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Chapter 1

Introduction

Interest in the corporate taxpayer has perhaps never been more pronounced. At the highest levels of international politics, initiatives are launched and measures undertaken to reform how firms are taxed.\(^1\) Mounting fiscal pressures in developed countries have no doubt contributed to this renewed interest in corporate tax practices.\(^2\) As have the ever more sophisticated ways in which firms reduce their tax burden, often with surprising efficacy. Google reportedly paid 2.4% in taxes on non-US profits from 2007 to 2009,\(^3\) General Electric no taxes at all on US profits of $5.1bn in 2010,\(^4\) and Starbucks remitted a mere £8.6m in income taxes to UK tax authorities on more than £3bn in coffee sales over the period from 1998 to 2011.\(^5\) Citing the legality of tax planning and their duty to shareholders, corporate narratives are no less compelling than the arguments advanced by policymakers to counter them. Whatever the normative merits of such justifications, economic interests clearly play a key role on both sides of the debate. Corporations want to increase their net profits by paying fewer taxes. Governments want to fill public coffers in order to go about their sovereign duties. In this thesis, we use rational choice theory to analyze how these economic motives interact - and collide.

Two important differences between corporations and individuals, and how they influence the tax evasion game between government and taxpayer, are the topic of the first two chapters of this thesis. First, there is corporate governance. Individuals simply decide for themselves, for instance whether or not to evade taxes. Corporate decisions are typically more complex. They are the outcome of the interaction of many individuals bound by contracts, governance structures, and the need for cooperation. Often, conflicts of interest emerge between the man-

\(^1\)See for instance recent OECD (2013) and G-20 initiatives (G-20 2014, Buergin and Vina 2013).
\(^2\)See for example IMF Fiscal Monitor (October 2013)
\(^3\)Drucker (2010)
\(^4\)Kocieniewski (2011)
\(^5\)Bergin (2012)
ifold individuals involved in corporate decision-making. Diverging agendas of shareholders and managers, for instance, have long been the subject of economic analysis. Surprisingly, such agency considerations have received little attention in analyzing corporate tax avoidance, although they are clearly relevant. Who decides on evasion and what is the exact nature of the task? What should a remuneration contract incentivizing a tax planner look like? Does it matter who is liable in case corporate wrongdoing is detected, and what are the implications for tax enforcement policy more widely? These are some of the questions addressed by the nascent agency-theoretic literature on corporate tax evasion and avoidance.6 The first chapter of this thesis contributes to this strand of the economic literature. We consider a multitask principal-agent model of corporate tax evasion, where a specialized tax manager is hired by a firm-owner to determine both the quantity and quality of tax evasion. Quantity means the amount of underreporting. Quality influences how tax evasion is treated by the tax authority. It is a form of self-insurance: if the firm is audited by the tax authority, higher quality will lower the penalty for evasion. Investing in quality can be seen as resorting to more sophisticated tax sheltering techniques, for example. We find that once the quality of a firm’s tax evasion is taken into account, asymmetric information inside firms may enhance the efficacy of tax enforcement. That is because high quality tax evasion may be just as difficult for the non-specialist shareholder to understand and incentivize as it is for the tax authority to fine. As a result of asymmetric information between shareholder and tax manager, both the quantity and quality of a firm’s tax evasion are thus reduced. If the quality reduction dominates, tax enforcement becomes more effective and, in equilibrium, is stricter where firms enter principal-agent relationships to evade taxes, rather than more lenient as suggested by the earlier literature, in particular by Crocker and Slemrod (2005).

A second key distinction between corporate and individual tax evasion concerns the interaction of corporate financing with the tax evasion game. This interaction is the subject of chapter 3 of this thesis. In it, we extend the classic costly state verification model of financial contracting due to Townsend (1979) and Gale and Hellwig (1985) to allow for tax evasion by the borrower. We fully characterize the constraints on financial contracting arising from tax evasion and then proceed to deriving the optimal financial contract. Tax evasion poses an interesting challenge for financial contracting. Its gains due to a lower tax bill are not contractible, because they arise from an illegal activity. But the potential losses associated with tax evasion, namely the fines payable upon detection, may reduce the borrower’s ability to repay. This occurs when the government’s claims are sufficiently senior relative to those of the investor. We find that in

6The earliest contributions in this literature are by Crocker and Slemrod (2005) and Chen and Chu (2005).
this context, and in contrast to the original model, standard debt contracts are no longer optimal. Instead, the optimal financial contract features elements of both debt and equity and is less efficient than a standard debt contract. It is debt-like only for very low and very high profit realizations. For intermediate project returns, it stipulates rent-sharing between entrepreneur and investor and verification of returns. This is because an optimal contract in this model, in addition to minimizing verification as in the standard model, needs to take into account the investor’s potential liability for fines. This liability matters in case the fines for evasion deplete the entrepreneur’s funds so that the contractually agreed-upon repayment to the investor cannot be made in full. To forestall such limited liability protection for the entrepreneur with regard to tax evasion, the entrepreneur has to be left with a sufficiently large rent for all but the very lowest profit realizations. This prevents her from abusing her limited liability protection for excessive tax evasion activities and renders the contract feasible and incentive-compatible, both of which a standard debt contract is not if tax evasion by entrepreneurs is possible.

In the final chapter of this thesis, we analyze more closely the government’s optimal tax audit policy. Though relevant also in a corporate context, the analysis presented in chapter 4 does not explicitly refer to predominantly corporate features such as corporate governance or financing, and their interaction with tax evasion. Rather, we are concerned here with how the perception of audit risk by taxpayers influences tax evasion, and what the implications are for optimal tax auditing. We begin with the observation that both overconfidence and underconfidence with respect to tax audits are likely present in the taxpayer population. Overconfidence is a well-established bias in human behavior, economic and otherwise. It is therefore plausible to assume that when preparing their tax report and underreporting income, some taxpayers will underestimate the likelihood of being detected. On the other hand, the economic literature on tax evasion has also hinted at the possibility that taxpayers overestimate the true audit probability. Rather than subscribing to the dominance of either overconfidence or underconfidence among taxpayers, we examine a population that is heterogeneous in its perception of the audit probability. Some people overestimate the likelihood of a tax audit, while others underestimate it. Using the principal-agent approach to tax auditing pioneered by Reinganum and Wilde (1985), we find that such heterogeneity in the perception of audit risk significantly alters a government’s optimal audit policy. More specifically, the tax authority’s audit policy choice depends on the extent of perception heterogeneity among taxpayers. If taxpayers are relatively homogeneous in their perception of the audit probability, the government chooses to induce full compliance like in

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7 For a survey on overconfidence, see Moore and Healey (2008).
8 See for instance the survey by Slemrod and Yitzhaki (2002), especially p.1431.
the standard model of optimal auditing without perception heterogeneity. When heterogeneity is large, however, both evasion and compliance are part of the equilibrium. The equilibrium audit intensity then increases with heterogeneity if audit costs are small, but decreases with heterogeneity if audit costs are large. In the chapter, we provide the exact conditions deciding between these cases and explain the underlying mechanisms. We also conduct a welfare analysis, and find that high levels of social welfare are associated with either very homogeneous or very heterogeneous taxpayer populations, while intermediate levels of perception heterogeneity are associated with lower levels of equilibrium welfare.
Chapter 2

Multitasking in corporate tax evasion

2.1 Introduction

A few fairly recent contributions to the theory of tax evasion have called attention to how principal-agent relationships in firms may alter the analysis of corporate as opposed to individual tax evasion.\(^1\) They share a common implication regarding tax enforcement policy targeting firms: less is more, for two reasons. First, agency costs\(^2\) reduce the extent of corporate tax evasion. Second, tax enforcement becomes less effective as a result of agency costs. Allowing for the internal organization of firms, rather than seeing them as individuals, they suggest, both reduces the scope of the problem of corporate tax evasion and makes what is left of it more resistant to enforcement. Because tax enforcement is a costly activity, then, it will be lower where taxpayers enter principal-agent relationships to evade taxes. In this chapter, we argue instead that asymmetric information inside corporations enhances the effectiveness of tax enforcement and may therefore be associated with stricter optimal tax enforcement. This is because the previous approach overlooks an important point: tax evasion has various elements, and informational asymmetry affects each of them differently. Indeed, like other executive jobs, corporate tax planning involves not one but a multitude of tasks. For the purpose of this analysis, we focus on two of them: deciding how much to evade and in which way to evade. Put differently, a tax manager determines both the quantity and the quality of tax evasion for the firm. Quality, or how to evade, influences how tax evasion is treated by the tax authority. From the firm’s perspective, investing in higher quality is a form of self-insurance: if the firm is audited, higher quality will lower the penalty for evasion. Tax evasion

\(^1\)See Crocker and Slemrod (2005) and Chen and Chu (2005).

\(^2\)In the principal-agent literature, “agency cost” means the cost accruing to a principal of hiring an agent to perform a task instead of performing the task herself.
often goes hand in hand with efforts to reduce the potential downside of being fined should an audit occur. In a classic example of such activities\(^3\), taxpayers in 18th century England are reported to have temporarily bricked up fireplaces as part of their effort to evade hearth taxes. A present-day corporate tax evader may self-insure, for instance, by resorting to more sophisticated tax sheltering techniques or by investing in the justification of a particular legal interpretation of the tax code to prepare for an audit.\(^4\) Yet, for the same reason a tax authority might not be able to fine high quality tax evasion, a non-specialist firm-owner may have difficulties commissioning it. When evasion schemes are so sophisticated as to be either technically legal or not fully detectable, even upon audit,\(^5\) there is likely a large informational asymmetry between a shareholder and the tax specialist she hires to perform such sophisticated tax evasion. A principal-agent firm therefore lowers the quality as well as the quantity of tax evasion as a result of agency costs, we find. This may enhance the government’s tax enforcement efficacy, because a less sophisticated tax evasion scheme is easier to punish and fully detect upon audit. Unlike earlier contributions, we explicitly model the tax authority as a welfare maximizing player and fully characterize its optimal tax enforcement policy. Because of the effect of agency on the quality dimension of tax evasion, we find that the earlier result of lower optimal tax enforcement due to agency costs is reversed, negated, or at least mitigated. We also provide the condition that decides between these three cases. This chapter is connected to three strands of literature.

First, there is the large economic literature on tax evasion, and in particular the aforementioned literature of corporate tax evasion with agency costs. It is well known since at least Jensen and Meckling (1976) that the separation of ownership and control in corporations systematically influences corporate decision making, as a privately interested manager may not always act in the shareholders’ best interest. Yet such agency considerations are largely absent from academic studies of tax evasion. The extensive theoretical literature on tax evasion beginning with Allingham and Sandmo (1972) mainly deals with tax evading individuals, rather than firms,\(^6\) although corporate tax evasion is clearly very relevant both as a way for firms to increase their bottom line and from a policy perspective.\(^7\)

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\(^3\)See Skinner and Slemrod (1985) and Lee (2001).

\(^4\)Indeed, as Lee (2001, p 74) states, referring to Slemrod and Sorum (1984), “outright evasion is rare. Instead, taxpayers plan and do research evasion/avoidance in preparation for possible audits”.

\(^5\)According to Feinstein (1991), typical audits fail to detect the full extent of evasion, as Lee (2001) notes.

\(^6\)For surveys of the literature on individual tax compliance, see Andreoni et al. (1998) and Slemrod and Yitzhaki (2002).

\(^7\)Slemrod (2007) estimates that 17 per cent of due corporate income taxes were not paid or collected in the US in 2001. There exist numerous well documented cases of corporations drastically reducing their effective tax rates. See, for instance, Drucker (2010) for the case of Google or Kocieniewski (2011) for the case of General Electric. Corporate tax evasion is a key
Where corporate tax evasion is explicitly studied, the focus is mostly on firms’ external activities in the market rather than on their internal organization. Abstracting from agency considerations, this literature therefore seems to be most applicable in the context of self-employment or small, sole-proprietor businesses. When analyzing the tax compliance behavior of widely held corporations, however, additional issues arise because of the separation of ownership and control, as Slemrod (2004) has noted. The closest work to our analysis is by Crocker and Slemrod (2005), who consider a principal-agent firm with exogenous income and two risk-neutral parties. Their main focus is the relative effectiveness of punishing the agent compared to punishing the principal as a means to curtail tax evasion. Under asymmetric information, they find, punishing a firm-owner is a less effective enforcement policy than punishing the agent or tax manager directly. This is due to the second-best contract that dilutes the impact of punishing the principal under asymmetric information. Agency therefore makes a tax authority’s enforcement policy directed at the firm less effective in their model. Chen and Chu (2005) consider a principal-agent firm, where as in the classic model by Holmstrom (1979), a risk-averse agent is hired to produce output while the risk-neutral principal may decide whether or not to evade taxes. If only the principal is liable for tax evasion, there is no efficiency loss in production due to tax evasion. If, however, the agent is also liable for tax evasion, the efficiency of the contract between principal and agent decreases, because the agent requires additional ex-ante compensation for conducting tax evasion activities. Unlike Chen and Chu (2005) and in line with Crocker and Slemrod (2005), this chapter is not concerned with production efficiency. The agent decides purely on the firm’s tax strategy. Desai and Dharmapala (2006) consider a model in which an agent is hired to perform the risk-free yet costly task of legal tax avoidance for the principal. They posit a complementary relationship between managerial rent diversion and corporate tax avoidance activities and, in the light of corporate scandals such as at Enron, where tax avoidance schemes featured prominently, examine the relationship between high-powered incentive contracts and tax sheltering activities. Desai and Dharmapala (2006) to some extent anticipate an agent performing two tasks, though only one of them pertains to tax avoidance.

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8 For overviews of this literature, see Cowell (2004), Slemrod (2004) and Nur-tegin (2008). For recent contributions, see Bayer and Cowell (2009) and Goerke and Runkel (2011).

9 Besides taking into account agency considerations, recent research on tax evasion has also extended the traditional paradigm to allow for the impact of norms, such as patriotism (Konrad and Qari 2012) or religion (Torgler 2006), and stressed the importance of third party information reporting (Kopzuck and Slemrod 2006, Kleven et al. 2009).
The other concerns managerial self-dealing for the manager’s private gain. This chapter is also related to work by Lipatov (2012), who considers a market with price-setting tax specialists whose services are bought by tax evading firms. As opposed to Lipatov (2012), we follow the principal-agent approach to analyzing corporate tax evasion introduced by Crocker and Slemrod (2005) and Chen and Chu (2005).

Second, the present analysis encompasses an element of self-insurance against the extent of criminal persecution, an idea pioneered by Ehrlich and Becker (1972). In particular, an agent may exert sophistication effort to reduce a firm’s expected punishment by resorting to sophisticated means of tax evasion that legalize these tax reductions or, alternatively, make them harder to detect for tax authorities upon audit. Lee (2001) considers a model of (individual) tax evasion with self-insurance, though not in a principal-agent setting. Biswas et al. (2012) analyze a principal-agent model of corporate tax evasion in which a “gatekeeper” must be hired when evading taxes. The gatekeeper’s effort, observable to the principal, determines firm’s probability of being audited; however, the principal’s (costless) choice of the amount of tax evasion is not part of the contract, and the focus is on symmetric information between principal and agent. Neither model analyzes the situation examined in this chapter, where a tax specialist agent is hired by a generalist firm-owner to determine both the level and the quality of a firm’s tax evasion activities, which is arguably closest to what real-world tax specialists hired in such a context do.

Finally, since the tax manager multitasks to determine a firm’s overall tax evasion profile, this chapter also builds on the multitasking principal-agent literature started by Holmstrom and Milgrom (1991). Multitasking models have been used to analyze a wide variety of situations in which various aspects of the agent’s job are measurable to different extents, such as teacher pay (Holmstrom and Milgrom 1991), the provision of quality by regulated firms (Laffont and Tirole 1991), or contests in which players compete along several dimensions, for instance when engaged in political or labor market competition (Clark and Konrad 2007). A survey of the literature on multitask principal agent problems is provided by Dewatripont et al. (2000). In particular, we employ a multitasking adaptation of the the widely used linear-exponential-normal (LEN) model analyzed in the seminal work of Holmstrom and Milgrom (1987). A canonical version of this model is presented for instance in Bolton and Dewatripont (2005). A recent paper by Ewert and Niemann (2014) also picks up on the idea of using a multitasking model in the context of corporate tax avoidance. It does not consider the quality-quantity distinction analyzed in this chapter, however, and instead examines an agent exerting a classic production effort, to which a tax planning effort is added. Summing up, the main divergence from the related theoretical literature is two-
agents multitask over different elements of corporate tax evasion activity, and the tax authority is explicitly featured as a welfare-maximizing player. It should also be noted that a number of recent studies in accounting research have provided empirical evidence showing that corporate governance characteristics matter for a firm’s tax strategy. Armstrong et al. (2012) find evidence that tax directors are provided with incentives to reduce the firm’s effective tax rate. Moreover, Dyreng et al. (2010) show that individual executives significantly impact a firm’s tax planning activities. Minnick and Noga (2010) conclude that “[corporate] governance plays an important role in tax management”. A recent survey of empirical studies concerning corporate governance and corporate tax management is contained in Hanlon and Heitzman (2010).

The remainder of this chapter is organized as follows. Section 2.2 introduces the formal model and Section 2.3 presents the analysis of the model. In particular, Section 2.3.1 describes the contractual relationship between firm-owner and tax manager, while Section 2.3.2 derives and characterizes the tax authority’s optimal tax enforcement policy. Section 2.4 concludes.

2.2 Model

We consider a corporate tax evasion game with three players: the tax authority $G$, a firm-owner or principal $P$, and a tax manager or agent $A$. The tax authority chooses a tax enforcement policy$^{10}$ to maximize a social welfare criterion. Given this policy and several exogenous factors described below, a firm-owner may contract with a tax specialist to evade taxes in the following manner.

The principal hires an agent to reduce the firm’s tax burden. The agent can exert effort along two dimensions, the quantity and quality of tax evasion. The quantity effort $e_R \in \mathbb{R}^+_0$ represents the tax manager’s work toward claiming illegal tax reductions for the firm. It reduces the firm’s taxable income by underreporting profits and so increases the firm’s expected net profit. Effort $e_R$ reduces the firm’s taxable income by

$$R = e_R + \epsilon_R.$$ 

The principal cannot observe the effort $e_R$, but only the total reductions $R$ of taxable income. $e_R$ is a normally distributed noise term with zero mean and variance $\sigma^2_R$, so $\epsilon_R \sim N(0, \sigma^2_R)$. Since the reductions $R$ constitute illegal tax evasion, they are punishable if detected.

However, the agent may also exert the quality effort $e_S \in \mathbb{R}^+_0$. Doing so reduces the firm’s expected fine by finding sophisticated legal ways of avoiding taxes that

$^{10}$In line with the previous literature, the focus of this chapter is on the choice of the optimal tax enforcement policy, rather than the optimal tax rate $\tau$. 

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protect the firm’s tax reductions from punishment by the authority. Effort $e_S$ produces a level $S$ of sophistication given by

$$S = e_S + \epsilon_S.$$  

As before, the principal cannot observe the effort $e_S$ but only the realized level of sophistication $S$, and the noise term $\epsilon_S$ follows $\epsilon_S \sim N(0, \sigma^2_S)$. The enforcement process that combines detection probability and fines for evasion is characterized by the government’s enforcement policy parameter $\gamma \in [0, 1]$. An amount $R$ in tax reductions will be punished with an expected fine $\gamma(R - S)$. That is, a potential fine will be assessed only on the amount of claimed reductions not covered by sophisticated means, namely the remaining level of illegal tax evasion $R - S$. $e_S$ therefore represents a form of self-protection effort against punishment. The random components are independently distributed, so $\text{Cov}(\epsilon_R, \epsilon_S) = 0$.

The tax manager

The tax manager $A$ chooses the effort levels $e_R$ and $e_S$ and has constant absolute risk averse preferences. His utility function is given by

$$U = -e^{-\eta(w - C(e_R,e_S))}.$$  

where $\eta \in \mathbb{R}^+$ is the agent’s Arrow-Pratt measure of absolute risk aversion ($\eta = -\frac{U''}{U'}$), $w$ is the agent’s monetary compensation, and $C$ is the agent’s cost of effort, measured in monetary units. Effort cost takes the quadratic form

$$C(e_R,e_S) = \frac{1}{2}e^2_R + \frac{1}{2}e^2_S.$$  

Following the widely used assumptions of a linear wage contract, exponential agent utility, and normally distributed errors (LEN-model) based on Holmstrom and Milgrom (1987), the compensation contract between principal and agent has the linear form

$$w = \alpha + \beta_R R + \beta_S S,$$

where $\alpha \in \mathbb{R}_0$ is the fixed wage component and $\beta_R \in \mathbb{R}_0$ and $\beta_S \in \mathbb{R}_0$ are the incentive components relating to $R$ and $S$ respectively. The agent accepts the contract if his expected utility from signing the contract (weakly) exceeds the utility of his outside option $u$, which for simplicity is normalized to $u = -1$.\(^{11}\)

\(^{11}\)Given the assumptions on $U$, $u = -1$ ensures that the agent’s reservation certainty equivalent is equal to zero.
The firm-owner

The firm-owner $P$ is risk neutral and offers a take-it-or-leave-it wage contract $(\alpha, \beta_R, \beta_S)$ to the agent. The principal chooses the contract parameters $\alpha$, $\beta_R$, and $\beta_S$ to maximize her expected profits

$$\mathbb{E}\Pi = \mathbb{E}\left[ I - \tau(I - R) - \gamma\tau(R - S) - w \right].$$

Profits have four components: The firm’s exogenous gross income $I \in \mathbb{R}^+$, the actual tax payment $\tau(I - R)$, where $\tau \in [0, 1]$ is the exogenous corporate income tax rate, the expected fine for evasion $\gamma\tau(R - S)$, and the managerial wage $w = \alpha + \beta_R R + \beta_S S$.

Expected profits are maximized subject to the agent’s participation constraint

$$\mathbb{E}[-e^{-\eta(w-C(e_R,e_S))}] \geq u = -1 \quad (PC)$$

and an incentive compatibility constraint

$$(e_R, e_S) \in \arg\max_{e_R, e_S} \mathbb{E}[-e^{-\eta(w-C(e_R,e_S))}] \quad (IC)$$

As is common in the principal-agent literature, the participation constraint $(PC)$ ensures that in expectation, the agent is at least as well off when he accepts the contract as when he does not. The incentive compatibility constraint $(IC)$ ensures that the agent’s choices of $e_R$ and $e_S$ are indeed optimal for him.

The tax authority

The tax authority chooses an enforcement policy $\gamma \in [0, 1]$ to maximize expected social welfare $\mathbb{E}W$,

$$\mathbb{E}W = \lambda\mathbb{E}\Pi + (1 - \lambda)\mathbb{E}T.$$

Expected social welfare $\mathbb{E}W^{12}$ is the weighted sum of the firm’s expected profits $\mathbb{E}\Pi$, with weight $\lambda \in [0, 1]$, and expected net government revenue $\mathbb{E}T$, with weight $(1 - \lambda)$, which is the sum of tax payments $\tau(I - R)$ and fines $\gamma\tau(R - S)$ less enforcement cost $k(\gamma)$, which is assumed to be quadratic and has a cost parameter $g \in \mathbb{R}^+$

$$k(\gamma) = \frac{1}{2}g\gamma^2.$$

This model therefore contains the case of a (net) revenue maximizing tax authority that is widely analyzed in the tax compliance literature$^{13}$ as a special case in

$^{12}$Note that for the purpose of our analysis, we can exclude the agent’s expected utility $\mathbb{E}U$ from the welfare function $\mathbb{E}W$ without loss of generality, because in equilibrium, the agent will always be reduced to his constant outside option by the principal.

$^{13}$See for instance the survey of the tax compliance literature by Andreoni et al. (1998)
which $\lambda = 0$.

**Timing**

In the first stage of the game, the tax authority sets a policy $\gamma \in [0, 1]$ given its expectation of tax evasion outcomes $R$ and $S$ as well as the exogenous tax rate $\tau \in (0, 1)$ and gross income $I$. In the second stage, the firm-owner offers a wage contract $(\alpha, \beta_R, \beta_S)$ to the tax manager, given the policy $\gamma$, the tax rate $\tau$, and gross income $I$. The manager then decides whether or not to accept the contract. Upon acceptance, the agent exerts efforts $(e_R, e_S)$ in the third stage of the game. Finally, tax evasion outcomes $R$ and $S$ are realized, the contract is executed, and the wage, tax payment, and potential fines are paid.

**2.3 Analysis**

We begin with the contractual relationship between firm-owner and tax manager, who may contract to evade taxes given a tax enforcement policy $\gamma \in [0, 1]$ previously chosen by the tax authority.

**2.3.1 Contracting to evade taxes**

**First best** benchmark with observable efforts

As a benchmark case, consider first the situation in which efforts are observable to the principal. The firm-owner will offer a full insurance contract to the tax manager, that is, she offers a fixed wage giving the agent exactly his outside option while inducing $e^{FB}_R$ and $e^{FB}_S$ such that

$$
\max_{e_R, e_S} \left\{ I - \tau(I - e_R) - \gamma \tau(e_R - e_S) - \frac{1}{2} e^2_R - \frac{1}{2} e^2_S \right\}.
$$

The first-best effort levels $e^{FB}_R$ and $e^{FB}_S$ that solve this problem are given by

$$
e^{FB}_R = (1 - \gamma)\tau \quad \text{and} \quad e^{FB}_S = \gamma \tau. \quad (2.1)
$$

We see that if enforcement $\gamma$ is increased, the first best effort with respect to the quantity evaded will be lowered, whereas the effort concerning legal quality or sophistication will be higher. This dual effect of enforcement will carry over into the analysis under asymmetric information.

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14Following the literature on principal-agent models of optimal tax enforcement based on Reinganum and Wilde (1985), we assume that the tax authority can and does commit to its policy choice. For the discourse regarding this assumption, see the surveys by Andreoni et al. (1998) and Slemrod and Yitzhaki (2002).

15Note that the term “first best” used here strictly refers to the principal-agent relationship and describes the situation in which efforts are observable by the principal. It does not concern the relationship between the tax authority and either principal or agent.
Contracting when efforts are unobservable

Suppose now that the firm-owner cannot observe the tax manager’s effort choices $e_R$ and $e_S$, but only the realized tax evasion outcomes $R = e_R + \epsilon_R$ and $S = e_S + \epsilon_S$. Then the agent is offered a linear incentive scheme $w = \alpha + \beta_R R + \beta_S S$ as described above. The tax manager now chooses the quantity of evasion effort $e_R$ and the quality of evasion effort $e_S$ to maximize his expected utility,

$$\max_{e_R, e_S} \mathbb{E}[-e^{-\eta(w-C(e_R,e_S))}].$$

The agent’s expected utility can be rewritten, using the properties of normally distributed error terms, as

$$\mathbb{E}[-e^{-\eta(w-C(e_R,e_S))}] = -e^{-\eta[\alpha + \beta_R e_R + \beta_S e_S - \frac{1}{2}e_R^2 - \frac{1}{2}e_S^2 - \frac{\eta}{2}(\beta_R^2 \sigma_R^2 + \beta_S^2 \sigma_S^2)]},$$

so that the bracketed term in the exponent is the agent’s certainty equivalent compensation,

$$\alpha + \beta_R e_R + \beta_S e_S - \frac{1}{2}e_R^2 - \frac{1}{2}e_S^2 - \frac{\eta}{2}(\beta_R^2 \sigma_R^2 + \beta_S^2 \sigma_S^2).$$

This is the certain amount of money that makes the agent exactly indifferent between receiving this certain amount and accepting the contract with uncertain payoffs. The agent’s certainty equivalent consists of his expected wage $\alpha + \beta_R e_R + \beta_S e_S$ less his cost of effort $\frac{1}{2}(e_R^2 + e_S^2)$ and a risk premium $\frac{\eta}{2}(\beta_R^2 \sigma_R^2 + \beta_S^2 \sigma_S^2)$, which is due to the unobservability of efforts in combination with the agent’s risk aversion. Maximizing the certainty equivalent compensation is mathematically equivalent to maximizing expected utility, so the agent’s problem may be written as

$$\max_{e_R, e_S} \{ \alpha + \beta_R e_R + \beta_S e_S - \frac{1}{2}e_R^2 - \frac{1}{2}e_S^2 - \frac{\eta}{2}(\beta_R^2 \sigma_R^2 + \beta_S^2 \sigma_S^2) \}.$$

The firm-owner, who offers a wage contract characterized by parameters $\alpha$, $\beta_R$, and $\beta_S$ to maximize her expected profits therefore solves

$$\max_{\alpha, \beta_R, \beta_S} \{ I - \tau(I - e_R) - \gamma \tau(e_R - e_S) - (\alpha + \beta_R e_R + \beta_S e_S) \}$$

subject to

$$\alpha + \beta_R e_R + \beta_S e_S - \frac{1}{2}e_R^2 - \frac{1}{2}e_S^2 - \frac{\eta}{2}(\beta_R^2 \sigma_R^2 + \beta_S^2 \sigma_S^2) \geq 0 \quad \text{(PC)}$$

\[16\] See Bolton and Dewatripont (2005) for the derivation in the single-task case, from which the dual-task case analyzed here follows by using a similar argument.
and

\[(e_R, e_S) \in \arg \max_{e_R, e_S} \left\{ \alpha + \beta_R e_R + \beta_S e_S - \frac{1}{2} e_R^2 - \frac{1}{2} e_S^2 - \frac{\eta}{2} (\beta_R^2 \sigma_R^2 + \beta_S^2 \sigma_S^2) \right\} \quad (IC)\]

In other words, the take-it-or-leave-it contract offered by the firm-owner to the tax manager is such that it maximizes the firm’s expected after-tax profits while ensuring that the agent accepts the contract by awarding him, in expectation, at least his outside option as described in condition \((PC)\), and it provides incentives that take into account the agent’s optimal effort choices, which yields the incentive compatibility constraint \((IC)\). Let us now solve for and characterize the equilibrium contract and effort choices that arise in this setup.

**Proposition 2.1.** [Optimal second best contract]

In the unique equilibrium, the contract offered by the firm-owner satisfies

\[
\beta_R^* = \frac{(1 - \gamma) \tau}{1 + \eta \sigma_R^2}, \quad \beta_S^* = \frac{\gamma \tau}{1 + \eta \sigma_S^2}, \quad \text{and} \quad \\
\alpha^* = \frac{1}{2} \left( \eta \sigma_R^2 - 1 \right) \left( 1 - \gamma \right)^2 \frac{\tau^2}{(1 + \eta \sigma_R^2)^2} + \frac{1}{2} \left( \frac{\eta \sigma_S^2}{1 + \eta \sigma_S^2} - 1 \right) \frac{\gamma^2 \tau^2}{(1 + \eta \sigma_S^2)^2}, \quad (2.2)
\]

and the equilibrium efforts \((e_R^*, e_S^*)\) exerted by the tax manager are given by

\[
e_R^* = \frac{(1 - \gamma) \tau}{1 + \eta \sigma_R^2} \quad \text{and} \quad e_S^* = \frac{\gamma \tau}{1 + \eta \sigma_S^2}, \quad (2.3)
\]

**Proof.** See appendix 2.5.1.

As is common in this type of principal-agent relationship\(^\text{17}\), the second best contract is inefficient because there exists a trade-off between risk and incentives. Optimal risk-sharing would require the risk-neutral principal to bear all risk and hence fully insure the risk-averse agent. Because efforts are not observable, however, such a full insurance contract would not provide optimal incentives to the agent. So, in the second best contract, the principal offers incentive components \(\beta_R^*\) and \(\beta_S^*\) based on the observable realizations of \(R\) and \(S\) as well as a fixed wage \(\alpha^*\), which is set such that, in expectation, the agent is reduced to his reservation utility, meaning his participation constraint \((PC)\) holds with equality. As can be seen from \((2.3)\), the agent’s tax evasion activities will always comprise some positive level of sophistication in equilibrium, except when there is no tax enforcement at all, i.e. when \(\gamma = 0\). It also emerges from \((2.3)\) that when efforts are unobservable, the quantity evaded is lower in expectation than

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\(^{17}\)See, for instance, Laffont and Martimort (2001).
under first best with observable efforts, namely
\[ e^*_R < e^{FB} = (1 - \gamma)\tau. \]

This is a version of the well-known argument that asymmetric information may lead to underprovision of effort compared to the full information case and is a finding first presented in the context of corporate tax evasion by Chen and Chu (2005) and Crocker and Slemrod (2005). Because the focus of our analysis is on the tax authority’s choice of an optimal enforcement policy, let us now examine how the tax authority’s enforcement policy \( \gamma \) affects equilibrium evasion activities \( e^*_R \) and \( e^*_S \).

**Corollary 2.1.** [Effect of enforcement on equilibrium efforts]

Higher enforcement reduces the quantity-of-evasion effort \( e^*_R \), but increases the quality-of-evasion effort \( e^*_S \).

\[ \frac{\partial e^*_R}{\partial \gamma} < 0 \quad \text{and} \quad \frac{\partial e^*_S}{\partial \gamma} > 0 \]

**Proof.** Follows directly from (2.3). \( \square \)

Corollary 2.1 shows the two effects that tax enforcement has when a form of self-insurance against potential punishment for tax evasion is available to the firm. On the one hand, higher enforcement decreases, in expectation, the quantity evaded. On the other hand, the legal quality of the firm’s tax evasion will rise in response to tougher enforcement. In some sense, the existence of sophisticated means of tax evasion that escape punishment therefore compromises the government’s ability to conduct an effective enforcement policy. Stricter enforcement, while reducing the quantity evaded, also drives firms into using more sophisticated means of tax evasion. This dual effect hints at a trade-off the tax authority faces in setting its optimal policy, as we explore in more detail below. Since, in this model, both types of evasion activities are produced in a principal-agent framework, the accuracy of the performance measures \( R \) and \( S \) used for contracting will impact the effect enforcement has on efforts \( e^*_R \) and \( e^*_S \) exerted in equilibrium, as the following corollary shows.
Corollary 2.2. [Performance measure accuracy and the effect of enforcement]

(i) A higher variance $\sigma^2_R$ diminishes the (negative) impact of enforcement on the quantity evaded

\[
\frac{\partial}{\partial \sigma^2_R} \left| \frac{\partial e^*_R}{\partial \gamma} \right| < 0. 
\]

(ii) A higher variance $\sigma^2_S$ diminishes the (positive) impact of enforcement on the quality of tax evasion

\[
\frac{\partial}{\partial \sigma^2_S} \left| \frac{\partial e^*_S}{\partial \gamma} \right| < 0. 
\]

Proof. See appendix 2.5.2.

Part (i) of Corollary 2.2 states that when efforts are unobservable, a less accurate performance measure for the quantity effort $e_R$ leads to less effective enforcement. This is the analogue in this model of the familiar result from Crocker and Slemrod (2005), which states that because enforcement directed at the principal impacts the agent only indirectly, through a second best wage contract, its impact on tax evasion activities is weakened compared to a situation with observable efforts. There exists, however, a second effect when efforts are unobservable by the principal. This second effect is stated in part (ii) of Corollary 2.2: A less accurate performance measure for the quality effort $e_S$ lowers the extent to which stricter enforcement (higher $\gamma$) drives up sophistication effort. Because a high variance $\sigma^2_S$ makes sophistication effort $e_S$ costly to incentivize, the principal is less inclined to counter stricter enforcement by commissioning a more sophisticated way of evading taxes than she would be in a first best world with observable efforts. The following section examines how the principal-agent contracting between firm-owner and tax manager characterized here shapes the tax authority’s enforcement policy.

2.3.2 Optimal enforcement policy

Consider now the tax authority’s problem of choosing an enforcement policy $\gamma \in [0, 1]$ to maximize expected social welfare $\mathbb{E}W$. Recall from above that the tax authority’s objective function is given by

\[
\mathbb{E}W = \lambda \mathbb{E}II + (1 - \lambda) \mathbb{E}T \quad (2.4)
\]

That is, the tax authority chooses a policy $\gamma$ to maximize a weighted average of expected profits and expected tax revenue. We proceed to examining the components of the government’s objective function (2.4) in the light of the equilibrium tax evasion contract between firm-owner and tax manager as obtained in Proposition 2.1.
Expected profits carry the welfare weight $\lambda \in [0, 1]$ and are given in the firm-owner’s maximization problem above. Using the expressions for the tax manager’s equilibrium effort choices $e^*_R$ and $e^*_S$ from Proposition 2.1, expected profits may be written as

$$E\Pi = (1 - \tau)I + \frac{1}{2}(1 - \gamma)^2 \frac{\tau^2}{\phi_R} + \frac{1}{2} \gamma^2 \frac{\tau^2}{\phi_S}$$

where we defined

$$\phi_R = 1 + \eta \sigma^2_R \quad \text{and} \quad \phi_S = 1 + \eta \sigma^2_S.$$

In the absence of any evasion activities, the firm’s net profits are given by $(1 - \tau)I$. It follows from (2.5) that since two non-negative terms are added to $(1 - \tau)I$, the firm’s expected profits increase due to tax evasion in equilibrium. Also notice that expected net profits are decreasing in both $\phi_R$ and $\phi_S$. This means in particular that as the performance measures $R$ and $S$ become less accurate ($\sigma^2_R$ and $\sigma^2_S$ increase), the firm owner’s benefit from tax evasion decreases. This is intuitively convincing: The more costly it is to incentivize tax evasion activities due to difficulties in measuring the agent’s performance accurately, the less profitable it is for the principal to do so. Conversely, expected net tax revenue $ET$, which includes tax payments and fines less enforcement cost, carries the welfare weight $(1 - \lambda)$ and is given by

$$ET = \tau(I - e_R) + \gamma \tau(e_R - e_S) - \frac{1}{2} g \gamma^2$$

Plugging in the tax manager’s equilibrium effort choices $e^*_R$ and $e^*_S$, expected tax revenue becomes

$$ET = \tau I - (1 - \gamma)^2 \frac{\tau^2}{\phi_R} - \gamma^2 \frac{\tau^2}{\phi_S} - \frac{1}{2} g \gamma^2.$$  

(2.6)

Expected tax revenue is lowered by tax evasion activities, as can be seen from (2.6). Two non-negative terms associated with the two dimensions of tax evasion, quantity and quality, are subtracted from $\tau I$, which is the level of tax revenue absent any tax evasion. In analogy to the case of expected profits, we now find that expected tax revenue increases in $\phi_R$ and $\phi_S$. The more costly it is to incentivize tax evasion efforts due to agency problems, the less of a dent such activities will make on the public purse. The last term $\frac{1}{2} g \gamma^2$ in (2.6) accounts for the monetary cost of tax enforcement, which is increasing and quadratic in the intensity of tax enforcement.

Combining the equilibrium expressions for expected profits (2.5) and expected
net tax revenue (2.6), the government chooses \( \gamma \in [0, 1] \) to maximize
\[
\lambda I + (1 - 2\lambda)\tau I - \frac{1}{2} (2 - 3\lambda)(1 - \gamma)^2 \frac{\tau^2}{\phi_R} - \frac{1}{2} (2 - 3\lambda)\gamma^2 \frac{\tau^2}{\phi_S} - (1 - \lambda)\frac{1}{2} g^2 \gamma^2.
\] (2.7)

It follows from (2.7) that if \( \lambda < \frac{2}{3} \), tax evasion leads to an overall loss in expected welfare. To see this, note that \( \lambda < \frac{2}{3} \) implies \( 2 - 3\lambda > 0 \); thus, the two terms in (2.7) associated with the two dimensions of tax evasion negatively impact expected social welfare. So, if tax revenue is a sufficiently important component of social welfare, namely if \( \lambda \in [0, \frac{2}{3}) \), tax evasion is socially harmful. If on the other hand \( \lambda > \frac{2}{3} \), that is profits are heavily weighted in social welfare compared to tax revenues, we see from (2.7) that tax evasion leads to a welfare gain. If \( \lambda = \frac{2}{3} \), the effects of tax evasion on profits and on tax revenue exactly cancel each other out and social welfare is not affected by tax evasion. From (2.7), it is seen that for a given policy \( \gamma \), the unobservability of efforts diminishes the effects of tax evasion on social welfare. As \( \phi_R \) and \( \phi_S \) become very large, social welfare approaches the level it would attain absent any tax evasion activities, as these activities become prohibitively costly to incentivize and equilibrium efforts \( e^{*}_R \) and \( e^{*}_S \) approach zero. Let us now consider the tax authority’s optimal choice of tax enforcement policy.

**Proposition 2.2.** [Tax enforcement policy in equilibrium]

Suppose \( \sigma^2_S > \sigma^2_R \). Then the equilibrium level of tax enforcement is given by
\[
\gamma^{*} = \begin{cases} 
\frac{\phi_S}{\phi_S + \phi_R + \frac{(1 - \lambda)^2 \phi_R}{(2 - 3\lambda)^2 \phi_S}} & \text{if } \lambda \in [0, \frac{2}{3}) \\
0 & \text{if } \lambda \in \left[ \frac{2}{3}, 1 \right]
\end{cases}
\]

*Proof.* See appendix 2.5.3. \( \square \)

At the optimum, the marginal change in social welfare in response to a change in tax enforcement activity equals the marginal cost of tax enforcement so that the first order condition of the government’s problem is satisfied. Proposition 2.2 shows that if profits are heavily weighted in the social welfare function, namely when \( \lambda \in \left[ \frac{2}{3}, 1 \right] \) so that tax evasion is socially beneficial, the government’s optimal policy is to conduct no enforcement at all and set \( \gamma^{*} = 0 \). Intuitively, since enforcement is costly and reduces a socially beneficial activity, it can only be optimal to enforce as little as possible.

The more interesting and relevant case is when tax revenues matter enough to social welfare for tax evasion to be a socially harmful activity. This is the case when \( \lambda < \frac{2}{3} \), which we assume in what follows. Before proceeding to analyze how the principal-agent relationship between firm-owner and tax manager influences tax enforcement policy, let us briefly note some properties of the equilibrium enforcement policy that emerge immediately from Proposition 2.2 and describe
the equilibrium effects of the cost of enforcement $g$, the welfare weight $\lambda$, and the tax rate $\tau$. Unsurprisingly, the optimal enforcement intensity decreases in the cost parameter of enforcement, $g$. Moreover, the level of enforcement $\gamma^*$ is decreasing in the welfare weight $\lambda$, meaning that if profits are a more heavily weighted component of social welfare, optimal enforcement will correspondingly be lower. This follows immediately from the fact that firms benefit from tax evasion. Optimal enforcement $\gamma^*$ is also increasing in the tax rate $\tau$, suggesting that tax rates and the level of tax enforcement are complementary policy choices.

The main focus of this chapter is the effect of the principal-agent relationship inside the firm on the tax authority’s enforcement policy choice. We see from Proposition 2.2 that the equilibrium enforcement intensity is decreasing in $\phi_R = 1 + \eta \sigma^2_R$ but increasing in $\phi_S = 1 + \eta \sigma^2_S$. This result is crucial for the subsequent analysis and is stated formally in the following corollary.

**Corollary 2.3.** [Performance measure accuracy and equilibrium tax enforcement]

Suppose $\lambda < \frac{2}{3}$. Then

$$\frac{\partial \gamma^*}{\partial \phi_R} < 0 \quad \text{and} \quad \frac{\partial \gamma^*}{\partial \phi_S} > 0.$$ 

*Proof.* Follows directly from Proposition 2.2 by taking partial derivatives. 

Corollary 2.3 shows the two countervailing effects of asymmetric information inside the firm on optimal tax enforcement. A less accurate performance measure $R$ for the quantity task (i.e. a higher $\sigma^2_R$) will lower optimal enforcement $\gamma^*$. This is in line with our finding from Corollary 2.2 (i) above: When the quantity effort $e_R$ is costly to incentivize because it is difficult to measure, enforcement will be less effective at reducing the quantity effort, and so the equilibrium level of enforcement $\gamma^*$ will be lower. By contrast, equilibrium enforcement $\gamma^*$ increases in $\phi_S = 1 + \eta \sigma^2_S$. This means that as the quality or sophistication task becomes more costly to incentivize for the firm-owner due to a less accurate performance measure $S$ (i.e. a higher $\sigma^2_S$), the tax authority’s optimal enforcement policy choice $\gamma^*$ increases. Again, this corresponds to Cororally 2.2 (ii) above: Agency frictions in the quality task $e_S$ make enforcement more effective, as they make it more costly for the firm-owner to respond to enforcement by incentivizing the agent to pursue sophisticated means of tax evasion. The observations formalized in Corollary 2.3 provide the intuition for the following analysis. We ask how the unobservability of efforts and the resulting second best nature of the tax evasion contract between firm-owner and tax manager influence the tax authority’s enforcement policy compared to the first best case where efforts are observable and the contract between firm-owner and tax manager is not distorted by the tax
manager’s private information. Denote by $\gamma^{FB}$ the tax authority’s equilibrium policy choice when the firm-owner can observe the tax manager’s effort choices, so that their contract is a first-best contract. We want to know how the tax authority’s second-best policy choice $\gamma^*$ relates to $\gamma^{FB}$. Therefore, we deduce the effect of agency frictions between a firm-owner and her tax manager on the government’s tax enforcement policy.

**Corollary 2.4.** [Observability of efforts and equilibrium enforcement policy] Suppose that $\lambda < \frac{2}{3}$. Then

$$\gamma^* > \gamma^{FB} \text{ if and only if } \phi_R < \frac{\phi_S + \rho \phi_S}{1 + \rho \phi_S},$$

where $\rho = \frac{(1 - \lambda)g}{(2 - 3\lambda)\tau^2}$.

**Proof.** See appendix 2.5.4.

According to Corollary 2.3 above, there are two countervailing effects of the agent’s private information on the tax authority’s equilibrium policy choice. On the one hand, enforcement becomes less effective due to the tax manager’s private information in the quantity dimension. But, on the other hand, enforcement becomes more effective due to private information in the quality dimension. Corollary 2.4 provides the condition that decides which of these effects dominates when we compare the optimal second-best policy $\gamma^*$ to the optimal tax enforcement policy in the first-best situation with observable efforts. If efforts are unobservable, both dimensions of effort become more costly to incentivize because of inefficient risk sharing in the second-best contract. The extent of these effects is captured by the levels of $\phi_R$ and $\phi_S$. If $\phi_S$ is sufficiently much larger than $\phi_R$ so that the condition in Corollary 2.4 is satisfied, equilibrium enforcement $\gamma^*$ in second best is stricter than in the full information case $\gamma^{FB}$. The following figure illustrates how the relative sizes of $\phi_R$ and $\phi_S$ impact the tax authority’s choice of optimal tax enforcement for given parameters $g$, $\lambda$, and $\tau$. 

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The concave curve in the above figure depicts the equilibrium level of enforcement chosen by the tax authority when efforts are observable, as it passes through the origin, which represents this first best case. As $\phi_S$ and $\phi_R$ rise from the origin in proportions along this curve, that is the performance measures $R$ and $S$ become less accurate in these proportions, the optimal tax enforcement policy does not change. This is because the two opposing effects of the tax manager’s private information on tax enforcement policy described in Corollary 2.3 exactly cancel each other out, leaving us with $\gamma^* = \gamma^{FB}$. If $\phi_S$ relative to $\phi_R$ rises by more than the proportions along the curve, optimal tax enforcement in equilibrium will be in the region below this curve and therefore stricter than if efforts were observable. Similarly, the region above the curve (but below the 45°-line) contains the combinations $(\phi_S, \phi_R)$ for which optimal tax enforcement policy in second best is less strict when the firm-owner cannot observe the tax manager’s effort. Our result stands in stark contrast to the existing literature. Crocker and Slemrod (2005) find that agency frictions inside the firm unambiguously lower the effectiveness of tax enforcement directed at the firm-owner. If enforcement is costly, this implies a lower level of tax enforcement as a result of asymmetric information inside the firm. We find that this effect of lowering enforcement effectiveness is reversed, exactly cancelled out, or at least mitigated by a second, counteracting effect which enhances enforcement effectiveness: Agency frictions due to contracting inside the firm impede the firm-owner’s ability to respond to stricter
enforcement by evading in a more sophisticated, unpunishable way instead of lowering the quantity evaded. In particular, Corollary 2.4 shows that if sophisticated means of tax avoidance are much less understandable, and hence less measurable, to the non-specialist firm-owner than the amount of tax reductions claimed in an unsophisticated manner, the effect of principal-agent contracting in corporate tax evasion is to make tax enforcement more effective, rather than less effective as previously thought.

2.4 Conclusion

This chapter investigates the optimal enforcement policy that a tax authority chooses to fight corporate tax evasion when such evasion activities take place in a principal-agent setting. Pioneered by Crocker and Slemrod (2005) and Chen and Chu (2005), this approach accounts for the fact that firm-owners are often less informed about the specifics of tax management than the specialist tax managers they employ to reduce a company’s tax burden. In contrast to the existing literature, we assume a firm’s tax evasion activities to consist of two dimensions, quantity and quality, and argue that these dimensions are affected by informational asymmetries to different extents. A higher quality of tax evasion reduces the firm’s expected fine for tax evasion. But highly sophisticated tax evasion schemes are difficult to understand for a non-specialist shareholder, thus the quality dimension might be where the informational asymmetry is largest. We analyze a multitasking model to allow for different levels of agency effects and find that tax enforcement may become more effective as a result of principal-agent contracting in firms because the firm chooses not only to evade less but also evades in a less sophisticated way due to asymmetric information. This suggests a stricter optimal corporate tax enforcement policy where firms enter principal-agent relationships to evade taxes, rather than a more lenient one as suggested by the earlier literature.
2.5 Appendix

2.5.1 Proof of Proposition 2.1

We begin with the tax manager’s problem. The agent chooses efforts $e_R, e_S$ to solve

$$\max_{e_R, e_S} \alpha + \beta_R e_R + \beta_S e_S - \frac{1}{2}e_R^2 - \frac{1}{2}e_S^2 - \eta \left(\frac{\beta^2_R\sigma^2_R + \beta^2_S\sigma^2_S}{2}\right)$$

The first-order conditions for the agent’s problem are given by

w.r.t. $e_R$:

$$\beta_R - e_R = 0 \quad (2.10)$$

and

w.r.t. $e_S$:

$$\beta_S - e_S = 0 \quad (2.11)$$

which since both equations are linear in $e_R$ and $e_S$ respectively has the unique solution

$$(e^*_R, e^*_S) = (\beta_R, \beta_S)$$

Since the agent’s objective function is strictly concave in $e_R$ and $e_S$ (negative definite Hessian), $(e^*_R, e^*_S) = (\beta_R, \beta_S)$ is the unique maximum.

Now consider the principal’s problem. The agent’s participation constraint $(PC)$ is binding at the optimum, for if it were not, the principal could marginally lower $\alpha, \beta_R,$ or $\beta_S$ (or any linear combination thereof) to increase her profits while still satisfying $(PC)$. Replacing the incentive compatibility constraint $(IC)$ by the agent’s unique optimal effort choice $(e^*_R, e^*_S) = (\beta_R, \beta_S)$, the principal’s unconstrained maximization problem becomes

$$\max_{\beta_R, \beta_S} I - \tau(I - \beta_R) - \gamma\tau(\beta_R - \beta_S) - \frac{1}{2}(\beta^2_R + \beta^2_S) - \frac{\eta}{2}(\beta^2_R\sigma^2_R + \beta^2_S\sigma^2_S)$$

The first-order conditions for the principal’s problem are given by

w.r.t. $\beta_R$:

$$\tau - \gamma\tau - \beta_R - \eta\sigma^2_R\beta_R = 0 \quad (2.12)$$

and

w.r.t. $\beta_S$:

$$\gamma\tau - \beta_S - \eta\sigma^2_S\beta_S = 0 \quad (2.13)$$

which since both equations are linear in $\beta_R$ and $\beta_S$ respectively has the unique solution

$$(\beta^*_R, \beta^*_S) = \left(\frac{(1 - \gamma)\tau}{1 + \eta\sigma^2_R}, \frac{\gamma\tau}{1 + \eta\sigma^2_S}\right)$$

The principal’s objective function is strictly concave in $\beta_R$ and $\beta_S$ (negative definite Hessian) and so $(\beta^*_R, \beta^*_S)$ is the unique maximum.
From the binding participation constraint \((PC)\), it follows that
\[
\alpha^* = \frac{1}{2} (\eta \sigma^2_R - 1) (1 - \gamma)^2 \tau^2 + \frac{1}{2} (\eta \sigma^2_S - 1) \frac{\gamma^2 \tau^2}{(1 + \eta \sigma^2_S)^2}.
\]

Finally, plugging \((\beta^*_R, \beta^*_S)\) into the agent’s unique best response, we obtain equilibrium efforts as
\[
(e^*_R, e^*_S) = (\beta^*_R, \beta^*_S) = \left( \frac{(1 - \gamma) \tau}{1 + \eta \sigma^2_R}, \frac{\gamma \tau}{1 + \eta \sigma^2_S} \right)
\]

\[\square\]

2.5.2 Proof of Corollary 2.2

(i) \[
\frac{\partial}{\partial \sigma^2_R} \left| \frac{\partial e^*_R}{\partial \gamma} \right| = \frac{\partial}{\partial \sigma^2_R} \left| \frac{-\tau}{1 + \eta \sigma^2_R} \right| = - \frac{\tau \eta}{(1 + \eta \sigma^2_R)^2} < 0
\]

(ii) \[
\frac{\partial}{\partial \sigma^2_S} \left| \frac{\partial e^*_S}{\partial \gamma} \right| = \frac{\partial}{\partial \sigma^2_S} \left| \frac{\tau}{1 + \eta \sigma^2_S} \right| = - \frac{\tau \eta}{(1 + \eta \sigma^2_S)^2} < 0
\]

\[\square\]

2.5.3 Proof of Proposition 2.2

**Suppose first that** \(\lambda \in [0, \frac{2}{3})\).

The first-order condition of the tax authority’s problem of choosing \(\gamma \in [0, 1]\) to maximize \(\mathbb{E}W\) is given by
\[
(2 - 3\lambda)(1 - \gamma) \frac{\tau^2}{\phi_R} - (2 - 3\lambda) \gamma \frac{\tau^2}{\phi_S} = (1 - \lambda)g \gamma \quad (2.8)
\]

Solving this equation for \(\gamma\) yields the unique solution
\[
\gamma^* = \frac{\phi_S}{\phi_S + \phi_R + \frac{(1 - \lambda)g}{(2 - 3\lambda)\tau^2} \phi_R \phi_S}
\]

The second-order condition for \(\gamma^*\) to be a maximum is given by
\[
-(2 - 3\lambda) \frac{\tau^2}{\phi_R} - (2 - 3\lambda) \frac{\tau^2}{\phi_S} - (1 - \lambda)g < 0
\]

and rearranging, it is seen that the second-order condition holds if and only if
\[
\lambda < \frac{2 + \frac{g \phi_R \phi_S}{\tau^2 \phi_R + \phi_S}}{3 + \frac{g \phi_R \phi_S}{\tau^2 \phi_R + \phi_S}} \equiv \lambda' \quad (2.14)
\]

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Thus a sufficient condition for \( \gamma^* = \frac{\phi_S}{\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)^2}\phi_R\phi_S} \) to be the optimal choice is

\[ \lambda < \frac{2}{3}. \]

**Now suppose** \( \lambda = \frac{2}{3} \).

Then the government’s objective function is obtained by using \( \lambda = \frac{2}{3} \) in (2.7) as

\[ \frac{1}{3}(2-\tau)I - \frac{1}{6}g\gamma^2 \]

and so social welfare strictly decreases in \( \gamma \) for \( \gamma \in [0, 1] \).

Therefore, \( \gamma^* = 0 \) if \( \lambda = \frac{2}{3} \).

**Now suppose** \( \lambda \in \left(\frac{2}{3}, \bar{\lambda}\right) \), where \( \bar{\lambda} = \frac{2+\frac{g}{\tau^2}\phi_R\phi_S}{3+\frac{g}{\tau^2}\phi_R\phi_S} \) as in (2.14).

Then since \( \lambda < \bar{\lambda} \), the tax authority’s objective function (2.7) is strictly concave in \( \gamma \) and so the unique maximum of the objective function is found at \( \frac{\phi_S}{\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)^2}\phi_R\phi_S} \). However, one obtains that

\[ \frac{2}{3} < \lambda < \bar{\lambda} \Rightarrow \frac{\phi_S}{\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)^2}\phi_R\phi_S} < 0 \]

and so by the property that a quadratic function has only one extremum, and the government chooses \( \gamma \in [0, 1] \), it follows that the tax authority’s objective function (2.7) is strictly decreasing on \([0, 1] \). Therefore, \( \gamma^* = 0 \) if \( \lambda \in \left(\frac{2}{3}, \bar{\lambda}\right) \).

**Now suppose** \( \lambda = \bar{\lambda} = \frac{2+\frac{g}{\tau^2}\phi_R\phi_S}{3+\frac{g}{\tau^2}\phi_R\phi_S} \).

Then the tax authority’s objective function given by (2.7) is linear in \( \gamma \) with negative slope

\[ -\frac{g\tau^2\phi_S}{g\phi_R\phi_S + 3\tau^2(\phi_R + \phi_S)} \]

as can be seen by plugging \( \lambda = \bar{\lambda} \) into (2.7) and taking the derivative of the resulting expression with respect to \( \gamma \). Since welfare is strictly decreasing in \( \gamma \), it follows that \( \gamma^* = 0 \) is the optimal choice when \( \lambda = \bar{\lambda} \).

**Finally, suppose** \( \lambda \in (\bar{\lambda}, 1] \).

Then the government’s objective function (2.7) is strictly convex in \( \gamma \), and so \( \frac{\phi_S}{\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)^2}\phi_R\phi_S} \) marks the global minimum of the objective function (2.7). This implies that the maximum \( \gamma^* \) has to be a corner solution, i.e. either \( \gamma^* = 0 \) or \( \gamma^* = 1 \). Since the objective function (2.7) is quadratic in \( \gamma \), we know it is symmetric about its minimum. Since \( \gamma \in [0, 1] \), symmetry implies that \( \gamma^* = 0 \) is
the optimal choice if the minimum is found to the right of \( \gamma = \frac{1}{2} \), i.e. when

\[
\frac{\phi_S}{\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}\phi_R\phi_S} > \frac{1}{2}
\]

This is equivalent to

\[
\phi_S - \phi_R > \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}\phi_R\phi_S
\]

And since we assumed \( \sigma_S^2 > \sigma_R^2 \), the left hand side of this inequality is positive while the right hand side is negative (recall that \( \lambda > \frac{2}{3} \)) and so this inequality always holds. So we have \( \gamma^* = 0 \) for \( \lambda \in (\bar{\lambda}, 1] \).

### 2.5.4 Proof of Corollary 2.4

We first derive the equilibrium tax enforcement policy \( \gamma^{FB} \) chosen by the tax authority when the tax manager’s efforts \( e_R \) and \( e_S \) are observable to the firm-owner. Recall from (2.1) that the first best effort levels induced in this case are given by

\[
e^{FB}_R = (1 - \gamma)\tau \quad \text{and} \quad e^{FB}_S = \gamma\tau.
\]

Using \( e^{FB}_R \) and \( e^{FB}_S \) in the social welfare function from (2.4), we obtain the tax authority’s problem in first best as

\[
\max_{\gamma} \{ \lambda I + (1 - 2\lambda)\tau I - \frac{1}{2}(2 - 3\lambda)(1 - \gamma)^2\tau^2 - \frac{1}{2}(2 - 3\lambda)\gamma^2\tau^2 - (1 - \lambda)\frac{1}{2}g\gamma^2 \}
\]

This problem is strictly concave in \( \gamma \) and has the unique solution

\[
\gamma^{FB} = \frac{1}{2 + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}}.
\]

Comparing \( \gamma^{FB} \) to the second-best optimal enforcement intensity \( \gamma^* \) when \( \lambda < \frac{2}{3} \) stated in Proposition 2 gives

\[
\gamma^* = \frac{\phi_S}{\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}\phi_R\phi_S} > \frac{1}{2 + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}} = \gamma^{FB}
\]

if and only if

\[
\phi_S + \phi_R + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}\phi_R\phi_S < \phi_S(2 + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2})
\]

if and only if

\[
\phi_R < \frac{\phi_S + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}\phi_S}{1 + \frac{(1-\lambda)g}{(2-3\lambda)\tau^2}\phi_S}.
\]

\( \square \)
Chapter 3

Financial contracting with tax evaders

3.1 Introduction

Tax evasion poses an interesting challenge for financial contracting. Since it is illegal, the potential gains from evading taxes (a lower tax bill) are typically not contractible. However, its potential losses (fines) may reduce a borrower’s ability to repay investors, as long as the government’s claims are sufficiently senior. An entrepreneur protected by limited liability, who borrows money to finance a project, may use precisely this asymmetry to her advantage. She might evade more, for instance, than she would if she were fully self-financed, because the gains from evasion are entirely hers if tax evasion goes undetected, while the loss in case of detection is, at least partially, borne by the investors.

This chapter analyzes financial contracting under such circumstances. If an entrepreneur may evade taxes, we ask, what will the optimal financial contract between an investor and this potentially tax-evading entrepreneur look like? Using a model of costly state verification, our analysis builds on the seminal work of Townsend (1979) and Gale and Hellwig (1985), who assume verification of an entrepreneur’s private information (e.g. about a venture’s financial success) to be costly. Their optimal financial contract - standard debt - is optimal because it minimizes the cost of verification. In our model, rather than assuming verification to be costly per se, we observe that it influences the entrepreneur’s ability to evade taxes. Verification likely delivers information about the true state of the world not only to the investor, but also to the government, as it involves courts and other third parties with informational duties such as auditing firms. Additional information for the government, however, makes tax evasion less profitable in expectation for the entrepreneur. This decreased profitability of tax evasion is the cost of investor verification in our model. An entrepreneur seeking to evade
taxes thus proposes a financial contract that minimizes verification, rather like in Townsend (1979) and Gale and Hellwig (1985). However, we find that the optimal contract in this setting is not standard debt. Instead, the optimal contract is debt-like only for very low and very high profit realizations. For intermediate realizations, it demands a constant repayment to the investor and verification of returns, thus leaving some rent for the entrepreneur. This intermediate range of the contract therefore combines elements of equity - leaving some rent to the entrepreneur despite verifying the true state of the world - with elements of debt, namely a constant repayment.

This is because an optimal contract in this setting, besides minimizing investor verification, faces the particular constraints imposed by tax evasion mentioned in the first paragraph above. Gains from evasion are not contractible, but an entrepreneur’s limited liability may make the investor partially liable for fines in case tax evasion is detected. This occurs when fines for evasion exhaust the entrepreneur’s funds so that the repayment stipulated in the contract cannot be made in full. An optimal financial contract hence stipulates only repayments that are feasible even if the entrepreneur’s tax evasion activities are detected. In particular, this means that except for very low profit realizations, the repayment is always sufficiently far below the entrepreneur’s available funds. This leaves the entrepreneur with a positive rent that prevents her from abusing her limited liability protection for excessive tax evasion activities. The present chapter is mainly related to two strands of the economics literature.

First, it builds on and contributes to the large literature on financial contracting and security design. Surveys of this literature include Hart (2001), Harris and Raviv (1995), and Franklin and Winton (1995). More specifically, we use a model of costly state verification pioneered by Townsend (1979) and Gale and Hellwig (1985). Other early contributions include Diamond (1984) and Williamson (1986). Canonical treatments of this model are found in Tirole (2005), Freixas and Rochet (2008), and Bolton and Dewatripont (2005). The basic costly state verification framework has been extended into a variety of directions such as allowing for random auditing (Mookherjee and Png 1989), multiple investors (Winton 1995), and multiple periods (Chang 1990 and Webb 1992). However, no study to date has examined the impact of tax evasion on financial contracting. We propose to extend the basic costly state verification model in this direction. Like for several other extensions, we find that the chief result of the basic costly state verification framework - the optimality of standard debt contracts - is not robust to allowing for tax evasion by borrowers.

Besides the original contributions of Townsend (1979) and Gale and Hellwig (1985), who provide the basic framework that we extend by allowing for an entrepreneur’s tax evasion, a work related to our analysis is Povel and Raith
(2004). They consider financial contracting when both an entrepreneur’s investment choice and the revenue realization are unobservable and not verifiable to outside investors. Tax evasion can be interpreted as an unobservable investment by the entrepreneur, and in this sense the present analysis is related to Povel and Raith (2004). However, in our model, tax evasion is verifiable upon audit. Similarly, revenues can be verified in our model by the investor, and so we operate in a very different framework from that of Povel and Raith, namely a framework of costly state verification.

Second, the present chapter is connected to the literature on the economics of tax evasion. For general surveys of this literature, see Slemrod and Yitzhaki (2002), Andreoni et al. (1998), or Slemrod (2007). In particular, we consider an entrepreneur’s tax evasion choice and its ramifications for corporate financing. Within the economics of tax evasion, our analysis is therefore located in the much smaller part of the literature concerning corporate tax evasion and avoidance, as surveyed by Slemrod (2004) and Nur-Tegin (2008). A key difference between individual and corporate tax evasion is the existence of informational asymmetries between different stakeholders of the corporation, rather than just between taxpayer and government. Crocker and Slemrod (2005) and Chen and Chu (2005) initiated this approach by examining the impact of informational asymmetries between managers and shareholders of the tax evading firm. Surveys of the fairly recent integration of the theory of tax evasion with principal-agent analysis are contained in Hanlon and Heitzman (2010) and Armstrong et al. (2014). While related, our focus is on the interaction of corporate financing and tax evasion, rather than on corporate governance and tax evasion. And indeed, although the general literature on taxation and corporate finance is large (see Auerbach 2002 or Graham 2003 for surveys), the impact of tax evasion on optimal security design that we analyze here has not previously been studied.

The remainder of this chapter is organized as follows. In section 3.2, we introduce the setup and timing of our formal model. Section 3.3 presents the analysis. In particular, section 3.3.1 derives the entrepreneur’s tax evasion choice and section 3.3.2 her reporting behavior toward the investor. In section 3.3.3 we derive the optimal financial contract in this setting. Section 3.4 concludes.

### 3.2 Model

Consider a risk-neutral, zero-wealth entrepreneur, $E$, who requires funding from a risk-neutral investor, $I$, to finance a project. The project turns a unit investment provided by $I$ into a random return $x$, which is uniformly distributed on the interval $[x, \pi]$. The timing of the game is as follows (see Figure 3.1).

In stage 0 of the game, the entrepreneur offers a take-it-or-leave-it financial
contract, \((R, \beta)\), to the investor. This contract combines a repayment function, \(R(\hat{x}, x)\), with a verification function, \(\beta(\hat{x})\), as is standard in the literature on costly state verification. That is, \(E\) proposes to issue a security. \(I\) accepts the offer if his expected payoff from it is at least \(v\), his reservation utility. At the time of contracting, both players have symmetric information.

![Figure 3.1: Timing of the game](image)

In stage 1, the entrepreneur *privately* observes the realized return \(x\) and makes a report, \(\hat{x}\), about it to the investor, which may or may not be truthful.

In stage 2 of the game, the investor either verifies \(E\)'s report and learns the true return \(x\), or he accepts the report and does not verify. We assume deterministic and contractible verification, so that the verification probability \(\beta(\hat{x})\) is a function \(\beta : [x, \overline{x}] \rightarrow \{0, 1\}\).

In stage 3, the entrepreneur needs to make a tax report, \(y\), to the tax authority, \(G\).\(^1\) The true return \(x\) is subject to taxation at rate \(\tau \in [0, 1]\). We denote by \(e = x - y\) the amount of underreporting, or evasion. Tax evasion is costly to the entrepreneur, and we denote this cost by \(c(e)\), assumed to take the quadratic form \(c(e) = ke^2\), where \(k \geq 0\) is a given cost parameter.

In stage 4, the tax authority may learn about the true realization \(x\). Without verification (i.e. if \(\beta(\hat{x}) = 0\)), it does so with a fixed probability, \(p\), its baseline audit probability. If, however, a report is verified (i.e. if \(\beta(\hat{x}) = 1\)) by the investor, this entails public revelation\(^2\) and so the tax authority learns the true \(x\) with certainty. If the tax authority learns the true \(x\), then in addition to the tax payment \(\tau y\) based on \(E\)'s reported profit, it collects a fine \(\lambda \tau e\) on evaded taxes, where \(\lambda > 1\).

In the last stage of the game, the repayment \(R(\hat{x}, x)\) stipulated in the financial contract is transferred to the investor.\(^3\) Under non-verification of a report, this

---

\(^1\)The tax authority \(G\) is assumed to be a static entity, not a player of the game.

\(^2\)In the classic interpretation due to Gale and Hellwig (1985), verification is interpreted as the initiation of bankruptcy proceedings. Involving the authorities, courts, and third parties with strictly regulated informational duties such as auditing firms, it is plausible to assume that such proceedings deliver information about the true state of the world to the government. For simplicity, we assume this information to be perfect. In this sense the chapter is also connected to a recent literature on third-party information reporting in tax enforcement (see, for instance, Kopczuk and Slemrod (2006), Gordon and Li (2009), or Kleven et al. (2009)).

\(^3\)Note that this implies absolute priority of government claims over creditor claims as was
renewal can only depend on the report \( \hat{x} \), whereas under verification, since the investor learns the true \( x \), the repayment can depend on both \( \hat{x} \) and \( x \). To clearly distinguish these cases, we denote by \( R_{nv}(\hat{x}) \) the repayment function if \( \beta(\hat{x}) = 0 \) and by \( R_v(\hat{x}, x) \) the repayment function in case \( \beta(\hat{x}) = 1 \). So the contractual repayment specification \( R(\hat{x}, x) \) can be written as

\[
R(\hat{x}, x) = \begin{cases} 
R_{nv}(\hat{x}) & \text{if } \beta(\hat{x}) = 0 \\
R_v(\hat{x}, x) & \text{if } \beta(\hat{x}) = 1 
\end{cases}
\]

\( R_{nv} \) maps reports onto real numbers, \( R_{nv} : [\underline{x}, \overline{x}] \rightarrow \mathbb{R}_0 \), and \( R_v \) is a mapping from the set of report-realization combinations onto the real numbers, \( R_v : [\underline{x}, \overline{x}] \times [\underline{x}, \overline{x}] \rightarrow \mathbb{R}_0 \).

The entrepreneur’s utility is quasi-linear in her monetary payoff and the cost of tax evasion. Her utility function, further specified below, therefore generally takes the form

\[
U = \Pi - c(e)
\]

where \( \Pi \) is an expected monetary payoff and \( c(e) \) is the cost of tax evasion. The investor’s utility is just his monetary payoff.

\section*{3.3 Analysis}

We want to characterize the financial contract offered by the entrepreneur \( E \) in the initial stage of this game. So we analyze the decisions made by \( E \) backwards, beginning with his tax reporting decision in stage 3 and continuing with \( E \)’s report to \( I \) in stage 1 of the game. These stages will imply a set of constraints on contracting required for \( I \)’s acceptance of the entrepreneur’s proposal at the initial contracting stage.

\subsection*{3.3.1 The entrepreneur’s tax reporting decision}

Consider first the entrepreneur’s choice of tax evasion, \( e \), in stage 3 of the game,\(^4\) given a contract \( (R, \beta) \), a realization \( x \), and a report \( \hat{x} \) made to \( I \) in stage 1 of the game. Two cases require distinction.

First, the case where the entrepreneur’s report has been verified, i.e. \( \beta(\hat{x}) = 1 \).

---

\(^4\)Since \( e = x - y \), choosing a level of evasion \( e \) implies a tax report \( y = x - e \). The analysis is more intuitive, though of course equivalent, when \( E \)’s choice is modeled in terms of \( e \), rather than \( y \).
This entails public revelation of the true \( x \) and so, in particular, \( \beta(\hat{x}) = 1 \) means
the government learns the true project success \( x \). As should be expected, this
will imply no tax evasion takes place.
Second, if the report \( \hat{x} \) has not been verified by the investor, i.e. \( \beta(\hat{x}) = 0 \),
evasion is detected only with probability \( p \), which will imply some tax evasion is
profitable in expectation. But let us formally analyze the two cases of verification
and non-verification in turn.

**Tax evasion when the investor verifies, i.e. when \( \beta(\hat{x}) = 1 \)**

If the investor verifies \( E \)’s report, the true \( x \) is publicly revealed. So the
entrepreneur’s utility in the final stage of the game is given by

\[
U_v(e; \hat{x}, x) = \max \left\{ x - \tau(x - e) - \lambda \tau e - R_v(\hat{x}, x), 0 \right\} - c(e) \tag{3.1}
\]

The subscript \( v \) is meant to indicate the case of investor verification examined
here. Utility \( U_v \) weighs the entrepreneur’s payoff, which is non-negative because
of her limited liability, against the utility cost of tax evasion. But tax evasion in
case of verification is always detected and implies a fine \( \lambda \tau e \), which is larger than
the tax savings from underreporting \( \tau e \), since \( \lambda > 1 \). So it is seen immediately
from (3.1) that the entrepreneur’s maximizing choice is to not evade taxes at all,
which we denote as

\[ e^*_v = 0. \]

**Tax evasion when the investor does not verify, i.e. when \( \beta(\hat{x}) = 0 \)**

By assumption, a tax audit now occurs with probability \( p \), since the report \( \hat{x} \)
has not been verified by the investor. We first state the entrepreneur’s respective
utilities in case of a tax audit and in absence of a tax audit. Her expected utility
is then the sum of these two utilities weighted by the audit probability \( p \). The
entrepreneur chooses a level of tax evasion that maximizes this expected utility.

*With a tax audit, \( E \)’s utility in the final stage of the game is given by

\[
U_{nv,a}(e; \hat{x}, x) = \max \left\{ x - \tau(x - e) - \lambda \tau e - R_{nv}(\hat{x}), 0 \right\} - c(e) \tag{3.2}
\]

The subscript \( nv \) indicates the case of non-verification that we analyze in this
section, and the subscript \( a \) indicates the case of a tax audit. \( E \)’s payoff is then
the true realization \( x \), minus the tax payment on reported profits, \( \tau(x - e) \),
minus a fine for evaded taxes, \( \lambda \tau e \), and the contractual repayment under non-
verification, \( R_{nv}(\hat{x}) \). Bearing in mind that the entrepreneur is protected by
limited liability, this difference is her payoff only if it is non-negative, however.
Otherwise it is just 0. This payoff is weighed against the utility cost of tax
evasion, \( c(e) \), to yield the utility \( U_{nv,a} \).
Without a tax audit, E’s utility in the final stage of the game is given by

$$U_{nv,na}(e; \hat{x}, x) = \max \{ x - \tau(x - e) - R_{nv}(\hat{x}), 0 \} - c(e)$$  \hspace{1cm} (3.3)

Again, the subscript \(nv\) denotes the case of non-verification and the subscript \(na\) indicates the absence of an audit by the tax authority. The utility in this case differs from \(U_{nv,a}\) only in that no fine for evasion is levied, because the tax authority does not learn the true \(x\) and so tax evasion goes undetected.

The entrepreneur’s expected utility in stage 3 of the game under non-verification is thus

$$U_{nv}(e; \hat{x}, x) = p U_{nv,a} + (1 - p) U_{nv,na}$$  \hspace{1cm} (3.4)

The entrepreneur chooses a level of tax evasion to maximize this expected utility, and we define

$$e_{nv}^* \in \arg\max_{e} \{ U_{nv} \}$$  \hspace{1cm} (3.5)

as the entrepreneur’s utility-maximizing choice of tax evasion under non-verification.

Let us now deduce \(e_{nv}^*\).

\(U_{nv}\) from (3.4) can be written as

$$U_{nv}(e; \hat{x}, x) = \begin{cases} (1 - \tau)x + (1 - p\lambda)\tau e - R_{nv}(\hat{x}) - c(e) & \text{if} \hspace{1cm} (a) \\ (1 - p)(1 - \tau)x + \tau e - R_{nv}(\hat{x}) - c(e) & \text{if} \hspace{1cm} (b) \end{cases}$$

where

$$x - \tau(x - e) - \lambda \tau e - R_{nv}(\hat{x}) \geq 0$$  \hspace{1cm} (a)

and

$$x - \tau(x - e) - \lambda \tau e - R_{nv}(\hat{x}) < 0 \leq x - \tau(x - e) - R_{nv}(\hat{x})$$  \hspace{1cm} (b)

Writing out expected utility in this way reveals two distinct ranges for \(U_{nv}\), associated with conditions \((a)\) and \((b)\). They arise depending on whether the payoff terms of the form \(\max \{ \cdot, 0 \}\) in (3.2) and (3.3) above bind at zero, or are slack and thus positive. That is, on whether the repayment \(R_{nv}\) can be made in full, or the entrepreneur’s limited liability binds and prevents a full repayment of \(R_{nv}\).

Condition \((a)\) implies that the payoff terms in both (3.2) and (3.3) are slack and so the repayment to \(I\) can be made both with and without a tax audit. As will become clear, an evasion choice satisfying condition \((a)\) does not cause any
problems for the investor, as even after a tax audit and fines, he will be repaid in full.

Condition \((b)\) says that the payoff term in \((3.2)\) binds at 0 while the payoff term in \((3.3)\) is slack. This means the repayment to \(I\) is only feasible if there is no tax audit, whereas if there is a tax audit, the fine for evasion exhausts the entrepreneur’s funds and prevents a full repayment of \(R_{nv}\) due to the entrepreneur’s limited liability protection. We will argue below that an evasion choice satisfying condition \((b)\) cannot be supported in an optimal contract, as with probability \(p\) of a tax audit, the investor will not be repaid what was stipulated in the contract.\(^5\)

We are now able to characterize the entrepreneur’s tax evasion choice in stage 3 of the game as follows.

**Proposition 3.1.** [Entrepreneur’s best response tax evasion choice]

Given any contract \((R, \beta)\), realization \(x\), and report \(\hat{x}\), the entrepreneur’s best response tax evasion choice in stage 3 of the game is given by

\[
\begin{align*}
e_{nv,(a)}^* &= (1 - p\lambda)\tau_k \quad \text{if} \quad \beta(\hat{x}) = 0 \quad \text{and} \quad R_{nv}(\hat{x}) \leq (1 - \tau)x - \phi_{nv} \\
e_{nv,(b)}^* &= (1 - p)\tau_k \quad \text{if} \quad \beta(\hat{x}) = 0 \quad \text{and} \quad R_{nv}(\hat{x}) > (1 - \tau)x - \phi_{nv} \\
e_v^* &= 0 \quad \text{if} \quad \beta(\hat{x}) = 1
\end{align*}
\]

where

\[
\phi_{nv} = \left[ (1 - p)^2 - (1 - p\lambda)^2 \right] \frac{\tau^2}{2pk}.
\]

**Proof.** See appendix 3.5.1.

We already stated in section 3.3.1 above that under verification of the entrepreneur’s report, her best response tax evasion choice is zero, \(e_v^* = 0\), since verification entails public revelation of the true project success \(x\).

The entrepreneur’s best response tax evasion choices under non-verification reflect the two regions of her expected utility \(U_{nv}\) characterized by conditions \((a)\) and \((b)\) above. If condition \((a)\) holds, the entrepreneur’s limited liability condition does not bind even after a tax audit. So in deciding how much to evade, \(E\) marginally weighs the expected tax savings against both the expected fine in case of audit and the cost of evasion, yielding \(e_{nv,(a)}^* = (1 - p\lambda)\tau_k\).

If, however, condition \((b)\) holds, the entrepreneur is protected by limited liability in case of a tax audit. This means she will evade only with a view to the non-

\(^5\)Technically, \(U_{nv}\) has a third region where both payoff terms bind at 0, i.e. where \(x - \tau(x - e) - R_{nv}(\hat{x}) < 0\). Then expected utility is just the utility cost of tax evasion, \(-c(e)\). This range will obviously never be attained since its maximizing choice would be \(e = 0\) yielding utility 0, which is always dominated by a choice of \(e\) that satisfies conditions \((a)\) or \((b)\).
audit case. She then marginally weighs expected tax savings against only the cost of tax evasion. The fine does not matter in this case, since it is only incurred in case of a tax audit, where \( E’\)’s payoff is 0 anyway. This yields a higher level of tax evasion, \( e^{\ast, (b)}_{nv} = (1 - p) \frac{\tau}{k} \). \(^6\)

Under non-verification, the entrepreneur thus chooses between \( e^{\ast, (a)}_{nv} \) and \( e^{\ast, (b)}_{nv} \) the level of evasion that yields a higher expected utility. The lower level \( e^{\ast, (a)}_{nv} \) is chosen if the entrepreneur is always left a sufficiently high rent under non-verification so that the fine for evasion still marginally matters to her, i.e. if \( R_{nv}(\hat{x}) \leq (1 - \tau)x - \phi_{nv} \). If the rent left to the entrepreneur is less than that, i.e. if \( R_{nv}(\hat{x}) > (1 - \tau)x - \phi_{nv} \), she is better off in expectation by choosing the high level of tax evasion \( e^{\ast, (b)}_{nv} \) that implies limited liability protection in case a tax audit occurs.

But in that latter case, the contractually agreed upon repayment, \( R_{nv}(\hat{x}) \), can only be made if there is no audit by the tax authority, i.e. with probability \((1 - p)\). Such a contract is therefore not feasible, for with probability \( p \), its stipulations cannot be met. The investor would not sign such a contract. We conclude that a feasible contract must satisfy the the constraints summarized in the following corollary.

**Corollary 3.1.** [Feasibility constraints on financial contracting]

A contract \((R, \beta)\) is feasible if it satisfies

\[
R_{nv}(\hat{x}) \leq (1 - \tau)x - \phi_{nv} \quad (F_{nv})
\]

\[
R_{v}(\hat{x}, x) \leq (1 - \tau)x \quad (F_{v})
\]

where

\[
\phi_{nv} = \frac{[(1 - p)^2 - (1 - p\lambda)^2] \tau^2}{2pk}
\]

**Proof.** Follows from Proposition 3.1. \(\square\)

Note that condition \((F_{nv})\) is stricter than condition \((F_{v})\) imposed when the investor verifies the entrepreneur’s report, in which case the entire statutory after-tax income \((1 - \tau)x\) is contractible.\(^7\)

Tax evasion thus imposes a constraint on financial contracting if it is such that the entrepreneur’s limited liability binds in case of a tax audit. Because the entrepreneur is protected by limited liability, she will evade as if there were no fine for evasion when the contractual repayment under non-verification exceeds \((1 - \tau)x - \phi_{nv}\).

---

\(^6\)For simplicity, let us assume that \( x > (1 - p) \frac{\tau}{k} \), so that the entrepreneur never evades the full amount of taxes due, but only a share thereof.

\(^7\)Note that agreeing to a repayment larger than the statutory net income amounts to contracting on the gains of tax evasion, which is illegal and thus not enforceable in a court of law. \((1 - \tau)x\) is therefore the legal maximum of contractible income.
A simple illustration of the problem that arises in this case is to consider $R_{nv}(\hat{x}) = (1 - \tau)x$, clearly a violation of ($F_{nv}$). Then with no evasion at all, $E$’s utility will be 0, since she pays $\tau x$ in taxes and $(1 - \tau)x$ to the investor.

But now consider a minimally positive amount of evasion, say $\epsilon > 0$. Now in case of a tax audit, $E$’s utility is $-c(\epsilon)$. If there is no tax audit, $E$’s utility is $\tau \epsilon - c(\epsilon)$. The entrepreneur’s expected utility is therefore $(1 - p)(\tau \epsilon - c(\epsilon)) - pc(\epsilon) = (1 - p)\tau \epsilon - c(\epsilon) = (1 - p)\tau \epsilon - \frac{k}{2} \epsilon^2 > 0$ for $\epsilon < (1 - p)\frac{2\tau}{k} > 0$. So the entrepreneur is better off in expectation when evading the positive amount $\epsilon$. But this means that in case of a tax audit, the funds available for repaying the investor are only $(1 - \tau)x - (\lambda - 1)\tau \epsilon < (1 - \tau)x = R_{nv}(\hat{x})$, and so the contractually stipulated repayment cannot be made. Such a repayment can therefore not be part of a feasible contract. Instead, feasibility requires that under non-verification, some rent is left to the entrepreneur with zero evasion (namely $\phi_{nv}$), so that she always evades in such a way that fines still marginally matter to her. This ensures repayment to the investor is feasible even in case of a tax audit. In the following, we will only consider feasible contracts.

### 3.3.2 The entrepreneur’s report to the investor

In this section, we look at the entrepreneur’s reporting behavior toward the investor in stage 1 of the game. Given a feasible contract $(R, \beta)$ and the realization $x$, the entrepreneur makes a report, $\hat{x}$, to the investor. $E$ will also take into account the implications of her reporting behavior on tax evasion choices as derived in the previous section. Let us distinguish two cases.

First, the case where $\beta(x) = 0$, i.e. contract $(R, \beta)$ and realization $x$ are such that a truthful report, $\hat{x} = x$ would not be verified by the investor. Second, the case where $\beta(x) = 1$, i.e. a truthful report would be verified. We consider the entrepreneur’s reporting behavior in each of these cases and derive incentive compatibility constraints required for contracting.

**Reporting $\hat{x}$ when $\beta(x) = 0$, i.e. a truthful report would not be verified by $I$**

Consider the entrepreneur’s possibilities for reporting to the investor if $\beta(x) = 0$. $E$ can either report truthfully, $\hat{x} = x$, in which case, assuming feasibility ($F_{nv}$) holds, her expected utility would be

$$U_{nv}(\epsilon^{*}_{nv,(a)},x,x) = (1 - \tau)x + \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(x)$$
Alternatively, the entrepreneur may choose to misrepresent true earnings and report some \( \hat{x} \neq x \) with \( \beta(\hat{x}) = 0 \). Then \( E \)'s expected utility is given by

\[
U_{nv}(e_{nv,(a)}^*, \hat{x}, x) = (1 - \tau)x + \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(\hat{x})
\]

Truthful reporting when \( \beta(x) = 0 \) is therefore preferred by the entrepreneur if and only if

\[
U_{nv}(e_{nv,(a)}^*, x, x) \geq U_{nv}(e_{nv,(a)}^*, \hat{x}, x) \quad \forall x, \hat{x} \text{ with } \beta(\hat{x}) = 0, \beta(x) = 0 \text{ and } \hat{x} \neq x
\]

But comparing the two expected utility terms above, this implies

\[
R_{nv}(x) \leq R_{nv}(\hat{x}) \quad \forall x, \hat{x} \text{ with } \beta(\hat{x}) = 0, \beta(x) = 0 \text{ and } \hat{x} \neq x
\]

Put differently, the entrepreneur has an incentive to misreport true earnings if some other, non-verified report \( \hat{x} \neq x \) induces a lower repayment. We therefore obtain the following incentive-compatibility constraint familiar from the literature on costly state verification.

\[
R_{nv}(\hat{x}) = D \quad \forall \hat{x} \text{ with } \beta(\hat{x}) = 0 \quad (IC1)
\]

where \( D \in \mathbb{R} \) is a constant. Since under non-verification, a lie will not be detected by the investor, the repayment to \( I \) under non-verification cannot depend on the report. It has to be constant.

**Reporting \( \hat{x} \) when \( \beta(x) = 1 \), i.e. a truthful report would be verified by \( I \)**

There are two possibilities the entrepreneur has when making her report to the investor. Either she can tell the truth and report \( \hat{x} = x \), or she may lie and make a report that does not induce verification, i.e. report some \( \hat{x} \neq x \) with \( \beta(\hat{x}) = 0 \).\(^9\)

Denoting, as above, her best-response evasion choices under these two options as \( e_{v}^* \) and \( e_{nv}^* \), we compare \( E \)'s resulting expected utilities and hence deduce \( E \)'s reporting behavior.

If \( E \) reports \( \hat{x} = x \), the report will be verified and so \( e_{v}^* = 0 \) will be chosen by

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\(^8\)Following the convention in the literature, we exclude the case of a lie that induces verification, i.e. reporting some \( \hat{x} \neq x \) with \( \beta(\hat{x}) = 1 \). Such a lie would be found out and can be arbitrarily punished as part of the contract. This exclusion is without loss of generality, as any contract inducing such reporting can be rewritten to induce truthful reporting under verification. See for instance Bolton and Dewatripont (2005), or Freixas and Rochet (2008).

\(^9\)Again, without loss of generality, we exclude from our analysis the case of misreporting that induces verification, i.e. reporting some \( \hat{x} \neq x \) with \( \beta(\hat{x}) = 1 \).
the entrepreneur in stage 3 of the game. Her expected utility is then given by

\[ U_v(e^*_v, x, x) = (1 - \tau) x - R_v(x, x) \]

If on the other hand, \( E \) chooses to misrepresent and report \( \hat{x} \neq x \) with \( \beta(\hat{x}) = 0 \), she will subsequently choose \( e^*_{nv(a)} = (1 - p\lambda) \frac{\tau^2}{k} \). \( E \)'s expected utility is then

\[ U_{nv}(e^*_{nv(a)}, \hat{x}, x) = (1 - \tau) x + \frac{1}{2} (1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(\hat{x}) \]

Truthful reporting is therefore preferred if and only if

\[ (1 - \tau) x - R_v(x, x) \geq (1 - \tau) x + \frac{1}{2} (1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(\hat{x}) \quad \forall x, \hat{x} \text{ with } \beta(x) = 1 \text{ and } \beta(\hat{x}) = 0 \]

The entrepreneur’s payoff from truthful reporting on the left-hand side of the inequality must be at least \( E \)'s expected utility from reporting a non-verified report. This is given on the right-hand side as the statutory net income \((1 - \tau) x \) minus the repayment \( R_{nv}(\hat{x}) \), plus the expected gain from tax evasion, \( \frac{1}{2} (1 - p\lambda)^2 \frac{\tau^2}{k} \). Rearranging this condition allows us to summarize the conditions for incentive compatibility in the following proposition.

**Proposition 3.2.** [Incentive compatibility constraints on financial contracting]

A feasible contract \((R, \beta)\) is incentive compatible if and only if

\[ R_{nv}(\hat{x}) = D \quad \forall \hat{x} \text{ with } \beta(\hat{x}) = 0 \quad (IC1) \]

for some constant \( D \in \mathbb{R} \) and

\[ R_v(x, x) \leq R_{nv}(\hat{x}) - \phi_T \quad \forall x, \hat{x} \text{ with } \beta(x) = 1 \text{ and } \beta(\hat{x}) = 0 \quad (IC2) \]

where \( \phi_T = \frac{1}{2} (1 - p\lambda)^2 \frac{\tau^2}{k} \) is the entrepreneur’s expected gain from tax evasion.

**Proof.** Follows from the arguments above.

As will become evident in the subsequent section, the incentive compatibility constraints in Proposition 3.2 are key determinants of the shape of the optimal financial contract. Constraint \((IC1)\) requires that whenever a report is not verified by the investor, the repayment is constant. This constant payment under non-verification is familiar from the literature on costly state verification and is typically associated with the face-value payment of a debt-contract. \((IC1)\) thus induces a debt-like property of the optimal contract.

The constraint \((IC2)\) requires that the repayment under non-verification always exceeds the repayment under verification by at least the entrepreneur’s expected gain from tax evasion, \( \phi_T \). This constraint is a variation of the incentive-compatibility constraint of Gale and Hellwig (1985). To prevent misreporting
by the entrepreneur in this setting, however, it is not simply enough to stipulate a (weakly) higher repayment under non-verification as in Gale and Hellwig’s (1985) standard debt contract. Instead, a strictly larger (by the amount $\phi T$) repayment is required because the entrepreneur has an additional incentive to lie and claim a non-verified project success so as to be able to engage in tax evasion.\(^{10}\) The jump in the optimal contract thus induced by $(IC2)$ due to the entrepreneur’s possibility to engage in tax evasion is a key novelty in the present chapter.

3.3.3 The optimal contract

We are now in a position to derive the optimal contract offered by the entrepreneur in the initial stage of the game. In addition to the feasibility constraints $(F_v)$ and $(F_{nv})$ from Corollary 3.1 and the incentive-compatibility constraints $(IC1)$ and $(IC2)$ from Proposition 3.2, the optimal contract also has to satisfy the investor’s participation constraint. That is, the financial contract has to yield, in expectation, at least $v$ to the investor, his reservation utility. Formally, we have

$$\int_{\tilde{x}}^{\mathcal{T}} [ (1 - \beta(\tilde{x})) R_{nv}(\tilde{x}) + \beta(\tilde{x}) R_v(\tilde{x}, x) ] dF(x) \geq v \quad (PC)$$

The optimal contract $(R, \beta)$ therefore solves the following problem of maximizing the entrepreneur’s expected utility subject to the constraints $(F_v)$, $(F_{nv})$, $(IC1)$, $(IC2)$, and $(PC)$.

$$\max_{R, \beta} \left\{ \int_{\tilde{x}}^{\mathcal{T}} [ \beta[(1 - \tau)x - R_v] + (1 - \beta)[(1 - \tau)x - R_{nv} + \phi_T] ] dF(x) \right\}$$

subject to

$$R_v(\tilde{x}, x) \leq (1 - \tau)x \quad (F_v)$$

and

$$R_{nv}(\tilde{x}) \leq (1 - \tau)x - \phi_{nv} \quad (F_{nv})$$

and

$$R_{nv}(\tilde{x}) = D \text{ for some } D \in \mathbb{R} \quad \forall \tilde{x} \text{ with } \beta(\tilde{x}) = 0 \quad (IC1)$$

\(^{10}\) Note that the expected gains from tax evasion, and hence the rigidity of $(IC2)$ and the discontinuity in any incentive-compatible financial contract, depend in expected ways on the parameters that are traditionally thought to govern tax evasion behavior, namely $p$, $\lambda$, $\tau$, and $k$. As a consequence, tax enforcement policy directly influences financial contracting, making it more standard debt-like as audit probability and fines increase, and less so as the tax rate $\tau$ increases.
and
\[ R_v(x, \hat{x}) \leq R_{nv}(\hat{x}) - \phi_T \quad \forall \ x, \hat{x} \text{ with } \beta(x) = 1 \text{ and } \beta(\hat{x}) = 0 \quad (IC2) \]

and
\[ \int_x^{\infty} \left[ (1 - \beta)R_{nv} + \beta R_v \right] dF(x) \geq \nu \quad (PC) \]

where \( \phi_T = \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k} \) is the expected gain from tax evasion and \( \phi_{nv} = \left[ (1 - p)^2 - (1 - p\lambda)^2 \right] \frac{\tau^2}{2pk} \) is the feasibility constant derived above. Before we derive the optimal contract \((R, \beta)\) solving this problem, let us consider two aspects of the maximization problem that will provide intuition regarding its solution.

First, we may rewrite the objective function of the problem, the entrepreneur’s expected utility, as
\[ \int_x^{\infty} (1 - \tau)x dF(x) - \int_x^{\infty} [(1 - \beta)R_{nv} + \beta R_v] dF(x) + \int_x^{\infty} (1 - \beta)\phi_T dF(x) \]

The first term is just the expected statutory net income, on which the choice of contract has no influence. The second term is the expected repayment to the investor, familiar from the participation constraint \((PC)\). Since the investor’s participation constraint binds at the optimum\(^\text{11}\), however, the second term reduces to \(\nu\). This focuses the entrepreneur’s contracting challenge on the third term, her expected gain from tax evasion, in a reasoning familiar from the literature on costly state verification.\(^\text{12}\) Since evasion only happens under non-verification, (i.e. when \(\beta(\hat{x}) = 0\), whereas the third term above becomes 0 if \(\beta(\hat{x}) = 1\)), the optimal contract will maximize the range of non-verification subject to satisfying the constraints. Equivalently, the range of realizations that are verified by the investor, thus leaving no room for a tax evasion gain, ought to be minimized by an optimal contract.

Second, notice that \((F_v)\) and \((IC2)\) may be combined into the following constraint on repayment under verification, \(R_v\),
\[ R_v \leq \min \left\{ (1 - \tau)x, R_{nv} - \phi_T \right\} \]

So repayment under investor verification needs to be weakly smaller than the statutory maximum contractible income, \((1 - \tau)x\), but also than repayment under

\(^{11}\)To see why the participation constraint \((PC)\) is binding at the optimum, consider towards a contradiction a non-binding \((PC)\) at the optimum. Then \(R_v\) could be lowered, thereby strictly increasing the entrepreneur’s profits while relaxing all constraints it affects and still satisfying \((PC)\). So the original situation cannot have been an optimum.

\(^{12}\)See, for instance, Tirole (2005), Ch.3
non-verification, $R_{nv}$, less the expected gain from tax evasion $\phi_T$. This points at the intuitive discontinuity discussed in the previous section. If repayment under verification, $R_v$, gets “too close” (closer than $\phi_T$) to repayment under non-verification, $R_{nv}$, the entrepreneur would prefer to misrepresent her true earnings and promise the slightly higher repayment to be able to take a chance at tax evasion. To be incentive-compatible, a contract therefore needs to demand from $E$ a repayment under non-verification that exceeds the repayment under verification by at least the expected gain from evading taxes. This combined constraint on $R_v$ is responsible for the characteristic shape of the optimal contract that we derive in the following proposition.

**Proposition 3.3.** [Optimal contract]

a) Every contract satisfying the constraints $(F_v)$, $(F_{nv})$, $(IC1)$, $(IC2)$, and $(PC)$ is weakly dominated by a contract of the following form, denoted by $\chi$,

$$R(x, \hat{x}) = \begin{cases} 
D & \text{if } \beta(\hat{x}) = 0 \\
\min\{(1 - \tau)x, D - \phi_T\} & \text{if } \beta(\hat{x}) = 1
\end{cases}$$

where

$$\beta(\hat{x}) = 0 \quad \text{iff} \quad \hat{x} \in B_{nv} = \{x \mid x \geq \tilde{x}\}$$

and

$$\beta(\hat{x}) = 1 \quad \text{iff} \quad \hat{x} \in B_v = \{x \mid x < \tilde{x}\}$$

and $\tilde{x}$ is such that

$$D = (1 - \tau)\tilde{x} - \phi_{nv}$$

where $\phi_T = \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k}$ and $\phi_{nv} = [(1 - p)^2 - (1 - p\lambda)^2] \frac{\tau^2}{2pk}$.

b) Contracts of the form $\chi$ are uniquely optimal.

*Proof. See appendix 3.5.2.*

The optimal contract minimizes verification by charging the maximum possible repayment inside the verification region. Crucially, this does not lead to a standard debt contract in this setting, which is neither feasible nor incentive-compatible when entrepreneurs can evade taxes. Instead, while sharing some characteristics with standard debt, our optimal contract differs from it in important respects, as the following figure illustrates.
First, if a report is not verified (range \textit{III} in Figure 3.2), the optimal contract stipulates a constant repayment of size $D = (1 - \tau)\tilde{x} - \phi_{nv}$ to the investor. This constant face-value, upon repayment of which no verification takes place, is a debt-like characteristic of our optimal contract. However, as we argued in section 3.3.1, the repayment $R_{nv} = D$ cannot charge up to the limited liability $(1 - \tau)x$ (dotted line) anywhere in the non-verification range \textit{III}. Instead, the repayment has to always be $\phi_{nv}$ below this level. This is required for a feasible contract because otherwise the entrepreneur could profitably use her limited liability protection when evading taxes, making the repayment unfeasible in case a tax audit occurs.

Second, consider region \textit{I}, where verification takes place, i.e. $\beta(\hat{x}) = 1$, and the repayment is the entire contractible income, $R_v = (1 - \tau)x$, leaving no rent to the entrepreneur. Here, as in a standard debt-contract, the repayment charges up to the agent’s limited liability.

Third, consider region \textit{II}, where verification takes place but the repayment is constant at $R_v = D - \phi_T$. This is a hybrid region combining elements of debt (constant repayment) and equity (leaving some rent to the entrepreneur under verification). As argued in section 3.3.2, $\phi_T$ is the entrepreneur’s expected gain from tax evasion. To induce incentive compatibility on part of the entrepreneur, the repayment under verification, $R_v$, (where no tax evasion is possible) has to always be at least $\phi_T$ below the repayment under non-verification, $R_{nv}$, where
evasion is possible. Otherwise, the entrepreneur would simply misrepresent her true earnings as being in the non-verification region III of the contract and take a chance at tax evasion, which, in expectation, would then be profitable for her.

3.4 Conclusion

This chapter analyzes a model of costly state verification with tax evading entrepreneurs. We posit an informational externality between investor verification and tax auditing. When investors choose to verify the true state of the world, this delivers information to the government, making tax evasion less profitable. The optimal financial contract therefore minimizes investor verification to provide the entrepreneur with a maximum of tax evasion possibilities. Yet the contract achieving this is not a standard debt contract, as the original work on costly state verification by Townsend (1979) and Gale and Hellwig (1985) might suggest. Instead, we find an optimal contract which is debt-like only for very low and very high realizations, and combines elements of debt and equity in an intermediate range. This is because tax evasion represents a special challenge for financial contracting. Since it is illegal, the gains from evasion cannot be contracted on. But if tax evasion is detected, fines may reduce the entrepreneur’s ability to repay the investor. Except for very low realizations, the optimal contract in this setting always leaves the entrepreneur with a positive rent to prevent her from abusing her limited liability protection for excessive tax evasion activities. This makes the optimal contract less efficient at minimizing investor verification than a standard debt contract, which is neither feasible nor incentive compatible when borrowers may evade taxes.
3.5 Appendix

3.5.1 Proof of Proposition 3.1

As argued in section 3.3.1 above, \( e_v^* = 0 \) follows immediately from the public revelation of \( x \) in case \( \beta(\hat{x}) = 1 \).

Let us now focus on the entrepreneur’s evasion choice under non-verification, \( e_{nv}^* \). So suppose \( \beta(\hat{x}) = 0 \).

The first order conditions for a maximum are given by

\[
(1 - p\lambda)\tau - c_e(e_{nv,(a)}^*) = 0 \quad \text{if} \quad (a)
\]

and

\[
(1 - p)\tau - c_e(e_{nv,(b)}^*) = 0 \quad \text{if} \quad (b)
\]

The assumptions on \( c(e) \) imply that the second order conditions for a maximum are satisfied.

From the first order conditions for a maximum, and the fact that \( c(e) = ke \), we obtain

\[ e_{nv,(a)}^* = (1 - p\lambda)\frac{\tau}{k} \]

and

\[ e_{nv,(b)}^* = (1 - p)\frac{\tau}{k} \]

The entrepreneur now chooses \( e_{nv}^* = e_{nv,(a)}^* \) if and only if

\[ U_{nv}(e_{nv,(a)}^*, \hat{x}, x) \geq U_{nv}(e_{nv,(b)}^*, \hat{x}, x) \]

or equivalently

\[
(1 - \tau)x + (1 - p\lambda)\tau e_{nv,(a)}^* - R_{nv}(\hat{x}) - c(e_{nv,(a)}^*) \geq (1 - p)[(1 - \tau)x + \tau e_{nv,(b)}^* - R_{nv}(\hat{x})] - c(e_{nv,(b)}^*)
\]

Using the expressions for \( e_{nv,(a)}^* \) and \( e_{nv,(b)}^* \) above and the fact that \( c(e) = \frac{k}{2}e^2 \), this inequality becomes

\[
(1 - \tau)x + (1 - p\lambda)^2 \frac{\tau^2}{2k} - R_{nv}(\hat{x}) \geq (1 - p)[(1 - \tau)x - R_{nv}(\hat{x})] + (1 - p)^2 \frac{\tau^2}{2k}
\]

And rearranging, we obtain

\[ R_{nv}(\hat{x}) \leq (1 - \tau)x - [(1 - p)^2 - (1 - p\lambda)^2] \frac{\tau^2}{2kp} \equiv (1 - \tau)x - \phi_{nv} \]

as required. □
3.5.2 Proof of Proposition 3.3

a)

Consider any arbitrary contract \( C : (R, \beta) \) satisfying \((F_v), (F_{nv}), (IC1), (IC2), \) and \((PC)\).

Since \( C \) satisfies \((IC1)\), repayment under non-verification is constant and we denote this constant by \( D \).

Now construct a contract of the form \( \chi \), denoted \( C^{\chi} : (R^{\chi}, \beta^{\chi}) \) with the same constant repayment \( D \) under non-verification as the original contract \( C \).

We have thus

\[
R^\chi(x, \hat{x}) = \begin{cases} 
D & \text{if } \beta^\chi(\hat{x}) = 0 \\
\min\{(1-\tau)x, D - \phi_T\} & \text{if } \beta^\chi(\hat{x}) = 1
\end{cases}
\]

where

\[
B^{\chi}_{nv} = \{x | \beta^\chi(x) = 0\} = \{x | x \geq \hat{x}\}
\]

and

\[
B^{\chi}_v = \{x | \beta^\chi(x) = 1\} = \{x | x < \hat{x}\}
\]

and \( \hat{x} \) is such that

\[
D = (1-\tau)\hat{x} - \phi_{nv}
\]

By construction, the new contract satisfies \((F_v), (F_{nv}), (IC1), \) and \((IC2)\).

Denote by \( B_v = \{x | \beta(x) = 1\} \) the verification set under the original contract, and by \( B_{nv} = \{x | \beta(x) = 0\} \) its complement, the non-verification set.

First, we show that \( B^{\chi}_v \subseteq B_v \), meaning the verification region is weakly smaller under the new contract.

So suppose \( x \in B^{\chi}_v \). Then in particular, \( x < \hat{x} \). We need to show that this implies \( x \in B_v \).

Toward a contradiction, suppose this were not the case, i.e. \( x \in B_{nv} \).

Then since \( x \) is not verified under the old contract, the repayment under the old contract is \( R_{nv}(x) = D = (1-\tau)\hat{x} - \phi_{nv} \).

However, since \( x < \hat{x} \), we have

\[
R_{nv}(x) = D = (1-\tau)\hat{x} - \phi_{nv} > (1-\tau)x - \phi_{nv}
\]

This is a violation of \((F_{nv})\), contradicting our assumption that the original contract satisfies \((F_{nv})\). Therefore, it cannot be true that \( x \in B_{nv} \) and so we must have \( x \in B_v \).

Thus we have shown that \( B^{\chi}_v \subseteq B_v \). The verification set under the new contract \( C^{\chi} \) is weakly smaller than under the original contract \( C \).
Note that this result implies \( B_{nv} \subseteq B_{n}^{\chi} \), that is, the non-verification set under the new contract is weakly larger than under the original contract \( C \).

Now compare the repayment to the investor under the two contracts.

If \( x \in B_{nv} \), then \( x \in B_{n}^{\chi} \) and so the repayment is the same in both contracts, 
\[
R_{nv}(x) = R_{n}^{\chi}(x) = D.
\]

If \( x \in B_{v} \), there are two cases, since \( B_{v}^{\chi} \subseteq B_{v} \). Either \( x \) induces verification also under the new contract, i.e. \( x \in B_{v}^{\chi} \). Or \( x \) does not induce verification under the new contract, i.e. \( x \in B_{nv}^{\chi} \).

Consider first the case where \( x \in B_{v}^{\chi} \). Then the repayment under the new contract \( C^{\chi} \) is 
\[
R_{v}^{\chi}(x) = \min\{ (1 - \tau) x, D - \phi T \}.
\]

Under the old contract, which satisfies \( (F_{v}) \) and \( (IC2) \), the repayment satisfies 
\[
R_{v}(x) \leq \min\{ (1 - \tau) x, D - \phi T \}.
\]

So the repayment to the investor weakly increases under the new contract.

Now consider the second case, i.e. \( x \in B_{nv}^{\chi} \). The new repayment is 
\[
R_{nv}^{\chi}(x) = D,
\]

which is larger than the repayment under the old contract for such \( x \), which satisfies 
\[
R_{v}(x) \leq \min\{ (1 - \tau) x, D - \phi T \} < D.
\]

The repayment to the investor therefore increases in this case.

The new contract \( C^{\chi} \) thus increases the expected repayment to the investor, making the participation constraint \( (PC) \) slack.

It also increases, however, the overall surplus for the contracting parties, since the non-verification region \( B_{nv}^{\chi} \) weakly increases under the new contract. This means the expected tax evasion gain is larger under the new contract \( C^{\chi} \):
\[
\int_{\frac{\tau}{2}}^{\frac{\tau}{2}} (1 - \beta^{\chi}) \phi T dF(x) \geq \int_{\frac{\tau}{2}}^{\frac{\tau}{2}} (1 - \beta) \phi T dF(x)
\]

But then we can decrease the threshold \( \bar{\hat{x}} \) and payments \( R \) until the investor’s participation constraint is binding again. This makes the entrepreneur strictly better off, since the overall surplus of the contract increases due to a smaller verification region but the expected repayment to \( I \) stays the same at \( v \), meaning the entire gain from less verification accrues to the entrepreneur through a higher expected tax evasion gain. \( \square \)

b)

So suppose the two contracts \( C : (R, \beta) \) and \( C^{\chi} : (R^{\chi}, \beta^{\chi}) \) are both optimal contracts, where \( C^{\chi} \) is a contract of the form \( \chi \) derived from \( C \) as part a) of the proposition. Since they are both optimal, this means they maximize the entrepreneur’s expected utility subject to the constraints \( (F_{v}), (F_{nv}), (IC1) \),

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$(IC2)$, and $(PC)$. But this implies
\[
\int_{\underline{x}}^{\bar{x}} (1 - \beta^X) \phi_T dF(x) = \int_{\underline{x}}^{\bar{x}} (1 - \beta) \phi_T dF(x)
\]
and so
\[
\int_{\underline{x}}^{\bar{x}} (\beta^X - \beta) \phi_T dF(x) = 0
\]
Since $B^\chi_X \subseteq B_v$, we have $\beta^X(x) \leq \beta(x)$ for all $x \in [\underline{x}, \bar{x}]$. But then the above equality can only hold if
\[
\beta^X(x) = \beta(x) \quad \forall x
\]
We will show that this implies the contracts have to be the same.
Consider first the case where $\beta^X(x) = \beta(x) = 0$. Then the repayment under both contracts is the same, $R_{nv}(x) = R_{nv}^X(x) = D$.
Now consider the other case, where $\beta^X(x) = \beta(x) = 1$. Under the new contract, the repayment is $R^X_v(x, x) = \min\{ (1 - \tau)x, D - \phi_T \}$. But under the original contract, since it satisfies $(F_v)$ and $(IC2)$, the repayment satisfies $R_v(x, x) \leq \min\{ (1 - \tau)x, D - \phi_T \}$. If, however, there exists an $x$ with $\beta^X(x) = \beta(x) = 1$ and $R_v(x, x) < \min\{ (1 - \tau)x, D - \phi_T \}$, this implies a strictly higher payoff under the original contract than under the new contract $C^X$. This contradicts our assumption that both contracts are optimal. So we must have $R_v(x, x) = R^X_v(x, x) = \min\{ (1 - \tau)x, D - \phi_T \}$ for all $x$ with $\beta(x) = \beta^X(x) = 1$.
This proves that both contracts are exactly equal and of the form $\chi$, and so any optimal contract is necessarily of the form $\chi$, as required. \[\square\]
Chapter 4

Optimal auditing with heterogeneous audit perceptions

4.1 Introduction

At its core, tax evasion is a bet on escaping detection, for being found out typically leaves the evader worse off than the honest taxpayer. Only undetected tax evasion attains the evader’s goal: paying fewer taxes. But just how likely is detection? In this chapter, we argue that people differ in their answer to this question, due to heterogeneous perceptions of audit risk. Such heterogeneity is well in line with the observation that people tend to disagree in many situations requiring an assessment of risky prospects, be they investments, elections, driving a car, or travel planning, to name but a few obvious examples. Tax evasion is a prime example of such a situation. For the purpose of this analysis, we focus on two perceptional biases that capture the essence of heterogeneity in audit perceptions. Taxpayers may be overconfident, thinking that tax evasion will most likely remain undetected. Maybe there is a history of successful evasion. Or a (perceived) knack for hiding money. Or a generally panglossian leaning.\footnote{There exists a large literature on the origins of overconfident behavior, a trait which has been shown to affect decision making under numerous circumstances. It is plausible to believe that at least some people also exhibit this bias when filing their tax reports. See Moore and Healy (2008) for a survey on overconfidence.} Whatever the reason, these intrepid types underestimate the likelihood of being audited by the tax authority.\footnote{We assume that audits perfectly reveal a taxpayer’s true income and use the terms “audit” and “detection” interchangeably throughout the chapter.} But taxpayers can also be underconfident. Overestimating the true probability of detection, underconfident taxpayers are less
prone to evasion than they should be from a purely probabilistic perspective.\(^3\) Being overly cautious may similarly be rooted in personal experience, or, for some people, simply be a general response to uncertainty. For the purpose of this analysis, we remain agnostic as to which of these biases ultimately prevails and where they come from. Instead, we perform a descriptive analysis that allows for a heterogeneous taxpayer population containing both overconfident and underconfident taxpayers.

We examine how such heterogeneity impacts taxpayer behavior, and how this influences the government’s optimal tax audit policy. We then proceed to analyzing the consequences for social welfare in the resulting equilibrium. Following the principal-agent approach to optimal tax auditing pioneered by Reinganum and Wilde (1985), we find that heterogeneity in audit perceptions substantially changes the government’s optimal audit policy. In particular, the tax authority’s audit policy choice critically depends on the extent of heterogeneity among taxpayers. If taxpayers are relatively homogeneous in their perception of the audit probability, the government’s optimal choice is to induce full compliance, as in the standard model of optimal auditing without heterogeneity. If, however, taxpayers differ significantly in their perception of the audit probability, both evasion and compliance are part of the equilibrium. The equilibrium audit probability may then either be increasing or decreasing with heterogeneity, depending on how costly tax audits are to the government. If audit costs are low, the equilibrium audit intensity increases with heterogeneity. This is because low audit costs make it relatively attractive, in welfare terms, to catch tax evaders, whose contribution to social welfare upon detection is the fines they pay less the cost of audit. But catching tax evaders with a high probability requires auditing with a high probability. Now, if taxpayers are relatively homogeneous and do not stray too far from the true audit probability, such high audit probabilities will induce full compliance, thus invalidating the opportunity of catching and fining evaders with a high likelihood. But when the extent of heterogeneity is large, some taxpayers will evade taxes even at very high audit probabilities, due to misperception. Then the equilibrium audit probability increases in the extent of heterogeneity, because heterogeneity allows detecting and fining evaders with a high likelihood without inducing too much honesty in the taxpayer population. Matters are different when audits are relatively costly to the government. Then the government’s main tradeoff is between inducing honesty among taxpayers on the one hand, and economizing on audit costs on the other hand. Stricter auditing induces a higher share of taxpayers to report honestly as well as increasing the expected audit cost. Perception heterogeneity weakens the first, honesty-inducing, effect, but leaves the second, audit-cost-increasing, effect un-

\(^3\)See Slemrod and Yitzhaki (2002), especially p.1431, for this often-voiced conundrum in the literature on tax evasion.
affected. When heterogeneity in audit perceptions is large, an increase in the audit intensity will lead to a smaller corresponding increase in the share of honest taxpayers than it would have if taxpayers were more homogeneous. And yet the increase in expected audit cost as a result of stricter auditing remains unchanged. When audit costs are high, a very heterogeneous taxpayer population is therefore associated with auditing less strictly in equilibrium.

We also consider the welfare effects of heterogeneity in audit perceptions, and find a non-monotonic, U-shaped relationship. Small levels of heterogeneity unambiguously decrease social welfare in equilibrium. That is because, as mentioned above, small levels of heterogeneity make inducing full compliance the government’s optimal choice. As heterogeneity increases, however, inducing full compliance becomes more expensive. It requires a higher audit probability, without changing anyone’s reporting behavior. The only welfare effect of more dispersed audit perceptions is thus to conduct more wasteful audits of honest taxpayers, an unambiguous welfare loss. But when the extent of heterogeneity is sufficiently large, a second effect enters to counteract this welfare loss. Working through the equilibrium audit probability, the nature of this second, welfare-enhancing, effect of heterogeneity in audit perceptions depends on the level of audit costs, mirroring the argument presented in the previous paragraph. When audit costs are low, the equilibrium audit intensity increases in the extent of heterogeneity. For large levels of heterogeneity, this is welfare-enhancing because it allows the tax authority to detect and fine evaders with a high probability without inducing too much honesty. When audit costs are high, the equilibrium audit probability decreases in the extent of heterogeneity. This is welfare-enhancing because it allows economizing on audit costs while still inducing a significant amount of honest tax reporting in the population.

This chapter is chiefly related to two strands of literature, on the economics of tax evasion and on behavioral biases in economic decision making, a topic which has received ample attention in both economics and psychology. Following the seminal work by Allingham and Sandmo (1972), a large economic literature has analyzed various aspects of tax evasion. Excellent surveys are provided by Andreoni et al. (1998) and Slemrod and Yitzhaki (2002). This chapter belongs to a slightly more recent literature on optimal tax auditing, and uses a principal-agent approach with commitment as pioneered by Reinganum and Wilde (1985). In this class of models, a tax authority first announces and commits to an audit policy, in response to which taxpayers make their tax reports. Other seminal publications in this vein include Scotchmer (1987), Sanchez and Sobel (1993), Macho-Stadler and Perez-Castrillo (1997), and Macho-Stadler and Perez-Castrillo (2002). For completeness, it should be noted that the optimal auditing literature has also considered models without commitment, for instance
by Graetz et al. (1986) or Erard and Feinstein (1994). The present chapter is most closely related work by Macho-Stadler and Perez-Castrillo (1997), by Cabañé and Panadés (2005), and by Cronshaw and Alm (1995). Macho-Stadler and Perez-Castrillo (1997) consider optimal auditing with heterogeneous income sources, which are categorized by an exogenously fixed and commonly known detection probability in case an audit occurs and income of a particular source was evaded. The idea is that some sources of income (say wages and salaries) are much easier to monitor than others (say income from owning a restaurant). However, since the income source is assumed to be observable by the tax authority, and the audit strategy thus made conditional on it, the model of Macho-Stadler and Perez-Castrillo (1997) does not address heterogeneity along two privately known types, as is the case here. It also assumes, plausibly enough in the case of income sources, that the source of income determines the actual detection probability. In our model, however, it is only the perceptions of the audit probability that vary, while the true audit probability is the same for every evader. Cabañé and Panadés (2005) consider a model of two-sided cost uncertainty, introducing a varying, idiosyncratic, and privately known audit cost on the part of auditors and a similarly heterogeneous, privately known cost of suffering a tax inspection on the part of taxpayers. They extend earlier work by Reinganum and Wilde (1988), who consider a setup with only a varying cost of audit as the private information of tax auditors, about which taxpayers are uncertain. In our model, there is no uncertainty about the cost of audit, which is commonly known to all. The focus of these papers, including also the work by Cronshaw and Alm (1995), is to explore the impact of taxpayer uncertainty on tax compliance, and what the implications are for the government’s audit policy. That is, whether uncertainty about audit policies should be fostered or reduced. In the present work, we abstract from such considerations and instead examine the impact of heterogeneity in audit perceptions itself on optimal auditing by the government, rather like in the work by Macho-Stadler and Perez-Castrillo (1997), but with a different and privately known source of heterogeneity. Relevant to but further removed from this chapter is a series of fairly recent works applying various non-expected utility approaches to tax compliance. These approaches include ambiguity aversion (Snow and Warren 2005), prospect theory (Yaniv 1999, al-Nowaihi and Dahmi 2007), rank dependent expected utility (Arcand and Graziosi 2005, Eide 2002), and reference dependence (Bernasconi and Zanardi 2004). A recent survey by Hashimzade et al. (2013) summarizes work undertaken along these lines. Although these studies do not concern themselves with optimal auditing, and, by using different notions of utility, stray quite far from the standard expected utility approach employed in this chapter, they aim to address puzzles not unlike the ones motivating this work, and should thus be included for completeness.
A second field of research this chapter builds on is the literature on probability misperceptions. To focus the argument, we use for the purpose of this chapter the terms overconfidence as in overestimating the likelihood of a good outcome (non-detection) and underconfidence as in overestimating the likelihood of a bad outcome (tax audit), although both terms have been used to describe different (but related) things throughout the literature, as the survey by Moore and Healy (2008) illustrates. Overconfidence is a well documented bias in human behavior, economic and otherwise. It has been drawn upon to explain entrepreneurial excess-entry (Camerer and Lovallo 1999), excessive M&A activity by managers (Malmendier and Tate 2008), overly high rates of stock trading (Odean 1998), and the politics of warfare (Howard 1983, Johnson 2004). People also seem to be overconfident in various aspects of their daily lives. Students overestimate their exam-performance (Clayson 2005), workers overestimate their job-performance relative to colleagues (Zenger 1992), and drivers overestimate their driving skills (Svenson 1981). It seems plausible that at least some taxpayers are thus overconfident when it comes to their tax evasion strategies, overestimating their ability to successfully claim deductions and escape punishment for evasion. Overconfidence has also recently found a wide range of applications in traditional issues of applied economic theory. Sandroni and Squintani (2007) examine the effect on insurance contracts when a (varying) share of the population underestimate their risk category and find that the effects of overconfidence depend both qualitatively and quantitatively on the prevalence of overconfidence in the population. Further examples of such applications are studies on optimal pricing with overconfident consumers by Grubb (2009) and Eliaz and Spiegler (2008) and on the effect of managerial overconfidence on corporate investment by Malmendier and Tate (2005) and on corporate innovation (Hirshleifer et al. 2012). De la Rosa (2011) considers a classic moral-hazard model along the lines of Holmstrom (1979), but allows for agent overconfidence regarding the relationship between managerial effort and the likelihood of success. Spinnewijn (2014, forthcoming) considers the optimal design of unemployment insurance when insurers overestimate the likelihood of finding work after a job loss. A survey of overconfidence and its contract-theoretic implications is contained in Koszegi (2014, forthcoming). The idea that individuals may also be underconfident, or overly cautious, seems particularly pertinent in the context of tax evasion. A recurring puzzle in the literature on tax evasion is why people evade so little, given the relatively low detection probability and fine rates observed in reality. As Slemrod and Yitzhaki (2002, p.1431) note, “the intriguing question becomes why people pay taxes rather than why people evade.” Although various explanations have been proposed to address this conundrum (for more details, see the surveys by Andreoni et al. 1998, Slemrod and Yitzhaki 2002 and Hashimzade et
al. 2013), one simple approach is to infer that some taxpayers overestimate the true audit probability, and therefore evade less than what a model based solely on this probability would suggest. Although ultimately an empirical question beyond the scope of this chapter, we take the large number of existing studies in both directions as an indication that both overconfidence and underconfidence are likely present in the taxpayer population. Rather than passing judgement on the direction of the perception bias, we therefore examine a population that exhibits heterogeneity in its perception of the audit probability. This approach is related to analyzing heterogeneous risk perceptions in insurance models, as a recent paper by Spinnewijn (2013) has done to explain heterogeneity in insurance choices. Indeed, we agree with Spinnewijn’s (2013, p.606) statement that “people have very different beliefs about the risks they face,” and take this observation as a starting point for the present analysis.

The remainder of this chapter is organized as follows. Section 4.2 introduces the formal model. Section 4.3.1 derives taxpayers’ best-response tax reporting choices. Section 4.3.2 analyzes the tax authority’s optimal audit policy. Section 4.3.3 considers the implications of heterogeneous audit perceptions for social welfare. Section 4 concludes.

### 4.2 Model

Consider a tax evasion game between a government and a population of taxpayers. In the first stage of the game, the government decides on an audit policy with the objective of maximizing social welfare. Given their privately known income and their belief about the government’s audit policy, taxpayers then file a tax report. This report is subsequently audited according to the previously specified policy.\(^4\) Taxes and potentially fines for evasion are paid in the final stage of the game. The following figure illustrates the timing and structure of the game.

\(^4\)We assume the tax authority commits to a chosen audit policy. This principal-agent analysis of optimal auditing follows the tradition of Reinganum and Wilde (1985), Scotchmer (1987), and Sanchez and Sobel (1993). For models without commitment, see for instance Graetz et al. (1986) or Erard and Feinstein (1994). While we assume commitment throughout the chapter, Appendix 4.5.1 provides an analysis of interim incentive compatibility, a property in which commitment and non-commitment models of optimal auditing often differ.
Figure 4.1: Timing of the game

Taxpayers

The risk neutral taxpayer population differs in income and in their belief about the government’s audit policy.

Income is denoted by $y \in \{0, 1\}$ and can be either low or high. We normalize the low income to $y = 0$ and the high income to $y = 1$. Taxpayers know their income, but the government learns a taxpayer’s true income only upon audit, and is otherwise dependent on a tax report $r \in \{0, 1\}$ made by the taxpayer in assessing its taxes. The distribution of income is common knowledge and follows $P(y = 1) = \nu$, $P(y = 0) = 1 - \nu$. Taxpayers also differ in their belief about the government’s audit policy. Denoting this probability belief by $\theta$, a taxpayer believes the probability of a tax audit is given by

$$\theta = \begin{cases} 
0 & \text{if } p + \epsilon < 0 \\
p + \epsilon & \text{if } p + \epsilon \in [0, 1] \\
1 & \text{if } p + \epsilon > 1 
\end{cases} \quad (4.1)$$

where $\epsilon$ is a measure of misperception uniformly distributed on $[-a, a]$ around the true audit probability $p$. The heterogeneity parameter $a \geq 0$ determines the extent to which taxpayers differ in their perception of the government’s audit policy. Negative values of $\epsilon$ mean a taxpayer underestimates the risk of detection, for instance due to overconfidence or optimism. The converse bias of underconfidence or undue caution with regard to tax audits is the case when $\epsilon > 0$. To focus the analysis on heterogeneity itself rather than on perceptual biases in one or the other direction, we assume that misperception is symmetrically distributed around the true audit probability $p$, so that on average, taxpayers are correct about the government’s policy.

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5Note that we restrict the message space to the set of possible incomes, $r \in \{0, 1\}$. Since the government knows the set of possible incomes, any report $r \notin \{0, 1\}$ would immediately be identifiable as a lie.

6Assuming a uniform distribution of the perception bias ensures tractability, without overly restricting our analysis. All mechanisms carry over to a triangular distribution, for instance. What is needed is that the bias is symmetrically distributed around the true audit probability with a strictly positive density throughout, giving rise to a continuous cumulative distribution function.
Government

In the initial stage of the game, the government chooses an audit probability \( p : \{0, 1\} \mapsto [0, 1] \) for each possible tax report, so as to maximize expected social welfare.\(^7\) Denoting expected welfare by \( W \), we obtain a social welfare function of the general form

\[
W = \Pi + (1 + \lambda) T
\]

(4.2)

where \( \Pi \) denotes taxpayers’ expected payoffs and \( T \) is expected net tax revenue composed of taxes and fines net of audit cost, which is a constant \( c > 0 \). We follow the convention of assuming \( \lambda > 0 \) to account for the shadow value of public funds, usually assumed to be larger than one.\(^8\)

To simplify the ensuing analysis, let us state right away a classic result in the literature on optimal auditing that also holds in this framework.

**Lemma 4.1.** It is never optimal to audit a high income report. Every optimal audit policy therefore satisfies \( p(1) = 0 \).

As will become clear, taxpayers have no incentive to overreport income, and so high reports \( (r = 1) \) will be filed only by honest, high-income taxpayers. Auditing them would not produce additional revenue, or change anyone’s behavior. Since audits are costly, auditing a high report therefore constitutes mere waste. In what follows, we shall hence refer to the probability \( p(0) \) that a low income report is audited simply as \( p \). Let us now proceed to the analysis of the tax evasion game presented in this section.

4.3 Analysis

Solving the game backwards, we begin with the decisions made by the taxpayer and then characterize the government’s optimal audit policy.

4.3.1 Tax reporting

Given income \( y \) and their belief \( \theta \) about the government’s audit policy, taxpayers choose a tax report \( r \) to maximize their expected payoff. This payoff is composed of their income net of the expected tax payment, which includes expected fines for evasion.\(^9\) The government does not pay taxpayers for overstating their income. Low-income individuals therefore always file a truthful tax report. High-income

\(^7\)We focus strictly on the choice of an optimal tax audit policy, and treat the tax rate \( \tau \) as given. See Appendix 4.5.2 for an analysis of the impact of \( \tau \) in this model.

\(^8\)See Laffont and Tirole (1993) for a discussion and application of this concept in the theory of regulation.

\(^9\)As in Becker (1968), maximum fines are optimal in this setup. We therefore assume that the government sets fines optimally and a tax evader loses her entire income upon detection.
taxpayers face a classic tax evasion gamble. Honest reporting implies, with certitude, a tax payment of exogenous size \( \tau \in [0, 1] \). Filing a low report, on the other hand, entails losing one’s entire income with (perceived) probability \( \theta \), but paying no taxes at all with the converse probability, \( 1 - \theta \). Formally, high-income taxpayers thus solve

\[
\max_{r \in \{0, 1\}} \{ 1 - \tau r - \theta(1 - r) \}
\] (4.3)

An honest high-income taxpayer reports \( r = 1 \) and receives her statutory net income

\[ \Pi_H = 1 - \tau \]

A tax evader reports \( r = 0 \) and expects her payoff to be

\[ \Pi_E = 1 - \theta \]

Tax evasion is thus a taxpayer’s preferred choice if and only if

\[ \theta < \tau , \] (4.4)

meaning the gain from evasion (tax savings \( \tau \)) exceeds the expected punishment for evasion (a fine of size 1 payable with perceived probability \( \theta \) of a tax audit). Recall from above that taxpayers differ in their perception \( \theta \) of the audit probability, unobservable to the government. Rewriting the evasion condition (4.4) using \( \theta = p + \epsilon \), we obtain that taxpayers evade whenever the true audit probability \( p \) is low enough relative to the gains of evasion \( \tau \) adjusted by the individual perception bias \( \epsilon \). Let us now formally state taxpayers’ best-response tax reports in the following proposition.

**Proposition 4.1.** [Best-response tax reports]

Given any audit probability \( p \) of a low report, income \( y \in \{0, 1\} \), and perception type \( \theta = p + \epsilon \), a taxpayer’s best response tax report in stage 1 of the game is given by

\[
\begin{align*}
  r^* &= \begin{cases} 
    0 & \text{if } y = 0 \quad \text{and} \quad p < \tau - \epsilon \\
    0 & \text{if } y = 1 \quad \text{and} \quad p \geq \tau - \epsilon \\
    1 & \text{if } y = 1 \quad \text{and} \quad p \geq \tau - \epsilon 
  \end{cases}
\end{align*}
\]

**Proof.** Follows directly from (4.4) and the fact that taxpayers do not overreport income. \( \square \)

In essence, Proposition 4.1 states that low-income taxpayers always report honestly, whereas high-income taxpayers report honestly only when the audit prob-
ability is sufficiently high, and otherwise evade taxes. Note that we follow the literature in assuming that when taxpayers are indifferent regarding their expected payoffs, they report honestly. In what follows, we furthermore assume that the maximum extent of misperception among taxpayers is bounded. That is, we assume \( 0 \leq \tau - a \leq \tau + a \leq 1 \), so that it is possible to induce honesty in the most overconfident taxpayer and that there exist audit probabilities low enough for the most underconfident taxpayer to evade taxes. This assumption focuses the analysis on a realistic range of misperceptions. No taxpayer is so overconfident as to evade even when audits are certain, or so underconfident as to report honestly even if the audit probability equals zero. Let us now proceed to analyzing the tax authority’s audit policy choice.

4.3.2 Audit policy choice

In the initial stage of the game, the government chooses an audit probability \( p \) for low reports \((r = 0)\). Bearing mind that high reports \((r = 1)\) are never audited according to Lemma 4.1, the choice of \( p \) fully determines a tax audit policy in this setup. As described in Section 4.2, the government maximizes an additive social welfare function that weighs expected taxpayer income at unity and expected tax revenue at \( 1 + \lambda > 1 \) to account for the shadow value of public funds. The government’s problem can be written as

\[
\max_{p \in [0,1]} \begin{pmatrix}
\nu Q(p) \left[ (1 - p) + p (1 + \lambda - c) \right] \\
\nu (1 - Q(p)) \left[ 1 - \tau + (1 + \lambda) \tau \right]
\end{pmatrix}
\]

Expected social welfare consists of three terms, associated with the two different income types and their respective reporting behavior. Since all low-income earners report their low income truthfully, we need to distinguish between honest and dishonest reporting function that weighs expected taxpayer income at unity and expected tax revenue at \( 1 + \lambda > 1 \) to account for the shadow value of public funds. The government’s problem can be written as

The first term describes the expected social welfare arising from low-income taxpayers (\( y = 0 \)), who make up a share \( 1 - \nu \) of the population. If they are not audited, their welfare contribution is zero. If low-income types are audited (w.p. \( p \)), however, the government has to pay the audit cost \( c > 0 \) without generating any revenue, and so this term enters negatively in expectation. The first term reflects the welfare cost of mistaking an honest low-income taxpayer for a tax evader.

Now consider the two terms relating to high-income taxpayers, who make up a share \( \nu \) of the population. They will either evade or report honestly, depending
on the audit policy $p$ and their audit perception type $\theta$. $Q(p)$ denotes the share of tax evaders among high-income earners as a function of the audit probability $p$. Let us first describe the respective welfare contributions of the evading and honest high-income taxpayers, and subsequently elaborate further on the shape of $Q(p)$.

The second line of the social welfare function gives, in square brackets, the expected welfare contribution of a high-income tax evader. If there is no audit (w.p. $1 - p$), this equals 1, the undetected evader’s private income. If there is an audit (w.p. $p$), however, private income is zero due to maximum fines. Social welfare is then the tax revenue of size 1, valued at $1 + \lambda$, minus the cost $c$ of a tax audit.

In third line of the above expression, we find in square brackets the welfare contribution term of the honest high-income taxpayer. Since high-income reports are not audited, this term does not contain the audit probability $p$ or audit costs $c$. Instead, it sums up the taxpayer income of 1 minus tax payment $\tau$ and the tax revenue $\tau$ valued at the shadow value of public funds $(1 + \lambda)$.

Now consider $Q(p)$, the share of tax evaders in the high-income population. Recall from Proposition 1 that tax evasion occurs when the audit probability is sufficiently small relative to the gains from evasion (tax savings $\tau$) adjusted by a taxpayer’s perception bias $\epsilon$. Specifically, a high-income taxpayer underreports income if $p < \tau - \epsilon$. Equivalently, evasion occurs if a taxpayer’s random perception component is small enough, or $\epsilon < \tau - p \equiv \tilde{\epsilon}$. For a uniformly distributed perception bias, $\epsilon \sim U(-a,a)$, we obtain for the share of tax evaders that

$$P(\epsilon < \tilde{\epsilon}) = Q(p) = \begin{cases} 
1 & \text{if } p \leq \tau - a \\
\frac{\tau - p + a}{2a} & \text{if } p \in (\tau - a, \tau + a) \\
0 & \text{if } p \geq \tau + a 
\end{cases} \quad (4.5)$$

The least confident taxpayer in the population has a bias of $\epsilon = +a$, meaning she overestimates the true audit probability by $a$. Since evasion occurs when $p < \tau - \epsilon$, an audit probability smaller than $\tau - a$ means that even the most underconfident high-income taxpayer still considers evasion optimal, implying that all high-income earners evade, or $Q(p) = 1$. If, on the other hand, the audit probability is sufficiently large to induce honesty in even the most overconfident taxpayer, characterized by $\epsilon = -a$, we have the converse result that all high-income taxpayers report truthfully, or $Q(p) = 0$. For any audit probability located between these extremes of full evasion and full compliance, a share $\frac{\tau - p + a}{2a}$ of high-income taxpayers will underreport income, while the converse share $1 - \frac{\tau - p + a}{2a}$ chooses to report honestly. The following figure illustrates the share of tax evaders among high-income earners as a function of the audit probability.
Figure 4.2: Share of tax evaders among high-income earners as a function of the audit probability, for \( a = 0.4 \) (solid) and \( a = 0.15 \) (dotted), with \( \tau = 0.5 \).

We see immediately from (4.5) and Figure 4.2 that setting \( p = \tau \) induces half the population to evade, and the other half to report honestly, a starkly different result from the standard model without perceptional biases, where \( p = \tau \) optimally induces full compliance. Notice also that in the intermediate range, the share of evaders is continuously decreasing in the audit probability \( p \).

When setting its audit policy to maximize social welfare, the government faces a three-way tradeoff between minimizing audit costs (requiring a low audit intensity), inducing honest tax reporting (requiring a high audit intensity), and catching evaders (requiring the audit intensity to be high enough to make detection likely, but low enough so as to still find some misreporting in the population). Detecting and fining some evaders, rather than inducing complete honesty, may be optimal. This is because an evader turns over her entire income to the government upon detection, where it is valued at the shadow value of public funds \( 1 + \lambda \). An honest taxpayer, on the other hand, only turns over her statutory tax payment \( \tau \) to the government, and keeps a net income of size \( 1 - \tau \) in private hands, where it is valued at unity. Whether or not tax evaders, via fines, contribute more to expected welfare than honest taxpayers depends on the audit costs \( c \) and the likelihood of detection \( p \). Let us now formally characterize the tax authority's optimal response to this tradeoff.
**Proposition 4.2.** [Equilibrium audit policy]

a) Suppose \( c < \lambda \nu \). Then the equilibrium audit probability of a low report is uniquely given by

\[
p^* = \begin{cases} 
\tau + a & \text{if } a \leq a_L \\
\left( \frac{\nu \lambda - (2 - \nu) c}{\nu \lambda - (3 \nu - 2) c} \right) \cdot \frac{\nu \tau (2 \lambda - c)}{2 \nu (\lambda - c)} & \text{if } a > a_L
\end{cases}
\]

where \( a_L = \frac{\nu \tau c}{\nu \lambda - (3 \nu - 2) c} > 0 \) is a threshold level of heterogeneity.

b) Suppose \( c \geq \lambda \nu \). Then the equilibrium audit probability of a low report is given by \( p^* = 0 \).

**Proof.** See appendix 4.5.3.

Part a) of Proposition 4.2 shows that, provided audit costs are sufficiently low, the tax authority’s audit policy choice critically depends on the extent of heterogeneity in audit perceptions among taxpayers. If taxpayers are relatively homogeneous, i.e. for small \( a \), their perception does not stray very far from the true audit probability. The tax authority then maximizes welfare by inducing full compliance, as in the standard model without perceptional biases. Full compliance is achieved setting \( p^* = \tau + a \), which is required to induce honesty in even the most overconfident taxpayer, whose perception of the audit probability is \( \theta = p - a \). Since all other taxpayers are less confident than the most overconfident one, \( p = \tau + a \) entails that they, too, will report truthfully. Because \( p^* = \tau + a \) already induces full compliance, auditing any stricter would not change anyone’s reporting behavior. At the same time, audit costs would increase due to more wasteful audits of low-income taxpayers, who do not generate any revenue. No audit probability higher than \( \tau + a \) can therefore be optimal.

Now consider the case where heterogeneity in audit perceptions is relatively large, i.e. \( a > a_L \), meaning taxpayers differ significantly in their assessment of the true audit probability. Then the tax authority chooses an interior audit probability in the sense that both tax evasion and tax compliance are part of the equilibrium. That is, the equilibrium share of tax evaders in the high-income population satisfies \( Q(p^*) \in (0, 1) \). When heterogeneity is large, the most overconfident taxpayers severely underestimate the true audit probability. Inducing full compliance is thus too expensive relative to the additional welfare cost of stricter auditing.

Lastly, part b) of Proposition 4.2 states for completeness the well-known result that if audit costs are sufficiently large, it is optimal not to audit at all, meaning \( p^* = 0 \).
The focus of this chapter is to examine the impact of heterogeneity in audit perceptions on the government’s optimal audit policy choice. Let us formally state the relationship between the equilibrium audit intensity $p^*$ and heterogeneity $a$ in the following corollary.

**Corollary 4.1.** [Impact of heterogeneous audit perceptions on equilibrium auditing]

a) Suppose $a < a_L$. Then the equilibrium audit probability increases in $a$.

b) Suppose $a > a_L$. Then the equilibrium audit probability increases in $a$ if and only if audit costs are sufficiently low. Formally,

$$\frac{\partial p^*}{\partial a} > 0 \iff c < \frac{\nu}{2 - \nu}$$

**Proof.** Follows directly from Proposition 4.2. \qed

First, consider the impact of heterogeneous audit perceptions when the extent of heterogeneity is small, i.e. when $a < a_L$. From Proposition 4.2 a), we know that inducing full compliance via $p^* = \tau + a$ is optimal. As heterogeneity $a$ increases, the most overconfident taxpayer’s perception of the true audit probability, $\theta = p - a$, decreases. So inducing honest tax reporting by the most overconfident taxpayer requires a higher audit probability as $a$ increases. We therefore immediately see that the equilibrium audit probability increases in the extent of heterogeneity.

If the extent of heterogeneity is large, i.e. when $a > a_L$, heterogeneous audit perceptions can either increase the equilibrium audit intensity, or decrease it, depending on how costly tax audits are to the government. To analyze the effects of a change in heterogeneity $a$ on the optimal audit probability, it is instructive to look at the first order condition that determines the optimal interior audit probability $p^*$. This condition is obtained by taking the partial derivative of the social welfare function with respect to $p$, and is formally given by

$$\frac{\nu}{2a} \lambda \tau + \nu \frac{\tau - p + a}{2a} (\lambda - c) = (1 - \nu) c + \frac{\nu}{2a} p(\lambda - c) \quad (4.6)$$

The left-hand side of equation (4.6) describes the marginal gain in social welfare from stricter auditing, while the right-hand side describes the marginal welfare cost of auditing. The uniquely optimal interior audit probability $p^*$ satisfying equation (4.6) exactly balances the marginal welfare gain and marginal welfare cost of tax auditing.

A change in the heterogeneity of audit perceptions has three effects, visible in equation (4.6).
First, the marginal gain in social welfare from a higher share of honest taxpayers ($\nu^2 \lambda \tau$) is lowered if $a$ increases. This is because a larger support $[-a, a]$ of possible audit perceptions reduces the marginal increase in the share of honest taxpayers in response to stricter auditing.\textsuperscript{10} We see immediately from the first order condition (4.6) that when audit cost $c$ is relatively large, this effect of lowering marginal revenue is the dominant effect of an increase in heterogeneity. It follows that when audit costs are large, an increase in the support of audit perceptions decreases the audit probability chosen in equilibrium. This case is shown on the right-hand side panel of Figure 4.3 below. Intuitively, large audit costs imply that the welfare contribution of evaders, $p(\lambda - c)$, is relatively small. The main tradeoff in choosing an optimal audit policy is therefore between inducing honesty among high-income types and conducting wasteful audits on low-income types. As $a$ increases, the honesty-inducing effect of stricter auditing is lowered, while the marginal cost of conducting wasteful audits remains unaffected by $a$. Consequently, then, as audit perceptions become more heterogeneous and hence inducing honesty more expensive, the optimal response for the tax authority is to audit less strictly.

Second, a change in $a$, the heterogeneity of audit perceptions, affects the marginal gain from detecting more evaders in response to stricter auditing, $\nu \frac{\tau - p + a}{2a} (\lambda - c)$.\textsuperscript{11} If the audit probability $p$ increases, more evaders are caught and fined in expectation. How this effect is impacted by a change in $a$ depends on how $a$ changes the share of tax evaders, $Q = \frac{\tau - p + a}{2a}$. We see that $Q_a < 0$ if and only if $\tau - p > 0$. That is, whenever $p < \tau$, meaning the audit probability is sufficiently low for more than half the population to evade, or $Q > \frac{1}{2}$, the result of an increase in $a$ is to lower the share of tax evaders, or $Q_a < 0$. Conversely, if $p > \tau$, less than half the (high-income) population evades, or $Q < \frac{1}{2}$, implying that $Q_a > 0$. This property of symmetric distributions, that majority shares decrease toward the mean when the support expands whereas minority shares increase toward the mean as the support expands, here implies that the effect of an increase in $a$ on the marginal revenue from detecting more evaders can go both ways, depending on how $a$ changes the share of evaders in the population. If an expanded support increases the share of evaders, the marginal revenue from detecting more evaders goes up. If an expanded support decreases the share of evaders, marginal revenue from detecting more evaders goes down.

Third, the marginal welfare cost of having fewer tax evaders as a result of stricter auditing, $\frac{1}{2a} p(\lambda - c)$, decreases as $a$ increases. This is because a larger support

\textsuperscript{10} The marginal increase in the share of honest taxpayers in response to stricter auditing corresponds the slope of the graph in Figure 4.2. Increasing the support $a$ flattens this slope.

\textsuperscript{11} The marginal gain in social welfare from detecting more evaders is composed of the share of evaders in the high-income population, $\nu \frac{\tau - p + a}{2a}$, whose expected welfare contribution is $p(\lambda - c)$, and the marginal increase in their expected welfare contribution in response to stricter auditing, $(\lambda - c)$. 

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$[-a, a]$ of audit perceptions lowers the marginal decrease in the share of tax evaders in response to stricter auditing, in analogy to the first effect described above. A large $a$ means that even at high audit probabilities, there will still be some evasion, by the most overconfident taxpayers.

If audit costs are low enough, specifically if $c < \frac{\mu}{2\sigma^2}\lambda$, the expected welfare contribution of tax evaders, $p(\lambda - c)$, is relatively important for expected social welfare. This means the second and third effects described above, namely the marginal gain from catching more evaders and the marginal cost of having fewer evaders, are key in determining the impact of a change in $a$ on the optimal audit probability. In fact, we can show that for such low $c$, the marginal cost of stricter auditing due to fewer evaders always decreases by more in response to a change in $a$ than does the marginal revenue (which might even increase). This means that if audit costs are low enough, the equilibrium audit intensity increases as heterogeneity $a$ increases. This case is shown in the left-hand side panel of Figure 4.3 below. Intuitively, when audit costs are low, the government maximizes social welfare by not inducing too much honesty, so as to still detect and fine some evaders. At the same time, detection is higher the higher is the audit probability. A higher $a$ eases this tradeoff between inducing too much honesty and detecting as many evaders as possible. If the spread of perceptions is large, the government makes relatively few evaders report honestly by auditing more strictly. This allows it to audit a still sizable share of very overconfident evaders at a high audit intensity. If audit costs are low, this amounts to a welfare gain to be had from stricter auditing. The following figure illustrates the equilibrium audit probability as a function of heterogeneity in audit perceptions $a$.

Figure 4.3: Equilibrium audit probability as a function of heterogeneity in audit perceptions, for low audit cost (left panel) and high audit cost (right panel)
Next, we consider the welfare effects of heterogeneity in audit perceptions.

4.3.3 Welfare and heterogeneous audit perceptions

The effect of heterogeneity in audit perceptions on social welfare is non-monotonic. Equilibrium welfare is U-shaped in the degree of heterogeneity $a$: Very low and very high levels of $a$ are associated with relatively high equilibrium welfare, whereas intermediate levels of heterogeneity are associated with a low level of social welfare in equilibrium. Let us consider these cases in turn. When heterogeneity is relatively small, or $a < a_L$, we know from the previous section that the tax authority optimally induces full compliance setting $p^* = \tau + a$, which is increasing in $a$. Since there is full compliance as long as $a \leq a_L$, increasing $a$ in this range does not change anyone’s tax reporting behavior, despite the higher audit probability. The only welfare effect of an increase in heterogeneity $a$, when $a$ is small, is therefore to conduct more wasteful audits of truthful low-income reports filed by low-income taxpayers. Equilibrium welfare thus decreases in the extent of heterogeneity $a$ for small levels of misperception up to $a_L$. This insight makes up part a) of Proposition 4.3 below. The following figure illustrates equilibrium welfare as a function of the degree of heterogeneity in audit perceptions.

![Figure 4.4: Social welfare in equilibrium as a function of heterogeneity in audit perceptions](image)

The precise shape of equilibrium welfare for large levels of heterogeneity ($a > a_L$) depends on the cost of tax audits. Both for high and low audit costs, however, the U-shaped social welfare pattern depicted in Figure 4.4 is retained. Let us now
formally state the relationship between equilibrium welfare and heterogeneous audit perceptions.

**Proposition 4.3.** [Social welfare and heterogeneous audit perceptions]

a) Suppose $a < a_L$. Then equilibrium social welfare is decreasing in $a$.

b) Suppose $a > a_L$. Then equilibrium social welfare is decreasing (increasing) in $a$ if $a < a_H$ ($a > a_H$),

where $a_H = \begin{cases} \frac{\nu \tau c}{\nu \lambda - (2 - \nu) c} > a_L & \text{if } c < \frac{\nu}{2 - \nu} \lambda \\ \frac{\nu \tau c}{(2 - \nu) c - \nu \lambda} > a_L & \text{if } c > \frac{\nu}{2 - \nu} \lambda \end{cases}$ and $a_L = \frac{\nu \tau c}{\nu \lambda - (3\nu - 2)c}$.

**Proof.** See appendix 4.5.4.

Consider now part b) of Proposition 4.3, which describes the welfare effects of heterogeneity in audit perceptions for larger levels of heterogeneity, i.e. for $a > a_L$. As shown in the previous section, larger levels of $a$ imply that both evasion and honest reporting are part of the equilibrium, i.e. $Q(p^*) \in (0, 1)$. Applying the envelope theorem, we find that the change in equilibrium social welfare with respect to heterogeneity is given by

$$\frac{dW^*}{da} = \nu \left( \frac{T - p^*}{2a^2} \right) \times (\lambda T - p^*(\lambda - c)) \quad (4.7)$$

Using equation (4.7), the welfare effects described in Proposition 4.3 b) become evident. Two cases require distinction, namely those of high and low audit costs.

**First,** suppose audit costs are small, i.e. $c < \frac{\nu}{2 - \nu} \lambda$. Recall from Corollary 4.1 above that small audit costs imply that the equilibrium audit intensity is increasing in the extent of heterogeneity, i.e. $\frac{\partial p^*}{\partial a} > 0$. This is because when audit costs are small, the social welfare contribution of tax evaders is relatively large. Since higher heterogeneity allows stricter auditing without inducing too much honesty, an increase in heterogeneity raises the audit intensity chosen in equilibrium. Furthermore, when audit costs are low, we always have $p^* > \tau$, meaning a majority of high-income taxpayers reports honestly. As argued above, this implies that an increase in the support of possible audit perceptions decreases the share of honest taxpayers in the population. The first term on the right-hand side of (4.7) is therefore always negative when audit costs are small. Put differently, an increase in heterogeneity $a$ always increases the share of tax evaders in the population if audit costs are small. This means an increase
in a induces a welfare gain when evaders contribute more to expected welfare than honest taxpayers, or $\lambda\tau - p^*(\lambda - c) < 0$, and vice versa. Since the audit probability $p^*$ increases in $a$, this is equivalent to saying that for intermediate levels of $a$, i.e. $a_L < a < a_H$ where honest taxpayers always contribute more, an increase in heterogeneity $a$ induces a decrease in social welfare. For high levels of heterogeneity, i.e. $a > a_H$, however, evaders contribute more to social welfare in expectation since their probability of being detected is relatively high in equilibrium, yet they still evade due to high levels of overconfidence. Then, an increase in heterogeneity $a$ causes equilibrium welfare to rise, which explains the U-shaped social welfare curve depicted in Figure 4.4 above.

Second, consider the case of large audit costs, i.e. $c > \frac{\nu}{2-\nu}\lambda$. From Corollary 4.1, we know that large audit costs imply that the equilibrium audit intensity is decreasing in heterogeneity, because inducing honesty becomes more expensive on the margin as the support of possible audit perceptions increases, while tax evaders contribute relatively little to social welfare. Indeed, when audit costs are large, the welfare contribution of honest taxpayers always exceeds the expected welfare contribution of tax evaders, or $\lambda\tau - p^*(\lambda - c) > 0$. This means the second term on the right-hand side of equation (7) is always positive. It follows that the effect of an increase in $a$ on equilibrium welfare depends on how the share of honest taxpayers changes with $a$. That is, on whether the first term on the right hand side of (7) is positive or negative. Honest taxpayers yield more welfare in expectation than evaders. So an increase in heterogeneity $a$ is associated with a welfare gain if the share of honest taxpayers increases due to $a \uparrow$, and with a welfare loss if the share of tax evaders increases due to $a \uparrow$. Because the audit probability decreases in $a$, then, intermediate levels of heterogeneity, $(a_L < a < a_H)$ are associated with decreasing equilibrium welfare, while large levels of heterogeneity $(a > a_H)$ cause social welfare to rise with heterogeneity in audit perceptions, yielding the U-shaped pattern of social welfare familiar from Figure 4.4. This concludes the analysis of the social welfare effects of heterogeneity in audit perceptions.

13 To see this, note that at $a_L$, $\lambda\tau > p^*(\lambda - c)$. But for $a > a_L$ and large audit costs, we have $\frac{\partial p^*}{\partial a} < 0$ by Corollary 4.1 b). So this inequality must hold for all $a > a_L$. 

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4.4 Conclusion

In this chapter, we examine how heterogeneity in taxpayers’ audit perceptions affects tax compliance, the tax authority’s audit policy, and social welfare. If taxpayers differ sufficiently in their perception of audit risk, both tax evasion and truthful reporting are part of the equilibrium. This contrasts with the full compliance equilibrium often found in the literature on optimal auditing. In our model, full compliance obtains only when audit perceptions are very homogeneous, close to identical. Moreover, we find that heterogeneity in audit perceptions substantially changes the government’s optimal audit policy. The precise nature of this change depends on how costly tax audits are to the government. If audit costs are sufficiently low, heterogeneity in audit perceptions is associated with stricter auditing in equilibrium. Intuitively, this is because low audit costs decrease the cost of detecting and fining evaders, which, if done with a high enough probability, can be welfare-enhancing. But such a high enough audit probability is not optimal when taxpayers are relatively homogeneous, for then it would induce full compliance, defying its purpose of catching and fining tax evaders with a high likelihood. Stricter auditing without inducing “too much” honesty (so as to still detect and fine some evaders) is only possible when taxpayers differ significantly in their audit perceptions, so that there are enough very overconfident taxpayers who evade even at high audit probabilities. If, on the other hand, tax audits are rather costly to the government, heterogeneity in audit perceptions is associated with auditing less strictly in equilibrium. This is because when audit costs are high, the government’s main tradeoff is between minimizing audit costs and inducing honesty, rather than focusing on detecting tax evaders. Heterogeneity in audit perceptions, however, weakens the honesty-inducing effect of stricter auditing and thus leads to a lower audit probability in equilibrium. Lastly, we consider the welfare effects of heterogeneous audit perceptions and find a non-monotonic, U-shaped relationship between the extent of heterogeneity and social welfare. Very homogenous or very heterogeneous populations are associated with relatively high levels of social welfare, whereas a moderately heterogeneous population corresponds to a relatively low level of social welfare in equilibrium.
4.5 Appendix

4.5.1 Interim incentive (in-)compatibility

This chapter follows the principal-agent approach to optimal auditing first developed by Reinganum and Wilde (1985). Characteristic of this class of models is that the tax authority commits to an audit policy before taxpayers file their reports. A point sometimes found wanting in these models\(^{14}\) is that after taxpayers have filed their reports, the tax authority may want to deviate from its previously announced audit policy. In the standard model of auditing without heterogeneity, where the equilibrium audit probability under commitment induces full compliance, this reconsideration takes an extreme form: after receiving only honest reports, the tax authority would rather not audit at all, because auditing is costly and, at this interim stage, does not change anyone’s reporting behavior or generate any revenue. Behavior is not changed because all reports have already been filed and revenue is not generated because all taxpayers have reported truthfully, meaning audits will not generate any fines. In our model, too, the government would want to deviate from its commitment audit policy, but with an interesting twist. Instead of not auditing at all after receiving tax reports, the tax authority may want to maximally increase its audit probability and audit with certainty. This occurs when audit costs are sufficiently low. Our model also contains, for sufficiently large audit costs, the standard result that redeciding at an interim stage yields an audit probability of zero. Let us now formally state and derive this result.

Proposition. [Interim incentive incompatibility]

Suppose that, unbeknownst to taxpayers, the tax authority can deviate from its announced audit policy \(p^*\) after taxpayers have filed their reports. Then the optimal audit choice at this interim stage is given by

\[
p_{\text{int}} = \begin{cases} 
1 & \text{if } a > a_L \text{ and } c < \frac{a\nu\lambda}{(2-\nu)a+\nu\tau} \\
0 & \text{if } a > a_L \text{ and } c \geq \frac{a\nu\lambda}{(2-\nu)a+\nu\tau} \\
0 & \text{if } a \leq a_L 
\end{cases}
\]

Proof. Suppose the tax authority has announced an audit probability \(p^*\) according to Proposition 4.2 and taxpayers have filed their best-response tax reports according to Proposition 4.1. Now the tax authority may set anew its probability of auditing a low report.

\(^{14}\)For a detailed discussion, see for instance the survey by Andreoni et al. (1998). Interim incentive incompatibility is also a property of many costly state verification models more widely, going back to Townsend (1979) and Gale and Hellwig (1985).
If $a \leq a_L$, we know from Proposition 4.2 that inducing full compliance via $p^* = \tau + a$ is optimal. All taxpayers therefore truthfully report their income, and so, as in the standard model without heterogeneity, auditing is wasteful and the tax authority optimally chooses $p_{int} = 0$.

The more involved case is when $a > a_L$. Then the equilibrium audit intensity under commitment, $p^* = \frac{(\nu \lambda - (2 - \nu) c) a + \nu \tau (2 \lambda - c)}{2 \nu (\lambda - c)}$, gives rise to an equilibrium where both evasion and honest reporting by high-income earners occurs, i.e. where $Q(p^*) \in (0, 1)$. A low income report may come from an honest low-income earner or from a high-income tax evader. Using Bayes’ rule, the probability that, given $p^*$, a low-income report was filed by a taxpayer who actually has high income is given by

$$\mu(p^*) = \frac{\nu Q(p^*)}{(1 - \nu) + \nu Q(p^*)} \quad (4.8)$$

The government’s problem is then equivalent to

$$\max_{p \in [0, 1]} \{ p \left[ (1 - \mu)(-c) + \mu (1 + \lambda - c) \right] + (1 - p) \left[ (1 - \mu) 0 + \mu 1 \right] \}$$

which simplifies to

$$\max_{p \in [0, 1]} \{ \mu + p (\mu \lambda - c) \}$$

Since the objective function is linear in the interim audit probability $p$, it is seen immediately that a corner solution obtains, and that the solution to this maximization problem is thus given by $p = 0$ if $\mu \lambda < c$ and $p = 1$ if $\mu \lambda > c$. Using the expressions for $p^*$ and $Q(p^*)$ in $\mu(p^*)$ then gives the conditions provided in the Proposition above. This concludes the proof.

4.5.2 The role of the tax rate $\tau$

Although we confine ourselves to focusing purely on the problem of how to optimally audit taxpayers given an exogeneous tax rate $\tau$, it is obvious that a related question is how to optimally tax them in the first place. Aside from a few exceptions, for instance the seminal paper by Chander and Wilde (1998), optimal auditing and optimal taxation have usually been treated separately, and it is beyond the scope of the present chapter to attempt this challenging integration here. However, a few insights about the role of the tax rate $\tau$ emerge from our analysis. We shall first consider the impact of $\tau$ on the equilibrium audit probability, and subsequently examine the impact of the tax rate on equilibrium welfare. To focus the analysis, consider the following corollary.

**Corollary.** (i) [Impact of $\tau$ on equilibrium auditing]

a) The equilibrium audit probability increases in $\tau$, but more so when $a > a_L$
b) The effect of heterogeneity on the equilibrium audit probability is not impacted by $\tau$.

c) The threshold level of heterogeneity required for an interior audit probability, $a_L$, increases with $\tau$.

Proof. a) When $a \leq a_L$, we have $p^* = \tau + a$ and hence $\frac{\partial p^*}{\partial \tau} = 1$. If $a > a_L$, we have $p^* = \frac{\nu c - (1 - \nu) c}{\nu c - (2 - \nu) c}$ and hence $\frac{\partial p^*}{\partial \tau} = \frac{2 \lambda - c}{2 (\lambda - c)} > 1$.

b) For both $a \leq a_L$ and $a > a_L$, it holds that $\frac{\partial^2 p^*}{\partial a \partial \tau} = 0$.

c) We have from $a_L = \nu c - (3 - \nu) c$ that $\frac{\partial a_L}{\partial \tau} = \nu c - \frac{(2 - \nu) c - \nu \lambda}{\nu c - (3 - \nu) c} > 0$, as both numerator and denominator are positive since $c < \nu$ and $\nu < 1$.

Corollary (i) presents the comparative statics of equilibrium auditing with respect to the tax rate $\tau$. We find that an increase in the tax rate is associated with stricter auditing in equilibrium, suggesting that tax rate and audit probability are complementary policy instruments. Part a) also states that this complementarity is stronger in the equilibrium with only partial compliance due to large heterogeneity ($a > a_L$) that is a distinguishing feature of our model compared to the standard model without heterogeneity, where full compliance obtains in equilibrium. Part b) of the preceding Corollary shows that the impact of heterogeneity on auditing as described in Corollary 4.1 is not affected by the tax rate. Finally, we find that the extent of heterogeneity required for the partial compliance equilibrium increases with $\tau$.

Let us now consider the relationship between the tax rate and social welfare. We arrive at two formal results, stated in the following Corollary.

**Corollary.** (ii) [Impact of $\tau$ on social welfare in equilibrium]

a) The threshold level of heterogeneity required for social welfare to increase in heterogeneity, $a_H$, increases with $\tau$.

b) If $a \leq a_L$, equilibrium welfare strictly increases in $\tau$.

Proof. a) From Proposition 4.3, we have $a_H = \left\{ \begin{array}{ll} \frac{\nu c}{\nu c - (2 - \nu) c} & \text{if } c < \frac{\nu}{2 - \nu} \lambda \\ \frac{\nu c}{\nu c - (2 - \nu) c} & \text{if } c > \frac{\nu}{2 - \nu} \lambda \end{array} \right.$

So $\frac{\partial a_H}{\partial \tau} = \left\{ \begin{array}{ll} \frac{\nu c}{\nu c - (2 - \nu) c} & \text{if } c < \frac{\nu}{2 - \nu} \lambda \\ \frac{\nu c}{\nu c - (2 - \nu) c} & \text{if } c > \frac{\nu}{2 - \nu} \lambda \end{array} \right.$, implying that $\frac{\partial a_H}{\partial \tau} > 0$ throughout.

b) Suppose $a \leq a_L$, so $p^* = \tau + a$. Then welfare is given by

$$W_{a \leq a_L} = \nu + \nu \lambda \tau - (1 - \nu) c \tau - (1 - \nu) ca$$

Therefore, $\frac{\partial W_{a \leq a_L}}{\partial \tau} = \nu \lambda - (1 - \nu) c$. So we have $\frac{\partial W_{a \leq a_L}}{\partial \tau} > 0$ if and only if $c < \frac{\nu}{2 - \nu} \lambda$. The latter condition is implied by our assumption that $c < \lambda \nu$. \qed
This Corollary shows that as the tax rate $\tau$ increases, the level of heterogeneity in audit perceptions that corresponds to a welfare minimum, $a_H$, increases. This means that the higher the tax rate $\tau$, the higher the extent of heterogeneity $a$ required to make further increases in heterogeneity welfare-increasing. In part b), we show that in the full compliance equilibrium, which obtains for very homogeneous populations in our model ($a \leq a_L$), setting a maximal tax rate is the welfare optimum. As would be expected due to the complex interactions between optimal taxation and optimal auditing, matters are less tractable when heterogeneity is large and the partial compliance equilibrium obtains. We therefore leave the joint-determination of optimal tax and audit policies in this case for future research.

4.5.3 Proof of Proposition 4.2

Consider three different ranges of the audit probability $p$ and the respective shape of the social welfare function in these ranges.

**First, suppose** $p \geq \tau + a$.

From (4.5), we know this implies $Q = 0$, meaning all high-income taxpayers report honestly as the audit probability is sufficiently high. Using $Q = 0$ in the social welfare function, one obtains

$$W_{p \geq \tau + a} = \nu(1 + \lambda\tau) - (1 - \nu)cp$$

So social welfare is strictly decreasing in $p$ for $p \geq \tau + a$. The tax authority’s equilibrium choice of $p$ will therefore never be higher than $\tau + a$.

**Second, suppose** $p \leq \tau - a$.

From (4.5), we know this implies $Q = 1$, meaning all high-income taxpayers underreport income because the audit probability is sufficiently low. Plugging $Q = 1$ into the social welfare function, we get

$$W_{p \leq \tau - a} = \nu + p(\lambda\nu - c)$$

So provided $c < \lambda\nu$, social welfare is strictly increasing in $p$ for $p \leq \tau - a$. The tax authority’s equilibrium choice of $p$ will therefore not be lower than $\tau - a$ provided $c < \lambda\nu$. We also see that if $c > \lambda\nu$, social welfare strictly decreases in $p$, so that the optimal policy is not to audit at all, or $p^* = 0$, proving part b) of Proposition 4.2.
Third, suppose \( p \in (\tau - a, \tau + a) \).

From (4.5), we know this implies \( Q = \frac{\tau - p + a}{2a} \in (0, 1) \). Using this expression in the social welfare function, one obtains

\[
W_{p \in (\tau - a, \tau + a)} = \nu - (1 - \nu) c p + \nu \frac{\tau - p + a}{2a} p (\lambda - c) + \nu (1 - \frac{\tau - p + a}{2a}) \lambda \tau
\]

Social welfare is concave in \( p \) if \( c < \lambda \), which is implied by \( c < \lambda \nu \) since \( \nu \in [0, 1] \). So the first-order condition given by (4.6) above defines a unique social welfare maximum, which, solving (4.6) for \( p \), is found at

\[
p^* = \frac{(\nu \lambda - (3\nu - 2) c) a + \nu \tau (2\lambda - c)}{2 \nu (\lambda - c)}.
\]

Let us now proceed to analyzing for which values of heterogeneity \( a \) the interior solution \( p^* \) is indeed optimal. That is, we need to show when \( p^* \in (\tau - a, \tau + a) \) holds.

First, consider \( p^* < \tau + a \).

Plugging in the expression for \( p^* \) above, the inequality \( p^* < \tau + a \) is equivalent to

\[
a (\nu \lambda - (3\nu - 2) c) > \nu c \tau
\]

Now, let us distinguish two cases.

a) Suppose \( \nu \leq \frac{2}{3} \). Then \( 3\nu - 2 \leq 0 \) and so inequality (4.8) is equivalent to

\[
a > \frac{\nu c \tau}{\nu \lambda - (3\nu - 2) c} \equiv a_L
\]

b) Suppose \( \nu > \frac{2}{3} \). Then we have

\[
\nu \lambda - (3\nu - 2) c > 0 \iff c < \frac{\nu \lambda}{3\nu - 2} \quad (4.9)
\]

But \( \nu \in (\frac{2}{3}, 1] \) implies \( \frac{\nu \lambda}{3\nu - 2} > \nu \lambda \). So from our assumption that \( c < \nu \lambda \), we know \( c < \frac{\nu \lambda}{3\nu - 2} \) and hence that, again, inequality (4.8) is equivalent to

\[
a > \frac{\nu c \tau}{\nu \lambda - (3\nu - 2) c} \equiv a_L
\]

This provides a necessary and sufficient condition on \( a \) for the interior solution \( p^* \) to be optimal indeed, namely that the extent of heterogeneity \( a \) is large enough.
Second, consider \( p^* > \tau - a \).

The inequality \( p^* > \tau - a \) is equivalent to

\[
(3\nu \lambda - (2 + \nu) c) a > -\nu c \tau
\]

which is satisfied if

\[
3\nu \lambda - (2 + \nu) c > 0
\]

or equivalently,

\[
c < \frac{3\nu \lambda}{2 + \nu}
\]

But since \( 2 + \nu < 3 \), our initial assumption of \( c < \nu \lambda \) ensures that we always have \( c < \frac{3\nu \lambda}{2 + \nu} \). It follows that \( p^* > \tau - a \) is implied by \( c < \nu \lambda \).

This concludes the proof of Proposition 4.2. \( \square \)

4.5.4 Proof of Proposition 4.3

We begin with part a), supposing \( a < a_L \).

From Proposition 4.2, we know that \( p^* = \tau + a \) when \( a < a_L \). This implies full compliance, or \( Q = 0 \). Social welfare is thus given by

\[
W_{a < a_L} = \nu(1 + \lambda \tau) - (1 - \nu) c (\tau + a)
\]

And it is seen immediately that \( \frac{\partial W_{a < a_L}}{\partial a} < 0 \), i.e. social welfare is decreasing in the extent of heterogeneity \( a \) for \( a < a_L \).

Next, consider part b), supposing \( a > a_L \).

As indicated in the proposition, two cases require distinction.

First, suppose \( c < \frac{\nu}{2 - \nu} \lambda \).

Applying the envelope theorem yields the change in equilibrium social welfare with respect to heterogeneity as

\[
\frac{dW^*}{da} = \nu \left( \frac{\tau - p^*}{2a^2} \right) \times (\lambda \tau - p^*(\lambda - c))
\]

Since \( c < \frac{\nu}{2 - \nu} \lambda \), we have \( \nu \left( \frac{\tau - p^*}{2a^2} \right) < 0 \) for all \( a > a_L \). To see this, note that by Corollary 4.1 b), we know that \( \frac{\partial p^*}{\partial a} > 0 \) for \( a > a_L \) and at \( a = a_L \), we have \( p^* = \tau + a > \tau \). So we have

\[
\frac{dW^*}{da} < 0 \iff (\lambda \tau - p^*(\lambda - c)) > 0
\]
Plugging \( p^* = \left( \frac{\nu \lambda - (2 - \nu) c}{2 \nu} \right) a + \frac{\nu \tau (2 \lambda - c)}{2 \nu (\lambda - c)} \) into the right-hand side, the inequality is equivalent to

\[
a < \frac{\nu \tau c}{\nu \lambda - (2 - \nu) c} \equiv a_H
\]

The last step of the argument for small audit cost is to show that \( a_L < a_H \). We have

\[
a_L = \frac{\nu \tau c}{\nu \lambda - (3 \nu - 2) c} < \frac{\nu \tau c}{\nu \lambda - (2 - \nu) c} = a_H
\]

or equivalently,

\[
3\nu - 2 < 2 - \nu
\]

or equivalently,

\[
4\nu < 4
\]

which is always satisfied since \( \nu \in (0, 1) \). This concludes the argument for small audit cost.

**Second, suppose** \( c > \frac{\nu}{2 - \nu} \lambda \).

As before, applying the envelope theorem yields the change in equilibrium social welfare with respect to heterogeneity as

\[
dW^* = dW = \nu \left( \frac{\tau - p^*}{2a^2} \right) \times (\lambda \tau - p^* (\lambda - c))
\]

Since \( c > \frac{\nu}{2 - \nu} \lambda \), we have \( \lambda \tau - p^*(\lambda - c) > 0 \) for all \( a > a_L \). To see this, note that by Corollary 4.1 b), we have \( \frac{\partial p^*}{\partial a} < 0 \) for \( a > a_L \). Moreover, we have that at \( a = a_L \), \( \lambda \tau - p^*(\lambda - c) > 0 \). So we have that

\[
dW^* = dW < 0 \iff \nu \left( \frac{\tau - p^*}{2a^2} \right) < 0
\]

And using \( p^* = \left( \frac{\nu \lambda - (2 - \nu) c}{2 \nu} \right) a + \frac{\nu \tau (2 \lambda - c)}{2 \nu (\lambda - c)} \), the inequality on the right-hand side is equivalent to

\[
\tau < \left( \frac{\nu \lambda - (2 - \nu) c}{2 \nu} \right) a + \frac{\nu \tau (2 \lambda - c)}{2 \nu (\lambda - c)}
\]

or equivalently,

\[
a < \frac{\nu \tau c}{(2 - \nu) c - \nu \lambda} \equiv a_H
\]
The last step of the argument is to show that $a_L < a_H$. We have

$$a_L = \frac{\nu \tau c}{\nu \lambda - (3\nu - 2)c} < \frac{\nu \tau c}{(2 - \nu)c - \nu \lambda} = a_H$$

Rearranging, this inequality is equivalent to

$$\nu \lambda - (3\nu - 2)c > (2 - \nu)c - \nu \lambda$$

or equivalently,

$$c < \lambda$$

which always holds due to our assumption that $c < \nu \lambda$ and the fact that $\nu \in (0, 1)$.

This concludes the proof of Proposition 4.3. □
Chapter 5

Conclusion

This thesis aims to deepen the theoretical understanding of tax evasion. It focuses particularly, but not exclusively, on corporate tax evasion. Using microeconomic theory, especially contract-theoretic methods, chapters 2 and 3 analyze two features distinguishing corporate from individual tax evasion, namely the separation of ownership and control and corporate financing. In chapter 2, we consider how contracting between a non-specialist shareholder and a specialized tax manager influences the tax evasion game when the manager decides not only on how much taxes to evade, but also in what way to evade them. That is, she may decide on both qualitative and quantitative aspects of evasion. We find that taking this quality dimension into account, earlier results about the effect of asymmetric information on corporate tax evasion change. For instance, as the quality of tax evasion is reduced by informational asymmetries between the specialized tax manager and the non-specialist shareholder who hires her, the efficacy of tax enforcement increases. Tax enforcement is then stricter in equilibrium where firms enter principal-agent relationships to evade taxes, rather than more lenient as suggested by earlier findings. Chapter 3 considers the interaction of corporate financing and tax evasion. We extend the classic costly state verification model of financial contracting to allow for tax evasion by the borrowing entrepreneur. Because tax evasion is illegal, the potential proceeds from it are not contractible. The downside in case of detection, however, is potentially harmful to the investor. That is because fines for evasion may exhaust the entrepreneur’s funds and thus constrain her ability to repay the investor. In this context, we find that standard debt contracts are no longer optimal. Instead, the optimal contract combines elements of debt and equity and is less efficient than a standard debt contract. In chapter 4 of this thesis, we examine how heterogeneous perceptions of audit risk by taxpayers influence the government’s optimal tax audit policy. We find that a taxpayer population exhibiting such heterogeneity requires a substantially different optimal audit strategy. Whether the equilibrium audit intensity increases or decreases as a result of heterogeneity in
audit perceptions depends on how costly audits are to the government. We also conduct a welfare analysis to gauge the impact of such heterogeneity on society. When the extent of heterogeneity in audit perceptions is either very small or very large, equilibrium social welfare is high, we find. By contrast, intermediate levels of heterogeneity are associated with lower levels of social welfare.
Chapter 6

English Summary

In this thesis, we use microeconomic theory, in particular contract-theoretic methods, to analyze various aspects of tax evasion. It contains three chapters. The first chapter (Ch. 2 in this document) analyzes the impact of the separation of ownership and control in firms on corporate tax evasion. We consider a principal-agent model with multitasking, where a non-specialist shareholder hires a specialized tax manager who decides on both the quantity and quality of tax evasion. Quantity simply means the extent of underreporting. Quality is a form of self-insurance that reduces the expected fine for evasion. Higher quality may be interpreted as resorting to more sophisticated or complex ways of evasion, for example. We first characterize the optimal second best contract between tax manager and firm-owner (Proposition 2.1). We find that compared to the first-best case of symmetric information, equilibrium efforts are lower due to asymmetric information along both the quantity and quality dimensions. We then derive the government’s optimal enforcement policy (Proposition 2.2) and examine how it changes with the extent of asymmetric information or noisiness along each dimension. There are two countervailing effects of asymmetric information on equilibrium tax enforcement. Along the quantity dimension, asymmetric information reduces the equilibrium audit intensity, whereas along the quality dimension, asymmetric information increases the equilibrium audit intensity (Corollary 2.3). The overall effect is ambiguous and we provide, in Corollary 2.4, the conditions deciding when the equilibrium enforcement intensity is higher and when it is lower than in first-best. We therefore find that earlier results, notably by Crocker and Slemrod (2005), on how equilibrium tax enforcement is affected by asymmetric information inside the firm reverse or at least have to be qualified. Tax enforcement may be stricter where firms enter principal-agent relationships to evade taxes, rather than more lenient as suggested in their work. In the second chapter (Ch. 3 in this document), we analyze how financial contracting and tax evasion interact. In particular, we extend the classic costly state verification model of financial contracting due to Townsend (1979) and...
Gale and Hellwig (1985) to allow for tax evasion by the borrowing entrepreneur. We first characterize the entrepreneur’s tax evasion behavior in Proposition 3.1, and derive feasibility constraints emerging from it in Corollary 3.1. Tax evasion violates feasibility unless some rent sharing occurs. So under non-verification by the investor, when tax evasion is possible, a feasible contract cannot exhaust the entrepreneur’s funds. Otherwise, the entrepreneur would be protected by limited liability regarding her tax evasion choice, and evade in such a way that the contractual repayment could not be made in full. We then find, in Proposition 3.2, that tax evasion also alters the incentive-compatibility constraints of the traditional framework. Even under investor verification, i.e. when tax evasion does not occur, some rent sharing is required to ensure incentive-compatibility. We thus arrive at an optimal contract that combines elements of debt and equity, and fully characterize this contract in Proposition 3.3. It is debt-like in the sense that very low project returns still lead to liquidation by the investor, and in that under non-verification repayment is constant, albeit lowered relative to a standard debt contract due to the feasibility constraint described above. For intermediate profit realizations, the optimal contract stipulates verification and rent-sharing, which is a feature more akin to equity contracts.

In the final chapter of this thesis (Ch. 4 in this document), we examine the impact of heterogeneity in taxpayers’ perception of audit risk on the government’s optimal audit policy. The equilibrium audit policy characterized in Proposition 4.2 depends on the extent of heterogeneity in audit perceptions, we find. If the spread of perceptions around the true audit probability is relatively small, the government chooses to induce full compliance, as in the standard model of optimal auditing without perception heterogeneity. Yet for larger heterogeneity in audit perceptions, the equilibrium audit intensity changes substantially, as shown in Corollary 4.1. It may decrease or increase as a result of perception heterogeneity, depending on the cost of auditing. When audit costs are large, the government’s main tradeoff is between inducing honesty and economizing on audit costs. Heterogeneity in audit perceptions weakens the honesty-inducing effect of auditing, since more taxpayers severely underestimate the audit probability when heterogeneity is large. But it does not impact the cost of auditing, thus tilting the tradeoff toward a lower audit intensity in equilibrium. A related argument can be made for low audit costs to find that the equilibrium audit intensity increases with heterogeneity in audit perceptions if audit costs are low.

We then proceed to analyzing the effects of heterogeneous audit perceptions on social welfare and present our findings in Proposition 4.3. A U-shaped pattern emerges: very low and very high levels of heterogeneity are associated with high levels of social welfare, with lower levels of equilibrium welfare for intermediate extents of perception heterogeneity among taxpayers.
Chapter 7

German Summary

metrische Information zu einer geringeren Qualität der Steuerhinterziehung, somit
t zu einer höheren Effektivität der Prüfung, und darüber schließlich zu einer höheren
gleichgewichtigen Prüfungsintensität (s. Korollar 2.3). Der Gesamteffekt asym-
metrischer Information auf die Prüfungsintensität im Gleichgewicht kann somit
sowohl eine Erhöhung als auch eine Verminderung der Prüfungsintensität sein.
Korollar 2.4 stellt die hierfür entscheidende Bedingung zwischen diesen beiden
Fällen dar. Die vorliegenden Ergebnisse erfordern daher eine Qualifizierung
der früheren Ergebnisse von Crocker und Slemrod (2005). Die optimale Prü-
fungsintensität kann in Folge einer Prinzipal-Agenten Beziehung innerhalb der
Unternehmung höher sein, anstatt niedriger wie von Crocker und Slemrod (2005)
suggieriert.

Im zweiten Kapitel der vorliegenden Arbeit (Kapitel 3 dieses Dokuments) wird
das Zusammenspiel von Steuerhinterziehung und der Wahl des optimalen Fi-
nanzkontrakts einer Unternehmung untersucht. Dabei wird ein klassisches Mod-
ell, das insbesondere von Townsend (1979) und Gale und Hellwig (1985) en-
twickelte “costly state verification” Modell, um die Möglichkeit der Steuerhin-
terziehung durch den Unternehmer bzw. Kreditnehmer erweitert. In Proposi-
tion 3.1 wird zunächst das Hinterziehungsverhalten des Unternehmers charak-
terisiert. Daraus ergeben sich unmittelbar Möglichkeitsbedingungen an den Fi-
nanzkontrakt, die wir in Korollar 3.1 darstellen. Es wird deutlich, dass Steuer-
hinterziehung nur in Verbindung mit einer Aufteilung der sich aus dem Projekt
ergebenen Renten möglich ist. Verifiziert der Investor den Projektvertrag nicht
und macht somit Steuerhinterziehung möglich, kann der Vertrag also an keiner Stelle
den vollen Projektvertrag dem Investor zuschreiben. Wäre dies der Fall, so ergäbe
sich für den Unternehmer eine Situation beschränkter Haftung im Hinblick auf
die Steuerhinterziehung. Er hinterzögere dann - ohne Rücksicht auf den Entdeck-
ungsfall - in einem Maße, welches bei Aufdeckung eine vollständige Rückzahlung
an den Investor verhindern würde. Ein solcher Vertrag wäre also nicht erfüllbar
und verletzt mithin die Möglichkeitsbedingungen aus Korollar 3.1. Weiterhin
beschreiben wir in Proposition 3.2 die im Gegensatz zum Standardmodell verän-
derten Anreizkompatibilitätsbedingungen. So ist selbst im Falle der Verifizierung
durch den Investor, wenn Steuerhinterziehung nicht möglich ist, eine Aufteilung
 der Projektrenditen notwendig, um Anreizkompatibilität zu gewährleisten. Der
optimale Finanzvertrag, den wir in Proposition 3.3 beschreiben, kombiniert somit
Elemente der Kreditfinanzierung mit Elementen der Eigenkapitalfinanzierung.
Er ähnelt einem Kreditvertrag, da sehr kleine Projektreträge nach wie vor zur
Liquidierung der Unternehmung führen, und da große Projektreträge mit Nicht-
verifizierung und einer konstanten Rückzahlung verbunden sind, wenngleich diese
aufgrund der veränderten Möglichkeitsbedingungen aus Korollar 3.1 geringer
ausfällt als in einem Standardkreditvertrag. An Eigenkapital erinnert der op-
timale Vertrag für mittlere Projekterträge, da hier zwar Verifizierung durch den Investor stattfindet, der Projektertrag aber aufgeteilt wird.

Bibliography


