

Boundary Value Problems for Higher Order Complex Partial Differential Equations

DISSERTATION

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Abstract

In this dissertation, we study some Dirichlet boundary value problems for higher order complex partial differential equations in the unit disc. It mainly consists of four chapters.

In Chapter 1, an introduction is given to the background of the investigations of the problems discussed in this thesis and the different methods to study the problems as well as the key tools used.

In Chapter 2, we sketchily introduce the theory of polyanalytic and of polyharmonic functions as well as poly-analytic-harmonic functions. Specially, we establish a new decomposition theorem for polyharmonic functions. Using the decomposition, we find the polyharmonic analogues of the classical Poisson kernel which are called higher order Poisson kernels. They are expressed in terms of some vertical sums with nice structure.

In Chapter 3, we discuss some Dirichlet problems for higher order homogeneous complex partial differential equations in the unit disc. First, we review the Dirichlet problem for analytic functions. Next, we consider the Dirichlet problem for polyharmonic functions (simply, PHD problem). Finally, we consider three kinds of Dirichlet problems for poly-analytic-harmonic functions by the decompositions for polyanalytic functions and polyharmonic functions as well as poly-analytic-harmonic functions.

In Chapter 4, we study some Dirichlet problems for higher order inhomogeneous complex partial differential equations in the unit disc. We begin with the higher order Pompeiu operators and then use these operators to study four classes of Dirichlet problems for the inhomogeneous equations which are correspondingly discussed in Chapter 3 for the homogeneous equations.

Key words: Dirichlet problem, polyanalytic function, polyharmonic function, decomposition, higher order Poisson kernels, higher order Pompeiu operators.

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