H}, respectively. For any $t_i \in (0, -\bar{c}/2 + T]$, i's expected payoff is

$$v - p_L \int_0^{t_i} x \frac{1 - p_L}{p_L} \frac{1}{\bar{c}} e^{-\frac{1}{2} + \frac{T-x}{\bar{c}}} dx - (1 - p_L) e^{-\frac{1}{2} + \frac{T-t_i}{\bar{c}}} (\bar{c} + t_i)$$

which is equal to $v - (1 - p_L) \exp(-1/2 + T/\bar{c}) \bar{c}$. If i concedes in T, he gets

$$v - p_L \int_0^{-\bar{c}/2+T} x \frac{1 - p_L}{p_L} \frac{1}{\bar{c}} e^{-\frac{1}{2} + \frac{T-x}{\bar{c}}} dx - (1 - p_L) \left(\frac{\bar{c}}{2} + T \right)$$

which again is equal to $v - (1 - p_L) \exp(-1/2 + T/\bar{c}) \bar{c}$. Hence, i is indifferent to all $t \in (0, -\bar{c}/2 + T] \cup \{T\}$. Any $t_i \in (-\bar{c}/2 + T, T)$ leads to a lower payoff.

Now turn to j and suppose that i follows F_i. The equilibrium strategy of j_H follows from Lemma 6.1. For j_L, a concession in $t \in [0, -\bar{c}/2 + T]$ yields an expected payoff of

$$\int_0^t (v - x) \frac{1}{c_L} e^{-\frac{x}{c_L}} dx + e^{-\frac{t}{c_L}} (v - c_L - t) = v - c_L.$$

Hence, j_L is indeed indifferent to all $t \in [0, -\bar{c}/2 + T]$. For all $t > -\bar{c}/2 + T$, j_L's expected payoff is strictly lower. The ex ante expected payoffs in (6.7) and (6.8) follow directly from these calculations.

[12] To be precise, if $T = \bar{c}/2 - \bar{c} \ln p_H$, we get $b_{j_L} = 0$ and $b_i \geq 0$ is not uniquely determined. Hence, there exists a continuum of mixed strategy equilibria where i's payoff is $v - \bar{c}$, as in the pure strategy equilibrium. We omit this case in order to simplify the exposition.

6.A.4 Proof of Proposition 6.4

By (A) and Lemma 6.1, $F_{i_H}(t) = 0$ for all $t < T$ and $F_{i_H}(t) = 1$ otherwise, $i = 1, 2$. Thus, for i_L, a concession in T is strictly preferred to any $t \in (-c_L/2 + T, T)$. Suppose that j_L follows F_{j_L}. For any $t_i \in [0, \bar{t})$, i_L's payoff is

$$p_L \int_0^{t_i} (v - x) \frac{1}{p_L c_L} e^{-\frac{x}{c_L}} dx + e^{-\frac{t_i}{c_L}} (v - t_i - c_L) = v - c_L.$$

By choosing $t_i = T$, i_L gets

$$\begin{aligned} & p_L \int_0^{\bar{t}} (v - x) \frac{1}{p_L c_L} e^{-\frac{x}{c_L}} dx + e^{-\frac{\bar{t}}{c_L}} \left(v - T - \frac{c_L}{2}\right) \\ = \;& v - c_L + \exp\left(-\frac{\bar{t}}{c_L}\right)\left(\bar{t} - T + \frac{c_L}{2}\right). \end{aligned}$$

If

$$-\frac{c_L}{2} + T < -c_L \ln(1 - p_L), \tag{6.15}$$

$\bar{t} = -\frac{c_L}{2} + T$, and i_L is indifferent to all $t_i \in [0, \bar{t}) \cup \{T\}$: (6.15) implies that $F_{i_L}(\bar{t}) < 1$ and i_L waits until T with strictly positive probability. If (6.15) is violated, $\bar{t} = -c_L \ln(1 - p_L)$ and $F_{i_L}(\bar{t}) = 1$, that is, i_L concedes with probability one before $\bar{t} < T$. Indeed, waiting until T would lead to a payoff lower than $v - c_L$. Since any symmetric equilibrium must be in mixed strategies, this is the only symmetric equilibrium. Expected payoff of i_L is $v - c_L$, and expected payoff of i_H is

$$\begin{aligned} & p_L \int_0^{\bar{t}} (v - x) \frac{1}{p_L c_L} e^{-\frac{x}{c_L}} dx + e^{-\frac{\bar{t}}{c_L}} \left(v - T - \frac{c_H}{2}\right) \\ = \;& v - c_L + \exp\left(-\frac{\bar{t}}{c_L}\right)\left(c_L - \frac{c_H}{2} + \bar{t} - T\right). \end{aligned}$$

Hence, the ex ante expected payoff is

$$E(\pi_i) = v - c_L + p_H \exp\left(-\frac{\bar{t}}{c_L}\right)\left(c_L - \frac{c_H}{2} + \bar{t} - T\right).$$

Inserting $\bar{t}$ leads to (6.9).

6.A.5 Proof of Lemma 6.2

(i) Suppose that $T < \bar{c}/2$. In case (N, N), expected payoffs are $v - \bar{c}/2 - T$. Together with (6.5),

$$V_i^N = -p_L c_L - p_H \left(\frac{c_H}{2} + T\right) + \frac{\bar{c}}{2} + T = -p_L \frac{c_L}{2} + (1 - p_H) T > 0.$$

If $T > \bar{c}/2$, expected payoff is $v - \bar{c}$ in case (N, N) which is the payoff an informed individual i can ensure by conceding immediately for both possible contribution costs. Since for high contribution cost, i strictly prefers waiting until T, his payoff must be strictly higher. Thus, $V_i^{\sigma_j = N} > 0$ for all $T \in (c_L/2, c_H/2)$.

(ii) If $T < c_L/2 - c_L \ln p_H$, subtracting (6.4) from (6.9) leads to

$$V_i^I(T) = -c_L - \frac{p_H}{2}(c_H - c_L)\, e^{\frac{1}{2} - \frac{T}{c_L}} + p_H \left(\frac{\bar{c}}{2} + T\right). \tag{6.16}$$

For $T \to c_L/2$, (6.16) converges to

$$-c_L - \frac{p_H}{2}(c_H - c_L) + \frac{p_H}{2}(\bar{c} + c_L) = -p_L c_L - \frac{p_H}{2}(c_H - \bar{c}) < 0.$$

Moreover, deriving (6.16) with respect to T yields

$$\frac{\partial V_i^I(T)}{\partial T} = \frac{1}{2}\frac{p_H}{c_L}(c_H - c_L)\, e^{\frac{1}{2} - \frac{T}{c_L}} + p_H > 0.$$

If $c_L/2 - c_L \ln p_H < T < \bar{c}/2$, we have

$$V_i^I(T) = -c_L - p_H^2 \left(T + \frac{c_H}{2} - c_L + c_L \ln p_H\right) + p_H \left(\frac{\bar{c}}{2} + T\right) \tag{6.17}$$

and

$$\frac{\partial V_i^I(T)}{\partial T} = -p_H^2 + p_H > 0.$$

(iii) With (6.6), i gets $v - \bar{c}$ if he does not become informed. By the same argument as in (i) for $T > \bar{c}/2$, i's ex ante payoff in case (I, I) must be strictly larger than $v - \bar{c}$, and thus $V_i^I > 0$ for all $T \in (\bar{c}/2, c_H/2)$.
(iv) In (ii), monotonicity has been shown for $T < \bar{c}/2$. Now suppose that $T > \bar{c}/2$. Consider first the case where T is smaller than $c_L/2 - c_L \ln p_H$, i.e. $T \in (\bar{c}/2, c_L/2 - c_L \ln p_H)$. (For sufficiently large p_H, this interval is empty.) Then, with (6.7) and (6.9),

$$V_i^I(T) = -c_L - \frac{p_H}{2}(c_H - c_L)\, e^{\frac{1}{2} - \frac{T}{c_L}} + p_H \bar{c} e^{-\frac{1}{2} + \frac{T}{\bar{c}}} \tag{6.18}$$

and

$$\frac{\partial V_i^I(T)}{\partial T} = \frac{p_H}{2c_L}(c_H - c_L)\, e^{\frac{1}{2} - \frac{T}{c_L}} + p_H e^{-\frac{1}{2} + \frac{T}{\bar{c}}} > 0.$$

Now suppose T is larger than $c_L/2 - c_L \ln p_H$, but smaller than $\bar{c}/2 - \bar{c} \ln p_H$, i.e. $T \in (c_L/2 - c_L \ln p_H, \min\{\bar{c}/2 - \bar{c} \ln p_H, c_H/2\})$. (This interval may be empty for a small p_H.) We get

$$V_i^I(T) = -c_L - p_H^2 \left(T + \frac{c_H}{2} - c_L + c_L \ln p_H\right) + p_H \bar{c} e^{-\frac{1}{2} + \frac{T}{\bar{c}}} \tag{6.19}$$

and hence

$$\frac{\partial V_i^I(T)}{\partial T} = -p_H^2 + p_H e^{-\frac{1}{2} + \frac{T}{\bar{c}}} > -p_H^2 + p_H > 0.$$

Continuity of V_i^I follows directly from continuity of the expected payoffs.

6.A.6 Proof of Proposition 6.5

From Lemma 6.2(i), it follows that best response to $\sigma_j = N$ is to become informed. Now suppose that in case (N, I) the pure strategy equilibrium is selected. With Lemma 6.2(ii)-(iii), there exists a unique $\tilde{T} \leq \bar{c}/2$ such that the best response to $\sigma_j = I$ is to remain uninformed if and only if $T < \tilde{T}$. In this case, there are two asymmetric equilibria where one individual acquires information and the other individual remains uninformed. In addition, there is a symmetric equilibrium where the individuals randomize their information choice and learn their provision cost with probability $V_i^N / \left(V_i^N - V_i^I\right) \in (0,1)$. If $T > \tilde{T}$, there is a unique equilibrium where both individuals find out about their provision cost.

For the mixed strategy equilibrium in case (N, I), this result follows from monotonicity of V_i^I (Lemma 6.2ii+iv). Note that $\tilde{T} < \bar{c}/2 - \bar{c} \ln p_H$ as, for $T \to \bar{c}/2 - \bar{c} \ln p_H$, V_i^I converges to the value of information in the pure strategy equilibrium and hence is strictly positive.[13] Therefore, whenever $\bar{c}/2 - \bar{c} \ln p_H < c_H/2$, there exists an interior $\tilde{T} \in (c_L, \bar{c}/2 - \bar{c} \ln p_H)$ such that the best response to $\sigma_j = I$ is to remain uninformed if and only if $T < \tilde{T}$, and information acquisition is strictly dominant if $T > \tilde{T}$. If $\bar{c}/2 - \bar{c} \ln p_H > c_H/2$, the interval where in equilibrium both individuals acquire information can be empty which is the case if $\lim_{T \to c_H/2} V_i^I$ is negative.

6.A.7 Proof of Corollary 6.1

Part (i) follows from Lemma 6.2(iii). For part (ii), the threshold $\tilde{T}$ is larger than $\bar{c}/2$ if and only if the value of information is negative as T approaches $\bar{c}/2$. Moreover, V_i^I may even be negative for all T. Note that $\bar{c}/2 - \bar{c} \ln p_H > c_L/2 - c_L \ln p_H$, and for small p_H, we have $c_L/2 - c_L \ln p_H > c_H/2$. With monotonicity of (6.18), it follows that, for all $T \in (c_L/2, c_H/2)$, V_i^I is smaller than

$$\lim_{T \to c_H/2} V_i^I(T) = -c_L - \frac{p_H}{2}(c_H - c_L) e^{\frac{1}{2} - \frac{c_H}{2c_L}} + p_H \bar{c} e^{-\frac{1}{2} + \frac{c_H}{2\bar{c}}}$$

which is negative for small p_H since the second and the third term approach zero if $p_H \to 0$. For intermediate values of p_H, we have $\tilde{T} \in (\bar{c}/2, c_H/2)$.

[13] This follows from the convergence of expected payoffs in case (N, I) for $T \to \bar{c}/2 - \bar{c} \ln p_H$.

Chapter 7

Conclusion

The acquisition of information prior to choosing a decision that involves uncertain cost or benefit is a characteristic of very different situations. Information acquisition can consist of more or less complex and time-consuming activities, from simply reading the news to ordering reports, creating special research groups, or even engaging in illegal activities such as spying. In strategic interactions such as conflicts, the question of how much information should be acquired does not only depend on the cost of the information. There is a strategic aspect itself involved in the decision on information acquisition that can lead to incentives that cause inefficient behavior regarding information acquisition or information disclosure.

This thesis has studied the strategic role of information in different conflictual settings, including standard models of contests and problems of private provision of public goods. The results have shown that factors such as the observability of the information acquisition and the nature of the conflict are important in determining the contestants' willingness to acquire information. Whenever possible, welfare implications have been derived, although general welfare statements depend on the nature of the conflict. In military conflict, effort can be destructive, but, for example in the context of R&D, effort may also be socially valuable. Therefore, the thesis has to a large extent concentrated on a positive analysis. Policy implications can be derived in concrete applications as, for instance, in the case of global warming where the identified inefficiencies which result from the strategic use of information may be a rationale for the provision of information by a supranational institution. In the context of information disclosure, the possible spillover effects of firms' efforts in research competition can justify a legal intervention in the firms' decisions on information sharing.

The theoretical results derived in this thesis establish a basis for the analysis of the actual behavior of individuals with respect to information acquisition. Since the cost of information and the knowledge of contestants typically are difficult to observe, controlling for these factors in a laboratory experiment could provide interesting evidence on the behavior of individuals with regard to the use of information. In particular, strategic motives could be isolated that are incorporated in decisions to acquire or to share information. Related to the issue of commitment to future actions, for instance, interesting questions emerge if the contestants are able to influence the degree of observability of the information acquisition.

Chapter 8

Zusammenfassung / Summary (in German)

Die vorliegende Dissertation untersucht die strategische Bedeutung, die Informationsbeschaffung im Vorfeld von Konflikten zukommt. Konflikte beschreiben in diesem Zusammenhang Interaktionen, in denen Kontrahenten unterschiedliche Interessen verfolgen (wie zum Beispiel in militärischen Auseinandersetzungen), aber auch Situationen, in denen die Akteure eigentlich ein gemeinsames Ziel verfolgen (wie etwa beim Umweltschutz).

Wenn ein Konflikt sich anbahnt, versuchen die Kontrahenten in der Regel, sich eine günstige Ausgangsposition zu verschaffen. Oft kann es für einen Akteur von Vorteil sein, sich auf ein bestimmtes (aggressives) Verhalten im Konflikt festzulegen. Strategische Maßnahmen im Vorfeld des Konfliktes können eine solche Selbstbindung glaubhaft machen. Ein Beispiel hierfür ist das Zerstören von Brücken und damit Rückzugsmöglichkeiten, wenn man direkt mit einem Feind konfrontiert ist (Schelling 1980). Auch für Firmen, die potentielle Konkurrenten abschrecken wollen, ist es entscheidend, ob sie glaubhaft machen können, dass sie zu einem Preiskampf bereit sind. Im Allgemeinen kann eine solche Festlegung dadurch erreicht werden, dass der eigene Strategieraum oder die Kosten bzw. der Nutzen einer bestimmten

Aktion beeinflusst werden. Diese Arbeit untersucht die Anreize, Information als strategisches Instrument einzusetzen.

Das Verhalten in Konflikten wird entscheidend durch die Information beeinflusst, die Wettbewerber über ihre Kontrahenten, aber auch über den eigenen Nutzen und die eigenen Kosten bestimmter Entscheidungen besitzen. Typischerweise sind die Akteure nicht vollständig über alle entscheidenden Gegebenheiten eines Konfliktes informiert und wenden deshalb Ressourcen auf, um sich zusätzliche Information zu beschaffen. In militärischen Konflikten kommt neben der Information über den Feind auch der Kenntnis des (wirtschaftlichen) Nutzens eines Sieges, der geographischen Bedingungen und der Kosten der Truppenentsendung eine wichtige Bedeutung zu. Ähnliches gilt für den Bereich der Forschung und Entwicklung; beispielsweise werden Automobilhersteller Informationen über Marktpotential und Produktionskosten einholen, bevor sie in Umwelttechnologien investieren.

In vielen Fällen können die Kontrahenten entscheiden, ob sie sich Information beschaffen, die nur ihnen selbst von Nutzen ist, oder Information, die auch ihre Mitbewerber betrifft. Auch können sie diese Information entweder unter Verschluss halten oder aber sie ihren Konkurrenten mitteilen, und sie können beeinflussen, ob ihre Konkurrenten beobachten können, dass sie zusätzliche Information eingeholt haben. Hinter solchen Entscheidungen stehen häufig strategische Überlegungen. Wenn eine Firma zum Beispiel weiß, dass sich ein Konkurrent präzise Informationen über die Marktbedingungen und die Präferenzen der Konsumenten beschafft hat, wird dies das eigene Investitionsverhalten verändern. Ebenso kann das Veröffentlichen von unabhängigen Marktstudien einen Einfluss auf die Gewinnerwartungen der konkurrierenden Firmen haben. Andererseits könnte beispielsweise im Falle des Umweltschutzes die Unkenntnis von Kosten und Nutzen des Handeln als Rechtfertigung dafür genutzt werden, nicht oder nur wenig beizutragen. In diesem Sinne kann die Informationslage als glaubhafte Festlegung auf eine bestimmte Handlungsweise eingesetzt werden. Sobald die Entscheidung über

Informationsbeschaffung das Konfliktverhalten der Kontrahenten beeinflusst, erhält die Information einen strategischen Wert, und dasselbe kann für die Unkenntnis bestimmter Sachverhalte gelten. Informationsbeschaffung sollte deshalb nicht nur in einem Vergleich von unmittelbaren Kosten der Information und deren Nutzen betrachtet werden; die Analyse von Informationsbeschaffung hat ebenso den strategischen Effekten Rechnung zu tragen, die mit ihr verbunden sind.

Diese strategischen Anreize stehen in der vorliegenden Dissertation im Mittelpunkt, und sie müssen berücksichtigt werden, um belastbare Aussagen über das Konfliktverhalten treffen zu können. In fünf Kapiteln werden unterschiedliche Konfliktsituationen analysiert, und es wird gezeigt, wie Anreize für Informationsbeschaffung und Informationsweitergabe das Handlungsergebnis beeinflussen können. Die einzelnen Kapitel tragen somit sowohl zur Literatur über strategisches Verhalten im Vorfeld von Konflikten als auch zur Literatur über Information und Unsicherheit bei. Der Fokus liegt auf der strategischen Bedeutung der Informationsentscheidung an sich, und es wird vernachlässigt, dass Kontrahenten nicht nur entscheiden können, ob sie sich Information beschaffen, sondern auch, wie viel Ressourcen sie für zusätzliche Information aufwenden wollen. Darüber hinaus geht der Großteil der Arbeit davon aus, dass die Entscheidung durch die Kontrahenten beobachtbar ist, da dies den Effekt der Selbstbindung hervortreten lässt.

Nach einer einleitenden Darstellung der Forschungsfrage und der wichtigsten Ergebnisse untersucht Kapitel 2 der Arbeit den Einfluss der Informationsstruktur auf die Profite von Akteuren, die in einer Standardauktion konkurrieren. Standardauktionen kennzeichnen sich dadurch, dass die Teilnehmer simultan ein Gebot abgeben und der Bieter mit dem höchsten Gebot die Auktion gewinnt. Solche Situationen ähneln Konflikten in dem Sinne, dass die Akteure versuchen, durch Aufwendung von Ressourcen die Kontrahenten zu überbieten und das Handlungsergebnis zu ihren Gunsten zu beeinflussen.

Zwei häufig analysierte Informationsstrukturen werden verglichen: Situationen, in denen der Wert des Gewinnens aller Kontrahenten allgemein bekannt ist (*vollständige Information*), und Situationen, in denen jeder Akteur lediglich seinen eigenen Wert des Gewinnen kennt (*private Information*). Es wird gezeigt, dass für eine Reihe von Auktionen die Profite der Akteure unter vollständiger Information und ihre Profite bei privater Information identisch sind. Dies gilt für die Erst- und die Zweitpreisauktion, für Kombinationen aus diesen beiden Auktionsformaten sowie für die All-Pay-Auktion, bei der jeder Akteur sein Gebot zu zahlen hat. In der Erst- und der Zweitpreisauktion ist auch der Erlös des Auktionators bei privater und vollständiger Information identisch. In der All-Pay-Auktion ist dessen Erlös bei vollständiger Information jedoch niedriger als bei privater Information. Auch gilt das Ergebnis der Äquivalenz der Profite nicht für alle Standardauktionen: in einer Klasse von All-Pay-Auktionen, bei denen der Gewinner eine Kombination aus seinem Gebot und dem zweithöchsten Gebot zahlt und alle anderen Bieter ihr eigenes Gebot, sind die Profite bei vollständiger Information höher als bei privater Information. Diese Untersuchung kann Aufschluss darüber geben, wie bestimmte Informationsstrukturen entstehen, und ein Teil der Ergebnisse bildet eine Grundlage für die Analyse der Anreize, Information weiterzugeben.

Während in Kapitel 2 von einer symmetrischen Informationsstruktur ausgegangen wird, trägt das dritte Kapitel der Tatsache Rechnung, dass sich Kontrahenten im Umfang der Information, die sie besitzen, unterscheiden können. In manchen Situationen steht mehr Information über einen bestimmten Akteur zur Verfügung, zum Beispiel, wenn alteingesessene Firmen sich gegen Marktneulinge zur Wehr setzen müssen. Asymmetrische Informationsstrukturen können aber auch entstehen, wenn sich nur ein Teil der Kontrahenten entscheidet, in Informationsbeschaffung zu investieren.

Der erste Teil des dritten Kapitels untersucht einen perfekt diskriminierenden Wettkampf (All-Pay-Auktion) mit zwei Kontrahenten, wobei der

Wert des Gewinnens des einen Kontrahenten allgemein bekannt ist, der Wert des anderen Kontrahenten jedoch dessen private Information ist. Darauf aufbauend analysiert der zweite Teil des dritten Kapitels Anreize zur Informationsbeschaffung, und es wird gezeigt, dass asymmetrische Informationsstrukturen im Gleichgewicht entstehen können, wenn die Informationsbeschaffung mit Kosten verbunden ist. Dies ist dadurch begründet, dass der Wert zusätzlicher Information höher ist, wenn der Gegenspieler entscheidet, keine Information zu akquirieren, als wenn der Gegenspieler ebenfalls in Information investiert. Entscheidend für die Analyse der Zahlungsbereitschaft für Information ist, ob der Gegenspieler beobachten kann, dass man Information eingeholt hat. Auch kann es Situationen geben, in denen die Information selbst nicht geheim gehalten werden kann oder in denen man beeinflussen kann, ob sie geheim gehalten oder öffentlich zugänglich wird. Deshalb werden in der Analyse drei unterschiedliche Fälle unterschieden: (1) die Entscheidungen über Informationsbeschaffung sind beobachtbar, nicht jedoch die Information selbst, (2) sowohl die Entscheidungen als auch die Information sind beobachtbar, und (3) weder die Entscheidungen noch die Information sind durch den Kontrahenten beobachtbar. Während in den Fällen (2) und (3) die Informationsbeschaffung effizient ist (im Sinne eines sozialen Planers), kommt es im Fall (1) zu übermäßiger Informationsbeschaffung. Dies bedeutet, dass die Beobachtbarkeit der Entscheidung, aber nicht der Information selbst, einen strategischen Vorteil in der Konfliktsituation darstellt.

Neben der Möglichkeit, sich Information zu beschaffen, können Kontrahenten oft auch darüber entscheiden, ob sie Information an ihre Mitbewerber weitergeben wollen. Studien zu oligopolistischem Wettbewerb haben sich intensiv mit dieser Frage beschäftigt. Wenn der Wettbewerb jedoch umfangreiche Werbe- oder Forschungsausgaben einbezieht, weist er Strukturen auf, die einem Wettkampf ähneln. Kapitel 4 beschäftigt sich mit den Anreizen von Firmen zur Informationsweitergabe in Strukturen, die durch einen perfekt diskriminierenden Wettkampf beschrieben werden können. Hier muss unterschieden werden, ob die Information sich auf firmenspezifische Größen wie Produktionskosten bezieht (*private Werte*) oder ob die Information allgemeiner Natur ist und beispielsweise Nachfragebedingungen betrifft (*gemeinsame Werte*). Zudem geht die Analyse darauf ein, wie über die Frage des Teilens von Information entschieden wird. Wenn solche Entscheidungen unabhängig getroffen werden, ist es strikt dominiert, Information weiterzugeben. Dies gilt sowohl für private als auch für gemeinsame Werte. Wenn die Informationsweitergabe als industrieweites Abkommen vereinbart wird, kann ein solches Abkommen im Gleichgewicht auftreten, wenn es sich um Information über private Werte handelt, nicht jedoch bei Information über gemeinsame Werte. Die Wohlfahrtswirkung solcher Entscheidungen hängt davon ab, ob die Aufwendungen der Firmen im Wettkampf positive externe Effekte auf die Gesamtwirtschaft aufweisen, wie dies etwa bei Forschungsaufwendungen der Fall sein kann. Das Vorhandensein von positiven externen Effekten kann aus Wohlfahrtsgesichtpunkten ein Verbot des Teilens von Information über private Werte erstrebenswert machen. Dagegen kann im Fall gemeinsamer Werte ein gesetzlicher Zwang zur Informationsweitergabe eine Wohlfahrtssteigerung mit sich bringen, wenn die externen Effekte der Aufwendungen im Wettkampf hinreichend groß sind.

Der strategische Charakter zusätzlicher Information kann in vielen Fällen bewirken, dass ein Anreiz besteht, Ressourcen zur Informationsbeschaffung aufzuwenden. Strategische Motive können jedoch auch dazu führen, dass sich Akteure entscheiden, auf zusätzliche Information zu verzichten, selbst wenn diese nicht mit direkten Kosten verbunden wäre. Die letzten beiden Kapitel der Dissertation untersuchen Konflikte, bei denen die Akteure ein gemeinsames Ziel verfolgen und die Interaktion der privaten Bereitstellung eines öffentlichen Gutes gleicht. Es werden Fälle identifiziert, bei denen Akteure aus rein strategischen Gründen zusätzliche Information ignorieren.

In Kapitel 5 wird die Bereitstellung eines globalen öffentlichen Gutes anhand des Beispiels der Bekämpfung des Klimawandels untersucht. Hier spielt

Informationsbeschaffung eine wesentliche Rolle, da in der Frage der Bewertung von Kosten und Nutzen von Klimaschutz immer noch große Uneinigkeit besteht. Länder können in Information über den länderspezifischen Wert von Klimaschutz investieren, indem sie wissenschaftliche Gutachten in Auftrag geben; sie müssen jedoch bedenken, dass zusätzliche Information häufig nicht unter Verschluss gehalten werden kann. Hier entsteht ein strategischer Anreiz, auf zusätzliche Information zu verzichten, da das Offenbaren starker Präferenzen für die Bekämpfung des Klimawandels dazu führen kann, dass sich die eigene Beitragslast erhöht, da andere Länder ihren Beitrag zum Klimaschutz reduzieren. Diese Möglichkeit des "Trittbrettfahrens" kann dazu führen, dass im Gleichgewicht das Land mit dem potentiell höchsten Nutzen aus einer Reduzierung der Erderwärmung entscheidet, keine Information einzuholen, und auf diese Weise seine Beitragslast verringert. Es werden Bedingungen abgeleitet, unter denen solch strategisches Verhalten zu einem Wohlfahrtsverlust führt, was eine Begründung für Informationsbeschaffung und Informationsbereitstellung durch internationale Organisationen liefert.

Darüber hinaus geht Kapitel 5 auf mögliche zusätzliche Wirkungsmechanismen ein, die entstehen können, wenn Forschungsergebnisse zum Klimawandel nicht unmittelbar verfügbar sind. In der Zukunft geleistete Beiträge zum Klimaschutz können jedoch höhere Kosten verursachen, da in der Zwischenzeit Schäden entstanden sind, die nicht wieder auszugleichen sind. In einer solchen Situation kann die Informationsbeschaffung eine Selbstbindung an eine Verzögerung des eigenen Beitrags bewirken, bis die zusätzliche Information verfügbar ist.

Bei der Analyse des Klimaschutzes als globales öffentliches Gut addieren sich die Beiträge einzelner Länder in Form von Reduzierung des CO2-Ausstoßes zu einer gesamten Bereitstellungsmenge. Andere öffentliche Güter kennzeichnen sich dadurch, dass ein einzelner und fixer Beitrag für die Bereitstellung benötigt wird. Beispiele hierfür sind anfallende Aufgaben, die der Allgemeinheit oder einer bestimmten Gruppe dienen, wie etwa an einer

Universität die Funktion eines Doktorandenvertreters oder Dekans. Bei der Suche nach einem Freiwilligen für die Erfüllung einer bestimmten Aufgabe versuchen Akteure typischerweise, durch Abwarten zu erreichen, dass sich eine andere Person vor ihnen meldet und die Kosten der Bereitstellung übernimmt. Solche Situationen lassen sich durch einen "Zermürbungskrieg" (*war of attrition*) beschreiben.

Kapitel 6 zeigt, dass Akteure durch das Verzichten auf zusätzliche Information über ihre Bereitstellungskosten erreichen können, dass mit höherer Wahrscheinlichkeit der Gegenspieler zuerst nachgibt und das öffentliche Gut bereitstellt. Durch den Informationsverzicht kann ein Akteur sich glaubhaft darauf festlegen, nicht zu früh nachzugeben, und als Reaktion darauf den Gegenspieler veranlassen, sich früh freiwillig zu melden, falls dessen Bereitstellungskosten niedrig sind. Infolgedessen kann im Gleichgewicht eine Situation auftreten, in der ein Kontrahent aus strategischen Gründen auf kostenlos verfügbare Information verzichtet. Hier wie auch in Kapitel 5 können sich die Akteure durch Unkenntnis der eigenen Kosten bzw. des eigenen Nutzens einer Bereitstellung in eine strategisch günstige Ausgangsposition im Bereitstellungsspiel bringen.

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