3 Portfolio Management

If all stock predictions were perfect, portfolio management would amount to the transfer of funds to the commodity that promises the highest return in the specified investment interval. Unfortunately, the future is not predictable to that degree of accuracy. Consequently, portfolio management requires a careful distribution of funds in various stocks so that any one single incorrect prediction does not dramatically and negatively affect the performance of the entire portfolio. On the other hand, spreading the risk between numerous stocks also implies that a dramatic upside gains by any one investment only helps the portfolio proportionally.

Due to this dynamic, portfolio selection is dependent on the risk adversity of the investor. Markowitz defined the theoretical concept of the perfect portfolio, on which NELION is based [Markowitz 1959]. After analyzing the concepts of return and risk in this chapter, I present the parameterization of the conflicting goals of high return with low risk in the optimal portfolio theory.

3.1 Return

The return of a stock in a specified period is the percentage increase of the value of the investment. It is defined as follows:

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \qquad \text{Equation 3.1.1}$$

In the equation above, P_t is the current price and P_{t-1} is the price at the beginning of the interval, while D_t is the dividend within that period. The dividend can never be negative. For periods, which do not coincide with the financial year of the underlying stock, the dividend is calculated as a percentage of the total accrued during the period. Following standard investment convention, we assume that the interval is one year. From Equation 3.1.1, it is clear that the return R_t is positive if P_t is larger than P_{t-1}, or the price of the commodity has increased.

The return of a portfolio is the weighted sum of the i individual stock returns.

$$R_{t,P}\left(\overline{X}\right) = \sum_i X_i R_{t,i} \qquad \text{Equation 3.1.2}$$

In this equation X_i denotes the fraction of the portfolio covered by each investment and therefore

$$\sum_{i=1}^{n} X_i = 1 \quad \text{where } X_i \geq 0 \qquad \text{Equation 3.1.3}$$

This requirement does not impose any restrictions on the allocation of funds, since it allows for money kept as cash. The return would then simply be the bank interest rate, which may be 0%, depending on the account type.

3.2 Risk

Unlike the return of an investment, the definition of risk is more subjective. Markowitz assumes a normal distribution of upside and downside potential around the return of a commodity, based on its volatility *ó*.

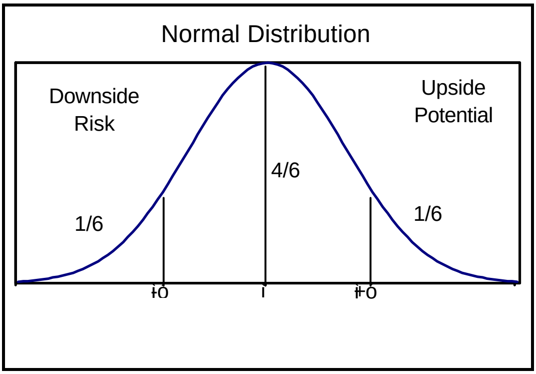

Figure 3.2.1: The One-in-Six Rule

In Figure 3.2.1, the expected return *ì* defines the peak of the normal distribution with *ì-s* and *ì+s* defining a *2/3* margin of return. The downside potential is *1/6*, hence the name of the rule. It is clear that a small s reduces potential loss thereby minimizing the associated risk of the equity. We therefore define the risk $V(X)$ of an investment of value X with a variance s as follows.

$$V(X) = X^2 s^2 \qquad \text{Equation 3.2.1}$$

Unlike the return, the risk can not simply be calculated as the weighted sum of the individual risks, since individual stocks can be dependent on similar external factors. Both Daimler-Chrysler and Ford are affected negatively by rising oil prices, for example, so that a portfolio consisting of these two stocks has a higher risk than one with Daimler-Chrysler and Microsoft, for example, assuming that Microsoft and Ford have the same volatility. Consequently, the systemic risk of a portfolio includes the covariance r_{ij} of the individual investments i and j as shown in Equation 3.2.2 below.

$$V_P\left(\overline{X}\right) = \sum_{i=1}^{n} X_i^2 r_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} X_i X_j r_{ij} \qquad \text{Equation 3.2.2}$$

The first term represents the inherent risk of every individual stock, while the second term captures the risk associated with the correlation between stocks. Given a portfolio where the correlation r_{ij} between all stocks i and j is zero, the risk V_P reduces to the simple sum of individual risks for each stock.

$$V_P\left(\overline{X}\right) = \sum_{i=1}^{n} X_i^2 \boldsymbol{r}_i^2 \qquad \text{Equation 3.2.3}$$

$$\text{for } \boldsymbol{r}_{ij} = 0$$

3.3 The Optimal Portfolio

The conditions of Equation 3.2.3 are virtually impossible to achieve for any $n>1$, but additionally, this approach ignored the return of the portfolio. In order to find the optimal stock distribution, we look at a sample portfolio with two stocks with an equal expected return ì, where r_{ij} = 0.2, s_1 = 0.6 and s_2 = 0.8. We can plot the risk of the portfolio as a function of the investment in the first stock.

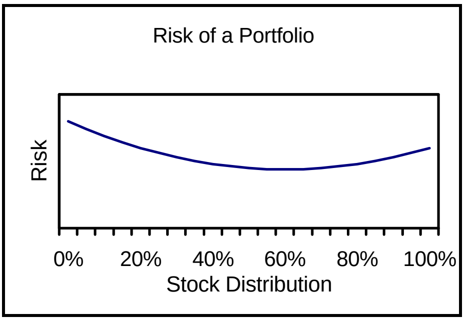

Figure 3.3.1: Risk of a Portfolio

If these two stocks were the only available choices, an investor would ideally distribute 60% of the available capital in stock 1 and the remaining 40% in stock 2. This example shows that the risk of a portfolio can be minimized without changing the expected return.

In order to calculate this optimal portfolio, we use Markowitz' approach. He defined the objective function, which assigns a weight between the desire for high returns and a low risk.

$$f = -A[E(R_P)] + V_P \qquad \text{Equation 3.3.1}$$

In this function, A represents the risk aversion of the investor and is dependant on his investment needs. A graph of this function highlights a region that satisfies the investor's requirements for return as well as risk. The edge of this region defines the portfolios with the highest return given a specific risk or conversely, the lowest risk give a defined return and is called the Efficient Frontier.

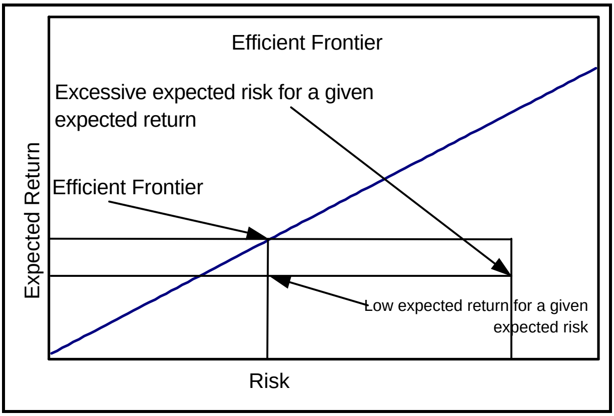

Figure 3.3.2: The Efficient Frontier

In the next step, Markowitz defined the Utility Function, which is also investor dependent and describes the utility $U(R)$ of a specific return R. This function is used to identify the desired return when optimizing a portfolio.

$$U(R_P) = a + bR_P - cR_P{}^2 \qquad \text{Equation 3.3.2}$$

The coefficients b and c are not negative so that the resulting graph will have a form as shown below.

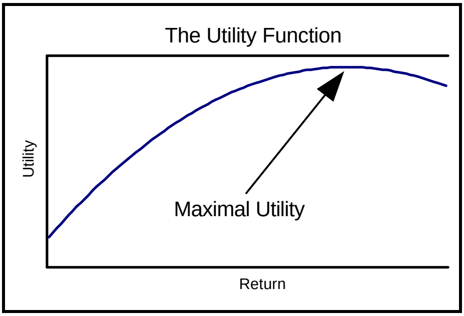

Figure 3.3.3: The Utility Function

A person at the beginning of his career can generally afford to take a larger risk since he will generally not depend on the savings in the near future but would benefit from higher returns later in life. Short-term downward fluctuations are tolerable to this group of persons but not for an investor who is close to retirement and will need his savings in the near future. Job security, the plans for a large purchase in the near future or personal risk aversion are other considerations, which will affect these parameters.

Applying the expectation operator E(.) on Equation 3.3.2 we get the following result.

$$E(U(R_P)) = a + bE(R_P) - cE(R_P{}^2)$$ Equation 3.3.3

Using the definition of the variance

$$V(R_P) = E(R_P{}^2) - [E(R_P)]^2$$ Equation 3.3.4

we can re-write Equation 3.3.4 as follows:

$$E(U(R_P)) = a + bE(R_P) + c[E(R_P)^2] - cV(R_P)$$ Equation 3.3.5

For a constant expected utility, C, we can solve this equation for the expected return $E(R_P)$.

$$E(R_P) = \sqrt{V(R_P) + C_1} - C_2$$ Equation 3.3.6

where

$$C_1 = \frac{C - a}{c} + \frac{b^2}{(2c)^2} \text{ and } C_2 = \frac{b}{2c}$$ Equation 3.3.7

This equation defines utility curves, which we can add to the graph shown in Figure 3.3.2, to arrive at the optimal portfolio as shown below.

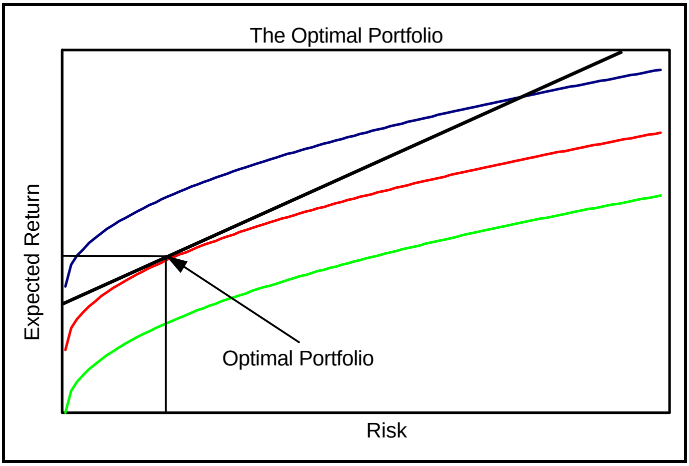

Figure 3.3.4: The Optimal Portfolio

The point of tangency between the utility curve and the efficient frontier defines the optimal portfolio for this investor. This point can be calculated by substituting Equation 3.3.6 into Equation 3.3.1 and solving for $V(R_P)$ or $E(R_P)$.

$$V(R_P)=\sqrt{\frac{1}{2A^2}\left[\frac{2}{A}\left(C_2-\frac{f}{A}\right)-1\right]^2-\frac{\left(C_2-\frac{f}{A}\right)^2-C_1}{A^2}}-\frac{1}{2A^2}\left[\frac{2}{A}\left(C_2-\frac{f}{A}\right)-1\right] \quad \text{Equation 3.3.8}$$

$$E(R_P)=\sqrt{f+C_1-C_2+\frac{(2C_2-A)^2}{4}}-\frac{2C_2-A}{2} \quad \text{Equation 3.3.9}$$

This expression uniquely specifies the optimal portfolio.

The challenge of this approach is the identification of the parameters in the utility and the objective functions since they are highly subjective and represent relative weights and cannot be attached to measurements in the physical world.

3.4 Applying the Theory

Most trading systems are extensions of financial prediction experiments and have the goal of measuring the real-world results that can be associated with the forecasts. The simplest form was already mentioned in the experiments from Rehkugler and Poddig: If an increase was predicted, the system purchased one additional fictitious unit of the DAX, if a decrease was predicted, one was sold. The system did not permit the ownership of negative numbers of the stock, or "short" positions.

Due to their mathematical simplicity, trading strategies based on moving averages are probably the most widely used technical rules. These models were prominently used by LeBaron and became the baseline for further comparison [LeBaron 1995].

In LeBaron's experiment, the single moving average indicator generated buying (selling) signals when its value was above (below) the current stock price. The adjustment of the model required identifying the optimal length of the data window. A slight improvement on the basic algorithm could be achieved if trading signals were only generated if the difference between the moving average and the current price exceeded a specified band. This reduced the number of trades and, by implication, the transaction costs that a real world investor has to bear.

Moving average oscillators compare a short term and a long term moving average of the stock price against each other. These models frequently use the commonly quoted moving averages of five, ten, 15, 50 and 200 days for their comparisons. Buy (sell) signals are generated only if the short (long) term moving average rises above that of the long (short) term. Again, frequently the difference between the two values has to exceed a specified value in order to trigger a trading signal, so that the number of transactions is kept at bay.

Using both of these moving average trading systems as a foundation, Dihardjo and Tan compared artificial neural network prediction models with an associated trading system to predict profitability opportunities in the Australian Dollar/US Dollar exchange rate [Dihardjo, Tan 1999]. Though they found that both systems were profitable in the period tested, the ANN models performed better (annualized returns between 13% and 19%) than the simple moving average approach (returns of between 8% and 13%). The experiments showed that both models were particularly successful in markets, which exhibited long term trends.

Kumar, Tan and Ghosh used the same Australian Dollar/US Dollar exchange rate data and built sophisticated financial forecasting models. These incorporated the chaotic components in numerous ways in an effort to optimize the predictive powers of the models and, by implication, the profitability of their system [Kumar et al, 1999]. The trading system worked with two different rule patterns:

Pattern 1:

```
If (Current Forecast – Previous Forecast) > Delta then
        Signal = “Buy”
Else If (Previous Forecast – Current Forecast) > Delta then
        Signal = “Sell”
Else
        Signal = “Hold”
```

Pattern 2:

```
If (Current Forecast – Current Close) > Delta then
        Signal = “Buy”
Else If (Current Close – Current Forecast) > Delta then
        Signal = “Sell”
Else
        Signal = “Hold”
```

The Delta value was used to provide a threshold, which eliminates excessive trades, since they were taken into account with 0.1% of the transaction value in this experiment.

Interestingly, though the forecasting models were considerably more complex than the ones used by Dihardjo and Tan, the profitability ranged between 11% and 20% annualized return and thus did not significantly help this goal much.

Notably missing from this list of trading strategies is one that addresses the realities of an individual investor, who has to decide not only which stocks offer good growth opportunities, but also how to distribute his investment between the numerous alternatives. Bookstaber describes a simple BASIC program that combines chart analysis with a simple risk calculation algorithm, but does not analyze the success or failure of this approach given historic data [Bookstaber 1985].

Programs with a similar focus exist with investment institutions or other professional investors who emphasize risk analysis, however, this work tends to not get published since it is considered the strategic advantage of the respective owner or user community. Jean Y. Lequarré voiced a similar sentiment in the conclusion of his article: “This inability to discuss their findings in the open is often frustrating for many of those involved in this activity and specially the ones who come from academia” [Lequarré 1993].

This thesis is an effort to combine the significant work on financial time series analysis and prediction with a coherent trading strategy that can be adjusted to the preferences and needs of the individual investor. The resulting system is designed to run on common PC hardware making it suitable for personal investment advice and as a portfolio management tool.