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Macro-Financial Linkages
in the Open Economy

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To Kora.
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Zusammenfassung


Der dritte Aufsatz trägt zur Erklärung von überhöhten Zinsen auf Staatsanleihen

**Schlagworte:** Konjunkturzyklen kleiner offener Volkswirtschaften, Finanzintermediation, Schwellenländer, Kapitalflüsse, Schuldenkrise, Interbankenmarkt, Unsicherheit, Ambiguitätsaversion

**JEL Klassifikation:** D81, E32, E4, F3, F4, H6.
Abstract

The global financial crisis of 2007/08 uncovered the need to integrate financial- and credit market frictions into macroeconomic models. This dissertation contributes to this research agenda by modelling and quantifying the role of financial market frictions in different topics in open economy macroeconomics through three essays.

The first essay provides a quantitative assessment of financial market frictions in emerging economies. It asks which frictions and which shocks are most suitable for the explanation of emerging market business cycle data? The contribution is to develop and estimate a quantitative business cycle model of a small-open economy with a leveraged and capital importing banking sector. Further, I model financial sector shocks which capture the idea of a sudden change in investor preferences in a reduced form. Using Mexican quarterly data in a Bayesian estimation approach, I find that financial sector shocks play a more important role compared to shocks to the foreign interest rate in the determination of investment and trade dynamics, in particular after the global financial crisis. Further, the financial accelerator related to bank balance sheets jointly with financial sector shocks contribute to the procyclicality of capital flows.

The second essay, which is joint work with Philipp Engler, asks how some advanced economies have become vulnerable to a simultaneous banking and sovereign debt crisis? The contribution is that we analyse the liquidity role of government debt for bank funding in a quantitative framework. We find that dynamics in sovereign risk premia and secured interbank intermediation during the European sovereign debt crisis can be captured by the concept of strategic default. While the high default penalty from a credit crunch extends the endogenous borrowing limit of an advanced economy such that higher debt levels are achieved, the recession following an unorderly sovereign default are more persistent due to the subsequent necessary reparations of bank balance sheets.

The third essay contributes to the explanation of excessive returns on sovereign debt in the course of the European sovereign debt crisis. Specifically, I show in an empirical analysis that yields on sovereign CDS contracts are positively affected by forecast disagreement about future GDP growth, a conventional proxy for macroeconomic uncertainty. I then build a simple model of optimal sovereign default in which international investors are ambiguity averse. Investors preferences are captured by multiple-prior utility, where the interval of priors under consideration is affected by uncertainty shocks. I find that uncertainty shock raises the cost for issuing public debt, which in turn distorts the optimal fiscal policy and default decision of the government. Within a crisis zone, uncertainty shocks may lead to non-fundamental default events. In a quantitative evaluation, the model attributes a sizeable share of sovereign yields to time-varying uncertainty.

Keywords: Open economy business cycles, Financial intermediation, Emerging markets, Capital flows, Debt crises, Interbank market, Uncertainty, Ambiguity aversion.

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CHAPTER 1

Introduction

1.1 Outline and Scope of the Thesis

Regarding the evolution of modern macroeconomic theory following the financial crisis of 2007/08, many questions remain unanswered, thus implying the need for better integration of financial intermediation and credit market frictions into macroeconomic analysis where these ingredients matter for allocations (Woodford, 2010). Although financial factors are integrated into the workhorse model of central bankers, the New Neoclassical Synthesis (Goodfriend and King, 1997), there are limitations to this research agenda that arise from methodological constraints and historical path dependencies. At the same time, quantitative macroeconomic models remain an important factor underlying policy advice given to policy makers about the impact of different policy scenarios and the state of the economy in the business cycle (Eichenbaum, 2011).

This dissertation contributes to this research agenda by modelling and quantifying the role of financial market frictions in the context of different questions of open economy macroeconomics. The point of departure in each chapter is the neoclassical frictionless dynamic stochastic general equilibrium (DSGE) model of a small open economy (Mendoza, 1991). This benchmark model in quantitative dynamic macroeconomic analysis is supplemented with a financial sector with different extensions in order to address the specific research question at hand.

¹Examples for macroeconomic models that account for shocks and frictions from within the financial system are Christiano et al. (2010), Gertler and Kiyotaki (2011) and Jermann and Quadrini (2012), among many others. See Brunnermeier, Eisenbach, and Sannikov (2012) for a survey of the fast growing literature.

²See Caballero (2010) for a very critical opinion on this line of research.

³One important area of future research is to analyse the interdependencies and reciprocal effects that monetary policy, fiscal policy and financial regulation have on maintaining price-, financial- and debt sustainability (Brunnermeier and Sannikov, 2012).
The second chapter studies the question of which financial market frictions are important for business cycle dynamics in emerging market economies? A second question is what are the most important shocks driving the business cycle in countries with procyclical capital flows that are intermediated by the financial sector? In order to answer these question, I augment a standard neoclassical real business cycle model with a stochastic trend and a debt elastic interest rate as presented by García-Cicco, Pancrazi, and Uribe (2010) by a capital importing and leveraged financial sector that is subject to financial frictions, following in the modelling approach Gertler, Kiyotaki, and Queralto (2012). The model is then estimated in a Bayesian approach with quarterly data from Mexico over the time period 1994:I to 2014:IV, i.e. comprising both the 1995 Mexican currency crisis and the 2007/08 global financial crisis.

In the third and fourth chapter, I analyse an endogenous optimal default decision by the central government of the small open economy under scrutiny, based on the quantitative literature on sovereign debt and default (Arellano, 2008; Aguiar and Gopinath, 2007). In chapter three, which is joint work with Philipp Engler, we ask how some advanced economies have become vulnerable to a simultaneous banking and sovereign debt crisis? We extend a version of the small open production economy with an optimal default decision of Mendoza and Yue (2012) by a heterogeneous banking sector. Motivated by funding patterns in European interbank markets, we assume that government debt plays an important role for funding liquidity of banks. Specifically, government debt serves as the only available collateral on the domestic interbank market. This creates a tight link between fiscal decisions and financial stability through a collateral channel on the interbank market that is not studied in a quantitative setting in the literature. Estimating the model with data from Spain, we quantify the endogenous and highly non-linear output costs that arise from an unorderly sovereign default in an advanced and financially developed economy.

The fourth chapter takes a look at the mutual feedback channel between uncertainty shocks and financial frictions for the optimal tax and borrowing decision governments. Uncertainty shocks follow the idea that there is time-varying uncertainty about macroeconomic fundamentals that shapes the process of expectation formation of ambiguity averse agents. This idea is put forward in the macroeconomic context by Bloom (2009) and Ilut and Schneider (2014), among others. With respect to the model environment, the chapter directly expands on chapter three. It then departs from the benchmark model by assuming that government debt is held by ambiguity averse international investors. Ambiguity aversion is introduced by the multiple priors model (Gilboa and Schmeidler, 1989) where investors consider various macroeconomic fundamentals for the evolution of
total factor productivity that are not attached to (subjective) probabilities. Regarding
the formation of expectations, this leads to a departure from the paradigm of subjective
expected utility (Savage, 1954).

The model is then used to study cases under which non-fundamental default events
occur, and how these are linked to the presence of financial frictions in the model. Further,
I quantify the amount of sovereign risk premia due to macroeconomic uncertainty by
estimating the model with Spanish data. The arising ambiguity premium is identified by
including data on GDP forecast dispersion in the estimation.

1.2 Motivation

Challenges for macroeconomic stabilisation policies shifted with the onset of the global
financial crisis. These are direct results from changes in market structures and the in-
stitutional environment. This section briefly lays out the most relevant tendencies and
questions that emerged following the crisis and that are related to the subsequent chapters in order to put the findings of this thesis into the context of the ongoing debate.
However, for the sake of clarity, which often goes in line with brevity, it does not attempt
to provide a survey of the literature which is vast and rapidly growing, thus failing short
to acknowledge all the contributions previously made.

1.2.1 Frictions in the Financial System: Lessons from the Crisis

The global financial crisis has uncovered the need to integrate financial market frictions better into macroeconomic models. Following Hall (2010), 'financial frictions' lead to "a
cost to one side of a transaction that is not a benefit to the other side. ... In debt markets,
where the intermediary is a bank or other financial institution, a friction drives up the
borrower’s cost of funds without raising the payoff that the supplier of funds receives." (p.
6).

Financial frictions can have devastating macroeconomic effects, in particular if they are
coupled with highly leveraged financial institutions, asset fire-sales and liquidity spirals.
The global financial crisis started with an adjustment of asset prices in the US subprime
mortgage market that affected the financial sector and induced a downward spiral due to
balance-sheet arithmetics that are the result of mainly two institutional features (Adrian
and Shin, 2010).4 First, bank managers are targeting a specific leverage ratio due to
regulatory and market requirements. Second, their balance sheets are usually marked to

4In particular, beginning in February 2007, prices for credit default swap prices started to fall in the
lower segment of the US mortgage market, followed in July 2007 by the highest segments (AAA).
market, i.e. assets and liabilities are valued at their current market price. In such an institutional setting, an adjustment in asset prices leads to a liquidation of assets in order to maintain a constant leverage ratio. If many market participants are affected at the same time and liquidate assets to a significant amount, such a 'fire sale' reduces asset prices further, leading to a downward spiral with aggregate consequences (Shleifer and Vishny, 1992, 2011). This mechanism is enforced by a tightening of margin requirements on secured debt contracts that reduces funding liquidity of financial intermediaries further and stimulates a erosion of market liquidity, thus causing 'liquidity spirals' (Brunnermeier and Pedersen, 2009).

With eroding asset prices and the dry-up in liquidity in the quantitatively important segment of mortgage loans, banks started to worry about counter-party risk at the inter-bank market where liquidity is traded between banks. As shown by Diamond and Dybvig (1983), asymmetric information about the solvency of banks gives rise to possible bank runs in the uninsured deposit market. When lenders expect other creditors to withdraw their funds, this increases the likelihood of a bank failure. Multiple equilibria arise in which some are inefficient since they produce a run on a viable bank. The repo money market was in a similar position in 2007/08. As liquidity became scarce and the situation more and more complex and uncertain, banks became worried about their own future liquidity needs and reduced lending to other financial institutions. Further, banks withdrew their funds due to an increased perception of counterparty risk that lead ultimately to a run on the repo market (Gorton and Metrick, 2012), a break down of private interbank intermediation, and the hoarding of liquidity (Heider et al., 2015).

The consequences for output and employment have been dramatic, producing the biggest recession in the US in post-war history (Reinhart and Reinhart, 2011). This was the case even though the monetary policy response to the crisis was benign by historic comparison in a sense that an extra-ordinary monetary expansion prevented a collapse in aggregate money supply that had reinforced the Great Depression (Friedman and Schwartz, 1963). However, in many ways the Great Recession that followed the global financial crisis can be generalized to recessions that follow a credit boom. In the historical context, Schularick and Taylor (2012) document that financial crises in the post-war era have similar real costs as in the pre-war era mainly due to the inflated size of the financial system. Further, they find that credit aggregates in relation to output are a strong predictor for financial crises.\footnote{A margin describes the difference between the market price of an asset and its collateral value, usually captured by the notion of a 'haircut'.}

\footnote{Gourinchas and Obstfeld (2012) also find evidence that local credit expansion is a strong predictor for crises in an analysis of twentieth century financial crises in advanced and emerging economies.}
So, what lead to the build-up of private sector debt? Gambacorta (2009), among others, argues that the low interest rate environment prior to the crisis encouraged bank risk-taking. Further, deregulation of financial markets in the United States and elsewhere during the 1970s and 1980s lead to significant weaknesses in the regulatory framework (Kane, 2012). While there are various institutional reasons that have contributed to the housing bubble in the United States, prolonged tranquil times are generally conducive for the build up of risks before an adjustment takes place. An important lesson from the global financial crisis that ended the preceding decades of moderate business cycle fluctuations known as the Great Moderation is, thus, that price and output stability by themselves do not ensure financial stability (Mishkin, 2011).

Due to the experience of the global financial crisis, academics and policy makers have come to the conclusion that better and stricter regulation of the financial system is required. Better regulation implies a macroprudential approach that goes beyond microstructures of individual institutions and integrates aggregate credit- and asset price dynamics over the business and credit cycle (Hanson et al., 2011). Stricter regulation requires higher capital ratios to be adopted (Admati and Hellwig, 2013), a solution for the too-big-too-fail problem to be found (Zhou et al., 2012), and the inclusion of the shadow banking system in the regulatory framework (Gorton and Metrick, 2010).

1.2.2 Financial Globalization and International Capital Flows

The financial crisis created new challenges for financially integrated economies with respect to the management of capital flows. In this context, bank intermediated capital flows turned out to be the most volatile component in recent times. Surprisingly, emerging market countries which are generally associated with weaker institutions fared relatively better than some advanced economies that were hit by the subprime crisis and, subsequently, by the European sovereign debt crisis. Specifically, the capital flows to emerging countries recovered relatively quickly compared to the most severely hit advanced countries (Milesi-Ferretti and Tille, 2011). Harsh internal adjustments in the form of a contraction in domestic demand were often required nevertheless to rebalance the contraction of the financial account due to a reversal in capital inflows (Lane and Milesi-Ferretti, 2012). Gross capital flows came under scrutiny given their rapid expansion in the run-up to the

\[ \text{footnote} \]

See, for example, Brunnermeier (2009) for a detailed description of the 'originate-to-distribute' model in the US mortgage market. Caballero and Krishnamurthy (2009) show how the US securitization industry was promoted due to increasing demand for dollar denominated and safe assets from emerging markets.

\[ \text{footnote} \]

In close analogy, Eichengreen and Hausmann (1999) show how the choice of a fixed exchange rate regime reduces volatility in the first place, but also gives rise to the creation of excessive amounts of foreign currency liabilities ('original sin') that might turn into great financial sector risks when the fixed exchange rate needs to be adjusted.
According to a report by Brunnermeier et al. (2012), the rise of procyclical gross capital flows is mainly due to international wholesale funding of banks. During an economic upswing, measured risks are low, leading to a reduction in credit spreads which further fuels the credit boom. In such a low risk environment (‘risk-off’), the domestic retail deposit base does not expand at the same pace as credit, leaving banks to search on international capital markets for funding. However, when the risk perceptions of investors change, access to these ‘non-core liabilities’ erodes and poses challenges to financial stability.

Thus, the ability of banks to raise funding on international capital markets is strongly affected by global factors. Specifically, the switch to a ‘risk-on’ environment in the present funding model of regional banks on international capital markets in advanced and emerging markets leads to strong spillovers. Bruno and Shin (2015b) show that the leverage cycle of globally active banks determines intermediated capital flows into regional banks.

There are other explanations for the presence of global factors driving capital flows. Allen and Gale (2000) raise the point that financial contagion might occur due to cross-holdings of deposits. If regions are only integrated to a limited extend, such that the interbank market is incomplete, liquidity shocks can generate contagion. Since each region only pursues its own interest by ring-fencing its pool of liquidity, a coordination failure arises, which causes inefficient liquidation in the crisis region and its closest neighbours. Other models refer to portfolio effects to create co-movement in capital flows across different countries (Fostel and Geanakoplos, 2008; Pavlova and Rigobon, 2008).

The common ground of this literature is that the procyclical nature and strong international comovement in capital flow dynamics is to some extent due to frictions in the financial sector that intermediates these capital flows. Early evidence for the presence of financial linkages being responsible for crisis contagion is reported in Kaminsky and Reinhart (2000).

The empirical evidence suggests that financial integration exposes countries to the global financial cycle. The consequences for the implementation and conduct of macroeconomic policies to achieve a stabilisation of the domestic economy are far reaching. Bruno and Shin (2015a) find that US monetary policy has an impact on emerging market bank risk taking. For many countries, an extra-ordinary loose monetary policy stance in western developed countries in response to the global financial crisis is conducive in order to

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9 As Obstfeld (2012b,a) argues, future challenges to global financial stability are most likely to arise from gross capital flows that have reached levels up to 15 percent of GDP in the US economy prior to the crisis. Other contributions to the distinction between gross and net capital flows are made by Broner et al. (2013) and Bruno and Shin (2015b).
cushion the consequences of asset fire-sales and liquidity spirals. However, these liquidity injections pose new challenges for policy makers in financially integrated economies due to monetary spillovers. Specifically, the impact of gross inflows on domestic credit conditions might counteract domestic requirements given the cyclical conditions. Therefore, it is questionable if monetary independence can be maintained with a fully liberalized capital account (Rey, 2013).

As a consequence, with the onset of the global financial crisis doubts have intensified about the benefits of financial integration, or if the downsides outweigh the upsides of being more interconnected (Stiglitz, 2010). This is even more the case in the light of missing clear-cut evidence for positive growth effects of financial integration (Kose et al., 2009). Although the literature does not achieve consensus here, one might argue that in regions already highly integrated in goods markets and pursuing a common monetary policy - as in the Euro area - ever closer financial integration is desirable (Brunnermeier et al., 2012). Under such conditions, the benefits of integrated liquidity markets may also fully play out and reduce the probability of future crises (Guembel and Sussman, 2014).

However, for the remaining regions, recent volatility in capital flows requires a tighter policy surveillance and active management of the capital account. This implies considering macroprudential policies that might extend to capital controls (Ostry et al., 2011). In order to formulate good policies, however, understanding what drives cyclical capital flows to open economies and how financial intermediaries contribute to the propagation of domestic and foreign shocks is required. Chapter 2 contributes to this question by providing a quantitative assessment on what drives cyclical capital flows to Mexico, a widely studied emerging country, and which financial frictions matter for emerging market business cycles.

1.2.3 Rethinking Sustainability of Public Finances

The question of sustainability of public finances is inherently complicated by the fact that it is not only the ability-to-pay of a government matters to ensure that governments service their debt obligations, but also its willingness-to-pay. Lending to a sovereign government is fundamentally different from domestic loan markets. Typically, there is no collateral pledged or assets seizable against a foreign sovereign entity. This naturally gives rise to an enforcement problem of debt contracts, as governments might strategically refuse to service their loan obligations. Therefore, public finances can be considered sustainable as long as government repayment incentives are in line with creditors proprietary interest that loans are repaid.

Economic theory suggests different answers to the question of why governments service
their debt. Most approaches are related to a sanctioning mechanism that is linked in one way or another to the loss of access to international capital markets conditional on outright sovereign default, either due to creditor retaliation (Eaton and Gersovitz, 1981) or legal sanctions (Pitchford and Wright, 2012). In the quantitative literature on sovereign debt, the evolution of sovereign credit risk is typically assessed under additional indirect default penalties, namely in the form of domestic output losses (Arellano, 2008). However, the existence of output losses conditional on historic sovereign default events is somewhat controversial as there remain endogeneity issues and a measurement problem in any empirical approach. Studies that look for the impact effect of sovereign default find relatively modest output losses in the range of 0.5 to 2.0 percent reductions in GDP growth (Sturzenegger and Zettelmeyer, 2007; Borensztein and Panizza, 2009). However, Levy-Yeyati and Panizza (2011) find that crises precede the actual default event, indicating that the costs of sovereign default are driven by the anticipation of debt repudiation.

The size of output losses determines in quantitative models of sovereign debt the endogenous borrowing limit since it is the presence of a default penalty that encourages debtor discipline (Dooley, 2000). The default penalty thereby helps to overcome the enforcement problem, leading to an improved ex ante efficiency of government borrowing. However, this comes at a lower ex post efficiency, given that actual default events are more costly. Gai et al. (2004) explore this trade-off in great detail and show that there is an interior optimum, meaning that neither too low penalties nor too high penalties are desirable.

The example of the European sovereign debt crisis demonstrates that this trade-off between a high penalty of sovereign debt restructuring and and adequate borrowing costs in normal times for the public sector has lost its balance. This is primarily due to an adverse feedback loop between sovereign credit risk and financial instability giving rise to a bank-sovereign nexus which induces a high penalty arising from a simultaneous twin crisis in the banking sector and public finances.

The adverse feedback loop operates in two directions, from banks to sovereigns and vice-versa. Since government securities serve as a safe asset in the financial system (Iorgova et al., 2012), the adjustment of sovereign credit risk during a sovereign debt crisis affects negatively the funding conditions of banks (BIS, 2011). Specifically, banks are battered by the bank capital channel, as government securities on the bank balance sheet lose value (Gennaioli et al., 2014). Other ways of transmission of sovereign credit risk to banks occur via the collateral and the liquidity channel, given that margin requirements and risk premia on secured interbank markets rise. As a result, interest rates rise, thus dampening aggregate production. A reduction in investment and consumption reduces
the tax base of the government, eroding further the prospects for debt repayment, thus pushing up sovereign credit risk.

The other direction of causality in the adverse feedback loop between governments and banks is based in governments’ contingent liabilities due to bailout guarantees, either explicit or implicit, for the domestic banking system. Thereby, a banking crisis can trigger a hike in public sector debt beyond the socially desirable level if these contingent liabilities are not closely monitored. As a consequence, countries may find themselves on the ‘wrong’ side of the Laffer curve where tax increases imply a reduction in total tax revenues due to incentive effects on households (Trabandt and Uhlig, 2011).

In such an environment, a fast and significant debt restructuring effort might not necessarily look like the best solution either given the *ex post* inefficiencies related to it. As a matter of fact, policy institutions dealing with the execution of sovereign debt restructurings have been concerned about the financial turmoil caused by a default event that creates high output costs and contagion (IMF, 2013). As a bottom line, the bank-sovereign nexus in advanced, financially developed, and integrated economies has contributed to a significant bias in the *ex ante* efficiency of public debt, leading to an expansion of the debt ceiling at the expense of a reduction of the *ex post* efficiency.\(^{10}\) Under such conditions, overborrowing is a possible outcome that leads to a debt overhang in the public sector (Buchheit et al., 2013). In order to rebalance the efficiency trade-off, it is necessary to come to a better understanding of which financial market institutional features contribute to the bank-sovereign feedback loop. Chapter 3 provides a first step in this direction by analysing the implications of government securities as collateral on European interbank markets for fiscal policy decisions and financial stability.

Emerging from the banking and sovereign debt crisis with high levels of legacy debt, Eurozone countries remain vulnerable to exogenous shocks that can trigger runs on public finances. In a seminal contribution, Calvo (1988) highlights the role of expectations for the sustainability of public debt. He shows that multiple equilibria arise in a model where the government can partially repudiate on debt. Given the possibility of a Pareto-ranking of equilibria, the full repayment equilibrium is dominant due to the absence of penalties. Cole and Kehoe (2000) come to a similar conclusion when analysing the conditions for self-fulfilling liquidity crises within a model of sovereign debt with multiple equilibria and sun-spot shocks. They find that the higher the amount of public debt, the more likely a country finds itself within a ‘crisis zone’ where sun-spot beliefs about future debt repayment cause a roll-over debt crisis.

\(^{10}\)Empirical findings that point into the direction of higher debt sustainability in countries with a more developed financial system can be found in Reinhart et al. (2003) and Kraay and Nehru (2006).
Given these findings on the role of expectations for sovereign debt markets, there remains the risk that the debt crisis will return to center stage in the Euro area. In the face of the prospect that debt levels will remain elevated for a long time, there is the possibility that a severe enough shock will push a Euro member country onto a bad trajectory. Indicators for substantial excess returns that are not related to fundamental macroeconomic developments in sovereign bond markets have been present throughout the crisis.\textsuperscript{11} Chapter 4 provides one explanation of how such excess premia occur, under which conditions non-fundamental defaults might arise, and how frictions in the financial sector interact with the foundation of expectations. The findings call for an urgent solution for the yet unresolved problem of legacy debt in the public sector (Corsetti et al., 2015).

1.3 Summary and Main Findings

1.3.1 Review: Business Cycles with Financial Intermediation in Emerging Economies

The second chapter presents a quantitative assessment of financial market frictions and shocks to the foreign interest rate for open economy business cycles and procyclical capital flows. These are considered by the literature to be the main explanation for a set of stylized facts related to emerging market business cycles (Neumeyer and Perri, 2005; Uribe and Yue, 2006; García-Cicco et al., 2010). However, leveraged and internationally active banks play an increasingly important role for the intermediation of capital flows (Broner et al., 2013). Further, interest rate dynamics are typically not included in the empirical assessment of financial market frictions, potentially leading to a bias in the role of interest rate shocks. In addition, empirical findings suggest that global factors are driving capital flows (Bruno and Shin, 2015b).

In order to come to a quantitative assessment of these competing views on financial market frictions and shocks driving capital flows, I augment a standard small open economy business cycle model with leveraged capital importing banks that are subject to financial shocks that mimic a sudden change in investor preference for emerging market assets. The model is estimated in a Bayesian approach using quarterly Mexican data.

There are two main findings in that chapter. First, once I control for data on the foreign interest rate in the estimation part, its role in explaining macroeconomic dynamics is reduced significantly. In particular during the global financial crisis, financial sector

\textsuperscript{11}For an econometric approach, see Aizenman et al. (2013), Grauwe and Ji (2012) and Beirne and Fratzsch (2013) who all show that yields during European sovereign debt crisis cannot be fully explained by macroeconomic fundamentals.
Introduction

shocks have played a larger role for the explanation of investment and current account dynamics.

Second, I compare the financial sector model with two popular alternative approaches to modelling financial frictions, namely a debt elastic interest rate model put forward by García-Cicco et al. (2010), and an endogenous country risk premium coupled with a working capital requirement as presented by Chang and Fernández (2013). I find that the financial sector model is preferred on the basis of the marginal data density hinting towards the importance of both the financial accelerator mechanism arising from bank balance sheets and the role of financial sector shocks.

1.3.2 Review: Sovereign Risk, Interbank Freezes, and Aggregate Fluctuations

Chapter three studies the bank-sovereign link in a dynamic stochastic general equilibrium set-up with strategic default on public debt. The European sovereign debt crisis highlighted the pivotal role of public debt held by the domestic financial sector in creating a bank-sovereign link that amplifies adverse shocks. This mechanism was previously analysed either within the framework of a bank capital channel (Gennaioli et al., 2014) or on a qualitative basis (Niemann and Pichler, 2013).

The contribution of this chapter is threefold. First, it analyses the role of government debt as collateral on the domestic interbank market in shaping sovereign default risk and aggregate output dynamics. We find that given a procyclical liquidity value of government debt for bank intermediation, there arises a highly procyclical default penalty due to the collapse of interbank intermediation and a subsequent credit crunch. We think that the mechanism for the endogenous default penalty is more suitable for advanced and financially developed economies than the trade channel discussed by Mendoza and Yue (2012).

Second, we find that *ex ante* spillovers from sovereign risk to interbank intermediation amplify aggregate shocks. Spillovers from sovereign credit risk to interbank allocations are due to a reduced form modelling of risk premia related to the underlying collateral. These spillovers impede interbank intermediation, driving up interest rates that are used to finance working capital loans. As a result, sovereign risk spillovers lead to similar consequences as distortionary taxation on labour in other quantitative models of sovereign default (Cuadra et al., 2010).

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12 This chapter is joint work with Philipp Engler.
13 Barro (1976) provides a microfoundation for such a risk premium.
Third, we quantify the default penalty in the context of the European sovereign debt crisis using Spanish data. We find that (i) recessions preceding a default event are more severe if the bank-sovereign amplification channel is at play; (ii) there is a stronger impact effect of default on output compared to related studies (Mendoza and Yue, 2012), which contributes to higher debt levels; and (iii) recessions are more persistent due to the destruction of collateral held by banks. Thereby, the model provides an explanation for more severe and longer lasting recessions after financial crises (Cerra and Saxena, 2008; Reinhart and Reinhart, 2011).

1.3.3 Review: Uncertainty Shocks and Non-fundamental Debt Crises: An Ambiguity Approach

The last chapter analyses the relationship between macroeconomic fundamental uncertainty and sovereign credit risk. There is a growing interest in the macroeconomic literature on the effects of time-varying uncertainty on aggregate allocations. Examples include, but are not limited to, Bloom (2009) and Ilut and Schneider (2014). This chapter can be understood as an extension of the analysis of uncertainty in the context of quantitative models of sovereign default.

In a first step, macroeconomic fundamental uncertainty is approximated by forecaster’s disagreement about GDP growth. In an empirical analysis using a structural VAR for a group of Euro area member countries, I find that a rise in macroeconomic fundamental uncertainty increases the yield on credit default swaps, indicating a positive and quantitatively significant relationship of fundamental uncertainty on sovereign credit risk.

I then develop a simple theory of sovereign debt crises driven by uncertainty shocks that are modelled as changes in investors’ confidence in the macroeconomic fundamental of the economy based on the multiple prior model in conjunction with maxmin preferences (Gilboa and Schmeidler, 1989). I find that due to defaultable government debt, uncertainty feeds into investors’ beliefs about the fiscal sustainability of public debt. The simple model is used to characterize a critical zone of indebtedness where uncertainty shocks are able to cause a non-fundamental default. The model thereby provides an alternative mechanism to the one documented by Cole and Kehoe (2000) that is based on self-fulfilling debt crises.

The simple dynamic three-period model is then extended to a quantitative DSGE setting. Additionally, a bank-sovereign feedback channel, like the one described in Chapter 3, gives rise to an amplification mechanism for adverse shocks. The quantitative model is estimated using Spanish data. Forecasters’ disagreement about GDP growth is used to
identify time-varying macro uncertainty and, thus, the non-fundamental share of sovereign yields.

I find that an uncertainty shock raises the price of issuing debt, which in turn affects the optimal tax and borrowing decision of the government. The presence of financial frictions increases the size of this 'crisis zone.' Further, the model attributes a sizeable share of sovereign yields to time-variation in uncertainty. Thereby, the results provide an explanation for excess returns that have been observed during the European sovereign debt crisis.
CHAPTER 2

Business Cycles with Financial Intermediation in Emerging Economies

2.1 Introduction

Recent studies of business cycle fluctuations in emerging economies have reached a preliminary consensus that financial frictions coupled with exogenous shocks to the country specific interest rate are best capable to explain patterns in the data that are specific to emerging market business cycles. Specifically, García-Cicco, Pancrazi, and Uribe (2010) and Chang and Fernández (2013) compare empirically the financial friction view, introduced by Neumeyer and Perri (2005) and Uribe and Yue (2006), against the competing hypothesis of a frictionless real business cycle model with transitory and permanent shocks to total factor productivity (Aguiar and Gopinath, 2007). Nesting the two hypothesis in an encompassing model of an estimated small open economy, these authors find that shocks to the foreign interest rate explain a larger share of macroeconomic dynamics than permanent productivity shocks. A major shortcoming of this literature is, however, that interest rate data is typically not included in the estimation, thereby leaving substantial degrees of freedom to changes in the foreign interest rate for the explanation of macroeconomic dynamics in general, and to movements in the current account in particular.¹

In this paper, I revisit the role of interest rate shocks and financial frictions for emerging market business cycles when interest rate dynamics are explicitly controlled for in the estimation part. To this end, I extend the data of Uribe and Yue (2006) on the world interest rate and the country specific spread from 1994 to 2014. Regarding the modelling of finan-

¹By including data on the trade balance, these studies identify changes in foreign interest rates mostly through variation in the trade balance to output ratio. Notably, Chang and Fernández (2013) control for interest rate data in a robustness analysis, finding that this substantially reduces the contribution of such shocks to aggregate dynamics. In particular, the explanation of interest rate shocks for trade dynamics in a forecast error variance decomposition collapses from 41.2 percent to 18.1 percent.
cial frictions, I make two extensions that are motivated by the experience of the Asian crisis (1997/98) and the global financial crisis (2007/08). First, I take into account that capital flows are largely intermediated by leveraged financial institutions that contribute to the volatility of gross capital flows with potentially destabilizing consequences for the macroeconomy (Brunnermeier et al., 2012). Second, I allow for an additional source of exogenous variation in capital flows, which is motivated by the observation that international investors frequently retrieve capital from emerging markets without necessarily being related to local economic developments (Fostel and Geanakoplos, 2008). Thereby, I implicitly assume that some periods of capital outflows cannot be identified from observed country premia alone. The knowledge on how such phases of a global anxious economy are transmitted to the domestic macroeconomy is still very limited.

To this end, the standard model of a small open emerging market economy with a stochastic trend and a debt elastic interest rate as presented in García-Cicco et al. (2010) is augmented by a capital importing banking sector, modelled in close analogy to Gertler, Kiyotaki, and Queralto (2012). The arising endogenous leverage constraint of banks establishes a financial accelerator that alters the propagation of conventional shocks such as changes in the foreign interest rate or transitory and permanent productivity shocks. The model is then used to analyse financial sector shocks that mimic a global anxious economy through a dry-up in funding markets of banks. Finally, the model is estimated in a Bayesian approach using Mexican quarterly data over the period 1994:I to 2014:IV, including the foreign and domestic interest rates. This period was marked by pronounced capital flow volatility, featuring the Mexican currency crisis of 1995 and the global financial crisis.

In the model, the funding sources of banks are domestic deposits and international capital markets. While the interest rate on domestic bank deposits is endogenously determined, banks face an exogenous foreign interest rate that is subject to stochastic shocks. It is assumed that the agency friction that limits bank borrowing in equilibrium is asymmetric regarding the two funding sources. Such asymmetry might be the result of heterogeneous information between the two investor classes (Dvořák, 2003; Tille and van Wincoop, 2014), or due to limitations in the regulatory framework that lead to concerns over discrimination of foreign investors (Broner et al., 2010). As a result, there arises an interior solution for the ratio of domestic deposits to international capital even in the presence of a positive interest rate differential between domestic over foreign funding. In addition, the asymmetry assumption makes the net foreign asset position stationary.

\[^2\]In the literature, there exist various alternative expressions for a global anxious economy, such as a ‘surge’ or a ‘stop’ in capital inflows (Forbes and Warnock, 2012), or ‘risk on’ and ‘risk off’ episodes Benes et al. (2013).
thereby ‘closing’ the small open economy.\(^3\) The intuitive explanation for this result is that additional foreign borrowing tightens the overall leverage constraint faster than domestic funding through bank deposits. At a positive excess return on lending, bankers face a trade-off between lower funding costs and smaller loan volumes that let’s them decide over the optimal combination of domestic and foreign funding sources. In equilibrium, there arise different interest rate differentials between foreign interest rates, the domestic deposit rate and the bank lending rate.

The financial sector shocks are modelled as exogenous changes in the agency friction of bankers. Specifically, I analyse a financial shock that leads to an aggregate tightening of the agency friction between banks and domestic and foreign investors, and an international funding shock that increases the degree of asymmetry in the agency problem. I thereby loosely follow the idea of financial shocks introduced by Jermann and Quadrini (2012). However, while these authors claim that a financial shock arises from within the financial system, the financial sector shocks considered here are not limited to such interpretation, but are also intended to capture phases of a global anxious economy, which are external to the emerging market economy and its financial sector.

In the estimation part, I use foreign interest rates, domestic deposit and lending rates alongside common observables from the national account statistics to identify the financial frictions in the model. The results allow for a number of conclusions on the sources of financial market frictions in emerging economies. First, the parameter governing the debt elastic interest rate is estimated to be close to zero once accounting for interest rate dynamics in the model and an alternative way of ‘closing’ the model via the agency friction of capital importing banks. This indicates that changes in the net foreign asset position not necessarily induce a change in the external interest rate as assumed in the model of a debt elastic interest rate.

Second, I find that financial sector shocks contribute significantly to the procyclical nature of capital flows. They reduce investment, output and consumption and lead to an increase in the trade balance to output ratio. In response to a financial shock, foreign funding is reduced relatively more than domestic deposits, thereby leading to a rebalancing of bank liabilities towards domestic sources. This is in line with recent empirical findings on gross capital flows by Broner et al. (2013) who show that foreign funds are retrieved more strongly during crisis times.

Third, foreign interest rate shocks lead to similar responses in macroeconomic developments as financial sector shocks. In particular, an increase in foreign funding costs is recessionary, thus maintaining the counter-cyclical nature of interest rates. Shocks are

\[^3\text{See Schmitt-Grohé and Uribe (2003) for alternative ways to closing small open economy models.}\]
amplified by the financial accelerator in the model, leading to higher amplitudes the more leveraged is the financial sector of the economy.

Fourth, a historical decomposition shows that once foreign interest rate changes are included in the estimation and financial sector shocks are allowed for, the latter account for a high degree of variation in total investment and the trade balance to output dynamics, while the contribution of shocks to the foreign interest rate is significantly reduced.

Finally, I compare the financial sector model proposed in this paper against two popular modelling approaches of financial frictions in emerging market economies when all models are estimated using the same data sample including the foreign interest rate. Specifically, I estimate a modified version of García-Cicco et al. (2010) that features a debt elastic interest rate parameter, and a version of Chang and Fernández (2013) with an endogenous country spread that is elastic to changes in domestic total factor productivity and that features a working capital requirement. Based on the evaluation of the marginal data density, I find that the financial sector model is better capable to explain the observed Mexican data.

Related literature. The results of the paper are most closely related to a literature that rationalises a set of stylized emerging market business cycle facts with the presence of financial frictions (García-Cicco et al., 2010; Chang and Fernández, 2013). Fernández and Gulan (2015) show that the counter-cyclical nature of the country premium emerges endogenously once accounting for a BBG-type financial accelerator and endogenous leverage in the entrepreneurial sector of the economy. This paper separates from their work since it puts the focus on the role of capital importing intermediaries for business cycle dynamics. Financial frictions originating in the financial sector as explanation of long-run emerging market business cycles facts have not been considered in the literature. One notable exception is Oviedo (2005), who introduces costly financial intermediation into the setup with shocks on the international interest rate. However, Oviedo (2005) does not put the financial sector at the center stage of emerging market business cycles. In fact, the financial sector with costly intermediation reduces aggregate volatility, whereas the here presented model with an agency problem enforces the cycle.

Previous literature on the financial sector in emerging markets relates to the concentration of currency crises during the second half of the 1990s, which often have been coupled with banking crises (Kaminsky and Reinhart, 1999). This phenomenon has spurred research in the form of so-called third generation crisis models that focus on the liability side of banks. Specifically, Chang and Velasco (2000, 2001) concentrate on the maturity mismatch problem that can cause bank runs by foreign creditors and domestic depositors alike. While this literature focusses on the reasons that endogenously lead to a sudden
investors run, this paper is agnostic about the actual source of a shock in the financial sector and focusses on its quantitative importance for macro variables at the business cycle frequency.

The idea of a stochastic shock within the agency problem between foreign investors and domestic banks is related to a strand of literature that investigates the relationship of global market liquidity and investors' appetite for emerging market assets, which leads to a comovement in capital flows (Fostel and Geanakoplos, 2008; Pavlova and Rigobon, 2008). As Brunnermeier and Pedersen (2009) show, asset market liquidity is closely related to funding liquidity of banks. Bruno and Shin (2015b) highlight the role of a global bank leverage cycle that is transmitted through procyclical capital flows. In a companion paper, Bruno and Shin (2015a) find empirical evidence for a risk-taking channel of US monetary policy in emerging markets. The here presented model takes account of this international transmission channel of US monetary policy, since a reduction in the world interest rate leads to an increase in bank leverage domestically. However, the historical decomposition shows that the quantitative effects are negligible. Thereby, the present model links the literature on macro financial developments and global liquidity (Rey, 2013) to the literature on endogenous sudden stops (Mendoza and Smith, 2006; Mendoza, 2010).

The introduction of financial sector shocks in this paper contributes also to the fundamental discourse on the question whether domestic developments or global factors drive the business cycle of emerging economies. Calvo et al. (1996), among others, have argued in favour of external factors as the dominant source of aggregate volatility, alluding to the experience of volatile international capital flows during the 1990s. This paper makes a strong case that shocks to the international interest rate alone cannot account for such volatile capital flows, but need to be accompanied by additional external factors. At the same time, I acknowledge the reduced form character of the financial sector shocks that might partly reflect other explanations such as changes in the volatility of country premia as put forward by Fernández-Villaverde et al. (2011).

Finally, the paper is related to the literature that investigates the question of excessive borrowing from abroad as in Jeanne and Korinek (2010), Bianchi (2010), Benigno et al. (2013) and Farhi and Werning (2014), among others. I find that the financial accelerator in the banking sector makes an economy more vulnerable to domestic and external shocks that account for an important share of aggregate fluctuations. However, this paper does not take a normative approach and I leave it for future research to evaluate different policy measures such as macroprudential regulation or capital controls to counteract the adverse consequences of such shocks.
The remainder of this paper is organized as follows. The next section documents stylized facts that elicit the pro-cyclical nature of gross capital flows and presents key Mexican business cycle moments from the real and financial sector. Section 2.3 describes the model. Section 2.4 presents the estimation strategy while the main findings are presented in Section 2.5. A comparison with alternative specifications of financial frictions is presented in section 2.6 before the final section concludes.

2.2 Cyclicality of capital flows and financial sector aggregates

This section documents the cyclical pattern of gross capital inflows for a range of emerging and developed small open economies. It is shown that the cyclicality of gross capital flows increases with the perceived riskiness of an economy. Then, business cycle statistics from the Mexican real and financial sector are presented. The patterns observed in gross capital flows become most apparent in the funding structure of banks that intermediate capital flows. In particular, the share of foreign liabilities in total liabilities decreases during recessions.

2.2.1 Gross capital inflows to emerging economies

For the extraction of cyclical information of gross capital inflows, I construct a panel of 33 countries of which there are 21 classified as emerging market economies and 12 as developed small open economies. I follow Aguiar and Gopinath (2007) in the classification where applicable and group European Union new member countries as emerging market economies. The longest series in the unbalanced panel range from 1980:I to 2011:I, the shortest from 1997q1 to 2011q1. Comprehensive details on the list of countries included in the panel and the series length can be obtained in Table A.1 in the Appendix.

I follow Broner et al. (2013) for the construction of gross capital inflows (GCI). Specifically, capital inflows are defined as the sum of three positions of the financial accounts: foreign direct investment in the country, portfolio investment liabilities, and other investment liabilities. Data on financial derivatives is left out due to their relatively small amount and their limited availability across countries. Since flows in reserve assets are operated via monetary authorities, they are also not considered here. GCI are reported in US dollars, deflated by the US deflator and detrended using the HP-filter for quarterly series. The data comes from the IMF’s Balance of Payments statistics.

Output data is taken either from the IMF’s International Financial Statistics or the
Figure 2.1: Lead-lag structure of cross-correlations.

(a) Emerging market economies

(b) Developed economies

(c) Risk classification

(d) Evolution over time

Notes: GDP in percentage deviations from HP-trend, GCI are in deviations from HP-trend. Panel (a) and (b): Cross-correlations for the period 2000q1-2010q4. Panel (c): 'High risk' sub-sample: Argentina, Brazil, Bulgaria, Indonesia, Latvia, Peru, Romania, Russia, South Africa, Turkey. 'Low risk' sub-sample: Chile, Czech Republic, Estonia, Hungary, Israel, Korea, Mexico, Poland, Slovak Republic, Thailand. Sample 2000q1 - 2010q4. Panels (d): The 'recent sample' covers 2000q1 - 2010q4. The 'maximum sample length' varies across countries due to data availability, see Table A.1 for details.

Country codes: AG=Argentina, AU=Australia, OE=Austria, BR=Brazil, BL=Bulgaria, CN=Canada, CL=Chile, CZ=Czech Republic, DK= Denmark, ET=Estonia, FN=Finland, HN=Hungary, ID=Indonesia, IR=Ireland, IS=Israel, KO=Korea, LV=Latvia, MX=Mexico, NL=Netherlands, NZ=New Zealand, NW=Norway, PE=Peru, PH=Philippines, PO=Poland, PT=Portugal, RM=Romania, RS=Russia, SX=Slovak Republic, SA=South Africa, ES=Spain, SD=Sweden, TH=Thailand, TK=Turkey.

Source: Risk classification according to OECD’s Historical Country Risk Classification. See data appendix.
OECD’s Quarterly National Accounts when available over the entire sample length. Output was de-seasonalized and deflated when necessary. I obtain the cyclical component of the logarithm of GDP by extracting the trend with the HP-filter for quarterly series. The data is plotted for each country in the Appendix (Figure A.1).

Figure 2.1 depicts the lead-lag structure of the cross-correlations of GDP with gross capital inflows. Gross capital inflows into emerging markets are pro-cyclical and lead the cycle by one quarter, as shown in panel (a). The average coefficient of correlation between output and lagged gross capital inflows is relatively high with 0.44. This pro-cyclicality is less pronounced in developed economies. Panel (b) shows a flat curve for developed economies that translates into lower cross-correlations and peak on average two periods before the cycle with a correlation coefficient of 0.35.

The pro-cyclical pattern of gross capital inflows gets more distinct if one divides the emerging market economies in a low risk sample and a high risk sample. For this purpose, I evaluated over the period 2001:I to 2010:IV the average OECD’s Historical Country Risk Classification that measures a country’s credit risk on a scale between 0 (lowest risk) and 7 (highest risk). The resulting high risk group includes Argentina, Brazil, Bulgaria, Indonesia, Latvia, Peru, Romania, Russia, South Africa and Turkey and exhibits an average rating of 4.6. The low risk group has an average rating of 2.1 and covers Chile, Czech Republic, Estonia, Hungary, Israel, Korea, Mexico, Poland, Slovak Republic and Thailand. This information is summarized in panel (c). A country’s credit risk increases significantly the pro-cyclical pattern of capital flows. High risk countries have a coefficient of correlation of 0.54 versus 0.35 for the low risk group. This result corresponds with the empirical findings of Meller (2013).

Finally, a observation on the evolution of the pro-cyclical pattern of gross capital inflows over time is presented in panel (d). The lead lag structure for the longest available series are confronted with the recent period of financial globalisation, i.e. the years 2000 to 2010. For both emerging and developed economies, the pro-cyclical pattern has increased over time. The stylized facts presented in Figure 2.1 lead me to the hypothesis that pro-cyclical capital inflows may drive the business cycle in emerging market economies, and that the recent period of financial globalisation may have assisted this channel beyond the role of counter-cyclical interest rates. The next section presents statistics on leveraged financial institutions that intermediate capital flows and might amplify its cyclicality.

2.2.2 Mexican business cycle statistics

Next, business cycle statistics for the Mexican economy are briefly presented. The novelty is that the co-movement of financial sector variables is documented alongside traditional
emerging market characteristics.

Data on output, private consumption, investment and the trade balance are taken from the OECD Quarterly National Accounts statistics. Specifically, I am extending the quarterly series on Mexican national accounts data as described by Aguiar and Gopinath (2007) until 2014:IV. I take the logarithm and first differences of the data to obtain output growth, consumption growth and investment growth, denoted by \( g_x \) for \( x \in \{c, i, y\} \) respectively. The trade balance to output ratio (\( \text{tby} \)) is computed as exports net of imports over output.

Regarding the foreign interest rate, I follow Uribe and Yue (2006) and update their data series up to the final quarter of 2014. Specifically, I approximate the world interest rate (\( R_t \)) by the US real interest rate. Data on \( R_t \) is obtained by taking the yield on US treasury bills with a remaining maturity of three months and subtracting the average deflator on US GDP in the previous four quarters. Further, I use data over Mexican EMBI+ stripped spreads from JPMorgan in order to construct a series on the country specific spread \( S_t^c \). Table 2.1 summarizes the results of the obtained business cycle statistics.

The foreign interest rate for Mexico is plotted in Figure 2.2a together with the quarterly rate of change in aggregate production. There are two outstanding recessions in the data sample, the first in the Mexican currency crisis of 1995 and the second the course of the global financial crisis of 2009. Remarkably, albeit a similar contraction in the business cycle, the interest rate change observed in 2009 does not replicate the spike in the country spread seen in 1995. Together with the low level of foreign interest rates for Mexico since 2004, this is a first indications that other explanations are needed to account for sudden movements in capital flows.

Next, balance sheet information for the Mexican financial sector is taken from the Mexican national supervisory authority (CNBV). It provides historical monthly balance sheet data on all banks belonging to the universal banking scheme, including foreign owned subsidiaries. These commercial banks control more than 50 percent of Mexico’s financial system (Banco de México, 2010). From this data I construct quarterly series from 2006:I to 2014:IV for the capital ratio of banks.\(^4\) Further, I use data from the Mexican central bank on the composition of bank liabilities that dates back to December 2000. I construct the share of foreign liabilities over total deposits, thus indicating the dependence of the Mexican financial system on access to international capital markets.

Figure 2.2b plots the quarterly rate of change in the capital ratio and the level of the funding ratio against output growth. The data shows that the global financial crisis was

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\(^4\)In particular, I use the ratio ICAP Total that measures bank capital over total risk weighted assets. See http://portafoliodeinformacion.cnbv.gob.mx/bm1/Paginas/alertas.aspx for further details.
accompanied by a modest deleveraging in the Mexican financial system, captured by a slight increase in the growth rate of the capital ratio in 2009. Further, the crisis heavily affected the composition of bank balance sheets. In particular, there is a pronounced reduction in the relative importance of foreign funding for Mexican banks.

Next, I investigate the second moments of the data series described before. In the real sector, Mexico exhibits typical emerging market business cycle patterns. These are a relative volatility of consumption over output greater than one, denoted as excess volatility in consumption, high volatility of the growth rate of investment with a standard deviation of 4.66 percent, and a strongly counter-cyclical current account. On the financial side, note that the bank capital ratio is counter-cyclical, whereas the funding ratio is procyclical. The next section presents a model of an emerging market with capital importing banks that is intended to capture the empirical regularities documented in this section by allowing financial frictions within banks play a major role in shock propagation and amplification.


2.3 The model

The core framework is a neoclassical model of a small open economy with a stochastic growth rate as introduced by Aguiar and Gopinath (2007). Further, I allow for a debt elastic interest rate as discussed in García-Cicco et al. (2010). I augment this framework with a financial sector related to Gertler, Kiyotaki, and Queralto (2012). The novelty is that banks\(^5\) simultaneously issue domestic deposits and borrow from international capital markets. An agency friction between lenders and banks poses a constraint on the leverage ratio in the financial sector. In the presence of an exogenous foreign interest rate, a corner solution in the composition of bank liabilities is avoided due to an asymmetry in the agency friction. This determines the asset positions of domestic households and foreign bank liabilities even in the presence of a non negligible funding spread, thereby closing the net foreign asset position of the small open economy without the need to impose additional assumptions on changes in the foreign interest rates (Schmitt-Grohé and Uribe, 2003). This setting is then used to analyse the propagation of shocks to the foreign interest rate and financial sector shocks reflecting a tightening of funding conditions in the banking sector.

2.3.1 Households

There is a measure one continuum of identical households in the economy. Households engage in consumption, saving and labour supply. In period \(t\), the representative household demands deposits \(D_t\) from domestic banks for consumption smoothing purposes. Deposits pay-off an interest rate \(R_{t+1}^d\) in period \(t + 1\).

In each household, there is a fraction \(1-f\) of workers and a fraction \(f\) of bankers. Workers supply labour to the non-financial sector and receive the wage rate \(w_t\) in return. Bankers own the financial intermediaries and contribute to the household’s income by transferring any profits from intermediation to the household. Within each household, there is perfect consumption insurance. Bankers face a finite time horizon in order to prevent that they can finance their entire activities from equity capital. Specifically, bankers have a non-contingent probability to exit the financial sector of \(1 - \sigma\) that entails an average survival time in the financial sector of \(1/(1 - \sigma)\) and leads to the amount \(f(1 - \sigma)\) of bankers leaving their sector each period. In order to keep a constant ratio of workers and bankers in the model economy, the same amount of workers switches to the financial sector. Bankers who exit give the accumulated net worth of their bank to the

\(^5\)In the following, I will use ‘financial intermediaries’ and ‘banks’ interchangeably, both meaning the same, i.e. the financial sector of the model economy.
household for consumption. New bankers will be provided with start-up funds, which are specified later.

Households choose the level of consumption \((C_t)\), labour supply \((h_t)\) and deposits \((D_t)\) to maximize expected utility

\[
\max_{\{C_t, h_t, D_t\}} E_t \left[ \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{C_t - \omega^{-1} \chi (\Gamma_{t-1} h_t)^{\omega} - 1}{1 - \gamma} \right) \right] \right]
\]

subject to a flow budget constraint

\[
D_t = D_{t-1} R^d_t + C_t - w_t h_t - \Pi_t.
\]

Preferences are non-separable in consumption and leisure (Greenwood et al., 1988) such that there is no income effect on labour supply. The parameter \(\beta \in (0, 1)\) denotes the time preference rate of the household while \(\zeta_t\) is a preference shock that introduces an additional demand-side variation in consumption. In the budget constraint, \(\Pi_t\) are the net profit transfers from capital goods producers and banks that are transferred in a lump sum to the household at the end of each period.

The model is non-stationary. Specifically, \(\Gamma_t\) denotes a permanent stochastic level in labour productivity that has a gross growth rate given by

\[
g_t \equiv \frac{\Gamma_t}{\Gamma_{t-1}}.
\]

Cumulative labour productivity enter household preferences at the disutility of hours worked mainly to insure a balanced growth path.\(^6\) To account for trend growth in the notation, let upper case letters denote variables that grow by the trend in equilibrium, whereas lower case letters are stationary with the exception of interest rate variables. Trending variables are transformed by using the definition \(x_t \equiv X_t / \Gamma_{t-1}\) to arrive at the stationary equilibrium.

The household’s optimality conditions for labour supply and saving in stationary form

\(^6\)As Aguiar and Gopinath (2004) note, the increase in disutility of work can be motivated by an increase in productivity of home production by a similar magnitude with a lag of order one.
are given by\footnote{A complete derivation of the stationarisation of the household problem and all following equilibrium conditions is provided in the Appendix.}

\begin{equation}
\chi h_t^{\omega-1} = \frac{w_t}{\Gamma_{t-1}} \tag{2.1}
\end{equation}

\begin{equation}
g_t = E_t \beta [\Lambda_{t+1} R^d_{t+1}] \tag{2.2}
\end{equation}

\begin{equation}
\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} = \frac{\zeta_{t+1} \left( c_{t+1} - \omega^{-1} \chi h_{t+1}^{\omega} \right)^{-\gamma}}{\zeta_t \left( c_t - \omega^{-1} \chi h_t^{\omega} \right)^{-\gamma}} \tag{2.3}
\end{equation}

The first equation represents optimal labour supply. The second condition is the standard Euler equation. Finally, \( \Lambda_{t,t+1} \) denotes the ratio of marginal utility of consumption as part of the stochastic discount factor of households.

The laws of motion for the growth rate \( g_t \) and the preference parameter \( \zeta_t \) are given by

\begin{equation}
\ln(g_t) = (1 - \rho_g) \ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon^g_t \tag{2.4}
\end{equation}

\begin{equation}
\ln(\zeta_t) = \rho_c \ln(\zeta_{t-1}) + \varepsilon^\zeta_t \tag{2.5}
\end{equation}

with \( i.i.d. \) and normally distributed disturbances of mean zero. The growth path has positive mean \( (g > 0) \).

### 2.3.2 Non-Financial Firms

There is a competitive non-financial sector in the model economy that produces a single tradable retail good, which serves as numeraire. Production takes place according to a Cobb-Douglas production technology

\begin{equation}
Y_t = a_t K_t^\alpha (\Gamma_t h_t)^{1-\alpha} \tag{2.6}
\end{equation}

where \( K_t \) denotes the capital stock, \( h_t \) labour input and \( a_t \) is a transitory shock on aggregate productivity. I assume that transitory productivity evolves according to

\begin{equation}
\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon^a_t, \quad \text{with} \ \varepsilon^a_t \sim N(0, \sigma^2_a) \tag{2.7}
\end{equation}

When output is available to firms at the end of the period, the wage bill \( w_t h_t \) is paid to the household. The non-depreciated physical capital stock \( (1 - \delta)K_t \) is sold to capital producers at the unitary price \( q_t \). The new capital stock \( K_{t+1} \) is purchased for production purposes in the subsequent period. It is financed by firms through the issuance of shares.
s_t that are bought by domestic banks according to the pricing equation

\[ q_t s_t = q_t k_{t+1} g_t. \]  

(2.8)

This condition equates the price of a unit of capital to the price of a financial claim. The arising equity contract between the bank and the non-financial firm yields the state-contingent real gross return \( R_k^t \). Thus, firms’ payments to their shareholders amount to \( R_k^t(q_{t-1} S_{t-1}) \), which implies zero profits in the non-financial sector. Firms maximize profits according to

\[
\max_{(K_{t+1}, h_t)} E_t \sum_{t=0}^{\infty} Y_t - w_t h_t - R_k^t q_{t-1} S_{t-1} + q_t (1 - \delta) K_t - q_t K_{t+1} + q_t S_t
\]

subject to the pricing of physical capital (2.8). The firm’s maximization problem yields the following first order conditions for factor demand in stationary form:

\[
(1 - \alpha) a_t k_t^\alpha g_t^{1-\alpha} h_t^{-\alpha} = \frac{w_t}{\Gamma_{t-1}} 
\]

(2.9)

\[
\alpha a_t k_t^{\alpha-1} g_t^{1-\alpha} h_t^{-1-\alpha} + q_t (1 - \delta) = R_k^t q_{t-1}
\]

(2.10)

The labour demand equation (2.9) sets the marginal product of labour equal to the wage rate. Capital demand equates the marginal product of capital to the rental rate of capital net of depreciation. From (2.10) follows the law of motion for return on capital as

\[
E_t [R_{t+1}^k] = E_t \left[ \frac{\alpha \left( \frac{y_{t+1}}{k_{t+1}} \right) + q_{t+1} (1 - \delta)}{q_t} \right]
\]

2.3.3 Financial Intermediaries

Banks provide finance to cash-in-advance constrained goods producers. To this end, they raise funds by issuing bank deposits to households \( (D_t) \) and by borrowing from international wholesale funding markets \( (B_t) \). Both contracts pay off a non-contingent interest rate of \( R_d^t \) and \( R_k^t \) in period \( t + 1 \), respectively.\(^8\)

By assumption, there are no frictions in lending between banks and goods producers. Specifically, firms receive funds \( s_t \) at price \( q_t \) at the beginning of the period and pay back a contingent rate of return \( R_k^t \) after selling the proceeds from production. This requires

\(^8\)Note that bank assets, liabilities and net worth all grow at the balanced growth rate. The notation adopted earlier with lower case letters therefore also applies in the banking section, following the definition \( b_t = B_t/\Gamma_{t-1} \).
a broader interpretation of the role of banks in the economy as the lending relationship is best compared to a banker being the shareholder that owns the firm.

The balance sheet constraint for an individual bank implies that the volume of lending \( q_t s_t \) is equal to bank net worth \( n_t \) and total liabilities consisting of deposits and international borrowing\(^9\)

\[
q_t s_t = n_t + b_t + d_t. \tag{2.11}
\]

The evolution of bank net worth at the beginning of period \( t \) can be described by the pay-off to loans funded in the previous period net of payments to lenders

\[
n_t = R_k^t q_{t-1} s_{t-1} g_{t-1} - R_b^t b_{t-1} g_{t-1} - R_d^t d_{t-1} g_{t-1},
\]

where \( d_t \) denote deposits and \( b_t \) international borrowing in stationary form. Let \( \phi_t \equiv b_t/(q_t s_t) \) define the fraction of bank assets that are financed through international capital markets, which I will henceforth refer to as the international funding ratio. Using the balance sheet constraint of banks and the definition of the funding ratio, the evolution of net worth can be reformulated as

\[
n_t = \left[ R_k^t - R_d^t + \phi_t^{-1} (R_d^t - R_b^t) \right] q_{t-1} s_{t-1} g_{t-1} + R_d^t n_{t-1} g_{t-1}. \tag{2.12}
\]

To understand the role of banks in the economy, one might think of specialists who provide their informational advantage over domestic non-financial firms to domestic and foreign investors.\(^10\) I follow Gertler, Kiyotaki, and Queralto (2012) in the adoption of an agency friction that gives rise to an endogenous leverage constraint. Such a friction can be motivated by the likely information asymmetry between banks and lenders that allows for the possibility of moral hazard, meaning that bankers do not act in the interest of their creditors. As a short-cut to implement this idea, let bankers have the possibility to divert a fraction of assets in order to enjoy a private benefit.\(^11\)

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\(^9\)There is no bank specific decision that would require to keep track of individual decisions. For the ease of notation I therefore omit the subscript \( i \) in the description of the model and highlight later where this result arises from.

\(^10\)According to Diamond (1984), it is optimal to delegate monitoring activities to local financial intermediaries if they have a comparative advantage of seeking information about investment projects.

\(^11\)This is similar to the idea of costly state verification due to asymmetric information (Townsend, 1979). Holmström and Tirole (1997) show in a model with uninformed investors, capital constrained banks and firms how bank monitoring helps firms to pledge a higher share of project returns to potential investors, establishing that monitoring and collateral are partial substitutes. Since intermediaries themselves have a moral hazard problem due to costly monitoring on their side, uninformed investors enforce market-determined leverage ratios. Hence, the amount of uninformed capital that a bank can attract depends on its capital.
With respect to the moral hazard problem, I further assume that the overall agency friction gets tighter the higher is the share of foreign liabilities of banks. This can be motivated in a model with heterogeneous information of domestic and foreign lenders (Dvořák, 2003; Tille and van Wincoop, 2014), or in the presence of poor regulatory quality that induces foreign lenders to think that they are discriminated in case a crisis occurs (Broner et al., 2010). Formalizing these ideas, let the share of bank assets subject to moral hazard be given by

\[ \Theta(\varphi_t, \vartheta_t, \nu_t) = \theta (\vartheta_t + \nu_t \varphi_t) . \] (2.13)

While the parameter \( \theta \) governs the overall tightness of the leverage constraint in the banking sector of the small open economy, the degree of asymmetry between domestic deposits and international funds in the agency friction is determined by \( \nu_t \). With \( \nu_t > 0 \), the higher the share of bank liabilities financed from abroad, the tighter is the overall financing constraint.

The funding conditions for banks are subject to two sorts of shocks originating in the financial sector, captured by the two exogenous components \( \vartheta_t \) and \( \nu_t \) in (2.13). First, a financial shock \( \vartheta_t \) leads to a general change of the financing constraint both vis-à-vis domestic and foreign liabilities. Second, an international funding shock \( \nu_t \) captures a change in the asymmetry between domestic and foreign funding sources. With an increase in \( \nu_t \) and everything else equal, banks are facing a tighter financing constraint when borrowing from abroad compared to domestic deposits. This source of time-variation in financing restrictions for emerging market economies on international capital markets is meant to capture times when international investors are anxious about investing into emerging economies, as has been described by Fostel and Geanakoplos (2008).\(^{12}\) Very strong changes in \( \vartheta_t \) and \( \nu_t \) are also capable to capture episodes of boom and bust cycles in capital flows or sudden stops (Reinhart and Reinhart, 2009; Calvo et al., 2008).

I normalize \( \vartheta_t \) to one and let the mean of \( \nu_t \) be given from the interval \( \nu \in (0, 1) \). The laws of motion of the financial shocks are

\[ \ln(\vartheta_t) = \rho_\vartheta \ln(\vartheta_{t-1}) + \varepsilon^\vartheta_t, \] (2.14)
\[ \ln(\nu_t) = (1 - \rho_\nu) \ln(\nu) + \rho_\nu \ln(\nu_{t-1}) + \varepsilon^\nu_t. \] (2.15)

The agency problem is known to domestic and foreign creditors. If bankers decide to divert assets, lenders will be left with the remaining assets \( 1 - \Theta(\varphi_t, \vartheta_t, \nu_t) \). Creditors

\(^{12}\)One can further think of changes in financial conditions induced by an anxious investor who perceives a deterioration of her monitoring capability in times of global distress and requires in response higher bank capital levels for monitoring-intense activities.
are only willing to lend up to an amount that insures that the franchise value of bankers \( V_t \) is at least as high as the private benefit enjoyed conditional on diverting assets. The incentive compatibility constraint aligns the objective of the banker with the interest of the lenders takes the form:

\[
V_t(s_t, \varphi_t, n_t) \geq \Theta(\varphi_t, \vartheta_t, \nu_t) q_t s_t
\]

(2.16)

Bankers maximize their terminal wealth by choosing the optimal funding structure \( \varphi_t \) and the loan volume \( s_t \). Being confronted with a financing constraint, it is optimal for them to accumulated retained earnings in order to relax this constraint until exiting and returning to the household. Hence, the maximization problem in recursive notation takes the form

\[
V_t(s_t, \varphi_t, n_t) = E_t \beta \Lambda_{t,t+1} \left[ (1 - \sigma) n_t + \sigma \max_{\{s_t, \varphi_t\}} \{ V_{t+1}(s_{t+1}, \varphi_{t+1}, n_{t+1}) \} \right],
\]

subject to the evolution of net worth (2.12) and the incentive constraint (2.16) while \( \beta \Lambda_{t,t+1} \) denotes the stochastic discount factor of the representative household who owns the banks.

In order to solve the maximization problem, it is useful to refer to the following definitions,

\[
\Omega_{t+1} = 1 - \sigma + \sigma \varpi_{t+1}
\]

(2.17)

\[
\eta^k_t = E_t \beta [ \Lambda_{t,t+1} \Omega_{t+1}(R^{k}_{t+1} - R^{d}_{t+1}) ] g_t
\]

(2.18)

\[
\eta^b_t = E_t \beta [ \Lambda_{t,t+1} \Omega_{t+1}(R^{d}_{t+1} - R^{b}_{t+1}) ] g_t
\]

(2.19)

\[
v_t = E_t \beta [ \Lambda_{t,t+1} \Omega_{t+1} R^{d}_{t+1} ] g_t
\]

(2.20)

The term \( \varpi_t \) denotes the value of a unit of net worth to the banker. Consequently, \( \Omega_{t+1} \) is the expectation of this value from the perspective of a banker that faces a probability \( (1 - \sigma) \) of exiting at the beginning of the consecutive period. Next, \( \eta^k \) denotes the excess value of assets over deposits. It is given by the difference between the return on assets and the domestic deposit rate, which will be referred to as the domestic spread henceforth, given by \( S^{\text{dom}}_t \equiv R^k_t - R^d_t \). On the other hand captures \( \eta^b \) the excess value to a bank of substituting international funding for domestic deposits. The cost advantage of the two funding sources is given by the international spread, \( S^{\text{int}}_t \equiv R^d_t - R^b_t \). Finally, \( v_t \) represents the savings for an additional unit of net worth given that a bank has to incur lower funding costs from issuing deposits.
Using the definitions above allows to re-write the objective function of the banker as\(^\text{13}\)

\[
\varpi_t n_t = \max_{\{s_t, \varphi_t\}} \left[ (\eta_t^s + \varphi_t \eta_t^b) q_t s_t + v_t n_t \right]
\]

s.t.

\[
(\eta_t^s + \varphi_t \eta_t^b) q_t s_t + v_t n_t \geq \Theta(\varphi_t, \vartheta_t, \nu_t) q_t s_t
\]

(2.21)

Assuming that banks have a non-negative excess return on loans, equation (2.21) holds with equality at all times. The combined first order conditions of this problem yield a relationship for the optimal funding structure of a bank as

\[
\varphi_t = \frac{1}{(s_t - 1)} \left[ \frac{\vartheta_t}{\nu_t} - \left( \frac{\eta_t^s}{\eta_t^b} \right) s_t \right].
\]

(2.22)

The liability side of a bank depends on the ratio between excess returns of assets over deposits and the excess value of substituting international wholesale finance for domestic deposits. Further, the funding structure is directly affected by a financial and the international funding shock. An increase in \(\nu_t\) leads to a reduction in foreign borrowing of banks. However, the direction of the effect of a sudden tightening of financial conditions on the composition of liabilities is \textit{a priori} unclear, since the ratio of excess returns adjusts endogenously to the new borrowing conditions.

Total bank liabilities are limited in the level of accumulated net worth. The maximum eligible leverage ratio of a bank that satisfies the incentive constraint is given by equation (2.16) holding with equality

\[
(\eta_t^s + \varphi_t \eta_t^b) \phi_t n_t + v_t n_t = \Theta(\varphi_t, \vartheta_t, \nu_t) q_t s_t \iff \phi_t = \frac{v_t}{\Theta(\varphi_t, \vartheta_t, \nu_t) - (\eta_t^s + \varphi_t \eta_t^b)}
\]

(2.23)

where the definition of the leverage ratio is given by

\[
\phi_t \equiv \frac{q_t s_t}{n_t}.
\]

(2.24)

Under the given assumptions, banks all behave identically such that the individual volumes of bank net worth can be simply aggregated by referring to a representative bank in the model. Given that a fraction of bankers exits every period, total net worth in the entire banking sector of the small open economy evolves as the sum of net worth accumulated by already existing banks that stay until next period \((n_t^e)\) and net worth of

\(^{13}\text{The detailed derivations of results are provided in the Appendix A.4.}\)
newly entering bankers \( (n^n_t) \),

\[
n_t = n^e_t + n^n_t.
\]

With \( \sigma \) denoting the probability of remaining banker until next period, and \( \varrho \) the fraction of bank assets of the exiting bankers being used as fresh start-up funds, total aggregate bank net worth evolves according to

\[
n_t = \sigma \left\{ \left[ R^k_t - R^d_t + \varphi_{t-1} (R^d_t - R^p_t) \right] q_t s_{t-1} g_{t-1} + R^d_t n_{t-1} \right\} + \varrho (1 - \sigma) R^k_t q_{t-1} s_{t-1} g_{t-1}.
\]

(2.25)

### 2.3.4 Capital Producers

There is a competitive sector of identical capital producing firms that is owned by households. Capital producers buy the non-depreciated physical capital stock \( q_t (1 - \delta) K_t \) from non-financial firms at the end of each period and invest the amount \( I_t \) that yields the gross newly built capital \( \Phi (I_t / K_t) K_t \). Only net investment is subject to quadratic adjustment costs, which are governed by the function \( \Phi (I_t / K_t) \) that satisfies \( \Phi (\delta) = \delta \) and \( \Phi' (\delta) = 1 \). Hence, adjustment costs are zero in the deterministic steady state. Capital producers sell the newly produced capital stock \( K_{t+1} \) at the competitive price \( q_t \) to non-financial firms. The related profit maximization problem takes the form

\[
\max_{\{K_{t+1}, I_t\}} E_t \sum_{t=0}^{\infty} \beta^t \{ \Lambda_{t,t+1} [q_t K_{t+1} - q_t (1 - \delta) K_t - I_t] \}
\]

subject to the law of motion of the capital stock

\[
K_{t+1} = (1 - \delta) K_t + I_t - \frac{\phi_k}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t
\]

(2.26)

This maximization problem yields the price of physical capital as the marginal cost of investment:

\[
q_t g_t = g_t + \phi_k \left( \frac{k_{t+1}}{k_t} g_t - g \right) g_t - E_t \beta \Lambda_{t,t+1} \left[ (1 - \delta) - q_{t+1} (1 - \delta) - \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) \right] - \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) - \frac{\phi_k}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2
\]

(2.27)
2.3.5 Equilibrium

As standard in models of a small open economy, the international interest rate is assumed to be exogenously given. Further, I decompose the foreign interest rate into its two components, namely the world interest rate and the country specific spread. The intention here is to come to an assessment of the relative importance of changes in these two components that are under focus in the light of exit strategies of major central banks from the extra-ordinary lax monetary policy stance since the onset of the global financial crisis. Let the foreign bank lending rate be given by

$$R^b_t = S^c_t R^*_t + \psi(e^{b_t-b} - 1).$$

As García-Cicco et al. (2010) discuss in detail, the parameter $\psi$ can be interpreted as a financial friction in reduced form that achieves to account for patterns in emerging market business cycles. In particular, it allows to generate an auto-correlation function in the trade balance to output ratio close to the one in the data. The world interest rate $R^*_t$ and the country spread $S^c_t$ each follow a moving average process with independent shock processes according to

$$\ln(R^*_t) = (1 - \rho_{R^*})\ln(R^*) + \rho_{R^*} R^*_{t-1} + \varepsilon^R_t,$$

$$\ln(S^c_t) = (1 - \rho_{S^c})\ln(S^c) + \rho_{S^c} S^c_{t-1} + \varepsilon^S_t,$$

where variables without a time-subscript denote steady state values.

Labour markets clear by equating the labour supply condition (2.1) with labour demand (2.9)

$$(1 - \alpha)g_t k_t^\alpha l_t^{1-\alpha} h_t^{-\alpha} = \chi h_t^{\alpha-1}. \quad (2.28)$$

Next, the trade balance-to-output ratio in this model amounts to aggregate production net of domestic absorption divided by output:

$$TB_t = Y_t - C_t - I_t$$

$$\Leftrightarrow \quad \frac{TB_t}{Y_t} \equiv lby_t = 1 - \frac{c_t + i_t}{y_t} \quad (2.29)$$

The aggregate resource constraint is given by

$$y_t = c_t + i_t - R^b_t b_{t-1} - b_t. \quad (2.30)$$
If markets for labour, capital and loans clear, also the goods market clears according to Walras’ law. Given the previous model description, the decentralized equilibrium is defined as a stochastic sequence of allocations \( \{ c_t, h_t, s_t, k_t, R^d_t, R^b_t, R^k_t, b_t, q_t, g_t, i_t, n_t, \varphi_t, \Omega_t, y^b_t, y^d_t, v_t, \omega_t, \phi_t, \lambda_t, \lambda_{t,t+1}, tby_t, \Theta_t \} \) that, given the initial capital stock \( k_0 \), the initial deposit and debt positions of households and banks, \( d_0 \) and \( b_0 \), and the exogenous processes \( \{ a_t, g_t, \zeta, \theta_t, \nu_t, R_t^*, S^c_t \} \), satisfies i) the optimality conditions of all agents in the model and ii) market clearing on all markets.\(^{14}\)

### 2.4 Estimation

#### 2.4.1 Calibration and prior distributions

The quantitative evaluation of the model focusses on the case of Mexico over the last two decades. This period was characterized by highly volatile capital flows due to two major crises, the currency crisis of 1995 and the global financial crisis of 2007/08.

To evaluate the quantitative fit of the model, I will refer to a partial calibration and estimation approach. Calibrated parameters are fitted either using long-run moments from Mexican data of the financial sector or values commonly used in the literature on emerging market business cycles. Further, parameters governing financial frictions (\( \theta, \nu, \psi \)), investment adjustment costs (\( \phi_k \)), the steady state growth rate (\( g \)), the time preference rate of households (\( \beta \)), as well as all parameters governing the shock processes are estimated with Bayesian techniques.

Calibrated parameters are summarized in Table 3.2. I follow Aguiar and Gopinath (2007) and set the capital share of output to 0.32 and the coefficient of risk aversion to 2. I also adopt the depreciation rate of the physical capital stock of 0.05 that yields a steady state ratio of investment over GDP of 20.8 percent. The parameter \( \omega \) that governs the wage elasticity of labour supply follows the calibration from García-Cicco et al. (2010) and is set to 1.6. Finally, \( \chi \) is determined endogenously such that steady state labour is one third (Aguiar and Gopinath, 2007).

The interest rate for international bank funding is calibrated according to long-run data. I follow Uribe and Yue (2006) and use the world rate \( R^* \) and the country specific spread \( S^c \) in order to calibrate the foreign interest rate in the long-run to \( R^b = R^* S^c \), as described in section 2.2.2.

The remaining parameters \( \sigma \) and \( \rho \) are calibrated using data on the Mexican financial sector. Parameter values have been chosen to fit the following two targets. First, an aver-

\(^{14}\)A complete list of the dynamic equilibrium conditions is provided in the Appendix.
Table 2.2: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences and production</strong></td>
<td></td>
</tr>
<tr>
<td>Capital share of output</td>
<td>$\alpha$ 0.32</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>$\delta$ 0.050</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$ 2</td>
</tr>
<tr>
<td>Wage elasticity of labor supply</td>
<td>$\omega$ 1.60</td>
</tr>
<tr>
<td>Disutility from labour</td>
<td>$\chi$ 2.37</td>
</tr>
<tr>
<td><strong>Financial sector</strong></td>
<td></td>
</tr>
<tr>
<td>World rate</td>
<td>$R^*$ 1.0005</td>
</tr>
<tr>
<td>Country spread</td>
<td>$S^c$ 1.0081</td>
</tr>
<tr>
<td>Country rate</td>
<td>$R^b$ 1.0104</td>
</tr>
<tr>
<td>Survival probability of bankers</td>
<td>$\sigma$ 0.70</td>
</tr>
<tr>
<td>Start-up capital of banks</td>
<td>$\varrho$ 0.082</td>
</tr>
</tbody>
</table>

Notes: The model is calibrated for a quarterly frequency.

age leverage ratio of approximately 6.7, which accounts for the capitalisation of Mexican banks over the period 2006:I to 2014:IV according to data from the national supervisory authority. Second, the calibration is set to achieve an average spread of the lending rate over domestic deposits ($\bar{R}_l - \bar{R}_d$) of 256 basis points at the quarterly frequency. The interest rate spread is taken from the data over the period 1994:I to 2014:IV. The Appendix provides a detailed description of the data used for calibration.

All remaining parameters are estimated using Bayesian methods. The prior distribution for the quarterly growth rate $\tilde{g}$ is assumed to follow a beta distribution with mean 0.6 and standard deviation of 0.05. The prior thereby covers the range of estimates found in previous work, e.g. by Aguiar and Gopinath (2007). The estimated parameter maps into the model’s growth parameter according to $g = 1 + \tilde{g}/100$. There is substantial variation in the literature regarding the calibration of the capital adjustment cost coefficient $\phi_k$. In order to account for a high degree of uncertainty, this parameter is assumed to follow a gamma distribution with mean 30 and a very high standard deviation of 20.

Turning next to the parameters governing financial frictions, I adopt a gamma distribution for the prior of $\psi$ with mean 2.8 and a standard deviation of 1.8. The mean is taken from the estimate in García-Cicco et al. (2010). The parameters genuine to the composition of bank liabilities, $\theta$ and $\nu$, are each assigned a gamma distribution as prior with mean 0.24 and a standard deviation of 0.03.

Next, turning to stochastic shocks, I follow Chang and Fernández (2013) in the assumption of prior distributions on the two productivity shocks. Specifically, I adopt a beta distribution for parameters $\rho_a$ and $\rho_g$ with means 0.95 and 0.72 and standard deviations 0.0112 and 0.0225, respectively. Choosing priors for the volatility of the two productivity shocks is not an easy task since the literature is somewhat ambiguous about the relative importance of these two shocks. Choosing identical priors for $\sigma_a$, and $\sigma_g ac-
Table 2.3: Prior densities

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distr.</th>
<th>Mean</th>
<th>[5%,95%]</th>
<th>S.D. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Beta</td>
<td>0.985</td>
<td>[0.982,0.988]</td>
<td>0.20</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>Gamma</td>
<td>0.60</td>
<td>[0.520,0.685]</td>
<td>5.00</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>Gamma</td>
<td>30.00</td>
<td>[6.134,68.59]</td>
<td>2000.00</td>
</tr>
<tr>
<td>Financial friction parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>Gamma</td>
<td>2.80</td>
<td>[0.620,6.261]</td>
<td>180.00</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Gamma</td>
<td>0.24</td>
<td>[0.193,0.291]</td>
<td>3.00</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Gamma</td>
<td>0.24</td>
<td>[0.193,0.291]</td>
<td>3.00</td>
</tr>
<tr>
<td>Stochastic shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Beta</td>
<td>0.72</td>
<td>[0.682,0.756]</td>
<td>2.25</td>
</tr>
<tr>
<td>$100\sigma_g$</td>
<td>Gamma</td>
<td>0.74</td>
<td>[0.110,1.833]</td>
<td>0.56</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.95</td>
<td>[0.930,0.967]</td>
<td>1.12</td>
</tr>
<tr>
<td>$100\sigma_a$</td>
<td>Gamma</td>
<td>0.74</td>
<td>[0.110,1.833]</td>
<td>0.56</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.335,0.665]</td>
<td>10.00</td>
</tr>
<tr>
<td>$100\sigma_c$</td>
<td>InvGamma</td>
<td>0.10</td>
<td>[0.004,0.125]</td>
<td>Inf</td>
</tr>
<tr>
<td>$\rho_{R_t}$</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.467,0.533]</td>
<td>2.00</td>
</tr>
<tr>
<td>$100\sigma_{R_t}$</td>
<td>InvGamma</td>
<td>0.10</td>
<td>[0.004,0.125]</td>
<td>Inf</td>
</tr>
<tr>
<td>$\rho_{S_c}$</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.467,0.533]</td>
<td>2.00</td>
</tr>
<tr>
<td>$100\sigma_{S_c}$</td>
<td>InvGamma</td>
<td>0.10</td>
<td>[0.004,0.125]</td>
<td>Inf</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.335,0.665]</td>
<td>10.00</td>
</tr>
<tr>
<td>$100\sigma_g$</td>
<td>InvGamma</td>
<td>0.25</td>
<td>[0.026,0.775]</td>
<td>Inf</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.50</td>
<td>[0.335,0.665]</td>
<td>10.00</td>
</tr>
<tr>
<td>$100\sigma_a$</td>
<td>InvGamma</td>
<td>0.25</td>
<td>[0.026,0.775]</td>
<td>Inf</td>
</tr>
<tr>
<td>Measurement errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$100\sigma_{mc}^y$</td>
<td>Uniform [0.01,0.33]</td>
<td>0.17</td>
<td>[0.026,0.313]</td>
<td>2.31</td>
</tr>
<tr>
<td>$100\sigma_{mc}^c$</td>
<td>Uniform [0.01,0.42]</td>
<td>0.21</td>
<td>[0.030,0.395]</td>
<td>2.60</td>
</tr>
<tr>
<td>$100\sigma_{mc}^i$</td>
<td>Uniform [0.01,1.16]</td>
<td>0.59</td>
<td>[0.068,1.107]</td>
<td>4.40</td>
</tr>
<tr>
<td>$100\sigma_{mc}^{tby}$</td>
<td>Uniform [0.01,0.92]</td>
<td>0.47</td>
<td>[0.056,0.879]</td>
<td>3.91</td>
</tr>
</tbody>
</table>

counts for this uncertainty. Both volatilities are assumed to follow a gamma distribution with mean 0.74 percent and a standard deviation of 0.56 percent.

There is little guidance regarding the remaining shock processes that govern the evolution of consumption preferences $\zeta_t$, the financial shock $\vartheta_t$, the international funding shock $\nu_t$, as well as the world interest rate $R^*_t$ and the country specific spread $S^*_t$. I therefore choose to adopt the highly uninformative priors of Smets and Wouters (2007) and let the prior of the persistence parameters be beta-distributed with a mean of 0.5 and a standard deviation of 0.1 percent. The priors of standard deviations follow an inverse gamma distribution with a mean of 0.1 percent and two degrees of freedom, except for the variance of the financial- and the international funding shock that are assumed to have a prior mean value of 0.25 percent and two degrees of freedom.

Finally, I allow for i.i.d. measurement errors in the observables output growth ($g_y$), consumption growth ($g_c$), investment growth ($g_i$) and the trade balance to output ratio ($tby$). The prior distributions regarding measurement errors are uniformly distributed with an upper bound of 25 percent of measured volatility in the data, in close analogy to
the assumptions made in García-Cicco et al. (2010). Given that there are seven observable time series compared to seven structural shocks in the baseline specification, the problem of stochastic singularity is of no concern here. Nevertheless I allow for measurement error due to general concerns on data accuracy.

2.4.2 Data and methodology

In order to estimate the model, I use Mexican data over the time period 1994:I to 2014:IV. In total, there are seven observable time series used for estimation, thereof four standard series from the national accounts statistics, namely aggregate output, private investment and consumption and the trade balance to output ratio, which have been used in previous studies\footnote{In particular, Aguiar and Gopinath (2007) and Chang and Fernández (2013) use these series for their analysis.} and that are described in detail in section 2.2.2. The series on output, consumption, and investment enter the estimation in log-differences. The trade balance to output ratio enters the estimation in levels as in García-Cicco et al. (2010).

Further, I include explicitly the evolution of interest rates as observed time series in the estimation procedure that is in contrast to the existing literature. Mexican interest rate data used for estimation is plotted in Figure 2.3. The foreign interest rate is decomposed into the world rate and the country spread, as described in section 2.2.2 (Fig. 2.3a).

Data on interest rates on domestic deposits and lending rates are taken from the IMF International Financial Statistics (IMF IFS). The lending rate is compiled from weighted returns placed on the securities exchange. Since these claims reflect a return on a con-

---

**Figure 2.3: Interest rates, Mexico (1994:I-2014:IV)**

(a) Composition of the country rate

(b) Lending and borrowing rates

Sources: IMF IFS, JPMorgan, St Louis Fed.
tingent asset, this lending rate seems to be reasonable in the light of the modelling of returns to bank assets. Data on the deposit rate contains a weighted average rate payable to individuals on 60-day time deposits, which more or less fits the duration of one quarter.

Figure 2.3b reveals that the funding costs to banks in the form of the domestic deposit rates are consistently above the foreign interest rate, while the lending rate features a positive spread with respect to both sources of funding. This is a strong indication for the presence of financial frictions as captured by the asymmetry parameter $\nu$ present in the financial sector model.

Since the financial sector model generates spreads between domestic deposit and lending rates as well as the foreign interest rate, the inclusion of interest rate data will allow me to take a deeper look into the frictions that determine financing conditions in emerging market economies. Specifically, the financial sector model takes into account the relative attractiveness of funding sources over the cycle, as captured by excess return on assets over deposits ($\eta^d_t$) from equation (2.18) and the excess value to a bank of substituting international funding for domestic deposits ($\eta^b_t$) from equation (2.19). These major determinants of the choice of liabilities from the perspective of the bank are approximated by the domestic spread ($R^d_t - R^k_t$) and the international spread ($R^d_t - R^b_t$), which are observed in the data and inform the estimation of structural parameters and shocks.

All interest rate series enter the estimation in levels even though there is a strong downward trend over the sample period. The observation equation that maps interest rates and the country spread into the respective model counterparts is given by

$$R^x_t = (\hat{R}^x_t - 1)/4 + 1, \quad \text{for } x \in (k, d, *)$$
$$S^c_t = (\hat{S}^c_t - 1)/4 + 1,$$

where the level of the interest rate is adjusted to the annualized rate reported in the data. Model variables with a hat (\(\hat{\cdot}\)) denote the actual data series.

The posterior distributions are obtained in two steps. First, the mode is computed using a numerical procedure based on Monte-Carlo simulations that combines prior information on parameter values and the likelihood of the data. In order to make sure that the obtained starting value for the Metropolis-Hastings algorithm is close to the true mode, I repeatedly call the numerical routine such that estimates of the posterior covariance matrix and the posterior mode are improved.

Second, based on a point with a high posterior density, the Metropolis-Hastings algorithm evaluates the marginal likelihood of the model (An and Schorfheide, 2007). I let the algorithm take 1,000,000 draws from which the first quarter is discarded.
2.5 Quantitative results

2.5.1 Estimation results

Table 2.4 presents the results from the estimation of the financial sector model. For convenience, I report basic statistics of the assumed prior distribution in columns two and three along the posterior distributions that are given in columns four to six. Overall, the data seems highly informative regarding all estimated parameters in the model. This is indicated by the reduction in the 10 percent probability interval of the posterior distribution compared to the initial prior.\textsuperscript{16} Notably, this holds also for the financial friction parameters in the banking sector.

The time preference rate and the average growth rate are tightly estimated. This is mostly due to the very close relationship of these two parameters in the deterministic steady state. Further, the domestic deposit rate together with the growth rates of output, consumption and investment aggregates are highly informative for these parameters.

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
Parameter & Mean & [5\%,95\%] & Mode & Mean & [5\%,95\%] \\
\hline
$\beta$ & 0.985 & [0.982,0.988] & 0.986 & 0.986 & [0.986,0.986] \\
$g$ & 0.600 & [0.520,0.685] & 0.555 & 0.567 & [0.558,0.578] \\
$\phi_b$ & 30.00 & [6.134,68.59] & 51.06 & 55.36 & [53.76,57.04] \\
Financial friction parameters & & & & & \\
$\psi$ & 2.800 & [0.620,6.261] & 0.020 & 0.05 & [0.033,0.061] \\
$\theta$ & 0.240 & [0.193,0.291] & 0.235 & 0.246 & [0.242,0.249] \\
$\nu$ & 0.240 & [0.193,0.291] & 0.264 & 0.269 & [0.264,0.275] \\
Stochastic shocks & & & & & \\
$\rho_g$ & 0.720 & [0.682,0.756] & 0.649 & 0.65 & [0.639,0.653] \\
$\rho_{\sigma_g}$ & 0.740 & [0.110,1.833] & 5.153 & 4.94 & [4.789,5.196] \\
$\rho_{\sigma_n}$ & 0.740 & [0.110,1.833] & 5.713 & 5.75 & [5.680,5.820] \\
$\rho_{\sigma_c}$ & 0.500 & [0.335,0.665] & 0.679 & 0.60 & [0.574,0.635] \\
$\rho_{\sigma_s}$ & 0.100 & [0.004,0.125] & 4.667 & 3.80 & [3.260,4.331] \\
$\rho_{\sigma_f}$ & 0.500 & [0.467,0.533] & 0.708 & 0.68 & [0.655,0.711] \\
$\rho_{\sigma_R}$ & 0.100 & [0.004,0.125] & 0.880 & 0.79 & [0.744,0.831] \\
$\rho_{\sigma_C}$ & 0.100 & [0.004,0.125] & 1.176 & 1.21 & [1.176,1.240] \\
$\gamma_{\sigma_C}$ & 0.100 & [0.004,0.125] & 1.767 & 1.80 & [1.767,1.800] \\
$\rho_{\sigma_f}$ & 0.500 & [0.335,0.665] & 0.858 & 0.96 & [0.935,0.977] \\
$\rho_{\sigma_R}$ & 0.100 & [0.004,0.125] & 1.281 & 1.30 & [1.281,1.300] \\
$\rho_{\sigma_C}$ & 0.100 & [0.004,0.125] & 0.788 & 0.71 & [0.688,0.743] \\
$\rho_{\sigma_f}$ & 0.500 & [0.335,0.665] & 0.678 & 0.71 & [0.688,0.743] \\
Measurement errors & & & & & \\
$\sigma_{me_0}$ & 0.170 & [0.026,0.313] & 0.108 & 0.09 & [0.065,0.107] \\
$\sigma_{me_1}$ & 0.213 & [0.030,0.395] & 0.099 & 0.12 & [0.097,0.136] \\
$\sigma_{me_2}$ & 0.187 & [0.068,1.107] & 0.332 & 0.50 & [0.439,0.566] \\
$\sigma_{me_3}$ & 0.467 & [0.056,0.879] & 0.360 & 0.66 & [0.584,0.740] \\
\hline
\end{tabular}
\caption{Estimation results}
\end{table}

The model seems to attribute a relatively high value to investment adjustment cost with a posterior mean estimate of 55. It is well known that small open economies tend
to overpredict the volatility of investment (Mendoza, 1991). Given the amplification channels of shocks present in the financial sector model, the value seems still to be within the ballpark of previous estimates, e.g. in Neumeyer and Perri (2005) who find a greatly varying capital adjustment cost parameter between 8 and 40.

The financial friction of a debt elastic interest rate seems to play almost no role in the model with a financial sector. While the prior attributes a comparatively high value to $\psi$, the posterior mean is close to zero (0.05). This can be explained by the asymmetric funding frictions that induces already a mean reversion in the net foreign asset position, thereby accounting for the autocorrelation function in the trade balance to output ratio. As García-Cicco et al. (2010) discuss in detail, the debt elastic interest rate parameter has the same role in their model.

In contrast, the parameters governing the overall tightness of the borrowing constraint $\theta$ and the parameter that addresses the asymmetry between foreign funds and domestic deposits $\nu$ are both well above zero with a tight posterior, indicating that these frictions play an important role for the explanation of the observed data. In this context, the inclusion of interest rate data is important to inform the estimation about the comparative strength of each of the frictions in the model since these determine the spreads between different funding sources.

### 2.5.2 Propagation of country spread- and financial sector shocks

In this section, I present the propagation of shocks to the country premium. Figure 2.4 shows impulse responses of the baseline model to one standard deviation shock in the country premium. This increases the funding cost at which banks can borrow from abroad. I compare the baseline specification with a model that features a lower leverage ratio in the financial sector.

The country premium shock induces a net capital outflow. The mechanism is due to the asymmetry in the financing constraint that leads to a reduction in the excess value to a bank of substituting international funding for domestic deposits from equation (2.19). Since the ratio of excess returns of assets and the excess value of substituting international funding ($\eta^*/\eta^b$) deteriorates, banks reduce their amount of international borrowing, as implied by equation (2.22). Overall, the reduction in bank borrowing from abroad dominates the increase in the country spread such that the trade balance to output ratio increases.

The shock on the country premium is recessionary as the drop in profitability of assets reduces aggregate investment and output. Confronted with lower wages, also households reduce consumption, which is partially offset by a reduction in savings activities, leading
to a decline in deposits. However, domestic residents reduce their deposits by less than foreign investors such that the foreign funding ratio declines. Since the financing constraint of banks depends on their aggregate net worth, financial frictions amplify the original recession, which is well known from the literature on the financial accelerator mechanism (Bernanke et al., 1999). This result in the present model can be seen from Figure 2.4 due to a weaker amplitude of the shock in the calibration that features a financial sector with a lower leverage ratio compared to the baseline specification.

Figure 2.4: IRF - Shocks to country premium

Turning next to the two newly introduced shocks in the financial sector model, Figure 2.5 shows impulse responses to a financial shock ($\theta_t$) and the international funding shock ($\nu_t$). The original shocks are reproduced in the first panel of the second row. Although the shocks have qualitatively similar implications, the quantitative differences are large. Most importantly, the financial sector shock is a lot more persistent and features a higher standard deviation compared to the international funding shock (See also results from Table 2.4).

In the case of the international funding shock ($\nu_t$), a deterioration of funding conditions
of banks on international capital markets leads to an imminent reduction in foreign capital inflows. However, since this increases the profitability of asset holding in the initial period, asset prices are boosted and the leverage ratio of banks drops. In total, the foreign funding shock remains recessionary, as the drop in international funding cannot be made up by an increase in domestic deposits. Therefore, total investment falls, leading to lower marginal productivity of labour and wages.

The tightening of the aggregate financial friction ($\theta_t$) leads to a fall in asset prices, as the non-depreciated physical capital stock cannot be financed at the previously higher price. The fire-sale effect in asset prices induces an initial jump in the leverage ratio, reducing funding possibilities from abroad and domestically. The consequences are recessionary, with a reduction in consumption, investment and output.

### 2.5.3 Decomposing the Mexican business cycle

In this section, I look at the relative contribution of shocks for the explanation for macroeconomic dynamics in Mexico over the period under consideration. The forecast error variance decomposition highlights the dominant role of transitory shocks to total fac-
tor productivity (Table 2.5). This is in line with previous findings that take account of financial frictions (García-Cicco et al., 2010; Chang and Fernández, 2013).

Table 2.5: Forecast error variance decomposition

<table>
<thead>
<tr>
<th>Structural shock</th>
<th>$g_y$</th>
<th>$g_c$</th>
<th>$g_i$</th>
<th>$tbg$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_\phi$</td>
<td>73.4</td>
<td>64.1</td>
<td>22.9</td>
<td>44.8</td>
</tr>
<tr>
<td>$\varepsilon_\theta$</td>
<td>26.3</td>
<td>33.3</td>
<td>63.3</td>
<td>29.6</td>
</tr>
<tr>
<td>$\varepsilon_\zeta$</td>
<td>0.0</td>
<td>1.8</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\varepsilon_{\mu}$</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varepsilon_{\sigma}$</td>
<td>0.0</td>
<td>0.4</td>
<td>0.5</td>
<td>2.6</td>
</tr>
<tr>
<td>$\varepsilon_{\varphi}$</td>
<td>0.2</td>
<td>0.1</td>
<td>12.9</td>
<td>20.5</td>
</tr>
<tr>
<td>$\varepsilon_{\nu}$</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.9</td>
</tr>
</tbody>
</table>

However, in contrast to their findings, the role of changes in the foreign interest rate nearly looses all its explanatory power. This might be due to three effects. First, the financial sector model alters the propagation of conventional productivity shocks, increasing their relative explanatory power vis-à-vis the interest rate shock. Second, accounting for interest rate data in the estimation reduces the degree of freedom for these shocks to drive aggregate macroeconomic dynamics. Third, the inclusion of financial sector shocks implies similar macroeconomic dynamics as changes in the foreign interest rate. In fact, the explanatory power of financial sector shocks is particularly high for investment and trade dynamics that have previously been explained by interest rate shocks.

The results are confirmed when looking at a historical decomposition of the national accounts data included in the estimation. Figure 2.6 visualizes the fitted shocks with the highest likelihood according to the Kalman filter in the Bayesian estimation. Financial sector shocks are important drivers of investment and the trade balance to output ratio. In particular, they make up for a huge reduction in investment during the Mexican currency crisis and the global financial crisis, as well as for the reversal in the Mexican current account in response to these crises. Remarkably, the country premium shock contributes to the current account reversal in 1995 and its aftermath, but has virtually no role during the global financial crisis. This is in line with the preliminary discussion of changes in the country premium in Figure 2.2a. Thus, in order to account for the significant current account reversals in the recent experience of the global financial crisis, shocks to the foreign interest rate played only a very minor role compared to alternative funding shocks that induce procyclical capital flows.

2.6 Comparison with benchmark models

This section compares the financial sector model discussed so far with two benchmark models with alternative specifications of financial frictions. Specifically, the model with
a debt elastic interest rate as presented by García-Cicco et al. (2010) and the model with an endogenous country spread and a working capital requirement from Chang and Fernández (2013) are estimated using data compiled over the time period 1994:I to 2014:IV and including the foreign interest rate. This restricts the degrees of freedom compared to previous analysis that left movements in foreign interest rates mostly unconstrained in order to explain the dynamics of the trade balance to output ratio.

### 2.6.1 Estimation with interest rate data

I assume a small modification to the benchmark model of a small open economy with a stochastic trend and a debt elastic interest rate as discussed by García-Cicco et al. (2010) for comparability reasons. Namely, I set the government spending share to zero and do not allow for domestic spending shocks. Since this shock plays no role in their variance decomposition of Argentinian data anyway, I consider this modification to be of minor relevance. All remaining changes made are not substantive but just for notational comparability with the financial sector model. The foreign interest rate rule is assumed
to be given by

\[ R^b_t = \tilde{R}^b + \psi(e^{h_t^b} - 1) + e^{\mu_t} - 1. \]

Otherwise, this model is a standard model of a small open economy with a stochastic trend (Aguir and Gopinath, 2007). The optimization problem of the household, optimality conditions and the full set of non-linear dynamic equilibrium conditions is provided in the Appendix.

The second benchmark model with an endogenous country premium and a working capital requirement closely follows Chang and Fernández (2013). I augment their model with a shock for consumption preferences (\( \zeta_t \)) to make the comparison across models easier.

The working capital requirement is introduced as a constraint to firms in financing a share of wages upfront. This friction has been proposed by Neumeyer and Perri (2005) and Uribe and Yue (2006) in order to make domestic production sensitive to changes in the foreign interest rate, since the standard RBC model of a small open economy does not achieve a transmission of country spread shocks to the domestic economy Mendoza (1991). Let \( \tau \) denote the financing requirement, then the working capital requirement changes the first order condition for labour demand of firms to

\[(1 - \alpha)a_t k_t^\alpha (g_t h_t)^{-\alpha} g_t = \frac{w_t}{1 + \tau(R^b_t - 1)}.\]

Thereby, the shock to the foreign interest rate dampens labour demand, since the marginal cost of labour is affected by \( \tau(R^b_t - 1) \) per unit employed.

Further, I adopt the baseline specification of Chang and Fernández (2013) for the formulation of the endogenous country premium. Let \( SR_t = a_t g_t^{1-\alpha} \) denote the Solow residual of the Cobb-Douglas production function under the specification as in the financial sector model. Then, Chang and Fernández (2013) assume an ad hoc negative relationship between the Solow residual and the country premium of the form

\[\ln \left( \frac{S^c_t}{S^c} \right) = -\eta E_t \left[ \ln \left( \frac{S R_{t+1}}{S R} \right) \right],\]

where variables without time subscript denote steady state values. The authors motivate their assumption by a counter-cyclical interest rate that is obtained from models with endogenous government default (Arellano, 2008). The parameter of interest is \( \eta \) that captures the elasticity of the country premium to the Solow residual. The law of motion
of the foreign interest rate is then given by

\[ R^b_t = S_c^r R^* + \psi \left( e^{d_t - \bar{d}} - 1 \right) + e^{\mu - 1} - 1, \]

where \( d_t \) denote household borrowing. In this case the coefficient on the elasticity of the interest rate towards deviations from steady state borrowing \( \psi \) is calibrated to a very low value that limits this parameter to induce a stationary net foreign asset position only. The optimization problems and all relevant equilibrium conditions for this benchmark model are provided in the Appendix.

Given the quarterly frequency of the Mexican data series, parameter values for calibration as well as prior distributions are set as in the financial sector model (Table 3.2). Steady state values for household debt \( \bar{d} \) in the two benchmark models are calibrated such that the steady state value of the trade balance to output ratio meets its empirical counterpart (-0.092 percent).

I re-estimate the financial sector model using only the foreign interest rate data and the four series from the national accounts in order to allow for a direct comparison of models based on the posterior likelihood. Therefore, the interest rate rule is adjusted to follow the one presented in this section for the benchmark model of a debt elastic interest rate. Further, as I leave out the domestic interest rates, which allowed for an identification of the financial friction parameters \( \theta \) and \( \nu \), I calibrate their values to the posterior means in Table 2.4. Finally, I also drop the international funding shock \( \varepsilon_t^f \) from the estimation, as the identification of two financial sector shocks is not feasible from the reduced set of observed time series.

For the estimation, I apply the same set of prior distributions on the parameters as in the previous section. For the elasticity of the country spread, I adopt identical priors as Chang and Fernández (2013). Thus, the elasticity of the endogenous country spread \( \eta \) is assumed to follow a gamma distribution with mean 1 and a standard deviation of 10.1 percent. The working capital requirement parameter \( \tau \) has a beta distribution as prior with mean 0.5 and a high standard deviation of 22.4 percent. All prior means and the 10 percent probability intervals are reported in column two of Table 2.6.

### Results

Turning first to the financial sector model, the estimation results are broadly consistent with estimates obtained previously when using additional interest rate series. In the model with a debt elastic interest rate, the coefficient \( \psi \) is more than two orders of magnitude higher compared with the financial sector model and close to the estimate obtained for
Table 2.6: Estimation results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean [5%, 95%]</th>
<th>Financial sector model</th>
<th>Debt elastic rate</th>
<th>Endog. country spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.600 [0.520, 0.685]</td>
<td>0.637</td>
<td>0.629 [0.554, 0.712]</td>
<td>0.601</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.800 [0.620, 0.261]</td>
<td>0.011</td>
<td>0.004 [0.000, 0.008]</td>
<td>2.877</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.000 [0.840, 1.172]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Stochastic shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>2.800 [0.620, 6.261]</td>
<td>0.011</td>
<td>0.004 [0.000, 0.008]</td>
<td>2.877</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>1.000 [0.840, 1.172]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>0.500 [0.840, 1.172]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Measurement errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{me}}$</td>
<td>0.170 [0.026, 0.775]</td>
<td>0.329</td>
<td>0.247 [0.141, 0.329]</td>
<td>0.134</td>
</tr>
<tr>
<td>$\sigma_{\text{mc}}$</td>
<td>0.170 [0.030, 0.395]</td>
<td>0.349</td>
<td>0.378 [0.334, 0.416]</td>
<td>0.416</td>
</tr>
<tr>
<td>$\sigma_{\text{br}}$</td>
<td>0.587 [0.068, 1.101]</td>
<td>1.655</td>
<td>1.031 [0.865, 1.164]</td>
<td>0.488</td>
</tr>
<tr>
<td>$\sigma_{\text{by}}$</td>
<td>0.467 [0.050, 0.879]</td>
<td>0.453</td>
<td>0.506 [0.399, 0.613]</td>
<td>0.512</td>
</tr>
</tbody>
</table>

Argentinian annual data over the period 1900-2005. In the model with an endogenous country spread and a working capital requirement, both parameters of interest have similar values as found by Chang and Fernández (2013).

Given that the models are estimated against the same data set, it is possible to evaluate the relative fit of the models with different approaches to financial frictions based on the likelihood and the marginal data density. Table 2.7 presents the Laplace approximation of the likelihood and Geweke’s (1999) modified harmonic mean estimate for the logarithm of the marginal data density. The results indicate that the financial sector model has the highest predictive accuracy of all three models under consideration.

Table 2.7: Comparing models with financial frictions

<table>
<thead>
<tr>
<th></th>
<th>Laplace Approximation</th>
<th>Harmonic Mean Estimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial sector model</td>
<td>1156.81</td>
<td>1176.14</td>
</tr>
<tr>
<td>Debt elastic rate</td>
<td>768.42</td>
<td>817.26</td>
</tr>
<tr>
<td>Endogenous country spread</td>
<td>1084.15</td>
<td>1082.58</td>
</tr>
</tbody>
</table>

2.7 Conclusion

In this paper, I revisit the role of shocks to foreign interest rates as an important driver of macroeconomic dynamics in emerging market economies. In particular, I address the shortcoming of some parts of the literature that do not include interest rate data in the
empirical estimation strategy. Arguably, this leaves a degree of freedom to the interest rate dynamics such that the observed trade balance dynamics can be explained by sufficient adjustment in the cost of foreign borrowing. The findings show that the effect of changes in country spreads was mostly missing during the global financial crisis in the case of Mexico, while capital flows have reverted nevertheless.

I then contrast the effects of interest rate shocks to a sudden change in investor preferences regarding emerging market assets that are modelled as financial sector shocks. These shocks have been of increasing concern for policy makers since they pose challenges with respect to an adequate policy response (Rey, 2013).

I present a model of a small open emerging market economy with capital importing banks that are modelled in close analogy to Gertler, Kiyotaki, and Queralto (2012). The model is estimated using Bayesian techniques and using Mexican data over the time period 1994:I to 2014:IV. I include the world interest rate, the country spread, and the domestic deposit and lending rate in order to identify the financial frictions parameters from the data.

Shocks in the agency friction of bankers and investors capture the observed response of macroeconomic and financial sector variables to a global anxious state of the economy. In particular, such shocks reduce the amount of international funding available to banks, leading to a crunch in investment, consumer spending and aggregate output. The reduction in foreign borrowing leads to a fast adjustment in the trade balance, which is characteristic for sudden stops. Notably, this adjustment is independent of changes in the country premium.

Given that capital inflow surges and sudden reversals pose a major challenge to policy makers in emerging market economies, the role of the financial sector in the intermediation of capital flows is still relatively unexplored. This paper shows that it is necessary to distinguish between the different sources that can cause the volatile access of emerging markets to international capital markets, as these might require different policy responses.

The here presented model is a first step to understand the role of a capital importing financial sector for the explanation of domestic aggregate volatility and financial stability in a quantitative framework. This suggests that future research should focus on the suitable policy response either in the form of capital control measures or macroprudential regulation of the financial sector to dampen the adverse consequences of financial sector shocks, or how monetary policy should accommodate these shocks.
CHAPTER 3

Sovereign Risk, Interbank Freezes, and Aggregate Fluctuations (with Philipp Engler)

3.1 Introduction

The European sovereign debt crisis has highlighted the pivotal role of public debt held by the domestic financial sector in creating a bank-sovereign link that amplifies adverse shocks in the presence of fiscal stress. The close interdependence between sovereign risk, financial stability and economic activity exposed the fragility of public debt positions in advanced economies. In particular, we make three observations during the crisis: (i) Banks in countries with an elevated risk premium on government debt underwent difficulties in tapping wholesale funding markets. (ii) The interest rates of the public and the private sector started to be highly correlated in countries under fiscal stress. (iii) Countries in the euro area periphery in very deep recessions experienced particularly strong increases in their yield spreads.

Until recently, the existing literature on strategic sovereign default has focused on emerging market debt.\textsuperscript{1} In this paper, we attempt to close the gap in the quantitative-theoretic literature by proposing a model of optimal sovereign default for advanced economies. We use the model to answer the following question: What makes debt positions in advanced and financially developed economies vulnerable? We find that the adverse feedback loop in Europe can be rationalised by the concept of strategic default and that ex ante spillovers from sovereign risk to financial market allocations are an important source of sovereign risk.

This paper makes three contributions. First, we nest a bank-sovereign link in a quantitative dynamic stochastic general equilibrium setting that gives rise to an endogenous

\textsuperscript{1} Arellano (2008) and Aguiar and Gopinath (2006) are seminal studies in this field.
penalty from defaulting. Based on the classic work of Eaton and Gersovitz (1981), a model of strategic sovereign default in a small open economy is augmented with a heterogeneous banking sector. Banks allocate financial resources on a secured interbank market where government bonds serve as collateral. The economic cost of default is due to an ensuing credit crunch in response to an interbank market freeze as collateral loses its market value. In line with previous findings, the penalty works as a powerful enforcement mechanism which is able to support higher debt levels compared to related quantitative frameworks. Thereby, the model is able to rationalise higher debt levels and lower frequencies of default that are typically observed in financially developed, advanced economies. While the described mechanism is not limited to advanced economies, we find, similar to Gennaioli et al. (2014), that the penalty is higher for financially more developed economies.

Second, we formalise an amplification mechanism that arises from endogenously evolving sovereign risk. A deterioration of the quality of underlying collateral assets gives rise to a risk premium that pushes up bank funding costs and dampens financial intermediation (Barro, 1976). Sovereign risk propagates into higher costs of external finance for the non-financial sector, in turn curbing private credit demand and depressing aggregate output. As a result, the penalty from the collapse of the interbank market turns less painful in relative terms during times of fiscal stress, feeding back into an increase of sovereign risk.

Third, we calibrate the model using Spanish data. The quantitative simulation exercise shows that the model is capable of reproducing key business cycle statistics alongside the stylised facts from above. Although the quantitative predictions of the model are broadly in line with the results from the literature on sovereign default in emerging markets, there are three differences in comparison to the Argentinian default of 2001 that has often served as an example of an emerging market sovereign default. Calibrated to an advanced and financially developed economy, we find that (i) recessions preceding a default event are more severe in advanced economies; (ii) there is a stronger impact effect of default on output; (iii) recessions last longer after a default. Thus, the model is supporting the view of long-lasting financial recessions (Cerra and Saxena, 2008) as opposed to a strong recovery observed in the follow-up to emerging market debt crises (Calvo et al., 2006).

There is a long academic debate on the enforcement problem of government debt. In recent work, endogenous penalties have gained more attention compared to reputational arguments as presented in Bulow and Rogoff (1989), Cole and Kehoe (1998), and Grossman and van Huyck (1988), among others. Empirical findings of Dell’ariccia et al. (2005) document a domestic credit crunch in response to a banking crisis which lowers aggregate production. See Dooley (2000) and Kumhof and Tanner (2008). Note that no bailout of the financial sector is needed in this set-up to amplify sovereign risk, contrary to the framework of Acharya et al. (2014).
High persistence and slow recovery are related to banks’ need to accumulate collateral from retained earnings during the post-default period.

The paper mainly contributes to three branches of the existing literature. First, we follow Mendoza and Yue (2012) in breaking the disconnect between sovereign default risk and the business cycle in a quantitative setting. The trade-off inherent to the default decision in their model is linked to the trade sector that depends on access to foreign finance. We extend their analysis by structurally modelling a banking sector that determines domestic borrowing conditions of the private sector. The model’s propagation mechanism leads to a co-movement of private and public sector interest rates during a debt crisis. This result sheds new insights on the countercyclical nature of interest rates, as previously analysed by Neumeyer and Perri (2005) and Uribe and Yue (2006). Fernández and Gulan (2015) and Kaas et al. (2014) explain counter-cyclical private sector interest rates by financial frictions on the side of entrepreneurs. We separate from their work by providing a different explanation through the interaction of the enforcement problem on the government side with frictions in the banking sector.

Second, our interpretation of the bank-sovereign link is motivated by the role of government debt in providing liquidity services to the banking sector, as stressed by Woodford (1990). According to this view, private agents are liquidity constrained insofar as they cannot pledge the entire future income stream from profitable projects in order to obtain external finance. It is the highly liquid claim on government bonds that enables these constrained firms to obtain additional external funds to increase the size of their portfolio. The resulting non-Ricardian effects of government debt are usually attributed with welfare improvements, since an expansion of government borrowing increases the amount of securities available to liquidity constrained firms as in Holmström and Tirole (1998) or Gorton and Ordoñez (2013). However, this body of literature typically assumes that the government can perfectly commit to repay its outstanding debt. Recent events in the euro area have illustrated that this is not necessarily the case. In this paper, we allow for limited enforcement of government debt that turns liquidity services from government securities into a state dependent general equilibrium outcome.

There have been a number of recent studies on the bank-sovereign link, mostly in non-stochastic models without reference to the business cycle. Within this literature, we are closest to Brutti (2011), Bolton and Jeanne (2011), and Niemann and Pichler (2013) who also account for a liquidity role for government bonds under sovereign risk. We extend this strand of literature by providing a quantification of the liquidity channel in a calibrated and simulated framework at the backdrop of the European sovereign debt crisis. Further, we are more specific on the propagation mechanism by structurally modelling
an interbank market. This gives rise to several channels that have been discussed by policymakers during the crisis.

The spillover mechanism from sovereign risk to financial intermediaries is related to the bank capital channel that also creates an ex post penalty from sovereign default. Gennaioli et al. (2014) propose a model where the size of the default penalty is a function of the quality of domestic financial institutions, which allow domestic agents to increase the leverage of their balance sheets and to accumulate more government debt. Closely related is the analysis by Sandleris (2014). Acharya and Rajan (2013) study a setting where government myopia helps to overcome the enforcement problem in the presence of endogenous default penalties. Short-termism induces policymakers to service debt today, whereas the adverse consequences of a write-down on the domestic financial system are shifted to the next political generation. Padilla (2013) proposes a stochastic model of optimal sovereign default with an endogenous default penalty due to a bank capital channel that is calibrated to the case of Argentina in 2001. The liquidity channel analysed here separates from the bank capital channel in that it induces an ex ante cost of sovereign risk which is important for the dynamic and quantitative results as it lowers the penalty and renders sovereign debt fragile even in the presence of high endogenous penalties.

Third, the framework features bond market trading that gives rise to simultaneous domestic and external debt positions in equilibrium. This is novel in the quantitative literature on sovereign default, which has so far focused on either external or domestic debt. Both the bank-sovereign link and the possibility of bond market trading affect default incentives and aggregate fluctuations through domestic accumulation of government securities. Since the government cannot discriminate between individual bond holders, bond market trading gives rise to non-penalty related incentives to repay external creditors, as the undesirable consequences of defaulting on domestic bond holders are internalised, similar to the political economy model of external and domestic sovereign debt by Guembel and Sussman (2009). More generally, the bond market in our framework works in support of the repayment equilibrium and is therefore in line with the secondary bond market hypothesis developed by Broner et al. (2010).

The next section presents a novel set of stylised facts regarding sovereign risk and interbank markets in the euro area and discusses its parallels with business cycles in emerging markets. Section 3 contains a detailed description of the model. We discuss the model mechanics of the bank-sovereign link and spillovers to feasible allocations in

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6Technical motives might be the main reason for this neglect, as the global solution technique restricts the amount of endogenous state variables. At the same time, this is surprising as Reinhart and Rogoff (2011) document that domestic public debt ranges between 40 and 80 percent of total public debt in a broad sample of advanced and emerging market countries.
3.2 Stylized facts on the bank-sovereign link in the euro area

In this section we highlight three stylized facts about the European sovereign debt crisis. The first stylized fact focuses on the bank-sovereign channel in the euro area in the light of structural changes in bank financing patterns. As a consequence of the global financial crisis, European interbank short-term funding increasingly shifted from unsecured to secured (repo) interbank markets (ECB, 2012). From the peak volume of unsecured lending traded in 2007, total market turnover shrank by more than 30 percent in the four years through 2011 (Figure 3.1a). Over the same time period, the repo market was comparatively stable such that it developed into the major interbank market for wholesale funding (ECB, 2013). The behavioural changes undergoing bank funding structures in Europe since the burst of the subprime bubble were primarily related to an elevated level of counter-party risk. In the presence of asymmetric information about banks’ idiosyncratic risk characteristics, collateral is a way to overcome agency problems underlying the interbank market. In a nutshell, a repo agreement replaces the counter-party default risk by the less likely event that the counter-party defaults and, simultaneously, the underlying collateral loses its value. For this reason, collateral used in private repo markets are usually high quality and liquid assets. In fact, more than 90 percent of collateral assets in European repo arrangements are securities backed by central governments (ICMA, 2013).

With an intensification of the euro area debt crisis in the course of 2009-10, government securities from countries under an assistance programme were no longer accepted in European repo markets (Figure 3.1b). The increase in sovereign default risk triggered an adverse collateral channel at a time when structural shifts toward more secured funding were taking place in the European banking sector. As a result, banks in countries with fiscal stress underwent difficulties in tapping wholesale funding markets (fact 1). Some countries suffered practically an interbank freeze for both, secured and unsecured funding, and had to revert to central bank liquidity (Figure 3.1c).

In the second half of 2011, repo market conditions for remaining euro area countries under fiscal stress tightened as well, as repo rates against collateral issued in core and

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7Financial sector rescue packages in October 2008 contributed to the initial increase in public sector default risk in the European sovereign debt crisis as documented by Ejsing and Lemke (2011).

8Gorton and Metrick (2012) find evidence that the U.S. financial crisis similarly unfolded through a "run on repo" markets, where previously as safe regarded assets were subject to re-evaluations of risk.

9Giannone et al. (2012) provide a detailed empirical analysis on how the ECB’s non-standard monetary policy interventions were replacing private intermediation on the interbank market.
Figure 3.1: The collateral channel of sovereign risk

(a) Interbank turnover

(b) Accepted collateral

(c) Public intermediation

(d) GC repo rate

(e) Growth and default risk

Notes: Panel (a) - Turnover formed as sum over lending and borrowing from ECB’s money market survey. Panel (c) - Degree of public intermediation evaluated as percentage share of 'Lending to credit institutions related to monetary policy operations' and 'Other claims on euro area residents in euros' from NCB balance sheets over 'Total liabilities' from aggregate MFI balance sheets. Panel (e) - Dotted line includes data point for Greece. Source: ECB, ICMA, Bloomberg, NCBs, OECD, Thomson Reuters Datastream.
periphery member countries started to diverge (Figure 3.1d). Specifically, the spread between the EONIA swap rate and the general collateral (GC) repo rate from France and Germany turned negative in the second half of 2011, while they were surging in Spain and Italy. Thus, the lower credit quality of government securities translated into more costly liquidity services on repo markets.¹⁰

Figure 3.2: Interest rates in the euro area

Note: Interest rates in levels. Black lines with dot = Government bond yields from 10 year benchmark bond; Green lines with cross = bank lending rates to non-financial firms, maturity up to 1 year. Correlations computed over the period 2009q3-2011q4. Source: ECB, Thomson Reuters Datastream.

The second stylized fact relates to an amplification mechanism arising from the collateral channel: With a deteriorating collateral value of government bonds in the presence of a strong home-bias in government bond portfolios (Arslanalp and Tsuda, 2012), the collateral channel contributed to the overall increase in funding costs in the euro area periphery (BIS, 2011). Higher funding costs for banks in the periphery translated into higher interest rates on loans to the non-financial sector.

¹⁰This phenomenon is related to flight-to-liquidity effects as described by Beber et al. (2009) for European government bond markets. However, while Beber et al. focus on asset liquidity in terms of its resaleability, we concentrate on liquidity services generated through the collateral value of an asset.
The interest rates of the public and the private sector started to be highly correlated with the onset of the Greek debt crisis in the second half of 2009 (fact 2, Figure A.1). This correlation was previously documented by Mendoza and Yue (2012) in connection with foreign interest rates for a sample of 18 emerging market economies over the entire business cycle. In the context of the euro area sovereign debt crisis, it is the domestic bank lending rate that turned out to be positively correlated with sovereign default risk due to the bank-sovereign link which unfolds, among others, via the collateral channel.

The third stylized fact is addressing the interaction of the bank-sovereign link with macroeconomic fluctuations. Overall economic performance seems to have played a major role in the further escalation of the sovereign debt crisis. Figure 3.1e) shows a strong negative correlation between average quarterly growth rates over the 2010 to 2011 period and average CDS yields. Countries in the euro area periphery in very deep recessions experienced particular strong increases in sovereign default risk (fact 3). This comes as little surprise given that the sovereign debt crisis developed into a systemic crisis. In the next section, we present a model that explains the stylized facts presented here by emphasising the collateral role of government securities in the financial system of an economy.

### 3.3 Model environment

#### 3.3.1 Overview

The core of the model is a small open economy real business cycle model with a benevolent government subject to a limited commitment friction that gives rise to endogenous risk of sovereign default. The novel block is a heterogeneous banking sector that intermediates financial resources on domestic interbank and credit markets, thereby providing working capital loans to non-financial firms. Government bonds are used as collateral on the interbank market.

Time is infinite and discrete \( t = (0, 1, 2, \ldots) \). The state space is given by \( s \in (B_t, B^D_t, A_t) \), where \( B_t \) denotes total outstanding government debt, \( B^D_t \) is domestically accumulated collateral on bank balance sheets, and \( A_t \) an exogenous aggregate productivity state. Endogenous states \( \epsilon_t \in (B_t, B^D_t) \) are given from period \( t - 1 \) decisions. When the productivity shock realizes at the beginning of the period, the government takes its binary default decision \( \delta_t \in \{0, 1\} \) as the outcome of an optimization problem. In case of default, the economy falls into financial autarky with a stochastic probability of re-entering capital markets in period \( t + 1 \). There is no debt trading in the autarky state. In the
no-default equilibrium, domestic banks intermediate funds on interbank and credit markets to non-financial firms. Interbank lending is subject to financial frictions that interact with sovereign risk. For simplicity, we assume that lending from banks to firms occurs without frictions with non-contingent debt contracts. Primary and secondary bond market trading evolves simultaneously and determines the asset position of the government and banks in the consecutive period.\footnote{We use bond market trading and secondary bond market trading interchangeably in this paper. A detailed overview on the timing of events is provided in the Appendix B.2.}

### 3.3.2 Households

A representative household derives utility from consumption and leisure. It owns domestic firms and banks while receiving profits and dividends in a lump sum fashion at the end of each period. The optimization problem of the household is static, choosing optimal period $t$ consumption and labour supply. We assume that the household consumes all available resources according to its flow budget constraint. Consumption smoothing over time can only be obtained by government transfers. We thereby follow the motivation typically provided for government transfers through international borrowing in the literature on optimal sovereign default.\footnote{Among others, Aguiar and Gopinath (2006) and Arellano (2008) use household consumption smoothing to motivate government borrowing. An alternative motivation for international borrowing is to smooth government expenditures, see Cuadra et al. (2010).} The household maximizes lifetime utility subject to a budget constraint

\[
\max_{\{C_t, L_t\}} E_t \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t),
\]

s.t. \[ C_t = W_t L_t + \Pi_t + D_t + T_t, \tag{3.1} \]

where $E_t$ is the rational expectations operator, $C_t$ denotes consumption and $\beta$ is the discount factor. Time available to households is normalised to one unit such that $1 - L_t$ and $L_t$ denote leisure and hours worked, respectively. The utility function $U: \mathbb{R}_+^2 \to \mathbb{R}$ is twice continuously differentiable in both its arguments, and satisfies $U_C > 0$, $U_{CC} < 0$, $U_L < 0$ and $U_{LL} < 0$. $W_t$ is the real wage that is exogenous from the perspective of the representative household. $\Pi_t$ denote non-financial firms’ profits, $D_t$ are banking sector dividend payments, and $T_t$ lump-sum government transfers or taxes. Labour supply is pinned down by the marginal rate of substitution between consumption and labour equated with the real wage:

\[
- \frac{U_L(C_t, 1 - L_t)}{U_C(C_t, 1 - L_t)} = W_t \tag{3.2}
\]
3.3.3 Non-financial firms

There is a competitive non-financial sector in the economy which produces a tradeable final good with a Cobb-Douglas production function. Profits are maximized by the choice of labour input at a constant capital stock \( K \). A fraction \( \eta > 0 \) of the wage bill needs to be paid to workers up-front at the beginning of each period prior to production. Since non-financial firms are liquidity constrained and do not have the possibility to save, they obtain credit \( \kappa_t \) from domestic banks at the endogenous interest rate \( r_t \).\(^{13}\) The representative non-financial firm’s profit maximisation problem takes the form:

\[
\max_{\{L_t, \kappa_t\}} \Pi_t
\]

with

\[
\Pi_t = Y_t - W_t L_t - \kappa_t r_t^\kappa,
\]

\( Y_t = e^{A_t} K^\alpha L_t^{(1-\alpha)} \), \( (3.3) \)

s.t. \( \kappa_t \geq \eta W_t L_t \). \( (3.4) \)

Since working capital is costly, the constraint in equation (4.6) holds with equality in equilibrium. The resulting first-order condition for labour demand equates the marginal product of labour to the marginal cost which consists of the wage rate plus financing costs from working capital borrowing.\(^{14}\)

\[
(1 - \alpha)e^{A_t} K^\alpha L_t^{-\alpha} = W_t(1 + \eta r_t^\kappa)
\]

\( (3.5) \)

The only aggregate uncertainty is due to a stationary autoregressive process of order one for total factor productivity \( A_t \)

\[
A_t = \rho A_{t-1} + \varepsilon_t,
\]

\( (3.6) \)

with \( 0 < \rho < 1 \) and white noise process \( \varepsilon_t \sim \mathcal{N}(0, \sigma^2) \).

3.3.4 Banks

Financial intermediation is motivated by the need for external finance of the non-financial sector.\(^{15}\) The banking sector is assumed to be of measure one and populated by an infinite

\(^{13}\)The implied assumption is that domestic firms do not access international capital markets for credit. This seems justified by the fact that (i) firms in the euro area are typically bank financed, and (ii) the banking sector is predominantly domestically owned (ECB, 2013).

\(^{14}\)See Uribe and Yue (2006) for an extensive discussion of the interest rate on working capital as an intra-period loan contract.

\(^{15}\)We abstract from a microfoundation of the intermediary. Diamond (1984) provides a theory for the existence of financial intermediaries due to a cost advantage in monitoring.
number of banks $i \in [0, 1]$. Banks enter period $t$ with previously accumulated government bonds $B^D_t$ as well as a constant and exogenous amount $N$ of household deposits. From an accounting identity perspective, it follows that collateral assets form bank equity $K^b_t$. Banks are endowed with retail deposits $N$ in the initial period and pass them from one period to the next as cash holdings $C^m_t$.

We follow the idea of temporary market segmentation in order to motivate an interbank market (e.g. Gertler and Kiyotaki, 2011). Banks exhibit within-period heterogeneity of two different types $\tau \in \{p, u\}$. At an exogenously given probability $\pi^p$, a bank turns into a productive bank of type $p$. This bank is matched with a non-financial firm to which it is able to extend credit $\kappa_t$ at the endogenously determined interest rate $r^p_t$. To this end, bank $p$ uses its own liquid liabilities $N^p$. Additionally, it can borrow funds at the domestic interbank market $M_t$. Working capital and interbank funding are intra-period loans that mature at the end of period $t$.

In case a bank borrows from the interbank market, we assume that it accumulates excess reserves $R^e_t$. This captures the self-insurance motive against costly liquidity shortfall due to roll-over risk in reduced form, formally:\footnote{As the focus of the paper is the adjustment of bank balance sheets through changes in wholesale funding from the interbank market, we leave the adjustment of retail deposits for future research. Further, we are restricted in the amount of state variables due to the numerical solution method.}

$$R^e_t = \frac{M_t}{\phi},\quad (3.7)$$

with $\phi > 1$. Demand for excess reserves induces a spread in the credit rate over bank funding costs, as they convey opportunity costs to productive banks.

A bank turns into an unproductive bank, or type $u$, with complement probability $\pi^u = 1 - \pi^p$. Type $u$ banks do not have the opportunity to provide loans to the non-financial sector. Instead, they offer their resources as interbank loans to type $p$ banks at the interbank market rate $r^M_t$. As productive banks are short of funding in equilibrium for many states of the world, the interbank market gives rise to a reallocation of funds across banks.

Finally, banks of both types may store cash within a period in the central bank’s deposit facility, $R^d_t$. Excess reserves and cash stored in the deposit facility are remunerated at a constant exogenous rate $r^R$. Bank balance sheets are summarized in Table 3.1, where total reserves are denoted by $R^t_t$.

\footnote{However, we do not model a maturity mismatch that would give rise to roll-over risk on the interbank market here. Costly liquidity shortfall in the banking sector under stochastic liability withdrawals has been modelled in Poole (1968) or Baltensperger (1980), among others. Chari et al. (1995) use a similar reduced form for demand in excess reserves in a macro setting.}
Table 3.1: Bank balance sheets

<table>
<thead>
<tr>
<th>productive ((p))</th>
<th>unproductive ((u))</th>
</tr>
</thead>
<tbody>
<tr>
<td>intra-period (t)</td>
<td></td>
</tr>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>(R^p_t)</td>
<td>(\kappa_t)</td>
</tr>
<tr>
<td>(q_t B^D_p)</td>
<td>(K^b_p)</td>
</tr>
<tr>
<td>end of period (t)</td>
<td></td>
</tr>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>(C^{m,p}_t)</td>
<td>(N^p)</td>
</tr>
<tr>
<td>(q_t B^D_{t+1})</td>
<td>(K^b_p)</td>
</tr>
</tbody>
</table>

Interbank loans need to be collateralised at a constant haircut of size \(\chi\).\(^18\) The only available collateral in the model economy are government securities which take the form of one-period discount bonds.\(^19\) The government cannot commit to repay. A bond issued in period \(t - 1\) contains a promise by the government to repay one unit of the numeraire good in period \(t\), conditional on not defaulting. Let \(\delta_t \in \{0, 1\}\) denote the period \(t\) default decision of the government, where repayment is given by \(\delta_t = 0\). The government’s default decision is taken as exogenous from the bank perspective. Further, let \(B^D_{t,\tau}\) denote collateral accumulated on bank balance sheet of type \(\tau\). Then, the period \(t\) value of government bonds held by banks is \((1 - \delta_t)B^D_{t,\tau}\) such that the collateral constraint takes the form:

\[
M_t \leq \frac{(1 - \delta_t)B^D_{t,p}}{\chi}
\]  

(3.8)

We use the result of Barro (1976) who shows that the price for a collateralised debt contract accounts for the quality of the underlying collateral.\(^20\) To this end, let \(\Psi(q_t)\) denote a risk premium on secured interbank contracts that is assumed to be decreasing in the price of government bonds \(q_t\).

Banks purchase government securities on bond markets at price \(q_t\). In line with the

---

\(^{18}\)The collateral requirement on the interbank market is introduced ad hoc for simplicity. It is possible to derive an incentive compatibility constraint similar to equation (3.8) from an agency problem. One possibility is that the borrowing bank \(p\) is limited to pledge future income for repayment on its liabilities \(M_t\) to type \(u\) banks due to moral hazard as shown by Holmström and Tirole (1998).

\(^{19}\)This assumption is in line with empirical regularities in the euro area as discussed in Section 3.2. Further, Krishnamurthy and Vissing-Jorgensen (2012) document that US-Treasuries are close substitutes to money due to their high liquidity and safety. With this property, government debt features low information sensitivity which makes it a preferred choice for collateral (Dang et al., 2013).

\(^{20}\)In Barro (1976), contract parties account for the effective interest rate by internalising the probability of collateral exchange in the event of a counter-party default. However, allowing for equilibrium interbank default that leads to the exchange of collateral is beyond the scope of this paper.
literature, we define government assets as net borrowing of the sovereign, \( B_t < 0 \). Government bonds are either held domestically by the banking sector, or internationally by financial investors, \( B_t^* \). The market clearing condition on the bond market reads

\[
B_{t+1} + B_{t+1}^D + B_{t+1}^* = 0, \quad \text{with } B_{t+1}^D, B_{t+1}^* \geq 0
\]  

(3.9)
such that \(-q_t B_{t+1}^* = q_t (B_{t+1} + B_{t+1}^D)\) denotes the net foreign asset position of the economy.

Each bank maximizes discounted expected lifetime dividend payments to the representative household using the bank discount factor \( \beta^b \),

\[
\max E_t \left[ \sum_{j=0}^{\infty} \beta^b E_t \left[ D_{t+j}^p \right] \right],
\]

where the choice variables depend on the idiosyncratic realization of types \( \tau \in \{p, u\} \). We exploit the recursive structure of the model to rewrite the problem in Bellman form. The maximization problem for a bank of type \( p \) reads

\[
W^p(B_t, B_{t}^D, A_t) = \max \left\{ D_{t}^p(B_t, B_{t}^D, A_t) + \beta^b E_t \left[ W(B_{t+1}, B_{t+1}^D, A_{t+1}) \right] \right\},
\]

with dividends defined as

\[
D_{t}^p = (1 - \delta_t)B_{t}^{D,p} + (1 + r_t^c)\kappa_t - (1 + r_t^M)M_t + (1 + r_t^R)R_{t}^p - (1 - \delta_t)\tilde{q}_t B_{t+1}^{D,p} - N^p.
\]  

(3.10)

The bank \( p \) objective function is subject to (3.7), (3.8), and the following constraints:

\[
N^p + M_t = \kappa_t + R_{t}^p
\]  

(3.11)

\[
R_{t}^p = R_{t}^c + R_{t}^{d,p}
\]  

(3.12)

\[
R_{t}^{d,p}, D_{t}^p \geq 0
\]  

(3.13)

The flow of funds constraint (3.11) illustrates that an increase in credit \( \kappa_t \) beyond the liquid resources \( N^p \) requires interbank borrowing. Total reserves held by a productive bank \( R_{t}^p \) are either excess reserves, or cash stored in the deposit facility (3.12). The non-negativity constraint on the deposit facility (4.12) implies that there is no direct funding available from the central bank.

The collateral constraint (3.8) constitutes an occasionally binding borrowing constraint. Productive banks can obtain additional funding only up to a multiple \( \chi \) of the market

\[\text{Using the stochastic discount factor of the representative household would distort the collateral accumulation decision of banks, since models with sovereign default feature low values for } \beta \text{ to account for default in equilibrium, see Mendoza and Yue (2012).}\]
value of government securities currently on their balance sheet. As financial sector holding of government debt is an endogenous state variable, this constraint establishes the inter-temporal dimension of the banks’ problem. Particularly, banks are trading-off dividend payments today against future dividend payments. The bank cannot purchase more public securities than there are cash-flows available in period \( t \) (4.12).

The maximization problem of type \( u \) banks takes the form

\[
W_u(B_t, B_D^t, A_t) = \max_{\{B_D^u, M_t, R_t^u\}} \left\{ D_t^u(B_t, B_D^t, A_t) + \beta^h E_t \left[ W(B_{t+1}, B_D^{t+1}, A_{t+1}) \right] \right\},
\]

with dividends defined as

\[
D_t^u = (1 - \delta_t)B_D^t + (1 + r^M)M_t + (1 + r^R)R_t^u - (1 - \delta_t)\tilde{q}_tB_D^{p,t} - N^u \tag{3.14}
\]

subject to the constraints

\[
\begin{align*}
N^u &= M_t + R_t^u \tag{3.15} \\
R_t^u &= R_d^u \tag{3.16} \\
R_d^u, D_t^u &\geq 0 \tag{3.17}
\end{align*}
\]

Equation (3.15) constitutes the flow of funds constraint. Bank \( u \) reserves are defined by its use of the central bank deposit facility (3.16). As in the problem of type \( p \) banks, additional funds from the central bank are not available, and collateral purchases are constrained by the non-negativity constraint on dividends (4.16).

As period \( t + 1 \) types are unknown and probabilities are identically and independently distributed, expected future dividend payments are formed using the unconditional probability for each type in the consecutive period:

\[
E_t \left[ W(B_{t+1}, B_D^{t+1}, A_{t+1}) \right] = E_t \left[ \pi^p W^p(B_{t+1}, B_D^{t+1}, A_{t+1}) + \pi^u W^u(B_{t+1}, B_D^{t+1}, A_{t+1}) \right]
\]

Forming the Lagrangian for both types of banks, the first-order conditions for credit, excess reserves and collateral of productive banks read\(^{22}\)

\[
\begin{align*}
r^K_t &= r^R + \mu^p_t, \tag{3.18} \\
r^K_t &= r^M + \frac{r^K_t - r^R}{\phi} + \lambda_t \chi \tag{3.19} \\
\tilde{q}_t &= \beta^h E_t \left[ W_B D(B_{t+1}, B_D^{t+1}, A_{t+1}) \right] + \mu_t^{D,p} \tag{3.20}
\end{align*}
\]

\(^{22}\)See Appendix B.3 for a detailed derivation of the results.
Optimal supply of working capital credit from (3.18) implies that the type \( p \) bank wants to increase its lending to non-financial firms as long as it earns at least a return of \( r_R \), which constitutes the opportunity investment within period \( t \). \( \mu_t^p \) denotes the Lagrange multiplier on the flow of funds constraint. The optimal amount of wholesale funding (3.19) accounts for the cost of funding \( r_t^M \) and the foregone profits due to additional excess reserves which lower funds available for credit by \( 1/\phi \). The Lagrange multiplier on the collateral constraint \( \lambda_t \) captures whether further interbank borrowing is feasible. Equation (3.20) is the pricing equation for government bonds at secondary bond markets which is discussed below.

Optimization of type \( u \) banks yield as first-order conditions

\[
\begin{align*}
\tilde{q}_t &= \beta^b E_t \left[ \mathcal{W}_{B^D}(B_{t+1}, B^D_{t+1}, A_{t+1}) \right] + \mu_t^{D,u}, \\
(3.21) \\
(3.22)
\end{align*}
\]

Optimal supply of interbank loans in (3.21) states that the interbank rate \( r_t^M \) makes type \( u \) banks indifferent between investing in excess reserves or interbank loans. In equilibrium, unproductive banks are competing for loan demand on the interbank market such that \( \mu_t^u = 0 \) and \( r_t^M = r_R + \Psi(q_t) \). The bond pricing equation (3.22) is discussed below.

The complementary slackness conditions for the inequality constraints of the banking sector are the following set of equations

\[
\begin{align*}
\lambda_t \left( 0 - \chi M_t + (1 - \delta_t)B_t^{D,p} \right) &= 0 \quad (3.23) \\
\mu_t^\tau (0 + P_t^{D,\tau}) &= 0 \quad \forall \tau \in \{p, u\} \quad (3.24) \\
\mu_t^{D,\tau} (0 + D_t^{u,\tau}) &= 0 \quad \forall \tau \in \{p, u\} \quad (3.25) \\
\lambda_t, \mu_t^\tau, \mu_t^{D,\tau} &\geq 0 \quad \forall \tau \in \{p, u\} \quad (3.26)
\end{align*}
\]

where \( \lambda_t, \mu_t^\tau, \mu_t^{D,\tau} \forall \tau \in \{p, u\} \) denote the Lagrange multipliers on the collateral constraint, the non-negativity constraints on central bank deposits and dividends, respectively.

In order to arrive at the aggregate equilibrium allocations in the model economy across banking types, we use the following proposition:

**Proposition 1.** (Aggregation) Under the assumption of equal collateral holdings of banks in the initial period \( t = 0 \), there exists a representative bank for each type \( \tau \in \{p, u\} \) such that the equilibrium allocations can be derived from the representative bank. In particular,
the following relationship holds:

$$\sum_{i=1}^{\pi^r} B_i^{D,r} = \pi^r B_i^D, \quad \tau \in \{p, u\}$$

**Proof.** See Appendix B.1.

From Proposition 1 follows that no subscripts for individual banks are required in equations (4.13)-(4.12), and (4.17)-(4.16). Aggregate dividend payments to households are obtained by summing over bank types \(D_t = D_t^p + D_t^u\) and, accordingly, for central bank reserves \(R_t = R_t^p + R_t^u\).

Turning now to the dynamic part of the bank decision problem, note that the pricing decisions (3.20) and (3.22) for collateral at government bond markets are identical across bank types due to the independent and identical probability of turning into a productive bank in the consecutive period. The envelope condition for \(W_{BD}(B_{t+1}, B_{t+1}^D, A_{t+1})\) yields

$$W_{BD}(B_{t+1}, B_{t+1}^D, A_{t+1}) = \pi^p(1 - \delta_{t+1} + \lambda_{t+1}) + \pi^u(1 - \delta_{t+1} + \mu_{t}^D)$$

$$= 1 - \delta_{t+1} + \pi^p \lambda_{t+1} + \mu_{t}^D$$

such that the pricing equation for collateral assets from both types of banks takes the form

$$\tilde{q}_t = \beta^b(1 - \pi^\delta) + \pi^p \beta^b E_t(\lambda_{t+1}) + \mu_{t}^D + \mu_{t}^{BD}, \quad (3.27)$$

where \(E_t(\delta_{t+1}) = \pi_t^\delta\) denotes the default probability of government bonds in period \(t + 1\), evaluated at the period \(t\) information set. The pricing equation (3.27) is composed of four parts. First, the bank prices the bond according to its discounted expected pay-offs from repayment in period \(t + 1\). Second, with probability \(\pi^p\) a bank will be of type \(p\) in period \(t + 1\) and, hence, is able to increase lending to non-financial firms if it holds additional collateral. We restate this analytical result in the following proposition:

**Proposition 2.** (Liquidity premium) Due to liquidity services derived from holding government bonds as collateral on the interbank market, banks’ asset pricing equation on public debt yields a liquidity premium, denoted as \(\tilde{\lambda}_t\), in the form:

$$\tilde{\lambda}_t \equiv \pi^p \beta^b E_t(\lambda_{t+1})$$

**Proof.** See Appendix B.1.

The two last terms in equation (3.27) relate to corner solutions on the government bond market in case of binding constraints, either due to non-negative dividend payments, or due to total outstanding public securities from (3.9). Banks take the total outstanding
amount of government debt as given and may run into the problem of low supply of
government bonds such that $\mu_t^B > 0$.

As international investors have unlimited funds, banks take the price for public securities $q_t$ as given. The following proposition describes the supply and demand of government bonds from international investors:

**Proposition 3.** Risk-neutral international investors exhibit a perfectly elastic supply and demand function of government bonds on secondary bond markets at price $q_t$.

**Proof.** See Appendix B.1. ■

Let $F(B_t, B_t^D, A_t) = B_t^{D^*} | B_{t+1}$ denote the demand function for domestically held debt conditional on the total amount of newly issued government securities. Demand from banks is pinned down by the no-arbitrage condition from equation (3.27) while taking prices as given from Proposition 3, hence

$$\tilde{q}_t - q_t = 0 \iff \beta^b(1 - \pi_t^\delta) + \pi^b \beta^b E_t(\lambda_{t+1}) + \mu_t^D + \mu_t^{B^D} - q_t = 0,$$

Banks purchase government debt up to the amount where they expect zero liquidity premium on additional collateral holding, if unconstrained by their respective flow budget constraint or the short-selling constraint.

![Figure 3.3: Bond market equilibria](image-url)

Figure 3.3 illustrates the possible equilibria on the market for government debt. International investors’ supply curve is perfectly elastic at the market price $q_t$ and given by $S^*$ (Proposition 3). Domestic banks take the bond price as given and have a downward sloping demand for collateral ($D^A$), which is due to a decreasing expected liquidity value for collateral, $\frac{\partial^2 \lambda_t}{\partial B_{t+1}^D} < 0$. An interior solution on the interval $B_{t+1}^D \in (0, B_{t+1})$ is found if $\tilde{q}_t = q_t$ and no constraint is binding ($A$), or if the accumulation of collateral is limited by
the non-negativity requirement for dividend payments \((A')\). Corner solutions are realized if demand for collateral assets is exceeding total supply \((B)\). In this case, total public debt is held domestically. The opposite case is given if domestic banks have a very low expected collateral value in the consecutive period such that \(\bar{q} < q\) \((C)\).

\subsection{3.3.5 Central bank}

A central bank collects bank deposits and excess reserves from the domestic banking system. All reserves are remunerated at a fixed and exogenously given rate \(r^R\). To keep a clear focus of the analysis, we do not consider any form of central bank intervention. Period \(t\) central bank profits amount to

\(\Pi_t^b = -R_t r^R,\) \hspace{1cm} (3.28)

which are transferred lump-sum to the government.

\subsection{3.3.6 Partial equilibrium}

We need to define two types of partial equilibria. In the repayment equilibrium absent sovereign default, the partial equilibrium consists of market clearing quantities and prices for the labour market, credit market, and the interbank market variables \(\{L_t, W_t, r_t^*, r_t^M, \kappa_t, M_t, R_t^e, R_t^{d*}, R_t^{p*}, \lambda_t, \mu_t^p\}\). Taking as given the aggregate state \(s \in (B_t, B_t^D, A_t)\) and the price of government debt \(q_t(B_t+1, B_t^D+1, A_t)\), the equations \((3.2), (4.6), (3.5), (3.7), (3.8), (3.11), (3.12), (4.12), (3.15), (3.16), (4.16), (3.18), (3.19), (3.21)\), in combination with the complementary slackness conditions \((3.23), (3.24), (3.26)\) solve the non-linear system.

The inter-temporal collateral decision is derived as a reaction function conditional on the government decision on total outstanding debt, \(F(B_t, B_t^D, A_t)\). The unknowns \(\{B_{t+1}^o, \mu_t^D, \mu_t^D\}\) are obtained from equations \((3.25), (3.26), (3.27)\). Finally, dividend payments are derived from \((4.13)\) and \((4.17)\).

In the partial equilibrium under sovereign default, interbank market trading breaks down as \(q_t = 0\), such that \(M_t = R_t^e = 0\) follows jointly from the collateral constraint and the liquidity preference. Hence, \(r_t^M\) and \(\lambda_t\) remain undefined and all resources of unproductive banks are deposited at the central bank, \(R_t^{d,u} = N_u\). The solution to the remaining partial equilibrium variables \(\{L_t, W_t, r_t^*, \kappa_t, R_t^{d,p}, R_t^p, \mu_t^p\}\) is given by equations \((3.2), (4.6), (3.5), (3.11), (3.12), (4.12), (3.18)\), in combination with the complementary slackness conditions \((3.24), (3.26)\) for \(\tau = p\).
3.3.7 Government

There is a benevolent government in analogy to Eaton and Gersovitz (1981) that cannot commit to repay its debt. Specifically, the government chooses to default on the total amount of outstanding debt if this is welfare maximizing in terms of household utility given the aggregate state of the economy $s \in (B_t, B_t^D, A_t)$. We look at equilibria where the government cannot discriminate between foreign and domestic bond holders when defaulting. Technically, the optimal default decision consists of maximizing the value function

$$V_t(B_t, B_t^D, A_t) = \max_{\delta \in \{0, 1\}} (1 - \delta)V^{rd}_t(B_t, B_t^D, A_t) + \delta V^d_t(B_t, B_t^D, A_t),$$

where $V^{rd}_t, V^d_t$ denote the value of repayment and default, respectively.

In case the government decides to repay its debt obligations, it borrows from capital markets by selling one period discount-bonds $B_{t+1} < 0$ at the market price $0 < q_t < 1$. This bond pays back $B_{t+1}$ units of consumption goods in period $t + 1$, conditional on not defaulting. Government borrowing serves a consumption smoothing purpose for private households via direct transfer payments $T_t$ according to the implicit government flow budget constraint

$$T_t = \Pi^{cb}_t + (B_t - q_tB_{t+1})(1 - \delta_t),$$

where central bank profits $\Pi^{cb}_t$ from equation (3.28) are consolidated in the government budget constraint.

The government decides on its optimal debt policy and internalises the decentralised collateral decision of the financial sector by taking account of the reaction function $F(B_t, B_t^D, A_t)$. Optimal new borrowing conditional on not defaulting maximizes the following value function

$$V^{nd}_t(B_t, B_t^D, A_t) = \max_{\{C_t, B_{t+1}\}} \{U(C_t, 1 - L_t)$$

$$+ \beta \int_{A_{t+1}} V_{t+1}(B_{t+1}, B_{t+1}^D, A_{t+1})f(A_{t+1})dA_{t+1}\},$$

subject to the aggregate resource constraint in the economy

$$C_t = e^{A_t}K^\alpha L_t^{1-\alpha} + (B_t + B_t^D) - q_t(B_{t+1} + B_{t+1}^D)$$

$$= Y_t - B_t^* + q_tB_{t+1}^*$$

$^{23}$A formal derivation of the aggregate resource constraint is provided in Appendix B.3.
and the set of partial equilibrium conditions under repayment as laid out in Section 3.3.6.

The trade-off for the optimal government debt policy is implied in the latter equation, where benefits in terms of household utility from increases in net external debt need to be weighted against the spillovers to aggregate production. Specifically, the collateral channel introduces an ambiguous sign on the reaction of output to an increase in the total amount of outstanding debt. At very low levels of government debt, an expansion of public borrowing leads to an increase in securities available to the banking sector, thereby stimulating output. On the other side weighs the interaction of sovereign risk with financial frictions on the cost of working capital and output if the level of public debt turns risky from the perspective of bond holders.

The government internalizes these externalities of its borrowing decision, which might arise either from a drop in the bond price $q_t$ as a consequence of choosing high debt levels, or from an overall scarcity of collateral from choosing very low debt levels. Given that the government cannot remove the deep frictions inherent to the domestic financial sector, it achieves a constrained efficient outcome under the repayment regime.

In line with the literature, investors penalise the government for defaulting by forcing the economy into financial autarky with a stochastic probability $\theta > 0$ of re-entering capital markets. When regaining market access, the economy starts with zero government bonds. The value of choosing default is defined by

$$V^d_t(B_t, B^D_t, A_t) = \max \{U(C_t, 1 - L_t) + \beta \int_{A_{t+1}} \left( \theta V_{t+1}^{nd}(0, 0, A_{t+1}) + (1 - \theta) V_{t+1}^d(0, 0, A_{t+1}) \right) f(A_{t+1}, A) dA_{t+1} \}$$

subject to the set of partial equilibrium conditions without interbank trading from Section 3.3.6 and the resource constraint under autarky:

$$C_t = Y_t$$

(3.31)

The default set $\Gamma^d_t$ is defined as the subset of the productivity state $A_t$ for which the value of defaulting is strictly higher than the value of repayment, given the endogenous state $\epsilon_t \in (B_t, B^D_t)$:

$$\Gamma^d_t(B_t, B^D_t) = \{ A_t \in s : V^d_t(B_t, B^D_t, A_t) > V^{nd}_t(B_t, B^D_t, A_t) \}$$

Note that the default set is two-dimensional, meaning that the distance between the value of repayment $V^{nd}$ and the value of default $V^d$ are affected not only by the level of
government debt $B_t$, but also by the amount of public debt held domestically, $B_t^D$.

We find that the model with external and domestic debt preserves standard properties of quantitative models of sovereign default. Specifically, the model confirms a positive relationship between the total amount of outstanding debt with the probability of default, given a productivity state $A_t$ and domestic debt $B_{t+1}^D$.

**Proposition 4.** If default is optimal in state $(A_t, B_t^D)$ for $B_t^2 \geq |\tilde{B}_t^D|$, default is also optimal for $B_t^1 < B^2 \geq |\tilde{B}_t^D|$ given the same state $(A_t, \tilde{B}_t)$, i.e. $\Gamma^\delta_t(B_t^1, B_t^D) \subseteq \Gamma^\delta_t(B_t^2, B_t^D)$.

**Proof.** See Appendix B.1.

Further, we extend this finding by the observation that the default set $\Gamma^\delta_t$ is shrinking in the amount of debt held by the domestic banking sector.

**Proposition 5.** If default is optimal in state $(A_t, \tilde{B}_t)$ for $B_{t+1}^{D,2} \leq |\tilde{B}_t|$, default is also optimal for $B_{t+1}^{D,1} < B_{t+1}^{D,2} \leq |\tilde{B}_t|$ given the same state $(A_t, \tilde{B}_t)$, i.e. $\Gamma^\delta_t(B_{t+1}, B_t^{D,1}) \subseteq \Gamma^\delta_t(B_{t+1}, B_t^{D,2})$.

**Proof.** See Appendix B.1.

The default probability is defined as the conditional cumulative probability density over the productivity state in period $t+1$ from the default set $\Gamma^\delta_t$:

$$\pi^\delta_t(B_{t+1}, B_{t+1}^D, A_t) = \int_{\Gamma^\delta_t(B_{t+1}, B_{t+1}^D)} f(A_{t+1}, A_t) dA_{t+1}$$

### 3.3.8 International investors

International investors are risk neutral. The expected returns on the one-period discount bond are denoted by $E_t(1 - \delta_{t+1})$, which are discounted by an exogenously given risk-free investment opportunity with constant return $r^f$. Government bonds are priced according to the no-arbitrage condition

$$q_t(B_{t+1}, B_{t+1}^D, A_t) = \frac{1 - \pi^\delta_t(B_{t+1}, B_{t+1}^D, A_t)}{1 + r^f}.$$  (3.32)

From the price of the one-period discount bond we extract the period $t$ rate of return on public debt as $r_t^g \equiv q_t^{-1} - 1$.

The effect of domestic government debt on the bond price follows jointly from Proposition 5, the definition of the default probability and the asset pricing equation of international investors (3.32):

**Proposition 6.** (i) Higher shares of total outstanding government debt purchased by the domestic banking sector on the secondary bond market lowers the probability of a sovereign
default in period $t+1$, or formally $\pi_t^\delta(\bar{B}_{t+1}, B_{t+1}^{D,2}, A_t) \leq \pi_t^\delta(\bar{B}_{t+1}, B_{t+1}^{D,1}, A_t)$ for $0 < B_{t+1}^{D,1} < B_{t+1}^{D,2} \leq |\bar{B}_{t+1}|$. (ii) A higher share of total outstanding government debt held by domestic banks lowers the spread on government bonds, i.e. $q_t(\bar{B}_{t+1}, B_{t+1}^{D,2}, A_t) \geq q_t(\bar{B}_{t+1}, B_{t+1}^{D,1}, A_t)$.

Proof. See Appendix B.1.

3.3.9 Equilibrium

We examine a Markov perfect equilibrium where the state vector $s = (B_t, B_{t+1}^D, A_t)$ is sufficient to define the optimal policy by all agents in the model.

Definition. The model’s recursive equilibrium is given by

1. Value functions $V, V^{rd}$, and $V^d$ for the government;
2. Policy functions of the government’s default decision $\delta_t \in \{0, 1\}$, and future borrowing $B_{t+1}$;
3. Policy functions for the private sector decisions on $C_t, L_t, M_t, \kappa_t, R_t$;
4. Bank policy function $F$ for the inter-temporal decision on collateral purchases and dividend payments $B_{t+1}^{D,r}, D_t^r$;
5. A bond pricing equation from international investors for $q_t$;

such that given the government policies and the bond pricing equation, the household policies for consumption and labour solve the household’s problem; given productivity and credit supply, the policies for labour and credit solve the profit maximizing problem of non-financial firms; given the bond pricing equation and credit demand, the banking sector decisions maximize financial sector dividends and satisfy the market clearing condition on secondary bond markets; the consumption plan $C_t(B_t, B_{t+1}^D, A_t)$ satisfies the resource constraint of the economy; the transfer policy $T_t(B_t, B_{t+1}^D, A_t)$ satisfies the government budget constraint; given $\Gamma_\delta^e(B_t, B_{t+1}^D)$ and $\pi_\delta^e(B_{t+1}, B_{t+1}^{D,2}, A_t)$, the bond pricing function $q_t(B_{t+1}, B_{t+1}^{D,2}, A_t)$ satisfies the no-arbitrage condition of foreign lenders.

3.4 Analysis of the bank-sovereign link

3.4.1 Calibration and solution method

We use Spanish data at a quarterly frequency over the period 2000q1 to 2011q4 in order to calibrate the model for the quantitative analysis. Although Spain did not default on its securities during the European sovereign debt crisis, it was among the group of countries that exhibited strong increases in country default risk that spilled over to the
domestic banking sector (Section 3.2). Therefore, we think that the case of Spain provides a valuable testing ground to evaluate our model’s propagation mechanism. We focus on the time period of membership within the European Monetary Union (EMU) until the end of 2011 when the ECB took non-standard policy measures at an unprecedented scale to stimulate the money market in order to alleviate the segmentation of funding conditions along national borders within the EMU. Limiting the analysis to this time period seems appropriate given that interventions from the central bank are absent in our model environment.

For the calibration of the household utility function, we use preferences proposed by Greenwood et al. (1988),

\[
U_t(C_t, 1 - L_t) = \left( \frac{C_t - \frac{1}{\omega} L_t^\omega}{1 - \gamma} \right)^{1-\gamma} - 1,
\]

and we take common values from the literature to calibrate the degree of risk aversion \( \gamma = 2 \), and the wage elasticity of labour supply \( \omega = 1.455 \). For the production function, we set the capital share of output to \( \alpha = 0.36 \) which is a standard value for models calibrated to the euro area (Smets and Wouters, 2002). The risk-free rate \( r_f \) is set to 1.0 percent at a quarterly frequency. This value is widely used in the literature and roughly in line with the average yield of German government bonds at a maturity of 10 years. Banks’ discount factor is set to \( \beta^b = 1/(1 + r_f) \) such that the difference in valuation of government bonds between foreign investors and domestic banks is due to liquidity services only. The probability of re-entry to international capital markets after default is set to \( \theta = 0.083 \), which is taken from Richmond and Dias (2009), who find an average historical exclusion spell of three years for a cross-section of default episodes since 1980 in emerging market economies.

We take the parameters related to the process of aggregate productivity directly from Mendoza and Yue (2012), setting \( \rho = 0.95 \) and \( \sigma_e = 1.7 \) percent. We interpret the model predictions for aggregate volatility with caution, as the volatility properties of our model are influenced by the solution method as shown by Hatchondo et al. (2010). Therefore, we do not target the standard deviation and autocorrelation of the cyclical component of logged GDP, as it is common in the literature, but favour instead a closer comparability with the existing literature on endogenous output penalties.

The central bank deposit rate is calibrated by taking the average difference of the ECB rate on the deposit facility and the EONIA-swap rate. This yields a rate on central bank

\footnote{Primarily, the ECB launched two long-term refinancing operations with a maturity of three years which took a volume of about 1000 billion euros). See the corresponding ECB announcement from December 8\textsuperscript{th} 2011, http://www.ecb.europa.eu/press/pr/date/2011/html/pr111208_1.en.html}
reserves that lies, on average, 23 basis points below the risk-free rate, which yields a value for \( r^R \) of 0.77 percent.

The relationship of collateral quality to the interest rate on interbank markets is governed by the risk premium \( \Psi(q_t) \). We assume a linear relationship and set the functional form to

\[
\Psi(q_t) = \psi \left( r^g_t - r^f \right)
\]

where the term in parentheses is the spread of the government bond yield over the risk-free rate, \( r^g_t \equiv r^g_t - r^f \). To calibrate \( \psi \), we estimate the sensitivity of the interbank rate \( r^M \) towards sovereign default risk from the data, using equation (3.21) and data at daily frequency over the period 31 March to 7 December 2011. The relatively short sample period is due to data availability. For the spread on the secured interbank market rate, we take the general collateral (GC) repo rate of Spain and subtract the 3-month EONIA-swap rate to obtain the interbank premium of Spanish collateral over the risk-free rate. The government risk premium is taken from the spread of a 10-year Spanish government benchmark bond over the German Bund. OLS regressions yield an average marginal effect of \( \psi = 0.245 \).

Table 3.2: Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
<th>Data(^{(a)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share of output</td>
<td>( \alpha )</td>
<td>0.36</td>
<td>Literature</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>( \gamma )</td>
<td>2</td>
<td>Literature</td>
</tr>
<tr>
<td>Wage elasticity of labor supply</td>
<td>( \omega )</td>
<td>1.455</td>
<td>Literature</td>
</tr>
<tr>
<td>Riskfree rate</td>
<td>( r^f )</td>
<td>0.01</td>
<td>Literature</td>
</tr>
<tr>
<td>Bank’s discount factor</td>
<td>( \beta^b )</td>
<td>0.99</td>
<td>( 1/(1 + r^f) )</td>
</tr>
<tr>
<td>Probability of reentry</td>
<td>( \theta )</td>
<td>0.083</td>
<td>Richmond and Dias (2009)</td>
</tr>
<tr>
<td>Persistence of TFP shock</td>
<td>( \rho )</td>
<td>0.95</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>Std.dev. of TFP shock</td>
<td>( \sigma_\rho )</td>
<td>0.017</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>Rate on central bank reserves</td>
<td>( r^R )</td>
<td>0.0077</td>
<td>( r^f - r^R )</td>
</tr>
<tr>
<td>Cost function collateral</td>
<td>( \psi )</td>
<td>0.245</td>
<td>OLS estimate</td>
</tr>
</tbody>
</table>

Simulated Method of Moments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
<th>Data(^{(a)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household’s discount factor</td>
<td>( \beta )</td>
<td>0.92</td>
<td>Default frequency</td>
</tr>
<tr>
<td>Capital stock</td>
<td>( K )</td>
<td>11.92</td>
<td>( K/Y )</td>
</tr>
<tr>
<td>Working capital requirement</td>
<td>( \eta )</td>
<td>1.074</td>
<td>( \kappa/Y )</td>
</tr>
<tr>
<td>Liquid liabilities</td>
<td>( N )</td>
<td>2.97</td>
<td>( r^e - r^f )</td>
</tr>
<tr>
<td>Efficiency parameter</td>
<td>( \phi )</td>
<td>7.52</td>
<td>( R^e/(\kappa + M + qB^D) )</td>
</tr>
<tr>
<td>Probability for lending bank</td>
<td>( \pi^p )</td>
<td>0.568</td>
<td>( M/L^{banks} )</td>
</tr>
<tr>
<td>Collateral requirement</td>
<td>( \chi )</td>
<td>0.257</td>
<td>( B^D/B )</td>
</tr>
</tbody>
</table>

Notes: \(^{(a)}\)Details regarding data sources and available sample periods are provided in the Appendix.

The remaining parameter values \( \{\beta, K, \eta, N, \pi^p, \chi, \phi\} \) are found using the simulated
method of moments (SMM) with the following calibration targets in the stochastic stationary state. The historical default frequency of Spain is 0.65 percent according to data from Sturzenegger and Zettelmeyer (2007) covering the period 1820 to 2011. The capital-to-output ratio from the European Commissions’ Ameco database takes the value of 3.47 over the period of interest ranging from 2000 to 2011. The working capital need at a quarterly frequency is approximated using data from the Bank of Spain on the nominal volume of short-term bank loans to the non-financial sector with a maturity of up to one year over nominal GDP, which yields a target for $\kappa/Y$ of 68 percent. Further, using the bank lending rate on credit to non-financial corporations with a maturity below one year, we find an average credit premium of 50 basis points at a quarterly frequency over the EONIA-swap rate.

The parameters $\phi$, $\pi^p$, and $\chi$ are chosen to target long-run relationships in the financial sector. In order to approximate the liquidity preference of banks, we use the bank liquid reserves to bank asset ratio from the World Bank. According to the data, it takes an average value of 1.53 percent over the period 2000 to 2011 and we target the long run average of the ratio $R_e/(\kappa + M + qB^D)$ from the model to match this statistic.

A central value in our calibration is the ratio of wholesale funding to aggregate bank liabilities in the stochastic steady state ($M/L_{banks}$), where we derive aggregate bank liabilities in the model from bank balance sheets as $L_{banks} = N + M + qB^D$. The ratio $M/L_{banks}$ has implications for the size of the default penalty through the freeze on the interbank market. In the baseline calibration, we approximate the aggregate use of wholesale funding in the Spanish banking sector by taking the ratio of deposits by domestic credit institutions over total domestic liabilities available from the Bank of Spain at a monthly frequency. Taking the long-run average of this ratio, we set the calibration target to $M/L_{banks} = 0.20$. We consider this to be a conservative assumption on the dependence of Spanish banks on wholesale funding given that we do abstract from cross-border flows within the euro area.

Finally, regarding the ratio of domestic to total government debt we set the target to roughly 55 percent using the sovereign investor base estimates by Arslanalp and Tsuda (2012) starting in the first quarter of 2004.

From the SMM, we obtain a relatively low value for the household discount factor, $\beta = 0.92$, which is common in the literature on endogenous sovereign default. The

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<sup>25</sup>Spain defaulted four times in the 19<sup>th</sup> century and exhibits a historical default frequency that is very close to that of Argentina.

<sup>26</sup>From an accounting perspective, government bonds constitutes bank equity according to the balance sheet equation $R^p + \kappa + qB^D,p = N^p + M + K^{b,p}$, where type $p$ bank equity $K^{b,p} \equiv qB^D,p$ is the residual to equate the asset with the liability side.

<sup>27</sup>For a comparison, Giannone et al. (2012) estimate that wholesale funding makes up to 60 percent of banking sector liabilities in the euro area.

<sup>28</sup>E.g. Mendoza and Yue (2012) use $\beta = 0.88$. 

---
value obtained for the working capital requirement $\eta = 1.074$ is not directly comparable to the literature on emerging market business cycles\(^{29}\) or to Mendoza and Yue (2012), since working capital in our model is financed by domestic banks while the literature on emerging market business cycles refers to a borrowing requirement on international capital markets. This modelling difference by itself justifies a higher parameter value for $\eta$, in particular given that external finance in Europe is traditionally bank-dominated.\(^{30}\)

For the remaining parameters, we find $K = 11.92$, $N = 2.97$, $\pi^p = 0.568$, $\chi = 0.257$, and $\phi = 7.52$.

In order to find a numerical solution to the equilibrium described in Section 3.3.9, we follow the literature on quantitative models of sovereign default using value function iteration on the government problem on a discrete state space. The exogenous productivity state variable is defined on a grid with 15 nodes. We use Tauchen’s 1986 procedure to discretise the state space of total factor productivity according to its law of motion from equation (3.6). The endogenous state variables are discretised over a grid with equidistant nodes, and intervals $B \in [B_{\text{min}}, 0]$ and $B^D \in [0, B^D_{\text{max}}]$, where $B^D_{\text{max}} = -B_{\text{min}}$. We make sure that the boundary of total government debt is non-binding along the simulated equilibrium path. Detailed explanations regarding the computational strategy are laid out in Appendix B.4.

3.4.2 Sovereign risk and financial frictions

The financial frictions in the model propagate endogenous sovereign default risk to financial market conditions, which in turn determine real sector allocations. In this section, we spell out the contributions of the main assumptions regarding these ex ante costs of sovereign risk in a numerical example before proceeding to the quantitative evaluation of the dynamic properties of the set-up. Specifically, we show partial equilibria as laid out in Section 3.3.6 for different calibrations of reduced form financial frictions.

Figure 3.4 illustrates the interaction of financial frictions and sovereign risk in the repayment equilibrium. Default risk is shown in terms of a decreasing bond price according to investors’ pricing equation (3.27). The parameter $\psi > 0$ governs the cost of wholesale bank funding according to the risk premium on secured interbank debt $\Psi(q_t)$. The lower the value of $\psi$, the lower is the elasticity of the interbank rate on sovereign risk (Fig. 3.4a). The increase in the interbank rate propagates into lower interbank loan volumes

\(^{29}\)Neumeyer and Perri (2005) and Uribe and Yue (2006) use values in the range of 1 to 1.2 in order to generate an effect of a stochastic interest rate on output.

\(^{30}\)Using the approximation of working capital needs following Schmitt-Grohé and Uribe (2007), i.e. two thirds of the Spanish contribution to euro area M1 over Spanish GDP, yields a similarly high parameter value for $\eta$, which is in line with higher observed working capital needs in the euro area compared to the U.S. (Christiano et al., 2010).
Figure 3.4: Interaction of default risk and financial frictions

through the interaction with the need for liquid reserves on interbank loans (Fig. 3.4b).

How does the precautionary motive for liquidity accumulation influence the decline in intermediated funds? This friction introduces a spread over the interbank rate $r^M_t$ in its relationship to the credit rate $r^C_t$. The optimality condition on interbank loans from equation (3.19) illustrates this point. Assume that the collateral constraint is non-binding, we can approximate equation (3.19) by the expression:

$$r^C_t = r^M_t - \frac{r^R_t}{1 - \frac{1}{\phi}}$$

An exogenous increase in sovereign risk results in a rise in $r^M_t$ according to the risk premium $\Psi(q_t)$, which in turn leads to an even stronger increase in the credit rate in order for the relationship to hold.\(^{31}\) The intuition follows from the opportunity cost of liquidity buffers that increase with higher funding costs on the interbank market.

Figure 3.4(c) shows variations in the parameter $\phi$ from its baseline value. With an increase in $\phi$, the interbank loans react less sensitive to an increase in default risk, as the need to build liquidity buffers for precautionary motives is reduced. This renders interbank borrowing more attractive, since a higher fraction of interbank loans can be channelled to profitable credit.

Next, we investigate the effects of an occasionally binding collateral constraint on the equilibrium allocations. In Figure 3.5, the allocations with a non-binding constraint are compared to those under a binding constraint, both under the baseline calibration.

---

\(^{31}\)This follows from $1/[1 - (1/\phi)] > 1$. Further, note that $\phi > 1$ is a necessary condition for an equilibrium on the interbank market.
Interbank loans, credit, labour and output are shown in percentages compared to the case of a least restricted economy, that is, given a non-binding collateral constraint and absent the risk of sovereign default. The amount of collateral is exogenously chosen in this example to induce a drop in interbank loan volume of 10 percent absent sovereign risk.

While the decline in interbank intermediation with rising sovereign risk is known from the previous explanations, the shortage of collateral assets imposes, on top of that, a borrowing constraint on the interbank market. In the constrained equilibrium, the amount of credit available to the non-financial sector is thus limited from the supply side. Through the working capital requirement, this has an immediate impact on labour input, which needs to be financed with credit from banks. With cash demand of non-financial firms remaining unmet, firms need to cut back on labour input for production. At some degree of default risk, the effect on interbank loans from financial frictions is stronger than the original scarcity of collateral assets, such that the collateral requirement is not binding any longer.

### 3.4.3 Default, credit crunch, and domestic debt

In this section, we characterise the optimal default decision in general equilibrium. We show that the default decision is affected by the size of the disciplining penalty and by domestic holdings of debt. Collateral purchases depend on the productivity state and propagate aggregate shocks over time.

The penalty from defaulting on sovereign debt results from a breakdown of interbank
intermediation when the market value of government bonds collapses to zero. Thus, financial resources can no longer be channelled to their most effective use and type $p$ banks are limited in their funds to the existing stock of liquid assets when extending credit to non-financial firms. The associated output cost of sovereign default is a consequence of the resulting credit crunch in the default period that shrinks the feasible set of allocations for the production sector of the model economy. The working capital assumption imposes an upper bound to the amount of labour which can be employed in the economy under autarky. From equation (4.6) we get the equilibrium condition $L_t = \kappa_t / (\eta W_t)$. Hence, as $\kappa_t$ drops from the supply side, output falls as wage bills cannot be financed. However, the output cost of a sovereign default induced credit crunch are not independent of the stance of the business cycle.

Figure 3.6: Cost of default and domestic debt accumulation

(a) Cost of sovereign default over the cycle

(b) Secondary bond market pricing

Note: Panel (a). Percentage reductions in the default regime compared to the repayment regime absent sovereign default risk. Dotted lines illustrate the scenario with a binding collateral constraint in the repayment equilibrium. Panel (b) - $A_N$ denotes a neutral productivity realisation $\exp(A_t) = 1$, and $A_L < A_N < A_H$.

Figure 3.6a shows the difference between allocations in the repayment equilibrium in comparison to the default equilibrium as a function of total factor productivity. For the repayment equilibrium, we assume that there is no default risk. In the unconstrained repayment equilibrium, a credit crunch associated with a sovereign default is more than six times higher in a high productivity state compared to a low productivity state. The reason is that credit demand is larger during a boom, leading to more interbank trading. With elevated labour productivity, non-financial firms are willing to pay higher wages which actuates labour supply. Hence, credit demand varies quite strongly over the cycle. The interbank freeze induced by a sovereign default shrinks available credit by up to 60
percent. This supply-side reduction of credit causes output to drop by more than 20 percent of GDP in high productivity states. At the same time, low demand for credit make a credit crunch less painful in terms of output losses during deep recessions, when the costs of default in terms of output losses are below 10 percent.

The occasionally binding collateral constraint introduces asymmetry in the form of downward skewness into the business cycle, as can be seen from the constrained allocations in Figure 3.6a. Since collateral holding is an endogenous state variable that is determined on the basis of expected future productivity realisations, unexpected large productivity realisations might not necessarily lead to a boom, as credit and output remain constrained through the interbank market. Domestically held government debt is, thus, not only shaping default incentives in the dynamic model, but also an important determinant of business cycle fluctuations.

Furthermore, the credit crunch is relatively less costly in the constrained regime when financial intermediation is limited by domestic holding of government debt. As a consequence, default incentives are directly affected by financial market allocations. This illustrates that causality is running in both direction in the model, from sovereign risk to the banking sector, but also from a dysfunctional interbank market to default risk. It is worth noting that this *reverse causality* is not the result of an explicit or implicit banking sector bailout guarantee from the public sector.\(^{32}\)

Figure 3.6b presents the pricing of collateral assets by banks in comparison to the pricing of government debt by international investors on the bond market. Pricing vectors \(\tilde{q}\) are the optimal policy functions extracted from the solved model for different states of productivity, \(A_t\). Specifically, we take the price vectors at three levels of productivity. To begin with, we take a neutral productivity state \(\tilde{q}(A_N)\), with \(\exp(A_N) = 1\). In line with the stylized representation of the secondary bond market in Figure 3.3, demand for government bonds from domestic banks is downward sloping, since the liquidity premium on government debt is falling with increasing collateral positions. The equilibrium quantity of bank purchases of government debt are determined where the no-arbitrage condition \(\tilde{q} = q\) is satisfied. Next, we are interested to see how current productivity realisations change incentives for domestic debt accumulation. Considering a higher (lower) productivity state today shifts demand for collateral upward (downward). This is in line with expected credit demand in the following period which is based on expected productivity, \(E_t(A_{t+1}) = \rho A_t\). Given higher expected profitability of intermediation, banks have an incentive to accumulate domestic debt in a pro-cyclical manner, as the liquidity premium \(\lambda_t\) from Proposition 2 is also pro-cyclical. In general equilibrium, this has implications

\(^{32}\) Acharya et al. (2014) model the link between public bailout policies and sovereign credit risk.
for the cyclicality of sovereign risk premiums, as from Proposition 6 domestically held debt is by itself lowering the incentive to opt for debt repudiation, contributing to overall pro-cyclicality of the financial sector.

Figure 3.7: Decision to default in the state space

The model exposes two channels that contribute to the counter-cyclical behaviour of the government bond risk premium, namely the pro-cyclical output cost of sovereign default and domestic debt accumulation in the decentralised equilibrium. Figure 3.7 shows the combined effect of the two channels on the enforceability of total government debt. Technically, this shows the value functions under default $V^d$ and repayment $V^{nd}$. The light grey shaded area represents the region where the value of repayment dominates the value of default in the government problem, whereas in the dark shaded area the government decides to default.

In a framework with non-Ricardian output effects of government debt, we find that transitory productivity shocks have a strong effect on the borrowing limit that is more than five times as large in high productivity states compared to very low productivity states (Figure 3.7a). This result is contrary to the findings of Aguiar and Gopinath (2006) who show that transitory productivity shocks in an endowment economy cannot generate a significant steepness of the indifference line which separates the default region from the repayment region. The sensitivity of the debt ceiling to aggregate shocks translates into a quantitatively important region of risky borrowing, which is defined over the interval of
government indebtedness where foreign investors charge a risk premium over the risk-free rate (Figure 3.8a).

Further, it is the net foreign asset position that determines default incentives in the presented setting (Figure 3.7b). The reason is that domestic government debt does not directly affect the aggregate resource constraint of the economy, as it is consolidated over dividend payments with the budget constraint of the representative household.\footnote{Notwithstanding, domestic debt does affect the resource constraint indirectly through aggregate production.} By contrast, benefits from defaulting are due to foreign owned government debt as debt repayment leads to wealth transfers abroad. In the extreme, government debt is always enforced if total outstanding debt is held domestically.\footnote{The 45-degree line in Figure 3.7b indicates the set of states where \( -B_t = B^D_t \). Our result rests on the assumption that taxation is non distortionary.}

This finding is in line with qualitative models on sovereign default with external and domestic debt. Guembel and Sussman (2009) develop a political economy model where the household sector is heterogeneous in its holdings of government assets. They show that an electoral system with majority voting leads to a conflict of interest which is able to enforce foreign-owned sovereign debt even if penalties are absent. For this mechanism to work, the inability to discriminate among bond holders is a crucial assumption. Similarly, Broner et al. (2010) show that the possibility of secondary market trading alone gives rise to full enforcement of sovereign debt contracts if domestic bond holders are unable
to collude.

We introduce secondary bond market trading under the assumptions that (i) the government cannot discriminate and that (ii) the aggregate amount of domestically held debt is public information. This allows banks to form expectations on default probabilities in period $t + 1$ by taking account of their own decision to accumulate domestic debt. Domestic debt thereby constitutes a coordination device for the government, domestic and foreign investors alike. However, this device itself is closely linked to the business cycle, making it weaker during recessions when domestic banks have lower incentives to hold government debt. Differently said, although there exists a repayment equilibrium for very high levels of government debt, the economy cannot coordinate to this point along the equilibrium path as banks do not have the incentive to accumulate a high enough share of outstanding government debt.\footnote{Thereby, the model does not support the crowding out hypothesis in the course of the European sovereign debt crisis as formulated in Broner et al. (2014). This is due to the fact that secondary market trading does not include arbitrage trading as in Broner et al. (2010). Besides, government debt and credit do not enter the same flow of funds constraint in this model as they have different maturities.}

Finally, there is propagation of the productivity shock over time. Since the productivity process is persistent, the domestic banking sector will raise its demand for collateral as it expects an increased demand for credit and intermediation in case of an above average productivity realisation. An improved net foreign asset position reduces the interest rate on government debt (Proposition 3). This has a positive feedback effect over time, since a higher share of domestic debt lowers default incentives in the consecutive period (Proposition 6). At lower levels of sovereign risk, aggregate output is higher since the cost of finance to non-financial firms is not inflated by financial frictions. Thus, the repayment equilibrium turns out to be self-enforcing.

However, the same mechanism induces a downward spiral in response to adverse shocks. After a series of bad shocks to productivity, domestic banks reduce their investment in collateral and pay out dividends to households. Credit demand drops, which lowers the penalty from a credit crunch. Under these conditions, the burden from repaying international investors is high, rendering default an attractive option from the planner’s perspective in bad states of the world.\footnote{While the theoretical literature stresses the possibility of partial insurance against very bad shocks through default (Zame, 1993), the empirical literature finds a weaker relationship (Tomz and Wright, 2007).} Figure 3.8b visualises the set of states $s \in \{A_t, B_t, B^D_t\}$ for which default from an ex post perspective turns out to be the optimal policy of the benevolent government.
3.5 Quantitative results

3.5.1 Stochastic stationary state and cyclical behaviour

We simulate the model in order to extract the stochastic stationary state and cyclical co-movement generated by the bank-sovereign link in our framework. To this end, we simulate the stochastic process for aggregate productivity 3000 times for 864 periods in each simulation round. We feed the generated series into the model and delete the first 100 elements such that we are left with 191 years of observations, which captures the 1820-2011 time span used to calibrate the default frequency. Data from 48 periods prior to a sovereign default generated by the model is extracted to calculate the moment statistics that cover the 2000q1 to 2011q4 era. The underlying assumption is that Spain most likely would have lost capital market access and defaulted as a consequence in the course of 2012 if non-standard measures taken by the ECB to calm secondary markets would not have been taken.  

We evaluate the model given moments from available Spanish data, as described in the baseline calibration. Additionally, data for output and consumption is taken from the OECD’s Quarterly National Accounts. We use the GDP deflator in order to transform nominal loan volumes into real volumes and we extract the cyclical component of logged variables using the hp-filter with the standard smoothing parameter for quarterly data.

Table 3.3 column (2) summarises the results under the baseline calibration. The model is able to generate a comparatively high debt-to-GDP ratio with an average of 21.4 percent. Although this is still below the debt level observed in the data, models with ad hoc default penalties on output endowment realisations have typically difficulties to generate such high public debt levels. The asymmetric default penalty from the interbank freeze contributes to our result, similar to the findings of Mendoza and Yue (2012). The planner faces two separate incentives to borrow when deciding on the optimal debt policy. First, there is the marginal benefit of borrowing from capital markets based on the consumption smoothing preferences in the light of risk-averse and impatient households, which is common to existing models of sovereign default. A novel borrowing incentive arises due to the liquidity services that are required to facilitate the creation of credit, which is used for production in the economy. Thereby arises simultaneously the incentive to borrow large amounts from capital markets in order to accommodate underlying financial frictions, while at the same time being disciplined by the endogenous penalty.

\footnote{This does not seem to be a very unrealistic assumption given that Spain and Italy suffered further increases in government bond yields in the first half of 2012. It was the ECB’s announcement of unlimited interventions in secondary sovereign bond markets under the OMT programme in August 2012 that prevented a further escalation of the debt crisis in the euro area.}
mechanism to adjust the debt level over the business cycle such that default is prevented. As a result, the amount of outstanding government debt is more sensitive to transitory productivity shocks and high debt levels are sustained in the stochastic stationary state. While high debt-to-GDP ratios can also be generated at low default frequencies (Aguiar and Gopinath, 2006), our model yields a realistic default frequency of 0.63 percent.

Table 3.3: Baseline results and robustness

<table>
<thead>
<tr>
<th></th>
<th>(1) Data</th>
<th>(2) Baseline</th>
<th>(3) $\psi = 0$</th>
<th>(4) $\phi = 10^3$</th>
<th>(5) $\psi = 0$, $\phi = 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stochastic stationary state</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt-to-GDP ratio</td>
<td>39.1</td>
<td>21.4</td>
<td>21.6</td>
<td>20.1</td>
<td>20.1</td>
</tr>
<tr>
<td>Dom. debt share</td>
<td>54.9</td>
<td>61.7</td>
<td>61.9</td>
<td>59.1</td>
<td>59.0</td>
</tr>
<tr>
<td>Sovereign premium</td>
<td>0.122</td>
<td>0.754</td>
<td>0.734</td>
<td>0.747</td>
<td>0.738</td>
</tr>
<tr>
<td>Credit premium</td>
<td>0.504</td>
<td>0.158</td>
<td>0.073</td>
<td>0.061</td>
<td>-0.120</td>
</tr>
<tr>
<td>Repo premium</td>
<td>0.194</td>
<td>-0.045</td>
<td>-0.230</td>
<td>-0.047</td>
<td>-0.230</td>
</tr>
<tr>
<td>Frequency of default</td>
<td>0.65</td>
<td>0.63</td>
<td>0.624</td>
<td>0.626</td>
<td>0.626</td>
</tr>
<tr>
<td>Output drop at default</td>
<td>-</td>
<td>19.6</td>
<td>19.9</td>
<td>19.8</td>
<td>20.0</td>
</tr>
<tr>
<td><strong>Relative volatility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std.consumption/Std.output</td>
<td>1.10</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td><strong>Correlations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A.) with output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lending</td>
<td>0.77</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Sovereign premium</td>
<td>-0.14</td>
<td>-0.44</td>
<td>-0.42</td>
<td>-0.43</td>
<td>-0.41</td>
</tr>
<tr>
<td>Credit premium</td>
<td>-0.32</td>
<td>-0.03</td>
<td>0.27</td>
<td>-0.03</td>
<td>0.26</td>
</tr>
<tr>
<td>Repo premium</td>
<td>-</td>
<td>-0.44</td>
<td>0.0</td>
<td>-0.43</td>
<td>0.0</td>
</tr>
<tr>
<td>(B.) with sovereign premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit premium</td>
<td>0.84</td>
<td>0.40</td>
<td>-0.17</td>
<td>0.44</td>
<td>-0.16</td>
</tr>
<tr>
<td>Lending</td>
<td>-0.26</td>
<td>-0.46</td>
<td>-0.41</td>
<td>-0.44</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

Notes: Details regarding the data are available in Appendix B.5.

We find a close link between the average default frequency and the sovereign risk premium, which is a common finding in the literature given the direct link of the default probability with the bond pricing equation of international investors. In this context, the no-arbitrage condition of international investors from Proposition 3 prevents that the liquidity premium on bond prices from Proposition 2 translates overall into a lower risk premium on government debt. The sovereign risk premium generated under the baseline calibration takes a value of 0.754 percent on average, which is substantially higher than the observable average risk premium on Spanish government bonds over German Bund. As shown by Bernoth and Erdogan (2012), the yield on Spanish debt over the period 2000-11 is affected by the convergence period after entering EMU when investors charged a historically low risk premium and is not necessarily a representative sample period.

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38 Models that generate a higher risk premium compared to the default frequency use modifications of the utility function of the international investor, see Lizarazo (2013) and Pouzo and Presno (2013).
CHAPTER 3. SOVEREIGN RISK AND THE INTERBANK MARKET

The credit premium and the interbank premium as targeted in the SMM procedure are both functions of the parameters governing the spillover of sovereign risk to financial frictions. Note that the interbank market premium \( r^M - r^f \) is below zero in the stochastic stationary state of the model since the opportunity investment for unproductive banks is \( r^R < r^f \). With low default risk in the majority of states along the equilibrium path, we obtain \( r^M < r^f \). Further, an average spread is also maintained between the secured interbank rate \( r^M \) and the credit rate \( r^c \) which is in our model due to the liquidity preference.

The data shows parallels in Spanish business cycle moments with stylised facts from emerging markets. Namely, the volatility in consumption exceeds that of output, and the sovereign risk premium is negatively correlated with the cycle. The model is able to reproduce both moments, although it overestimates the negative correlation of output with the sovereign risk premium. The counter-cyclical sovereign risk premium is a result of the endogenous default penalty which makes default more attractive during recessions, while there are virtually no defaults during boom periods. As a consequence, the counter-cyclical risk premium renders public borrowing less attractive during recessions, while simultaneously leading to a consumption boom during expansions given loose credit conditions on international capital markets and impatient households. On top of that, the planner is trading-off the expected future distortions on domestic financial markets when taking his optimal borrowing decision. In models with exogenous endowment processes, the trade-off on borrowing higher amounts takes only the interest rate effect into consideration.

In Mendoza and Yue (2012), increases in the cost of borrowing for the public sector translate, by assumption, into a higher cost of credit for the private sector, which induces an inefficient substitution into domestic goods in the production of intermediate goods that weighs on output. In our framework, the propagation of higher sovereign risk into rising cost of private credit is explicitly modelled. The credit rate \( r^c \) depends primarily on aggregate productivity and the funding situation of domestic banks. As the funding conditions are tied to sovereign risk during times of fiscal stress, the collateral channel of sovereign risk is able to account for a positive correlation on public and private sector risk premiums. The model underestimates the correlation coefficient with 0.40 while the data exhibits a stronger correlation of 0.84.

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39 Variables with a bar denote steady state values.
40 This is in line with the data, as the secured interbank rate EUREPO usually trades below the unsecured interbank rates EURIBOR and EONIA swap. Only the short time period of elevated sovereign risk available for the Spanish GC repo rate indicates a slightly positive spread.
41 The authors rationalise this result by showing that a government diverts the outstanding private loans on international capital markets in the event of a default. This leads to coinciding risk of default in the public and the private sector.
The same mechanism leads to an overall mildly counter-cyclical credit premium of -0.03. There are two off-setting effects in the model that shape the correlation of the credit premium with output. While the credit premium is demand-determined and pro-cyclical in normal times absent fiscal stress, it turns counter-cyclical in the model during crisis periods, as it is discussed in the next section.

The data shows a high correlation of lending to non-financial firms with output. This pro-cyclicality of credit is overestimated by the model, as working capital lending is perfectly correlated with output. With a fixed capital stock, output has only two sources of fluctuations, labour input and productivity, which are both highly correlated, too. As labour input requires working capital lending, credit is directly linked to production. With endogenous capital accumulation, this link may well be weaker.

Finally, Spanish data shows that the sovereign risk spread of government bonds over German Bunds is negatively correlated with short-term lending to non-financial firms. The model closely matches this negative correlation in the data and rationalises the result by providing a theory of endogenous sovereign risk and the supply of credit in general equilibrium.

We conduct a range of robustness checks in order to validate our findings. In column (3), we eliminate the risk premium on secured interbank debt. This implies a repo premium of -23 basis points from the calibration since $r^M_t = r^R_t$, independent of sovereign risk. Since the bank funding conditions do not react to the level of default risk any longer, the credit premium turns strongly pro-cyclical as the demand effect dominates credit conditions over the business cycle. Also, the co-movement of interest rates can no longer be replicated by the model. This underlines that supply side effects of bank credit are important for the explanation of cross-correlations and long-run averages of different interest rates in the model. However, since the ex post penalty from defaulting remains in place, the stochastic steady state levels of government debt are not affected. The marginal increase of the debt-to-GDP ratio follows from the fact that there are less adverse consequences of sovereign risk, which allows the government to marginally increase its level of indebtedness.

In column (4), we eliminate the liquidity preference and set $\phi = 10^3$. As a direct consequence, the credit premium in the steady state drops as the opportunity cost vanishes from accumulating liquid reserves when productive banks borrow from the interbank market since $R^e_t \to 0$. However, the remaining results stay qualitatively and quantitatively the same as under the baseline calibration.

Finally, we eliminate both frictions simultaneously in column (5). The results are similar as in column (3), suggesting that the risk premium is more important for the quantitative results than the liquidity preference.
3.5.2 Debt crisis, collateral squeeze, and macroeconomic dynamics

Against the background of the stylised facts from the euro area sovereign debt crisis, we are interested in the qualitative and quantitative predictions of the model around default events. We present the average behaviour of time series produced by our model simulations in a default window including 4 years prior and after a default event (Figure 3.9).

The effect of the sovereign risk channel unfolding in the run-up to a sovereign debt crisis event are presented in Figure 3.9a. The risk premium on sovereign debt rises steadily from its long-term mean before a default takes place. In the pre-crisis period, lower than average productivity realisations dampen the demand for credit, which translates into a slightly lower credit premium on private debt. The initial decline in the credit rate is demand-driven, reflecting the below average labour productivity, which has to compensate for the cost of credit according to the optimal labour demand condition (3.5). With rising sovereign risk and a further deterioration in the quality of collateral, funding costs of banks start to increase. In response, lending banks charge higher credit rates on working capital loans to non-financial firms even though demand is falling. Additionally, the rise in the cost of interbank loans interacts with the preference for liquidity, that requires lending banks to build excess reserves on each unit of borrowed funds on the interbank market. This preserves a spread between the interbank rate and the credit rate, bearing an additional rise in the credit rate. As a result, the interest rates on public and private sector debt are highly correlated shortly before a default takes place.

In comparison to the Argentinian default of 2001, the macroeconomic dynamics around default events as characterised by the bank-sovereign link are different in that our model predicts (i) a more severe recession related to a sovereign debt crisis; (ii) the impact effect of default on output to be much stronger; and (iii) that recessions last longer (Figure 3.9b).

Quantitative models of sovereign default typically predict a recession prior to default as debt repayment gets more painful during these times (Arellano, 2008). However, the interaction of sovereign risk with financial frictions in the presence of the collateral channel of sovereign risk forms an amplification mechanism that leads to output losses beyond the fall in aggregate productivity prior to a sovereign default. The deep recessions observed in the euro area in the aftermath of the global financial crisis that preceded the outbreak of the sovereign debt crisis complement this view.

\footnote{This result is in line with empirical findings by Levy-Yeyati and Panizza (2011) who analyse a cross-section of historical default events.}
In the period of government default, a freeze on the interbank market leads to an imminent credit crunch and a deep recession. Under the baseline calibration, the model predicts a drop in credit of about 27 percent in response to a complete breakdown of interbank intermediation. As a consequence of the credit crunch and rising cost of capital, output drops by about 19 percent compared to the long-run mean on impact. Although this is a dramatic decline in output, it is not unprecedented in economic history. Reinhart and Rogoff (2009) document that financial crises lead to output losses of 9.3 percent on average, but have also significantly exceeded this number. Otherwise, given that the model does not account for financial assistance programmes from EMU member countries, the output losses generated by the model probably poses an upper bound in the Spanish case.

In the post-crisis period, output recovers slowly. Persistence of production is due to the highly auto-correlated exogenous process of productivity and an endogenous propagation mechanism, as collateral needs to be accumulated by banks out of retained earnings over time. This prevents a fast recovery in lending conditions. Remember from the bank balance sheet identity that collateral assets represent bank capital from an accounting perspective. After a complete write-off of government debt, there is no bank capital left. This poses an extremely large shock on bank balance sheets, which have a difficult time recovering during the low productivity environment following a default. Therefore, the

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43In particular, the US cumulative decline in per capita GDP in response to the Great Depression of 1929 from peak to trough was 29 percent.

44See Fink and Scholl (2011) and Juessen and Schabert (2013) for models of sovereign default with a bailout mechanism.
model is supporting the view of long-lasting financial recessions (Cerra and Saxena, 2008) as opposed to a strong recovery observed in the follow-up to emerging market crises (Calvo et al., 2006).

### 3.5.3 Sensitivity analysis and discussion

In this section we provide further sensitivity analyses in order to underpin the robustness of the quantitative results. Besides, a more detailed discussion of the model propagation mechanism helps to understand the complex interactions of different default incentives at play in the model.

Table 3.4 presents the results of the sensitivity analysis. Lowering the parameter $\pi^p$ to 0.45 increases the need for reallocation of funds across banks on the interbank market, as fewer banks are matched with non-financial firms. With an increasingly important interbank market, a default-related credit crunch gets more destructive, as there are fewer financial resources available to lending banks in the autarky state. As a consequence, the debt-to-GDP ratio more than doubles and exceeds, at 48.5 percent, the observable level in the data, while default occurs less often.

The parameter $\pi^p$ can be interpreted as a short cut for financial development or financial institutions which allow a banking system to generate more intermediation. The here presented model with a collateral channel of sovereign risk confirms the findings of Gennaioli et al. (2014) in that an economy with more developed financial institutions is more vulnerable to sovereign defaults.

A reduction in the available financial resources $N$ increases the penalty of default as fewer financial resources are left for credit in the autarky period. The average sustainable level of government indebtedness increases to 26.2 percent. This is the case even though a lower share of total outstanding debt is held domestically, given that fewer financial resources are intermediated on the interbank market. At the same time, a lower value for $N$ pushes up the interest rate on credit as financial resources are more scarce in equilibrium and the spread $\bar{r}^k - \bar{r}^M$ almost doubles compared to the baseline calibration. When sovereign risk rises due to low productivity realisations, the demand-effect in the premium on credit is now stronger, leading to a less strong correlation of private and public sector interest rates.

Turning to the working capital assumption, we reduce the fraction that needs to be financed up-front to $\eta = 0.95$. Note that this change leaves all other parameters from the baseline calibration unaffected. Under these conditions, lowering $\eta$ has a similar effect as increasing $N$, as more financial resources are available in the system in relation to a fall in credit demand. The default penalty from an output drop decreases, making default
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more likely on average. This, in turn, lowers the ratio of debt-to-GDP in the model.

The parameter $\omega$ governs the Frisch elasticity of labour supply according to $1/(1 - \omega)$. Thereby, increasing $\omega$ translates into lower substitution effects of labour supply in response to changes in the equilibrium wage rate. Remember that the bank-sovereign link unfolds in conjunction with the working capital constraint on labour input, as sovereign risk drives up the price of labour from the perspective of the non-financial firm. Consequently, setting $\omega$ to 1.6, we find that the output cost of default drops, leading to lower levels of debt. As labour supply is less sensitive to a fall in the wage rate given higher values for $\omega$, the distortions arising from the sovereign risk channel can be compensated by lower equilibrium wages.

Lifting the parameter $\psi$ from the baseline calibration of 0.245 to 0.4 intensifies the spillovers from sovereign risk to the cost of funding on domestic interbank markets when the quality of collateral deteriorates. This drives the correlation of risk premiums on public and private sector debt to 0.55 and lightly enforces the counter-cyclical correlation with output.

Increasing $\phi$ reduces the need for excess reserves when borrowing funds on the interbank market. Less severe frictions in the model lead to marginally more financial resources available for working capital credit, as there are fewer productive reserves required to self-insure against roll-over risk. This lowers the negative correlation of the sovereign risk premium with output to -0.26.

Table 3.4: Sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>(i) Debt/GDP</th>
<th>(ii) Domestic debt share</th>
<th>(iii) default freq.</th>
<th>(iv) $\delta^c - \delta^M$</th>
<th>(v) $\rho(\hat{\delta}^c_t, \delta^M_t)$</th>
<th>(vi) $\rho(\hat{\delta}^c_t, \hat{\delta}^M_t)$</th>
<th>(vii) Output drop</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>49.5</td>
<td>49.0</td>
<td>0.654</td>
<td>0.459</td>
<td>-0.08</td>
<td>0.84</td>
<td>-</td>
</tr>
<tr>
<td><strong>Baseline</strong></td>
<td>21.4</td>
<td>61.7</td>
<td>0.628</td>
<td>0.210</td>
<td>-0.44</td>
<td>0.40</td>
<td>19.6</td>
</tr>
<tr>
<td>$\pi^f = 0.70$</td>
<td>7.0</td>
<td>86.7</td>
<td>0.397</td>
<td>0.320</td>
<td>0.21</td>
<td>0.43</td>
<td>8.3</td>
</tr>
<tr>
<td>$\pi^f = 0.45$</td>
<td>48.5</td>
<td>47.1</td>
<td>0.151</td>
<td>0.250</td>
<td>-0.30</td>
<td>0.28</td>
<td>28.4</td>
</tr>
<tr>
<td>$N = 3.15$</td>
<td>16.5</td>
<td>72.0</td>
<td>0.656</td>
<td>0.140</td>
<td>-0.15</td>
<td>0.48</td>
<td>16.2</td>
</tr>
<tr>
<td>$N = 2.80$</td>
<td>26.2</td>
<td>55.3</td>
<td>0.437</td>
<td>0.350</td>
<td>-0.51</td>
<td>0.33</td>
<td>22.3</td>
</tr>
<tr>
<td>$\eta = 1.25$</td>
<td>35.0</td>
<td>48.9</td>
<td>0.151</td>
<td>0.122</td>
<td>-0.30</td>
<td>0.13</td>
<td>24.8</td>
</tr>
<tr>
<td>$\eta = 0.95$</td>
<td>12.1</td>
<td>78.7</td>
<td>0.684</td>
<td>0.180</td>
<td>0.02</td>
<td>0.51</td>
<td>13.5</td>
</tr>
<tr>
<td>$\omega = 1.6$</td>
<td>16.2</td>
<td>73.5</td>
<td>0.616</td>
<td>0.140</td>
<td>-0.05</td>
<td>0.55</td>
<td>15.0</td>
</tr>
<tr>
<td>$\omega = 1.3$</td>
<td>27.1</td>
<td>54.0</td>
<td>0.313</td>
<td>0.580</td>
<td>-0.39</td>
<td>0.24</td>
<td>24.5</td>
</tr>
<tr>
<td>$\psi = 0.40$</td>
<td>21.5</td>
<td>61.6</td>
<td>0.613</td>
<td>0.220</td>
<td>-0.45</td>
<td>0.55</td>
<td>19.7</td>
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<tr>
<td>$\psi = 0.10$</td>
<td>21.6</td>
<td>61.8</td>
<td>0.630</td>
<td>0.180</td>
<td>-0.43</td>
<td>0.22</td>
<td>19.9</td>
</tr>
<tr>
<td>$\phi = 15$</td>
<td>19.5</td>
<td>63.0</td>
<td>0.529</td>
<td>0.150</td>
<td>-0.26</td>
<td>0.42</td>
<td>18.8</td>
</tr>
<tr>
<td>$\phi = 3$</td>
<td>24.1</td>
<td>65.5</td>
<td>0.619</td>
<td>0.740</td>
<td>-0.46</td>
<td>0.24</td>
<td>19.5</td>
</tr>
<tr>
<td>$\theta = 0.25$</td>
<td>16.0</td>
<td>82.6</td>
<td>0.491</td>
<td>0.180</td>
<td>-0.14</td>
<td>0.49</td>
<td>18.5</td>
</tr>
<tr>
<td>$\chi = 0.35$</td>
<td>25.7</td>
<td>67.7</td>
<td>0.626</td>
<td>0.270</td>
<td>-0.43</td>
<td>0.24</td>
<td>19.7</td>
</tr>
<tr>
<td>$\chi = 0.15$</td>
<td>16.6</td>
<td>50.8</td>
<td>0.622</td>
<td>0.160</td>
<td>-0.42</td>
<td>0.56</td>
<td>19.6</td>
</tr>
</tbody>
</table>

Notes: A hat denotes the spread of the corresponding interest rate over the risk-free rate $r^f$. 
The parameter governing the stochastic probability of re-entering capital markets $\theta$ might be lower in the Spanish case than in the baseline calibration fitted to a cross-section of emerging market economies, as international investors might put more confidence into the institutional setting in the euro area.\footnote{For Greece, it took more than two years after the pre-emptive debt restructuring in February 2012 until the first bond issuance in April 2014.} To check for the effects of a faster re-access to capital markets, we set the re-entry probability to $\theta = 0.25$, which represents an average exclusion spell of one year. As this constitutes a direct reduction of costs of default, the debt-to-GDP level shrinks in the long-run average. With fewer government securities available in the economy, the share of domestic debt rises.

At very low levels of $\chi$, domestic banks draw higher liquidity services per unit of government bonds. At the same time, the demand for liquidity services is limited in the model given a downward sloping demand curve for credit and interbank loans. In sum, the domestic share of total government debt drops in the stochastic stationary state compared with the baseline calibration. Non-penalty related repayment incentives on the side of the government are reduced, which translates into an overall smaller ratio of sustainable public debt to output. We conclude that non-penalty related incentives to repay creditors might be an important explanation for high debt-to-GDP ratios observed in the data of advanced economies.\footnote{Anecdotal evidence points into this direction, as the examples of Italy and Japan indicate, which both exhibit very high ratios of debt-to-GDP and depend on a domestic investor base. Moreover, there is evidence in the European sovereign debt crisis that the repatriation of debt is lowering default risk (Brutti and Sauré, 2013).}

### 3.6 Conclusion

In this paper, we propose a model of strategic default for advanced, financially developed economies that accounts for the bank-sovereign link in a dynamic stochastic general equilibrium setting. The model is calibrated to Spanish data and capable of reproducing business cycle statistics alongside stylized facts from the European sovereign debt crisis.

The results show that the adverse feedback loop in peripheral euro area countries between a dysfunctional interbank market, a weak growth performance, and a rise in sovereign bond yield spreads can be rationalised by the concept of strategic default. In the presence of spillovers from sovereign risk to financial market outcomes, an otherwise high default penalty loses its disciplinary role and introduces fragility into advanced countries’ public debt positions. The model is able to explain the impairment of credit conditions along national borders due to the collateral channel of sovereign risk on the secured interbank market.
Domestically held government debt introduces a motive for government borrowing by providing liquidity services to the economy, which has not been considered in the quantitative literature on optimal sovereign default. As a result, the borrowing limit becomes more responsive to transitory productivity shocks, while at the same time sustaining higher aggregate levels of debt. Further, we find that it is the net foreign asset position of an economy that determines incentives under strategic default considerations.

There are limitations to this framework. Specifically, the model abstracts from many relevant questions surrounding the decision to default, as debt renegotiation (Yue, 2010; Bai and Zhang, 2012), the maturity structure of debt (Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012; Chatterjee and Eyigungor, 2012), the effects of international financial assistance programs (Boz, 2011; Fink and Scholl, 2011; Juessen and Schabert, 2013), international spillovers (Arellano and Bai, 2013), and redistributive effects of default on domestic debt (D’Erasmo and Mendoza, 2012). Further research is also required to deepen our understanding of the interactions of sovereign risk with other sources of bank funding. Finally, we leave it for future research to investigate financial sector stabilisation policies through the central bank.
CHAPTER 4

Uncertainty Shocks and Non-Fundamental Debt Crises: An Ambiguity Approach

4.1 Introduction

Macroeconomic uncertainty increased considerably during the European sovereign debt crisis of 2011/12. Over a short seven month period in the second half of 2011 through January 2012, the amount of disagreement between professional forecasters regarding GDP growth projections increased in the periphery Euro area countries by up to 4.5 standard deviations.\textsuperscript{1} Looking at the co-movement between forecast disagreement and sovereign credit risk, it turns out that these uncertainty shocks are positively correlated with yields on sovereign credit default swaps (CDS).

Given a recent debate of policy makers and academics on potential 'mispricing' or 'over-pricing' of debt in the course of the European sovereign debt crisis,\textsuperscript{2} it seems surprising that, to the best of my knowledge, there is no model that analyses the implications of macroeconomic uncertainty shocks on sovereign credit risk. Based on the concept of multiplicity of equilibria in sovereign debt markets, it is argued that the problem of indeterminacy may give rise to 'bad' equilibria that lead to self-fulfilling debt crises, eventually imposing dead weight losses due to outright sovereign default.\textsuperscript{3} This view seems to be backed by a growing strand of empirical literature that finds that yields on public debt are partly unexplained by macroeconomic fundamentals.\textsuperscript{4} While sunspots are

\textsuperscript{1}This is according to survey data from Consensus Economics and Focus Economics, as presented in detail in Section 4.2.

\textsuperscript{2}See IMF (2012), European Commission (2012), Corsetti et al. (2013) and Grauwe and Ji (2013), among others. The notion of 'over-pricing' refers to yields on sovereign debt as used e.g. by Aizenman et al. (2013).

\textsuperscript{3}Calvo (1988) is a classic reference for multiplicity of equilibria in sovereign debt markets that might lead to a Pareto-inferior equilibrium.

\textsuperscript{4}See Aizenman et al. (2013), Beirne and Fratzscher (2013) and D’Agostino and Ehrmann (2013).
a widely used approach to tackle the arising problem of indeterminacy, previous criticism constitutes that this leaves the shift of investors’ beliefs unexplained (Morris and Shin, 2000).

Driven by uncertainty shocks, this paper develops a theory of sovereign debt crises that is able to rationalise a non-fundamental increase in the price of issuing government debt. The derived equilibria are unique despite the presence of multiple beliefs over economic fundamentals. For the definition of uncertainty shocks, I follow Keynes (1936) in the distinction between risk and uncertainty. He notes on the nature of expectation that “The state of long-term expectations, upon which our decisions are based, does not solely depend ... on the most probable forecast we can make. It also depends on the confidence with which we make this forecast” (p. 148). In close analogy, I assume that agents have a probabilistic view of the future macroeconomic fundamentals of an economy, but that there are uncertainty shocks that change the perceived confidence due to Knightian uncertainty attached to their own assessment of the fundamentals. The contribution of the paper is to analyse the role of such uncertainty shocks for the optimal fiscal policy decision of a benevolent government under limited commitment.

For the sake of the exposition, the basic mechanism is first discussed in a three period setting. Formally, an uncertainty shock increases the range of priors considered by agents over a set of forecasting models for productivity growth. Even though each forecasting model in the set of priors contains a probabilistic view of the path for productivity, there are no (subjective) probabilities available that could provide a basis to evaluate the cross-section of models a priori. As a result, the evolution of productivity becomes ambiguous since a probabilistic assessment over models contained in the set of prior beliefs is not feasible.

A central assumption in this paper is that investors who purchase public debt are ambiguity averse. That is, investors have a strict preference for knowing probabilities (Ellsberg, 1961). When the servicing of sovereign debt is subject to strategic default considerations, uncertainty shocks that increase the level of ambiguity about the fundamentals feed into investors’ beliefs about the fiscal sustainability of public debt. Being disciplined by a cyclical default penalty, the government’s willingness to pay depends on the state of the economy. Since servicing the debt is relatively more costly during recessions, default probability is counter-cyclical (Arellano, 2008).

Default is possible in the intermediate period, when debt needs to be rolled over, and in the final repayment period. For simplicity, I assume that productivity is subject to ambiguity only in the final period. The realisation of an uncertainty shock in the intermediate period determines the level of ambiguity surrounding productivity in the final period. The
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higher the level of uncertainty, the less confident investors are about their own model forecast, and the more priors are considered *a priori*. I follow Gilboa and Schmeidler (1989) and their formulation of maxmin preferences in the multiple-priors environment. These lead investors to act under their worst case prior when confronted with ambiguity. As a result, Savage’s 1954 representation theorem no longer holds, under which there exists a subjective probability measure guiding agent behaviour. From the default decision of the government that is counter-cyclical, maxmin preferences allow for a strict ordering of priors over the fundamental state and, consequentially, default expectations which prevents indeterminacy due to an otherwise inherent problem of multiplicity of equilibria.

In this setting, it is welfare improving for the government to borrow from international capital markets in order to smooth consumption of representative households over the life-cycle. An uncertainty shock raises the cost for issuing debt for the sovereign, which in turn distorts the optimal fiscal policy and default decision of the government. In times of high uncertainty, the benefits from international borrowing are relatively lower, such that governments issue less debt. Uncertainty shocks can lead to *non-fundamental* default due to the interaction of the limited commitment friction of the government with the formation of worst case priors of ambiguity averse investors in the roll-over period: for the same macro fundamentals, both default and repayment equilibria, are possible outcomes. It is the realisation of the uncertainty shock that determines the outcome of the sovereign’s default decision. Thereby, worst case beliefs of investors can give rise to a roll-over debt crisis.

Non-fundamental defaults share important features with sunspot driven self-fulfilling crises. Specifically, the formation of worst case beliefs over future debt repayment partly endogenizes the sunspot shock in Cole and Kehoe (2000). In their framework, the realisation of an exogenous variable declares a liquidity crisis where the price of debt falls to zero for an exogenous reason. A self-fulfilling default occurs if the economy is within a crisis zone where, conditional on the realisation of the sunspot variable, debt repudiation is the utility-maximizing choice. In the set-up presented in this paper, investors’ beliefs are the result of the equilibrium mapping from the model with limited commitment and ambiguity aversion. Further, uncertainty shocks allow for a continuous distortion of investors’ beliefs. In particular, the probability of a non-fundamental default and ambiguity premia are increasing in the amount of government debt even within the crisis zone, a feature that is not present in the framework with sunspot shocks.

This paper is the first to document two novel stylized facts, namely that (i) forecast disagreement regarding GDP growth is positively correlated with yields on sovereign CDS; and (ii) the correlation is strongest for disagreement lagging CDS yields for a month, which
is due to the collection of survey data. In the empirical part of the paper, the quantitative effect of uncertainty shocks on sovereign credit risk are explored in two steps. First, I estimate a structural vector autoregressive (VAR) model for a panel of five Euro area countries. Disagreement among professional forecasters is used as an uncertainty measure. In the VAR analysis, I investigate the effect of forecast disagreement on sovereign credit risk jointly with other possible drivers for sovereign credit risk. I use monthly data over the 2007 to 2014 period which exhibits both elevated levels and high volatility of uncertainty. The empirical findings can be summarized in three main observations. First, there is an overall positive effect of uncertainty shocks on sovereign credit risk that lasts about one year. Second, the effect is about twice as strong in countries of the periphery of the Euro area. Third, a shock on macroeconomic uncertainty mildly pushes up bank lending rates. The empirical results imply that uncertainty shocks matter for sovereign debt pricing and that these shocks might be propagated through the banking sector. However, the cross-country responses are not symmetric in amplitude but feature a core-periphery dichotomy.

As a further quantitative evaluation of the relationship between uncertainty shocks and sovereign default risk, I develop a quantitative business cycle model with strategic sovereign default, ambiguity averse investors, and an endogenous default penalty due to a bank-sovereign nexus. I estimate the model using Spanish time-series over forecast disagreement in order to discipline the degree of ambiguity by the data. The starting point is a real business cycle (RBC) model of a small open economy. I follow Ilut and Schneider (2014) in the formalisation of a process for ambiguous aggregate factor productivity in a stochastic, infinite horizon economy. Agents do not know the exact underlying probabilistic model of the evolution of the aggregate productivity state while uncertainty increase the set of priors considered for a deterministic component of future productivity.

Investors purchase government securities that are subject to endogenous default risk due to limited commitment (Eaton and Gersovitz, 1981). In line with the quantitative literature on optimal sovereign default, the government inherits the previously accumulated stock of public debt. If the government decides to service the debt, it is able to issue new bonds, rolling over existing debt. The bank-sovereign nexus follows Engler and Große Steffen (2014), who build a quantitative model of strategic default in advanced economies. Heterogeneous banks use government bonds as collateral on the domestic interbank market to obtain wholesale funding. Non-financial firms need working capital loans provided by banks in order to produce. Outright sovereign default involves an \textit{ex post} penalty through a freeze on the interbank market, which triggers a credit crunch. Further, adverse shocks are amplified in this setting. When the quality of collateral falls in response to increased levels of sovereign risk, the risk premium for secured debt on
the interbank market, which is modelled in reduced form, rises and makes the allocation of financial resources in the economy less efficient, leading to a further decline in output. Since less intense interbank intermediation implies a lower penalty from defaulting, sovereign default risk rises further.

In a dynamic setting with default expectations that depend on the future evolution of the fundamental state of the economy, uncertainty shocks translate into ambiguous pay-offs from holding government debt. As a result, investors ask for higher yields in the form of an \textit{ambiguity premium} to roll-over the existing stock of government debt that is labelled as the \textit{pricing channel} of uncertainty shocks.

Uncertainty shocks can be an important source of economic fluctuations in this setting. However, the propagation of uncertainty is distinct from other business cycle models (Angeletos et al., 2014; Bloom et al., 2012; Fajgelbaum et al., 2014; Ilut and Schneider, 2014). In the model with endogenous sovereign default risk, uncertainty shocks amplify frictions in the domestic interbank market. At elevated levels of fundamental ambiguity, investors’ worst case beliefs lead, via the pricing channel, to a loss in the quality of collateral used for secured lending on the interbank market. Lower bond prices induce a higher interest rate on wholesale loans that render borrowing on the interbank market less attractive. The arising \textit{spillover channel} of uncertainty shocks to the financial sector lowers available credit to non-financial firms, thus dampening aggregate production.

The model is used to decompose the sovereign yield premium into a fundamental share and an uncertainty premium. Using Spanish data of forecast disagreement from a survey of professional forecasters allows for the identification of the uncertainty process, which is assumed to be orthogonal to fundamental productivity shocks. Model simulations assign a sizeable share of sovereign yield premia to non-fundamental shocks which can reach up to 60 percent of the total yield premium. In a comparative static approach, I find that countries more prone to uncertainty shocks accumulate less debt compared to those countries which are less affected by fundamental uncertainty. The intuition for this result is that uncertainty lowers the marginal benefit from borrowing internationally as costs for issuing debt are higher in an environment with elevated fundamental uncertainty due to the pricing channel. Further, there is a strong \textit{precautionary motive} to prevent high levels of public debt if the economy is more prone to non-fundamental default. Similarly, a stronger spillover effect due to the bank-sovereign nexus reduces the optimal amount of public indebtedness \textit{ceteris paribus} since it expands the size of the crisis zone where non-fundamental default is possible, thereby reinforcing the precautionary motive of the government.

In order to derive the overall effect of uncertainty shocks on sovereign credit risk, one
must consider a second general equilibrium effect due to the endogenous determination of public debt held by domestic banks. The *debt composition effect* increases the share of domestic debt due to a liquidity premium attached to government bonds, which serve as collateral on the interbank market. Thus, there is a downward effect of uncertainty related to the composition of investors as the incentives for default decreases with domestic-ownership of public debt.

The findings of the model are related to a number of crisis models in the literature with multiplicity of equilibria, many following the seminal bank-run model of Diamond and Dybvig (1983) in which asymmetric information gives rise to a coordination problem among investors. Implications for sovereign debt markets are studied in Calvo (1988), Alesina et al. (1990), and Giavazzi and Pagano (1990). The role of expectations was re-iterated by Navarro et al. (2014) for a widely used framework that nests the commitment friction as formalised in the seminal paper by Eaton and Gersovitz (1981) in a quantitative dynamic general equilibrium framework. Morris and Shin (2006) and Corsetti et al. (2006) apply global games methods to solve the arising indeterminacy problem in sovereign debt markets due to strategic complementarity in the presence of uncertain economic fundamentals. Morris and Shin (2000) discuss the problem of equilibrium selection under incomplete information more generally. The mechanism of ambiguity aversion and multiple-prior preferences in a setting with limited commitment presented here contributes to this line of literature by adding another approach to address the inherent problem of indeterminacy in sovereign debt markets.

Corsetti et al. (2013) analyse the spillover channel of sovereign risk in a New Keynesian model at the zero lower bound. They find that spillovers from sovereign risk exacerbate the problem of indeterminacy, concluding that this makes an economy more prone to self-fulfilling beliefs. In my model there is a similar implication of the adverse feedback mechanism between domestic banks and sovereign risk. By expanding the size of the crisis zone, the spillover channel of sovereign risk to the banking sector makes a country more vulnerable for uncertainty shocks. Since the model remains fully determined, additional results are obtained by comparing the *ex ante* versus *ex post* efficiency of public debt in the light of uncertainty shocks.

There are a number of papers analysing sunspot driven self-fulfilling debt crises in a quantitative setting. Chatterjee and Eyigungor (2012) show that issuing long-term debt is successful in preventing some self-fulfilling crises. Roch and Uhlig (2014) and Kirsch and Rühmkorf (2013) analyse bailout policies in the presence of suns-spot induced roll-over crises. The prediction of the ambiguity model with respect to the effect of an increase of average uncertainty on the long-run accumulation of debt is equivalent to an increase
in the frequency of sunspot shocks. Both settings imply a lower aggregate level of public
debt due to the pricing channel and precautionary motives.

There are other studies attempting to decompose sovereign yields over the course of
the European sovereign debt crisis. Kriwoluzky et al. (2014) use Greek data to identify
the amount of re-denomination risk present in sovereign bond spreads if countries belong
to a currency union. For the identification of ‘exit premia,’ they use the assumption that
private domestic debt is not subject to re-denomination risk and find that re-denomination
risk played a minor role during the Greek debt crisis. Bocola and Dovis (2015) exploit
the implications of sunspot shocks for the optimal maturity structure of debt for the
identification of self-fulfilling beliefs. They use Italian data on the maturity structure of
debt and sovereign risk premia in order to decompose sovereign yields. They find that
the risk of a self-fulfilling roll-over crisis was relatively small at the peak of the debt crisis
in the case of Italy.

The modelling of preferences with ambiguity aversion is closely related to the concept
of multiplier preferences from the robust control theory (Hansen et al., 1999; Hansen and
to an optimal sovereign default model with endowment shocks. They find, similar to
the present study, that concerns about model misspecification generate higher yields on
sovereign debt. However, the concept of multiplier preferences is not able to accommodate
the analysis of uncertainty shocks. Since uncertainty premia arise from the degree of ro-
 robustness required by investors specified by a penalty function, probability distortions are
endogenously related to the fundamental state and not independent orthogonal structural
shocks. Further, the model presented in this paper provides an endogenous propagation
mechanism of uncertainty shocks on aggregate production that is not present in the pre-
viously analysed endowment settings.

Time-varying uncertainty is distinct from risk shocks as characterised by a sudden
increase in volatility. An increase in fundamental volatility will induce a risk averse
lender to impose higher returns on sovereign borrowing, as has been studied by Borri
and Verdelhan (2011) and Lizarazo (2013). Uncertainty shocks as presented here differ
from risk shocks since they have a first-order effect on asset pricing only. The pricing of
government debt does not depend on the wealth level of investors or the correlation of
investors’ income with sovereign default risk, as in the paradigm of risk aversion. The
effect of an uncertainty shock is also distinct from a shock to general risk aversion. While
the latter would affect assets with similar riskiness symmetrically, an uncertainty shock
has target-oriented implications for the pricing of public debt.\footnote{This prediction of the model is an empirical question that is out of scope of the present study.}
Durdu et al. (2013) analyse how news shocks affect the pricing of sovereign debt. They show that news shift and reshape the probability density function for the productivity shock. As news is an informative signal, its effect is symmetric in the sense that good news implies higher bond prices while bad news leads to lower bond prices. Uncertainty shocks cannot generate such symmetric responses, as only worst case beliefs are considered by ambiguity averse agents.

Finally, there is a growing literature focussing on the role of uncertainty shocks for the business cycle (Bloom et al., 2012; Bachmann et al., 2013; Ilut and Schneider, 2014). In this paper, a novel propagation mechanism of uncertainty shocks on the business cycle, due to the liquidity role of defaultable government debt in the economy, is laid out.

The paper is structured as follows. Section 4.2 contains the empirical VAR analysis of uncertainty shocks in the Euro area. Section 4.3 presents the basic theoretical mechanism in a three-period setting. A quantitative business cycle model with optimal sovereign default, ambiguity averse investors and an endogenous default penalty is presented in Section 4.4. A discussion of simulation results, the propagation mechanisms and the identification of uncertainty premia follows in Section 4.5. The final section concludes.

### 4.2 Empirical link between uncertainty and sovereign credit risk

#### 4.2.1 Determinants of sovereign credit risk and the role of uncertainty

What is the empirical relationship between uncertainty about macroeconomic fundamentals and sovereign credit risk? While, since the start of the global financial crisis in 2008, a fast growing strand of empirical literature investigating the pricing of sovereign debt in advanced economies has emerged, the role of fundamental uncertainty is studied only to a very limited extent. This section briefly summarizes the major empirical findings on the determinants of sovereign credit risk and outlines the conceptual framework for the subsequent empirical analysis.

To what extent is sovereign credit risk explained by macroeconomic fundamentals? Laubach (2009) finds a positive effect of the government budget deficit and the debt-to-GDP ratio on long-run US Treasury yields using a 30-year sample. Borgy et al. (2011) estimate an affine term structure model for a panel of eight EMU countries. They also underline the importance of the fiscal position as a major fundamental variable of a country that matters for the country yield spread. At the same time, Aizenman et al.
(2013) find in a cross country panel that a part of the premium on euro area periphery debt during the crisis, especially in 2010, cannot be explained by measures of fiscal space or other macro fundamentals. Haan et al. (2014) show that the explanatory power of macro fundamentals and the degree of ‘mispricing’ in empirical work is very sensitive to modelling assumptions. Evaluating different empirical specifications, the finding of ‘over-pricing’ is most robust in the case of Greece, Portugal and Ireland, but less so for Italy or Spain.

Are country interest rates affected by common factors? Many empirical studies find that the Lehman crash of 2008 gave rise to higher risk aversion with significant spillovers to sovereign debt markets. On the basis of a cross-country principal component analysis, Longstaff et al. (2011) point to the dominance of a global market factor relative to country-specific fundamentals. Hagen et al. (2011) show that coefficients to fiscal imbalances increased following the Lehman shock and explain this with a shift in general risk aversion. Further, Bernoth and Erdogan (2012) find evidence for time-varying coefficients in European sovereign debt markets. D’Agostino and Ehrmann (2013) confirm this finding using a panel of G7 countries. They conclude that time variation in risk perception lead to over-pricing of risk in European peripheral countries during the crisis period, which is interpreted as re-denomination risk stemming from the possibility of a collapse of the European monetary union.

There are a number complementary explanations for an intensified co-movement in sovereign CDS yields across countries since the onset of the global financial crisis. First, contagion has been identified as a source for regional spillovers. Favero and Missale (2012) find evidence for contagion in Europe in response to an increase in global risk aversion. Beirne and Fratzscher (2013) distinguish between ‘wake-up call’-, regional- and pure ‘herding’ contagion. They find only a minor role for pure contagion during the European sovereign debt crisis and stress that there has been an ‘under-pricing’ of risk in the run-up to the crisis. Second, there is the concept of systemic sovereign risk, as investigated by Ang and Longstaff (2013), which compares the degree of common credit risk between European sovereigns and US states. They find that the extent of systemic sovereign risk is substantially lower in the US than in Europe, and that financial market variables are an important source for these differences. This indicates to a third source for higher sovereign credit risk in Europe, the existence of a bank-sovereign nexus.

Dieckmann and Plank (2012) argue that financial sector variables matter to explain CDS spreads in all advanced countries due to implicit or explicit bailout guarantees, as raised by Ejsing and Lemke (2011) for European countries. Fratzscher and Rieth (2015) confirm the interconnectedness between sovereign and bank sector risk in a time-series approach, while showing that monetary policy intervention achieved to moderately ease the adverse
feedback loop.

Figure 4.1: Disagreement of real GDP forecasts in the Euro area

![Graphs showing disagreement of real GDP forecasts for France, Germany, Italy, Spain, and the Netherlands. The x-axis represents months from 2007m1 to 2014m7, and the y-axis represents percentage points. The graphs show the range of forecast disagreement over time.]

Note: 'Disagreement' captures the interdecile range of the distribution of point forecasts over GDP growth provided by a panel of professional forecasters. Source: Consensus Economics, Focus Economics, Author’s calculations.

The conceptual approach chosen here to explain sovereign credit risk builds on the hypothesis that part of the unexplained share in CDS premia in European sovereign debt markets during the crisis period might be due to uncertainty about the macroeconomic fundamental of an economy. Since investors need to form expectations about the likelihood of future debt repayment, there could be a link between the confidence investors have in their own evaluation of the macroeconomic fundamental that varies over time (Keynes, 1936, p. 148), and the pricing of sovereign debt.

As a proxy for fundamental uncertainty, I use survey data to construct a series of disagreement among professional forecasters over GDP growth for five member states of the Euro area: France, Germany, Italy, Spain and the Netherlands.\(^6\) This data includes,

\(^6\)Due to data availability, I use two different sources. From January 2007 to July 2010 I use data from Consensus Economics, while the data from August 2010 to August 2014 is from Focus Economics. The surveys contain identical questions and share many panellists such that a problem of structural breaks...
on a monthly basis, a point forecast for the current year and the following year, as well
as the standard deviation of the point forecasts as collected from the panellists. As the
data is collected on a monthly basis, the forecast horizon changes with each survey round.
In order to derive a rolling-window projection of one-year-ahead GDP growth, denoted
by $\hat{x}_{t+12|t}$, I follow Dovern et al. (2012) and combine the two mean point forecasts by a
weighting procedure according to

$$\hat{x}_{t+12|t} = \frac{k}{12} x_{t+k|t} + \frac{12-k}{12} x_{t+12+k|t},$$

where $x_{t+k|t}$ denotes the mean point forecast for the current year, and $x_{t+12+k|t}$ is the point
forecast for the consecutive year, as collected from the survey panellists in month $t$. The
forecast horizon is then $k \in \{1, 2, ..., 12\}$ and $k + 12$ months, respectively. The weighting
procedure adjusts for the shortening of time horizons when advancing in a calendar year
and shifts weight towards forecasts for next year.

I proceed accordingly with the standard deviation of point forecasts to obtain a rolling-
window measure for forecast disagreement, denoted by $\hat{\sigma}_{t+12|t}$. In a next step, I compute
the interdecile range $D_t$ from the inverse of the normal c.d.f. with mean $\hat{x}_{t+12|t}$ and
variance $\hat{\sigma}^2_{t+12|t}$. The resulting series for forecast disagreement are shown in Figure 4.1,
with the yields on sovereign CDS with a remaining maturity of 5 years. As
can be seen, forecast disagreement varies strongly over time. Standard deviations range
between 0.26 percentage points for France up to 0.38 percentage points for Germany.
Further, forecast disagreement spikes in times of crises, as around the global financial
crisis in late 2008 in the case of France and Germany, and at the peak of the European
sovereign debt crisis in 2011/12 for Spain and Italy. Indeed, there is a positive correlation
between forecast disagreement and sovereign default risk in the data which I describe in
more detail below.

### 4.2.2 Model specification and data

What was the impact of uncertainty shocks on sovereign credit risk during the European
sovereign debt crisis? This section presents a time-series approach to the dynamic be-

haviour of credit risk in response to uncertainty shocks in the Euro area over the period
2007 to 2014. The empirical model follows a structural VAR. The choice of variables is
guided by three objectives. First, the model should contain determinants for sovereign
default risk and aggregate production as I am interested in the response of these variables
to uncertainty shocks. Second, in order to account for severe disruptions in financial

...
markets in the sample period, a measure is included that addresses exogenous financial shocks. Third, given the relatively short sample period, a parsimonious setting with few variables is preferred.

The considered empirical model has \( n = 5 \) endogenous variables which include a measure for central government debt, uncertainty, industrial production, sovereign credit risk, and the spread of the bank lending rate over the money market rate, summarised in the vector \( y_t \). The spread of the bank lending rate is included in the analysis in order to account for potential propagation channels of macroeconomic uncertainty to sovereign credit risk. Further, I include \( m = 2 \) exogenous variables comprising of the real US interest rate as well as the spread of Moody’s corporate bond yields rated Baa over those rated Aaa, summarized in the \( x_t \) vector.

The model is estimated separately for each country. The country panel is chosen due to data availability. The frequency of observations is monthly, starting in January 2007 and ending in August 2014 such that there are 92 observations for each time-series.\(^7\) The system takes the form

\[
y_t = c + B(L)y_t + \Gamma(L)x_t + \epsilon_t,
\]

(4.1)

with matrices of polynomials in the lag operator \( L \), \( B(L) = B_0L^0 + \ldots + B_pL^p \) and \( \Gamma(L) = \Gamma_0L^0 + \ldots + \Gamma_qL^q \). Further, note that \( c \) is an \( n \times 1 \) vector of constants, \( B_i \) are \( n \times n \) matrices and \( \Gamma_i \) are matrices of dimension \( n \times m \).

In the system, endogenous variables depend on contemporaneous and past values of endogenous and exogenous variables. Identification of structural shocks is obtained through impact restrictions by the ordering of endogenous variables in the stacked vector \( y_t \).\(^8\) Specifically, I assume that the impact matrix \( B_0 \) is lower triangular such that variables

\[^7\]An exception is the debt-to-GDP ratio which is available only at a quarterly frequency. I assume that there are no changes within a quarter and keep the ratio constant between quarters.

\[^8\]For the identification of the model (4.1), define \( \Psi(L) = \Psi_0 + \Psi_1L + \ldots = [I - B(L)]^{-1} \) with \( \Psi_0 = I \), i.e. an infinite polynomial on the lag operator. This allows the VMA-X representation of the model (4.1) as \( y_t = \Psi(1)c + \Psi(L)\Gamma(L)x_t + \Psi(L)\epsilon_t \) to be computed. The structural shocks \( \epsilon_t \) can then be estimated from the equation \( \mu + \Delta(L)x_t + C(L)\epsilon_t = \Psi(1)c + \Psi(L)\Gamma(L)x_t + \Psi(L)\epsilon_t \), where \( \epsilon_t \sim \mathcal{N}(0, I) \) such that parameters can be estimated by OLS.
ordered at lower ranks are assumed to not have a contemporaneous effect on the higher ranked variables (Sims, 1980, 1986). In the specification considered, a lag length of order one is chosen for endogenous and exogenous variables. As an important fundamental macro variable to determine sovereign credit risk, the debt-to-GDP ratio at a quarterly frequency enters the set of endogenous variables first. As there are only a few movements at the low frequency, it seems appropriate to assume that there is no contemporaneous effect of the other variables ranked below on the debt ratio.

Figure 4.2: Correlation pattern

Notes: Coefficients of correlation between forecast disagreement and sovereign CDS with a remaining maturity of five years. The original time series are plotted in Figure 4.1.

At the second position enters the uncertainty variable. I use the interdecile range of forecast disagreement as plotted in Figure 4.1 as a proxy for macroeconomic fundamental uncertainty. Disagreement is a widely used measure for uncertainty and is discussed in detail in Zarnowitz and Lambros (1987) and Bomberger (1996). An augmented Dickey-Fuller test (ADF) declines the presence of a unit-root within the sample period for most countries in the panel. I use three leads of the disagreement variable given the special nature of the survey. While the data is collected monthly, it is unlikely that all participants in the panel update their individual growth projections at such a high frequency. Therefore, the survey collected at period $t$ might also contain uncertainty.

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9Lag lengths are validated using various statistical criteria, such as the Akaike Information Criterion (AIC), the likelihood ratio test, and the Bayesian Information Criterion (BIC).
10A critical assessment can be found in Rich and Butler (1998) and Abel et al. (2015).
11Coibion and Gorodnichenko (2012) find substantial information stickiness in the U.S. survey of professional forecasters. Sheng and Wallen (2014) quantify information rigidities to an interval of two to three months.
as of period \( t - 1 \) up to \( t - 3 \), assuming a quarterly frequency of growth forecasts. In fact, the correlation structure between forecast disagreement and sovereign CDS yields also indicates that the relationship is slightly stronger when considering leads of the disagreement variable (Figure A.1).

I use yields on sovereign CDS with a maturity of five years as a measure for sovereign credit risk. Although the ADF cannot reject the null of a unit root, CDS rates enter the system in levels for two reasons. First, on theoretical grounds it can be argued that interest rates do not follow a non-stationary process over a sufficiently long time period. Second, the level information is the objective for the underlying analysis where I am interested in the intensity of sovereign default risk.

Real industrial production as a proxy for GDP is seasonally adjusted and enters the model in first differences. It is assumed that industrial production has no contemporaneous effect on sovereign credit risk, the debt ratio and macroeconomic uncertainty.

Finally, the spread of the short-term bank lending rate enters the system as the last variable. It is constructed as follows: I subtract the 3-month Eonia rate from the national bank lending rate to non-financial firms of loans with a volume of more than one million euros and a maturity up to one year. Similar to developments in CDS markets, the ADF test cannot reject the null of a unit-root. As this is again strongly related to the sample period, the bank lending spread enter the empirical model in levels.

As exogenous variables, I construct the real US interest rate by subtracting the average inflation rate in the previous four months from the three month interest rate on US Treasury bills. Further, I include a typical credit risk measure by constructing the spread of yields on US corporate bonds with a Moody’s rating of Baa over those with a Aaa-rating. Both indicators are meant to capture exogenous global shocks in the sample period, which are identified in previous studies as changes in general risk aversion.

In the Appendix, I provide a robustness analysis by estimating the VAR using different specifications. In particular, I allow for fewer leads in forecast disagreement, I use a different ordering of endogenous variables, and I include the European money market spread as a separate exogenous variable to control for tensions in European interbank markets as a source for spillovers between banks and sovereigns.

### 4.2.3 Uncertainty shocks, sovereign credit risk, and spillovers

The dynamic effects of structural uncertainty shocks identified in the empirical model are analysed next. Figure 4.3 presents the impulse responses of a one standard deviation increase in the uncertainty measure on sovereign default risk, i.e. yields on sovereign CDS. There is a significant and long-lasting response of default risk in Italian, French
and Spanish CDS yields that lasts up to one year. Between June 2011 and January 2012, forecast disagreement for Spanish GDP growth increased by 4.5 standard deviations. Multiplied with the maximum amplitude of the impulse response from the VAR, which reaches an increase of 5 basis points three to four months after the shock, there is an overall effect of 0.22 percentage point increase during the crisis due to uncertainty only.

Figure 4.3: Effect of uncertainty shock on sovereign credit risk (IRFs)

![Graphs showing the effect of uncertainty shock on sovereign credit risk for France, Germany, Italy, Spain, and the Netherlands.](image)

Note: Orthogonalized impulse response functions. Dashed lines are 66 percent confidence intervals. The impulse is a one standard deviation increase in uncertainty (Interdecile range of forecast disagreement), the response the yield on sovereign CDS (in basis points).

On the contrary, there is practically no response of sovereign credit risk in the cases of Germany and the Netherlands. This clear separation of Euro area member countries in a core and a periphery is remarkable. It highlights that there is an underlying relationship between economic uncertainty and sovereign default risk which affects the country groups asymmetrically. While financial markets anticipate a significant increase in sovereign default risk in response to uncertainty in periphery countries, there is only a minor increase in the core countries.

Next, I am interested in the underlying propagation channel between uncertainty and sovereign credit risk in the Euro area periphery. In the context of the European sovereign
In the context of the hypothesis of a downward spiral in the presence of a bank-sovereign nexus, I consider the effect of an uncertainty shock on the bank lending rate. If the bank-sovereign nexus is strong, there should be an increase in the bank lending rate in response to uncertainty.

As expected, there is a positive reaction of private sector interest rates around five months after a one-off increase in fundamental uncertainty, as shown in Figure 4.4. However, in France and Germany, there is a reduction in bank lending rates on impact. Although the separation of the sample in core and periphery countries is not as clear as before, the amplitude of the impulse response is significantly higher in Italy in the medium term compared to Germany and France.

In the next section, I present a theoretical model that investigates the interaction of macroeconomic uncertainty and sovereign credit risk in a simple three-period setting.
CHAPTER 4. NON-FUNDAMENTAL DEBT CRISES

4.3 Non-fundamental default: a simple model

This section develops a theory of non-fundamental default in the presence of uncertainty shocks when investors are ambiguity averse. It is shown that uncertainty shocks partly endogenize sunspot driven self-fulfilling debt crises as analysed by Cole and Kehoe (2000).

4.3.1 Environment

I assume a small open economy that lasts for three periods, \( t = 0, 1, 2 \). There are three types of agents in the economy, households, a government, and international investors.

*Households.* Households are of the producer-consumer type. At period \( t = 1 \) and \( t = 2 \), they produce a final tradable good \( y_t \) with a technology \( e^{z_t} F(l) \), where \( l \) denotes fixed labour supply and \( z_t \) is the aggregate productivity state. Goods cannot be stored and households consume the returns from their production technology net of lump-sum taxes. In the initial period, there is no production and consumption is based on government transfers made possible through borrowing on international capital markets.\(^{12}\) The lifetime utility of a representative household is given by a quadratic utility function

\[
u(c_0, c_1, c_2) = \sum_{t=0}^{2} \left( c_t - \frac{\psi}{2} (c_t)^2 \right),\]

where \( \psi < 1 \) is a constant. The period \( t \) budget constraints are given by

\[
c_0 = -\tau_0, \\
c_1 = y_1 - \tau_1, \\
c_2 = y_2 - \tau_2,
\]

where \( \tau_t \in \mathbb{R} \) denotes a lump-sum tax or transfer chosen by the government.

*Technology and uncertainty.* The productivity parameter \( z_t \) can take two values, \( z_t \in \{ z^l, z^h \} \), with \( z^h > 0 \) and \( z^l < 0 \). For simplicity, I assume that aggregate productivity at \( t = 1 \) is deterministic and takes the lower value, \( z_1 = z^l \). In contrast, productivity at \( t = 2 \) is uncertain and can take either the low or the high value. Whether the high value realises is determined by a draw of a random variable \( x \) from a uniform distribution

\(^{12}\text{This can be thought of as initial investment in human capital through training which requires time but no capital.}\)
$x \sim U(x_{lb}^*, x_{ub}^*)$, according to

$$z_2 = \begin{cases} 
  z^h & \text{if } x > \bar{x} \\
  z^l & \text{if } x \leq \bar{x} 
\end{cases},$$

where $\bar{x}$ denotes a threshold value for the high productivity state. Although agents know the threshold $\bar{x}$ and that $x$ is drawn from a uniform distribution, it is assumed that agents do not know the exact parameters $(x_{lb}^*, x_{ub}^*)$ that govern the true data generating process (DGP) for productivity. This can be seen as a special case of the Ellsberg-urn (Ellsberg, 1961) where no probabilities can be attached over a bet. In this case, there are no known probabilities for the specific outcome $x > \bar{x}$, which would indicate the high productivity state. Thereby, period $t = 2$ productivity is ambiguous.

**Government.** The government maximizes the representative household utility by smoothing consumption through taxation and household transfers. To this end, the government borrows from international investors by selling one period discount bonds $B_{t+1} < 0$ at the price $q_t < 1$.\(^{13}\) Note that the subscript of bonds denotes the period when these bonds become due, such that $B_1$ represent bonds sold in the initial period $t = 0$. The period $t = 0, 1, 2$ government budget constraints read

$$\tau_0 = q_0 B_1,$$
$$\tau_1 = q_1 B_2 - B_1,$$
$$\tau_2 = -B_2.$$

There is imperfect commitment to repay the debt. In periods $t = 1, 2$, the government has the possibility to repudiate the total amount of outstanding debt if this is optimal in terms of welfare.\(^{14}\) Default is denoted by the indicator variable $\delta_t = \{0, 1\}$.

In case the government decides to repudiate the debt, there is an exogenous penalty, formalized by a proportional output loss for the remaining periods (Arellano, 2008) that takes the form:\(^{15}\)

$$h(y) = \begin{cases} 
  \hat{y} & \text{if } z_t = z^h \\
  y & \text{if } z_t = z^l 
\end{cases} \quad (4.2)$$

Thus, the default penalty is pro-cyclical and there is no punishment in the low produc-

\(^{13}\)I am only interested in international borrowing. However, international savings are not ruled out \textit{a priori}. The assumption regarding the utility function and increasing production over time make borrowing the preferred asset decision.

\(^{14}\)For uniqueness of equilibria, it is essential that there is no contingent haircut, see Calvo (1988) for a model with multiple equilibria.

\(^{15}\)Quantitatively, the penalty function sets $\hat{y} = \phi(y)$, where $\phi < 1$ is an exogenous parameter.
tivity state. Further, the government is excluded from borrowing on international capital markets conditional on defaulting (Eaton and Gersovitz, 1981).

International Investors. International investors purchase government debt by taking into account the risk of sovereign default. It is assumed that investors know the underlying decision problem of the government. In addition, investors are ambiguity averse. Confronted with uncertainty surrounding the probabilistic model from which date \( t = 2 \) productivity is drawn, they gather additional information from less tangible sources.

This process is formalised as follows. At the beginning of period \( t = 1 \), an uncertainty shock, denoted by \( a \), realises and determines the aggregate level of ambiguity in the model. This shock is drawn from a uniform distribution, \( a \sim \mathcal{U}[0, \bar{a}] \). All agents have full information regarding the probability model for uncertainty \( a \).

The level of ambiguity determines the range of investors’ prior beliefs about possible distributions from which \( x \) is drawn and that determines \( z_2 \). In particular, let \( \text{supp}^P(\mathcal{U}) = (x_{lb}^P, x_{ub}^P) \) denote the support of an arbitrary uniform distribution. Then, \( \text{supp}^P(\mathcal{U}) \in \mathcal{P} \) denotes the set of probability models considered for \( t = 2 \) productivity. One way to think of a prior \( \text{supp}^P(\mathcal{U}) \) is a forecasting model for GDP growth for which some uncertainty remains. The relationship between the ambiguity level \( a \) and the set of prior beliefs \( \mathcal{P} \) is such that

\[
\text{supp}^P(\mathcal{U}) \in \mathcal{P} = \left\{ \begin{array}{l}
x_{lb}^P \in [\tilde{x}_{lb} - a, \tilde{x}_{lb} + a] \\
x_{ub}^P \in [\tilde{x}_{ub} - a, \tilde{x}_{ub} + a]
\end{array} \right.
\]

Thus, the realised level of ambiguity spans the range of prior beliefs about the true probabilistic model \( \mathcal{U}(x_{lb}^*, x_{ub}^*) \) which are centred around the parameters \( \tilde{x}_{lb} \) and \( \tilde{x}_{ub} \).\(^{16}\) There is no relationship assumed of \( \tilde{x}_{lb} \) being close or distant to the true value \( x_{lb}^* \), so a high realisation of uncertainty cannot be interpreted in a way that agents generally consider worse models.

Further, note that within this setting there is true uncertainty in the Knightian or Keynesian sense over the appropriate forecasting model. In particular, there are no subjective probabilities that guide agents’ evaluation of prior beliefs over possible forecasting models for the fundamental. As a result, this setting deviates from the paradigm of subjective expected utility (SEU) (Savage, 1954). Instead, investors with multiple priors exhibit maxmin preferences (Gilboa and Schmeidler, 1989), i.e. they maximize returns conditional on each prior in the set and act under the prior that delivers the worst outcome. Ambiguity averse investors act as if there is just one belief in the set \( \mathcal{P} \).

\(^{16}\)The parameters \( \tilde{x}_{lb}, \tilde{x}_{ub} \) can be thought of as the part of the DGP that can be gathered through mechanisms not explained in the model, e.g. Bayesian learning. Further, for consistency of the model note the parameter restriction \( a \leq \tilde{x}_{ub} - \tilde{x} \).
4.3.2 Equilibrium

An equilibrium to the model is characterized by

i. Consumption plans \( \{c_0, c_1, c_2\} \) that satisfy the household budget constraints;

ii. Government borrowing decision rules \( \{B_1, B_2\} \), as well as tax- and transfer schedules \( \{\tau_0, \tau_1, \tau_2\} \) that maximize household lifetime-utility \( ex \ ante \) and that respect the government budget constraints;

iii. Government default decision rules \( \{\delta_1, \delta_2\} \) that maximize the household lifetime-utility conditional on the uncertainty shock \( a \) and the productivity shock \( x \);

iv. Pricing schedules for debt \( \{q_0, q_1\} \) that satisfy the no-arbitrage condition from international investors at the time of contracting.

4.3.3 Solution

The model is solved backwards. First, we look at the conditions for default at \( t = 2 \).

In the final period, repayment can only be enforced through the exogenous penalty \( h(y) \). Absent any penalty mechanism, it would be optimal for the government to repudiate the debt as this would increase household consumption. The condition for default at \( t = 2 \) can be phrased in terms of utility at \( t = 2 \), where \( V_2^{d} \) and \( V_2^{nd} \) denote utility under default and repayment, respectively:

\[
V_2^{nd} < V_2^{d} \\
\Leftrightarrow \quad u(y_2 + B_2) < u(h(y_2))
\]

The default definition can be used to derive two threshold values for borrowing at \( t = 1 \).

First, let \( B_2 \) denote the value up to which repayment is sure. This is given up to the borrowing level where the value of default and repayment are identical under the low
Considering only solutions to this lower or equal then zero, this condition can be rewritten as\(^{17}\)

\[
B_2 - \frac{\psi}{2}(B_2)^2 = 0,
\]
\[
\iff B_2(B_2 - \frac{\psi}{2}) = 0
\]
\[
\implies B_2 = 0,
\]

Second, let \(B_2\) denote the threshold level for default with certainty, given by the borrowing level at which default is optimal in the high productivity state:

\[
V^{nd}_2(z_2 = z_h) = V^d_2(z_2 = z_h)
\]
\[
\iff (e^{z_l} + B_2) - \frac{\psi}{2}(e^{z_l} + B_2)^2 = h(e^{z_l}) - \frac{\psi}{2}(h(e^{z_l}))^2
\]

Again, focusing on the solution lower or equal to zero, I obtain

\[
B_2 - \frac{\psi}{2}(B_2)^2 = h(e^{z_h}l) - e^{z_h}l - \frac{\psi}{2}(h(e^{z_h}l))^2 + \frac{\psi}{2}(e^{z_h}l)^2
\]
\[
\iff (B_2)^2 - \frac{2}{\psi}B_2 + \frac{2\bar{e}}{\psi} = 0
\]
\[
\implies B_2 = \frac{1}{\psi} - \sqrt{\left(-1\right)^2 - \frac{2\bar{e}}{\psi}}
\]

International investors anticipate the default decision of the government. They maximize expected returns from lending to the government. Investors have a risk-free investment alternative that yields a return \(r^f\). Investors profit maximization takes the form

\[
\max_{\{B_{t+1}\}} \Pi = -q_t B_{t+1} + \min_{\{supp(t)\} \in \mathcal{P}} \mathbb{E}_t^p \left[ \frac{1 - \delta_{t+1}}{1 + r^f} \right] B_{t+1},
\]

where \(E_t^p\) denotes the mathematical expectation operator with multiple priors given by the period \(t\) information set. From the first order condition, we obtain the asset pricing

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\(^{17}\)Positive solutions for \(B_2\) from the quadratic equation are not considered as debt is defined in negative terms. Remember also that there is no default penalty in the low productivity case.
CHAPTER 4. NON-FUNDAMENTAL DEBT CRISIS

Under maxmin preferences, investors choose the prior \( \text{supp}^p(U) \in \mathcal{P} \) that minimizes the expected returns. As returns are zero in the case of outright default, investors pin down the prior from the set of beliefs that maximizes expected default, \( E^p_t[\delta_{t+1}] \). As it is known from the default decision at \( t = 2 \), default is given for sure in the low productivity state for any positive lending. Maxmin preferences thus lead to the adoption of the prior that yields the lowest probability for the high productivity state, i.e.

\[
x^p_{lb} = \bar{x}_{lb} - a, \\
x^p_{ub} = \bar{x}_{ub} - a.
\]

Ambiguity averse investors’ \( t = 1 \) expectations on productivity in the final period are formed using the worst case prior which yields the probability model \( x \sim \mathcal{U}(x^p_{lb}, x^p_{ub}) \) with c.d.f. \( F(x) \). Let \( \pi^I_p \) denote the probability of a low productivity state at \( t = 2 \) as perceived by an ambiguity averse agent,

\[
\pi^h_p = 1 - \int_{x^p_{lb}}^{x} F(x)dx = 1 - \frac{\bar{x} - x^p_{lb}}{x^p_{ub} - x^p_{lb}}, \\
\pi^l_p = 1 - \pi^h_p.
\]

From the perspective of an ambiguity averse investor, the prior belief about the low productivity state coincides with her default expectations for \( B_2 \in (B_2, \bar{B}_2) \):

\[
E^p_1[\delta_2] = \pi^I_p
\]

Combining the default rule at \( t = 2 \) with the asset pricing equation of investors, government debt issuance at \( t = 1 \) is priced as:

\[
q_1(B_2, a) = \begin{cases} 
\frac{1}{1+r^f} & \text{if } \bar{B}_2 \leq B_2 \\
\frac{1 - \pi^I_p}{1+r^f} & \text{if } B_2 \in (B_2, \bar{B}_2) \\
0 & \text{if } B_2 < B_2
\end{cases}
\]

In order to pin down the default decision rule at \( t = 1 \), the government solves a related optimisation problem. Denote the expected value of repayment at \( t = 1 \) conditional on
previous debt accumulation as $V^{nd}_1(B_1)$, then formally

$$V^{nd}_1(B_1) = u(c_1) + E[u(c_2)]$$

$$= (y_1 - q_1(B_2, a)B_2 + B_1) - \psi \left( y_1 - q_1(B_2, a)B_2 + B_1 \right)^2$$

$$+ E_1[\delta_1(h(y_2)) + (1 - \delta_1)(y_2 + B_2)] - \frac{\psi}{2} (E_1[\delta_1 h(y_2) + (1 - \delta_1)(y_2 + B_2)])^2,$$

where the government internalises the optimal default decision conditional on the productivity realisation at $t = 2$. Similarly, the value of default can be formalized as

$$V^d_1 = h(y_1) - \frac{\psi}{2} (h(y_1))^2 + E_1[h(y_2)] - \frac{\psi}{2} (E_1[h(y_2)])^2.$$

The value of default is independent of the previously accumulated debt. Note that expectations about productivity are heterogeneous by assumption in this model. The not-ambiguity averse government takes the mean of the interval $\text{supp}^\mu$ as prior belief. Let $\tilde{\pi}^h$ denote the probability for a high productivity state and $\tilde{\pi}^l$ the probability of a low state under the probability model $x \sim \mathcal{U}(\tilde{x}_{lb}, \tilde{x}_{ub})$. Then, the government’s expectations about ambiguous productivity are given by

$$\tilde{\pi}^h = 1 - \int_{\tilde{x}_{lb}}^{\tilde{x}_{ub}} F(x)dx = 1 - \frac{\tilde{x} - \tilde{x}_{lb}}{\tilde{x}_{ub} - \tilde{x}_{lb}},$$

The value of $t = 1$ repayment, $V^{nd}_1(B_1)$, depends on the price schedule the government faces, $q_1(B_2, a)$, which itself is a function of aggregate uncertainty $a$. The higher the aggregate level of uncertainty, the higher the cost to roll-over debt accumulated in the initial period and the lower is household consumption, and thus

$$\frac{\partial V^{nd}_1(B_1)}{\partial a} < 0.$$

In close analogy to the pricing of debt at $t = 1$, let’s define two threshold values for period $t = 0$ borrowing. First, denote $B_1$ the endogenous borrowing limit for positive values of debt $q_0 > 0$. $B_1$ is given by

$$V^{nd}_1([B_1]|a = 0) = V^d_1$$

This is the maximum borrowing limit as for any level of debt $B_1 < B_1$, there is guaranteed default at $t = 1$. Further, let us denote by $\bar{B}_1$ the risk-free borrowing limit, up to which

\footnote{This assumption on heterogeneous beliefs about productivity does not change the results.}
the government will repay the debt with certainty at \( t = 1 \). The condition is

\[
V^{nd}_1[(B_1)|a = \bar{a}] = V^d_1.
\]

**Proposition 7.** For levels of borrowing \( B_1 \in (B_1, \bar{B}_1] \), there exists a critical threshold \( a^*(B_1) \) beyond which default becomes optimal along the realisation of aggregate ambiguity about \( t = 2 \) productivity.

**Proof.** See Appendix. ■

The critical threshold \( a^*(B_1) \) describes the default rule at \( t = 1 \) and is given by

\[
V^{nd}_1(a^*(B_1)) = V^d_1
\]

The probability of default at \( t = 0 \) can then be formalised as

\[
E_0[\delta_1] = \pi_0^\delta = \int_{a^*(B_1)}^{\bar{a}} F(a)da = 1 - \frac{a^*(B_1)}{\bar{a}}
\]

for any debt level \( B_1 \in (B_1, \bar{B}_1] \). The pricing of debt at \( t = 0 \) is given by

\[
q_0(B_1) = \begin{cases} 
\frac{1}{1+r_f} & \text{if } B_1 \geq \bar{B}_1 \\
\frac{1-\pi_0^\delta}{1+r_f} & \text{if } B_1 \in (B_1, \bar{B}_1] \\
0 & \text{if } B_1 > B_1
\end{cases}
\]

We are now able to derive the optimal policy. The benevolent government takes pricing schedules \( q_0, E[q_1] \) as given and puts up optimal borrowing plans as viewed from the initial period by maximizing household lifetime utility

\[
\max_{(B_1, B_2, \delta_1, \delta_2)} E_0 \left[ \sum_{t=0}^{2} \left( c_t - \frac{\psi}{2} (c_t)^2 \right) \right],
\]

subject to the constraints

\[
c_0 = -q_0B_1, \\
c_1 = y_1 - E_0[q_1]B_2 + B_1, \\
c_2 = E_0[y_2] + B_2.
\]

At the beginning of period \( t = 1 \), aggregate uncertainty \( a \) realises. The government
adjusts its plan according to
\[
\max_{\{\delta_1, B_2, \delta_2\}} E_1 \left[ \sum_{t=1}^{2} \left( c_t - \frac{\psi}{2} (c_t)^2 \right) \right],
\]
subject to
\[
c_1 = y_1 - q_1 B_2 + B_1, \quad c_2 = y_2 + B_2,
\]
where default is given if \( a > a^* (B_1) \). In the final stage, the government repays any debt \( B_2 \in (B_2, \bar{B}_2] \) only in the high productivity state, thus if \( x > \bar{x} \).

### 4.3.4 Results and discussion

The pricing schedules for debt at dates \( t = 0, 1 \) are illustrated in Figure 4.6. From an \textit{ex ante} perspective, it is welfare improving for the government to borrow from international capital markets to smooth household consumption through transfer payments and taxation over the life-cycle. However, due to limited commitment, the yields on government debt are increasing in the amount of debt issued, as the probability for exceeding the default threshold \( a^* \) is also increasing in debt issued as long as \( B_1 < \bar{B}_1 \) (Fig. 4.6a).

At date \( t = 1 \), the aggregate level of uncertainty realises and determines worst case
beliefs of ambiguity averse investors, $\pi_p^t$ (Fig. 4.6b). Note that there is no subjective probabilistic evaluation of investors over the DGP or forecasting models over $t = 2$ productivity. Every prior $\text{supp}^p(\mathcal{U}) \in \mathcal{P}$ has no attached probability. Therefore, in principle, each prior could constitute an equilibrium in the model. However, there is no multiplicity of equilibria conditional on the realisation of the uncertainty shock. In fact, it is the ordering over priors obtained through maxmin preferences that prevent indeterminacy in this set-up, as each prior can be ranked in terms of expected returns from holding government bonds. Investors achieve this through the equilibrium mapping from the optimal default decision of the government, which is based on the fundamental state of the economy. Thereby, maxmin preferences serve the role of equilibrium selection under incomplete information.

Within the area $B_1 \in (\mathcal{B}_1, \mathcal{B}_1]$, uncertainty surrounding the forecasting model regarding the fundamental state $z_2$ can have dramatic consequences. Given the same fundamental at $t = 1$, high uncertainty may lead to outright sovereign default if $a > a^*(B_1)$, as shown in Figure 4.7. I will refer to such default events as non-fundamental default.

In analogy to Cole and Kehoe (2000), denote the region where non-fundamental default is possible as the crisis zone. Cole and Kehoe assume a sunspot variable that declares a liquidity crisis for which the price of debt is set to zero for exogenous reasons. In the ambiguity framework, maxmin preferences of investors interact with the limited commitment friction on the side of the government. Given that investors anticipate the worst case outcome, they form pessimistic beliefs about future debt repayment as a function of the ambiguous fundamental, $E_1^p [\delta_2] = \pi_p^t$, and price debt issued in $t = 1$ accordingly. Thereby, pricing of debt in the roll-over stage is determined by beliefs of market participants. The investor beliefs induced by uncertainty shocks therefore partly endogenize sunspot shocks analysed in the model of self-fulfilling debt crises. Specifically, be reminded that in Cole and Kehoe, a self-fulfilling crisis occurs if a sunspot variable $\zeta_t$ exceeds an
exogenous threshold value $\pi$, conditional on being in the crisis zone. The condition for default derived in the ambiguity framework is given for a cut-off rule $a > a^*(B_1)$, which is very similar.

Note that the critical threshold $a^*(B_1)$ is an endogenous equilibrium object influenced by previous borrowing decisions. This is not the case in a setting with sunspot shocks where the threshold for a liquidity crisis is exogenous. Therefore, the probability of exceeding the critical threshold for a liquidity crisis is constant in Cole and Kehoe while it depends on the debt issued in the case of uncertainty shocks. One implication of the threshold $a^*(B_1)$ is a decreasing pricing schedule in the amount of debt issued. This seems closer to observed liquidity crises where there is usually still a price for debt substantially above zero. However, if the government is only offered relatively costly conditions to roll-over the previously accumulated debt, it might optimally decide to default on the outstanding debt. As investors anticipate the default decision of the government conditional on $a > a^*$, the price schedule drops to zero in this specific case but remains positive for $a < a^*$.

The timing assumption constitutes the main conceptual difference between non-fundamental default in the presence of uncertainty shocks and self-fulfilling default. In Cole and Kehoe, a government issues new debt at the beginning of the period. If the economy is within the crisis zone, a sunspot liquidity crises induces outright default at the end of the period, which is correctly anticipated by investors, therefore the self-fulfilling character of the initial exogenously induced beliefs about future default. In the ambiguity framework, I assume that the government can commit to not default on newly issued debt within the same period. This is implied by the timing assumptions where optimal borrowing and default decisions are taken simultaneously. As a result, the implications of uncertainty shocks for the self-fulfilling character of the default decision are less clear-cut.

Most importantly, note that there is no resolution of uncertainty after productivity is realised. Agents can observe $z_2$ at period $t = 2$, but not the true DGP from which the stochastic variable $x$ was drawn and which determines productivity in the final period. Therefore, it is impossible to declare whether the adoption of the worst case prior that lead to default was justified initially.

### 4.4 Quantitative business cycle model

#### 4.4.1 Overview

This section nests the mechanism of non-fundamental default as described in the three-period setting in a quantitative DSGE model. The objective is to estimate the quantitative
model and disciplining the uncertainty shock by data over forecast disagreement. Finally, model simulations are used to decompose model generated yields on sovereign debt. This enables a quantitative evaluation of uncertainty shocks for the Euro area.

The point of departure is an RBC model of a small open economy with a benevolent government. The novelty is that I apply a process of aggregate productivity which is subject to time-varying levels of ambiguity to a framework with endogenous sovereign default. In line with the literature on strategic default (Eaton and Gersovitz, 1981), the government cannot commit to service its obligations from previous debt issuances but takes an optimal default decision each period. I follow Engler and Große Steffen (2014) in modelling endogenous output costs of default due to disruptions in the domestic interbank market where government bonds serve as collateral.

International investors have preferences that exhibit ambiguity aversion. This leads to pricing decisions different from the paradigm of subjective expected utility maximization (SEU). Following the recursive multiple-priors model, ambiguity averse agents form a worst case belief that pins down their optimal inter-temporal decisions when confronted with ambiguity about the true data generating process of aggregate productivity.

Additional to a benevolent government and a central bank, the domestic economy is populated by a representative household, a unit mass of non-financial firms and heterogeneous banks. Time is infinite and discrete \( t = (0, 1, ...) \). Endogenous states, i.e. total government debt \( (B_t) \) and domestic debt held by the domestic banking sector \( (B^D_t) \), are given from decisions made in period \( t - 1 \) decisions. Exogenous states are stochastic and given by aggregate total factor productivity \( (z_t) \) and the degree of ambiguity about the future fundamental state of the economy \( (a_t) \), which constitutes the uncertainty shock. After the government learns about the aggregate state \( s \in (B_t, B^D_t, z_t, a_t) \) at the beginning of each period when shocks realise, it decides to default \( (\delta_t = 1) \) or repay \( (\delta_t = 0) \) its debt. In case of repayment, non-financial firms produce a tradeable final good. They receive working capital loans from heterogeneous banks that re-allocate financial resources on the domestic interbank market. Borrowing in the interbank market is collateralised with government debt and is affected by changes in sovereign risk perceptions.

If the government defaults on its debt, the economy falls into financial autarky with an exogenous probability of re-accessing capital markets. Since public debt cannot be issued in autarky states, there is no collateral available. Interbank intermediation breaks down and the economy suffers an endogenous credit crunch. An overview of the sequence of events is presented in the Appendix, Figure C.2.
4.4.2 Households

A representative household derives utility from consumption ($c_t$) and leisure ($1 - l_t$). It provides labour ($l_t$) to non-financial firms and receives a wage ($w_t$) in return. The household owns non-financial firms and banks in the economy and receives profits ($\Pi_t$) and dividend payments ($D_t$) in a lump-sum transfer at the end of each period. The inter-temporal savings decision to smooth consumption is done by the benevolent government through transfer payments ($T_t$). The household maximizes life-time utility subject to a budget constraint

$$\max_{\{c_t, l_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t),$$

$$s.t. \quad c_t = w_t l_t + \Pi_t + D_t + T_t,$$

(4.3)

where $E_t$ denotes the rational expectations operator and the utility function $u(\cdot)$ satisfies the inada conditions. Optimal labour supply is determined from the combined first order conditions for consumption and labour:

$$-\frac{u_t(c_t, 1 - l_t)}{u_c(c_t, 1 - l_t)} = w_t$$

(4.4)

4.4.3 Non-financial firms and technological change

The non-financial sector produces a tradeable final good $Y_t$ with a Cobb-Douglas production function with variable labour input $l_t$ and a constant capital stock $K$, thus $Y_t = e^{zt} F(l_t, K)$ with $z_t$ denoting aggregate total factor productivity. Within-period working capital loans $\kappa_t$ are required to finance production input factors up-front. These loans pay an interest rate $r^c_t > 0$. Let $\eta$ denote the share of the wage bill which needs to be financed before production starts. Then, the firm’s profit maximisation problem reads

$$\max_{\{l_t, \kappa_t\}} e^{zt} F(l_t, K) - w_t l_t - \kappa_t r^c_t,$$

$$s.t. \quad \kappa_t \geq \eta w_t l_t.$$

(4.5)

(4.6)

Optimal labour demand is pinned down by the condition

$$F_t = w_t (1 + \eta r^c_t)$$

(4.7)

Technological change is stationary, stochastic and persistent. I closely follow Ilut and Schneider (2014) and introduce ambiguity about the true data generating process that
drives aggregate productivity in the set-up. The true law of motion for productivity is described by

\[ z_t = \rho z_{t-1} + u_t + \mu^*_t, \]  

(4.8)

where \( u_t \) is a mean zero iid shock with variance \( \sigma^2_u \). Further, let \( \mu^*_t \) denote a deterministic sequence of changes in productivity. It is assumed that its long-run empirical distribution converges to an iid normal distribution with mean zero and variance \( \sigma^2_z - \sigma^2_u > 0 \) which is independent of stochastic shocks \( u_t \).

Since only aggregate productivity \( z_t \) is observable, there is no probabilistic assessment for the deterministic component \( \mu^*_t \). An econometrician would find, by observing the sequence of productivity realisations \( z^t = (z_1, \ldots, z_t) \), that innovations to \( z_t \) are iid normal with mean zero and variance \( \sigma^2_z \). Therefore, there is no possibility of learning since the processes \( \mu_t \) and \( u_t \) cannot be disentangled.\(^{19}\)

As a result, agents are not only confronted with ambiguity about the future realisation of productivity, which they infer from iterating equation (4.8) one period forward, they also face ambiguity about the underlying probabilistic model, since the law of motion (4.8) covers a hole class of different DGPs. This requires additional structural assumptions on individual preferences for the formation of expectations as will be explained in detail in the optimisation problems of each class of agents in the subsequent sections.\(^{20}\)

### 4.4.4 Banks

The modelling of the heterogeneous banking sector follows the setting in Engler and Große Steffen (2014). Let there be an infinite amount of banks of measure one in the domestic economy. Banks are endowed with previously accumulated household deposits \( N \). Further, they carry over government bonds \( B^D_t \) from period \( t - 1 \) decisions. There are two types of banks, \( \tau \in \{p, u\} \). At an exogenous probability \( \pi^p \), a bank turns into a productive bank \( (\tau = p) \), which is matched with a non-financial firm. At the complement probability \( \pi^u = 1 - \pi^p \), a bank is of the unproductive type \( (\tau = u) \). These banks are not matched, but offer their financial resources as interbank loans \( (M_t) \) on the domestic interbank market to type \( p \) banks at the interest rate \( r^M_t \).

Productive banks use deposits \( N^p \) and interbank loans to extend working capital loans to non-financial firms. Two frictions are assumed to be present in the domestic interbank

\(^{19}\)See Epstein and Schneider (2007) for a formalisation of learning under ambiguity. They specify processes which prevent the full resolution of uncertainty.

\(^{20}\)In particular in Section 4.4.6 on the preferences of ambiguity averse investors which is deferred after the discussion of the government problem for a better comprehensibleness.
market. First, type \( p \) banks borrowing wholesale funds want to self-insure against potential roll-over risk given the inherent maturity mismatch. Banks build excess reserves that are a constant fraction of interbank loans, \( R_t^e = M_t / \phi \), with \( \phi > 1 \).

Second, interbank loans are collateralised with government debt. Unproductive banks who lend in the interbank market thereby lower counterparty risk in the market for wholesale funding. The collateral constraint takes the form

\[
M_t \leq \frac{(1 - \delta_t) B_t^{D,p}}{\chi},
\]

where \( \chi \) denotes a constant haircut parameter. The term \( (1 - \delta_t) \) alludes to the riskiness of government debt. In case the government decides to default, government debt loses its collateral value. We apply a short-cut and calibrate a risk-premium on collateralised debt with risky underlying assets. Specifically, let \( \Psi(q_t) \) denote a risk premium on secured interbank debt that is assumed to be decreasing in the price of government bonds \( q_t \) (Barro, 1976).

Each bank maximizes a discounted infinite stream of dividend payments to the representative household, using the bank discount factor \( \beta^b \),

\[
\max E_t \left[ \sum_{j=0}^{\infty} \beta^b D_{t+j}^p \right].
\]

Additional to interbank loans, working capital and excess reserves, banks of both types may also invest in the deposit facility of a central bank. Central bank deposits \( R_t^{d,\tau} \) are remunerated at a constant exogenous rate \( r^R \).

Domestic banks form expectations under the paradigm of subjective expected utility (SEU), hence they are assumed to be not ambiguity averse. Banks treat the evolution of TFP from the perspective of an econometrician, i.e. as an AR(1) mean zero process,

\[
z_{t+1} = \rho z_t + \varepsilon_t^z \quad \text{with iid innovations } \varepsilon_t^z \sim \mathcal{N}(0, \sigma^2_z).
\]

Having discussed the available investment opportunities and the objective function of banks \( \tau = \{p, u\} \), we next derive the efficiency conditions for optimal intra-temporal loan decisions and inter-temporal collateral holdings. We start with a representative productive bank. The maximization problem for a type \( p \) banks is

\[
\mathcal{W}^p(s(t)) = \max_{\{B_t^{D,p}, M_t, R_t^e, R_t^{d,p}\}} \quad D_t^p(s(t)) + \beta^b E_t [\mathcal{W}(s(t + 1))],
\]

---

21 This can be interpreted according to domestic banks having more confidence in their own forecasting model such that they are less inclined to consider less tangible information in contrast to foreign investors.
subject to a flow of funds constraint, and a non-negativity constraint for central bank deposits and dividend payments

\[ N^p + M_t = \kappa_t + R_t^p, \]
\[ R_t^{d,p}, D_t^p \geq 0. \]

The term \( s(t) \) is a short cut representation of the state \( s = (B_t, B_t^D, z_t, a_t) \) at period \( t \). Dividend payments of type \( p \) banks amount to

\[ D_t^p = (1 - \delta_t)B_t^{D,p} + (1 + r_t^M)M_t + (1 + R_t^R)R_t^p + (1 + r_t^M)M_t + (1 + R_t^R)R_t^p \]
\[ - (1 - \delta_t)\tilde{q}_tB_{t+1}^{D,p} - N^p, \]  
\[ \text{with } R_t^p = R_t^e + R_t^{d,p}. \]

Writing down the corresponding maximization problem of type \( u \) banks, one obtains

\[ W^u(s(t)) = \max_{\{B_t^{D,u}, M_t, R_t^{d,u}\}} D_t^u(s(t)) + \beta^u E_t[W(s(t + 1))] \]

subject to the flow of funds constraint and non-negativity requirement on central bank deposits and dividend payments

\[ N^u = M_t + R_t^{d,u}, \]
\[ R_t^{d,u}, D_t^u \geq 0. \]

Type \( u \) banks' dividend payments amount to

\[ D_t^u = (1 - \delta_t)B_t^{D,u} + (1 + r_t^M)M_t + (1 + R_t^R)R_t^{d,u} - (1 - \delta_t)\tilde{q}_tB_{t+1}^{D,u} - N^u \]

The continuation value for both types of banks is identical, since the probabilities of types in the consecutive period are independently and identically distributed. Therefore, we obtain

\[ E_t[W(s(t + 1))] = E_t[\pi^p W^p(s(t + 1)) + \pi^u W^u(s(t + 1))]. \]

The efficiency conditions for financial intermediation in the domestic banking sector can be derived from the respective Lagrangian to the optimization problems of banks.
\( \tau = \{p, u\} \). For productive banks, we obtain

\[
\begin{align*}
    r_t^p &= R + \mu_t^p, \\
    r_t^p &= r_t^M + \frac{r_p^c}{\phi} + \lambda_t \chi, \\
    \tilde{q}_t &= \beta^b E_t [W_{BP}(s(t + 1))] + \mu_t^{D,p}.
\end{align*}
\] (4.19)
(4.20)
(4.21)

For unproductive banks, I get

\[
\begin{align*}
    r_t^M &= R + \Psi(q_t) + \mu_t^u, \\
    \tilde{q}_t &= \beta^b E_t [W_{BP}(s(t + 1))] + \mu_t^{D,u}.
\end{align*}
\] (4.22)
(4.23)

With the envelope condition for (4.18), the pricing equation for collateral assets for both types of banks reads

\[
\tilde{q}_t = \beta^b (1 - \pi_t^\delta) + \pi^p \beta^b E_t(\lambda_{t+1}) + \mu_t^D + \mu_t^{BD}.
\] (4.24)

The equilibrium quantities of collateral are found using the no-arbitrage condition \( \tilde{q}_t - q_t = 0 \) where \( q_t \) is the price of government bonds at international capital markets as explained below. Using (4.24), this condition can be re-written as

\[
\beta^b (1 - \pi_t^\delta) + \pi^p \beta^b E_t(\lambda_{t+1}) + \mu_t^D + \mu_t^{BD} - q_t = 0.
\]

### 4.4.5 Government problem

The benevolent government maximizes the infinite life-time utility of the representative household. To do so, it has two decision variables. There is the binary decision to default \( \delta_t \in \{0, 1\} \). In case of debt repayment, the government also takes a borrowing decision, \( B_{t+1} \). Formally, the government objective function reads,

\[
\mathcal{V}_t(s(t)) = \max_{\{\delta_t\}} \left( V_{t}^{nd}, V_{t}^{d} \right),
\] (4.25)

where \( V_{t}^{nd}, V_{t}^{d} \) denote the values under repayment and default, respectively.

The value under repayment is given by

\[
V_{t}^{nd}(s(t)) = \max_{\{c_t, B_{t+1}\}} \left\{ U(c_t, 1 - l_t) + \beta E_t[V_{t+1}(s(t + 1))] \right\},
\] (4.26)
subject to the aggregate resource constraint in the economy
\[ c_t = e^{zt}K_t^{\alpha}t^{1-\alpha} + (B_t + B_{t+1}^D) - q_t(B_{t+1} + B_{t+1}^D) \]
\[ = Y_t - B_t^* + q_tB_t^{*} \]
(4.27)

and the set of partial equilibrium conditions under repayment.

Further, let the value under default be given by
\[ V_d^d(s(t)) = \max_{\{c_t\}} \{U(c_t, 1 - l_t) \}
+ \beta E_t \left[ \left( \theta V_{t+1}^{nd}(0, 0, z_{t+1}, a_{t+1}) + (1 - \theta)V_{t+1}^d(0, 0, z_{t+1}, a_{t+1}) \right) \right] \]
(4.28)

subject to the set of partial equilibrium conditions without interbank trading and the resource constraint under autarky:
\[ c_t = Y_t \]
(4.30)

Conditional on default, the government writes off the entire stock of government debt. The economy falls into autarky with a probability \( \theta \) to return to international and domestic capital markets. With the complement probability \( 1 - \theta \) it will remain in the autarky state in the consecutive period.

The government’s decisions characterise a repayment set \( \Gamma^{nd} \) and a default set \( \Gamma^d \) within the state space:

\[ \Gamma_t^{nd}(B_t, B_{t+1}^D) = \{ s^* = (z_t, a_t) \in s : V_t^{nd}(s(t)) \geq V_t^d(s(t)) \} \]
\[ \Gamma_t^d(B_t, B_{t+1}^D) = \{ s^* = (z_t, a_t) \in s : V_t^d(s(t)) > V_t^{nd}(s(t)) \} \]

The default probability is defined as the conditional cumulative probability density over the productivity and the ambiguity state in period \( t + 1 \) from the default set \( \Gamma^d \), given the future endogenous state \( (B_{t+1}, B_{t+1}^D) \):
\[ \pi_t^d(B_{t+1}, B_{t+1}^D, z_t, a_t) = \int_z^a \int_{\Gamma(B_{t+1}, B_{t+1}^D)} f(z_{t+1}, z_t, a_{t+1}, a_t)dz_{t+1}da_{t+1} \]
(4.31)

### 4.4.6 Ambiguity averse international investors and uncertainty shocks

International investors are modelled in line with the literature on optimal sovereign default with the exception that I assume that they are ambiguity averse. Confronted with ambiguity about the true value of the deterministic component of aggregate productivity...
\(\mu_t^*,\) investors gather less tangible information, e.g. newspaper articles, in order to arrive at an assessment of the deterministic component of aggregate productivity. The degree of ambiguity after considering all information is captured by a numeric value \(a_t\) and summarized in a belief set \(\mathcal{P}_t,\) which collects multiple priors on the conditional mean of \(\mu_t^*.\) This belief set is assumed to be symmetrically centred around zero, \(\mathcal{P}_t = [-|a_t|, |a_t|],\) thus the DGP of productivity from equation (4.8) is constrained by investors’ beliefs to

\[
z_{t+1} = \rho_z z_t + \mu_t + u_{t+1}, \quad \text{with } \mu_t \in [-|a_t|, |a_t|] \tag{4.32}
\]

The set of prior beliefs is affected by uncertainty shocks. Specifically, the boundaries of the belief set are widened as ambiguity increases. An increase in \(|a_t|\) can thereby be interpreted as a loss of confidence in the own forecasting model in response to either a deterioration of the quality of intangible information or an increase in forecast dispersion.\(^{22}\)

The level of uncertainty follows an exogenous AR(1) process, which is known to agents, taking the form

\[
a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \varepsilon_t^a, \tag{4.33}
\]

where \(\bar{a} \geq 0\) denotes the unconditional mean and \(\varepsilon_t^a\) is an iid disturbance with variance \(\sigma_a^2,\) which is uncorrelated with \(u_t\) from (4.32).

Each period, investors are required to choose a specific conditional mean \(\mu_t^p\) from the set of priors in order to form expectations and forecast productivity according to the law of motion of \(z_t\) from equation (4.32). I follow Gilboa and Schmeidler (1989) and Epstein and Schneider (2003), adopting maxmin preferences in recursive notation, which leads investors to select the worst case prior.\(^{23}\) Specifically, the international investor minimizes the expected continuation utility under ambiguity subject to the constraint that the prior \(\mu_t^p\) must lie in the period \(t\) belief set.

Applying the maxmin preference structure to the optimization problem of a risk-neutral representative international investor who is confronted with sovereign default risk yields

\(^{22}\)Note that the model does not structurally explain how \(a_t\) is determined. Instead, I follow Ilut and Schneider (2014) and use available data in order to discipline the evolution of uncertainty according to its law of motion (4.33).

\(^{23}\)The adoption of worst case beliefs, formalized by the maxmin representation of expected utility, can be derived from the axioms of uncertainty aversion and certainty independence, see Epstein (1999) and Epstein and Schneider (2003). The recursive formulation also allows for dynamic consistency of preferences.
the following optimization problem:

$$\max_{\{B_t^*: B_t^* \leq B_t+1\}} \Pi_t^* = -q_t B_t^* + \min_{\{\mu^p \in \mathcal{P}_t\}} E_{t}^p \left( \frac{1 - \delta_{t+1}}{1 + r_f} \right) B_t^*$$

(4.34)

The minimization step in (4.34) requires that the expectations operator with multiple priors $E_t^p$ is evaluated under the prior yielding the lowest outcome to the investor.

**Proposition 8.** Let $p_0$ denote the worst case belief from the perspective of an ambiguity averse international investor. Then, the prior from the set $\mathcal{P}_t$ that minimizes expected utility has mean $\mu^{p_0} = -a_t$.

**Proof.** See Appendix. $\blacksquare$

As a result, the international investor acts as if there is just one belief in the set $\mathcal{P}_t$. The investor uses equation (4.32) to form the forecasting rule for aggregate productivity

$$E_t^p(z_{t+1}) = \rho_z z_t - a_t,$$

(4.35)

where $\mu_t = -a_t$ follows from the worst case belief. The pricing condition for government debt with multiple priors yields

$$q_t(B_t+1, B_t^D, z_t, a_t) = \min_{\{\mu^p \in [-a_t, a_t]\}} E_{t}^p \left( \frac{1 - \delta_{t+1}}{1 + r_f} \right),$$

(4.36)

where the expected default probability $E_t^p(\delta_{t+1}) = \pi_t^p(B_t+1, B_t^D, z_t, a_t)$ is given by (4.31).

### 4.4.7 Equilibrium

The model’s recursive equilibrium is given by

1. Value functions $V, V^{nd},$ and $V^d$ for the government;
2. Policy functions of the government’s default decision $\delta_t \in (0, 1)$, and borrowing $B_t+1$ that maximize the welfare of a representative household;
3. Policy functions for the private sector decisions on $c_t, l_t, M_t, \kappa_t, R_t^\tau$ for $\tau \in (p, u)$;
4. Bank policy functions for the inter-temporal decision on collateral purchases and dividend payments, $B_t^{D,\tau}$, $D_t^{\tau}$ for $\tau \in (p, u)$; and

$^{24}$It can be shown that the default probability of the sovereign is monotonically decreasing in productivity, $\partial \delta_t / \partial z_t < 0$. This is a necessary and sufficient condition such that one can conclude that the minimizing prior is indeed the lowest element in the belief set, $\mu_t = -a_t$, as this minimizes the expected pay-off to the investor.

$^{25}$See the Appendix for a detailed description of the formation of a one period ahead conditional forecast in a discrete state space using numerical methods.
5. A bond pricing schedule $q_t$ of international investors;

such that given the government policies and the bond pricing equation, the household policies for consumption and labour solve the household’s optimization problem; given productivity and credit supply, the policies for labour and credit solve the profit maximization problem of non-financial firms; given the bond pricing schedule and credit demand, the banking sector decisions maximize financial sector dividends and satisfy the market clearing condition on secondary bond markets; the consumption plan $c_t(s(t))$ satisfies the resource constraints of the economy; the transfer policy $T_t(s(t))$ satisfies the government budget constraint; given $\Gamma^D_t(B_t, B^D_t)$ and $\pi^F_t(s(t))$, the bond pricing function $q_t(s(t))$ satisfies the no-arbitrage condition of ambiguity averse foreign investors.

4.5 Simulation results

4.5.1 Calibration

The parameters of the model are found partly by reference to standard values from the literature and partly by estimation through the simulated method of moments (SMM) using Spanish data at a quarterly frequency. In order to discipline the process for uncertainty in the model from equation (4.33), I follow Ilut and Schneider (2014) who use data of forecast disagreement about GDP growth projections. Since the model implies uncertainty about movements in aggregate TFP, I first define the model counterpart of the data series $D_t$. Let $\varepsilon_{gz}$ denote the elasticity of the growth in aggregate output ($y_t$) conditional on a change in aggregate TFP ($z_t$). Then, the model generates a measure of GDP growth forecast disagreement according to

$$D_t = \varepsilon_{gz} 2|a_t|.$$  

Computationally, I obtain $\varepsilon_{gz}$ by a simple log-log regression model on simulated data, $\ln(y_t) = c + \beta \ln(z_t)$, where $\varepsilon_{gz} = \beta$.

Further, note that the disagreement series $D_t$ from the data as in Figure 4.1 provides a measure for the one-year-ahead projection, while the model is calibrated at a quarterly frequency and disagreement arises on one-quarter-ahead growth projections. There might be a systematic difference with respect to disagreement between growth projections of different time horizons. Figure C.3 in the Appendix shows the interquartile range of one-quarter-ahead and four-quarter-ahead projections from the Survey of Professional Forecasters collected on a quarterly basis for the US economy. There is only a slightly higher level of disagreement in the short-run projections with a mean interquartile range
of 1.48 over the period 1964q4 to 2015q1, versus a mean of 1.37 over the longer-run projections. I conclude that there is no big quantitative difference in the disagreement among forecasters over different time horizons and use the moments obtained from the monthly series for the SMM estimation procedure. The disagreement series exhibits mean $\mu(D_t) = 1.039$, a standard deviation of $\sigma(D_t) = 0.3006$ and a persistence of $\rho(D_t) = 0.739$.

The remaining target values for the calibration of the model are taken from Engler and Große Steffen (2014) who use Spanish data for the quantitative model fit. A detailed description and discussion of calibration targets and estimated values can be found there, too. All parameter values are summarized in Table 4.1.

Table 4.1: Baseline calibration

<table>
<thead>
<tr>
<th>Simulated Method of Moments</th>
<th>Baseline</th>
<th>Target statistic</th>
<th>Target value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean level of uncertainty</td>
<td>$\bar{\alpha}$ 0.00645 $\mu(D_t)$</td>
<td>1.0284</td>
<td></td>
</tr>
<tr>
<td>Variance of uncertainty</td>
<td>$\sigma_{\alpha}$ 0.00183 $std(D_t)$</td>
<td>0.3047</td>
<td></td>
</tr>
<tr>
<td>Persistence of uncertainty</td>
<td>$\rho_{\alpha}$ 0.774 $Corr(D_t, D_{t-1})$</td>
<td>0.8659</td>
<td></td>
</tr>
<tr>
<td>Household’s discount factor</td>
<td>$\beta$ 0.92 Default frequency</td>
<td>0.65%</td>
<td></td>
</tr>
<tr>
<td>Capital stock</td>
<td>$K$ 12.07 $K/Y$</td>
<td>3.47</td>
<td></td>
</tr>
<tr>
<td>Working capital requirement</td>
<td>$\eta$ 1.10 $\kappa/Y$</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>Liquid liabilities</td>
<td>$N$ 2.97 $r^h - r^f$</td>
<td>50 bps</td>
<td></td>
</tr>
<tr>
<td>Efficiency parameter</td>
<td>$\phi$ 4.2 $R^e/(\kappa + M + qB^D)$</td>
<td>0.0153</td>
<td></td>
</tr>
<tr>
<td>Probability for lending bank</td>
<td>$\pi^p$ 0.568 $M/L^{banks}$</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Collateral requirement</td>
<td>$\chi$ 0.260 $-B^D/B$</td>
<td>0.55</td>
<td></td>
</tr>
</tbody>
</table>

Calibrated

| Capital share of output      | $\alpha$ 0.36 Standard | $b)$ |
| Risk aversion               | $\gamma$ 2 Standard | $c)$ |
| Wage elasticity of labor supply | $\omega$ 1.455 Standard | $c)$ |
| Risk-free rate              | $r^f$ 0.01 Standard | $c)$ |
| Bank’s discount factor      | $\beta^b$ 0.99 $1/(1 + r^f)$ | $a)$ |
| Probability of reentry      | $\theta$ 0.083 Richmond and Dias (2009) |
| Persistence of TFP shock    | $\rho$ 0.95 Mendoza and Yue (2012) |
| Std.dev. of TFP shock       | $\sigma_c$ 0.017 Mendoza and Yue (2012) |
| Rate on central bank reserves | $r^R$ 0.0077 $r^f - r^R$ |
| Cost function collateral    | $\psi$ 0.245 OLS estimate | $a)$ |

Notes: $a)$ Engler and Große Steffen (2014); $b)$ Smets and Wouters (2003); $c)$ Mendoza and Yue (2012).

### 4.5.2 Pricing and spillover effect of uncertainty shocks

In order to gauge the effect of uncertainty shocks, consider the model with uncertainty shocks as in the baseline calibration. Elevated levels of uncertainty translate through investors’ preferences monotonically into lower bond prices at a given debt level. Figure 4.8
illustrates the effect of uncertainty on bond prices for a fixed level of aggregate productivity. It confirms the typical pricing schedule for government borrowing in quantitative default models (Arellano, 2008). The more a government borrows today, the lower is the price at which a bond can be sold in primary markets, since the probability of repayment shrinks with higher debt levels. An increase in the level of uncertainty $a_t$ shifts the bond price schedule inwards. This has two effects. First, to roll-over the amount of public debt accumulated in previous periods, the government has to pay a higher interest rate. This pricing channel of uncertainty makes the option of servicing debt relatively less valuable today, as the government anticipates the higher burden of debt repayment in the future.

Figure 4.8: Pricing channel of uncertainty shocks

Second, lower bond prices affect equilibrium allocations through the sovereign risk channel on the interbank market. The spillover channel of uncertainty shocks is illustrated in Figure 4.9. At higher levels of ambiguity, the outlook for future debt repayment deteriorates, leading to lower prices for government debt (Panel a). As a consequence, risk premia on collateralised wholesale funding rise such that the interbank rate increases (Panel b). Since banks accumulate excess reserves that imply opportunity costs from interbank borrowing, risk premia on interbank loans translate into higher financing costs for non-financial firms through matched banks. Due to the working capital requirement (4.6), cash-in-advance constrained non-financial firms can hire labour only up to the point where wage bills can be financed through bank lending. Non-financial firms demand less labour at a given wage rate since the wedge between the marginal product of labour and the wage rate in equation (4.7) increases by $w_t \eta_t^x$. As a result, uncertainty shocks
that affect the level of ambiguity are a source of macroeconomic fluctuations in aggregate output.

Figure 4.9: Spillover channel of uncertainty shocks

(a) Collateral value

(b) Loan rates

Note: Values for $a = 0$ interpolated.

The propagation mechanism of uncertainty on sovereign default risk is linked to the endogenous default penalty. As ambiguity on future TFP realisations dampens financial intermediation today, it simultaneously lowers the costs implied by a freeze on the domestic interbank market. As it is less beneficial to service outstanding debt today, the spillover channel amplifies the effect of uncertainty shocks on sovereign default risk beyond its direct implication for the beliefs of international investors.

### 4.5.3 Non-fundamental roll-over crises

This section characterises under which conditions uncertainty is able to induce a non-fundamental roll-over crisis in sovereign debt markets. The previous section laid out how the pricing and spillover channel of uncertainty shocks lower the value in the repayment case, $V^{nd}(s(t))$. Figure 4.10 illustrates the case for which the joint effect of ambiguity leads to non-fundamental default. Note that the value of repayment is a strictly decreasing function in the absolute level of ambiguity, $|a_t|$, as belief sets $P_t$ are symmetric in the realisation $a_t$. According to the composition of the value of repayment today from equation (4.26), the spillover channel affects the contemporaneous utility in the repayment regime, while the pricing channel lowers the continuation value.

The default value $V^d(s(t))$ is equally a decreasing function in the absolute value of
uncertainty $|a_t|$. Intuitively, the continuation value conditional on re-accessing financial markets is higher if the level of uncertainty is lower, such that the component $V_{t+1}^{nd}(0, 0, z_{t+1}, a_{t+1})$ is affected by current realisations of $a_t$ due to the persistence in uncertainty, $\rho_a > 0$. In sum, the sensitivity in the contemporaneous realisation of ambiguity is significantly higher for repayment ($V^{nd}$) than for defaulting ($V^d$).

Figure 4.10 depicts the case where the value of repayment and default intersect for a given economic fundamental $z_t$. The intersection point is characterised by the critical level of uncertainty, which is a function of the fundamental state, $a^*_t(B_t, B^D_t, z_t)$. While the government decides to service the debt if $|a_t| \leq a^*_t$, it defaults if $|a_t| > a^*_t$ given that the default condition $V^d > V^{nd}$ is satisfied. I define non-fundamental default in the quantitative model as follows:

**Definition.** (Non-fundamental default) A non-fundamental default is given in case when there is optimal default under the ambiguity-restricted price schedule at non-zero levels of uncertainty (i), whereas the government services the debt in case there is zero uncertainty (ii). Technically, the conditions for a non-fundamental default in the model are:

\[
(i) \quad s^* = (z_t, a_t \leq 0) \in s : V^d_t(B_t, B^D_t, z_t, a_t) > V^{nd}_t(B_t, B^D_t, z_t, a_t),
\]

\[
\text{and} \quad (ii) \quad s^* = (z_t, a_t = 0) \in s : V^d_t(B_t, B^D_t, z_t, 0) \leq V^{nd}_t(B_t, B^D_t, z_t, 0).
\]

**Figure 4.10: Non-fundamental default**

Note: Value for $a = 0$ interpolated.

Allowing for ambiguity aversion leads to a structural interpretation of an investors’ run in the presence of multiple equilibria as discussed in Cole and Kehoe (2000). Typically,
models with strategic complementarities for investors’ behaviour as in the seminal bank-run model by Diamond and Dybvig (1983) require an exogenous sunspot event to determine which equilibrium materialises, the run or the roll-over equilibrium. One alternative to address the problem of equilibrium refinement is provided by the literature on global games (Corsetti et al., 2006; Morris and Shin, 2006). Maxmin preferences in the presence of multiple-priors present a further mechanism for equilibrium selection in sovereign debt markets. As shown, the problem of multiplicity of equilibria disappears conditional on the materialisation of uncertainty, although the multiple-prior model inherently alludes to the fact that different outcomes might be possible. However, maxmin preferences on the side of investors and the fact that the level of ambiguity $a_t$ is public information aligns default expectations of different agents in the model economy as investors act as if there is only one belief in their set of possible priors. As a result, there exists a unique Markov perfect equilibrium in the model with ambiguity aversion conditional on the level of uncertainty (Auclert and Rognlie, 2015).

Figure 4.11 illustrates the default set conditional on the level of uncertainty along the productivity dimension (Panel a), and the domestic debt dimension (Panel b). The dark shaded area indicate a default for sure at any level of uncertainty. The grey shaded area is the repayment set, again no matter what level of uncertainty materialises.

Figure 4.11: Crisis zone

![Crisis zone](image)

Note: Preliminary simulation results computed using a quite coarse grid.

In contrast, the yellow coloured area in between the default and the repayment sets is the region where default is pinned down by uncertainty: A high realisation of $|a_t|$ that exceeds the critical value $a^*_t$ triggers outright default, while lower levels of uncertainty make debt repayment the optimal policy choice (cf. Figure 4.10). Following Cole and Kehoe (2000), I
denote this area as the crisis zone. If the economy is in the crisis zone, a non-fundamental
default might occur in a sense that worst case beliefs of international investors induce
outright sovereign default on public debt, conditional on sufficiently elevated levels of
ambiguity regarding the fundamental state.

It is the presence of a crisis zone in a model of strategic sovereign default, augmented
with ambiguity averse investors, that allow for the rationalization of the empirical impulse
responses obtained in section 4.2. A country that is in or close to the crisis zone, as defined
in the theoretical model, will exhibit a much stronger response in sovereign credit risk
and bank lending rates to a one-off increase in the level of uncertainty than a country
far outside the crisis zone. Therefore, developments of sovereign yields in the Euro area
can partly be explained by distinguishing between countries in or close to the crisis zone,
typically labelled as periphery, versus the core countries that are further from the crisis
zone.

4.5.4 Quantitative results

This section analyses the quantitative implications of uncertainty shocks for business
cycle fluctuations. Using a production economy with financial frictions as in Engler and
Große Steffen (2014), it is possible to treat time-varying levels of uncertainty about the
fundamental state of the economy as a source for structural shocks that are propagated
into equilibrium allocations. This dimension of the analysis is absent in related studies,
e.g. in Costa (2009) and Pouzo and Presno (2013), who look at robust control preferences
in pure endowment economies.

I simulate the model 2000 times over a period of 864 quarters. I subtract the first
one hundred observations as a burn-in period. The remaining 191 years of simulated data
represent the time span for which historical default frequencies are available (Sturzenegger
and Zettelmeyer, 2007). Further, 48 periods prior to a default are isolated and used for
the computation of statistical moments produced by the model. This time-span covers the
membership of the Spanish economy to the European Monetary Union. Table 4.2 presents
the moments obtained by averaging over all numerical simulations. The first column
presents the results of the baseline model, which was estimated using the interdecile range
of point forecasts over GDP growth. For a comparative static analysis, columns (2) and
(3) present a regime with lower uncertainty and absent financial frictions, respectively,
leaving all remaining parameters unchanged.

The results show that the lower mean level of ambiguity $\bar{a}$ is related to a higher prob-
ability of default as reported by the default frequency. While this result might seem
counter-intuitive at first glance, it is due to four effects in the quantitative model that are
partly off-setting each other and that are being discussed next. First, the pricing channel of uncertainty shocks increases the probability of a roll-over crisis as the government finds it more attractive to default in times of high uncertainty. Second, the spillover channel lowers the default penalty from a freeze on the interbank market, hence amplifying sovereign default risk.

While these two effects are direct implications from the propagation mechanism of uncertainty shocks discussed in the previous section, there are two general equilibrium effects that both affect default risk in the opposite direction. Specifically, there is a debt composition effect related to the holding of public debt by domestic banks. Since domestic banks are not ambiguity averse, a higher share of total government debt is held domestically when uncertainty rises which can be seen by combining equation (4.24) with equation (4.36). Since government debt has a shadow value for domestic banks due to its collateral role to obtain funding liquidity, the difference in valuation of government debt between foreign and domestic bond holders increases even further. Thus, more debt is held domestically in economies with a higher mean level of uncertainty. The default decision of the government is affected by the debt composition effect which leads to an overall negative contribution to default risk. The second general equilibrium effect is due to a precautionary motive on the side of the government to lower total debt issuance when confronted with higher mean levels of uncertainty. Table 4.3 summarizes the effects at play in the model and their contribution to overall sovereign default risk.

Column (4) of Table 4.2 contains the case where spillovers from sovereign credit risk to the interbank market are shut down. This is obtained by setting the risk-premium $\psi$ to zero, as well as the need to build excess reserves on the side of banks. As discussed previously, the spillover channel renders an economy vulnerable for non-fundamental debt

\[ \text{Equation (4.24)} \]

\[ \text{Equation (4.36)} \]

\[ \text{(c.p.)} \]

\[ \psi = 0 \]

\[ \phi = 10^3 \]

Notes: Statistics with an asterisk denote calibration targets.

---

26 This is a feature discussed under the notion of re-patriation of sovereign debt, see e.g. Bratti and Sauré (2013).
Table 4.3: Uncertainty shocks and default risk

<table>
<thead>
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<th></th>
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<tr>
<td>Spillover channel</td>
<td>↑</td>
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<tr>
<td>Debt composition effect</td>
<td>↓</td>
</tr>
<tr>
<td>Precautionary motive</td>
<td>↓</td>
</tr>
</tbody>
</table>

Figure 4.12: Uncertainty premium in the run-up to default

(a) Levels

(b) Share of uncertainty

Note: Model averages over all simulation paths.

crises in the presence of uncertainty shocks by an expansion of the crisis zone. This mechanism gives rise to the precautionary motive on the side of the government. Consequently, shutting down the spillover channel incentives the government to accumulate more debt, leading to a higher frequency of default.

Next, the estimated model, in its baseline calibration, is used to decompose the yield on sovereign debt in a fundamental share and an uncertainty premium. Figure 4.12 analyses the uncertainty premium in the run-up to a default event. The illustration contrasts the annualised yields on sovereign debt over all model simulations with the counter-factual yield obtained when assuming that there is no uncertainty ($a = 0$), holding all other elements of the state space constant. Interestingly, the share of the sovereign yield that is due to macroeconomic uncertainty is higher in relatively tranquil times when still distant to a crisis, and gets smaller when approaching the default event. Quantitatively, the share of the yield attributed to uncertainty shocks drops in the simulations from around 60 percent in tranquil times to around 50 percent one quarter prior to default (Panel b).
CHAPTER 4. NON-FUNDAMENTAL DEBT CRISIS

4.6 Conclusion

Macroeconomic uncertainty, as measured by forecast disagreement about future GDP growth, is positively correlated with yields on sovereign credit default swaps (CDS). An empirical VAR analysis shows that uncertainty shocks have a positive and economically significant effect on sovereign credit risk. At the same time, there is a dichotomy in the effect of uncertainty shocks in the Euro area, as sovereign credit risk in core countries like Germany and the Netherlands do not react as much to uncertainty shocks.

At the backdrop of these facts, this paper develops a theory of sovereign debt crises driven by uncertainty shocks which are modelled as changes in investors’ confidence in the macroeconomic fundamental of the economy. Ambiguity averse investors form worst case beliefs in line with the multiple-priors model of Gilboa and Schmeidler (1989). In a setting with strategic sovereign default where repayment decisions of a benevolent government depend on the state of the business cycle, uncertainty shocks feed into investors’ beliefs about debt sustainability. As a result, uncertainty shocks increase the price of issuing debt, thus affecting the optimal fiscal plan of the government. Since the benefit from international borrowing is lowered, there is less debt issuance in times of high macroeconomic uncertainty.

At a critical levels of indebtedness, uncertainty shocks can induce non-fundamental roll-over crises. Given the same level of aggregate productivity, the government may find it optimal to refuse the costly contracts offered by international investors to roll-over debt and default in times of high uncertainty, while it would decide to repay at lower levels of uncertainty that coincide with more favourable financial contracts. It is shown that default expectations under ambiguity aversion are closely related and partly endogenize sunspot driven self-fulfilling debt crises as in Cole and Kehoe (2000).

The mechanism of non-fundamental default is further analysed in a quantitative DSGE model with strategic default, ambiguity averse international investors and an endogenous penalty mechanism through a bank-sovereign nexus. Spanish data at the quarterly frequency is used to estimate the model. In particular, the interdecile range of forecast disagreement over GDP growth projections allows to discipline the process of uncertainty shocks.

A novel propagation mechanism of uncertainty shocks to the macro economy arises from the spillover channel associated with a liquidity role of government debt in the financial sector of the domestic economy. Financial market outcomes, business cycle fluctuations and endogenous sovereign default risk are jointly affected by time-varying degrees of uncertainty on the future macroeconomic fundamental. Thereby, the spillover channel
makes the economy vulnerable to non-fundamental default risk. Simulation exercises show that higher levels of uncertainty lead to lower default risk due to a precautionary motive of the benevolent government as a general equilibrium outcome. A decomposition of sovereign yields assigns a sizeable share to an uncertainty premium.

The theoretical and empirical results suggest that taking account of macroeconomic uncertainty may be important for the explanation of sovereign debt crises. In particular, the existence of a crisis zone can rationalise non-fundamental uncertainty premia on sovereign debt.

In future research, I would like to address implications of uncertainty premia for the effectiveness of policy interventions.
Appendix to Chapter 2
A.1 Cyclicality of capital inflows

Figure A.1: Correlations of gross capital inflows with the business cycle.

(a) Emerging market economies
Figure A.1: [continued]
(a) Emerging market economies (ii)
Figure A.1: [continued]

(b) Developed economies

Notes: GDP in percentage deviations from HP-trend, GCI are in deviations from HP-trend. Source: See data appendix.
A.2 Prior and posterior plots

Figure A.2: Prior and posterior plots: Financial sector model
A.3 Data

A.3.1 Country sample

Table A.1: Country sample

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<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Classification</th>
<th>Source</th>
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<td>Advanced</td>
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<td>IMF</td>
</tr>
<tr>
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<td>IMF</td>
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<td>1989q1 - 2011q1</td>
<td>Emerging</td>
<td>IMF</td>
</tr>
</tbody>
</table>

Notes: Data comes from IMF’s International Financial Statistics and OECD’s Quarterly National Accounts. Classification of countries evolves according to Aguiar and Gopinath (2007) where applicable. European New Member States are grouped as emerging countries.
A.3.2 Gross capital flows

Gross capital flows for the panel of 33 countries are constructed as the sum over three positions of the IMF Balance of Payment (BOP) database, namely foreign direct investment in the country, portfolio investment liabilities, and other investment liabilities. All data is obtained via datastream (codes %I78BEDA, %I78BGDA, and %I78BIDA). Gross capital flows are in quarterly frequency and reported in millions of US dollars. Hence, I deflate all series with the US deflator.

I obtain output data and relevant GDP deflators for the country panel mainly from the IMF International Financial Statistics database. Data for output was deflated and de-seasonalized if necessary. If the series was not available for the sample length of gross capital inflows, I took output data in real terms from the OECD Quarterly National Accounts (see Table A.1 for details).

I extract the cyclical component of gross capital inflows and from the logs of output by applying the HP-filter with smoothing parameter $\lambda = 1600$.

The OECD’s Risk Classification are reported on quarterly basis and available online. For the sample of 33 countries, I construct an average country specific risk classification as a simple average over the period 2001:I to 2010:IV.

A.3.3 Mexican business cycle statistics

Data for output, consumption, investment and the trade balance are all from the OECD Quarterly National Accounts Statistics and obtained via datastream. They are available on a real and seasonally adjusted basis. Consumption is ”private consumption” and investment ”gross fixed capital formation”. The trade balance is constructed as ”exports” net of ”imports of goods and services”.

In order to replicate the interest rate series from Uribe and Yue (2006), I use data from the Federal Reserve of St Louis to replicate the world interest rate ($R_t$). Specifically, I use the average 3 month US Treasury bill rate and subtract the average implied inflation rate from the US GDP deflator over the previous four quarters. The series on the country premium ($S_{ct}$) is obtained from the EMBI+ stripped spreads which is provided by JPMorgan.

I obtain data for the Mexican financial system from the website of the Mexican banking and securities commission (CNBV). Data is chosen from the universal banking category. The bank capital ratio follows the classification ’ICAP total’ which is a capital ratio based

\footnotesize
\begin{itemize}
\item 27 \url{http://www.oecd.org/document/49/0,3746,en_2649_34169_1901105_1_1_1_1_00.html}
\item 28 \url{https://research.stlouisfed.org/fred2/categories/}
\item 29 \url{http://portafoliodeinformacion.cnbv.gob.mx/bm1/Paginas/alertas.aspx}
\end{itemize}
on risk weighted assets. This ratio translates into the model counterpart of the bank leverage ratio as $ICAP^{-1} \times 100 = \phi$. Data is available at a monthly frequency. I use end-of-period values to obtain a quarterly series. For the foreign funding ratio, I construct the ratio of total liabilities over liabilities from foreign financial institutions. Specifically, I take the ratio of data category code SF99049 over the category code SF99055.
A.4 Model appendix

A.4.1 The financial sector model

A.4.1.1 Derivation of optimality conditions in stationary form

Households

Optimization.

\[
\max_{\{C_t,h_t,D_t\}} \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{C_t - \omega^{-1} \chi (\Gamma_{t-1} h_t)^\omega}{1 - \gamma} \right)^{1-\gamma} - 1 \right]
\]

s.t.

\[
C_t + D_t = w_t h_t + D_{t-1} \Gamma_t^d + \Pi_t
\]

Stationarization.

\[
\max_{\{C_t,h_t,D_t\}} \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{C_t \Gamma_{t-1} - \omega^{-1} \chi (\Gamma_{t-1} h_t)^\omega}{1 - \gamma} \right)^{1-\gamma} - 1 \right]
\]

s.t.

\[
\frac{D_t}{\Gamma_{t-1}} + \frac{C_t}{\Gamma_{t-1}} = \frac{w_t}{\Gamma_{t-1}} h_t + \frac{D_{t-1}}{\Gamma_{t-2}} \Gamma_{t-1}^d + \frac{\Pi_t}{\Gamma_{t-1}}
\]

Lagrangian.\(^{30}\)

\[
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \zeta_t \left( \frac{C_t - \omega^{-1} \chi h_t^\omega}{1 - \gamma} \right)^{1-\gamma} + \lambda_t \left( \frac{w_t}{\Gamma_{t-1}^\omega} h_t - c_t - d_t + d_{t-1} g_{t-1} \Gamma_{t-1}^d + \pi_t \right) \right\}
\]

FOCs.

\[
\zeta_t \left( C_t \Gamma_{t-1} - \omega^{-1} \chi h_t^\omega \right)^{-\gamma} = \lambda_t \quad \text{(A.4.1.1)}
\]

\[
\zeta_t \left( C_t \Gamma_{t-1} - \omega^{-1} \chi h_t^\omega \right)^{-\gamma} ( -\chi h_t^{-\omega -1} ) = -\lambda_t \left( \frac{w_t}{\Gamma_{t-1}^\omega} \right) \quad \text{(A.4.1.2)}
\]

\[
\lambda_t g_t = \beta \mathcal{L}_t \left[ \lambda_{t+1} \Gamma_{t+1}^d \right] \quad \text{(A.4.1.3)}
\]

\(^{30}\)Using the definitions \( g_t \equiv \frac{\Gamma_t}{\Gamma_{t-1}} \) and \( x_t \equiv \frac{\Gamma_t}{\Gamma_{t-1}}. \)
Combined FOCs.

\[ \chi h_t^{w-1} = \frac{w_t}{\Gamma_{t-1}} \]  
(A.4.1.4)

\[ g_t = E_t \beta [ \Lambda_{t,t+1} R^d_{t+1} ] \]  
(A.4.1.5)

\[ \Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \]  
(A.4.1.6)

Non-Financial Firms Optimization.

\[ Y_t = a_t K_t^\alpha (\Gamma_t h_t)^{1-\alpha} \]
\[ q_t S_t = q_t K_{t+1} \]

\[
\max_{\{K_{t+1}, h_t\}} E_t \sum_{t=0}^{\infty} Y_t - w_t h_t - R^k_t q_{t-1} S_{t-1} + q_t (1 - \delta) K_t - q_t K_{t+1} + q_t S_t
\]

substituting \( Y_t \) and \( S_{t-1} \) from (A.4.1.1) and (A.4.1.1)

\[
\max_{\{K_{t+1}, h_t\}} E_t \sum_{t=0}^{\infty} a_t K_t^\alpha (\Gamma_t h_t)^{1-\alpha} - w_t h_t - R^k_t q_{t-1} K_t + q_t (1 - \delta) K_t - q_t K_{t+1} + q_t S_t
\]

Stationarization.

\[ \frac{Y_t}{\Gamma_{t-1}} = a_t \left( \frac{K_t}{\Gamma_{t-1}} \right)^\alpha \left( \frac{\Gamma_t}{\Gamma_{t-1}} h_t \right)^{1-\alpha} \]

\[ y_t = a_t k_t^\alpha (g_t h_t)^{1-\alpha} \]
\[ q_t S_t = q_t \frac{K_t + 1}{\Gamma_{t-1} \Gamma_t} \]

\[ s_t = k_{t+1} g_t \]

\[
\max_{\{K_{t+1}, h_t\}} E_t \sum_{t=0}^{\infty} a_t \left( \frac{K_t}{\Gamma_{t-1}} \right)^\alpha \left( \frac{\Gamma_t}{\Gamma_{t-1}} h_t \right)^{1-\alpha} - \frac{w_t}{\Gamma_{t-1}} h_t - R^k_t q_{t-1} K_t \\
+ q_t (1 - \delta) \frac{K_t}{\Gamma_{t-1}} - q_t K_{t+1} \frac{\Gamma_t}{\Gamma_{t-1} \Gamma_t} + q_t S_t
\]

\[ \Leftrightarrow \max_{\{K_{t+1}, h_t\}} E_t \sum_{t=0}^{\infty} a_t k_t^\alpha g_t^{1-\alpha} h_t^{1-\alpha} - \frac{w_t}{\Gamma_{t-1}} h_t - R^k_t q_{t-1} k_t + q_t (1 - \delta) k_t - q_t k_{t+1} g_t + q_t s_t \]
FOCs.

\[(1 - \alpha) a_t k_t^{\alpha} g_t^{1-\alpha} h_t^{-\alpha} = \frac{w_t}{\Gamma_{t-1}} \]  
(A.4.1.7)

\[\alpha a_t k_t^{\alpha-1} g_t^{1-\alpha} h_t^{1-\alpha} + q_t (1 - \delta) = R_t^k q_{t-1} \]  
(A.4.1.8)

Rewrite (A.4.1.8) and iterate one period forward to obtain the law of motion for the return on capital:

\[E_t \left[ R_{t+1}^k \right] = E_t \left[ \frac{\alpha \left( \frac{y_{t+1}}{k_{t+1}} \right) + q_{t+1} (1 - \delta)}{q_t} \right] \]  
(A.4.1.9)

Capital Producer Optimization.

\[
\max_{\{K_{t+1}, I_t\}} \ E_t \sum_{t=0}^{\infty} \beta^t \left\{ \Lambda_{t,t+1} \left[ q_t K_{t+1} - q_t (1 - \delta) K_t - I_t \right] \right\}
\]

s.t.

\[K_{t+1} = (1 - \delta) K_t + I_t - \frac{\phi_k}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t \]

Stationarization.

\[
\max_{\{K_{t+1}, I_t\}} \ E_t \sum_{t=0}^{\infty} \beta^t \left\{ \Lambda_{t,t+1} \left[ q_t \frac{K_{t+1}}{\Gamma_{t-1}} - q_t (1 - \delta) \frac{K_t}{\Gamma_{t-1}} - \frac{I_t}{\Gamma_{t-1}} \right] \right\}
\]

s.t.

\[\frac{K_{t+1} \Gamma_t}{\Gamma_{t-1} \Gamma_t} = (1 - \delta) \frac{K_t}{\Gamma_{t-1}} + \frac{I_t}{\Gamma_{t-1}} - \frac{\phi_k}{2} \left( \frac{K_{t+1} / \Gamma_{t-1}}{K_t / \Gamma_{t-1}} - g \right)^2 \frac{K_t}{\Gamma_{t-1}} \]

\[\Leftrightarrow \ k_{t+1} g_t = (1 - \delta) k_t + i_t - \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} - g \right)^2 k_t \]

\[\Leftrightarrow \ i_t = k_{t+1} g_t - (1 - \delta) k_t + \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} - g \right)^2 k_t \]

Substituting \(i_t\) into the objective function yields

\[
\max_{\{k_{t+1}\}} \ E_t \sum_{t=0}^{\infty} \beta^t \left\{ \Lambda_{t,t+1} \left[ q_t k_{t+1} g_t - q_t (1 - \delta) k_t - k_{t+1} g_t + (1 - \delta) k_t - \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} - g \right)^2 k_t \right] \right\}
\]
FOCs.

\[ q_t g_t = g_t + \phi_k \left( \frac{k_{t+1}}{k_t} g_t - g \right) \]

\[ - \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \]  

(A.4.1.10)

Financial Intermediaries

Bank balance sheets are given by

\[ q_t S_t = N_t + B_t + D_t \]

Stationarization.

\[ q_t \frac{S_t}{\Gamma_{t-1}} = \frac{N_t}{\Gamma_{t-1}} + \frac{B_t}{\Gamma_{t-1}} + \frac{D_t}{\Gamma_{t-1}} \]

\[ \Leftrightarrow q_t s_t = n_t + b_t + d_t \]  

(A.4.1.11)

Optimization.

Bank net worth evolves according to

\[ N_t = R_t^k q_{t-1} s_{t-1} - R_t^b B_{t-1} - R_t^d D_{t-1} \]

Stationarization.

\[ \frac{N_t}{\Gamma_{t-1}} = R_t^k q_{t-1} \frac{S_{t-1}}{\Gamma_{t-1}} - R_t^b B_{t-1} \frac{\Gamma_{t-1}}{\Gamma_{t-1}} - R_t^d D_{t-1} \frac{\Gamma_{t-1}}{\Gamma_{t-1}} \]

\[ \Leftrightarrow \frac{N_t}{\Gamma_{t-1}} = R_t^k q_{t-1} \frac{S_{t-1}}{\Gamma_{t-1}} \frac{\Gamma_{t-2}}{\Gamma_{t-2}} - R_t^b B_{t-1} \frac{\Gamma_{t-1}}{\Gamma_{t-1}} \frac{\Gamma_{t-2}}{\Gamma_{t-2}} - R_t^d D_{t-1} \frac{\Gamma_{t-1}}{\Gamma_{t-1}} \frac{\Gamma_{t-2}}{\Gamma_{t-2}} \]

\[ \Leftrightarrow n_t = R_t^k q_{t-1} s_{t-1} g_{t-1} - R_t^b b_{t-1} g_{t-1} - R_t^d d_{t-1} g_{t-1} \]  

(A.4.1.12)

Substituting deposits \( d_{t-1} \) from the balance sheet

\[ n_t = R_t^k q_{t-1} s_{t-1} g_{t-1} - R_t^b b_{t-1} g_{t-1} - R_t^d (q_{t-1} s_{t-1} - n_{t-1} - b_{t-1}) g_{t-1} \]

\[ = R_t^k q_{t-1} s_{t-1} g_{t-1} + (R_t^d - R_t^b) b_{t-1} g_{t-1} - R_t^d (q_{t-1} s_{t-1} - n_{t-1}) g_{t-1} \]

\[ = (R_t^k - R_t^d) q_{t-1} s_{t-1} g_{t-1} + (R_t^d - R_t^b) \frac{b_{t-1}}{q_{t-1} s_{t-1}} q_{t-1} s_{t-1} g_{t-1} + R_t^d n_{t-1} g_{t-1} \]

\[ = [R_t^k - R_t^d + \varphi_{t-1} (R_t^d - R_t^b)] q_{t-1} s_{t-1} g_{t-1} + R_t^d n_{t-1} g_{t-1} \]  

(A.4.1.13)
Recursive notation of bank optimization problem.

\[ V_{t-1}(s_{t-1}, \varphi_{t-1}, n_{t-1}) = E_t(1 - \sigma)n_{t-1} + \sigma \max_{n_{t-1}, \varphi_{t-1}} \{ V_t(s_t, \varphi_t, n_t) \} \]

s.t.

\[ n_t = [R^k_t - R^d_t + \varphi_{t-1}(R^d_t - R^k_t)] q_{t-1} s_{t-1} g_{t-1} + R^d_t n_{t-1} g_{t-1} \]

and the incentive constraint

\[ V_t(s_t, \varphi_t, n_t) \geq \Theta(\varphi_t, \vartheta_t, \nu_t) q_t s_t \]

with

\[ \Theta(\varphi_t) = \theta (\vartheta_t + \nu_t \varphi_t) . \]

Define the following terms:

\[ \Omega_{t+1} = 1 - \sigma + \sigma \omega_{t+1} \]

\[ \eta^s_t = E_t(\Lambda_{t+1} \Omega_{t+1}(R^k_t - R^d_t)) g_t \]

\[ \eta^b_t = E_t(\Lambda_{t+1} \Omega_{t+1}(R^d_t - R^k_t)) g_t \]

\[ \nu_t = E_t(\Lambda_{t+1} \Omega_{t+1} R^d_t g_t) \]

Using the definitions, the optimization problem of the bank can be rewritten as

\[ \omega_t n_t = \max_{\{n_t, \varphi_t\}} \left[ (\eta^s_t + \varphi \eta^b_t) q_t s_t + \nu_t n_t \right] \]

s.t.

\[ (\eta^s_t + \varphi \eta^b_t) q_t s_t + \nu_t n_t \geq \Theta(\varphi_t, \vartheta_t, \nu_t) q_t s_t \]

Lagrangian.

\[ \mathcal{L} = (1 + \lambda^B_t) \left[ (\eta^s_t + \varphi \eta^b_t) q_t s_t + \nu_t n_t \right] - \lambda^B_t \Theta(\varphi_t, \vartheta_t, \nu_t) q_t s_t \]

FOCs.

\[ (1 + \lambda^B_t) (\eta^s_t + \varphi \eta^b_t) = \lambda^B_t \Theta(\varphi_t, \vartheta_t, \nu_t) \]

\[ (1 + \lambda^B_t) \eta^b_t = \lambda^B_t \Theta'(\varphi_t, \vartheta_t, \nu_t) s_t \]

Combined FOCs.

\[ \frac{\Theta(\varphi_t, \vartheta_t, \nu_t)}{\eta^s_t + \varphi \eta^b_t} = \frac{\Theta'(\varphi_t, \vartheta_t, \nu_t) s_t}{\eta^b_t} \]
\[ \theta (\vartheta_t + \nu_t \varphi_t) + \eta_b \varphi_t + \nu_t s_t = \eta_b s_t \]

\[ \vdots \]

\[ \varphi_t = \frac{\eta_b^t \vartheta_t - \eta_b^t \nu_t s_t}{\nu_t \eta_b^t (s_t - 1)} \]

With incentive constraint at equality

\[ (\eta_b^t + \varphi_t \eta_b^t) q_t s_t + \nu_t n_t = \theta (\vartheta_t + \nu_t \varphi_t) q_t s_t \]

Substitute for \( q_t s_t \) from the definition of the leverage ratio twice and using the definition of the incentive constraint \( \Theta(\varphi_t, \vartheta_t, \nu_t) \) to obtain

\[ (\eta_b^t + \varphi_t \eta_b^t) \varphi_t n_t + \nu_t n_t = \Theta(\varphi_t, \vartheta_t, \nu_t) \varphi_t n_t \]

\[ \varphi_t = \frac{\nu_t}{\Theta(\varphi_t, \vartheta_t, \nu_t) - (\eta_b^t + \varphi_t \eta_b^t)} \]

**Value of a unit of net worth.** Substituting the definition of the leverage ratio into the recursive bank problem yields the value of a unit of net worth to the banker

\[ \omega_t n_t = (\eta_b^t + \varphi_t \eta_b^t) \varphi_t n_t + \nu_t n_t \]

\[ \varphi_t = (\eta_b^t + \varphi_t \eta_b^t) \varphi_t + \nu_t \]

**Aggregation.**

\[ n_t = n^e_t + n^n_t \]
with

\[ n_t^c = \sigma \left\{ \left[ R_t^k - R_t^d + \varphi_{t-1}(R_t^d - R_t^b) \right] q_t s_{t-1} g_{t-1} + R_t^d n_{t-1} g_{t-1} \right\} \]

\[ n_t^n = \varrho (1 - \sigma) R_t^b q_{t-1} s_{t-1} g_{t-1} \]

\[ = \varrho (1 - \sigma) \left[ \frac{\alpha y_t}{k_t} + (1 - \delta) q_t \right] q_{t-1} s_{t-1} g_{t-1} \]

\[ = \varrho (1 - \sigma) \left[ \alpha \frac{y_t}{k_t} + (1 - \delta) q_t \right] s_{t-1} g_{t-1} \]

such that aggregate net worth evolves according to

\[ n_t = \sigma \left\{ \left[ R_t^k - R_t^d + \varphi_{t-1}(R_t^d - R_t^b) \right] q_t s_{t-1} + R_t^d n_{t-1} \right\} + \varrho (1 - \sigma) R_t^b q_{t-1} s_{t-1} g_{t-1} \]

**Market clearing and further definitions**

*Labour market clearing.*

\[ (1 - \alpha) a_t k_t^{\alpha} g_t^{1-\alpha} h_t^{-\alpha} = \chi h_t^{\omega-1} \]

*Net profit transfers from capital producers and banks to households.*

\[ \pi_t = i_t (q_t - 1) - \frac{\phi_t}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t q_t + \varrho (1 - \sigma) q_t s_{t-1} \]

*Aggregate resource constraint.*

\[ y_t = c_t + i_t - R_t^b b_{t-1} g_{t-1} + b_t \]

*Trade balance to output-ratio.*

\[ tby_t = 1 - \frac{c_t + i_t}{y_t} \]

*Growth rate*

\[ g_t \equiv \frac{\Gamma_t}{\Gamma_{t-1}} \]

*Foreign interest rate*

\[ R_t^b = S_t^c R_t^a + \psi (e^{b_t - \bar{b}} - 1) \]

**Exogenous processes**

\[ \ln(\zeta_t) = \rho_\zeta \ln(\zeta_{t-1}) + \varepsilon_t^\zeta, \]

\[ \ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a, \]
\[
\begin{align*}
\ln(g_t) &= (1 - \rho_g)\ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g, \\
\ln(\vartheta_t) &= \rho_{\vartheta} \ln(\vartheta_{t-1}) + \varepsilon_t^\vartheta, \\
\ln(\nu_t) &= (1 - \rho_\nu)\ln(\nu) + \rho_\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu, \\
\ln(R_t^\star) &= (1 - \rho_{R^\star})R^\star + \rho_{R^\star}R_{t-1}^\star + \varepsilon_t^{R^\star}, \\
\ln(S_t^c) &= (1 - \rho_{S^c})S^c + \rho_{S^c}S_{t-1}^c + \varepsilon_t^{S^c},
\end{align*}
\]

with \(\varepsilon_t^{x,i.i.d.} \sim \mathcal{N}(0, \sigma_x^2)\) for \(x \in (a, g, \zeta, \nu, \vartheta R^\star, S^c, \).

### A.4.1.2 Non-linear dynamic equilibrium conditions

We are looking for a set of non-linear equations that define an equilibrium for the predetermined endogenous variables \(\{d_{t+1}, k_{t+1}, b_{t+1}\}\), the endogenous contemporaneous variables \(\{c_t, h_t, s_t, R_t^d, R_t^k, q_t, y_t, i_t, n_t, n_t^c, n^\psi_t, \varphi_t, \Omega_t, \eta^r_t, \eta^b_t, \nu_t, \omega_t, \vartheta_t, \lambda_t, \Lambda_t, tby_t, \Theta_t(\varphi_t, \vartheta_t, \nu_t)\}\), and the exogenous processes \(\{\zeta_t, a_t, g_t, \vartheta_t, \nu_t, R_t^\star, S_t^c\}\).

\[
\begin{align*}
\zeta_t (c_t - \omega^{-1} \chi h_t^\omega)^{-\gamma} &= \lambda_t \\
\lambda_t g_t &= \beta E_t [\lambda_{t+1} R_{t+1}^d] \\
\Lambda_{t,t+1} &= \frac{\lambda_{t+1}}{\lambda_t} \\
(1 - \alpha) a_t (k_t)^\alpha g_t^{1 - \alpha} h_t^{-\alpha} &= \chi h_t^{\omega - 1} \\
R_t^k &= \frac{\alpha \left( \frac{y_t}{R_t^k} \right) + q_t (1 - \delta)}{q_{t-1}} \\
q_t s_t &= q_t k_{t+1} g_t \\
y_t &= a_t k_t^\alpha g_t^{1 - \alpha} h_t^{-\alpha} \\
k_{t+1} g_t &= (1 - \delta) k_t + i_t - \frac{\phi_k}{2} \left( \frac{k_{t+1} g_t - g}{k_t} \right)^2 k_t \\
q_t g_t &= g_t + \phi_k \left( \frac{k_{t+1} g_t - g}{k_t} \right) g_t \\
-E_t \beta \Lambda_{t,t+1} \left[ (1 - \delta) - q_{t+1} (1 - \delta) - \phi_k \left( \frac{k_{t+2}}{k_t} g_{t+1} - g \right) \frac{k_{t+2}}{k_t} g_{t+1} - \frac{\phi_k}{2} \left( \frac{k_{t+2}}{k_t} g_{t+1} - g \right)^2 \right] \\
c_t &= y_t - i_t - R_t^b b_{t-1} g_{t-1} + b_t \\
tby_t &= \frac{c_t + i_t}{y_t} \\
R_t^b &= S_t^c R_t^\star + \psi(e^{b_t - b} - 1) \\
q_t s_t &= \phi_t n_t \\
\phi_t &= \frac{v_t g_t}{\Theta(\varphi_t) - (\eta^r_t + \varphi_t \eta^b_t) g_t}
\end{align*}
\]
\begin{equation}
\Theta(\varphi_t, \vartheta_t, \nu_t) = \theta (\vartheta_t + \nu_t \varphi_t) \nonumber
\end{equation}
\begin{equation}
q_t s_t = n_t + b_t + d_t \nonumber
\end{equation}
\begin{equation}
n_t = n_t^c + n_t^n \nonumber
\end{equation}
\begin{equation}
n_t^n = \sigma \left\{ \left[ R_t^k - R_t^d + \varphi_{t-1} (R_t^d - R_t^b) \right] q_{t-1} g_{t-1} + R_t^d n_{t-1} g_{t-1} \right\} \nonumber
\end{equation}
\begin{equation}
n_t^c = \varphi_t \left( n_t^c + n_t^n \right) \nonumber
\end{equation}
\begin{equation}
q_t s_t = n_t + b_t + d_t \nonumber
\end{equation}
\begin{equation}
\varpi_t = (n_t^c + \varphi_t n_t^b) \phi_t g_t + \nu_t g_t \nonumber
\end{equation}
\begin{equation}
\Omega_{t+1} = 1 - \sigma + \sigma \varpi_{t+1} \nonumber
\end{equation}
\begin{equation}
n_t^c = E_t \beta \left[ \Lambda_{t,t+1} \Omega_{t+1} (R_t^k - R_t^d) \right] g_t \nonumber
\end{equation}
\begin{equation}
n_t^b = E_t \beta \left[ \Lambda_{t,t+1} \Omega_{t+1} (R_t^d - R_t^b) \right] g_t \nonumber
\end{equation}
\begin{equation}
\nu_t = E_t \beta \left[ \Lambda_{t,t+1} \Omega_{t+1} \right] R_t^d g_t \nonumber
\end{equation}
\begin{equation}
\varphi_t = \frac{n_t^b \vartheta_t - n_t^c \nu_t s_t}{\nu_t n_t^b (s_t - 1)} \nonumber
\end{equation}
\begin{equation}
\varphi_t = \frac{b_t}{q_t s_t} \nonumber
\end{equation}
\begin{equation}
\ln(r_t) = \rho_r \ln(r_{t-1}) + \varepsilon^r_t \nonumber
\end{equation}
\begin{equation}
\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon^a_t \nonumber
\end{equation}
\begin{equation}
\ln(g_t) = (1 - \rho_g) \ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon^g_t \nonumber
\end{equation}
\begin{equation}
\ln(\delta_t) = \rho_\delta \ln(\delta_{t-1}) + \varepsilon^\delta_t \nonumber
\end{equation}
\begin{equation}
\ln(\upsilon_t) = (1 - \rho_\upsilon) \ln(\upsilon) + \rho_\upsilon \ln(\upsilon_{t-1}) + \varepsilon^\upsilon_t \nonumber
\end{equation}
\begin{equation}
\ln(R_t^c) = (1 - \rho_{R^c}) R^c + \rho_{R^c} R_{t-1}^c + \varepsilon^R_t \nonumber
\end{equation}
\begin{equation}
\ln(S_t^c) = (1 - \rho_{S^c}) S^c + \rho_{S^c} S_{t-1}^c + \varepsilon^S_t \nonumber
\end{equation}

**A.4.1.3 Steady state**

In the stochastic steady state, the following variables are normalised to one: $\bar{a}, \bar{\xi}, \bar{\vartheta}, \bar{\zeta}, \bar{\mu}$. Further, $\bar{g} = 1 + \bar{g}/100$ is a parameter and $\bar{A} = 1$ and $\bar{Q} = 1$ in the deterministic steady state.

Given are also the numerical values for calibration targets $\bar{h}$, the world interest rate and the country spread, $\bar{R}^*, \bar{S}^c$, and the long-run return on capital $\bar{R}^k$. The calibration targets are used to pin down the parameters $\chi$ and $\varphi$.

The following conditions arise directly from the non-linear equilibrium conditions:

\begin{equation}
\bar{R}^d = \frac{g}{\beta} \nonumber
\end{equation}
\begin{equation}
\bar{R}^b = \bar{S}^c \bar{R}^* \nonumber
\end{equation}
With steady state labour of $\bar{h} = 0.33$, the capital-labour ratio is derived from the equilibrium condition and the calibration target for the return on assets $\bar{R}^k$

$$\bar{R}^k = \alpha \left( \frac{\bar{k}}{\bar{g} \bar{h}} \right)^{\alpha - 1} + (1 - \delta)$$

$$\iff \left( \frac{\bar{k}}{\bar{g} \bar{h}} \right) = \left[ \frac{\bar{R}^k - (1 - \delta)}{\alpha} \right]^{\frac{1}{\alpha - 1}}$$

Hence, the parameter $\chi$ follows endogenously from

$$\chi = \left( \frac{\bar{g} \bar{h}}{\bar{h} \omega - 1} \right)^{\alpha} (1 - \alpha) \bar{g}$$

Further one gets

$$\bar{y} = \left( \frac{\bar{k}}{\bar{g} \bar{h}} \right)^{\alpha} \bar{g} \bar{h}$$

$$\bar{k} = \frac{\bar{k}}{\bar{g} \bar{h}} \bar{g} \bar{h}$$

$$\bar{s} = \bar{k} \bar{g}$$

$$\bar{i} = \bar{k} (g - 1 + \delta)$$

To obtain the liability structure of banks, we substitute equilibrium conditions for $\eta^b$, $\eta^s$ and $\Omega$ into the condition for the ratio of foreign funding which then takes the value

$$\bar{\varphi} = \frac{(\bar{R}^d - \bar{R}^b) - \bar{s} \bar{\varphi} (\bar{R}^k - \bar{R}^d)}{\bar{\nu} (\bar{s} - 1) (\bar{R}^d - \bar{R}^b)}$$

such that the steady state amount of foreign funding is given by $\bar{b} = \bar{\varphi} \bar{s}$ and the tightness of financial frictions yields $\bar{\Theta} = \theta (1 + \bar{\nu} \bar{\varphi})$.

Next, we use two equilibrium conditions for the variables $\{ \bar{\omega}, \bar{\phi} \}$ and substitute once more for $\eta^b$, $\eta^s$ and $\Omega$ which leads to the following system

$$\bar{\phi} = \frac{\beta [(1 - \sigma) + \sigma \bar{\omega}] \bar{R}^d \bar{g}}{\theta (1 + \nu \bar{\varphi}) - \{ \beta [(1 - \sigma) + \sigma \bar{\omega}] (\bar{R}^k - \bar{R}^d) \bar{g} + \bar{\varphi} \beta [(1 - \sigma) + \sigma \bar{\omega}] (\bar{R}^d - \bar{R}^b) \bar{g} \}$$

$$\bar{\omega} = \{ \beta [(1 - \sigma) + \sigma \bar{\omega}] (\bar{R}^k - \bar{R}^d) \bar{g} + \bar{\varphi} \beta [(1 - \sigma) + \sigma \bar{\omega}] (\bar{R}^d - \bar{R}^b) \bar{g} \} \bar{\varphi} + \beta [(1 - \sigma) + \sigma \bar{\omega}] \bar{R}^d \bar{g}$$

Plugging the condition for $\bar{\phi}$ into the second equation yields an equation which can be solved for $\bar{\omega}$ numerically and which is used to find the steady state value for $\bar{\phi}$. The
remaining steady state values are then obtained as follows

\[
\bar{n} = \frac{s}{\phi} \\
\bar{d} = \bar{s} - \bar{n} - \bar{b} \\
\bar{c} = \bar{y} - \bar{i} + (1 - \bar{R}^b \bar{g})\bar{b} \\
t by = 1 - \frac{\bar{c} + \bar{i}}{\bar{y}} \\
\bar{\lambda} = (\bar{c} - \omega^{-1}\bar{R}^\omega)\gamma^{-1} \\
\Omega = (1 - \sigma) + \sigma\bar{w} \\
\bar{v} = \beta\Omega\bar{R}^d \bar{g} \\
\bar{\eta}^s = \beta\Omega(\bar{R}^k - \bar{R}^d)\bar{g} \\
\bar{\eta}^b = \beta\Omega(\bar{R}^d - \bar{R}^b)\bar{g}
\]

Finally, the missing endogenous parameter \( \varrho \) is obtained from the law of motion of aggregate net worth

\[
\varrho = \frac{\bar{n} - \sigma \left\{ \left[ \bar{R}^k - \bar{R}^d + \bar{\varphi}(\bar{R}^d - \bar{R}^b) \right] \bar{s}\bar{g} + \bar{R}^d\bar{n}\bar{g} \right\}}{(1 - \sigma)\bar{R}^k\bar{s}\bar{g}}
\]

A.4.2 Debt elastic interest rate model

The following model builds upon the small open economy model by García-Cicco et al. (2010) with a debt elastic foreign interest rate.

A.4.2.1 Derivation of optimality conditions in stationary form

Household

Optimization.

\[
\max_{\{C_t,h_t,D_t,K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{[C_t - \omega^{-1}\chi(\Gamma_t h_t)\omega]^{1-\gamma} - 1}{1 - \gamma} \right) \right]
\]

s.t.

\[
Y_t + D_t = C_t + I_t + \frac{\phi_k}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t + D_{t-1} \bar{R}^b_t \\
Y_t = a_t K_t^\alpha (\Gamma_t h_t)^{1-\alpha} \\
K_{t+1} = (1 - \delta)K_t + I_t
\]
Stationarization.

\[
\max_{\{C_t, h_t, D_t, K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{[C_t - \omega^{-1} \lambda \chi_t h_t^\omega]^{1-\gamma} - 1}{1 - \gamma} \right) \right]
\]

s.t.

\[
Y_t \frac{d_t}{\Gamma_{t-1}} + D_t + \frac{I_t}{\Gamma_{t-1}} + \frac{\phi_k}{2} \left( \frac{K_{t+1}/\Gamma_{t-1} - \Gamma_{t-1}/\Gamma_t - g}{K_t/\Gamma_{t-1} - \Gamma_{t-1}/\Gamma_t} \right)^2 \frac{K_t}{\Gamma_{t-1}} + D_{t-1} \frac{\Gamma_{t-2} R_t^b}{\Gamma_{t-1} \Gamma_{t-2}} = \frac{Y_t}{\Gamma_{t-1}} = a_t \left( \frac{K_t}{\Gamma_{t-1}} \right)^\alpha \left( \frac{\Gamma_t h_t}{\Gamma_{t-1}} \right)^{1-\alpha}
\]

\[
\Leftrightarrow y_t + d_t = c_t + \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t + d_{t-1} g_{t-1} R_t^b
\]

\[
K_{t+1} \frac{\Gamma_t}{\Gamma_{t-1}} = (1 - \delta) \frac{K_t}{\Gamma_{t-1}} + I_t \frac{\Gamma_t}{\Gamma_{t-1}}
\]

\[
\Leftrightarrow k_{t+1} = (1 - \delta) k_t + i_t
\]

Lagrangian\(^{31}\)

\[
\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \zeta_t \left( \frac{[c_t - \omega^{-1} \lambda \chi_t h_t^\omega]^{1-\gamma} - 1}{1 - \gamma} \right) + \lambda_t \left( a_t k_t^\alpha (g_t h_t) - \right)^{1-\alpha} \right. + d_t - c_t - k_{t+1} g_t + (1 - \delta) k_t - \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t - d_{t-1} g_{t-1} R_t^b \right\}
\]

FOCs.

\[
\zeta_t (c_t - \omega^{-1} \chi_t h_t^\omega)^{-\gamma} = \lambda_t
\]

\[
\zeta_t (c_t - \omega^{-1} \chi_t h_t^\omega)^{-\gamma} \chi_t h_t^{-\omega-1} = \lambda_t (1 - \alpha) a_t k_t^\alpha (g_t h_t)^{-\alpha} g_t
\]

\[
\lambda_t = E_t \beta \left[ \lambda_{t+1} g_t R_{t+1}^b \right]
\]

\[
\lambda_t \left[ g_t + \phi_k \left( \frac{k_{t+1}}{k_t} g_t - g \right) g_t \right] =
\]

\[
\beta E_t \left[ \lambda_{t+1} \left\{ (1 - \delta) + \alpha a_t \left( \frac{g_{t+1} h_{t+1}}{k_{t+1}} \right)^{1-\alpha} + \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \phi_k \frac{2}{(\frac{k_{t+2}}{k_{t+1}} g_{t+1} - g)^2} \right\} \right]
\]

Further definitions

Trade balance to output-ratio.

\[
thy_t = 1 - \frac{c_t + i_t}{y_t}
\]

\(^{31}\)Using the definitions \(g_t \equiv \frac{R_t}{\Gamma_{t-1}}\) and \(x_t \equiv \frac{K_t}{\Gamma_{t-1}}\).
Growth rate

\[ g_t = \frac{\Gamma_t}{\Gamma_{t-1}} \]

Foreign interest rate rule

\[ R_t^b = R_t^b + \psi(e^{a_t - \bar{a}} - 1) + e^{\mu_t - 1} - 1 \]

Exogenous processes

\[
\begin{align*}
\ln(a_t) &= \rho_a \ln(a_{t-1}) + \varepsilon_t^a \\
\ln(g_t) &= (1 - \rho_g) \ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \\
\ln(\zeta_t) &= \rho_\zeta \ln(\zeta_{t-1}) + \varepsilon_t^\zeta \\
\ln(\mu_t) &= (1 - \rho_\mu) \ln(\mu) + \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu
\end{align*}
\]

with \( \varepsilon_t^x \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_x^2) \) for \( x \in (a, g, \zeta, \mu) \).

A.4.2.2 Non-linear dynamic equilibrium conditions

A solution to the non-linear system of equations in the variables \((y_t, h_t, d_t, c_t, k_{t+1}, i_t, R_t^b, \lambda_t, tby_t)\) and exogenously given processes for \((a_t, g_t, \zeta_t, \mu_t)\) is given by the following system of equations:

\[
\begin{align*}
y_t &= a_t k_t^\alpha (g_t h_t)^{1-\alpha} \\
y_t + d_t &= c_t + i_t + \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t + d_{t-1} g_{t-1} R_t^b \\
k_{t+1} g_t &= (1 - \delta) k_t + i_t \\
\zeta_t (c_t - \omega^{-1} \chi h_t^\omega)^{-\gamma} \chi h_t^{\omega-1} &= \lambda_t (1 - \omega) a_t k_t^\alpha (g_t h_t)^{-\alpha} g_t \\
\lambda_t &= E_t \beta \left[ \lambda_{t+1} g_t R^b_{t+1} \right] \\
\lambda_t \left[ g_t + \phi_k \left( \frac{k_{t+1}}{k_t} g_t - g \right) g_t \right] &= \\
\beta E_t \left[ \lambda_{t+1} \left\{ (1 - \delta) + \alpha a_{t+1} \left( \frac{g_{t+1} h_{t+1}}{k_{t+1}} \right)^{1-\alpha} + \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) \frac{k_{t+2}}{k_{t+1}} g_{t+1} - \frac{\phi_k}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \right\} \right] \\
tby_t &= 1 - \frac{c_t + i_t}{y_t} \\
R_t^b &= R_t^b + \psi(e^{a_t - \bar{a}} - 1) + e^{\mu_t - 1} - 1 \\
\ln(a_t) &= \rho_a \ln(a_{t-1}) + \varepsilon_t^a \\
\ln(g_t) &= (1 - \rho_g) \ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g
\end{align*}
\]
\[
\ln(\zeta_t) = \rho_{\zeta}\ln(\zeta_{t-1}) + \varepsilon^\zeta_t \\
\ln(\mu_t) = (1 - \rho_{\mu})\ln(\mu) + \rho_{\mu}\ln(\mu_{t-1}) + \varepsilon^\mu_t
\]

A.4.2.3 Steady state

In the deterministic steady state, we know that \(\bar{h} = 0.33\) and \(\bar{R}^b = R^*S^c\). Further, the shocks are standardized to \(\bar{a} = \bar{\zeta} = \bar{\mu} = 1\), whereas \(\bar{g} = 1 + \bar{g}/100\). Then, the steady state is obtained recursively as

\[
\frac{\bar{g} \bar{h}}{\bar{k}} = \left[ \frac{\bar{g} - (1 - \delta)}{\alpha} \right]^{1/\alpha}
\]

\[
\chi = \frac{(1 - \alpha) \left( \frac{\bar{g} \bar{h}}{\bar{k}} \right)^{-\alpha} \bar{g}}{\bar{h}^{\omega - 1}}
\]

\[
\bar{h} \bar{k} = \bar{g} \bar{h} \frac{1}{\bar{k}} \frac{\bar{g}}{\bar{y}}
\]

\[
\bar{k} = \left( \frac{\bar{h}}{\bar{k}} \right)^{-1} \bar{h}
\]

\[
\bar{y} = \bar{k}^{\alpha} (\bar{g} \bar{h})^{1 - \alpha}
\]

\[
\bar{i} = \bar{k} \left[ \bar{g} - (1 - \delta) \right]
\]

The steady state value of household debt is calibrated to yield a targeted value for the trade balance to output ratio, denoted by \(\bar{t}\bar{b}y\), which is obtained from combining the household budget constraint and the definition of the trade balance to output ratio:

\[
\bar{d} = \frac{\bar{t}\bar{b}y\bar{y}}{(1 - \bar{d}R^b)}
\]

Then, the remaining steady state values are given by

\[
\bar{c} = \bar{y} + \bar{d} - \bar{i} - \bar{d}\bar{g}\bar{R}^b
\]

\[
\bar{\lambda} = (\bar{c} - \omega^{-1} \chi \bar{h}^\omega)^{-\gamma}
\]

A.4.3 Endogenous country spread model

The following model builds upon the small open economy model by Chang and Fernández (2013) with an endogenous country spread and a working capital requirement.
A.4.3.1 Derivation of optimality conditions in stationary form

Households

Optimization.

$$\max_{\{C_t, h_t, D_t, K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{[C_t - \omega^{-1} \chi h_t]^{1-\gamma} - 1}{1-\gamma} \right) \right]$$

s.t.

$$R^h_t K_t + w_t h_t + D_t = C_t + D_{t-1} R^h_t + K_{t+1} - (1-\delta)K_t + \frac{\phi_k}{2} \left( \frac{K_{t+1}}{K_t} - g \right)^2 K_t$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

Stationarization.

$$\max_{\{C_t, h_t, D_t, K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t \left( \frac{[C_t - \omega^{-1} \chi h_t]^{1-\gamma} - 1}{1-\gamma} \right) \right]$$

s.t.

$$R^h_t \frac{K_t}{\Gamma_{t-1}} + \frac{w_t}{\Gamma_{t-1}} h_t + \frac{D_t}{\Gamma_{t-1}} = C_t + \frac{D_{t-1} \Gamma_{t-2}}{\Gamma_{t-1} \Gamma_{t-2}} R^h_t + \frac{K_{t+1}}{\Gamma_{t-1}} \Gamma_{t-1} - (1-\delta) \frac{K_t}{\Gamma_{t-1}} + \frac{\phi_k}{2} \left( \frac{K_{t+1}/\Gamma_{t-1}}{\Gamma_t} \Gamma_{t-1} - g \right)^2 \frac{K_t}{\Gamma_{t-1}}$$

$$\Leftrightarrow R^h_t k_t + \frac{w_t}{\Gamma_{t-1}} h_t + d_t = c_t + d_{t-1} g_{t-1} R^h_t + k_{t+1} g_t - (1-\delta)k_t + \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t$$

$$\frac{K_{t+1} \Gamma_{t-1}}{\Gamma_t} = (1-\delta) \frac{K_t}{\Gamma_{t-1}} + \frac{I_t}{\Gamma_{t-1}}$$

$$\Leftrightarrow k_{t+1} g_t = (1-\delta)k_t + i_t$$

Lagrangian.

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \zeta_t \left( \frac{[c_t - \omega^{-1} \chi h_t]^{1-\gamma} - 1}{1-\gamma} \right) + \lambda_t \left[ R^h_t k_t + \frac{w_t}{\Gamma_{t-1}} h_t + d_t - c_t ight. ight.$$

$$- \left. d_{t-1} g_{t-1} R^h_t - k_{t+1} g_t + (1-\delta)k_t - \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t \right\}$$
FOCs.

\[
\zeta_t (c_t - \omega^{-1} \chi h_t^\omega)^{-\gamma} = \lambda_t
\]

\[
\zeta_t (c_t - \omega^{-1} \chi h_t^\omega)^{-\gamma} \chi h_t^{\omega - 1} = \lambda_t \frac{w_t}{\Gamma_{t-1}}
\]

\[
\lambda_t = E_t \beta \left[ \lambda_{t+1} g_t R_{t+1}^b \right]
\]

\[
\lambda_t \left[ g_t + \phi_k \left( \frac{k_{t+1}}{k_t} g_t - g \right) g_t \right] = \beta E_t \left[ \lambda_{t+1} \left\{ R_{t+1}^k + (1 - \delta) \right\} 
\]

\[+ \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) \frac{k_{t+2}}{k_{t+1}} + \frac{\phi_k}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \right\}
\]

Non-Financial Firms

Optimization.

\[
\max_{\{K_t, h_t\}} E_t \sum_{t=0}^{\infty} Y_t - w_t h_t \left[ 1 + \tau (R_t^b - 1) \right] - R_t^k K_t
\]

s.t. \( Y_t = a_t K_t^\alpha (\Gamma_t h_t)^{1-\alpha} \)

Stationarization.

\[
\max_{\{K_t, h_t\}} E_t \sum_{t=0}^{\infty} Y_t \frac{w_t}{\Gamma_{t-1}} h_t \left[ 1 + \tau (R_t^b - 1) \right] - R_t^k K_t \frac{\Gamma_t}{\Gamma_{t-1}}
\]

\[\Leftrightarrow \max_{\{h_t, h_t\}} E_t \sum_{t=0}^{\infty} y_t \frac{w_t}{\Gamma_{t-1}} h_t \left[ 1 + \tau (R_t^b - 1) \right] - R_t^k k_t
\]

s.t. \( \frac{Y_t}{\Gamma_{t-1}} = a_t \left( \frac{K_t}{\Gamma_{t-1}} \right)^\alpha \left( \frac{\Gamma_t}{\Gamma_{t-1}} h_t \right)^{1-\alpha} \)

\[\Leftrightarrow y_t = a_t k_t^\alpha (g_t h_t)^{1-\alpha} \]

FOCs.

\[
(1 - \alpha) a_t k_t^\alpha (g_t h_t)^{-\alpha} g_t = \frac{w_t}{\Gamma_{t-1}} \left[ 1 + \tau (R_t^b - 1) \right]
\]

\[
\alpha a_t k_t^{\alpha - 1} (g_t h_t)^{1-\alpha} = R_t^k
\]

Further definitions

Labour market clearing.

\[
\chi h_t^{\omega - 1} = \frac{(1 - \alpha) a_t k_t^\alpha (g_t h_t)^{-\alpha} g_t}{\left[ 1 + \tau (R_t^b - 1) \right]}
\]
Trade balance to output-ratio.

\[ \text{tby}_t = 1 - \frac{c_t + i_t}{y_t} \]

Growth rate

\[ g_t \equiv \frac{\Gamma_t}{\Gamma_{t-1}} \]

Foreign interest rate rule

\[ R_t^b = S^c_t R^s + \psi \left( e^{d_t - \bar{d}} - 1 \right) + e^{\mu_t - 1} - 1 \]

Endogenous country spread

\[ \ln \left( \frac{S^c_t}{S^c} \right) = -\eta E_t \left[ \ln \left( \frac{SR_{t+1}}{SR} \right) \right] \]

Solow residual

\[ SR_t = a_t g_t^{1-\alpha} \]

Exogenous processes

\[ \ln(\zeta_t) = \rho_\zeta \ln(\zeta_{t-1}) + \varepsilon_t^\zeta \]
\[ \ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \]
\[ \ln(g_t) = (1 - \rho_g) \ln(g_t) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \]
\[ \ln(\mu_t) = (1 - \rho_\mu) \ln(\mu_t) + \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu \]

with \( \varepsilon_t^x \sim i.i.d. \mathcal{N}(0, \sigma_x^2) \) for \( x \in \{a, g, \zeta, \mu\} \).

### A.4.3.2 Non-linear dynamic equilibrium conditions

We are looking for a set of non-linear equations that define an equilibrium for the predetermined endogenous variable \( k_{t+1} \), the endogenous contemporaneous variables \( c_t, h_t, d_t, y_t, i_t, tby_t, R^b_t, S^c_t, SR_t \), and the exogenous processes \( a_t, g_t, \zeta_t, \mu_t \).

\[ \zeta_t \left( c_t - \omega^{-1} \chi h_t^{\omega-1} \right) - \gamma = \lambda_t \]
\[ \chi h_t^{\omega-1} = \frac{(1 - \alpha) a_t k_t^{\alpha} (g_t h_t)^{-\alpha} g_t}{[1 + \tau (R^p_t - 1)]} \]
\[ \lambda_t = E_t \beta \left[ \lambda_{t+1} g_t R^b_t \right] \]
\[ \lambda_t \left[ g_t + \phi_k \left( \frac{k_{t+1}}{k_t} g_t - g \right) g_t \right] = \beta E_t \left[ \lambda_{t+1} \left\{ \alpha a_{t+1} k_t^{\alpha-1} (g_{t+1} h_{t+1})^{1-\alpha} + (1 - \delta) \right\} \right] \]
APPENDIX CHAPTER 2

\[ + \phi_k \left( \frac{k_{t+2}}{k_{t+1}} \right) g_{t+1} - g \right) \frac{k_{t+2}}{k_{t+1}} + \phi_k \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \right] \]

\[ y_t = a_t k_t^\alpha (g_t h_t)^{1-\alpha} \]

\[ k_{t+1} g_t = (1 - \delta) k_t + \delta_t \]

\[ \alpha a_t k_t^{\alpha-1} (g_t h_t)^{1-\alpha} k_t + \frac{(1 - \alpha) a_t k_t^\alpha (g_t h_t)^{-\alpha} g_t}{1 + \tau(R_b^t - 1)} h_t + d_t \]

\[ = c_t + d_{t-1} g_{t-1} R_b^t + k_{t+1} g_t - (1 - \delta) k_t + \frac{\phi_k}{2} \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t \]

\[ R_b^t = S_t^c R^c + \psi \left( e^{d_t - \bar{d}} - 1 \right) + e^{\mu_t} - 1 \]

\[ \ln \left( \frac{S_t^c}{S^c} \right) = - \eta E_t \left[ \ln \left( \frac{SR_{t+1}}{S^c} \right) \right] \]

\[ SR_t = a_t g_t^{1-\alpha} \]

\[ tby_t = 1 - c_t + i_t \]

\[ \ln(\zeta_t) = \rho \varsigma \ln(\zeta_{t-1}) + \varepsilon_t^{\zeta} \]

\[ \ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \]

\[ \ln(g_t) = (1 - \rho_g) \ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \]

\[ \ln(\mu_t) = (1 - \rho_\mu) \ln(\mu) + \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu \]

### A.4.3.3 Steady state

In the deterministic steady state, we set \( \bar{h} = 0.33 \) and \( \bar{R}^b = R^c S^c \). Further, the shocks are standardized to \( \bar{a} = \bar{\zeta} = \bar{\mu} = 1 \), whereas \( \bar{g} = 1 + \bar{g}/100 \). The steady state is then computed recursively according to

\[ \frac{\bar{g} \bar{h}}{\bar{k}} = \left[ \left( \frac{\bar{g}}{\bar{g}} - (1 - \delta) \right) \frac{1}{\alpha} \right]^{1/\alpha} \]

\[ \chi = \frac{(1 - \alpha) \left( \frac{\bar{g} \bar{h}}{\bar{k}} \right)^{1-\alpha}}{\bar{k}^{\omega-1} \left[ 1 + \tau(R_b^t - 1) \right]} \]

\[ \bar{h} \]

\[ \bar{k} = \left( \frac{\bar{h}}{\bar{k}} \right)^{-1} \bar{h} \]

\[ \bar{g} = \bar{k}^\alpha (\bar{g} \bar{h})^{1-\alpha} \]

\[ \bar{i} = \bar{k} [\bar{g} - (1 - \delta)] \]

\[ \bar{d} = \frac{tby \bar{g}}{(1 - \bar{g} R_b^t)} \]
\[ \bar{c} = \alpha \bar{k}^{\alpha - 1}(\bar{g} \bar{h})^{1-\alpha} \bar{k} + \frac{(1 - \alpha) \bar{k}(\bar{g} \bar{h})^{-\alpha} \bar{g} - \bar{d}(1 - \bar{g} \bar{R}^b) + \bar{k} [(1 - \delta) - \bar{g}]}{1 + \tau (\bar{R}^b - 1)} \]

\[ \bar{\lambda} = (\bar{c} - \omega^{-1} \bar{k}^\omega)^{-\gamma} \]
Appendix to Chapter 3

B.1 Proofs

Proof of Proposition 1. In order that a representative bank for each type exists, the
proof has to show (i) that aggregated collateral holding is identical to the probability
mass \( \pi^\tau \); and (ii) that the distribution of collateral within each type \( \tau \) does not affect
individual decisions.

Step (i) By assumption it holds that each bank retains an identical amount of liquid
financial resources from dividend payments and collateral purchases, i.e. for \( i \neq j \) it holds
that \( N_i^\tau = N_j^\tau \) for all \( \tau \in \{p, u\} \). It follows that \( \int_0^{\pi^\tau} N_i^\tau \, di = \pi^\tau N \). Returns on investment
opportunities are equal across idiosyncratic banks, hence each bank takes identical choices
over credit and interbank allocations given identical holdings of collateral assets and en-
dowment of liquid assets \( N_j^\tau \). Although \( D^p_{t=0} \neq D^u_{t=0} \), it follows from the independent and
identically distributed funding shock \( \tau \in \{p, u\} \) jointly with the law of large numbers that
\( \int_0^{\pi^\tau} B_{t;i}^{D,\tau} \, di = \pi^\tau B^D_t \). Step (ii) From the linearity in the profit maximisation problem of
banks follows that individual decisions are independent of the distribution of collateral.

Proof of Proposition 2. The proof is fulfilled by the first-order-necessary condition on
bond purchases by banks in the model equilibrium according to equation (B.3.3).

Proof of Proposition 3. The proof is derived from the profit maximisation problem
of risk-neutral international investors. The expected pay-off on bonds is denoted by
\( E_t(1 - \delta_{t+1}) \), which is discounted by a risk-free interest rate \( r^f \). International investors
maximise expected returns on investment, subject to the market clearing condition on
secondary markets from equation (3.9)

$$\max_{\{B_{t+1}^D\}} \Pi^*_t = \tilde{q}_t B_{t+1} + B_{t+1}^* E_t \left( \frac{1 - \delta_{t+1}}{1 + r_f} \right),$$

s.t. 
\begin{align*}
B_{t+1} + B_{t+1}^D + B_{t+1}^* &= 0, \\
B_{t+1}^D &\geq 0.
\end{align*}

The first-order condition for bond supply of international investors yields the no-arbitrage condition

$$\tilde{q}_t = E_t \left( \frac{1 - \delta_{t+1}}{1 + r_f} \right) + \mu_t^{BD},$$

where the Lagrange multiplier on the constraint $\mu_t^{BD}$ is zero due to the price-taking behaviour of investors on secondary markets. Using equation (3.32) yields the condition $\tilde{q}_t = q_t$, which implies a perfectly elastic supply of bonds from international investors at a secondary market price $\tilde{q}_t$ which is identical to the primary market price $q_t$.

**Proof of Proposition 4.** Denote the set of possible productivity realisations by $A_t \in \mathcal{A}$ and an endogenous state by $\epsilon^* \equiv (B^n_t, \bar{B}^D_t)$, where $n$ indicates different values of total debt that satisfy the condition $|B^n_t| \geq \bar{B}^D_t$. From state $\epsilon^2 \in \Gamma^d_t$ follows

$$V^d_t(\epsilon^2, A_t) = U(C_t, 1 - L_t) + \beta E_t \left[ \theta V^{nd}_{t+1}(0, 0, A_{t+1}) + (1 - \theta) V^d_{t+1}(0, 0, A_{t+1}) \right]$$

$$> U(C_t, 1 - L_t) + \beta E_t \left[ V_{t+1}(B^2_{t+1}, \bar{B}^D_{t+1}, A_{t+1}) \right] = V^{nd}_t(\epsilon^2, A_t).$$

**Claim:** The default value is independent of the endogenous state, i.e.

$$V^d_t(\epsilon^1, A_t) = V^d_t(\epsilon^2, A_t) \forall A_t \in \mathcal{A}$$

**Proof of Claim:** The independence of equilibrium allocations under default from $\epsilon_t$ follows directly from the definition of the partial equilibrium under default in Section 3.3.6 in combination with the aggregate resource constraint (4.30).

Next, consider a state $\epsilon^1_t$ such that $B^1_t < B^2_t$. Trivially, this implies $B^1_t + \bar{B}^D_t < B^2_t + \bar{B}^D_t \leq 0$ from the market clearing condition on secondary markets (3.9). Conditional on the repayment equilibrium, it follows from the aggregate resource constraint that for any pair of choices $\{B_{t+1}, B^D_{t+1}\}$ that satisfies the conditions above one obtains $C_t(\epsilon^1_t, A_t) < C_t(\epsilon^2_t, A_t)$. This implies from the equilibrium condition for labour supply from equation
(3.2) that the marginal disutility from labour is higher in the state with low total debt:

\[
\frac{\partial U_t}{\partial L_t}(\epsilon_t^2) = -\frac{\partial U_t}{\partial L_t}(\epsilon_t^1) \quad \text{and} \quad \frac{\partial U_t}{\partial C_t}(\epsilon_t^1) = -\frac{\partial U_t}{\partial L_t}(\epsilon_t^1)
\]

Thus

\[
\frac{\partial U_t}{\partial C_t}(\epsilon_t^2) < \frac{\partial U_t}{\partial C_t}(\epsilon_t^1)
\]

\[
\Rightarrow -\frac{\partial U_t}{\partial L_t}(\epsilon_t^2) > -\frac{\partial U_t}{\partial L_t}(\epsilon_t^1)
\]

where the conclusion is derived from the decreasing marginal rate of utility from consumption given the standard properties of the utility function from Section 4.5.1. Hence, 

\[V_t^{nd}(B_t^d, \tilde{B}_t^D, A_t) > V_t^{nd}(B_t^1, \tilde{B}_t^D, A_t),\]

and therefore \(\epsilon_t^1 = (B_t^1, \tilde{B}_t^D) \in \Gamma_t^\delta \forall A_t \in A : \epsilon_t^2 \in \Gamma_t^\delta.

\[\text{Proof of Proposition 5.}\]

This proof closely follows the steps in the proof of Proposition 4.

Denote the set of possible productivity realisations by \(A_t \in A\) and an endogenous state by \(\epsilon_t^n = (\tilde{B}_t, B_t^{D,n})\) where \(n\) is an index for values of domestic debt. From state \(\epsilon_t^2 \in \Gamma_t^\delta\) follows

\[V_t^d(\epsilon_t^2, A_t) = U(C_t, 1 - L_t) + \beta E_t \left[\theta V_{t+1}^{nd}(0, 0, A_{t+1}) + (1 - \theta)V_{t+1}^{d}(0, 0, A_{t+1})\right] > U(C_t, 1 - L_t) + \beta E_t \left[V_{t+1}(\tilde{B}_{t+1}, B_{t+1}^{D,2}, A_{t+1})\right] = V_{t+1}(\epsilon_t^1, A_t).

Next, consider a state \(\epsilon_t^1\) such that \(B_t^{D,1} < B_t^{D,2} \leq |\tilde{B}_t|\), which implies \(\tilde{B}_t + B_t^{D,1} < \tilde{B}_t + B_t^{D,2} \leq 0\). Conditional on the repayment equilibrium, it follows that for any pair of choices \(\{B_{t+1}, B_{t+1}^{D}\}\) one obtains \(C_t(\epsilon_t^1, A_t) < C_t(\epsilon_t^2, A_t)\). This implies from the equilibrium condition for labour supply from equation (3.2) that the marginal disutility from labour is lower in the state with higher domestic debt:

\[
\frac{\partial U_t}{\partial L_t}(\epsilon_t^2) = -\frac{\partial U_t}{\partial L_t}(\epsilon_t^1) \quad \text{and} \quad \frac{\partial U_t}{\partial C_t}(\epsilon_t^1) = -\frac{\partial U_t}{\partial L_t}(\epsilon_t^1)
\]

Thus

\[
\frac{\partial U_t}{\partial C_t}(\epsilon_t^1) < \frac{\partial U_t}{\partial C_t}(\epsilon_t^2)
\]

\[
\Rightarrow -\frac{\partial U_t}{\partial L_t}(\epsilon_t^1) > -\frac{\partial U_t}{\partial L_t}(\epsilon_t^2)
\]

Hence, 

\[V_t^{nd}(\tilde{B}_t, B_t^{D,2}, A_t) > V_t^{nd}(\tilde{B}_t, B_t^{D,1}, A_t),\]

and therefore \(\epsilon_t^1 = (\tilde{B}_t, B_t^{D,1}) \in \Gamma_t^\delta \forall A_t \in A : \epsilon_t^2 \in \Gamma_t^\delta.\)

\[\text{Proof of Proposition 6.}\]

Given are the two choices over period \(t + 1\) endogenous states \(\epsilon_{t+1}^1 \equiv (\tilde{B}_{t+1}, B_{t+1}^{D,1})\), and \(\epsilon_{t+1}^2 \equiv (\tilde{B}_{t+1}, B_{t+1}^{D,2})\). From Proposition 5 follows directly that 

\[\Gamma_t^\delta(\epsilon_{t+1}^1) \subseteq \Gamma_t^\delta(\epsilon_{t+1}^2) \forall A_{t+1} \in A : \epsilon_{t+1}^2 \in \Gamma_t^\delta.\]

(i) Applying the definition of the default probability, one gets \(\pi_t^\delta(\epsilon_{t+1}^2, A_t) \leq \pi_t^\delta(\epsilon_{t+1}^1, A_t)\). (ii) Using the pricing equation of
international investors (3.27) yields $q_t(\epsilon_{t+1}^2, A_t) \geq q_t(\epsilon_{t+1}^1, A_t)$.
B.2 Timeline

Figure B.1: Timeline of events

1. Aggregate shock, default decision: $\delta_t \in \{0,1\}$; financial intermediation and production
2. Financial intermediation w/o interbank market; production
3. Bond market trading: govt debt issuance; collateral decision
4. Exclusion from financial market; change over to autarky if $\delta_t = 1$
5. Stochastic reentry in period $t+1$
B.3 Model Appendix

B.3.1 Bank optimization

B.3.1.1 Productive banks

\[
\begin{align*}
\max_{\{B^{D,p}_{j,t+1}, \kappa_{j,t}, M_{j,t}, R^{d,p}_{j,t}, R^{d,d}_{j,t}\}} & \quad E_t \left[ \sum_{i=0}^{\infty} \beta^i D^{p}_{j,t+i} \right] \\
\text{s.t.} & \quad N^p_j + M_{j,t} = \kappa_{j,t} + R^p_{j,t} \\
& \quad R^e_{j,t} = \frac{M_{j,t}}{\phi} \\
& \quad R^0_{j,t} = R^e_{j,t} + R^{d,p}_{j,t} \\
& \quad M_{j,t} \leq \frac{(1 - \delta_t) D^{D,p}_{j,t}}{\chi} \\
& \quad R^{d,p}_{j,t}, D^d_{j,t}, B^{D,p}_{j,t+1} \geq 0
\end{align*}
\]

with

\[
N^p_j + D^p_{j,t} = (1 - \delta_t) B^{D,p}_{j,t} + (1 + r^M_t) \kappa_{j,t} - (1 + r^R_t) M_{j,t} + (1 + r^R_t) R^p_{j,t} - (1 - \delta_t) q_t B^{D,p}_{j,t+1} \\
\equiv \text{type } p \text{ period } t \text{ cashflows, } X^p_{j,t}
\]

Note that subscripts \(j\) for individual banks can be ignored due to the existence of a representative bank for each type \(\tau \in \{p, u\}\), according to Proposition 1.

B.3.1.2 Unproductive banks

\[
\begin{align*}
\max_{\{B^{D,u}_{j,t+1}, M_{j,t}, R^{d,u}_{j,t}\}} & \quad E_t \left[ \sum_{i=0}^{\infty} \beta^i D^{u}_{j,t+i} \right] \\
\text{s.t.} & \quad N^u_j = M_{j,t} + R^u_{j,t} \\
& \quad R^u_{j,t} = \frac{R^{d,u}_{j,t}}{\phi} \\
& \quad R^{d,u}_{j,t}, D^d_{j,t}, B^{D,u}_{j,t+1} \geq 0
\end{align*}
\]

with

\[
N^u_j + D^u_{j,t} = (1 - \delta_t) B^{D,u}_{j,t} + (1 + r^M_t) M_{j,t} + (1 + r^R_t) R^u_{j,t} - (1 - \delta_t) q_t B^{D,p}_{j,t+1} \\
\equiv \text{type } u \text{ period } t \text{ cashflows, } X^u_{j,t}
\]
Rewrite the maximization problems of banks from (B.3.1) and (B.3.2) in recursive form as

\[
W_p(B_t, B_t^D, A_t) = \max_{\{B_{t+1}, B_t^D, \kappa_t, M_t, R_t^e, R_t^{D,p}\}} \{ D_t^p(B_t, B_t^D, A_t) + \beta b E_t [W(B_{t+1}, B_{t+1}^D, A_{t+1})] \}
\]

(B.3.3)

\[
W_u(B_t, B_t^D, A_t) = \max_{\{B_{t+1}, B_t^D, A_t\}} \{ D_t^u(B_t, B_t^D, A_t) + \beta b E_t [W(B_{t+1}, B_{t+1}^D, A_{t+1})] \}
\]

(B.3.4)

Denote the expected continuation value of banks in period \( t \) under \emph{i.i.d.} probability of taking type \( \tau \in \{p, u\} \) next period as

\[
E_t [W(B_{t+1}, B_{t+1}^D, A_{t+1})] = E_t [\pi_p W_p(B_{t+1}, B_{t+1}^D, A_{t+1}) + \pi_u W_u(B_{t+1}, B_{t+1}^D, A_{t+1})]
\]

(B.3.5)

B.3.1.4 Optimality conditions

B.3.1.5 Productive banks

Forming the Lagrangian by combining (B.3.1) and (B.3.3):

\[
L_t = \max_{\{B_{t+1}, B_t^D, \kappa_t, M_t, R_t^e, R_t^{D,p}\}} \{ (1 - \delta_t)B_t^{D,p} + (1 + r_t^e)\kappa_t - (1 + r_t^M)\phi R_t^e + (1 + r_t^R)(R_t^e + R_t^{D,p}) - (1 - \delta_t)q_t B_t^{D,p} - N^p + \eta_t^p(N^p + \phi R_t^e - \kappa_t - R_t^e - R_t^{D,p}) + \lambda_t((1 - \delta_t)B_t^{D,p} - \chi \phi R_t^e) + \mu_t^p(R_t^{D,p} - 0) + \beta b E_t [W(B_{t+1}, B_{t+1}^D, A_{t+1})] + \mu_t^{D,p}(0 + D_t^p) + \mu_t^{D,x}(0 + B_t^{D,x}) \}
\]

with Lagrange multipliers for the collateral constraint \( \lambda_t \), the balance sheet constraint \( \eta_t^p \), the non-negativity constraint on the deposit facility \( \mu_t^p \), dividends \( \mu_t^{D,p} \), and collateral purchases \( \mu_t^{D,x} \), respectively. Interbank loans have been substituted.

The first-order necessary conditions w.r.t. \( (\kappa_t, R_t^e, R_t^{D,p}, B_t^{D,p}) \) are

\[
1 + r_t^e = \eta_t^p
\]

(B.3.7)

\[
(1 + r_t^R) - \eta_t^p = \phi(1 + r_t^M + \lambda_t \chi - \eta_t^p)
\]

(B.3.8)

\[
(1 + r_t^R) + \mu_t^p = \eta_t^p
\]

(B.3.9)

\[
(1 - \delta_t)q_t = \beta b E_t [W_{BD}(B_{t+1}, B_{t+1}^D, A_{t+1})] + \mu_t^{D,p}
\]

(B.3.10)
with complementary slackness conditions

\[ \lambda_t(0 - \chi M_t - (1 - \delta_t) B_t^{D,p}) = 0 \quad \text{(B.3.11)} \]
\[ \mu_t^p(0 + R_t^{d,p}) = 0 \quad \text{(B.3.12)} \]
\[ \mu_t^{D,p}(0 + D_t^p) = 0 \quad \text{(B.3.13)} \]
\[ \mu_t^{B^{D,p}}(0 + B_t^{D,p}) = 0 \quad \text{(B.3.14)} \]
\[ \lambda_t, \eta_t^p, \mu_t^p, \mu_t^{D,p}, \mu_t^{B^{D,p}} \geq 0 \]

### B.3.1.6 Unproductive banks

Forming the Lagrangian by combining \((\text{B.3.2})\) and \((\text{B.3.4})\):

\[
\mathcal{L}^u = \max_{\{B_{t+1}^{D,u}, M_t, R_t^{d,u}\}} (1 - \delta_t) B_t^{D,u} + (1 + r^M) M_t + (1 + r^R) R_t^{d,u} - (1 - \delta_t) q_t B_{t+1}^{D,p} - N^u \\
+ \eta_t^u (N^u - M_t - R_t^{d,u}) + \mu_t^u (R_t^{d,u} - 0) + \beta^b E_t [W(B_{t+1}, B_{t+1}^{D}, A_{t+1})] \\
+ \mu_t^{D,u}(0 + D_t^u) + \mu_t^{B^{D,u}}(0 + B_t^{D,u}) \tag{B.3.15}
\]

with Lagrange multipliers for the balance sheet constraint \(\eta_t^u\), and the non-negativity constraints on central bank deposits \(\mu_t^u\), dividends \(\mu_t^{D,u}\), and collateral purchases \(\mu_t^{B^{D,u}}\), respectively. We use the following functional form for \(\Psi(q_t)\):

\[
\Psi(q_t) = \psi \left( r_t^q - r^f \right),
\]
with \(r_t^q \equiv q_t^{-1} - 1\)

From the definition of the interbank rate \(r_t^M \equiv 1 + r^R + \Psi(q_t)\), we get

\[ r_t^M = 1 + r^R + \psi(r_t^q - r^f) \]

The first-order necessary conditions w.r.t. \((M_t, R_t^{d,u}, B_{t+1}^{D,u})\) are

\[ 1 + r_t^M = \eta_t^u \quad \text{(B.3.16)} \]
\[ (1 + r^R) + \mu_t^u = \eta_t^u \quad \text{(B.3.17)} \]
\[ (1 - \delta_t) q_t = \beta^b E_t [W_{B^{D}}(B_{t+1}, B_{t+1}^{D}, A_{t+1})] + \mu_t^{D,u} + \mu_t^{B^{D,u}} \quad \text{(B.3.18)} \]
and the complementary slackness conditions

\[
\begin{align*}
\mu_t^u (0 + R_t^{d,u}) &= 0 \quad (B.3.19) \\
\mu_t^{D,u} (0 + D_t^u) &= 0 \quad (B.3.20) \\
\mu_t^{B,D,u} (0 + B_{t+1}^{D,u}) &= 0 \quad (B.3.21) \\
\eta_t^u, \mu_t^u, \mu_t^{D,u}, \mu_t^{B,D,u} &\geq 0
\end{align*}
\]

### B.3.2 Partial equilibrium

We find \{A_t, B_t, B_t^D\} from the state space. Bond prices \{q_t\} are taken as given. This yields the interbank rate as

\[
r_t^M = r^R + \psi (q_t^{-1} - 1 - r^f).
\]

Using all static optimality conditions derived above, the following nonlinear constrained system of equations yields a solution to the unknowns \{L_t, W_t, r_t^e, \kappa_t, M_t, R_t^e, R_t^{d,p}, R_t^{d,u}, \lambda_t, \mu_t^p, \mu_t^u\}:

**Equality constraints**

\[
\begin{align*}
M_t &= \phi R_t^e \quad (B.3.1) \\
W_t &= L_t^{-1} \quad (B.3.2) \\
\kappa_t &= \eta W_t L_t \quad (B.3.3) \\
(1 - \alpha)e^{A_t} K^\alpha L_t^{-\alpha} &= W_t (1 + \eta \kappa_t) \quad (B.3.4) \\
r_t^e - r^R &= \phi [r_t^e - r_t^M - \lambda_t \chi] \quad (B.3.5) \\
r_t^e &= r^R + \mu_t^p \quad (B.3.6) \\
N^p + M_t &= \kappa_t + R_t^e + R_t^{d,p} \quad (B.3.7) \\
N^u &= M_t + R_t^{d,u} \quad (B.3.8)
\end{align*}
\]

**Inequality constraints**

\[
\begin{align*}
\chi M_t &\leq (1 - \delta_t) B_t^{D,p} \\
R_t^{d,p} &\geq 0, \\
R_t^{d,u} &\geq 0,
\end{align*}
\]
complementary slackness conditions

\[
\lambda_t \left(0 - \chi M_t + (1 - \delta_t)B_t^{D,p}\right) = 0 \quad \text{(B.3.9)}
\]

\[
\mu_t^p(0 + R_t^{d,p}) = 0, \quad \text{(B.3.10)}
\]

\[
\mu_t^u(0 + R_t^{d,u}) = 0, \quad \text{(B.3.11)}
\]

\[
\lambda_t, \mu_t^p, \mu_t^u \geq 0, \quad \text{(B.3.12)}
\]

In case of default \((\delta_t = 1)\), we have \(q_t = 0\) such that our collateral constraint imposes \(M_t = R_t^c = 0\) and \(r_t^M\) is undefined. All resources of unproductive banks are deposited at the central bank, \(R_t^{d,u} = N^u\). The solution to the partial equilibrium in the unknowns under default \(\{L_t, W_t, r_t^c, \kappa_t, R_t^{d,p}, \mu_t^p\}\) is then derived from the system:

\[
W_t = L_t^{\omega-1} \\
\kappa_t = \eta W_t L_t \\
L_t = \left[\frac{1 + \eta \rho c}{(1 - \alpha) e^{\Lambda_t K_t}}\right]^{\frac{1}{1-\alpha-\omega}} \\
r_t^c = r^R + \mu_t^p \\
N^p = \kappa_t + R_t^{d,p}
\]

with the inequality constraint \(R_t^{d,p} \geq 0\), and complementary slackness condition

\[
\mu_t^p(0 + R_t^{d,p}) = 0 \\
\mu_t^p \geq 0
\]

### B.3.3 Collateral choice

We use the remaining dynamic equilibrium conditions to pin down the optimal amount of aggregate collateral holding for the consecutive period. We pin down the optimal choice over collateral \(B_t^{D+1}\) from the system of equations:

\[
N^p + D_t^p = X_t^p - (1 - \delta_t)q_t B_t^{D,p} \\
N^u + D_t^u = X_t^u - (1 - \delta_t)q_t B_t^{D,u} \\
(1 - \delta_t)q_t = \beta^E \frac{E_t}{B_t^{D}(B_t^{D+1}, B_t^{D,u}, A_t^{D+1})} + \mu_t^D \\
B_t^{D+1} = B_t^{D,p} + B_t^{D,u} \\
B_t + B_t^{D+1} + B_t^{D+1} = 0
\]
subject to the constraints

\[ B_{t+1}^{D, \tau} \geq 0, \quad \tau \in \{ p, u \} \]
\[ D_{t}^{\tau} \geq 0, \quad \tau \in \{ p, u \} \]

The envelope condition is derived from bank value functions (B.3.3)-(B.3.5) and (B.3.6, B.3.15) which yields

\[ W_{BD}(B_{t+1}, B_{t+1}^D, A_{t+1}) = \pi^p [1 - \delta_{t+1} + \lambda_{t+1}] + \pi^u (1 - \delta_{t+1}) \]
\[ = 1 - \delta_{t+1} + \pi^p \lambda_{t+1} \]

Since both types of banks have equal probabilities over funding needs in the consecutive period, the inter-temporal optimality conditions are identical. We aggregate over banks budget constraints in order to obtain the system of equations that pin down the optimal collateral choice:

\[ N + D_t = X_t^p + X_t^u - (1 - \delta_t)q_t B_{t+1}^D \quad (B.3.1) \]
\[ D_t = (1 - \delta_t)B_t^D + (1 + r_t^e)\kappa_t + (1 + r_R)R_t - (1 - \delta_t)q_t B_{t+1}^D - N \]
\[ (1 - \delta_t)q_t = \beta^b E_t [1 - \delta_{t+1} + \pi^p \lambda_{t+1}] + \mu_t^D \quad (B.3.2) \]
\[ B_{t+1}^D + B_{t+1}^* + B_{t+1} = 0 \quad (B.3.3) \]
\[ B_{t+1}^D, D_t \geq 0 \quad (B.3.4) \]

Note that \( E_t[1 - \delta_{t+1}] = 1 - \pi_t^d \). Further, denoting aggregate cashflows in the financial sector by \( X_t = X_t^p + X_t^u \), and noting the fact that in absence of default we have \( \delta_t = 0 \), the optimal period \( t \) collateral choice in the unknowns \( \{ B_{t+1}^D, B_{t+1}^*, D_t, \mu_t^D, \mu_t^{BD} \} \) is fully described by the system

\[ N + D_t = X_t - q_t B_{t+1}^D \quad (B.3.5) \]
\[ q_t = \beta^b (1 - \pi_t^d + \pi^p E_t [\lambda_{t+1}]) + \mu_t^D \]
\[ B_{t+1} + B_{t+1}^D + B_{t+1}^* = 0 \quad (B.3.6) \]

with the associated complementary slackness conditions

\[ \mu_t^D (0 + D_t) = 0 \quad (B.3.7) \]
\[ \mu_t^{BD} (0 + B_t^D) = 0 \quad (B.3.8) \]
\[ \mu_t^D, \mu_t^{BD} \geq 0 \]

Under default we have \( \delta_t = 0 \), hence \( q_t = 0 \). It follows that \( B_{t+1} = B_{t+1}^D = B_{t+1}^* = 0 \).
\section*{B.3.4 Resource constraint}

The agents’ budget constraints are used to form the aggregate resource constraint in the model economy:

\[ C_t = W_t L_t + D_t + T_t + \Pi_t^{NF} \]
\[ \Pi_t^{NF} = Y_t - W_t L_t - \kappa_t r_t^c \]
\[ D_t = D_t^p + D_t^u \]
\[ = (1 - \delta_t)B_t^{D,p} + (1 + r_t^c)\kappa_t \left( 1 + r_t^M \right) M_t + (1 + r_t^R)R_t^p - (1 - \delta_t)q_t B_{t+1}^{D,p} - N^p \]
\[ + (1 - \delta_t)B_t^{D,u} + (1 + r_t^M)M_t + (1 + r_t^R)R_t^u - (1 - \delta_t)q_t B_{t+1}^{D,u} - N^u \]
\[ = (1 - \delta_t)B_t^D + (1 + r_t^c)\kappa_t + (1 + r_t^R)R_t - (1 - \delta_t)q_t B_{t+1}^D - N \]
\[ \Pi_t^{cb} = - R_t r_t^R \]
\[ T_t = \Pi_t^{cb} + B_t - q_t B_{t+1} \]

Summing over all agents’ budget constraints, period \( t \) consumption is determined in

\[ C_t = Y_t + (\kappa_t + R_t - N) + (1 - \delta_t)(B_t + B_t^D) - (1 - \delta_t)q_t (B_{t+1} + B_{t+1}^D) \]
\[ = Y_t + (1 - \delta_t)(B_t + B_t^D) - (1 - \delta_t)q_t (B_{t+1} + B_{t+1}^D) \]

\section*{B.4 Computational strategy}

The following algorithm proposes value function iteration in the discrete state space (DSS) as a solution to this numerical problem in a two-loop algorithm.\textsuperscript{32}

1. Initiate the system. Form discrete grids over the three state variables with productivity \( A \in A = G_A\{A_1, A_2, ..., A_i\} \) of size \( i \), government bonds \( G_B \in B = \{B_1, B_2, ..., B_j\} \) of size \( j \), and bonds held domestically as collateral \( G_D \in D = \{B_1^D, B_2^D, ..., B_p^D\} \) of size \( p \) with \( B \in B = [B_{\text{min}}, B_{\text{max}}], B^D \in F = [B^D_{\text{min}}, B^D_{\text{max}}]. \) Form a matrix \( S \) that represents the discrete state space and contains \( s = i \times j \times p \) elements along the rows that represent the current state, \( j \) elements along the columns for choice over \( B^o \), and \( p \) elements along the pages for choice over \( B^D \), hence \( S = s \times j \times p \). Form a second matrix, \( S^o = s \times 1 \), that describes the state under default when choices are \( B^o = B^D = 0 \). Note that the state space matrices \( S, S^o \) do not have full rank, as all combinations that violate the condition \( |B| > B^D \) from condition (3.9) need to be eliminated from the state space. Initialize the government’s value functions \( V_{(0)}(B, B^D, A), V_{(0)}^d(B, B^D, A), \) and \( V_{(0)}^{nd}(B, B^D, A) \) of size

\textsuperscript{32}The quasi-code is presented in recursive notation in this appendix. Variables for the consecutive period are denoted by a prime.
s \times 1, and the bank pricing schedule for government debt of size s \times j.

2. Calculate period t default allocations for each element in S^δ using the system of partial equilibrium equations under default. Note that default allocations can be computed outside the pricing-loop as they are based on the assumption q_t = 0 if δ_t = 1.

3. Take q(0)(B^t, B^D^t, A) = 1/(1 + r_f^t) for all combinations of current and future states S as the initial guess for the pricing-loop.

(a) Calculate period t static allocations under repayment conditional on the state matrix S and the bond price q(0) by using the system of partial equilibrium equations in the repayment regime.

(b) Collateral policy:

   i. Use the transition matrix Ω on the exogenous state to form expectations about future liquidity value of government bonds, E_t(\lambda_{t+1}).

   ii. Back out the optimal collateral decision for each potential choice of total government debt B^t, given current state s, denoted by B^{F^s\delta} | B^t = F(B, B^D, A) of size s \times j. The optimal collateral policy follows the optimality condition E_t(\hat{q}_t - q_t) = 0 + \mu_t^D + \mu_t^{BD}.

(c) Government’s value functions:

   i. Compute the value of household utility under repayment for each element of S. Use the optimal collateral policy of banks F to form the continuation value of the government.

   ii. Take default decision to form the default set \Gamma^d(B, B^D, A) = s \times 1. In case of repayment, pin down optimal level of public debt B^{*s}. Obtain updated value functions V_{(1)}(B, B^D, A), V_{(1)}^{d}(B, B^D, A), and V_{(1)}^{nd}(B, B^D, A).

   iii. Iterate on government value functions until convergence is achieved.

   iv. Back out the optimal debt policy of the government, consisting of the default decision δ_t ∈ \{0, 1\}, and the optimal borrowing decision B^t in case of repayment. The vector of optimal government debt policy is of size s \times 1.

4. Use the default set \Gamma^d(B, B^D, A) and the transition matrix Ω to form default probabilities π^d_{(1)}. Update the bond price equation. Convergence is achieved if |q(0) - q(1)| < \varepsilon. Else, update the bond price schedule q(0) = q(1) and go back to step 3a).
B.5 Data


- **Private consumption, Spain**: Nominal quarterly consumption, deflated by the GDP deflator, 2001q1-2011q4. Source: National Statistics Institute (INE), Spain.

- **Credit, Spain**: Outstanding loans to non-financial corporations, up to 1 year, deflated by GDP deflator, 2003q1-2011q4. Source: Bank of Spain.

- **Interbank loans, Spain**: Deposits from domestic monetary financial institutions, Jan-2000 to Dec-2011 as a share in total liabilities. Source: Bank of Spain, Monetary and Financial Statistics.


- **Sovereign spread, Spain**: Spanish benchmark bond yield, 5 year, over German Bund, 20001q1 - 2011q4. Source: Datastream.

- **Credit rate, Spain**: Bank lending rate to non-financial corporations, up to 1 year, Mar-2003 to Dec-2012. (also: Belgium, France, Germany, Greece, Italy, Ireland, Netherlands, Portugal). Source: ECB.

- **Interbank rate**: GC repo rate, 3 month, Germany, France, Italy, Spain; 31-Mar-2011 to 07-Dec-2011; Germany starts on 25-May-2011. Source: Bloomberg.

- **Government debt**: Total and domestic government debt, Spain, 2004q1 to 2013q1. Source: Arslanap and Tsuda (2012).

- **Bank liquid reserves to bank assets ratio, Spain**: Ratio of domestic currency holdings and deposits with the monetary authorities to claims on other governments, nonfinancial public entities, the private sector, and other banking institutions, 2000-2011. Source: World Bank, World Development Indicators.
Appendix to Chapter 4
C.1 Data and robustness

Data


- **Industrial production**: Monthly index of total industrial production excluding construction, seasonally adjusted, Jan 2007-Aug 2014. Source: OECD.

- **Bank lending rate**: Monthly interest rate on loans to non-financial corporations, over 1 million Euro, up to 1 year rate (in percent), Jan 2007-Aug 2014. Source: ECB.

- **Eonia swap rate**: Monthly, 3-month, Jan 2007-Aug 2014. Source: European Banking Federation/AIC.


VAR robustness

The robustness analysis regarding the empirical model from equation (4.1) in Section 4.2 presents alternative specifications. The orthogonalized impulse response functions (IRF) from an increase in uncertainty on the yield on sovereign CDS are plotted jointly with the baseline specification in Figure C.1.
First, I consider a different ordering of endogenous variables. One concern might be that an uncertainty shock needs time to pass-through on bank lending rates and industrial production. I re-order the variables according to \( y_t = [\text{debt}/\text{gdp}_t, \hat{r}_t, \hat{i}_t, D_t, \text{cds}_t]^\prime \). The main change consists of ordering the uncertainty shock at the second to last position.

Second, one might be concerned about a measure for stress in European banking sector as an exogenous variable in the system of equation (4.1). I estimate the system using the European money market spread as a control for tensions in European interbank markets as a source for spillovers between banks and sovereigns. It is constructed as the difference between the three month Euribor and the overnight index swap (OIS).

Figure C.1: Robustness of VAR analysis

![Graph showing IRFs for different countries](image)

Note: Orthogonalized impulse response functions.

Third, I use one lead instead of three leads of forecast disagreement about GDP growth in Figure 4.1. Although the results are similar for France and Spain, there is a quite different path of the IRF in the cases of Germany, Italy and the Netherlands. The typical hump-shape response of yields on sovereign CDS is no longer present. This can be explained by information stickiness in surveys of professional forecasters. Coibion and Gorodnichenko (2012) and Sheng and Wallen (2014) have shown that the usual pass-through of new information into a forecast usually takes two to three months. This information rigidity can be confirmed by the correlation pattern between forecast disagreement and sovereign CDS in Figure A.1.
Finally, I replace the country specific forecast disagreement by the CBOE volatility index (VIX), which measures the volatility of the SP 500. The results show that the effect of a ‘global’ volatility shock on European sovereign CDS yields is rather limited and substantially lower than a shock to country specific disagreement. I take this result as supportive evidence for the use of forecast disagreement as the measure for (fundamental) macroeconomic uncertainty (Zarnowitz and Lambros, 1987; Bomberger, 1996).
C.2 Quantitative model, Appendix

Figures

Figure C.2: Timing assumptions in the quantitative model

![Timing assumptions in the quantitative model](image)

Figure C.3: Interquartile range at different forecast horizons in the US

![Interquartile range at different forecast horizons in the US](image)

Notes: The average level of GDP forecast dispersion as measured by the interquartile range in US data over the one-quarter-ahead forecast horizon ($\mu_{iqr}^{t+1} = 1.48$) versus the one-year-ahead forecast horizon ($\mu_{iqr}^{t+4} = 1.37$). Source: Federal Reserve Bank of Philadelphia.

Forming default expectations under uncertainty

I interpolate the default probability conditional on the expected productivity states $E_t(z_{t+1})$ and endogenous choices, in order to determine the distorted default expectation:
1. Conditional on the level of uncertainty \( (a_t) \), form default expectations \( E_t(\delta_{t+1}) \) and expectations on the future productivity state \( E_t(z_{t+1}) \) using the current default decision \( \delta_t \in (0, 1) \) and the transition matrix \( \Omega \) on the grid of the exogenous states \( z \in \mathbb{Z} = G_z \{ z_1, z_2, \ldots, z_i \} \) and \( a \in A = G_a \{ a_1, a_2, \ldots, a_i \} \), obtained via Tauchen’s method (Tauchen, 1986).

2. Get new expectations on the aggregate productivity state derived under multiplier preferences:

\[
E_t^p(z_{t+1} \mid z_t, a_t) = E_t^p(z_{t+1} \mid z_t) - a_t.
\]

3. We can use the three objects \( E_t(z_{t+1}) \), \( E_t(\delta_{t+1}) \), and \( E_t^p(z_{t+1}) \) in order to get the default expectation under the maxmin representation \( E_t^p(\delta_{t+1}) \) by interpolation.\(^{33}\)

4. Match the derived default expectation with the current state, \( E_t^p(\delta_{t+1} \mid z_t, a_t) \).

As a result, current bond prices and allocations depend on the current level of uncertainty \( a_t \).

Figure C.4: Interpolating the default expectation

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\(^{33}\) I use a piecewise cubic hermite interpolating polynomial for the interpolation (Fritsch and Carlson, 1980).
C.3 Proofs

Proof of Proposition 7. The condition for the risk-free borrowing limit can be characterised as

\[ V^{nd}_1(B_1 \mid a = 0) = V^d_1 \]
\[ \iff \left( y_1 - \frac{1 - \bar{\alpha}}{1 + r} B_2 + \bar{B}_1 \right) - \frac{\psi}{2} \left( y_1 - \frac{1 - \bar{\alpha}}{1 + r} B_2 + \bar{B}_1 \right)^2 \]
\[ + E_1 [\delta_1 (h(y_2)) + (1 - \delta_1)(y_2 + B_2)] - \frac{\psi}{2} (E_1 [\delta_1 h(y_2) + (1 - \delta_1)(y_2 + B_2)])^2 \]
\[ = h(y_1) - \frac{\psi}{2} (h(y_1))^2 + E_1 [h(y_2)] - \frac{\psi}{2} (E_1 [h(y_2)])^2. \] (C.3.1)

This is an equation in the unknown \( \bar{B}_1 \) such that a solution to this problem exists. The endogenous borrowing limit \( B_1 \) is given by the condition \( a \rightarrow \bar{a} = \bar{\alpha}_{eb} - \bar{x} \), which gives a worst case prior belief \( E_1^p[\bar{z}_2 = z^l] = E_1^p(\delta_2) \rightarrow 1 \) and thus \( q_1 \rightarrow 0 \):

\[ V^{nd}_1(B_1 \mid a = \bar{a}) = V^d_1 \]
\[ \iff (y_1 + B_1) - \frac{\psi}{2} (y_1 + B_1)^2 \]
\[ + E_1 [\delta_1 (h(y_2)) + (1 - \delta_1)(y_2 + B_2)] - \frac{\psi}{2} (E_1 [\delta_1 h(y_2) + (1 - \delta_1)(y_2 + B_2)])^2 \]
\[ = h(y_1) - \frac{\psi}{2} (h(y_1))^2 + E_1 [h(y_2)] - \frac{\psi}{2} (E_1 [h(y_2)])^2. \] (C.3.2)

For every intermediate level of borrowing \( B_1 \in (B_1, \bar{B}_1] \), we obtain at the optimal borrowing decision \( B^*_2 \)

\[ \frac{\partial c_1}{\partial a} < 0, \]
with \( c_1 = y_1 - q_1 B^*_2 + B_1 \),

since the price of debt \( q_1 \) is decreasing with the level of uncertainty \( a \). Thus,

\[ \frac{\partial V^{nd}_1}{\partial a} < 0 \]

such there exists a unique intersection point on the interval \( a^* \in [0, \bar{a} = \bar{\alpha}_{eb}] \) for which \( V^{nd}_1(B_1) = V^d_1 \).

\[ \blacksquare \]

Proof of Proposition 8. The international ambiguity averse investor maximizes expected returns under the worst case prior. It needs to be shown that the probability of repayment conditional on the endogenous state variables \( (B_{t+1}, B^D_{t+1}) \) is (weakly) decreas-
ing in period $t + 1$ productivity. For this, I refine the definition of the default set and condition it on the productivity state:

**Claim** If default is optimal in state $s^1 = (B_t, B_t^D, z_t^1, a_t)$, then default is also optimal in state $s^2 = (B_t, B_t^D, z_t^2, a_t)$ for $z^2 < z^1$. That is,

$$
\Gamma_t^d(B_t, B_t^D) \mid z^1 = \{ s^t = (z^1, a_t) \in s : V_t^d(s^1(t)) > V_t^{nd}(s^1(t)) \}
\subseteq
\Gamma_t^d(B_t, B_t^D) \mid z^2 = \{ s^t = (z^2, a_t) \in s : V_t^d(s^2(t)) > V_t^{nd}(s^2(t)) \}.
$$

**Proof of claim:** By looking at the margin of a case of optimal default where $V^{nd} = V^d$, the effect of a higher productivity realisation increases the demand for labour due to a higher marginal product, see equation (4.7). The working capital requirement (4.6) pushes up credit demand. This pushes up the social benefit from interbank lending. In contrast, there is no such effect in the regime of default, where higher marginal labour productivity does not increase credit supply due to the freeze on the interbank market. As a consequence, it holds that

$$
\frac{V^{nd}}{\partial z_t} \geq \frac{V^d}{\partial z_t} < 0
$$

such that one can conclude that

$$
V^{nd}(s^1) \geq V^{nd}(s^2)
$$

and

$$
\Gamma_t^d(B_t, B_t^D) \mid z^1 \subseteq \Gamma_t^d(B_t, B_t^D) \mid z^2
$$

Using the definition of the default probability from equation (4.31), there is a (weakly) monotonically decreasing probability of default in productivity, $\partial \delta_t / \partial z_t < 0$, such that the expected returns conditional on each prior in the set are minimized when using the prior $E^p_t(z_{t+1}) = \rho_z - a_t$. 

\[\blacksquare\]


BIBLIOGRAPHY


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